Exchange Rates and Interest Rates:
Levels and Changes of the Price of Foreign Currency

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There are two well-known puzzles concerning exchange rates and interest rates:

1. The “Fama” puzzle: the foreign exchange “risk premium” on a country’s short term interest bearing assets covaries positively with its interest rate.

2. The excess volatility in levels of the exchange rate: When a country’s interest rate rises, its currency appreciates, but much more than can be accounted for in Dornbusch-style models that assume uncovered interest parity (i.e., no risk premium.)

These two puzzles both involve the foreign exchange “risk premium” and its relationship to interest rates.

Are they really capturing the same phenomenon?
The answer is NO.

They say, in a sense, the opposite.

What has been heretofore unnoticed is that we have a puzzle squared: the solutions put forth to account for one of the puzzles go in the wrong direction for the other puzzle.

This paper:

1. Documents the two puzzles in a simple unified framework

2. Explains why our models of the Fama puzzle don’t explain the volatility puzzle (and really cannot be easily modified so they will.)

3. Sketches a simple model that can account for both based on the unmeasured liquidity return to short-term assets
Fama puzzle:

Define the excess return on “foreign” short-term deposits (in this study, “home” is the U.S. and “foreign” are the other G7 countries):

\[ \rho_{t+1} \equiv i_t^* + s_{t+1} - s_t - i_t \]

We tend to think of the deposit rates as riskless. The risk is from the foreign exchange rate.

Fama puzzle: \[ \text{cov}(\rho_{t+1}, r_t^* - r_t) > 0 \]

That is, whatever is driving the risk from foreign exchange (covariance risk, e.g.) covaries with whatever drives the interest differential. Like a covariance of covariances.

→ Note that I use \( r_t^* - r_t \). All of the theories call for real interest rates
Recent Models of the Fama Puzzle

It has been hard to explain, though recent work has offered explanations based on (1) non-standard preferences, or on (2) rational inattention.

Models of risk premiums with standard expected utility don’t work: Bekaert et. al. (1997); Backus et. al. (2001)

(1) Models based on Campbell-Cochrane preferences or Epstein-Zin-Weil preferences: Verdelhan (2010); Colacito and Croce (2011); Bansal and Shaliastovich (2013); Lustig, Roussanov, Verdelhan (2011)

(2) Models based on “delayed overshooting”: Froot and Thaler (1990); Eichenbaum and Evans (1995); Bacchetta and van Wincoop (2010)
“Excess Volatility”

Let $s^T_t$ be the transitory (in Beveridge-Nelson sense) component of the exchange rate.

$$s^{ip}_t = E_t \sum_{j=0}^{\infty} \left( i^*_t - i_t - (\overline{i^*} - \overline{i}) \right)$$ -- the “interest parity” level of the stationary component of the exchange rate. That is, if interest parity held, $s^T_t = s^{ip}_t$.

In Dornbusch-style models, $\text{cov} \left( s^T_t (= s^{ip}_t), r^*_t - r_t \right) > 0$

We find $\text{cov} \left( s^T_t - s^{ip}_t, r^*_t - r_t \right) > 0$

That is, exchange rate comoves in the right direction but is excessively volatile.
\[ s_t^T - s_t^{lp} = -E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho}). \]

So, the excess volatility puzzle can be expressed as:

\[ -\text{cov}(s_t^T - s_t^{lp}, r_t^* - r_t) = \text{cov}(E_t \sum_{j=0}^{\infty} \rho_{t+j+1}, r_t^* - r_t) < 0 \]

The high-interest rate currency is less risky

But the Fama puzzle was:

\[ \text{cov}(\rho_{t+1}, r_t^* - r_t) > 0 \]

The high-interest rate is more risky

For some \( j > 0 \), \( \text{cov}(E_t \rho_{t+j+1}, r_t^* - r_t) \) switches sign. That is the challenge for our models.
Data

U.S., Canada, France, Germany, Italy, Japan, U.K., and “G6”

G6, a weighted average of the six non-U.S. countries, smooths out some of the idiosyncratic movements

Exchange rates – last day of month (noon buy rates, NY)
Prices – consumer price indexes
Interest rates – 30-day Eurodeposit rates (last day of month)

Monthly, June 1979 – October 2009
Fama Regressions:  
\[ \rho_{t+1} = \zeta_s + \beta_s (i_t^* - i_t) + u_{s,t+1} \]

1979:6-2009:10

| Country   | \( \hat{\beta}_s \) | 90% c.i.\(( \hat{\beta}_s \) | |
|-----------|----------------------|-----------------------------|
| Canada    | 2.271                | (1.186, 3.355)              |
| France    | 1.216                | (-0.171, 2.603)             |
| Germany   | 2.091                | (0.599, 3.583)              |
| Italy     | 0.339                | (-0.680, 1.359)             |
| Japan     | 3.713                | (2.390, 5.036)              |
| U.K.      | 3.198                | (1.170, 5.225)              |
| G6        | 2.467                | (0.769, 4.164)              |
Empirical procedure

Estimate a VECM in nominal exchange rates, relative prices and relative interest rates:

\[ s_t - s_{t-1} = c_1^0 + g_{11} (s_{t-1} - (p_{t-1} - p_{t-1}^*)) + g_{13} i_{t-1}^R + \sum_{j=1}^{3} \left( c_{11}^j \Delta s_{t-j} + c_{12}^j \pi_{t-j} + c_{13}^j \Delta i_{t-j}^R \right) \]

\[ \pi_t^R = c_1^0 + g_{21} (s_{t-1} - (p_{t-1} - p_{t-1}^*)) + g_{23} i_{t-1}^R + \sum_{j=1}^{3} \left( c_{21}^j \Delta s_{t-j} + c_{22}^j \pi_{t-j} + c_{23}^j \Delta i_{t-j}^R \right) \]

\[ i_t^R - i_{t-1}^R = c_1^0 + g_{31} (s_{t-1} - (p_{t-1} - p_{t-1}^*)) + g_{33} i_{t-1}^R + \sum_{j=1}^{3} \left( c_{31}^j \Delta s_{t-j} + c_{32}^j \pi_{t-j} + c_{33}^j \Delta i_{t-j}^R \right) \]
First, is real exchange rate stationary? Test of $g_{11} - g_{21} < 0$:

Estimate and bootstrap critical values:

<table>
<thead>
<tr>
<th>Country</th>
<th>$g_{11} - g_{21}$</th>
<th>Critical value 5%</th>
<th>Critical value 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-0.0209</td>
<td>-0.0382</td>
<td>-0.0318</td>
</tr>
<tr>
<td>France</td>
<td>-0.0305*</td>
<td>-0.0352</td>
<td>-0.0279</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.0364**</td>
<td>-0.0328</td>
<td>-0.0257</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.0258</td>
<td>-0.0339</td>
<td>-0.0266</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.0250*</td>
<td>-0.0289</td>
<td>-0.0207</td>
</tr>
<tr>
<td>U.K.</td>
<td>-0.0408**</td>
<td>-0.0333</td>
<td>-0.0272</td>
</tr>
<tr>
<td>G6</td>
<td>-0.0328**</td>
<td>-0.0299</td>
<td>-0.0235</td>
</tr>
</tbody>
</table>

1. Null rejections are always stronger for G6 in this study.
2. It is interesting that including interest rates as a “covariate” increases power of test for unit root in real exchange rate.
Using standard projection methods, we can calculate estimates of $r_t^* - r_t$ and $E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho})$.

There are two senses in which we measure $r_t^* - r_t$ and $E_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho})$ with error:

1. Estimation error (for VECM coefficients.) To handle this, I bootstrap all standard errors.
2. The VECM does not contain all information that markets use in forming expectations.
   a. There is no way to eliminate this problem. For robustness, I add variables to VECMs that contain “information” (stock returns, gold price, oil price, long-term bond yields.)
   b. Also, try longer lags in VECM (though AIC and BIC argue for very short lags.)
Fama Regression in Real Terms:  

\[ \rho_{t+1} = \zeta_s + \beta_s (\hat{r}_t^* - \hat{r}_t) + u_{s,t+1} \]

1979:6-2009:10

<table>
<thead>
<tr>
<th>Country</th>
<th>( \hat{\beta}_s )</th>
<th>90% c.i. ( \hat{\beta}_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.722</td>
<td>(-0.670,2.665)</td>
</tr>
<tr>
<td>France</td>
<td>1.482</td>
<td>(0.076,3.004)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.733</td>
<td>(0.643,4.531)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.431</td>
<td>(-0.881,2.227)</td>
</tr>
<tr>
<td>Japan</td>
<td>2.360</td>
<td>(0.985,4.320)</td>
</tr>
<tr>
<td>U.K.</td>
<td>1.850</td>
<td>(0.654,3.771)</td>
</tr>
<tr>
<td>G6</td>
<td>1.983</td>
<td>(0.644,3.969)</td>
</tr>
</tbody>
</table>

1. G6 average is significant.
2. All coefficient estimates for individual countries are negative. Joint bootstrap test of null they are all \( \leq 0 \) is strongly rejected.
Regression $\hat{E}_t \sum_{j=0}^{\infty} (\rho_{t+j+1} - \bar{\rho}) = \beta_0 + \beta_1 (\hat{r}_t^* - \hat{r}_t) + u_{t+1}$

1979:6-2009:10

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\beta}_s$</th>
<th>90% c.i.( $\hat{\beta}_s$ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-24.762</td>
<td>(-52.700, -15.414)</td>
</tr>
<tr>
<td>France</td>
<td>-13.983</td>
<td>(-34.960, 0.200)</td>
</tr>
<tr>
<td>Germany</td>
<td>-33.895</td>
<td>(-58.804, -10.621)</td>
</tr>
<tr>
<td>Italy</td>
<td>-26.556</td>
<td>(-49.863, -10.649)</td>
</tr>
<tr>
<td>Japan</td>
<td>-15.225</td>
<td>(-37.617, -2.177)</td>
</tr>
<tr>
<td>U.K.</td>
<td>-10.717</td>
<td>(-27.130, 1.060)</td>
</tr>
<tr>
<td>G6</td>
<td>-30.890</td>
<td>(-56.359, -14.642)</td>
</tr>
</tbody>
</table>

1. Confidence intervals are wide, reflecting mostly serial correlation in residual.
2. G6 average is significant.
3. All coefficient estimates for individual countries are positive.
   Joint bootstrap test of null they are all $\geq 0$ is strongly rejected.
Slope coefficients and 90% confidence interval of the regression:

\[ \hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j(\hat{r}_t^* - \hat{r}_t) + u_t^j \]
Slope coefficients and 90% confidence interval of the regression:

\[ \hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j(i^*_t - i_t) + u_t^j \]
Slope coefficients and 90% confidence interval of the regression:

\[ \rho_{t+j} = \zeta_j + \beta_j \left( \hat{r}_t^* - \hat{r}_t \right) + u_t^j \]
Slope coefficients and 90% confidence interval of the regression:

$$\rho_{t+j} = \zeta_j + \beta_j \left( i_t^* - i_t \right) + u_t^j$$
Slope coefficients and 90% confidence interval of the regression:

\[ \hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j \left( \hat{r}^*_t - \hat{r}_t \right) + u_t^j \]

12-lag VECM
Slope coefficients and 90% confidence interval of the regression:

\[ \hat{E}_t(\rho_{t+j}) = \zeta_j + \beta_j (\hat{r}_t^* - \hat{r}_t) + u_t \]

Stock prices, gold price, oil price, long-term bond yields in VECM
An intuitive explanation of why the models built to explain the Fama puzzle cannot explain the excess volatility puzzle:

The key economic behavior in the models is underreaction. When $r_t^* - r_t$ rises, investors buy foreign assets, but they underreact.

In the risk premium story, they underreact because the foreign exchange risk has increased for home investors. In the rational inattention story, they underreact because not everyone rebalances their portfolio.

But the volatility puzzle, $\text{cov}(s_t^T - s_t^{lp}, r_t^* - r_t) > 0$, calls for overreaction. When $r_t^* - r_t$ rises, the exchange rate tends to rise more than it would under interest parity.

The stories for the volatility puzzle necessarily get things wrong for the level of the exchange rate.
Some algebraic intuition

Models of the Fama puzzle, \( \text{cov}(\rho_{t+1}, r_t^* - r_t) > 0 \), are not able to explain a switch in sign for \( \text{cov}(E_t \rho_{t+j+1}, r_t^* - r_t) \) as \( j \) increases.

Why not? They have a single economic factor driving \( \rho_{t+1} \) and \( r_t^* - r_t \). The economic logic of the models dictates those covary positively.

But \( \text{cov}(E_t \rho_{t+j+1}, r_t^* - r_t) = \text{cov}(E_t \rho_{t+1}, r_{t-j}^* - r_{t-j}) \).

Unless the single factor driving \( r_t^* - r_t \) has some funky dynamics, if \( \text{cov}(\rho_{t+1}, r_t^* - r_t) > 0 \) then we will also have \( \text{cov}(E_t \rho_{t+1}, r_{t-j}^* - r_{t-j}) > 0 \).
Review of foreign exchange risk premium

$m_{t+1}, m^*_{t+1}$ are logs of home, foreign stochastic discount factors

Under complete markets,

$$s_{t+1} + p^*_{t+1} - p_{t+1} - (s_t + p^* - p_t) = m^*_{t+1} - m_{t+1}$$

Since $r_t = -E_t m_{t+1} - \frac{1}{2} \text{var}_t (m_{t+1})$ and $r^*_t = -E_t m^*_{t+1} - \frac{1}{2} \text{var}_t (m^*_{t+1})$

Then $r^*_t - r_t = E_t (m_{t+1} - m^*_{t+1}) + \frac{1}{2} (\text{var}_t m_{t+1} - \text{var}_t m^*_{t+1})$

$$E_t \rho_{t+1} = \frac{1}{2} (\text{var}_t m_{t+1} - \text{var}_t m^*_{t+1})$$
1. Campbell-Cochrane preferences (Verdelhan, 2010)
2. EZ preferences, identical preferences (Bansal and Shaliastovich, 2013)
3. EZ preferences, asymmetric preferences (Lustig, et. al. 2011)

I will use (2) as an example. The others are similar.
Under EZ preferences, let $\theta$ be the coefficient of RRA. Assume $\theta > 1$.

Let $\varepsilon$ be the intertemporal rate of substitution, $>0$.

$g_{t+1}^H, g_{t+1}^F$ are innovations to growth rates of consumption.

Assume $\text{var}_t(g_{t+1}^i)$ is AR(1) with serial correlation of $\eta_i$ and mutually ind.

\[
E_t \rho_{t+1} = \frac{1}{2} \theta^2 \left( \text{var}_t(g_{t+1}^H) - \text{var}_t(g_{t+1}^F) \right)
\]

\[
r_t^* - r_t = \frac{1}{2} \left( \theta + (\theta - 1) / \varepsilon \right) \left( \text{var}_t(g_{t+1}^H) - \text{var}_t(g_{t+1}^F) \right)
\]

\[
\text{cov}(\rho_{t+1}, r_t^* - r_t) = \frac{1}{4} \theta^2 \left( \theta + (\theta - 1) / \varepsilon \right) \left[ \text{var}_t(g_{t+1}^H) + \text{var}_t(g_{t+1}^F) \right]
\]

But

\[
\text{cov}(\rho_{t+j+1}, r_t^* - r_t) = \frac{1}{4} \theta^2 \left( \theta + (\theta - 1) / \varepsilon \right) \left[ \eta_H^j \text{var}_t(g_{t+1}^H) + \eta_F^j \text{var}_t(g_{t+1}^F) \right]
\]

Obviously, there is no change in sign.
With “delayed overshooting” there is a monetary contraction in, for example, the foreign country. \( r_t^* - r_t \) rises.

The home currency should depreciate, so \( s_t^{IP} \) should rise. Some investors are inattentive, so \( s_t \) does not rise as much as it should. We can expect further depreciation of home currency.

In essence, because investors are inattentive, when \( r_t^* - r_t \) rises, it takes time for investors to shift to the foreign asset. As this shifting occurs over time, the value of the foreign asset keeps being driven up, and holders of the foreign asset receive an excess return.

When all the investors have shifted, the excess return disappears. But it never reverses. That is,

\[
\text{cov}(\rho_{t+1}, r_t^* - r_t) > 0 \quad \text{and also} \quad \text{cov}(E_t \rho_{t+j+1}, r_t^* - r_t) \geq 0.
\]
A sketch of a model that might work

The key is that there must be some two economic forces, one that leads to an underreaction and one to an overreaction. If the former force is more volatile in the short run, it can account for the Fama puzzle. If the latter is more persistent, it can account for the volatility puzzle.

Incorporating a liquidity return seems like a natural candidate. Certainly recently the demand for short term dollar assets, valued for their liquidity (usable as collateral, e.g.) seems to have had a role in driving the exchange rate.

It is also natural to think that there are factors that give the ex ante excess return and interest rates different correlations.

The paper lays out a simple model in which a liquidity premium is incorporated in a very standard New Keynesian open-economy model.
→ On the one hand, at the margin when the Fed raises the interest rate, liquid assets are more valuable on the margin. Dollar assets earn a liquidity return – which appears as a smaller pecuniary return.

This can account for the excess volatility when interest rates rise. The dollar is stronger both because of persistent interest rate increases and because of higher liquidity value.

→ But also, interest rates may respond endogenously to liquidity “shocks”. Suppose there is a shock to the financial system that reduces the liquidity value of foreign assets. There is a drop in demand for those assets, leading to a depreciation of the foreign currency. This increases inflationary pressure in the foreign country, leading the central bank to increase the interest rate.

As in the risk-premium and delayed overshooting stories, the desirability of the asset is lower from non-pecuniary factors when the interest rate is high.
Conclusions

The two puzzles concerning interest rates and exchange rates are not plausibly explained by a single economic force.

I propose an example of a model that could work, but there are other possibilities (peso problem, bandwagon effects,...), possibly in conjunction with the models of the Fama puzzle.

These puzzles have implications beyond international finance. The currency is the perhaps the only “national” asset. Its pricing is determined only by aggregate factors, and so may be important in understanding how aggregate shocks affect asset prices.