ONLINE APPENDIX: UNCERTAINTY SHOCKS ARE AGGREGATE DEMAND SHOCKS
(NOT FOR PUBLICATION)

SYLVAIN LEDUC AND ZHENG LIU

Abstract. We provide some additional empirical evidence about the macroeconomic effects of uncertainty shocks. We also describe some additional details of the DSGE model.

Date: July 1, 2016.

Key words and phrases. Uncertainty, Demand shocks, Unemployment, Inflation, Monetary policy, Survey data.

JEL classification: E21, E27, E32.

Leduc: Bank of Canada; Email: SLeduc@bank-banque-canada.ca. Liu: Federal Reserve Bank of San Francisco; Email: Zheng.Liu@sf.frb.org. The views expressed herein are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System.
Appendix A. Robustness of VAR evidence

A.1. Measuring uncertainty using the VIX. The aggregate demand effects of uncertainty are not an artifact of our measure of consumer uncertainty. They are also present when we replace the consumer uncertainty measure with the VIX, as shown in Figure A1. In particular, following a shock to uncertainty (measured by the VIX index), unemployment rises significantly for about two years, inflation declines significantly for about 15 months, and monetary policy accommodates by lowering the nominal interest rate.

A.2. Monetary policy and the zero lower bound. Our sample covers the post-2008 periods, during which the U.S. monetary policy has been constrained by the zero lower bound (ZLB) on the short-term nominal interest rates. The inability of monetary policy to accommodate the negative effects of uncertainty when the ZLB is binding implies that our BVAR model could be misspecified. In reality, however, the Fed used unconventional monetary tools after the short-term nominal interest rate reached the zero lower bound. In particular, the Fed has conducted three rounds of large-scale asset purchases and provided systematic forward guidance about the future path of monetary policy.

To examine the importance of this issue for our results, we estimate a BVAR model with an alternative indicator of monetary policy. In particular, we use the two-year Treasury bond yields as an indicator of monetary policy instead of the three-month Treasury bills rate. Swanson and Williams (2014) argue that the Federal Reserve’s forward guidance policy typically attempts to influence the two-year Treasury bond yields. Gertler and Karadi (2015) also argue for the use of a long-term interest rate as an indicator of monetary policy in a VAR. Unlike the three-month Treasury bills rate, the two-year Treasury yields did not reach the zero lower bound. More importantly, the use of this longer-term interest rate helps capture the effects of unconventional monetary policy, which is designed to lower yields on long-term securities. Thus, we examine a BVAR model that is identical to the baseline four-variable BVAR model, except that the short-term nominal interest rate series is replaced by a long-term interest rate series.

Figure A2 displays the impulse responses to an uncertainty shock in the BVAR model with the survey-based consumer uncertainty, the unemployment rate, the CPI inflation rate, and the two-year Treasury yields. As in our baseline model, the shock that raises uncertainty raises the unemployment rate and lowers both the inflation rate and the two-year Treasury yields. These effects are persistent and statistically significant at the 90-percent level.

When we use the VIX instead of the survey-based measure of uncertainty, we obtain similar results. See Figure A3.
A.3. **Uncertain future or bad economic times?** The consumer uncertainty measure rises in recessions. To assess the extent to which consumer uncertainty might reflect households’ perceptions of bad economic times, we follow an approach similar to Baker et al. (2015) and estimate a five-variable BVAR model that includes a consumer sentiment index as an additional variable to control for potential effects from movements in consumer confidence.

Figure A4 displays the impulse responses to an uncertainty shock in the five-variable BVAR model that includes a consumer sentiment index. The impulse responses show that the macroeconomic effects of uncertainty shocks are qualitatively similar to those estimated from the benchmark BVAR. A shock to uncertainty raises the unemployment and lowers consumer sentiment, the inflation rate, and the short-term nominal interest rate. The responses of the macroeconomic variables are statistically significant at the 90 percent level. The inflation response remains negative and significantly different zero between 12 and 24 months following the shock. Thus, the macroeconomic effects of consumer uncertainty shocks do not seem to reflect responses of macroeconomic variables to changes in consumer confidence.

When we use the VIX instead of the survey-based measure of uncertainty, we obtain similar results. See Figure A5.

**Appendix B. Summary of equilibrium conditions in the DSGE model**

A search equilibrium is a system of 16 equations for 16 variables summarized in the vector

\[
[C_t, \Lambda_t, \pi_t, m_t, q_t^u, q_t^v, N_t, u_t, U_t, Y_t, R_t, v_t, q_t, J_t^F, w_t^N, w_t].
\]

We write the equations in the same order as in the dynare code.

1. **Household’s bond Euler equation:**

\[
1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\pi_{t+1}}, \quad (A1)
\]

2. **Marginal utility of consumption**

\[
\Lambda_t = \frac{1}{C_t - hC_{t-1}} - E_t \beta h.
\]

3. **Retail firm’s optimal pricing decision:**

\[
q_t = \eta - 1 \eta + \frac{\Omega_p}{\eta} \left[ \frac{\pi_t}{\pi} \left( \frac{\pi_t}{\pi} - 1 \right) - E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \frac{\pi_{t+1}}{\pi} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \right], \quad (A3)
\]

4. **Matching function**

\[
m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (A4)
\]

5. **Job finding rate**

\[
q_t^u = \frac{m_t}{u_t}, \quad (A5)
\]
(6) Vacancy filling rate

\[ q^v_t = \frac{m_t}{v_t}, \]  

(A6)

(7) Employment dynamics:

\[ N_t = (1 - \rho) N_{t-1} + m_t, \]  

(A7)

(8) Number of searching workers:

\[ u_t = 1 - (1 - \rho) N_{t-1}, \]  

(A8)

(9) Unemployment:

\[ U_t = 1 - N_t, \]  

(A9)

(10) Aggregate production function:

\[ Y_t = Z_t N_t \]  

(A10)

(11) Taylor rule:

\[ R_t = r^\pi (\frac{\pi_t}{\pi^*})^{\phi_\pi} (\frac{Y_t}{Y})^{\phi_y}, \]  

(A11)

(12) Aggregate Resource constraint:

\[ C_t + \kappa v_t + \frac{\Omega_p}{2} (\frac{\pi_t}{\pi} - 1)^2 Y_t = Y_t, \]  

(A12)

(13) Match value

\[ J^F_t = q_t Z_t - w_t + E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1 - \rho) J^F_{t+1}, \]  

(A13)

(14) Vacancy posting

\[ J^F_t = \frac{\kappa}{q^v_t}, \]  

(A14)

(15) Nash bargaining wage:

\[ \frac{b}{1 - b} J^F_t = w^N_t - \phi - \frac{X}{\Lambda_t} + (1 - \rho) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} (1 - q^u_{t+1}) \frac{b}{1 - b} J^F_{t+1}, \]  

(A15)

(16) Actual real wage (with real wage rigidity)

\[ w_t = w^\gamma_{t-1} (w^N_t)^\gamma, \]  

(A16)
Appendix C. Equilibrium system scaled by steady state (used in Dynare)

Denote by $\hat{X}_t \equiv \frac{X_t}{X}$ the scaled value of the variable $X_t$ by its steady-state level. The system of equilibrium conditions can be reduced to the following 16 equations to solve for the 16 endogenous variables summarized in the vector

$$[\hat{C}_t, \hat{\Lambda}_t, \hat{R}_t, \hat{\pi}_t, \hat{q}_t, \hat{m}_t, \hat{u}_t, \hat{q}^u_t, \hat{q}^v_t, \hat{N}_t, \hat{U}_t, \hat{J}_t^F, \hat{w}_t^N, \hat{w}_t,]$$

(1) Household’s bond Euler equation:

$$1 = E_t \hat{\Lambda}_{t+1} \frac{\hat{R}_t}{\hat{\pi}_{t+1}}, \quad (A17)$$

(2) Marginal utility of consumption

$$\frac{1 - \beta h}{1 - h} \hat{\Lambda}_t = \frac{1}{C_t - hC_{t-1}} - E_t \frac{\beta h}{C_{t+1} - hC_t}. \quad (A18)$$

(3) Retail firm’s optimal pricing decision:

$$\hat{q}_t = 1 + \frac{\Omega_p}{\eta - 1} \left[ \hat{\pi}_t (\hat{\pi}_t - 1) - E_t \beta \frac{\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \hat{Y}_{t+1} \hat{\pi}_{t+1} (\hat{\pi}_{t+1} - 1) \right], \quad (A19)$$

(4) Matching function

$$\hat{m}_t = \hat{u}_t^\alpha \hat{u}_t^{1-\alpha}, \quad (A20)$$

(5) Job finding rate

$$\hat{q}^u_t = \frac{\hat{m}_t}{\hat{u}_t}, \quad (A21)$$

(6) Vacancy filling rate

$$\hat{q}^v_t = \frac{\hat{m}_t}{\hat{v}_t}, \quad (A22)$$

(7) Employment dynamics:

$$\hat{N}_t = (1 - \rho) \hat{N}_{t-1} + \rho \hat{m}_t, \quad (A23)$$

(8) Number of searching workers:

$$u\hat{u}_t = 1 - (1 - \rho)N\hat{N}_{t-1}, \quad (A24)$$

(9) Unemployment:

$$U\hat{U}_t = 1 - N\hat{N}_t, \quad (A25)$$

(10) Aggregate production function:

$$\hat{Y}_t = \hat{Z}_t \hat{N}_t \quad (A26)$$

(11) Taylor rule:

$$\hat{R}_t = (\pi^*)^{1-\phi_x} \hat{\pi}_t^{\phi_x} \hat{Y}_t^{\phi_y}, \quad (A27)$$
(12) Aggregate Resource constraint:
\[
\frac{C}{Y} \dot{C}_t + \frac{\kappa v}{Y} \dot{v}_t + \frac{\Omega_p}{2} (\hat{\pi}_t - 1)^2 \ddot{Y}_t = \ddot{Y}_t,
\]  
(A28)

(13) Match value:
\[
\hat{J}_t^F = qZ_t \hat{Z}_t - w_t \hat{w}_t + \beta E_t (1 - \rho) \frac{\hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \hat{J}_{t+1}^F,
\]  
(A29)

(14) Vacancy posting:
\[
\hat{J}_t^F = \frac{1}{\hat{q}_t},
\]  
(A30)

(15) Nash wage (after substitutions):
\[
\frac{b}{1 - b} J^F \dot{J}_t^F = w_t \hat{w}_t^N - \phi - \frac{\chi}{\hat{\Lambda}_t} (1 - q^u) \frac{b}{1 - b} J^F \dot{J}_{t+1}^F,
\]  
(A31)

(16) Actual real wage (with real wage rigidity)
\[
\hat{w}_t = \hat{w}_{t-1}^N (\hat{w}_t^N)^\gamma,
\]  
(A32)

APPENDIX D. IMPULSE RESPONSES OF UNEMPLOYMENT IN ALTERNATIVE DSGE SPECIFICATIONS WITHOUT REAL WAGE RIGIDITIES

In the main text, we show that the macro effects of uncertainty shocks are amplified through the interactions between the option-value channel related to search frictions in the labor market and the demand channel associated with nominal rigidities in the goods market. In the baseline model there, we assume that real wages are rigid. Here we consider the case with flexible real wages. In particular, we set the real wage rigidity parameter to \( \gamma = 0 \) instead of the value of 0.8 assumed in the baseline calibration.

Figure A6 shows the impulse responses of unemployment in alternative DSGE models without real wage rigidities. The patterns are very similar to those in the baseline model shown in Figure 7 of the main text. In particular, both search frictions and nominal rigidities are important for amplifying the effects of uncertainty shocks on unemployment. When search frictions are reduced or when prices are flexible, the responses of unemployment to an uncertainty shock become more muted. In addition, habit formation further amplifies the effects of uncertainty shocks.
Figure A1. The effects of a one-standard deviation shock to the VIX in the BVAR model. The time series data are monthly, ranging from 1986:01 to 2013:10. The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The dashed lines around each solid line represent the 90-percent error bands for the estimated median impulse responses.
Figure A2. The effects of a one-standard deviation shock to perceived uncertainty in the Michigan Survey of Consumers in BVAR model with long-term interest rates. The time series data are monthly, ranging from 1978:01 to 2013:10. The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The dashed lines around each solid line represent the 90-percent confidence bands of the estimated median impulse responses.
Figure A3. The effects of a one-standard deviation shock to VIX in BVAR model with long-term interest rates. The time series data are monthly, ranging from 1986:01 to 2013:10. The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The dashed lines around each solid line represent the 90-percent confidence bands of the estimated median impulse responses.
Figure A4. The effects of a one-standard deviation shock to perceived uncertainty in the Michigan Survey of Consumers in BVAR model augmented with consumer sentiment about current economic conditions. The time series data are monthly, ranging from 1978:01 to 2013:10. The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The dashed lines around each solid line represent the 90-percent confidence bands of the estimated median impulse responses.
Figure A5. The effects of a one-standard deviation shock to VIX in BVAR model augmented with consumer sentiment about current economic conditions. The time series data are monthly, ranging from 1986:01 to 2013:10. The solid lines represent median responses of the variables to a one-standard-deviation increase in the innovations to uncertainty. The dashed lines around each solid line represent the 90-percent confidence bands of the estimated median impulse responses.
Figure A6. Amplification mechanisms for technology uncertainty in the model with no real wage rigidities.
References

