Robust Self-Insurance

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Abstract

A robust agent facing an income fluctuation problem is informed by a 'worst case distribution' represented by a state dependent distorted transition matrix for his income process that shifts probability towards adverse income realizations. When his income features two risk components the worst case exhibits positive correlations between them - even though no such correlation necessarily exists. When the agent faces a time varying interest rate, the worst case features conditional correlation between the interest rate, the income shock and the agent's asset position. The agent's pessimism is more pronounced when his wealth is low. This hints at a promising approach to endogenizing a form of disagreement. We also raise concerns regarding the identification of latent orthogonal risk factors based on the behavior of robust households.

1. Motivation

- How does pessimism of a robust agent depend on endogenous idiosyncratic states?
- How does pessimism feed back into decisions that affect these idiosyncratic states?
- Why might we suddenly see increases in disagreement in times of stress?
- What happens if an econometrician ignores a desire for robustness in Bewley models?

2. Robustness

An agent possesses a 'benchmark' model of the economy but fears it is misspecified in some unknown way. The agent expresses doubts of his model by considering alternative distributions that are distorted versions of the distribution implied by the benchmark. In order to construct a robust policy, the agent considers adverse distributions and balances the damage that an implicit misspecification would cause, against the plausibility of the misspecification. The distribution that emerges from this problem can be thought of as a 'worst case distribution' encoding these concerns.

Working in a recursive framework with multiplier preferences ([1]), the conditional distribution of the state next period under the benchmark and worst case are related by a state dependent likelihood ratio, m, where

$$m\left(x'\right) = \frac{e^{-\frac{W(x')}{\theta}}}{E\left[e^{-\frac{W(x')}{\theta}}|x\right]}$$

3. Model(s)

We assert a CRRA felicity function

$$u\left(c\right) = \frac{c^{1-\varphi} - 1}{1-\varphi}$$

and a borrowing constraint,

$$a \geq -\phi$$

We consider three cases: $1)$ a single factor in income, $2)$ two (independent) factors in income and $3)$ a single factor in income but a time varying bond rate. In the latter case, negative asset holdings are allowed, whereas an *ad hoc* limit of $\phi = 0$ is imposed in the first two. The budget constraint constraint in the single risk factor case is

$$\frac{a}{1+r} + c = w \exp\left\{z\right\} + a_{-1}$$

We model z as a first order Markov chain with transition matrix $T_z \equiv \left\{p_{ij}\right\}_{i,j=1,n_z}$. The two shock case is similar. The rate shock takes only two values and follows a Markov chain characterized by p_{hh} and p_{ll}.

We use the minimizing likelihood ratio to twist each element of T_z. This yields a state dependent worst case transition matrix $\tilde{T}_z\left(a_{-1}\right) \equiv \left\{\tilde{p}_{ij}\left(a_{-1}\right)\right\}_{i,j=1,n_z}$ where

$$\tilde{p}_{ij}\left(a_{-1}\right) \equiv p_{ij} \cdot m\left(z_j, a^*\left(z_i, a_{-1}\right)\right)$$

We choose to restrict the optimal asset choice to be on the asset grid so evaluating the likelihood under the worst case is simply a question of working with a first order Markov chain in (z, a) pairs, for which the likelihood is very tractable. The detection error probability in the one shock case below is $\approx 20\%$ using 50 periods (years) of data for each likelihood evaluation.

4. Results: One shock case

In table 1 we observe T_z and $\tilde{T}_z(a)$ at different wealth percentiles. We see pessimistic reallocations of probability, with the probabilities of receiving an adverse (desirable) income realization increased (decreased).

The distortions increase in severity as the agent's wealth level declines. Contrast this with the dependence on an *exogenous* aggregate (volatility) state discussed in [2] and [3]. Thus we generate a phenomenon akin to disagreement. Also, note that the existence of model uncertainty may not be so obvious (the w.c. conditional is quite close to benchmark) in calm and secure times - but may become prominent when a crisis hits and a fraction of a population have experienced a painful shock to their wealth.

(a)

0.12443	0.70284	0.17154	0.00118
0.03737	0.58606	0.36862	0.00795
0.00795	0.36862	0.58606	0.03737
0.00118	0.17154	0.70284	0.12443

(b)

0.18981	0.69473	0.11493	0.00053
0.0585	0.65007	0.28724	0.00419
0.01262	0.44531	0.51899	0.02308
0.00189	0.22015	0.68963	0.08833

(c)

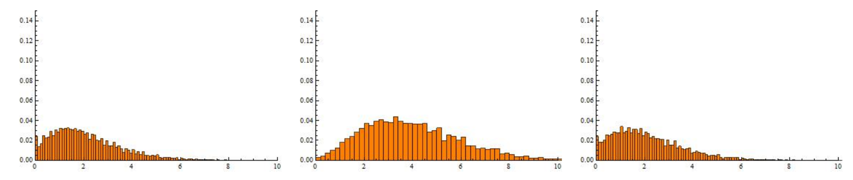
0.16037	0.70933	0.12967	0.00063
0.05102	0.63847	0.30575	0.00475
0.01141	0.43106	0.53257	0.02496
0.00176	0.21251	0.69321	0.09252

(d)

0.14196	0.7094	0.14781	0.00083
0.04433	0.61705	0.33272	0.0059
0.00979	0.40511	0.55577	0.02933
0.00151	0.19615	0.6989	0.10344

Table 1: *Transition matrices for the labor shock. a) Benchmark, b) W.C. with a at 1^{st} %-ile, c) W.C. with a at 10^{th} %-ile, W.C. with a at 90^{th} %-ile. Shock values are ordered from lowest to highest. Some illustrative elements are in red.*

The main behavioral implication is that the robust agent saves more, relative to expected utility - see figure 1. Asset holdings under the worst case are comparable to EU, because despite the extra resources devoted to saving, the adverse twists to the income shocks provide an offsetting effect.



(a) (b) (c)

Figure 1: *Asset simulations under a) benchmark with EU b) benchmark with RC c) worst case with RC.*

5. Results: Two shock case

In the two shock case many of the insights from the one shock case above still apply. In addition, as shown in table 2 we observe evidence of the worst case featuring correlation between the two income components, z_1 and z_2, even though under the benchmark they are uncorrelated. Intuitively, the agent fears misspecifications that can be represented by a loss of diversification.

(a)

0.17347	0.70074	0.12518	0.00061
0.05372	0.63988	0.30175	0.00464
0.0114	0.43015	0.53357	0.02488
0.00167	0.20877	0.69619	0.09337

(b)

0.14416	0.71108	0.14399	0.00077
0.04518	0.62236	0.32698	0.00549
0.01	0.41097	0.55133	0.02768
0.00153	0.19948	0.69973	0.09928

(c)

0.13525	0.70876	0.15508	0.00091
0.04177	0.60832	0.34359	0.00633
0.00915	0.39468	0.56525	0.03092
0.0014	0.18931	0.70196	0.10733

(d)

0.13393	0.70816	0.15697	0.00094
0.04122	0.60573	0.34654	0.00651
0.009	0.39162	0.56779	0.03159
0.00137	0.18722	0.70226	0.10915

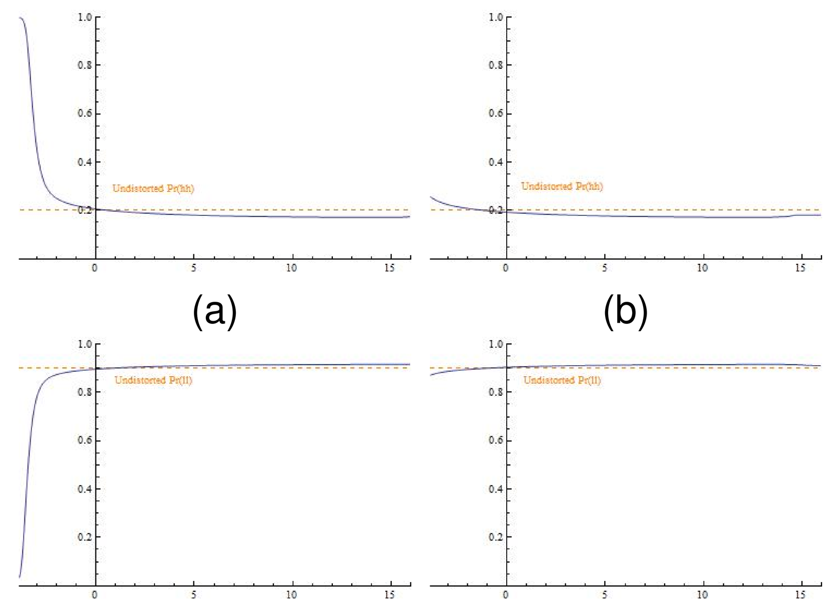
Table 2: *Transition matrices for the z_1 shock. a) W.C. with a at 1^{st} %-ile and z_2 low, b) W.C. with a at 1^{st} %-ile and z_2 high, c) W.C. with a at 90^{th} %-ile and z_2 low, W.C. with a at 90^{th} %-ile and z_2 high.*

The magnitude of the correlation is small (especially at higher wealth levels) but qualitatively it hints at a cautionary tale for those who use the methods of [4] to extract latent income factors. If agents behave as if income components are correlated (interpret them as, say, idiosyncratic and systemic, or temporary and permanent) then identifying factors that are assumed orthogonal may be problematic if the econometrician ignores robustness.

6. Results: Rate shock case

The bond rate $r \in \left\{r_{high}, r_{low}\right\} \equiv \left\{0.08, 0.02\right\}$ with transition probabilities $p_{hh} = 0.2$ and $p_{ll} = 0.9$. Thus, sometimes it is expensive to borrow, and sometimes cheap. The cheaper borrowing regime is more persistent.

Figure 2 plots the distorted p_{hh} (1^{st} row) and p_{ll}, conditional a low z realization (1^{st} column) and high, as functions of the agent's asset level. What constitutes pessimism depends on the asset position. If $a < 0$ then the worst case implies that there is a higher probability of a high cost of rolling over their debt next period. Intuitively, the opposite is true when $a > 0$. Comparing the two columns, we see that if the agent is in a high income state, then the distortions are very mild. But if the agent receives an adverse income shock (and is deep in debt) he is more vulnerable and this is expressed in a greater distortion.



(a) (b)

(c) (d)

Figure 2: *(a) and (b) are the distorted p_{hh} (given low and high income realizations respectively) as functions of the agent's assets. (c) and (d) are the distorted p_{ll}.*

This is not GE but it is suggestive. Since the agent in a GE Bewley model is atomistic, we can think of the time varying bond rate as capturing the effects of aggregate shocks. Thus, the agent may behave as if idiosyncratic and systemic shocks are correlated, even though they are not in the benchmark. Intuitively, the agent really doesn't want borrowing to become more expensive after a bad income draw and especially not when 'everyone else' is also trying to borrow (which is associated with a higher equilibrium r).

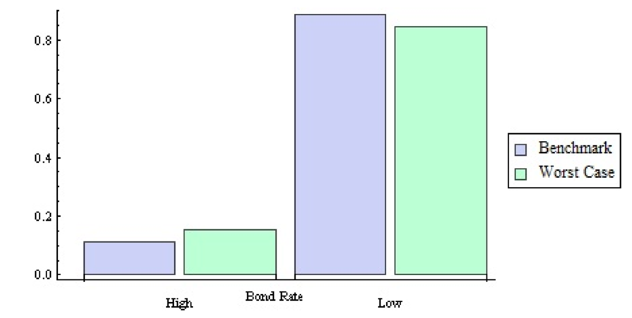


Figure 3: *Stationary distribution of the bond rate under the benchmark and worst case.*

The distorted stationary distribution of r has greater probability of $r = r_{high}$ than in the benchmark (see figure 3). This is because the agent is typically in debt under the worst case. If, in contrast, the benchmark is actually generating the data, the agent typically will have positive savings so the distorted transition matrices would generally feature the opposite distortion.

7. Summary

- Preliminary exploration of self-insurance problems with robustness
- Construction of worst case transition matrices
- Fear misspecifications represented by lower income and greater correlations of shocks (econometric problems?)
- Pessimism more pronounced at lower wealth (route to endogenous disagreement - especially in times of stress?)
- Induces a precautionary saving motive

References

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