

Eaton Comments on

“Technological Diversification”

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- Complex model
- Connects Producer Level and Aggregate Shocks in a very clever way

- Connection to Klette Kortum
  - Continuum of products
  - Continuum of firms
  - Heterogeneity across firms in number of products
  - Over time firms take products from other firms or lose products to them

- Here
  - Continuum of firms
  - Firms combine an integer number of inputs
  - In the static version of the model we can think of each firm using its own set of inputs
  - Firms add or lose inputs but we don't need to think of them taking inputs away from other firms or losing inputs to other firms.

- Each firm  $j$  combines inputs to produce its output with elasticity of substitution  $\varepsilon$

$$y(j) = \left[ \sum_{i=1}^{n(j)} l(i, j)^{(\varepsilon-1)/\varepsilon} \right]^{\varepsilon/(\varepsilon-1)}$$

- Consumers combine outputs of a measure 1 of firms to produce utility with elasticity of substitution  $\varepsilon$ :

$$Y = \left[ \int_0^1 y(j)^{(\varepsilon-1)/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)}$$

so that:

$$Y = \left[ \int \sum_{i=1}^{n(j)} l(i, j)^{(\varepsilon-1)/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)}$$

- It's easy to show that each firm will employ the same measure of workers  $\tilde{l}$  on each activity so that  $\tilde{l} = L/N$  where:

$$N = \int n(j) dj$$

so that:

$$Y = LN^{1/(\varepsilon-1)e}$$

- Progress occurs as firms add inputs

- Up to this point we haven't had to say anything about whether one firm's input is the "same" as another's or not.
- All that matters is the number of inputs that each firm is using

- But now rank inputs so that all firms using 1 input use the input 1, all firms using 2 inputs use the input 1 and 2, etc.
- Allow for an input to “die” with hazard  $\gamma$ .
- If input 1 dies every firm loses it:  $N$  falls by the measure of firms 1.
- If input  $i$  dies then all the firms using  $i' \geq i$  are affected. If the measure is  $m(i)$  then  $N$  falls by  $m(i)$ . Note that  $m(i)$  is decreasing in  $i$ .
- Hence the death of an input creates an aggregate shock.

- The range of different inputs in use can increase when the frontier firm adds a new input.
- But passing the frontier isn't any harder than acquiring an input that is already used by other firms.

- Much of the workings of the model can be described without the need for any dynamic optimization
  - A firm adds an input with hazard  $\lambda$  which is independent of the number of firms using or that have ever used that input
  - All firms using input  $i$  lose it with hazard  $\gamma$
- But, as in KK, the authors introduce an activity that increases the probability of finding a new input for the firm
- The added difficulty here is that, since the economy is subject to aggregate shocks, the dynamic optimization is much more challenging

## Here is where I got lost

- The authors specify demand for firm  $j$  as:

$$Y p(j)^{-\varepsilon}$$

where “aggregate output is taken as the numeraire.”

- But I would have specified it as:

$$\frac{p(j)^{-\varepsilon}}{P^{-\varepsilon}} Y$$

where  $P$  is the CES price index

$$P = \left[ \int_0^1 p(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)}$$

(See, e.g., Atkeson Burstein)

- In Koren Tenreyro

$$p(j) = \bar{m}w n(j)^{1/(1-\varepsilon)}$$

where:

$$\bar{m} = \frac{\varepsilon}{\varepsilon - 1}$$

so that:

$$P = \bar{m}w N^{1/(1-\varepsilon)}$$

- With this difference I get that the profit of firm  $j$  is:

$$\frac{wL}{(\varepsilon - 1)N}n(j)$$

as opposed to their:

$$\frac{1}{\varepsilon}N^{(2-\varepsilon)/(\varepsilon-1)}Ln(j)$$

- So I can't find a value of  $\varepsilon$  that eliminates feedback from  $N$  to profits

- Whatever the resolution any flaw is not fatal
- I don't think that the optimizing dynamics are playing that much of a role.
- The dynamics can be summarized in terms of the probability of gaining a new input
- I checked most of the rest of the paper and we agreed
- The key contribution, a connection between firm-level and aggregate shocks, remains

# Suggestions

- Do much more with the simulations
- Compare the moments generated by the model with firm and aggregate correlations
- Get through regressions at the front faster: get to the model and simulations