Discussion of

Futures Prices as Risk-Adjusted Forecasts of Monetary Policy

Frank Schorfheide
Department of Economics, University of Pennsylvania
Summary of Paper

• Question: is it true that

\[ \tilde{F}_t^{(n)} = \mathbb{E}_t[\tilde{R}_{t+n}] \] (1)

Here: \( \tilde{R}_t \) is Fed Funds Rate, \( \tilde{F}_t^{(n)} \) is Fed Funds Future Rate.

• Answer: no! In order to obtain \( \mathbb{E}_t[\tilde{R}_{t+n}] \) futures rates have to be risk-adjusted.

• Findings:
  
  – Adjustments are substantial: \( \tilde{F}_t^{(4)} = \mathbb{E}_t[\tilde{R}_{t+4}] + 50 \) basis points. (1 quarter horizon)
  
  – Adjustments increase in recessions (negatively correlated with employment).
  
  – Risk adjustments improve Fed Funds forecasts

• Very interesting empirical work!
Context (I)

- Reduced Form VAR:
  \[ y_t = \Phi_0 + \Phi_1 y_{t-1} + u_t, \quad u_t \sim (0, \Sigma_u) \]  \hfill (2)

- Structural VAR:
  \[ y_t = \Phi_0 + \Phi_1 y_{t-1} + \Sigma_u^{chol} \Omega \epsilon_t, \quad \epsilon_t \sim (0, I) \]  \hfill (3)
  
  $\epsilon_t$'s are “structural”, $\Omega$ is orthonormal and cannot be estimated.

- Alternative:
  \[
  \text{Monetary Policy Shock } = \epsilon_{R,t} \equiv R_t - IE_{t-\delta}[R_t] \equiv F_{t-\delta}^{\delta}
  \]  \hfill (4)
Suppose monetary policy follows:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{X_t}{X^*}\right)^{\psi_2}\right]^{(1-\rho_R)} e^{\epsilon_{R,t}}. \quad (5)$$

Then (take log deviations: $\tilde{R}_t = \ln(R_t/R^*)$):

$$\tilde{R}_t - I\mathbb{E}_{t-1}[\tilde{R}_t] = \epsilon_{R,t} + (1 - \rho_R)\psi_1(\tilde{\pi}_t - I\mathbb{E}_{t-1}[\tilde{\pi}_t]) + (1 - \rho_R)\psi_2(\tilde{X}_t - I\mathbb{E}_{t-1}[\tilde{X}_t]) \quad (6)$$

Ideally we want

$$\tilde{R}_t - I\mathbb{E}[\tilde{R}_t|\mathcal{F}_{t-1}, \pi_t, \tilde{X}_t] \quad (7)$$

to identify policy shock. Generates some delicate timing issues.

E.g., Faust, Swanson, and Wright (JME, 2004). But Monika and Eric’s paper does not focus on VAR identification...
Outline for Remainder of Discussion

• What does a prototypical monetary DSGE model have to say about the adjustment?

• Introduce Fed Funds Futures

• Derive a pricing formula, assuming log-normality: conditional covariance of real rates and output growth / consumption growth matter.

• Estimate VAR and calculate adjustment, compare to Monika and Eric’s estimates.

• Outlook
A Simple Monetary DSGE Model (I)

• Households maximize:

\[ E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{C_s^{1-\tau} - 1}{1 - \tau} + \chi \log \frac{M_s}{P_s} - h_s \right) \right], \quad (8) \]

• Consumption \( C_t \) is composed of differentiated products:

\[ C_t = \left[ \int_{0}^{1} C_t(i) \frac{\nu-1}{\nu} \right]^{\frac{\nu}{\nu-1}} \quad (9) \]

• Budget constraint:

\[ C_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} + \frac{T_t}{P_t} = W_t h_t + \frac{M_{t-1}}{P_t} + R_{t-1} \frac{B_{t-1}}{P_t} + Q_{t-1} \frac{F_{t-1} - R_t}{P_t} + D_t. \quad (10) \]

Moreover, households have access to a complete set of state-contingent claims.
A Simple Monetary DSGE Model (II)

- Household has access to Fed Future contracts. Budget constraint:

\[ \ldots = \ldots + Q_{t-1} \frac{F_{t-1} - R_t}{P_t} + \ldots \]

- At \( t - 1 \) choose quantity \( Q_{t-1} \), but no payments yet. Contracts are settled in period \( t \). \( F_{t-1} \) is Futures rate.

- Production side: monopolistic competition, Calvo-style price rigidities.

- Monetary policy:

\[
\frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho_R} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\psi_1} \left( \frac{X_t}{X^*} \right)^{\psi_2} \right]^{(1-\rho_R)} e^{\epsilon_{R,t}}. \tag{11}
\]
A Simple Monetary DSGE Model (II)

- First-order conditions imply

\[ \mathbb{I}E_{t-1} \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\tau} F_{t-1} \right] = \mathbb{I}E_{t-1} \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\tau} R_t \right] \]  \hspace{1cm} (12)

- which yields

\[ F_{t-1} = \frac{\mathbb{I}E_{t-1} \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\tau} R_t \right]}{\mathbb{I}E_{t-1} \left[ \left( \frac{C_t}{C_{t-1}} \right)^{-\tau} \frac{1}{\pi_t} \right]} \]  \hspace{1cm} (13)

- Define \( M_t^C = \frac{(C_t/C_{t-1})^{-\tau}}{\pi_t} \), then

\[ F_{t-1} = \frac{\mathbb{I}E_{t-1}[M_t^C R_t]}{\mathbb{I}E_{t-1}[M_t^C]} \]  \hspace{1cm} (14)
A Simple Monetary DSGE Model (II)

• Assume $M^C_t$ and $R_t$ are jointly log-normally distributed conditional on time $t-1$ information.

• Use $\tilde{F}_{t-1}$, $\tilde{M}^C_t$, and $\tilde{R}_t$ to denote percentage deviations from steady state.

• Then,

$$
\tilde{F}_{t-1} = \mathbb{E}_{t-1}[\tilde{R}_t] + \mathbb{E}_{t-1}[\tilde{M}^C_t] + \frac{1}{2} \left( var_{t-1}[\tilde{R}_t] + var_{t-1}[\tilde{M}^C_t] + 2cov_{t-1}[\tilde{R}_t, \tilde{M}^C_t] \right) \\
- \mathbb{E}_{t-1}[\tilde{M}^C_t] - \frac{1}{2} var_{t-1}[\tilde{M}^C_t] \\
= \mathbb{E}_{t-1}[\tilde{R}_t] + \frac{1}{2} var_{t-1}[\tilde{R}_t] + cov_{t-1}[\tilde{R}_t, \tilde{M}^C_t]
$$

Risk-Adjustment, positive according to estimates

• Roughly speaking, we need negative conditional correlation between real rate and consumption growth.
Risk Adjustments (I)


- In my simple framework I cannot explain the time variation in the adjustment that Monika and Eric find unless I introduce heteroskedasticity:

  Recession $\implies$ stronger negative correlation between consumption growth and real rate.

- Sample period: 1989:I to 2003:III. 4 lags (selected using Bayesian posterior odds),

  - Output growth, inflation (GDP deflator), Fed Funds Rate
  
  - Consumption growth, inflation (GDP deflator), Fed Funds Rate

All rates are quarter-to-quarter percentages.
Risk Adjustments (II)

- Stochastic discount factor:

\[ \tilde{M}_t^X = -\tau(\tilde{X}_t - \tilde{X}_{t-1}) - \tilde{\pi}_t \]  

(16)

where \( \tilde{X}_t \) is consumption or output.

- Risk aversion: \( \tau = 1, 2, 5, 10 \).
## Posterior for Adjustment (I)

<table>
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<tr>
<th>$\tau$</th>
<th>Mean</th>
<th>CI(low)</th>
<th>CI(high)</th>
<th>P &amp; S</th>
</tr>
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</table>

*Note:* I report posterior mean 90% probability intervals based on the output growth VAR.

P & S value: Table 1, $n = 3$. 

### Posterior for Adjustment (II)

<table>
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<th>$\tau$</th>
<th>Mean</th>
<th>CI(low)</th>
<th>CI(high)</th>
<th>P &amp; S</th>
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<td>1.351</td>
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</tbody>
</table>

*Note:* I report posterior mean 90% probability intervals based on the consumption growth VAR. P & S value: Table 1, $n = 3$. 
Further Comments and Outlook

• I enjoyed reading the paper...

• it has very interesting empirical results - certainly relevant for VAR studies that identify policy shocks using Futures data.

• Next step: study pricing models for Fed Funds Futures – fit VAR or DSGE model to obtain estimates of conditional moments needed for pricing formula and check whether we can explain magnitude of adjustment and time-variation.