Technical Appendix: Correction to “Assessing the Behavior of Recent Inflation.”

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Abstract

This appendix contains the details for computing the standard deviation of the 12-month mean inflation rate.
This appendix contains the details for computing the standard deviation of the 12-month mean inflation rate. To simplify the expressions, I employ the arithmetic mean in place of the geometric mean but the quantitative effect of this approximation is negligible.

Stochastic process for monthly inflation:

\[ \pi_t - E(\pi_t) = \rho [\pi_{t-1} - E(\pi_t)] + \varepsilon_t \]

12-month mean inflation:

\[ \bar{\pi}_t = \frac{1}{12} \sum_{i=0}^{11} \pi_{t-i} \]

Variance of monthly inflation:

\[ Var(\pi_t) = \frac{1}{(12 - 1)} \sum_{i=0}^{11} (\pi_{t-i} - \bar{\pi}_t)^2 \]

Variance of the 12-month mean:

\[
\begin{align*}
Var(\bar{\pi}_t) &= Var\left[\frac{1}{12} \sum_{i=0}^{11} \pi_{t-i}\right] \\
&= \frac{1}{12^2} Var[\pi_t + \pi_{t-1} + \pi_{t-2} + \ldots + \pi_{t-11}] \\
&= \frac{1}{12^2} \left[ 12 Var(\pi_t) + 11 \times \underbrace{Cov(\pi_t, \pi_{t-1})}_{=\rho Var(\pi_t)} + 10 \times \underbrace{Cov(\pi_t, \pi_{t-2})}_{=\rho^2 Var(\pi_t)} + \ldots \right] \\
&\approx \frac{1}{12^2} \left[ 12 Var(\pi_t) + 13.8 Var(\pi_t) \right], \text{ when } \rho = 0.4
\end{align*}
\]

Standard deviation of the 12-month mean:

\[
SD(\bar{\pi}_t) = \sqrt{Var(\bar{\pi}_t)} = \frac{\sqrt{1+13.8/12}}{\sqrt{12}} \sqrt{Var(\pi_t)} = \frac{1}{2.36} \sqrt{Var(\pi_t)}
\]