Discussion of

The Term Structure of Real Rates and Expected Inflation

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Overview

- The paper presents a “No Arbitrage” model of nominal interest rates that incorporates regime switching.

- Model estimates allow us to decompose movements in the nominal term structure into variations in (i) real yields, (ii) expected inflation, and (iii) (inflation) risk premia.

- The model estimates imply:
  - the real term structure is fairly flat ~1.4%,
  - real rates are negatively correlated with inflation (expected and unexpected),
  - the inflation risk premium rises with maturity,
  - expected inflation and inflation risk account for ~80% of the variation in nominal rates.

Main Comment: This is a nice paper with a clear and important goal.
A Nominal Yield Decomposition

\[ y_t^k = E_t \pi_{t+k,k} + \hat{y}_t^k + \frac{1}{k} E_t \sum_{i=0}^{k-1} \left\{ \theta_{t+i}^k - \hat{\theta}_{t+i}^k + \varphi_{t+i} \right\} \]

where

- **Nominal Term Premia:** \( \theta_t^k \equiv E_t \left[ p_{t+1}^{k-1} - p_t^k \right] - y_1 \)
- **Real Term Premia:** \( \hat{\theta}_t^k \equiv E_t \left[ \hat{p}_{t+1}^{k-1} - \hat{p}_t^k \right] - \hat{y}_1 \)
- **Inflation Risk Premia:** \( \varphi_t \equiv y_t^1 - \hat{y}_t^1 - E_t \pi_{t+1,1} \)

**Observations:**

- The absence of arbitrage opportunities only places weak restrictions on \( \theta_t^k, \hat{\theta}_t^k, \) and \( \varphi_t. \)
- Decomposing movements in \( y_t^k \) will depend critically on how \( E_t \pi_{t+k,k}, \hat{y}_t^k \) and the risk premia are identified.
- Previous work on the nominal term structure suggests that \( \theta_t^k \) is time-varying.
Alternative Identification/Estimation Strategies

**Regression Method** (Mishkin 1990): Assume
\[ \theta_{t+i}^{k-i} - \hat{\theta}_{t+i}^{k-i} + \varphi_{t+i} \]
and
\[ \pi_{t+k,k} = E_t \pi_{t+k,k} + \eta_{t+k,k} \]
where \( \eta_{t+k,k} \) is a RE forecast error. Estimates of
\( \hat{y}_t^k \) (and \( E_t \pi_{t+k,k} \)) are obtained from projecting
\( y_t^k - \pi_{t+k,k} \) on variables that span the time \( t \) information set.

**Model-Based 1** (Evans 2003) Identify \( \theta_{t+i}^{k-i} \), \( \hat{\theta}_{t+i}^{k-i} \) and \( \varphi_{t+i} \) via “No Arbitrage” + other assumptions. Use UK data on \( y_t^k \) and \( \hat{y}_t^k \) to estimate
\( E_t \pi_{t+k,k} \).

**Model-Based 2** (A&B): Identify \( \theta_{t+i}^{k-i} \), \( \hat{\theta}_{t+i}^{k-i} \) and \( \varphi_{t+i} \) via “No Arbitrage” + other assumptions. Use US data on \( y_t^k \) and \( \pi_{t+k,k} \) + RE to estimate \( E_t \pi_{t+k,k} \) and \( \hat{y}_t^k \).
A&B’s Model

\[ 1 = E_t [\exp (\hat{m}_{t+1}) \mathcal{R}_{t+1}^i] \]

\[ \hat{m}_{t+1} = -\delta_0 - \delta'_1 X_t - \lambda'_{t+1} \lambda_{t+1} - \lambda'_{t+1} \varepsilon_{t+1} \]

\[ \delta'_1 = [1 \ 1 \ \delta_\pi] \quad \lambda'_{t+1} = \begin{bmatrix} \gamma_1 q_t & \lambda_f (s_{t+1}^f) & 0 \end{bmatrix} \]

\[ X_{t+1} = \mu(s_{t+1}) + \Phi X_t + \Sigma(s_{t+1})^{1/2} \varepsilon_{t+1} \]

\[ X'_{t+1} = [q_{t+1} \ f_{t+1} \ \pi_{t+1}] \quad \mu(s_{t+1})' = [0 \ 0 \ \mu_\pi(s_{t+1}^\pi)] \]

\[ \text{diag} [\Sigma(s_{t+1})]' = \begin{bmatrix} \sigma_q^2 & \sigma_f^2(s_{t+1}^f) & \sigma_\pi^2(s_{t+1}^\pi) \end{bmatrix} \]

\[ s_{t+1}^i = \{1, 2\}, \quad \text{Independent, Markov Chains} \]
Observation 1: The model restricts the inflation risk premium. In particular

\[ Cov_t [\exp (\hat{m}_{t+1}), \exp (\pi_{t+1,1})] = 0, \]

so

\[ \varphi_t = -\ln [E_t \exp (-\pi_{t+1,1})] - E_t \pi_{t+1,1} \leq 0. \]

Observation 2: The real and nominal term premia can vary within a regime. For example, suppose there is no switching. Then.

\[ \hat{\theta}_{2,t} = Cov_t (\hat{m}_{t+1}, r_{t+1}) - \frac{1}{2} Var_t (r_{t+1}), \]

\[ = \lambda_f \sigma_f + \lambda_1 \sigma_q q_t - \frac{1}{2} \delta_1' \Sigma \delta_1. \]

A&B’s estimates \( \Rightarrow q_t \) is very persistent. (NB \( \lambda_{t+1} \) is not the price of risk when switching is present).
**Observation 3:** The model ignores reporting lags in the CPI. There is variable reporting lag in the CPI of approximately 2 weeks, which might be economically significant when inflation is high and variable.

**Observation 4:** Nominal Yields and Inflation display the same persistence across regimes:

\[
y_t^k = -\frac{1}{k} \left( A_k(s_t) + B_kX_t \right) \\
\pi_{t,1} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} X_t \\
X_{t+1} = \mu(s_{t+1}) + \Phi X_t + \Sigma(s_{t+1})^{1/2} \varepsilon_{t+1}
\]

- Evidence on state-dependent mean-reversion in inflation is reported in Evans and Wachtel (1993), and Evans and Lewis (1995).
**Comment 1:** Consider a representative consumer model with CRRA utility. With conditional log normality and no switching

\[
\psi_t = \text{Cov}_t (m_{t+1}, \pi_{t+1,1}) - \frac{1}{2} \text{Var}_t (\pi_{t+1,1}) \\
= -\gamma \text{Cov}_t (\Delta c_{t+1}, \pi_{t+1,1}) - \frac{1}{2} \text{Var}_t (\pi_{t+1,1})
\]

**How big is \( \text{Cov}_t (\Delta c_{t+1}, \pi_{t+1,1}) \)?** We can estimate \( \text{Cov} (\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t) \) where \( \Omega_t \) denotes the information set available to the researcher, but

\[
\text{Cov} (\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t) = E [\text{Cov}_t (\Delta c_{t+1}, \pi_{t+1,1}) | \Omega_t] \\
+ \text{Cov} [E_t \Delta c_{t+1}, E_t \pi_{t+1,1} | \Omega_t]
\]

\( \Rightarrow \text{Cov} (\Delta c_{t+1}, \pi_{t+1,1} | \Omega_t) \) may be biased.
Comment 2: The introduction of switching has more potential than the model allows. For example, we could have

\[ X_{t+1} = \mu(s_t) + \Phi(s_t)X_t + \Sigma(s_t)^{1/2}\varepsilon_{t+1} \]

A&B chose not to go this route because no closed form solution for yields is available with \( \lambda_{t+1} \) a function of \( q_t \). However, if we eliminated \( q_t \) from \( \lambda_{t+1} \) so that

\[ \hat{m}_{t+1} = -\delta_0 - \delta_1X_t - \lambda(s_t)\lambda_t(s_t) - \lambda_t(s_t)\varepsilon_{t+1} \]

we could solve for yields.
Comment 3: Switching makes “choosing the right specification” tricky. A&B favor version IV of their model because the inflation forecast errors are not serially correlated. This is certainly a large sample property of the model. But.....

- Clearly these forecast errors are serially correlated (beyond the forecast horizon).
Are the forecasters irrational, or was there a peso/learning problem?
The answer matters: Consider the alternative estimates of one-year real yields.
Summary

• Making inferences about the source of nominal term structure movements is HARD. The “No Arbitrage” model helps, but does not make up for the lack of data on real yields/inflation expectations/an economic model.

• Introducing regime-switching is a good idea because it has the potential to deliver a good deal of model flexibility (c.f. changing monetary policy as in Cogley and Sargent 2002).

• My preference would be to drop within-regime variation in the risk premia, and allow for state-dependent mean reversion (as in Evans 2003).

• Whatever the approach, relying on just nominal yields and realized inflation is not enough to establish “stylized facts”. We need:
  – to account for the data on inflation expectations, and/or,
  – a GOOD model for the discount factor $\hat{m}_{t+1}$. 