# COMPETITION DURING AND AFTER INNOVATION AND THE OPTIMAL R&D PORTFOLIO

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## I. STARTING QUESTION:

It is clearly preferable that different centers engaged in developing a technology to meet a given problem should take different approaches. (See Manhattan project, Nelson [1956]). But naively it might seem that every developer will choose the most likely strategy, so the market will not produce differentiation of approach. This is certainly not optimal.

It turns out that this question requires rethinking of the patent race literature. A typical model assumes the following: (1) each firm has a probability $p$ of solving a given problem, and the events that the different firms solve the problem are independent in probability; (2) the social surplus if successful is given, goes to the successful firm if there is only one and is divided among the successful firms if there are more than one; and (3) there is a cost $c$ of engaging in the problem solving.. .Then the socially optimal number of firms, $n$, who should engage in the problem-solving is obtained by maximizing,

$$1 - (1-p)^n - n\,c.$$

But competition will drive the number of firms to the point of zero profits and zero expected social surplus.

When examined closely, these assumptions are dubious. (1) Independence of success is odd. If the problem can be solved at all, then it is likely that many firms will

succeed, since they are probably pursuing similar paths; (2) if several firms succeed, then there is an oligopoly, but most theories of imperfect competition predict that aggregate profits will be lower than under monopoly, so that some of the social surplus leaks out to the consumers.

Finally, there is a broader critique of the standard patent race model. The probability of success of a firm is really a decision variable, at least in many contexts. There are ideas floating around, the result of general scientific progress. A firm can decide which idea to follow; it can of course follow several ideas and incur the corresponding costs. To be more explicit, the R&D program of the firm (and therefore its probability of success as well as its cost) is chosen by the firm from a set of alternatives given by general scientific knowledge and the previous state of technology.

## II. THE OPTIMAL R&D PORTFOLIO

Assume that there is, at any given time, a potential product, which, if it were technological feasible, would yield a social surplus of 1. There is a set of (untried) projects, each of which may be successful or unsuccessful in making the potential product feasible. The probabilities of success of any combination of the projects are known; no restrictive assumptions are made. To carry out project $x$ requires a known cost, $c(x)$.

First, consider the social planner's problem. This is to choose a set of projects, $S$, to maximize net social surplus. For convenience, let $c(S)$ be the cost incurred by choice of the set $S$, so that,

$$c(S) = \sum_{x \varepsilon S} c(x), \qquad (1)$$

Given the set of projects, *S*, let,

$S^e$ be the event that at least one project in S would succeed if tried. (2)

For later reference, also let,

$S^n$ be the event than no project in S would succeed if tried. (3)

Then the social surplus derived by trying the projects in *S* and no others is,

$$V(S) = P(S^e) - c(S), \qquad (4),$$

and therefore *S* is *optimal* if,

$$P(S^e) - c(S) >= P(T^e) - c(T) \text{ for all sets of projects, T.} \qquad (5)$$

Thus, if *S* satisfies (5), it is the socially optimal R&D portfolio. It has just the right amount of diversification considering the costs.

## III. R&D COMPETITION

We suppose there is a set of firms which can choose projects. The set of projects is assumed to be common knowledge (all firms have the same general knowledge based on science and previous technological achievement). Then firm *f* can choose a set of projects, $S_f$; it could, of course, be the empty set.

What is the payoff? Clearly, if $S_f$ contains one or more successful projects, while the sets $S_g$ (all *g* other than *f*) have no successful projects, then firm *f* has a monopoly on the project. I will assume that, as is usual in the patent race literature, the monopoly firm can extract the entire social surplus, taken to be 1, through some form of perfect discrimination.

What if two or more firms have successful projects within their chosen sets? I have argued above that the aggregate private surplus must be less than 1. Let me go to the opposite extreme, that represented by Bertrand competition. That is, none of the firms with successful projects will have any net operating revenue.

Thus, any firm receives a payoff of 1 if it has a monopoly and 0 otherwise. For simplicity of notation, let

$S_{-f}$ be the union of the sets $S_g$, g not equal to f. (6)

Then, in the notation of (2-3), firm *f* receives 1 if the conjunction of the events,, $S_f^e$ and $S_{-f}^n$, occurs and 0 otherwise. Therefore, the payoff to *f* is,

$$V_f\ (S_1\ldots, S_n) = P(S_f^e \ \&\ S_{-f}^n) - c(S_f), \qquad (7)$$

and an equilibrium point is characterized by the statement,

$$P(S_f^e \ \&\ S_{-f}^n) - c(S_f) >= P(T_f^e \ \&\ S_{-f}^n) - c(T_f), \text{ all } T_f. \qquad (8)$$

One observation is immediate. It is not optimal for the sets of projects chosen by any two firms to intersect. If the project does not succeed, it will yield zero operating profit. If it does succeed, it will succeed for both, and therefore again the payoff will be zero. In either case, the cost of engaging in the project will be lost. Hence, either firm will gain by dropping the project from its chosen set. We can therefore assume that at the equilibrium, the sets $S_f$ are disjoint. Further, in considering alternative choices $T_f$, we might as well assume they are disjoint from those chosen by the other firms, $S_{-f}$.

Some elementary uses of probability theory will make (8) more transparent. Let *S* be the union of the sets $S_f$. Note that,

$$P(S_f^e \ \&\ S_{-f}^n) = P(S_{-f}^n) - P(S_I^n \ \&\ S_{-f}^n) = 1 - P(S_{-f}^e) - P(S^n)$$

$$= P(S^e) - P(S_{-f}^e). \qquad (9)$$

Similarly,

$$P(T_f^e \,\&\, S_{-f}^n) = P(S_{-f}^n) - P(T_f^n \,\&\, S_{-f}^n) = 1 - P(S_{-f}^e) - [1 - P(T_f^e \text{ or } S_{-f}^e)]$$

$$= P[\,(T_f \text{ u } S_{-f})] - P(S_{-f}^e). \qquad (10)$$

Substitute (9-10) in (8) and subtract $c(S_{-f})$ from both sides.

THEOREM. $(S_1,\ldots, S_n)$ is an equilibrium point of the R&D competition if and only if,

$$P(S) - c(S) >= P(T) - c(T),$$

where S is the union of the sets $S_f$ and T is any other allocation which differs from the original in only one firm's choice.

COROLLARY: Let S be a socially optimal choice of projects. Then any partition $(S_1,\ldots, S_n)$ of S among the firms. is an equilibrium point.

We need only compare the criterion in the Theorem with (5).

The converse is not true, as can be shown by a counter-example. That is, an equilibrium point of the R&D competition does not necessarily yield an optimal R&D portfolio.

## IV. COMMENTS

(1) The outcome depends of course on strong assumptions, in particular, the strength of post-innovation competition when possible. This may have some implications for policy.

(2) The R&D competition will have many equilibrium points. Even those that yield the optimal portfolio will differ in the extent to which the surplus is appropriated by the sellers compared with that which leaks out to the consumers.