Natural Expectations,
Macroeconomic Dynamics,
and Asset Pricing

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Two starting assumptions
(cf. Fuster, Mendel, and Laibson 2010)

1. Assume that fundamentals are hump-shaped.
   - Momentum in the short-run.
   - Partial mean reversion in the long run.
Second assumption

2. Agents do not know that fundamentals are hump-shaped and base their beliefs on parsimonious high-frequency models that they fit to the data.

Assume that this preference for parsimonious high-frequency models is at least partially a psychological bias.
Economic reasons for parsimonious models

• Tradeoff between model flexibility (more parameters) and overfitting
• To avoid overfitting limit number of parameters, $k$
• Formalizations:
  ▪ Akaike Information Criterion (AIC)
  ▪ Bayesian (Schwarz) Information Criterion (BIC)
Psychological reasons for parsimonious models:

- Myopia: short-term predictions $\rightarrow$ low $k$
- Recency bias: small samples $\rightarrow$ low $k$
- Complexity aversion $\rightarrow$ low $k$
- Preference for tractibility $\rightarrow$ low $k$
- Anchoring and Representativeness, also lead agents to underestimate mean reversion, which is similar to low $k$
Consequences of parsimonious models:

1. Agents recognize the short-term momentum but miss some of the long-run mean reversion
   - Endogenous extrapolation bias and pro-cyclical excess optimism

2. Asset returns are excessively volatile and exhibit overreaction
   - Returns negatively predicted by lagged returns, P/E, and $\Delta \ln C$

3. Real economic activity has amplified cycles
   - $\Delta \ln C$ negatively auto-correlated in medium run

4. Equity premium is large, although long-run equity returns covary weakly with long-run consumption growth
   - If agents had RE, equity premium nearly vanishes

5. Agents with rational expectations hold large equity shares
   - Follow counter-cyclical asset allocation policy
Related Literature

Adam and Marcet (2011): learning and asset pricing
Barberis, Shleifer, and Vishny (1998): extrapolative dividend forecasts
Barsky and De Long (1993): extrapolation and excess volatility
Benartzi (2001): extrapolation and company stock
Black (1986): noise traders
Campbell and Mankiw (1987): shocks are persistent in low-order ARIMA
Campbell and Shiller (1988a,b): P/E ratio and return predictability
Choi (2006): extrapolation and asset pricing
Choi, Laibson, and Madrian (2009): positive feedback in investment
De Bondt (1993): extrapolation bias in surveys and experiments
Gabaix (2010): sparse representations
Hommes (2005, 2008): bubbles in the lab
Hong and Stein (1999): forecasting biases
Some Related Literature

Kahneman and Tversky (1973): representativeness
Keynes (1936): animal spirits
Lansing (2010): extrapolation and asset pricing in a macro model
LaPorta (1996): Growth expectations have insufficient mean reversion
LeBaron, Arthur, and Palmer (1999): agent-based modeling
LeBaron and Tesfatsion (2008): agent-based modeling
Leroy and Porter (1981): excess volatility in stock prices
Lo and MacKinlay (1988): variance ratio tests
Loewenstein, O’Donoghue, and Rabin (2003): projection bias
Malmendier and Nagel (2011): Recency bias bias and role of personal experience
Parker (2001): Cov of returns and $\Delta \ln C$ rises from short- to medium-run
Piazessi and Schneider (2009): extrapolative beliefs in the housing market
Previtero (2010): extrapolative beliefs and annuity investment
Shiller (1981): excess volatility in stock prices
Summers (1986): power problems in financial econometrics
Tortorice (2010): extrapolative beliefs in unemployment forecasts
Model

• Equity tree: earnings growth is an AR(40)
• CARA habit preferences (Alessie and Lusardi)

\[\sum_{t=0}^{\infty} \delta^t \left\{-\frac{1}{\alpha} \exp\left(-\alpha \left[ c_t - \gamma c_{t-1} \right]\right)\right\}\]

\(\alpha\) controls curvature of the utility function
\(\gamma\) represents strength of habit
• Dynamic budget constraint for wealth, $w_t$

$$w_t = \left( w_{t-1} - c_{t-1} - \theta_{t-1} P_{t-1} \right) R + \theta_{t-1} (d_t + P_t)$$

$\theta_t$ represents claims on the Lucas tree at date $t$

$d_t$ represents Lucas tree dividend at date $t$

$P_t$ represents price of Lucas tree at date $t$

• Elastic supply of foreign capital with gross return $R$

• Assume foreign agents don’t hold domestic capital
  – Home bias
  – Moral hazard
Natural expectations

\[ \Delta d_t = AR(40) \]
\[ \Delta d_t = AR(p) \quad p \leq 40 \]

Data generating process

Natural expectations

We will study cases \( 1 \leq p \leq 40 \).
Model matches the data for \( p \leq 20 \).
Consumption is a weighted average of $c_{t-1}$ and $Y_t$

$$c_t = \frac{\gamma}{R} c_{t-1} + \left(1 - \frac{\gamma}{R}\right) Y_t - Q$$

Permanent income

$$Y_t = \frac{R - 1}{R} \left[ -R B_t + \sum_{s=0}^{\infty} \frac{E_t d_{t+s}}{R^s} \right]$$

Shift term

$$Q = \frac{1}{R - 1} \left[ \frac{1}{\alpha} \ln (R \delta) + \frac{\alpha}{2} Var_t (c_{t+1}) \right]$$
Value function:

\[
V(c_{t-1}, B_t, d_t, d_{t-1}, \ldots) = \frac{-1}{\alpha (1 - \delta)} \exp\left( -\alpha [c_t - \gamma c_{t-1}] \right)
\]

Price of the equity tree:

\[
P_t = \sum_{s=1}^{\infty} \frac{E_t d_{t+s}}{R^s} - \frac{\alpha \times \text{Var}_t (c_{t+1})}{\left(1 - \frac{\gamma}{R}\right)(R - 1)^2}
\]
U.S. Log Real Capital Income
(1947q1-2010q3)

U.S. NIPA (BEA): net operating surplus of private enterprises.
IRF’s for real capital income

Quarters
## Calibration

<table>
<thead>
<tr>
<th>True DGP</th>
<th>Perceived DGP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 1.0025$</td>
<td>$\Delta \ln d \sim \text{AR}(40)$ estimated from NIPA</td>
</tr>
<tr>
<td>$\delta R = 1$</td>
<td>$\Delta \ln d \sim \text{AR}(p)$ estimated from NIPA</td>
</tr>
<tr>
<td>$\gamma = 0.9$</td>
<td>gross risk-free rate (quarterly)</td>
</tr>
<tr>
<td></td>
<td>discount factor</td>
</tr>
<tr>
<td>$\alpha = \frac{4}{c \left(1 - \frac{\gamma}{R}\right)}$</td>
<td>habit weight</td>
</tr>
<tr>
<td></td>
<td>local relative risk aversion of 4</td>
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</tbody>
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IRF’s for cumulative excess returns

Quarters after unit shock
IRF’s for consumption
Covariance of consumption growth and cumulative return at different horizons
Empirical evaluation

- Annual data (1929-2010)
- Real per-capita consumption: US NIPA
- Excess returns
- P/E ratios

- Simulations annualized for comparisons
- Simulations generated for 82 years of data
- Monte Carlo to generate confidence intervals
Correlation of Excess Returns in Year $\tau$ with Cumulative Excess Returns for Years $\tau + 2$ to $\tau + 5$, for Different AR(p) Models of Earnings
Correlation of P/E\textsubscript{40} in Year \(\tau\) with Cumulative Excess Returns for Years \(\tau + 2\) to \(\tau + 5\), for Different AR(\(p\)) Models of Earnings
Correlation $\Delta \ln C_\tau$ with Cumulative Excess Returns for Years $\tau+2$ to $\tau+5$, for Different AR($p$) Models of Earnings
Correlation of $P/E_{40}$ in Year $\tau$ with $(\ln C_{\tau+6} - \ln C_{\tau+2})$, for Different AR($p$) Models of Earnings.
Correlation of $\Delta \ln C_{\tau}$ with $(\ln C_{\tau+6} - \ln C_{\tau+2})$, for Different AR($p$) Models of Earnings
Application to equity premium puzzle

- Agents perceive equities to be very risky, since they don’t recognize the mean reversion

\[ \text{COV} (\Delta_h c_{t+h}, R_{t,t+h}) = \frac{1}{3} \times \frac{1}{3} \times \text{COV} (\Delta_h c_{t+h}, R_{t,t+h}) \]

- In other words, equities are about 9 times less risky than they are perceived to be.
Equity Premium for Different AR($p$) Models of Earnings
Standard deviation of equity returns for Different AR($p$) Models of Earnings
Standard Deviation of Consumption Growth for Different AR($p$) Models of Earnings
Covariance of consumption growth and cumulative return at different horizons

Simulated data

Empirical data
How would RE agents behave in this economy?

- Closed form solution for consumption function and asset allocation
- RE agents are relatively highly leveraged
- RE agents adjust their equity allocation counter-cyclically
Leverage of RE agents for Different AR($p$) Models of Earnings
Summary

1. Fundamentals follow hump-shaped dynamics:
   • Short-run momentum
   • Long-run (partial) mean reversion

2. Agents estimate simple models
   – Parsimonious, tractable
   – Typical models chosen in economics literature
Summary

1. Low order forecasting equations miss some of the mean reversion in fundamentals, so resulting asset prices exhibit excess volatility and long-run mean reversion
2. Cycles in consumption (and investment)
3. The covariance of returns and consumption growth rises and then falls with $h$
4. New explanation for the ability of $cay$ to predict returns.
5. Equity is perceived as many times riskier than it actually is
6. Rational Expectations investors hold far more equity than Natural Expectations investors