Thoughts on Event Forecasting:
Idiosyncratic and Systemic Aspects

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“Predicting Rare Events:
Evaluating Systemic and Idiosyncratic Risk”
September 28-29, 2012

September 21, 2012
A Difficult Problem (So Set Your Expectations Low)

Consider a currency collapse:

- Comparatively easy to identify overvalued exchange rates

- Notoriously difficult to predict whether/when a crash will occur
Consideration: The Forecast Object

Time series

Cross section

Event
  – Event timing
  – Event outcome or magnitude

e.g., default, loss given default
Financial and Economic Events

- Financial:
  - Corporate bond default
  - Sovereign bond default
  - Margin call
  - $\alpha\%$ VaR breach
  - Crisis: Stock market, currency, banking, current account, ...
  - Circuit breaker tripped
  - Yields hit zero lower bound

- Economic:
  - Two firms merge
  - Recession begins or ends
  - A firm fails to meet analysts’ earnings expectations
  - Country X leaves EMU
  - EMU collapses
  - Europe collapses and resumes feudalism

- Broader: Marketing, insurance, politics, ...
Consideration: The Forecast Statement

<table>
<thead>
<tr>
<th>Point</th>
<th>Interval</th>
<th>Density</th>
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<tbody>
<tr>
<td>Probability</td>
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- Event forecasts are probability forecasts
  
- Can be interpreted as a *point* forecast $p_t$ of a 0-1 indicator $I_t$

- Can be interpreted as a complete *density* forecast

$\implies$ Cross-fertilization possibilities with density forecasting
  (construction, evaluation, combining)
Consideration: The Loss function

Prob. forecast evaluation from a *point* forecasting perspective:

\[ QPS = \frac{1}{T} \sum_{t=1}^{T} (p_t - l_t)^2 \]

– Most relevant loss function? False alarms vs. missed calls.

Prob. forecast evaluation from a *density* forecasting perspective:

\[ \{p_t\}_{t=1}^{T} = \{p^*_t\}_{t=1}^{T} \implies PIT \sim iidU(0, 1) \]

– Subsumes measures of global calibration, local calibration, resolution, sharpness, etc.?
Evaluation of Rare-Event Forecasts is Challenging

Events come at different frequencies:

- **High**: Transactions in liquid financial assets, ...
- **Less high**: Annual (seasonal) peaks in retail sales
- **Medium**: Business cycle expansions and recessions

- **Low**: Government bond defaults, binding zero-lower-bound constraints, systemic financial collapses, depressions, apocalypse

It’s hard to assess the conditional calibration of rare event forecasts, precisely because of their rarity!

- Theory, Bayesian priors, introspection
Covariates hold special appeal for event forecasting:

- Beyond “lagged dependent variables”
- Useful “leading indicators”? (e.g., A leverage regime switch may presage a credit market regime switch)
A Model for Dynamic Event Forecasting

\[ p(y_t | \pi_t) = \pi_t^{y_t} (1 - \pi_t)^{1-y_t} \]

\[ P(y | \pi) = \prod_{t=1}^{T} p(y_t | \pi_t) \]

\[ \theta_t = \log \left( \frac{\pi_t}{1 - \pi_t} \right) \]

\[ P(y_t | \theta_t) = \exp[y_t \theta_t - \log(1 + \exp(\theta_t))] \]

\[ P(y | \theta) = \prod_{t=1}^{T} p(y_t | \theta_t) \]

\[ \theta_t = \mu + \chi_t^\prime \beta + \varepsilon_t \]

\[ (1 - L)^d \Phi(L) \varepsilon_t = \eta_t \]

\[ \eta_t \sim iid(0, \sigma_\eta^2) \]
Multivariate

Everything so far has been univariate

But we really want multivariate:
  – Idiosyncratic aspects
  – Common aspects
  – Connectedness and systemic behavior
Financial and Economic Connectedness

- Market Risk, Portfolio Concentration Risk (return connectedness)
- Credit Risk (default connectedness)
- Counterparty Risk, Gridlock Risk (bilateral and multilateral contractual connectedness)
- Systemic Risk (system-wide connectedness)
- Business Cycle Risk (local or global real output connectedness)
Factor Structure, Single Factor, Fully Orthogonal

\[ r_{it} = \lambda_i f_t + \varepsilon_{it} \]

\[ \implies \sigma_{it}^2 = \lambda_i^2 \sigma_f^2 + \gamma_i^2 \]

\[ i = 1, \ldots, N \]

Fraction of \( i \)'s variance coming from others:

\[ \frac{\lambda_i^2 \sigma_f^2}{\lambda_i^2 \sigma_f^2 + \gamma_i^2} = \frac{1}{1 + \frac{\gamma_i^2}{\lambda_i^2 \sigma_f^2}} \]

So take:

\[ C_i = \frac{\lambda_i^2 \sigma_f^2}{\gamma_i^2} \]

Obtain system total by adding over \( i \):

\[ C = \sigma_f^2 \sum_{i=1}^{N} \left( \frac{\lambda_i}{\gamma_i} \right)^2 \]
Now Move to Factor Structure for Event Indicator

Linear probability model analog:

\[ I(r_{it}) = \lambda_i I(f_t) + \varepsilon_{it} \]

\[ \sigma^2_{it} = \lambda_i^2 \sigma^2_{I(f)} + \gamma_i^2 \]

\[ i = 1, \ldots, N \]

– Connectedness measures remain intact

Logit/probit analog:

\[ I(r_{it}) = squash(\lambda_i I(f_t) + \varepsilon_{it}) \]

– What happens to connectedness measures?
What We Really Want

– General framework not necessarily assuming factor structure
– Based on conditional as opposed to unconditional variation
  – General connectedness and systemic risk measures
  – Links to stress testing?
A natural modeling question:

*What fraction of the H-step-ahead prediction-error variance of variable i is due to shocks in variable j, ∀i, j?*

Variance decomposition: $d_{ij}^H, \forall i, j$

A natural financial/economic connectedness question:

*What fraction of the H-step-ahead prediction-error variance of variable i is due to shocks in variable j, ∀j ≠ i?*

**Non-own** elements of the variance decomposition: $d_{ij}^H, \forall j \neq i$
## Variance Decompositions and the Connectedness Table

### $N$-Variable Connectedness Table

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>...</th>
<th>$x_N$</th>
<th>From Others to $i$</th>
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<td>$d_{11}^H$</td>
<td>$d_{12}^H$</td>
<td>...</td>
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<td>$\Sigma_{j=1}^N d_{1j}^H, j \neq 1$</td>
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<td>$\Sigma_{j=1}^N d_{2j}^H, j \neq 2$</td>
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<td>...</td>
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<tr>
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<td>$d_{N2}^H$</td>
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</table>

<table>
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<th>$\Sigma_{i=1}^N d_{i2}^H$</th>
<th>...</th>
<th>$\Sigma_{i=1}^N d_{iN}^H$</th>
<th>$\Sigma_{i,j=1}^N d_{ij}^H$</th>
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</thead>
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<td>$i \neq 1$</td>
<td>$\Sigma_{i=1}^N d_{i1}^H$</td>
<td>$\Sigma_{i=1}^N d_{i2}^H$</td>
<td>...</td>
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<td>$i \neq 2$</td>
<td>$\Sigma_{i=1}^N d_{i1}^H$</td>
<td>$\Sigma_{i=1}^N d_{i2}^H$</td>
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Upper-left block is variance decomposition matrix, $D$

Connectedness involves the **non-diagonal** elements of $D$. 

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[University of Pennsylvania logo]  

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Connectedness Measures

- **Pairwise Directional:** \( C_{i \leftarrow j}^H = d_{ij} \) ("i’s imports from j")
- **Net:** \( C_{ij}^H = C_{j \leftarrow i}^H - C_{i \leftarrow j}^H \) ("ij bilateral trade balance")

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- **Total Directional:**
  - From others to i: \( C_{i \leftarrow \bullet}^H = \sum_{j=1}^{N} d_{ij} \) ("i’s total imports")
  - To others from j: \( C_{\bullet \leftarrow j}^H = \sum_{i=1}^{N} d_{ij} \) ("j’s total exports")
- **Net:** \( C_{i}^H = C_{\bullet \leftarrow i}^H - C_{i \leftarrow \bullet}^H \) ("i’s multilateral trade balance")

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- **Total:** \( C^H = \frac{1}{N} \sum_{i,j=1}^{N} d_{ij}^H \) ("total world exports")
Networks I: Representation

Adjacency Matrix (Symmetric)

\[ A = \begin{pmatrix}
0 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 \\
\end{pmatrix} \]

\[ A_{ij} = 1 \text{ if nodes } i, j \text{ linked} \]

\[ A_{ij} = 0 \text{ otherwise} \]
Degree of node $i$, $d_i$:
\[ d_i = \sum_{j=1}^{N} A_{ij} \]

Discrete degree distribution, $P(d)$, on $0, ..., N - 1$

Mean degree, $E(d)$, is the key connectedness measure

Beautiful results (e.g., “small world”) involve the mean degree:
\[ \text{diameter} \approx \frac{\ln N}{\ln E(d)} \]
Networks II: Representation
(Weighted, Directed)

\[ A = \begin{pmatrix}
0 & .5 & .7 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & .3 & 0 \\
0 & 0 & 0 & .7 & 0 & .3 \\
.3 & .5 & 0 & 0 & 0 & 0 \\
.5 & 0 & 0 & 0 & 0 & 0 .3 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix} \]

“to \( i \), from \( j \)”
Networks II: Degree
(Weighted, Directed)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_i^{\text{from}} = \sum_{j=1}^{N} A_{ij}$$

$$d_j^{\text{to}} = \sum_{i=1}^{N} A_{ij}$$

Continuous “from” and “to” degree distributions on $[0, N - 1]$

Mean degrees $E(d)$ remain key for connectedness
Central Observation: $D$ is a Weighted, Directed Network

**Connectedness Table**

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<td>...</td>
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</tr>
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<td>$d_{N2}^H$</td>
<td>...</td>
<td>$d_{NN}^H$</td>
<td>$\sum_{j \neq N} d_{Nj}^H$</td>
</tr>
</tbody>
</table>

To Others

$\sum_{i \neq 1} d_{i1}^H$, $\sum_{i \neq 2} d_{i2}^H$, ... $\sum_{i \neq N} d_{iN}^H$, $\sum_{i \neq j} d_{ij}^H$

$C_{i \leftarrow \bullet}^H = \sum_{j=1}^{N} d_{ij}^H$, are the “from degrees”

$C_{\bullet \leftarrow j}^H = \sum_{i=1}^{N} d_{ij}^H$, are the “to degrees”

$C^H = \frac{1}{N} \sum_{i,j=1}^{N} d_{ij}^H$, is the mean degree (to or from)
Relationships to Other Market-Based Measures I: Marginal Expected Shortfall (MES)

\[ MES^{j|mkt}_{T+1|T} = E_T [r_{j,T+1}|C (r_{mkt,T+1})] \]

- Sensitivity of firm \( j \)'s return to extreme market event \( C \)
- Market-based “stress test” of firm \( j \)'s fragility
- Like “total directional connectedness from” (from degree)
Relationships to Other Market-Based Measures II: CoVaR and ∆CoVaR

\[ p = \Pr_T \left( r_{mkt,T+1} < -\text{CoVaR}_{T+1|T}^{mkt|i} \mid \mathbb{C}(r_{i,T+1}) \right) \]

- Measures tail-event linkages
- Leading choice of \( \mathbb{C}(r_{i,T+1}) \) is that firm \( i \) breaches its VaR
- Like “total directional connectedness to” (to degree)

\[ \Delta \text{CoVaR}_{T+1|T}^{mkt|i} = \text{CoVaR}_{T+1|T}^{mkt|\text{VaR}(i)} - \text{CoVaR}_{T+1|T}^{mkt|\text{Med}(i)} \]
Thus far we’ve worked under correct specification, in population:

\[ C(x, H, B(L)) \]

Now we want:

\[ \hat{C} \left( x, H, B(L), M(L; \hat{\theta}) \right), \]

and similarly for other variants of connectedness.
Many Interesting Issues

- x objects: Returns? **Return volatilities**? Real activities?
- x universe: How many and which ones?
  \((\approx 15 \text{ major financial institutions})\)
- x frequency: **Daily**? Monthly? Quarterly?

- \(H:\) **Match VaR horizon**? Holding period?

- \(M:\) **VAR**? Structural?

- Identification of variance decompositions:
  Cholesky? **Generalized**? Structural?

- Estimation: **Classical**? Bayesian?
Connectedness of Major U.S. Financial Institutions

\[ \hat{C} \left( x, H, B(L), M(L; \theta) \right) \]

- **x**: Thirteen daily realized stock return volatilities
  - Commercial banks: JP Morgan Chase (JPM), Bank of America (BAC), CitiGroup (C), Wells Fargo (WFC), Bank of New York Mellon (BK), U.S. BankCorp (USB), PNC Bank (PNC)
  - Investment Banks: Goldman Sachs (GS), Morgan Stanley (MS)
  - GSEs: Fannie Mae (FNM), Freddie Mac (FRE)
  - Insurance: AIG (AIG)
  - Specialized: American Express (AXP)

- **H**: 12 days

- **M(L; \theta)**: logarithmic VAR(3), generalized identification, 5/4/1999 - 4/30/2010
### Full-Sample Connectedness Table

<table>
<thead>
<tr>
<th></th>
<th>AXP</th>
<th>BAC</th>
<th>BK</th>
<th>C</th>
<th>GS</th>
<th>JPM</th>
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<td>7.4</td>
<td>17.6</td>
<td>29.6</td>
<td>70.4</td>
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</table>

| TO   | 92.9 | 99.7 | 71.3 | 106.1| 62.7 | 90.2 | 88.2 | 63.7 | 75.5 | 92.2 | 53.8  | 53.1  | 68.1  | 78.3  |

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Penn University of Pennsylvania
Estimating Time-Varying Connectedness

Before:
\[ C(x, H, B(L), M(L; \theta)) \]
\[ \hat{C}(x, H, B(L), M(L; \hat{\theta})) \]

Now:
\[ C_t(x, H, B_t(L), M(L; \theta_t)) \]
\[ \hat{C}_t(x, H, B_t(L), M(L; \hat{\theta}_t)) \]

- Time-varying parameters: **Rolling estimation**? Smooth TVP model? Regime-switching?

(100-day estimation window)
Rolling Total Connectedness

![Graph showing rolling total connectedness over the years 2000 to 2009. The graph indicates fluctuations in connectedness with peaks and troughs.](chart.png)
Net Pairwise Directional Connectedness: The Lehman Bankruptcy, September 17, 2008
Rare Event Forecasting: Swallow Hard and March Onward (There’s no Alternative)

- Probability forecasts and their evaluation
- Flexible multivariate modeling
- Systemic risk and network connectedness

[– Theory, Bayesian priors, model averaging]