Exiting from QE

by

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Some comments from Mark Watson

Questions:

- How can we estimate effects of monetary policy when the policy instrument changes endogenously from the short term interest rate to QE?

- How can we estimate the effect of changing instruments, i.e., exiting from QE?

Answer:

- Use standard recursive SVAR that allows for discrete breaks associated with changes in policy instruments.

  - Model breaks in terms of observables
  - Do some careful data work
  - Estimate parameters using appropriate methods (breaks, truncation associated with bounds).
  - Make some sensible empirical choices
  - Compute IRFs and counterfactuals using nonlinear methods.

Models and VARs

Notation: $\begin{bmatrix} X_t \\ P_t \end{bmatrix}$, $X_t = \begin{bmatrix} \text{Output gap} \\ \text{Inflation} \end{bmatrix}$ and $P_t = \begin{bmatrix} \text{Bank Rate} \\ \text{Excess Reserves} \end{bmatrix}$

Model:
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \ \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_P\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \ \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

Linearize model and solve

SVAR:
$$\begin{cases} X_t = \Phi X_{t-1} + \mathrm{B} P_{t-1} + \Gamma_{XX}\eta_{X,t} + \Gamma_{XP}\eta_{P,t} \\ P_t = \Lambda X_{t-1} + \Psi P_{t-1} + \Gamma_{PX}\eta_{X,t} + G_{PP}\eta_{P,t} \end{cases}$$

2 Models

Model 0:
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ \mid \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{0,P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ \mid \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

Model 1:
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ \mid \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{1,P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ \mid \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

SVAR 0:
$$\begin{cases} X_t = \Phi_0 X_{t-1} + \mathrm{B}_0 P_{t-1} + \Gamma_{0,XX}\eta_{X,t} + \Gamma_{0,XP}\eta_{P,t} \\ P_t = \Lambda_0 X_{t-1} + \Psi_0 P_{t-1} + \Gamma_{0,PX}\eta_{X,t} + G_{0,PP}\eta_{P,t} \end{cases}$$

SVAR 1:
$$\begin{cases} X_t = \Phi_1 X_{t-1} + \mathrm{B}_1 P_{t-1} + \Gamma_{1,XX}\eta_{X,t} + \Gamma_{1,XP}\eta_{P,t} \\ P_t = \Lambda_1 X_{t-1} + \Psi_1 P_{t-1} + \Gamma_{1,PX}\eta_{X,t} + G_{1,PP}\eta_{P,t} \end{cases}$$

1 Model, 2 Regimes ($s_t = 0$ or $s_t = 1$)

Model($s_t = 0$):
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{0,P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

Model($s_t = 1$):
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{1,P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

SVAR($s_t = 0$): ??
$$\begin{cases} X_t = \Phi_0 X_{t-1} + \mathrm{B}_0 P_{t-1} + \Gamma_{0,XX}\eta_{X,t} + \Gamma_{0,XP}\eta_{P,t} \\ P_t = \Lambda_0 X_{t-1} + \Psi_0 P_{t-1} + \Gamma_{0,PX}\eta_{X,t} + G_{0,PP}\eta_{P,t} \end{cases}$$

SVAR($s_t = 1$): ??
$$\begin{cases} X_t = \Phi_1 X_{t-1} + \mathrm{B}_1 P_{t-1} + \Gamma_{1,XX}\eta_{X,t} + \Gamma_{1,XP}\eta_{P,t} \\ P_t = \Lambda_1 X_{t-1} + \Psi_1 P_{t-1} + \Gamma_{1,PX}\eta_{X,t} + G_{1,PP}\eta_{P,t} \end{cases}$$

What matters: $E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t)$
which depends on forecasts of s_t, s_{t+1}, s_{t+2}, s_{t+3}, …

A special case: $P\left(s_{t+k} \middle| s_t, X_t, P_t, \left\{s_{t-j}, X_{t-j}, P_{t-j}\right\}_{j\geq 1}\right) = P\left(s_{t+k} \middle| s_t\right)$
That is, s_t is an exogenous first-order Markov process

Special Case: $P\left(s_{t+k} \middle| s_t, X_t, P_t, \left\{s_{t-j}, X_{t-j}, P_{t-j}\right\}_{j\geq 1}\right) = P\left(s_{t+k} \middle| s_t\right)$

Model: $$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{(s_t=0),P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \\ P_t = f_{(s_t=1),P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

Suppose $\Omega_t^X = \Omega_t^P$ (and both include s_t). Things are simple

SVAR($s_t = 0$): $$\begin{cases} X_t = \Phi_0 X_{t-1} + \mathrm{B}_0 P_{t-1} + \Gamma_{0,XX}\eta_{X,t} + \Gamma_{0,XP}\eta_{P,t} \\ P_t = \Lambda_0 X_{t-1} + \Psi_0 P_{t-1} + \Gamma_{0,PX}\eta_{X,t} + G_{0,PP}\eta_{P,t} \end{cases}$$

SVAR($s_t = 1$): $$\begin{cases} X_t = \Phi_1 X_{t-1} + \mathrm{B}_1 P_{t-1} + \Gamma_{1,XX}\eta_{X,t} + \Gamma_{1,XP}\eta_{P,t} \\ P_t = \Lambda_1 X_{t-1} + \Psi_1 P_{t-1} + \Gamma_{1,PX}\eta_{X,t} + G_{1,PP}\eta_{P,t} \end{cases}$$

Suppose Ω_t^X includes s_{t-1} but not s_t

Ω_t^P include s_t

$$\text{SVAR}(s_t = 0, s_{t-1} = 0): \begin{cases} X_t = \Phi_{0,0} X_{t-1} + \mathrm{B}_{0,0} P_{t-1} + \Gamma_{0,0,XX} \eta_{X,t} + \Gamma_{0,0,XP} \eta_{P,t} \\ P_t = \Lambda_{0,0} X_{t-1} + \Psi_{0,0} P_{t-1} + \Gamma_{0,0,PX} \eta_{X,t} + G_{0,0,PP} \eta_{P,t} \end{cases}$$

$$\text{SVAR}(s_t = 1, s_{t-1} = 0): \begin{cases} X_t = \Phi_{1,0} X_{t-1} + \mathrm{B}_{1,0} P_{t-1} + \Gamma_{1,0,XX} \eta_{X,t} + \Gamma_{1,0,XP} \eta_{P,t} \\ P_t = \Lambda_{1,0} X_{t-1} + \Psi_{1,0} P_{t-1} + \Gamma_{1,0,PX} \eta_{X,t} + G_{1,0,PP} \eta_{P,t} \end{cases}$$

$$\text{SVAR}(s_t = 0, s_{t-1} = 1): \begin{cases} X_t = \Phi_{0,1} X_{t-1} + \mathrm{B}_{0,1} P_{t-1} + \Gamma_{0,1,XX} \eta_{X,t} + \Gamma_{0,1,XP} \eta_{P,t} \\ P_t = \Lambda_{0,1} X_{t-1} + \Psi_{0,1} P_{t-1} + \Gamma_{0,1,PX} \eta_{X,t} + G_{0,1,PP} \eta_{P,t} \end{cases}$$

$$\text{SVAR}(s_t = 1, s_{t-1} = 1): \begin{cases} X_t = \Phi_{1,1} X_{t-1} + \mathrm{B}_{1,1} P_{t-1} + \Gamma_{1,1,XX} \eta_{X,t} + \Gamma_{1,1,XP} \eta_{P,t} \\ P_t = \Lambda_{1,1} X_{t-1} + \Psi_{1,1} P_{t-1} + \Gamma_{1,1,PX} \eta_{X,t} + G_{1,1,PP} \eta_{P,t} \end{cases}$$

Truth: SVAR($s_t = 0$, $s_{t-1} = 0$), SVAR($s_t = 0$, $s_{t-1} = 1$),
SVAR($s_t = 1$, $s_{t-1} = 0$),SVAR($s_t = 1$, $s_{t-1} = 1$)

Estimate: SVAR($s_t = 0$) , SVAR($s_t = 1$) model.

What will I find ?

SVAR($s_t = 0$) will be a mixture of SVAR($s_t = 0$, $s_{t-1} = 0$), SVAR($s_t = 0$, $s_{t-1} = 1$)

SVAR($s_t = 1$) will be a mixture of SVAR($s_t = 1$, $s_{t-1} = 0$), SVAR($s_t = 1$, $s_{t-1} = 1$)

but

$P(s_t = 0, s_{t-1} = 0) \gg P(s_t = 0, s_{t-1} = 1)$, so SVAR($s_t = 0$) $\approx$ SVAR($s_t = 0$, $s_{t-1} = 0$)

$P(s_t = 1, s_{t-1} = 1) \gg P(s_t = 1, s_{t-1} = 0)$, so SVAR($s_t = 1$) $\approx$ SVAR($s_t = 1$, $s_{t-1} = 1$)

Some questions for this misspecified model

(1) IRFs of Policy variables under $s = 1$ (R – shocks) or $s = 0$ (m – shocks)

$$\text{SVAR}(s_t = 1) \approx \text{SVAR}(s_t = 1, s_{t-1} = 1)$$
$$\text{SVAR}(s_t = 0) \approx \text{SVAR}(s_t = 0, s_{t-1} = 0)$$

Approximately correct "continuing regime" answers.

(2) Counterfactual "Exit from Regime"

	s_{t-1}	s_t	s_{t+1}	s_{t+2}
Factual	0	1	1	1
Counterfactual	0	0	1	1

Correct Answers from

	$t-1$	t	$t+1$	$t+2$
Factual		SVAR($s_t = 1$, $s_{t-1} = 0$)	SVAR($s_t = 1$, $s_{t-1} = 1$)	
Counterfactual		SVAR($s_t = 0$, $s_{t-1} = 0$)	SVAR($s_t = 0$, $s_{t-1} = 1$)	

Misspecfied Model Answers from

	$t-1$	t	$t+1$	$t+2$
Factual		**SVAR($s_t = 1$, $s_{t-1} = 1$)**	SVAR($s_t = 1$, $s_{t-1} = 1$)	
Counterfactual		SVAR($s_t = 0$, $s_{t-1} = 0$)	**SVAR($s_t = 1$, $s_{t-1} = 1$)**	

Special Case: $P\left(s_{t+k}\middle|s_t, X_t, P_t, \left\{s_{t-j}, X_{t-j}, P_{t-j}\right\}_{j\geq 1}\right) = P\left(s_{t+k}\middle|s_t\right)$

Hyashi-Koeda:

$$s_t = \begin{cases} 1 \text{ if } \begin{cases} s_{t-1} = 0, \rho r^*\left(X_t, \ldots, X_{t-11}\right) + (1-\rho) r_{t-1} > \bar{r}_t > 0 \text{ and } \pi_t > q_t \sim N(\bar{\pi}, \sigma^2) \\ s_{t-1} = 1, \rho r^*\left(X_t, \ldots, X_{t-11}\right) + (1-\rho) r_{t-1} > \bar{r}_t > 0 \end{cases} \\ 0 \quad \text{otherwise} \end{cases}$$

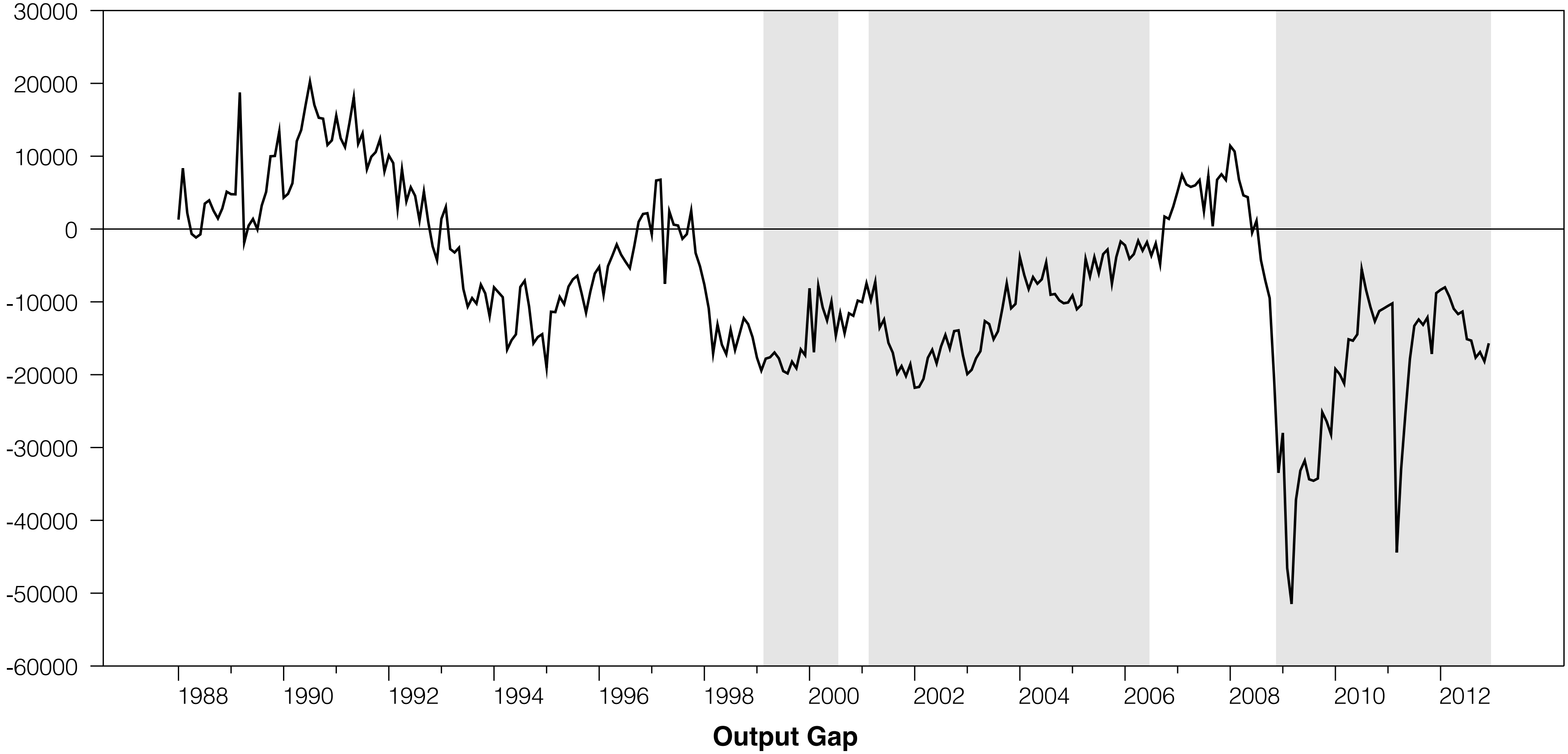
Model:

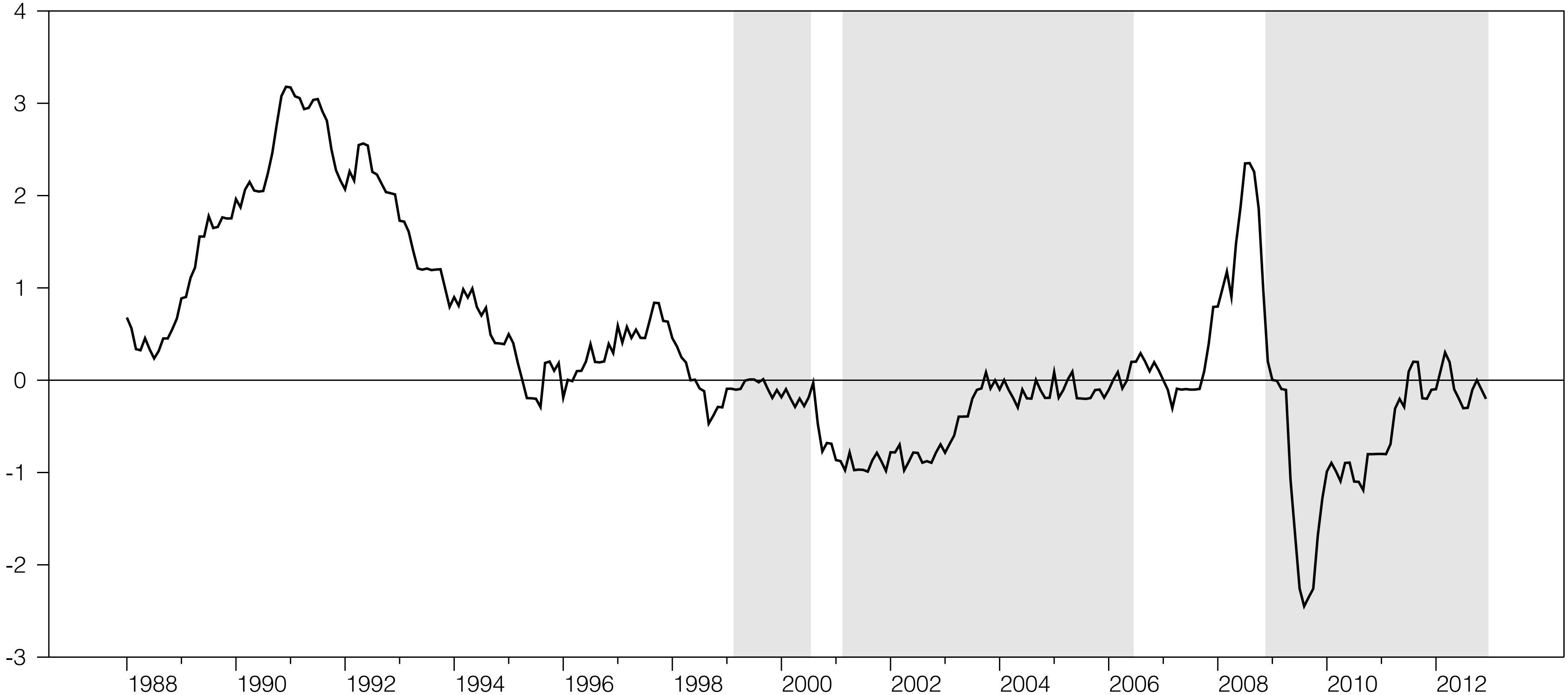
$$\begin{cases} X_t = f_X\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^X), \eta_{X,t}\right) \\ P_t = f_{(s_t=0),P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \\ P_t = f_{(s_t=1),P}\left(X_{t-1}, P_{t-1}, E(P_t, X_{t+1}, P_{t+1}, \ \ldots \ | \Omega_t^P), \eta_{P,t}\right) \end{cases}$$

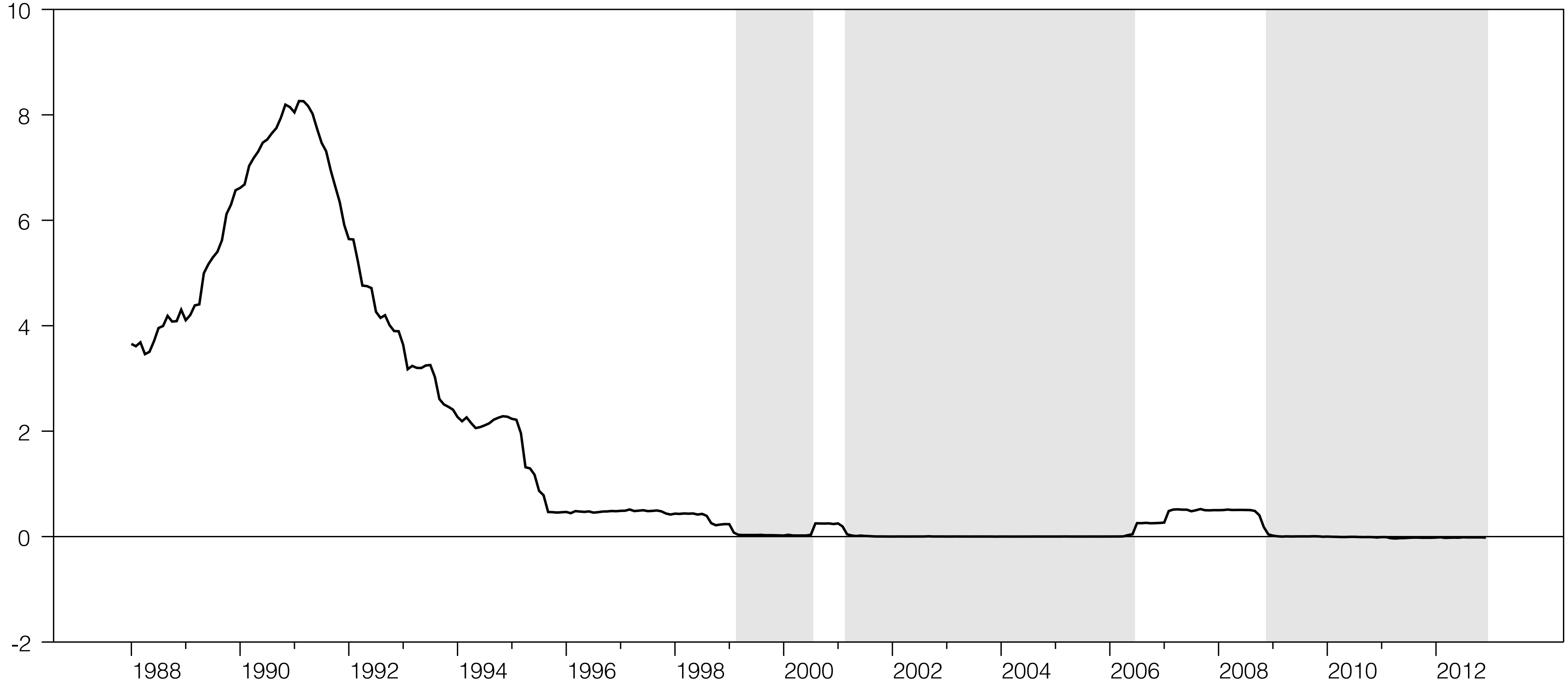
SVAR … more complicated (!)

Approximations: SVAR($s_t = 0$) and SVAR($s_t = 1$) are averages over these regimes

Data:


30000
20000
10000
0
-10000
-20000
-30000
-40000
-50000
-60000
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
Output Gap


4
3
2
1
0
-1
-2
-3
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
Inflation


10
8
6
4
2
0
-2
1988
1990
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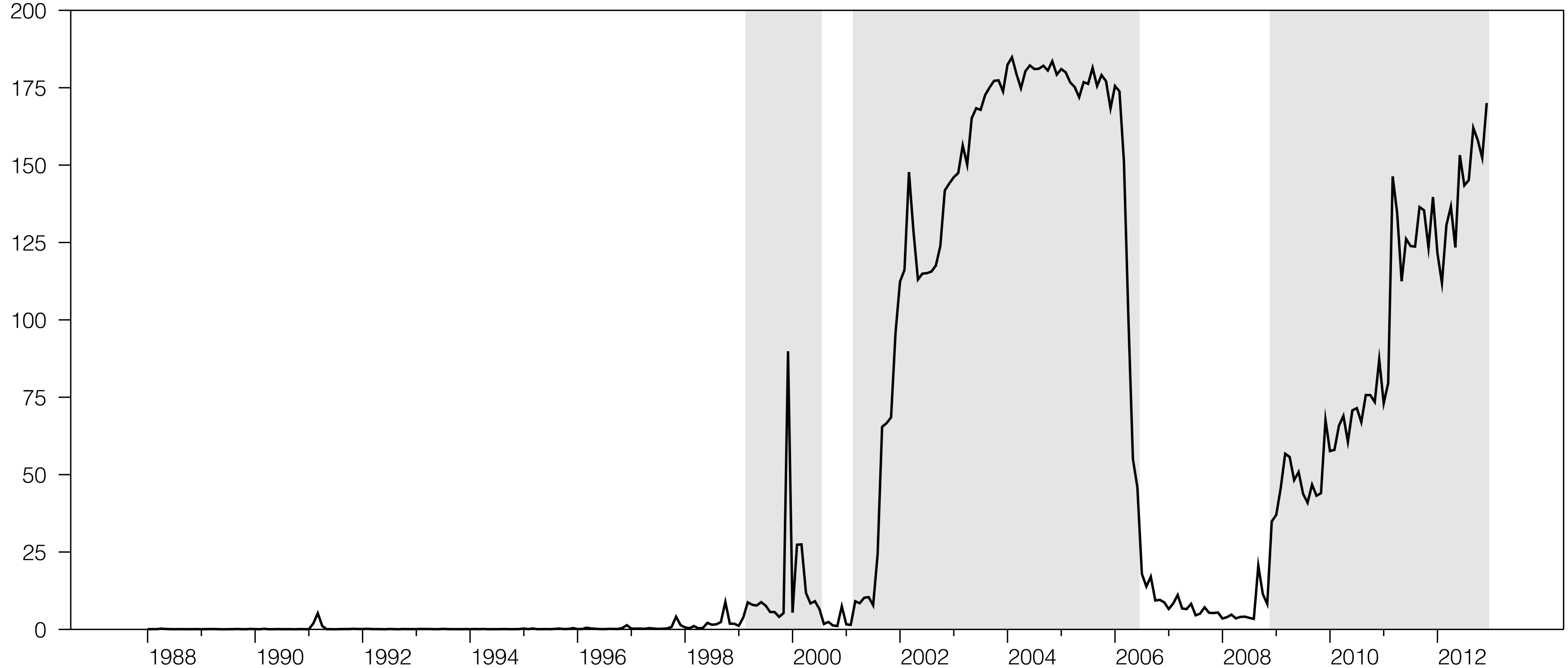
2012

R-RBar



Excess Reserves

Approximations: SVAR($s_t = 0$) and SVAR($s_t = 1$) are averages over these regimes

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(2) Counterfactual "Exit from Regime"

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