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Implications of Labor Market Frictions for Risk Aversion and Risk Premia

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Abstract

A flexible labor margin allows households to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to absorb shocks along both margins can greatly alter the household's attitudes toward risk, as shown in Swanson (2012). The present paper analyzes how frictional labor markets affect that analysis. Risk aversion is higher: 1) in recessions, 2) in countries with more frictional labor markets, and 3) for households that have more difficulty finding a job. These predictions are consistent with empirical evidence from a variety of sources. Traditional, fixed-labor measures of risk aversion show no stable relationship to the equity premium in a standard real business cycle model with search frictions, while the closed-form expressions derived in the present paper match the equity premium closely.

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1. Introduction

Recent research has made substantial progress bringing macroeconomic models into closer agreement with basic asset pricing facts, such as the equity premium or long-term bond premium.[1] In these studies, as in any consumption-based asset-pricing model, a crucial parameter is risk aversion, the compensation that households require to hold a risky asset. At the same time, a key feature of standard macroeconomic models is that households have some ability to vary their labor supply. A fundamental difficulty with this line of research, then, is that much of what is known about risk aversion has been derived under the assumption that household labor is fixed. For example, Arrow (1964) and Pratt (1965) define absolute and relative risk aversion, $-u''(c)/u'(c)$ and $-c\,u''(c)/u'(c)$, in a static model with a single consumption good. Similarly, Epstein and Zin (1989) and Weil (1989) define risk aversion for generalized recursive preferences in a dynamic model without labor (or, equivalently, in which labor is fixed).

Swanson (2012) considers this problem when households have standard expected utility preferences in a general, but frictionless, dynamic macroeconomic framework. That paper derives closed-form expressions for risk aversion and shows that risk aversion—and risk premia on assets in the model—can vary dramatically depending on how the household's labor margin is specified. Intuitively, a flexible labor margin gives households the ability to absorb shocks to asset values with changes in hours worked as well as changes in consumption. This ability to absorb shocks along either or both margins can greatly alter the household's attitudes toward risk. For example, with period utility $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t$, the quantity $-c\,u_{11}/u_1 = \gamma$ is often referred to as the household's coefficient of relative risk aversion, but in fact the household is *risk neutral* with respect to gambles over asset values or wealth (Swanson, 2012). Intuitively, the household is indifferent at the margin between using labor or consumption to absorb a shock to asset values, and the household in this example is clearly risk neutral with respect to gambles over hours. More generally, when $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l_t^{1+\chi}/(1+\chi)$, risk aversion is given by $(\gamma^{-1} + \chi^{-1})^{-1}$, a combination of the parameters on the household's consumption and labor margins, reflecting that the household absorbs shocks along both margins.

The present paper analyzes how those results are affected when labor markets are frictional, as in Mortensen and Pissarides (1994). In that case, risk aversion lies somewhere between the

[1] See, for example, Boldrin, Christiano, and Fisher (2001), Tallarini (2000), Rudebusch and Swanson (2008, 2012), Uhlig (2007), Van Binsbergen et al. (2012), Backus, Routledge, and Zin (2008), Gourio (2012, 2013), Palomino (2012), Andreasen (2012a,b), Colacito and Croce (2012), Dew-Becker (2012), Kung (2012), and Swanson (2014), which all consider asset pricing in dynamic macroeconomic models with a variable labor margin.

fixed- and flexible-labor cases—between γ and $(\gamma^{-1}+\chi^{-1})^{-1}$ in the example above. The present paper derives the corresponding closed-form expressions for risk aversion with frictional labor markets and shows that those expressions depend on the ratio of labor market flow rates to the household's discount rate. Intuitively, labor market frictions only delay, and do not prevent, the household's labor adjustment; thus, a lower discount rate implies that frictions are less of a concern to the household because this delay is less costly.

The closed-form expressions for risk aversion derived in the present paper have three main implications: First, risk aversion is higher in recessions, when unemployment is higher. Second, risk aversion is greater in more frictional labor markets, such as Continental Europe. And third, risk aversion is higher for households that are less likely to find jobs, such as retirees, the less educated, and households that face labor market discrimination. In all of these cases, it is more difficult for the household to vary its employment in response to shocks, and so more of the burden of asset fluctuations must pass through to consumption.

These predictions of the model are consistent with empirical evidence from a variety of sources. For example, Fama and French (1989) show that risk premia on stocks and bonds are higher in recessions, consistent with the first implication of the model. Campbell and Cochrane (1999) discuss a number of other studies that report similar findings,[2] and Guiso, Sapienza, and Zingales (2013) show that direct measures of household risk aversion from surveys increased during the 2008–09 recession. Consistent with the second implication of the model, Guiso, Haliassos, and Jappelli (2002) and Ynesta (2008) document that the portfolio holdings of European households are substantially more conservative than those of U.S. households. And consistent with the third implication, in all of these countries the portfolios of households near retirement are more conservative than those of younger households (Guiso et al., 2002).

More generally, there is substantial evidence that households vary their labor supply in response to financial shocks—i.e., that the wealth effect on labor supply is negative. Imbens, Rubin, and Sacerdote (2001) find that households who win a prize in the lottery reduce their labor supply significantly; Coile and Levine (2009) document that older workers are less likely to retire after the stock market performs poorly; and Coronado and Perozek (2003) find that households retire earlier when the stock market performs well. Pencavel (1986) and Killingsworth and Heckman (1986) survey estimates of the wealth effect on labor supply and find it to be

[2] See, e.g., Campbell (1999), Lettau and Ludvigson (2010), Piazzesi and Swanson (2008), and Cochrane and Piazzesi (2005).

significantly negative.

The mechanism introduced in the present paper is novel and promising for generating risk aversion and risk premia that vary over the business cycle, across countries, and across households. However, the stylized model of the present paper requires a high discount rate—about 10 to 15 percent per year—for the effects of labor market frictions on risk aversion to be quantitatively different from the frictionless labor market case considered in Swanson (2012). As mentioned above, labor market frictions only delay, rather than prevent, households' labor adjustment, and the cost of this delay is closely related to the discount rate. Although such high discount rates might seem implausible at first glance, they are in fact completely consistent with the behavior of the stock market, which Hall (2014) argues is the right framework for thinking about labor market frictions (because firm investment in a long-term employment relationship is similar to other types of firm investment). Explaining such high discount rates is beyond the scope of the present paper, but may be reasonable if viewed as coming from a household in a risky environment with Epstein-Zin (1989) preferences, a common framework in macroeconomic models of asset prices.[3] Alternatively, other costs of delayed labor adjustment—such as liquidity constraints, borrowing constraints, or skill depreciation—could be incorporated into the model.

There are a few previous studies that extend the Arrow-Pratt definition of risk aversion beyond the one-good, one-period case. Kihlstrom and Mirman (1974) provide an early example of the difficulties involved. In a static, multiple-good setting, Stiglitz (1969) measures risk aversion using the household's indirect utility function rather than utility itself, essentially a special case of Swanson (2012) and Proposition 1 of the present paper. Constantinides (1990) measures risk aversion in a dynamic endowment economy (i.e., with fixed labor) using the household's value function, another special case of Proposition 1. Boldrin, Christiano, and Fisher (1997) apply Constantinides' definition to some very simple endowment economy models for which they can compute closed-form expressions for the value function, and hence risk aversion. The present paper builds on these studies by deriving closed-form solutions for risk aversion in dynamic equilibrium models in general, demonstrating the importance of the labor margin, and showing how labor market frictions affect those results.

The remainder of the paper proceeds as follows. Section 2 defines a general dynamic equilibrium framework with labor market frictions. Section 3 derives closed-form expressions for risk

[3] See, e.g., Barillas, Hansen, and Sargent (2009), Bansal and Yaron (2004), Guvenen (2010), Barro (2006), and Swanson (2014).

aversion in that framework and presents a numerical example showing the importance of taking the labor margin into account. Section 4 derives the implications of labor market frictions for risk aversion described above. The quantitative importance of these results is explored in Section 5. Section 6 concludes. An Appendix provides details of the model, proofs, and numerical solution methods that are outlined in the main text.

2. Dynamic Equilibrium Framework with Labor Market Frictions

2.1 The Household's Optimization Problem and Value Function

Time is discrete and continues forever. At each time t, the household seeks to maximize the expected present discounted value of utility flows,

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right], \tag{1}$$

where E_t denotes the mathematical expectation conditional on the household's information set at time t, $\beta \in (0,1)$ is the discount factor, and c_τ, l_τ, and u_τ denote the household's state-contingent plans for future consumption, labor, and unemployment at time τ. The explicit state-dependence of these plans is suppressed to reduce notation. Let Ω_c denote the domain of c_τ and Ω_{lu} the set of possible values for $l_\tau + u_\tau$.

Assumption 1. *The function $U : \Omega_c \to \mathbb{R}$ is increasing, twice-differentiable, and strictly concave, and $V : \Omega_{lu} \to \mathbb{R}$ is increasing, twice-differentiable, and strictly convex.*

A detailed microfoundation for the household's preferences in (1) is tangential to the present discussion, but is provided in the Appendix. Briefly, the household consists of a unit continuum of individuals who pool their income. At each time t, an individual who is not employed can either search for a job or stay home and produce nonmarket goods and services (including "leisure"). The household's home production function is increasing and concave in the number of individuals staying at home; as a result, V in (1) is increasing and convex in the number of workers not at home, $l_t + u_t$.[4]

[4] The labor market search literature often assumes that household leisure or home production is linear in the number of workers staying at home (e.g., Shimer, 2010). Assumption 1 requires strict convexity of V in order to guarantee the uniqueness of the household's optimal choice of (c_t, u_t) at each time t, discussed below. Intuitively, the case of a linear V can be approximated with a V having infinitesimal convexity.

The labor market is characterized by search and matching. Household labor l_t is a state variable rather than a choice variable, evolving according to

$$l_{t+1} \; = \; (1-s)l_t + f(\Theta_t)u_t, \tag{2}$$

where $s \in [0,1]$ denotes a constant exogenous rate of job destruction, $\Theta_t \in \Omega_\Theta$ is a Markovian state vector that is exogenous to the household and characterizes the state of the aggregate economy at time t, and $f : \Omega_\Theta \to [0,1]$ is a function of the aggregate state that gives the measure of jobs found per unit of unemployed workers searching for a job.

In each period t, the household chooses c_t and u_t (and a state-contingent plan for future c_τ and u_τ) to maximize (1), subject to the labor market friction (2), the flow budget constraint

$$a_{t+1} = (1+r_t)a_t + w_t l_t + d_t - c_t, \tag{3}$$

and the no-Ponzi condition

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1+r_{\tau+1})^{-1} a_{T+1} \geq 0, \tag{4}$$

where a_t denotes the household's beginning-of-period assets and w_t, r_t, and d_t denote the real wage, interest rate, and net transfer payments to the household in each period t, respectively.

The exogenous Markov state vector Θ_t governs the processes for w_t, r_t, and d_t. Before choosing c_t and u_t in each period, the household observes Θ_t and hence w_t, r_t, and d_t. The household's information set and state vector at each date t is thus $(a_t, l_t; \Theta_t)$, where a_t and l_t are endogenous and Θ_t is exogenous to the household. Let X denote the domain of $(a_t, l_t; \Theta_t)$, Ω the domain of (c_t, u_t), and $\Gamma : X \to \Omega$ the set-valued correspondence of feasible choices for (c_t, u_t) for each given $(a_t, l_t; \Theta_t)$.

In addition to Assumption 1, a few more technical conditions are required to ensure the value function for the household's optimization problem exists and satisfies the Bellman equation (see Stokey and Lucas (1990), Alvarez and Stokey (1998), and Rincón-Zapatera and Rodríguez-Palmero (2003) for different sets of such sufficient conditions). The details of these conditions are tangential to the present paper, so I simply assume that:

Assumption 2. *The value function $\mathbb{V} : X \to \mathbb{R}$ for the household's optimization problem exists and satisfies the Bellman equation*

$$\mathbb{V}(a_t, l_t; \Theta_t) \; = \max_{(c_t,u_t)\in\Gamma(a_t,l_t;\Theta_t)} U(c_t) - V(l_t + u_t) \; + \; \beta E_t \mathbb{V}(a_{t+1}, l_{t+1}; \Theta_{t+1}), \tag{5}$$

where l_{t+1} is given by equation (2)*, and a_{t+1} by equation* (3)*.*

Together, Assumptions 1–2 guarantee the existence of a unique optimal choice for (c_t, u_t) at each point in time, given $(a_t, l_t; \Theta_t)$. Let $c_t^* \equiv c^*(a_t, l_t; \Theta_t)$ and $u_t^* \equiv u^*(a_t, l_t; \Theta_t)$ denote the household's optimal choices of c_t and u_t as functions of the state $(a_t, l_t; \Theta_t)$. Then $\mathbb{V}$ can be written as

$$\mathbb{V}(a_t, l_t; \Theta_t) \;=\; U(c_t^*) - V(l_t^* + u_t^*) \;+\; \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}), \tag{6}$$

where $a_{t+1}^* \equiv (1+r_t)a_t + w_t l_t^* + d_t - c_t^*$ and $l_{t+1}^* \equiv (1-s)l_t + f(\Theta_t)u_t^*$. To ensure c_t^* and l_t^* satisfy standard first-order conditions with equality, I assume these optimal choices are interior:

Assumption 3. *For any $(a_t, l_t; \Theta_t) \in X$, the household's optimal choice (c_t^*, u_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \Theta_t)$.*

Intuitively, Assumption 3 requires the partial derivatives of U and V to grow sufficiently large toward the boundary that only interior solutions for c_t^* and u_t^* are optimal for all $(a_t, l_t; \Theta_t) \in X$.

Assumptions 1–3 guarantee that $\mathbb{V}$ is continuously differentiable with respect to a, but in order to define risk aversion below, I require slightly more than this:

Assumption 4. *For any $(a_t, l_t; \Theta_t)$ in the interior of X, the second derivative of $\mathbb{V}$ with respect to its first argument, $\mathbb{V}_{11}(a_t, l_t; \Theta_t)$, exists.*

Santos (1991) provides relatively mild sufficient conditions for this assumption to be satisfied; intuitively, U and V must be strongly concave. Note that Assumption 4 also implies differentiability of the optimal policy functions, c^* and u^*, with respect to a_t.

2.2 Representative Household and Steady State Assumptions

Up to this point, the analysis has focused on a single household in isolation, leaving the other households of the model and the production side of the economy unspecified. Implicitly, the other households and production sector jointly determine the process for Θ_t (and hence w_t, r_t, and d_t), and much of the analysis below does not need to be any more specific about these processes than this. However, to move from general expressions for risk aversion to more concrete, closed-form expressions, I adopt the following three standard assumptions from the macroeconomics literature:[5]

[5] Alternative assumptions about the nature of the other households in the model or the production sector may also allow for closed-form expressions for risk aversion. However, the assumptions used here are standard and thus the most natural to pursue.

Assumption 5. *The household is* infinitesimal.

Assumption 6. *The household is* representative.

Assumption 7. *The model has a* nonstochastic steady state, *at which* $x_t = x_{t+k}$ *for all* $k = 1, 2, \ldots,$ *and* $x \in \{c, u, l, a, w, r, d, \Theta\}$.

Assumption 5 implies that an individual household's choices for c_t and u_t have no effect on the aggregate quantities w_t, r_t, d_t, and Θ_t. Assumption 6 implies that, when the economy is at the nonstochastic steady state, any individual household finds it optimal to choose the steady-state values of c and u given a and Θ. Throughout the text, a variable without a time subscript t denotes its steady-state value.[6]

It is important to note that Assumptions 6–7 do not prohibit offering an individual household a hypothetical gamble of the type described below. The steady state of the model serves only as a reference point around which the *aggregate* variables w, r, d, and Θ and the *other households'* choices of c, u, a and l can be predicted with certainty. This reference point is important because it is there that closed-form expressions for risk aversion can be computed.

Finally, many dynamic models do not have a steady state *per se,* but rather a balanced growth path. The results below carry through essentially unchanged to the case of balanced growth. For ease of exposition, Sections 3–5 restrict attention to the case of a steady state, while the Appendix shows the adjustments required under the more general:

Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

3. Risk Aversion

3.1 The Coefficient of Absolute Risk Aversion

The household's attitudes toward risk at time t generally depend on the household's state vector at time t, $(a_t, l_t; \Theta_t)$. Given this state, the household's aversion to a hypothetical one-shot gamble in period t of the form

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1} \tag{7}$$

[6] Let the exogenous state Θ_t contain the variances of any shocks to the model, so that $(a, l; \Theta)$ denotes the nonstochastic steady state, with the variances of any shocks (other than the hypothetical gamble described in the next section) set equal to zero; $c(a, l; \Theta)$ corresponds to the household's optimal consumption choice at the nonstochastic steady state, etc.

can be considered, where ε_{t+1} is a random variable representing the gamble, with bounded support $[\underline{\varepsilon}, \overline{\varepsilon}]$, mean zero, unit variance, independent of Θ_τ for all times τ, and independent of a_τ, l_τ, c_τ, and u_τ for all $\tau \leq t$. A few words about (7) are in order: First, the gamble is dated $t+1$ to clarify that its outcome is not in the household's information set at time t. Second, c_t cannot be made the subject of the gamble without substantial modifications to the household's optimization problem, because c_t is a choice variable under control of the household at time t. However, (7) is clearly equivalent to a one-shot gamble over net transfers d_t or asset returns r_t, both of which are exogenous to the household. Indeed, thinking of the gamble as being over r_t helps to illuminate the connection between (7) and the price of risky assets, which I will discuss further in Section 3.3, below.

Following Arrow (1964) and Pratt (1965), one can ask what one-time fee μ the household would be willing to pay in period t to avoid the gamble in (7):

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu. \tag{8}$$

The quantity μ that makes the household just indifferent between (7) and (8), for infinitesimal σ and μ, is the household's coefficient of absolute risk aversion.[7] Formally, this corresponds to the following definition:

Definition 1. *Let* $(a_t, l_t; \Theta_t)$ *be an interior point of* X. *Let* $\widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ *denote the value function for the household's optimization problem inclusive of the one-shot gamble* (7)*, and let* $\mu(a_t, l_t; \Theta_t; \sigma)$ *denote the value of* μ *that satisfies* $\mathbb{V}(a_t - \frac{\mu}{1+r_t}; \Theta_t) = \widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$. *The household's* coefficient of absolute risk aversion *at* $(a_t, l_t; \Theta_t)$*, denoted* $R^a(a_t, l_t; \Theta_t)$*, is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} \mu(a_t, l_t; \Theta_t; \sigma)/(\sigma^2/2)$.

In Definition 1, $\mu(a_t, l_t; \Theta_t; \sigma)$ denotes the household's "willingness to pay" to avoid a one-shot gamble of size σ in (7). As in Arrow (1964) and Pratt (1965), R^a denotes the limit of the household's willingness to pay per unit of variance as this variance becomes small. Note that $R^a(a_t, l_t; \Theta_t)$ depends on the economic state because $\mu(a_t, l_t; \Theta_t; \sigma)$ depends on that state. Proposition 1 shows that $\widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$, $\mu(a_t, l_t; \Theta_t; \sigma)$, and $R^a(a_t, l_t; \Theta_t)$ in Definition 1 are well-defined and that $R^a(a_t, l_t; \Theta_t)$ equals the "folk wisdom" value of $-\mathbb{V}_{11}/\mathbb{V}_1$:[8]

[7] Discussion of relative risk aversion is deferred until the next subsection because defining total household wealth is complicated by the presence of human capital—that is, the household's labor income.

[8] See, e.g., Constantinides (1990), Farmer (1990), Campbell and Cochrane (1999), and Flavin and Nakagawa (2008). For the more general case of Epstein-Zin (1990) preferences, equation (9) no longer holds and there is no folk wisdom; see Swanson (2013) for the more general expressions corresponding to that case.

Proposition 1. *Let* $(a_t, l_t; \Theta_t)$ *be an interior point of* X. *Given Assumptions 1–5,* $\widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$, $\mu(a_t, l_t; \Theta_t; \sigma)$, *and* $R^a(a_t, l_t; \Theta_t)$ *exist and*

$$R^a(a_t, l_t; \Theta_t) \;=\; \frac{-\,E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}\,, \tag{9}$$

where $\mathbb{V}_1$ *and* $\mathbb{V}_{11}$ *denote the first and second partial derivatives of* $\mathbb{V}$ *with respect to its first argument. Given Assumptions 6–7,* (9) *can be evaluated at the steady state to yield*

$$R^a(a, l; \Theta) \;=\; \frac{-\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)}\,. \tag{10}$$

Proof: See Appendix.

Equations (9)–(10) are essentially Constantinides' (1990) definition of risk aversion, and have obvious similarities to Arrow (1964) and Pratt (1965). Here, of course, it is the curvature of the value function $\mathbb{V}$ with respect to assets that matters, rather than the curvature of U with respect to consumption.[9]

A practical difficulty with Proposition 1 is that closed-form expressions for the value function $\mathbb{V}$ do not exist in general, even for the simplest dynamic models with labor. One can solve this problem by observing that $\mathbb{V}_1$ and $\mathbb{V}_{11}$ often can be computed even when closed-form solutions for $\mathbb{V}$ cannot be. For example, the Benveniste-Scheinkman equation,

$$\mathbb{V}_1(a_t, l_t; \Theta_t) = (1 + r_t)\, U'(c^*_t), \tag{11}$$

states that the marginal value of a dollar of assets equals the marginal utility of consumption times $1 + r_t$ (the interest rate appears here because beginning-of-period assets in the model generate income in period t). In (11), U' is a known function. Although a closed-form solution for the function c^* is not known in general, the point c^*_t often is known—for example, when it is evaluated at the nonstochastic steady state, c. Thus, one can compute $\mathbb{V}_1$ at the nonstochastic steady state by evaluating the right-hand side of (11) at that point.

The second derivative $\mathbb{V}_{11}$ can be computed by noting that equation (11) holds for general a_t; hence it can be differentiated to yield

$$\mathbb{V}_{11}(a_t, l_t; \Theta_t) \;=\; (1 + r_t)\, U''(c^*_t)\, \frac{\partial c^*_t}{\partial a_t}\,. \tag{12}$$

All that remains is to find the derivative $\partial c^*_t / \partial a_t$.

[9] Arrow (1964) and Pratt (1965) occasionally refer to utility as being defined over "money", so one could argue that they always intended for risk aversion to be measured using indirect utility or the value function.

Intuitively, $\partial c_t^*/\partial a_t$ should not be too difficult to compute: it is just the household's marginal propensity to consume today out of a change in assets, which can be deduced from the household's Euler equation and budget constraint. Differentiating the Euler equation

$$U'(c_t^*) \;=\; \beta E_t(1+r_{t+1})\, U'(c_{t+1}^*) \tag{13}$$

with respect to a_t yields[10]

$$U''(c_t^*)\,\frac{\partial c_t^*}{\partial a_t} \;=\; \beta E_t(1+r_{t+1})\, U''(c_{t+1}^*)\,\frac{\partial c_{t+1}^*}{\partial a_t}\,. \tag{14}$$

Evaluating (14) at steady state, $\beta = (1+r)^{-1}$ and the $U''(c)$ factors cancel, giving

$$\frac{\partial c_t^*}{\partial a_t} \;=\; E_t\frac{\partial c_{t+1}^*}{\partial a_t} \;=\; E_t\frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots \tag{15}$$

In other words, starting from steady state, whatever the change in the household's optimal consumption today, it must be the same as the change in the household's expected optimal consumption tomorrow, and the change in the household's expected optimal consumption at each future date $t+k$.[11]

The household's budget constraint is implied by asset accumulation equation (3) and the no-Ponzi condition (4). Differentiating (3) with respect to a_t, evaluating at steady state, and applying (4) gives

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k}\, E_t\left[\frac{\partial c_{t+k}^*}{\partial a_t} - w\frac{\partial l_{t+k}^*}{\partial a_t}\right] \;=\; 1 + r\,. \tag{16}$$

In other words, the present discounted value of the change in consumption equals the change in assets plus the present discounted value of the change in labor income.

To solve for $\partial c_t/\partial a_t$ using equations (15)–(16), it only remains to solve for $\partial l_{t+k}^*/\partial a_t$. This is done in two steps, using the household's Euler equation for unemployment and the transition equation (2) for labor. The details of this computation are tangential to the main points of this section, so the result is summarized in the following lemma:

Lemma 1. *Given Assumptions 1–7 and either $s < 1$ or $f(\Theta) < 1$, the household's expected marginal propensity to work at each future date $t+k$, $k = 1, 2, \ldots$, with respect to changes in*

[10] The notation $\dfrac{\partial c_{t+1}^*}{\partial a_t}$ is taken to mean $\dfrac{\partial c_{t+1}^*}{\partial a_{t+1}}\dfrac{da_{t+1}^*}{da_t} + \dfrac{\partial c_{t+1}^*}{\partial l_{t+1}}\dfrac{dl_{t+1}^*}{da_t} = \dfrac{\partial c_{t+1}^*}{\partial a_{t+1}}\left[(1+r_t) - \dfrac{\partial c_t^*}{\partial a_t}\right] + \dfrac{\partial c_{t+1}^*}{\partial l_{t+1}} f(\Theta_t)\dfrac{\partial u_t}{\partial a_t}$, and analogously for $\dfrac{\partial c_{t+2}^*}{\partial a_t}$, $\dfrac{\partial c_{t+3}^*}{\partial a_t}$, etc.

[11] Note that this equality does not follow from the steady state assumption. For example, in a model with internal habits, considered in Swanson (2009), the individual household's optimal consumption response to a change in assets increases with time, even starting from steady state.

assets at time t, evaluated at steady state, satisfies

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} \;=\; -\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\Theta)}{s+f(\Theta)}\left[1-\left(1-s-f(\Theta)\right)^k\right]\frac{\partial c^*_t}{\partial a_t}\,, \tag{17}$$

where $\gamma \equiv -cU''(c)/U'(c)$ is the elasticity of U' with respect to c, evaluated at steady state, and $\chi \equiv (l+u)V''(l+u)/V'(l+u)$ the elasticity of V' with respect to $l+u$, evaluated at steady state.

Proof: See Appendix.

Note that, in response to a change in assets, household consumption jumps instantly to a new steady-state level, but l responds only gradually, approaching a new steady-state level asymptotically as $k \to \infty$. The household adjusts along the labor margin by relatively more when χ is low (i.e., the marginal disutility of working is flat), γ is high (the marginal utility of consumption is curved), or the probability of finding a job $f(\Theta)$ is high.

Substituting (17) into the budget constraint (16) and solving for $\partial c^*_t/\partial a_t$ yields

$$\frac{\partial c^*_t}{\partial a_t} \;=\; \frac{r}{1+w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\Theta)}{r+s+f(\Theta)}}\,. \tag{18}$$

In response to a unit increase in assets, the household raises consumption in every period by the extra asset income, r (the "golden rule"), adjusted downward by $1+w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\Theta)}{r+s+f(\Theta)}$, which takes into account the household's decrease in hours worked and labor income. Thus, equation (18) represents a "modified golden rule" that accounts for variation in the household's labor supply. When $f(\Theta)$ is large relative to $r+s$, (18) converges to the modified golden rule derived in Swanson (2012) for a frictionless labor market. Alternatively, when $f(\Theta)=0$, labor is exogenously fixed and (18) equals r, the traditional golden rule.

The household's coefficient of absolute risk aversion can now be written in terms of known quantities. Substituting (11), (12), and (18) into (10) proves the following:

Proposition 2. *Given Assumptions 1–7, the household's coefficient of absolute risk aversion, $R^a(a_t, l_t; \Theta_t)$, evaluated at steady state, satisfies*

$$R^a(a,l;\Theta) \;=\; \frac{-U''(c)}{U'(c)}\;\frac{r}{1+w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\Theta)}{r+s+f(\Theta)}}\,. \tag{19}$$

There are several features of Proposition 2 worth noting. If labor supply is exogenously fixed, corresponding to $s=f(\Theta)=0$, then risk aversion in (19) reduces to $-rU''/U'$, the usual

Arrow-Pratt definition multiplied by a scale factor r, which translates assets into units of current-period consumption.[12] More generally, when $f(\Theta) > 0$, households can partially offset shocks to asset values through changes in hours worked. Note that even though consumption and labor are additively separable in (1), the household's consumption process is still connected to the labor market through the budget constraint. As a result, the household's aversion to a gamble over assets is related to its ability to offset asset fluctuations by varying hours of work.

A flexible labor margin implies that risk aversion is less than in the fixed-labor case:

Corollary 1. *The coefficient of absolute risk aversion,* $R^a(a_t, l_t; \Theta_t)$*, satisfies*

$$R^a(a, l; \Theta) \;\leq\; \frac{-rU''(c)}{U'(c)}. \tag{20}$$

If $r < 1$*, then* (18) *is also less than* $-U''/U'$*.*

Note that, since r is the net interest rate, $r \ll 1$ in typical calibrations.

I discuss the relationship between labor market flexibility, risk aversion, and risk premia below, after first defining relative risk aversion.

3.2 The Coefficient of Relative Risk Aversion

The distinction between absolute and relative risk aversion lies in the size of the hypothetical gamble faced by the household. If the household faces a one-shot gamble of size A_t in period t,

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1}, \tag{21}$$

or the household can pay a one-time fee $A_t\mu$ in period t to avoid this gamble, then it follows from Proposition 1 that $\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2$ for this gamble is given by

$$\frac{-\,A_t E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}\,. \tag{22}$$

The natural definition of A_t, considered by Arrow (1964) and Pratt (1965), is the household's wealth at time t. The gamble in (21) is then over a fraction of the household's wealth and (22) is referred to as the household's coefficient of relative risk aversion.

[12] A gamble over a lump sum of \$$X$ is equivalent here to a gamble over an annuity of \$$X/r$. Thus, even though $\mathbb{V}_{11}/\mathbb{V}_1$ is different from U''/U' by a factor of r, this difference is exactly the same as a change from lump-sum to annuity units. Thus, the difference in scale is essentially one of units. See Swanson (2012).

In models with labor, however, household wealth can be more difficult to define because of the presence of human capital (see Swanson (2012, 2013) for a discussion).[13] These issues are tangential to the present paper, so for simplicity I define human capital here to be the present discounted value of labor earnings, as suggested by the results in Swanson (2013).[14] Equivalently, from the budget constraint (3)–(4), household wealth equals the present discounted value of consumption.

Definition 2. *Let* $(a_t, l_t; \Theta_t)$ *be an interior point of* X. *The household's* coefficient of relative risk aversion, *denoted* $R^c(a_t, l_t; \Theta_t)$, *is given by* (22) *with wealth* $A_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c_\tau^*$, *the present discounted value of household consumption, where* $m_{t,\tau} = \beta U'(c_\tau)/U'(c_t)$ *denotes the household's stochastic discount factor.*

The factor $(1+r_t)^{-1}$ in the definition expresses wealth A_t in beginning- rather than end-of-period-t units, so that in steady state $A = c/r$ and relative risk aversion is given by

$$R^c(a, l; \Theta) \;=\; \frac{-A\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)} \;=\; \frac{\gamma}{1 + w\,\dfrac{\gamma}{\chi}\,\dfrac{l+u}{c}\,\dfrac{f(\Theta)}{r+s+f(\Theta)}}\,, \tag{23}$$

where (23) makes use of the definition $\gamma = -cU''(c)/U'(c)$. Note that if labor is exogenously fixed, so that $s = f(\Theta) = 0$, equation (23) reduces to the usual Arrow-Pratt definition. But as long as the household has some ability to vary its hours of work, risk aversion is reduced by the factor in the denominator of (23).

3.3 Numerical Example

The relationship between the labor margin, risk aversion, and risk premia can be seen in a simple real business cycle model with labor market frictions.[15] Let the economy consist of a unit continuum of representative households, each with optimization problem (1)–(4) and period utility function

$$U(c_t) - V(l_t + u_t) \;\equiv\; \frac{c_t^{1-\gamma}}{1-\gamma} \;-\; \chi_0 \frac{(l_t+u_t)^{1+\chi}}{1+\chi}\,. \tag{24}$$

There is a unit continuum of perfectly competitive firms, each with production function

$$y_t = Z_t k_t^{1-\alpha} l_t^{\alpha}, \tag{25}$$

[13] Note that the household's financial assets a_t are *not* a good measure of wealth A_t, since a_t for an individual household may be zero or negative at some points in time.

[14] Swanson (2013) shows that this measure of wealth and risk aversion seems to be more closely related to risk premia in standard macroeconomic models than if the value of leisure is included in human capital.

[15] Swanson (2012) computes risk aversion and the equity premium in several examples where households can vary their labor supply in a frictionless labor market.

where y_t, l_t, and k_t denote firm output, labor, and beginning-of-period capital, respectively, and Z_t denotes an exogenous aggregate productivity process that follows

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t. \tag{26}$$

The innovations ε_t in (26) are i.i.d. with mean zero and variance σ_ε^2. Firms rent capital from households in a frictionless competitive market at rental rate r_t^k. Households accumulate capital according to

$$k_{t+1} = (1 + r_t)k_t + w_t l_t - c_t, \tag{27}$$

where $r_t = r_t^k - \delta$ and δ denotes the capital depreciation rate.

Firms hire labor by posting vacancies v_t at a cost of κ per vacancy per period. The number of workers employed by each firm evolves according to

$$l_{t+1} = (1-s)l_t + h_t, \tag{28}$$

where l_t is the number of workers employed by the firm and h_t the number of new hires.[16] New hires are determined by the Cobb-Douglas matching function,[17]

$$h_t = \mu \, u_t^{1-\eta} v_t^{\eta}. \tag{29}$$

This implies the job-finding rate for households is

$$f(\Theta_t) \;=\; \frac{h_t}{u_t} \;=\; \mu \left(\frac{v_t}{u_t}\right)^{\eta}. \tag{30}$$

As is typical in these models, the job-finding rate depends only on the vacancy-unemployment ratio, v_t/u_t, which is often denoted by θ_t. In the present paper, the aggregate state vector Θ_t is more general than this, but $f(\Theta_t)$ nevertheless depends only on v_t/u_t in this example.

At the beginning of each period t, workers and firms who were matched in the previous period bargain over the wage w_t. If negotiations break down, the worker and firm each can search for a new match in period t. Let J_t denote the representative firm's surplus from hiring an additional worker in period t:

$$J_t \;=\; \alpha \frac{y_t}{l_t} \;-\; w_t + (1-s) E_t m_{t+1} J_{t+1}. \tag{31}$$

[16] Note that both firms and households are representative and have unit measure, so the number of workers employed by each firm and the number of household members who work is given by l_t.

[17] Some authors interpret the Cobb-Douglas matching function in (29) to be $h_t = \max\{\mu u_t^{1-\eta} v_t^{\eta}, u_t\}$ so that $h_t \leq u_t$. However, $h_t \leq u_t$ holds around the steady state in this example, so including this max operator does not affect numerical solutions local to the model's steady state.

The firm's surplus is the difference between the marginal product of labor and the wage this period, plus the expected discounted value of the firm surplus next period, if the match persists.

Let S_t denote the representative household's marginal surplus from employment,

$$S_t \;=\; w_t \;+\; \big(1-s-f(\Theta_t)\big)E_t m_{t+1} S_{t+1}. \tag{32}$$

The household's surplus, relative to being unemployed, is the wage plus the expected discounted value of the surplus next period, if the match persists. (For simplicity, I assume that there is no compensation for being unemployed; also note that the household incurs marginal disutility $V'(l_t + u_t)$ whether the individual works or is unemployed.)[18]

The wage w_t in each period is set by Nash bargaining, so that

$$(1-\nu)S_t = \nu J_t, \tag{33}$$

where $\nu \in [0,1]$ denotes the household's Nash bargaining weight.

In equilibrium, the marginal cost and marginal benefit to the firm of hiring a worker are equal, so

$$J_t \;=\; \kappa\frac{v_t}{h_t} \;=\; \frac{\kappa}{\mu}\Big(\frac{v_t}{u_t}\Big)^{1-\eta}. \tag{34}$$

Similarly, the marginal cost and benefit to the household of searching for a job are equal, giving the household's unemployment Euler equation (A14).

As discussed in Shimer (2010), models with frictional labor markets are more naturally calibrated to monthly than to quarterly data, because unemployed workers in the U.S. typically find jobs in much less than one quarter. Benchmark values for the model's parameters are reported in Table 1. The household's discount factor β is set to .996, as in Shimer (2010), implying an annual real interest rate of about 5 percent. The utility curvature parameters with respect to consumption and labor, γ and χ, are each set to 200 in order to generate a nontrivial equity premium in the model, but much lower values for these parameters are also considered in Figure 1, below. I set the utility parameter χ_0 governing the disutility of work to achieve a target value of $l + u = 0.3$ in steady state; that is, the houshold is assumed to devote about 30 percent of its

[18] Let $\mathbb{V}_t^E$, $\mathbb{V}_t^U$, and $\mathbb{V}_t^H$ denote the value to the household of an individual being employed, unemployed, and at home, respectively. Then $\mathbb{V}_t^E = w_t - V'(l_t+u_t)/U'(c_t) + (1-s)E_t m_{t+1}\mathbb{V}_{t+1}^E + sE_t m_{t+1}\mathbb{V}_{t+1}^U$. Because there is no compensation for unemployment, $\mathbb{V}_t^U = -V'(l_t+u_t)/U'(c_t) + f(\Theta_t)E_t m_{t+1}\mathbb{V}_{t+1}^E + (1-f(\Theta_t))E_t m_{t+1}\mathbb{V}_{t+1}^U$. Thus $S_t = \mathbb{V}_t^E - \mathbb{V}_t^U$. Note that individuals can move freely between unemployment and home production, so $\mathbb{V}_t^U = \mathbb{V}_t^H$. Thus, $\mathbb{V}_t^H$ does not need to be computed to derive S_t, although $\mathbb{V}_t^H = H'(h_t) + E_t m_{t+1}\mathbb{V}_t^H$, using the notation in the Appendix. It follows that $\mathbb{V}_t^H > 0$, $\mathbb{V}_t^U > 0$, and $\mathbb{V}_t^E > \mathbb{V}_t^U$.

Table 1: Benchmark Parameter Values

β	.996	α	.7	s	.02
γ	200	ρ	.98	η	.5
χ	200	σ_ε	.005	ν	.5
		δ	.0028		

The numerical example is calibrated to monthly data; χ_0, κ, and μ are set to achieve steady-state values of $l + u = 0.3$, $v/u = 0.6$, and $f(\Theta) = 0.28$, respectively. Benchmark values for γ and χ are high in order to achieve a nontrivial equity premium, but a wide range for these parameter values is considered in Figure 1, below. See text for details.

time endowment to market work and labor market search.

I calibrate labor's share of output, α, to 0.7. The exogenous productivity process is assumed to have a monthly persistence $\rho = 0.98$ and a shock standard deviation of $\sigma_\varepsilon = .005$, as in Shimer (2010). I set the capital depreciation rate δ to .0028, also following Shimer (2010), and implying a steady-state capital/annual output ratio of 3.2.

Following Shimer (2010), I set the exogenous job separation rate s to .02, the wage bargaining parameter ν to 0.5, and the matching function elasticity η to 0.5. Firms' cost of posting a vacancy κ is set to achieve a target ratio $v/u = 0.6$ in steady state, consistent with the estimates in den Haan, Ramey, and Watson (2000) and Hall (2005). I set the matching function productivity parameter μ to achieve a target of $f(\Theta) = 0.28$ in steady state, consistent with the estimate in Shimer (2012).

An equity security in the model is defined to be a claim on the aggregate consumption stream, where aggregate consumption $C_t = c_t$ in equilibrium. The ex-dividend price of the equity claim, p_t, satisfies

$$p_t \;=\; E_t m_{t+1}(C_{t+1} + p_{t+1}) \tag{35}$$

in equilibrium, where $m_{t+1} = \beta c_t^\gamma / c_{t+1}^\gamma$ denotes the household's stochastic discount factor. The equity premium, ψ_t, is defined to be the expected excess return

$$\psi_t \;\equiv\; \frac{E_t(C_{t+1} + p_{t+1})}{p_t} \;-\; (1 + r_t^f)\,, \tag{36}$$

where $(1 + r_t^f) \equiv 1/(E_t m_{t+1})$ denotes the one-period gross risk-free interest rate.

For any given set of parameter values, the model is solved numerically using perturbation methods, as in Swanson (2012). This involves computing a nonstochastic steady state for the model and an nth-order Taylor series approximation to the true nonlinear solution for the model's endogenous variables around the steady state. (Results in the figures below are for a fifth-order approximation, $n = 5$.) The numerical algorithm is described in more detail in Swanson (2012) and Swanson, Anderson, and Levin (2006). Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) solve a standard RBC model using a variety of numerical methods and find that the fifth-order perturbation solution is among the most accurate globally as well as being the fastest to compute.

Figure 1 graphs the equity premium and risk aversion as functions of χ, γ, and $f(\Theta)$, holding the values of the other model parameters fixed at their benchmark values in Table 1. In the top panel, χ ranges from 0.5 to 600; in the middle panel, γ varies from 0.5 to 500; and in the bottom panel, $f(\Theta)$ varies from 0.001 to 0.7 (which is achieved by varying the matching function productivity parameter μ). In each panel, the dotted red line graphs the traditional, fixed-labor measure of risk aversion, γ, while the dashed blue line graphs relative risk aversion R^c from equation (23). The solid black line in each panel plots the model-implied average equity premium against the right axis. As is typical in standard real business cycle models (e.g., Rouwenhorst 1995), the equity premium implied by the model is small, less than about 40 basis points per year, even when γ and χ are large.[19]

In Figure 1, the equity premium tracks relative risk aversion R^c closely and is essentially unrelated to the traditional, fixed-labor measure of risk aversion, γ. In the top panel, γ is constant at 200 while relative risk aversion R^c varies widely along with χ, consistent with the wide variation in the equity premium. In the middle panel, the equity premium does not increase linearly along with γ, but rather follows a concave trajectory that matches relative risk aversion R^c closely. In the bottom panel, γ is constant at 200 while R^c and the equity premium increases sharply with labor market rigidity (as the job-finding rate, $f(\Theta)$, approaches zero).[20]

The numerical example in this section thus illustrates three main points. First, the traditional, fixed-labor measure of risk aversion has essentially no relationship to the price of risky assets when households can vary their labor supply. Second, relative risk aversion R^c, defined in the present paper, is much more closely related to the equity premium. (There are good theoretical reasons to expect this to be the case; see Swanson (2013) and the Appendix for a derivation of the relationship between the equity premium and risk aversion in the model.) And third, the

[19] Epstein-Zin (1989) preferences solve this problem by separating risk aversion from the intertemporal elasticity of substitution—see, e.g., Rudebusch and Swanson (2012) and Swanson (2014).

[20] As $f(\Theta) \to 0$, relative risk aversion does not approach the fixed-labor measure γ because the parameters χ_0 and κ are also changing to keep $l + u = 0.3$ and $v/u = 0.6$ in steady state (see Table 1 and its discussion). As a result, l and c in equation (23) both vanish as $f(\Theta) \to 0$, so the limit of equation (23) is not γ.

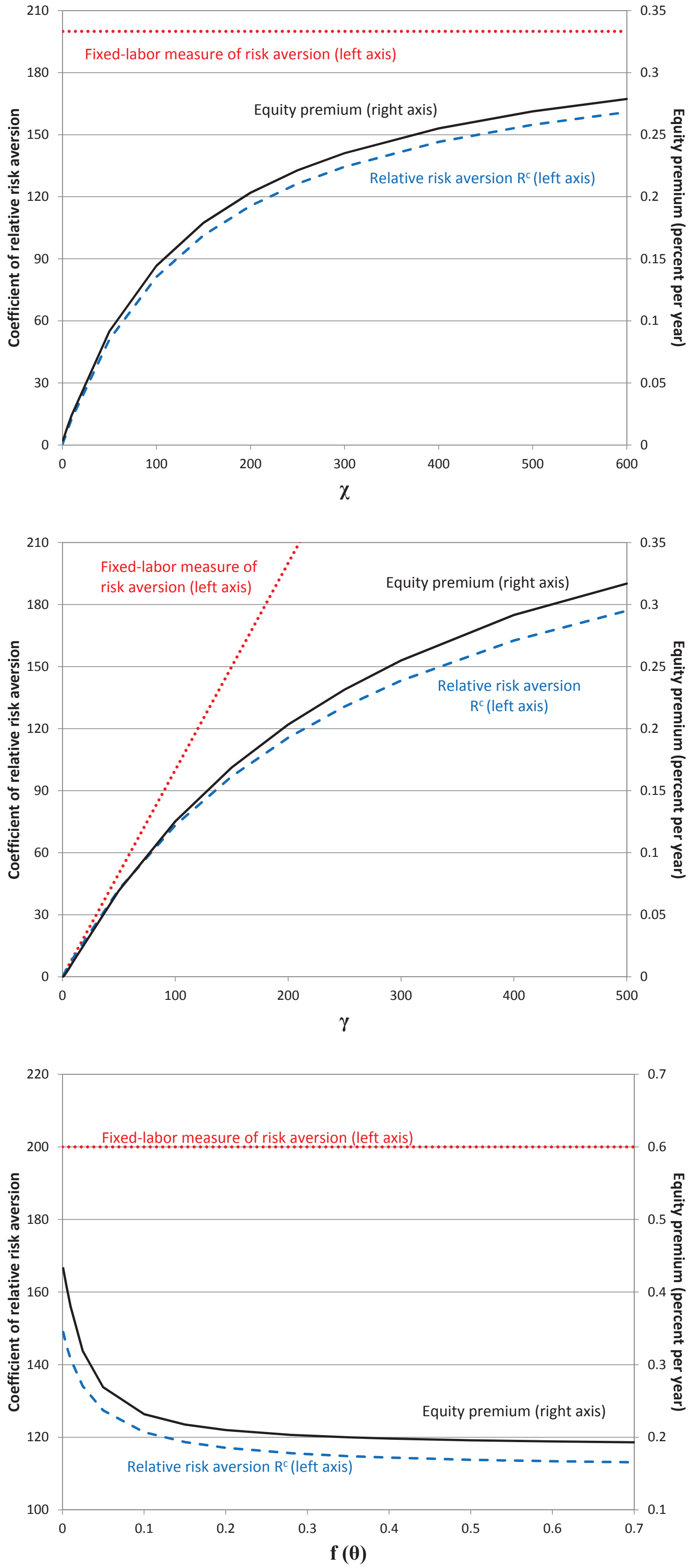

Figure 1. Coefficient of relative risk aversion R^c and the equity premium in a real business cycle model with labor market frictions. In each panel, one parameter is varied while the other model parameters are fixed at their benchmark values; in the top panel, χ is varied from 0.5 to 600; in the middle panel, γ ranges from 0.5 to 500; and in the bottom panel, $f(\Theta)$ ranges from 0.001 to 0.7. In each panel, the equity premium tracks relative risk aversion R^c closely and is generally unrelated to the fixed-labor measure of risk aversion, γ. See text for details.

difference between relative risk aversion R^c and the fixed-labor measure of risk aversion can be very large, as in the top panel of Figure 1, where relative risk aversion R^c can be arbitrarily small as χ becomes small even while γ remains fixed at 200.

4. Implications of Labor Market Frictions for Risk Aversion

Intuitively, labor market frictions make it more difficult for households to insure themselves from asset fluctuations by varying hours of work. Thus, a greater degree of labor market frictions should imply higher risk aversion, all else equal. That effect is evident in the bottom panel of Figure 1; in the present section, the implications of labor market frictions for risk aversion are analyzed more generally.

The transition equation (2) for labor, evaluated at steady state, implies

$$sl = f(\Theta)u. \tag{37}$$

Equation (37) can be used to substitute out $f(\Theta)u/l$ in (23) to obtain

$$R^c(a, l; \Theta) \;=\; \frac{\gamma}{1 + \dfrac{\gamma}{\chi}\,\dfrac{wl}{c}\,\dfrac{s + f(\Theta)}{r + s + f(\Theta)}}\,. \tag{38}$$

If labor is perfectly fixed, corresponding to the case $s = f(\Theta) = 0$, equation (38) reduces to the usual Arrow-Pratt definition, γ. As the ratio $(s + f(\Theta))/\big(r + s + f(\Theta)\big)$ approaches 1, equation (38) converges to the formula for risk aversion for the case where labor is perfectly flexible, reported in Swanson (2012).[21] Thus, $(s + f(\Theta))/\big(r + s + f(\Theta)\big)$ lies between 0 and 1 and can be thought of as an index of labor market flexibility, with 0 corresponding to perfect rigidity and 1 to perfect flexibility. The interest rate r appears in this index because labor frictions only *delay* the household's labor adjustment, rather than preventing it, and r is related to the cost of this delay. Households that are very patient (have a low r) view labor market frictions as less costly, because they can adjust their labor supply as desired given enough time to do so. This labor market flexilibity index features prominently in the quantitative analysis of the next section.

Equation (37) can also be used to substitute out $f(\Theta)$ in (23), giving

$$R^c(a, l; \Theta) \;=\; \frac{\gamma}{1 + \dfrac{\gamma}{\chi}\,\dfrac{wl}{c}\,\dfrac{s(1{+}(l/u))}{r + s(1{+}(l/u))}}\,. \tag{39}$$

[21] Technically, $|1 - s - f(\Theta)| < 1$ was required to solve equation (A17) forward, so $s + f(\Theta) \in (0, 2)$ and the ratio $(s + f(\Theta))/(r + s + f(\Theta))$ has a maximum of $2/(r + 2)$. However, for small enough r this ratio can be arbitrarily close to 1.

$R^c(a, l; \Theta)$ is decreasing in $s\big(1 + (l/u)\big)$, holding fixed the other quantities in equation (39). This suggests that greater labor market frictions (lower s) or a recession (lower l/u) should correspond to higher levels of risk aversion. The remainder of this section makes these two points more rigorously and investigates their quantitative importance.

The following assumption is not strictly necessary, but helps to simplify the discussion and intuition in the analysis below:

Assumption 8. *The elasticity $-c_t U''(c_t)/U'(c_t) = \gamma$ for all $c_t \in \Omega_c$, the elasticity $(l_t + u_t)V''(l_t + u_t)/V'(l_t + u_t) = \chi$ for all $l_t + u_t \in \Omega_{lu}$, and $wl = c$.*

Assumption 8 implies that U and V each have an isoelastic functional form, as in the numerical example above. The assumption $wl = c$ is equivalent to $ra + d = 0$, from the household's flow budget constraint (3); this will be the case, for example, if there are no assets in steady state and no transfers in the model, or alternatively if lump-sum taxes offset the household's asset income.

The crucial feature of Assumption 8 is that γ, χ, and wl/c in (39) can be regarded as stable compared to $s\big(1 + (l/u)\big)$. The intuition and basic results in the analysis below continue to hold if Assumption 8 is satisfied only approximately rather than exactly. However, if any of γ, χ, or wl/c vary substantially more than $s\big(1 + (l/u)\big)$, then the analytical results below may not hold and one would have to resort to numerical solutions of the model to determine the corresponding variation in (39).

4.1 Risk Aversion Is Higher in Recessions

Intuitively, a recession is a period in which employment is low and unemployment is high, or l/u is low. The following proposition characterizes the relationship between l/u and risk aversion:

Proposition 3. *Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, $R^c(a, l; \Theta)$ is decreasing in l/u.*

PROOF: Since $1 + r = 1/\beta$, r is independent of l/u. Assumption 8 then implies $R^c(a, l; \Theta)$ in equation (39) is decreasing in l/u.

Proposition 3 shows that risk aversion is higher in recessions. A lower ratio of employment to unemployment implies that it is harder for unemployed workers to find a job, because $f(\Theta) = sl/u$. As a result, it is more difficult for households to use the labor market to insure themselves from asset fluctuations.

Note that the source of the change in l/u in Proposition 3 is irrelevant. The ratio l/u could be lower because of a decrease in the efficiency of the matching function, a fall in firm productivity or government purchases, or a change in some other element of the economic state Θ. Proposition 3 also holds regardless of how the production side of the economy is specified, so long as Assumptions 1–8 for the household's problem are satisfied. The details of the production function and matching technology will generally affect the stochastic process for Θ_t and the functional form of f, but do not affect the conclusions of the proposition.

Although l/u is the ratio of *steady-state* employment to unemployment, low l/u is the standard way to model a recession in labor search models. Gross flows in and out of employment and unemployment are large in the U.S., so calibrated labor search models imply that employment and unemployment converge very rapidly to their steady states, in a matter of weeks rather than quarters (e.g., Shimer, 2012, Elsby, Hobijn, and Şahin, 2014). Thus, the interpretation of low l/u in Proposition 3 as a recession is typical in the literature.

Finally, the model's prediction that risk aversion is countercyclical is interesting because there is a great deal of empirical evidence that risk premia in financial markets are countercyclical (e.g., Fama and French, 1989, Campbell and Cochrane 1999, Cochrane 1999, Lettau and Ludvigson 2010, Piazzesi and Swanson 2008). Indeed, an important contribution of Campbell and Cochrane (1999) was to generate countercyclical risk aversion in an asset pricing model to better match the observed countercyclicality of risk premia in the data. Proposition 3 of the present paper shows that labor market frictions provide an additional or alternative source of countercyclical risk aversion to consumption habits. In Campbell and Cochrane (1999), risk aversion is high in recessions because consumption is lower than its long-run history; here, risk aversion is higher in recessions because it's harder for households to offset shocks to their portfolios. I investigate the quantitative importance of this effect below.

4.2 Risk Aversion Is Higher in More Frictional Labor Markets

Labor market frictions are greater when $f(\Theta)$ is lower—when it is harder for an unemployed worker to find a job. The following proposition characterizes the relationship between $f(\Theta)$ and risk aversion:

Proposition 4. *Let* $f_1, f_2 : \Omega_\Theta \to [0,1]$*, and let the other parameters of the household's optimization problem be held fixed. Given Assumptions 1–8, let* $(a_1, l_1; \Theta_1)$ *and* $(a_2, l_2; \Theta_2)$ *denote the steady-state values of* $(a_t, l_t; \Theta_t)$ *corresponding to* f_1 *and* f_2*, respectively, and let* $R_1^c(a_1, l_1; \Theta_1)$

and $R_2^c(a_2, l_2; \Theta_2)$ *denote the corresponding values of risk aversion from* (38)*. If* $f_1(\Theta_1) < f_2(\Theta_2)$*, then* $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$*.*

Proof: Since $1 + r = 1/\beta$, r is independent of $f(\Theta)$. Assumption 8 then implies $R^c(a, l; \Theta)$ in equation (38) is decreasing in $f(\Theta)$.

Consistent with the intuition presented earlier, Proposition 4 shows that a more rigid labor market implies greater risk aversion. The case $f(\Theta) = s = 0$ corresponds to complete labor market rigidity and leads to the maximal level of household risk aversion, γ.

Proposition 4 is interesting because it suggests that economies with more frictional labor markets should also have higher risk premia. For example, if labor markets in Europe are characterized by lower job-finding probabilities $f(\Theta)$ than labor markets in the U.S., then Propostion 4 implies risk aversion in those countries should be higher than in the U.S. I will explore the quantitative importance of this effect below.

As with Proposition 3, Proposition 4 holds regardless of the production side of the economy. The details of the production function and matching technology may affect the stochastic process for Θ_t and the functional form of f, but do not affect the conclusions of Proposition 4, so long as its conditions remain satisfied.

Finally, it may be tempting to conclude from (38) that R^c is decreasing in s as well as $f(\Theta)$, but that is not necessarily the case. Changes in s may affect the steady-state level of Θ and thus $f(\Theta)$—for example, a lower value of s tends to increase l/u, which increases $f(\Theta)$ in standard labor market search models. Thus, the effect of s on R^c in (38) is ambiguous. The following corollary correctly characterizes the relationship between s and risk aversion:

Corollary 2. *Let* $s_1, s_2 \in [0, 1]$*,* $f_1, f_2 : \Omega_\Theta \to [0, 1]$*, and let the other parameters of the household's optimization problem be held fixed. Given Assumptions 1–8, let* $(a_1, l_1; \Theta_1)$ *and* $(a_2, l_2; \Theta_2)$ *denote the steady-state values of* $(a_t, l_t; \Theta_t)$ *corresponding to* (s_1, f_1) *and* (s_2, f_2)*, respectively, and* $R_1^c(a_1, l_1; \Theta_1)$ *and* $R_2^c(a_2, l_2; \Theta_2)$ *the corresponding values of risk aversion* (38)*. If* $s_1 < s_2$ *and* $f_1(\Theta_1) \leq f_2(\Theta_2)$*, then* $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$*. More generally, if* $s_1 + f_1(\Theta_1) < s_2 + f_2(\Theta_2)$*, then* $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$*.*

Proof: Since $1 + r = 1/\beta$, r is independent of s and f. Assumption 8 then implies $R^c(a, l; \Theta)$ in equation (38) is increasing in $s + f(\Theta)$.

4.3 Risk Aversion Is Higher for Households that Are Less Employable

Propositions 3–4 and Corollary 2 all assumed a representative household. I now relax that assumption in order to consider the case where the economy is populated by a unit continuum of households divided into two types: a set of measure one of households of type 1, and a set of measure zero of households of type 2. Obviously, the aggregate equilbrium of the economy is determined by the type 1 households, which can be thought of as "representative". Type 2 households are assumed to be the same as those of type 1 except that it is harder for them to find a job—that is, $f_2(\Theta) < f_1(\Theta)$ at the steady-state Θ. (The aggregate state Θ is the same for both household types.)

Given that the aggregate equilibrium is determined by the type 1 households, it is straightforward to check that Propositions 1–4 continue to hold in this economy for both type 1 and type 2 households. In particular, Propositions 1 and 2 give the expressions for risk aversion for both type 1 and type 2 households, and the risk aversion of the two types of households can be compared using Proposition 4. Since $f_2(\Theta) < f_1(\Theta)$, Proposition 4 implies that $R_2^c(a_2, l_2; \Theta) > R_1^c(a_1, l_1; \Theta)$.

In other words, risk aversion is higher for less employable (type 2) households. These households face greater labor market frictions than those of type 1, so it is more difficult for them to insure themselves from asset fluctuations in the labor market, and they are correspondingly more risk averse. I explore the quantitative importance of this effect below.

Some examples of households that might fit the type 2, "less employable" classification are households nearing retirement age, households with less education, and households that suffer from discrimination in the labor market. According to the theory above, these households should be more risk averse than regular, type 1 households, and hold a smaller fraction of their wealth in risky assets such as stocks.

This result provides a formal justification for why households nearing retirement should hold a greater fraction of their wealth in bonds rather than stocks: they are more risk averse.[22] Of course, households near retirement also have a greater fraction of their wealth in financial assets rather than human capital, so retirees have a greater need to diversify their financial holdings in order to diversify their total wealth. Bodie, Merton, and Samuelson (1992) solve a calibrated life-cycle portfolio allocation model numerically and find that labor market flexibility is an important factor in the household's willingness to hold risky assets. For tractability, their analysis considers only the extreme cases of perfectly flexible and perfectly rigid labor markets, essentially the same two cases considered in Swanson (2012). The possibility that older households face greater labor market frictions than younger households is acknowledged by Bodie et al. (1992) as being

[22] Gollier (2002) notes that younger households should have lower absolute risk aversion because they can distribute a given fall in wealth over more periods of consumption. That effect is not present in the infinite-horizon model of the present paper, although it is present in the numerical analysis of Bodie et al. (1992).

potentially important, but is beyond the scope of their model.[23] The present paper shows how their analysis can be extended to the case of frictional labor markets, and provides some immediate insights into the nature and magnitude of these effects.

5. Empirical Evidence and Quantitative Implications

The theoretical predictions of the model above are consistent with a number of empirical observations. First, there is substantial evidence that households vary their labor supply in response to financial shocks. For example, Imbens, Rubin, and Sacerdote (2001) estimate that individuals who win a prize in the Massachusetts state lottery reduce their earnings by about 11 cents for every dollar won, and the effect on household earnings is potentially larger since spousal earnings are not included in their analysis. Coronado and Perozek (2003) find that individuals who held more stocks in the late 1990s retired on average about 7 months earlier than nonstockholders. And Coile and Levine (2009) find that older individuals are about 7 percent less likely to retire in a given year after the stock market falls 30 percent. More generally, the labor supply literature consistently estimates a significant negative effect of unearned income on hours worked (see, e.g., the surveys in Pencavel, 1986, and Killingsworth and Heckman, 1986).

As mentioned previously, there is also substantial evidence that risk premia in financial markets are higher in recessions (Fama and French, 1989, and Campbell and Cochrane, 1999).[24] A natural explanation for this finding is that investors' risk aversion itself is higher in recessions—indeed, one of the contributions of Campbell and Cochrane (1999) was to provide a model that generates countercyclical risk aversion. Guiso, Sapienza, and Zingales (2013) provide direct evidence for countercyclical risk aversion using survey data from a large panel of Italian households before and during the 2008–09 recession. They show that these households' risk aversion is substantially higher during the recession and, importantly, that the increase in risk aversion is independent of the household's portfolio performance during the 2008 financial crisis. In other words, the increase in risk aversion appears to be unrelated to whether the household incurred portfolio

[23] For example, "The ability to vary labor supply ex post induces the individual to assume greater risks in his portfolio ex ante." (Bodie et al. 1992, p. 427); "Obviously, the opportunity to vary continuously one's labor supply without cost is a far cry from the workings of actual labor markets. A more realistic model would allow limited flexibility in varying labor and leisure." (ibid., p. 448); and "It is reasonable to hypothesize that, for most individuals, the degree of labor flexiliby diminishes over the life cycle. For this reason, the effective human capital on which the individual can draw also declines." (ibid., p. 446).

[24] See also Campbell (1999), Lettau and Ludvigson (2010), Piazzesi and Swanson (2008), and Cochrane and Piazzesi (2005).

TABLE 2: INTERNATIONAL COMPARISON OF LABOR FLOWS AND HOUSEHOLD PORTFOLIOS

	s	$f(\Theta)$	percentage of households owning equities	percentage of households owning risky financial assets	share of household portfolios in currency and deposits
United States	.019	.282	48.9	49.2	12.4
United Kingdom	.009	.056	31.5	32.4	26.0
Germany	.006	.035	18.9	25.1	33.9
France	.007	.033	–	–	29.1
Spain	.012	.020	–	–	38.1
Italy	.004	.013	18.9	22.1	27.9

Labor market flows s and $f(\Theta)$ are from Hobijn and Şahin (2007, Table 4) and Shimer (2012); s is sum of monthly flow rates from employment (E) to unemployment (U) and to nonemployment (N); $f(\Theta)$ is monthly flow from U to E. Percentages of households owning equities (directly and indirectly) and risky financial assets are from Guiso et al. (2002, Table I.5) and are upper bounds for the U.K., Germany, and Italy (ibid., p. 7). Share of household portfolios in currency and deposits is from Ynesta (2008, Table 1). See text for details.

losses or whether consumption fell below its previous ("habit") level, presenting a challenge for habit-based models of countercyclical risk aversion.

The simple, stylized model of the present paper is consistent with all of these basic empirical facts. The remainder of the section considers the international evidence and the more detailed quantitative predictions of the model.

5.1 International Comparison of Labor Flows and Household Portfolios

Table 2 reports international evidence on labor market flows and household portfolio allocations across risky and safe assets. Hobijn and Şahin (2007) estimate average values of s and $f(\Theta)$ over time for a variety of OECD countries, and values for the six largest countries in their sample are reported in Table 2.[25] In the present paper, individuals may be out of the labor force, employed, or unemployed, so s in Table 2 is defined to be the sum of the monthly flow rates from employment into unemployment and into nonemployment. The job-finding rate $f(\Theta)$ is the monthly flow rate from unemployment into employment. A striking feature of these estimates is that the job-finding rate in Continental Europe is an order of magnitude lower than in the U.S., ranging from 1.3 to 3.5 percent as opposed to more than 28 percent in the U.S. By contrast, job separation rates are broadly similar across all the countries in the table.

[25] Hobijn and Şahin (2007) estimate a job separation rate for the U.S. of .006. For consistency with Shimer's (2012) results for the U.S. reported later, I use Shimer's estimates for the U.S. in Table 2 as well (for which $s = .019$ and $f(\Theta)$ is the same as in Hobijn and Şahin (2007)).

The last three columns of Table 2 provide some insight into household portfolio allocations between risky and safe assets in each country. Data on the percentage of households owning equities is from Guiso, Haliassos, and Japelli (2002) for the year 1998 and includes indirect holdings through mutual funds and retirement accounts as well as direct equity holdings.[26] Data on the percentage of households holding risky financial assets (including corporate bonds as well as equities) is from the same source. The share of household portfolios in each country held in currency and deposits is from Ynesta (2008) for the year 2006.

The theme in the last three columns of Table 2 is that households in the U.S. are more likely to hold stocks and allocate a greater fraction of their portfolios to riskier assets than households in the U.K. and Continental Europe. In fact, the difference between portfolio allocations in the U.S. and other countries in Table 2 are *understatements* because the U.K., German, and Italian data overestimate the degree of indirect stockholding in those countries, as discussed in the previous footnote. As shown by Guiso et al. (2002), these differences in portfolio holdings are robust to controlling for demographic characteristics such as household wealth or age; that is, U.S. households of a given wealth level and age are more likely to hold equities or risky financial assets than are European households of a similar wealth level and age.

The international evidence in Table 2 is thus consistent with the theoretical points raised in the previous section: Labor markets in Europe are more frictional than those in the U.S., and households in Europe are more risk averse in their portfolio allocations. There are, of course, other reasons why European households might be more reluctant than U.S. households to invest in stocks; for example, stock markets in Europe are generally less liquid and more volatile than in the U.S., intermediation fees for indirect stock holding are typically higher, and the costs of acquiring financial information may be higher in Europe (see, e.g., Guiso and Jappelli, 2002, p. 260). But the lower liquidity and higher intermediation costs in European equity and mutual fund markets are also partly an equilibrium outcome of the lower level of household participation and activity in those markets. Thus, it is not clear that these other factors are fundamental causes of low equity market participation in Europe, as opposed to an endogenous response to European households' apparently higher risk aversion and preference for holding safer assets. The point of the paper here is that this apparently higher level of risk aversion may be due in part to

[26] Data for France and Spain are not available in the survey. Data on indirect equity holdings for the U.K., Germany, and Italy are upper bounds because the portfolio surveys for those countries do not provide information on the type of mutual funds held, so that indirect stockholding in these countries cannot be separated from indirect holdings of other assets (Guiso et al., 2002, p. 7).

the greater labor market rigidity that European households face.

5.2 Quantitative Implications of Labor Market Frictions for Risk Aversion

Labor market frictions can potentially have a large effect on risk aversion. For example, Swanson (2012) considers the two extreme cases of perfect labor market rigidity and perfect flexibility, corresponding in the present paper to $f(\Theta) = s = 0$ and $\big(s + f(\Theta)\big)/\big(r + s + f(\Theta)\big) \to 1$, respectively. In the former case, relative risk aversion $R^c(a, l; \Theta) = -cU''(c)/U'(c) = \gamma$, the traditional fixed-labor Arrow-Pratt measure, while for perfect flexibility, $R^c(a, l; \Theta) = (\gamma^{-1} + \chi^{-1})^{-1}$.[27] As discussed in Swanson (2012), the difference between these two expressions can be very large for reasonable parameterizations; this can also be seen in the first two rows of Table 3, which report values of s, $f(\Theta)$, and risk aversion R^c for these two extreme cases, as benchmarks. The ratio $(s + f(\Theta))/\big(r + s + f(\Theta)\big)$ is also reported, taking the monthly interest rate r to be 0.4 percent, as in Shimer (2010); this ratio lies between 0 and 1 and can be thought of as an index of labor market flexibility, as discussed earlier.[28] The value of relative risk aversion R^c from equation (38) for different values of the utility curvature parameters γ and χ is reported in the last four columns. When labor markets are perfectly rigid, $R^c = \gamma$, while when labor markets are perfectly flexible, R^c is smaller by a factor of between three and ten for the parameterizations considered in the table.

The next panel of Table 3 focus on the case of labor market frictions that are intermediate between these two extremes, investigating how risk aversion varies over a range of empirically plausible values for s and $f(\Theta)$, such as those estimated by Hobijn and Şahin (2007) that were reported in Table 2. For the U.S., the labor market flexibility index $(s + f(\Theta))/\big(r + s + f(\Theta)\big) =$ .987, almost as high as the perfect flexibility benchmark. The values for Europe are lower (taking the monthly interest rate r for each country to be 0.4 percent, the same as for the U.S.), but even Italy's labor market flexibility measures about 81 percent by this metric. According to this index, the labor market in every country in the table is much closer to perfect flexibility than perfect rigidity. Mathematically, the job-finding rate $f(\Theta)$ is much larger than the monthly interest rate r in every country, so the ratio $(s + f(\Theta))/\big(r + s + f(\Theta)\big)$ is close to unity. Intuitively, households

[27] If $wl \neq c$, risk aversion in the flexible-labor case is given by $R^c(a, l; \Theta) = (\gamma^{-1} + (wl/c)\chi^{-1})^{-1}$. To simplify the exposition in this paragraph, the approximation $wl \approx c$ is used.

[28] The benchmark $s = f(\Theta) = 1$ yields a labor market flexibility index of .998, which is slightly less than 1. One cannot generate an index value higher than this without assuming a lower value for r or violating the assumption that $s \in [0, 1]$ and $f(\Theta) \in [0, 1]$.

Table 3: Quantitative Importance of Labor Market Frictions for Risk Aversion

				Relative Risk Aversion R^c			
				$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
Theoretical labor market benchmarks, $r = .004$:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.998	0.86	0.46	2.00	6.68
International comparison, $r = .004$:							
United States	.019	.282	.987	0.86	0.46	2.02	6.73
United Kingdom	.009	.056	.942	0.89	0.48	2.10	6.93
Germany	.006	.035	.911	0.90	0.49	2.15	7.09
France	.007	.033	.909	0.90	0.50	2.16	7.10
Spain	.012	.020	.889	0.92	0.51	2.20	7.20
Italy	.004	.013	.810	0.96	0.55	2.36	7.64
International comparison, $r = .0083$:							
United States	.019	.282	.973	0.87	0.47	2.04	6.79
United Kingdom	.009	.056	.887	0.92	0.51	2.20	7.21
Germany	.006	.035	.832	0.95	0.54	2.31	7.51
France	.007	.033	.828	0.95	0.54	2.32	7.53
Spain	.012	.020	.794	0.97	0.56	2.40	7.73
Italy	.004	.013	.672	1.05	0.65	2.71	8.53
International comparison, $r = .0167$:							
United States	.019	.282	.947	0.88	0.48	2.09	6.91
United Kingdom	.009	.056	.796	0.97	0.56	2.39	7.72
Germany	.006	.035	.711	1.03	0.62	2.60	8.26
France	.007	.033	.705	1.03	0.62	2.62	8.30
Spain	.012	.020	.657	1.07	0.66	2.76	8.64
Italy	.004	.013	.504	1.20	0.83	3.31	9.96

Relative risk aversion R^c from equation (38) for different values of γ, χ, and r, using estimated values of s and $f(\Theta)$ from Hobijn and Şahin (2007) and Shimer (2012), and estimated values of r from Hall (2014). See notes to Table 2 and text for details.

in the model are sufficiently patient that the extra time it takes to vary household employment is not very costly; the household is able to insure itself from asset fluctuations almost as well as if labor markets were perfectly flexible.

This observation is reflected in the last four columns of Table 3, which report the model-implied coefficient of relative risk aversion R^c across countries for different values of γ and χ. Labor market frictions in Italy cause risk aversion to be about 10 to 20 percent higher than for the U.S., a remarkably small difference given the large difference in $f(\Theta)$ across the two countries. Households would either have to be much more impatient, or labor markets much more rigid, for risk aversion to be more substantially affected.

The next two panels of Table 3 investigate the extent to which a higher household discount rate r would improve the model's performance. Intuitively, a higher discount rate makes the delays caused by labor market frictions more costly; in fact, in the extremely stylized model of the present paper, the *only* cost of labor market frictions is this adjustment delay. Thus, when bringing the model to the data, the interest rate r can also be viewed as a proxy for other costs of labor market frictions, such as the depreciation of human capital or stigmatization associated with longer unemployment spells. In this respect, higher calibrated values of r, as a cost parameter, are empirically plausible. Morewover, Hall (2014) emphasizes the similarities between firm investment in "match capital" in the labor market and other types of investment; based on his analysis of the stock market and the labor market, he finds that discount rates of 10 percent per year in expansions and 20 percent in recessions are implied by the data.

The bottom panels of Table 3 thus repeat the computations from the second panel, using monthly discount rates of .0083 percent and .0167 percent, respectively. The higher discount rates greatly improve the model's ability to generate differences in risk aversion across countries. For example, when r is about 10 percent per year, Italy's labor market flexibility index falls to 67 percent and its model-implied risk aversion is about 20 to 30 percent higher than that for the U.S. Differences in risk aversion of this magnitude seem much more consistent with the cross-country differences in portfolio allocation in Table 2. When $r = .0167$, the implied differences in risk aversion are even greater, about 30 to 60 percent.

5.3 Quantitative Implications for Cyclical Variation in Risk Aversion

Table 4 considers the quantitative implications of the model for cyclical variation in risk aversion. The first two rows of Table 4 report values of s and $f(\Theta)$ for the U.S. in expansions and recessions, as estimated by Shimer (2012).[29] In recessions, the job separation rate increases by almost a third, while the job finding rate falls by almost half. Nevertheless, labor markets are so flexible in the U.S. that even the lower value of $s + f(\Theta)$ in recessions has very little effect on the index of labor market flexibility (the third column). The resulting differences in risk aversion R^c between recessions and expansions in the last four columns are thus very small. Although risk aversion is higher in recessions, U.S. labor markets are sufficiently flexible that the extra time it takes to

[29] As for the international comparison in Table 3, the job separation rate s is here taken to be the sum of the monthly flow rate from employment to unemployment and to nonemployment, while the job-finding rate $f(\Theta)$ is the monthly flow rate from unemployment to employment.

Table 4: Quantitative Implications for Cyclical Variation in Risk Aversion

				Relative Risk Aversion R^c			
				$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
$r = .004$:							
United States, expansion	.017	.35	.989	0.86	0.46	2.02	6.71
United States, recession	.022	.20	.982	0.87	0.46	2.03	6.75
$r = .0083$:							
United States, expansion	.017	.35	.978	0.87	0.46	2.04	6.77
United States, recession	.022	.20	.964	0.88	0.47	2.06	6.83
$r = .0167$:							
United States, expansion	.017	.35	.956	0.88	0.47	2.07	6.87
United States, recession	.022	.20	.930	0.89	0.49	2.12	6.99

Relative risk aversion R^c from equation (38) for different values of γ, χ, and r, using estimated values of s and $f(\Theta)$ from Shimer (2012) and estimated values of r from Hall (2014). See notes to Table 3 and text for details.

vary employment is not significantly costly. Labor market flows in the U.S. are simply too large to make delay very costly for households, even in recessions. In contrast to the international comparisons in Table 3, here even the highest discount factor of 20 percent per year results in risk aversion varying by about 1 to 2 percent over the business cycle.

Of course, the importance of labor market frictions for business cycle variation in other countries could be larger; for example, if the job-finding rate in Italy were to fall by half in recessions—say from .013 to .007—the implied increase in risk aversion would be about 10 to 20 percent (not shown), much greater than in the U.S. More data on the cyclicality of job separation and finding rates in other countries would be needed to obtain better estimates of the importance of this effect.[30]

6. Conclusions

Traditional studies of risk aversion, such as Arrow (1964), Pratt (1965), Epstein and Zin (1989), and Weil (1989), assume that household labor supply is fixed. However, this assumption ignores

[30] Concrete evidence on labor market frictions faced by less employable households is less readily available, although a back-of-the-envelope calculation suggests that the effects in the model for these households should also be small. If the steady-state unemployment rate is roughly twice as large for less employable households as for the representative household, and job separation rates are about the same, then the job-finding rate $f(\Theta)$ would be roughly one-half or one-third as large as for the representative household (since l might also be lower). But even a fall in $f(\Theta)$ by a factor of three has relatively little effect on $(s + f(\Theta))/(r + s + f(\Theta))$ or risk aversion R^c.

households' ability to partially offset portfolio shocks by varying hours of work. As a result, fixed-labor measures of risk aversion are not representative of the household's aversion to holding risky assets when labor supply can vary. For reasonable parameterizations, traditional, fixed-labor measures of risk aversion can overstate the household's actual aversion to holding a risky asset by a factor of as much as ten, as in Table 3.

The closed-form expressions for risk aversion derived in the present paper lie between the fixed- and flexible-labor expressions derived in Swanson (2012). Traditional, fixed-labor measures of risk aversion are essentially unrelated to the equity premium in a standard real business cycle model with labor market search, while the expressions in the present paper match the equity premium closely. Thus, measuring risk aversion correctly—taking into account the household's labor margin—is necessary for it to correspond to risk premia in the model.

The formulas for risk aversion derived above imply that risk aversion should be higher: 1) in recessions, 2) in countries with more frictional labor markets, such as Continental Europe, and 3) for households that are less employable, such as retirees, less-educated households, and households facing labor market discrimination. In all of these cases, shocks to the household's financial assets are passed through to consumption to a greater extent.

These predictions are consistent with a wide variety of empirical evidence. In particular, numerous authors find that risk premia are higher in recessions (e.g., Fama and French, 1989), that Continental European households are more conservative in their portfolio allocations (e.g., Guiso et al., 2002), and that older households invest more conservatively (e.g., Guiso et al., 2002). The present paper demonstates the potential connection between these empirical facts and the labor market.

Quantitatively, the stylized model of the present paper requires a high discount rate—about 10 to 15 percent per year—for the effects of labor market frictions on risk aversion to be very different from the frictionless case considered in Swanson (2012). Intuitively, search frictions only delay, and do not prevent, households' labor adjustment, and the cost of this delay is closely related to the discount rate. Although such a high discount rate may seem implausible at first glance, it is standing in for all costs of delayed labor market adjustment in the very simple model of the present paper. Moreover, Hall (2014) estimates that the correct discount rate for labor market search models is 10 or 20 percent per year, based on data from the labor market and stock market. Since many studies find that the high state-contingent discount rates implied by Epstein-Zin (1989) preferences help macroeconomic models explain a variety of asset pricing puzzles (see, e.g., the studies cited in footnote 1), incorporating them into the framework of the present paper would be a promising avenue for future research. Alternatively, other costs of delayed labor adjustment, such as liquidity constraints, borrowing constraints, or skill depreciation, could be incorporated into the model to help it match the substantial variation in risk aversion across countries and over time that is seen in the data.

Appendix: Proofs of Propositions and Technical Details

Microfoundation for Household Preferences

The household consists of a unit continuum of individuals, indexed by $i \in [0,1]$. At each time t, each individual either works, searches for a job, or engages in home production of nonmarket goods and services, including "leisure". Let l_t denote the measure of household members who work in period t, u_t the measure of household members who search, and $h_t = 1 - l_t - u_t$ the measure of household members who engage in home production. The household's output of nonmarket goods and services, c_t^n, is given by

$$c_t^n = H(h_t), \tag{A1}$$

where H is an increasing, concave, twice-differentiable function of the measure of household members engaged in home production.

Each individual household member has a period utility function given by

$$u(c_{it}^m) + v(c_{it}^n), \tag{A2}$$

where c_{it}^m and c_{it}^n denotes individual i's consumption at time t of market and nonmarket goods, respectively, and u and v are increasing, strictly concave, and twice-differentiable functions. For simplicity, I assume that the disutility of working, searching, and home production are equal and enter additively into (A2). Since the disutility of work then equals a constant for each individual in each period t, that constant can be normalized to zero.

Household members pool their income and consumption of home-produced goods, so that each member consumes the same amount of market and nonmarket goods in each period t. Let c_t and c_t^n denote this common level of market and nonmarket good consumption, respectively.

The household's utility flow in each period t is given by the integral of its individual members' utility flows, which equals

$$u(c_t) + v(H(1 - l_t - u_t)) \tag{A3}$$

since each individual's consumption of market and nonmarket goods is the same. Let $U(c_t) \equiv u(c_t)$ and $V(l_t + u_t) \equiv -v(H(1 - l_t - u_t))$. Then the household's period utility function is given by

$$U(c_t) - V(l_t + u_t), \tag{A4}$$

where U is increasing, strictly concave, and twice-differentiable, and V is increasing, strictly convex, and twice-differentiable, as stated in preference specification (1).

Proof of Proposition 1

Since $(a_t, l_t; \Theta_t)$ is an interior point of X, $\mathbb{V}(a_t + \frac{\sigma\underline{\varepsilon}}{1+r_t}, l_t; \Theta_t)$ and $\mathbb{V}(a_t + \frac{\sigma\overline{\varepsilon}}{1+r_t}, l_t; \Theta_t)$ exist for sufficiently small σ, and $\mathbb{V}(a_t + \frac{\sigma\underline{\varepsilon}}{1+r_t}, l_t; \Theta_t) \leq \widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma) \leq \mathbb{V}(a_t + \frac{\sigma\overline{\varepsilon}}{1+r_t}, l_t; \Theta_t)$, hence $\widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$ exists. Moreover, since $\mathbb{V}(\cdot, \cdot; \cdot)$ is continuous and increasing in its first argument, the intermediate value theorem implies there exists a unique $-\mu(a_t, l_t; \Theta_t; \sigma) \in [\sigma\underline{\varepsilon}, \sigma\overline{\varepsilon}]$ satisfying $\mathbb{V}(a_t - \frac{\mu}{1+r_t}, l_t; \Theta_t) = \widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma)$.

For a sufficiently small fee μ in (8), the change in household welfare (6) is given to first order by

$$\frac{-\mathbb{V}_1(a_t, l_t; \Theta_t)}{1 + r_t} \, d\mu \,. \tag{A5}$$

Using the envelope theroem, we can rewrite (A5) as

$$-\beta E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \, d\mu \,, \tag{A6}$$

where $a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*$ and $l_{t+1}^* = (1 - s)l_t + f(\Theta_t)u_t^*$.

Turning now to the gamble in (7), note that the household's optimal choices for consumption and unemployment in period t, c_t^* and u_t^*, will generally depend on the size of the gamble σ—for example, the household may undertake precautionary saving when faced with this gamble. Thus, in this section we write $c_t^* \equiv c^*(a_t, l_t; \Theta_t; \sigma)$ and $u_t^* \equiv u^*(a_t, l_t; \Theta_t; \sigma)$ to emphasize this dependence on σ. The household's value function, inclusive of the one-shot gamble in (7), satisfies

$$\widetilde{\mathbb{V}}(a_t, l_t; \Theta_t; \sigma) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}). \tag{A7}$$

Because (7) describes a one-shot gamble in period t, it affects assets a_{t+1}^* and l_{t+1}^* in period $t+1$ but otherwise does not affect the household's optimization problem from period $t+1$ onward; as a result, the household's value-to-go at time $t+1$ is just $\mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})$, which does not depend on σ except through a_{t+1}^* and l_{t+1}^*.

Differentiating (A7) with respect to σ, the first-order effect of the gamble on household welfare is

$$\left[U' \frac{\partial c_t^*}{\partial \sigma} - V' \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_1 \cdot \big(- \frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \big) + \beta E_t \mathbb{V}_2 f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} \right] d\sigma, \tag{A8}$$

where the arguments of U', V', $\mathbb{V}_1$, and $\mathbb{V}_2$ are suppressed to reduce notation. Optimality of c_t^* and u_t^* implies that the terms involving $\partial c_t^*/\partial \sigma$ and $\partial u_t^*/\partial \sigma$ in (A8) cancel, as in the usual envelope theorem (these derivatives vanish at $\sigma = 0$ anyway, for the reasons discussed below). Moreover, $E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \varepsilon_{t+1} = 0$ because ε_{t+1} is independent of Θ_{t+1}, l_{t+1}^*, and a_{t+1}^*, evaluating the latter two at $\sigma = 0$. Thus, the first-order cost of the gamble is zero, as in Arrow (1964) and Pratt (1965).

To second order, the effect of the gamble on household welfare is

$$\begin{aligned}
\Bigg[& U'' \left(\frac{\partial c_t^*}{\partial \sigma} \right)^2 - V'' \left(\frac{\partial u_t^*}{\partial \sigma} \right)^2 + U' \frac{\partial^2 c_t^*}{\partial \sigma^2} - V' \frac{\partial^2 u_t^*}{\partial \sigma^2} \\
& + \beta E_t \mathbb{V}_{11} \cdot \left(- \frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right)^2 + \beta E_t \mathbb{V}_{12} \cdot \left(- \frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right) f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_1 \cdot \left(- \frac{\partial^2 c_t^*}{\partial \sigma^2} \right) \\
& + \beta E_t \mathbb{V}_{22} f(\Theta_t)^2 \left(\frac{\partial u_t^*}{\partial \sigma} \right)^2 + \beta E_t \mathbb{V}_{12} \cdot \left(- \frac{\partial c_t^*}{\partial \sigma} + \varepsilon_{t+1} \right) f(\Theta_t) \frac{\partial u_t^*}{\partial \sigma} + \beta E_t \mathbb{V}_2 f(\Theta_t) \frac{\partial^2 u_t^*}{\partial \sigma^2} \Bigg] \frac{d\sigma^2}{2} \,.
\end{aligned} \tag{A9}$$

The terms involving $\partial^2 c_t^*/\partial \sigma^2$ and $\partial^2 u_t^*/\partial \sigma^2$ cancel due to the optimality of c_t^* and u_t^*. The derivatives $\partial c_t^*/\partial \sigma$ and $\partial u_t^*/\partial \sigma$ also vanish at $\sigma = 0$ (there are two ways to see this: first, the linearized version of the model is certainty equivalent; alternatively, if ε is symmetric, the gamble in (7) is isomorphic for positive and negative σ, hence c^* and u^* must be symmetric about $\sigma = 0$, implying the derivatives vanish). Thus, for infinitesimal gambles, (A9) simplifies to

$$\beta E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \, \varepsilon_{t+1}^2 \frac{d\sigma^2}{2} \,. \tag{A10}$$

Finally, ε_{t+1} is independent of Θ_{t+1}, l_{t+1}^*, and a_{t+1}^*, evaluating the latter two at $\sigma = 0$. Since ε_{t+1} has unit variance, (A10) reduces to

$$\beta E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}) \frac{d\sigma^2}{2} \,. \tag{A11}$$

Equating (A6) to (A11) allows us to solve for $d\mu$ as a function of $d\sigma^2$. Thus, $\lim_{\sigma \to 0} 2\mu(a_t, l_t; \Theta_t; \sigma)/\sigma^2$ exists and is given by

$$\frac{- E_t \mathbb{V}_{11}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})}{E_t \mathbb{V}_1(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1})} \,. \tag{A12}$$

To evaluate (A12) at the nonstochastic steady state, set $a_{t+1} = a$, $l_{t+1} = l$, and $\Theta_{t+1} = \Theta$ to get

$$\frac{- \mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)} \,. \tag{A13}$$

Proof of Lemma 1

The computation in the lemma is performed in two steps, using the household's Euler equation for unemployment,

$$\frac{V'(l^*_\tau + u^*_\tau)}{f(\Theta_\tau)} \;=\; \beta E_\tau \left[w_{\tau+1} U'(c^*_{\tau+1}) \;-\; V'(l^*_{\tau+1}{+}u^*_{\tau+1}) \;+\; (1-s)\frac{V'(l^*_{\tau+1}{+}u^*_{\tau+1})}{f(\Theta_{\tau+1})} \right], \tag{A14}$$

and the transition equation (2) for labor. Equation (A14) is derived in the same way as the consumption Euler equation and is standard in the labor search literature. The left-hand side of (A14) represents the household's marginal cost of finding a job, while the right-hand side is the discounted marginal benefit of having one more employed worker, given by the wage times the marginal utility of consumption less the marginal disutility of work, plus the job retention rate $(1-s)$ times the marginal benefit of not having to search for a job next period.

Equation (A14) holds at each date $\tau \geq t$ for any initial asset stock a_t. Differentiating (A14) with respect to a_t and taking time-t expectations yields

$$\begin{aligned} E_t \frac{V''(l^*_\tau + u^*_\tau)}{f(\Theta_\tau)} \left(\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} \right) \;=\; \beta E_t \Bigg[& w_{\tau+1} U''(c^*_{\tau+1}) \frac{\partial c^*_{\tau+1}}{\partial a_t} \\ & + V''(l^*_{\tau+1}{+}u^*_{\tau+1}) \, \frac{1-s-f(\Theta_{\tau+1})}{f(\Theta_{\tau+1})} \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right) \Bigg]. \end{aligned} \tag{A15}$$

Evaluating (A15) at steady state gives

$$E_t \left(\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} \right) \;=\; \beta w \, \frac{U''(c) f(\Theta)}{V''(l+u)} \, E_t \frac{\partial c^*_{\tau+1}}{\partial a_t} \;+\; \beta(1-s-f(\Theta)) E_t \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right). \tag{A16}$$

Substituting for $\partial c^*_{\tau+1}/\partial a_t$ from (15), and for w from (A14) evaluated at steady state, this becomes

$$E_t \left(\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} \right) \;=\; -\frac{\gamma}{\chi} \, \frac{l+u}{c} \left[1 - \beta(1-s-f(\Theta)) \right] \frac{\partial c^*_t}{\partial a_t} + \beta(1-s-f(\Theta)) E_t \left(\frac{\partial l^*_{\tau+1}}{\partial a_t} + \frac{\partial u^*_{\tau+1}}{\partial a_t} \right), \tag{A17}$$

where $\gamma \equiv -cU''(c)/U'(c)$ and $\chi \equiv (l+u)V''(l+u)/V'(l+u)$. Since $|1-s-f(\Theta)| < 1$, equation (A17) can be solved forward to yield

$$E_t \left(\frac{\partial l^*_\tau}{\partial a_t} + \frac{\partial u^*_\tau}{\partial a_t} \right) \;=\; -\frac{\gamma}{\chi} \, \frac{l+u}{c} \, \frac{\partial c^*_t}{\partial a_t}. \tag{A18}$$

In response to a change in assets, household leisure $(1 - l_t - u_t)$ and consumption move in the same direction. (Intuitively, it is natural to think of both consumption and leisure as normal goods, so that $\partial c^*_t/\partial a_t > 0$ and $\partial(l^*_t + u^*_t)/\partial a_t < 0$, although this sign restriction is not required below.)

The transition equation (2) for labor implies

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} \;=\; (1-s)\, E_t \frac{\partial l^*_{t+k-1}}{\partial a_t} \;+\; f(\Theta_{t+k-1})\, E_t \frac{\partial u^*_{t+k-1}}{\partial a_t}. \tag{A19}$$

Evaluating (A19) at steady state and applying (A18) gives

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} \;=\; (1-s-f(\Theta))\, E_t \frac{\partial l^*_{t+k-1}}{\partial a_t} \;-\; \frac{\gamma}{\chi} \, \frac{l+u}{c} \, f(\Theta) \, \frac{\partial c^*_t}{\partial a_t}. \tag{A20}$$

Solving (A20) backward to the initial condition $\partial l_t/\partial a_t = 0$ gives

$$E_t \frac{\partial l^*_{t+k}}{\partial a_t} \;=\; -\,\frac{\gamma}{\chi} \, \frac{l+u}{c} \, \frac{f(\Theta)}{s+f(\Theta)} \left[1 - (1-s-f(\Theta))^k \right] \frac{\partial c^*_t}{\partial a_t}. \tag{A21}$$

Relationship between the Stochastic Discount Factor, Risk Premia, and Risk Aversion

The household's aversion to gambles over asset values depends on its ability to offset the outcome of those gambles with changes in hours worked. Here, the analysis is extended to show the relationship between risk aversion and risk premia in the Lucas-Breeden stochastic discounting framework. Risk premia in this framework are closely related to the definition of risk aversion in the present paper, and are generally unrelated to traditional, fixed-labor measures of risk aversion.

Let p_t^i denote the ex-dividend time-t price of an asset i that pays stochastic dividend d_t^i each period. In equilibrium, p_t^i satisfies

$$p_t^i \;=\; E_t m_{t+1}(d_{t+1}^i + p_{t+1}^i), \tag{A22}$$

where $m_{t+1} \equiv \beta U'(c_{t+1})/U'(c_t)$ denotes the household's stochastic discount factor.

Let $1 + r_{t+1}^i$ denote the realized gross return on the asset,

$$1 + r_{t+1}^i \;\equiv\; \frac{d_{t+1}^i + p_{t+1}^i}{p_t^i}\,, \tag{A23}$$

and define the risk premium on the asset, ψ_t^i, to be its expected excess return,

$$\psi_t^i \;\equiv\; E_t r_{t+1}^i - r_{t+1}^f\,, \tag{A24}$$

where $1 + r_{t+1}^f \equiv 1/E_t m_{t+1}$ denotes the risk-free rate. Then

$$\begin{aligned} \psi_t^i \;&=\; \frac{E_t m_{t+1} E_t(d_{t+1}^i + p_{t+1}^i) - E_t m_{t+1}(d_{t+1}^i + p_{t+1}^i)}{p_t^i E_t m_{t+1}} \\ &=\; \frac{-\mathrm{Cov}_t(m_{t+1}, r_{t+1}^i)}{E_t m_{t+1}}\,, \end{aligned} \tag{A25}$$

where Cov_t denotes the covariance conditional on information at time t.

Conditional on information at time t, the household's stochastic discount factor can be written to first order as

$$dm_{t+1} \;=\; -\frac{\beta\gamma}{c}\, dc_{t+1}^*, \tag{A26}$$

where $\gamma \equiv cU''/U'$. The first-order change in consumption, dc_{t+1}^*, in turn can be computed from the household's Euler equation and budget constraint, where it is now recognized that w_t, r_t, d_t, and Θ_t may vary in response to economic shocks. This computation is summarized in the following lemma:

Lemma A1. *Let* $\varphi \equiv \dfrac{s + f(\Theta)}{r + s + f(\Theta)}$, *and let* $\lambda \equiv \dfrac{\gamma}{\chi}\dfrac{l}{c}$, *the long-run sensitivity of labor relative to consumption in response to a change in assets. To first order, evaluated at steady state,*

$$\begin{aligned} dc_{t+1}^* = \frac{r}{1 + w\lambda\varphi} \Bigg[& da_{t+1} + \frac{w}{r{+}s{+}f(\Theta)}\, dl_{t+1} \\ & + E_{t+1} \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \left(l\, dw_{t+k} + dd_{t+k} + a dr_{t+k} + \frac{w}{r{+}s{+}f(\Theta)}\, u df_{t+k} \right) \Bigg] \\ & - \frac{c}{\gamma} E_{t+1} \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \left(\beta dr_{t+k+1} - \frac{rw\lambda\varphi}{1 + w\lambda\varphi}\, d\omega_{t+k} \right), \end{aligned} \tag{A27}$$

where df_{t+k} *denotes the first-order change in the job-finding rate* $f(\Theta_{t+k})$ *and* $d\omega_{t+k}$ *denotes the first-order change in the return to searching for a job at time* $t + k$*, defined in the Appendix.*

Proof: See below.

Note that if w, r, d, and $f(\Theta)$ do not change, as in the Arrow-Pratt gamble for a single household in Section 3, then equation (A27) reduces to (25), where $dl_t = 0$ in that example because labor at time t is given. More generally, (A27) includes the effects of changes in w, r, d, and $f(\Theta)$ on consumption. The term in square brackets in (A27) describes the change in household wealth—including nonfinancial wealth—and thus the first line of (A27) describes the wealth effect on consumption. The last line of (A27) describes the substitution effect: changes in consumption due to changes in current and future wages and interest rates. The ratio $\frac{w}{r+s+f(\Theta)}$ corresponds to the present discounted value of wages from an increase in labor at time t, over the lifetime of the job. The term udf_{t+k}, which represents the change in the number of jobs found each period, is multiplied by the same present value.

Let $d\hat{A}_{t+1}$ denote the quantity inside the square brackets in (A27), representing the change in household wealth, and let $d\Phi_{t+1} \equiv E_{t+1} \sum_{k=1}^{\infty} (1+r)^{-k} \big(\beta dr_{t+k+1} - \frac{r\lambda\varphi}{1+w\lambda\varphi}\, d\omega_{t+k}\big)$, the intertemporal substitution term from (A27). Then the stochastic discount factor and risk premium can be written as follows:

Proposition A1. *To first order, evaluated at steady state,*

$$dm_{t+1} \;=\; -R^a(a,l;\Theta)\,\beta\, d\hat{A}_{t+1} + \beta\, d\Phi_{t+1}\,. \tag{A28}$$

To second order, evaluated at steady state,

$$\psi_t^i \;=\; R^a(a,l;\Theta)\,\mathrm{Cov}_t(dr_{t+1}^i, d\hat{A}_{t+1}) \;-\; \mathrm{Cov}_t(dr_{t+1}^i, d\Phi_{t+1})\,. \tag{A29}$$

Proof: Note first that $R^a(a,l;\Theta) = \dfrac{\gamma}{c}\,\dfrac{r}{1+w\lambda\varphi}$. Substituting (A19) into (A18) yields (A28). Substituting (A28) into (A17) yields (A29). Finally, $\mathrm{Cov}(dx, dy)$ is accurate to second order when dx and dy are accurate to first order.

Equation (A29) characterizes the relationship between risk aversion and the risk premium on the asset. The first term in (A29) shows that ψ_t^i increases locally linearly with R^a, by an amount that depends on the covariance between the asset return and the household's wealth, including nonfinancial wealth. This link between risk premia and risk aversion should not be too surprising: Propositions 1–2 described the risk premium for extremely simple, idiosyncratic gambles over household wealth, while Proposition A1 shows that the same coefficient also appears in the household's aversion to more general financial market gambles that may be correlated with aggregate variables such as interest rates, wages, and transfers.

The second term in (A29) corresponds to Merton's (1973) "changes in investment opportunities" in the ICAPM framework. Even if $R^a = 0$—that is, even if households are risk-neutral in a cross-sectional or CAPM sense—ψ_t^i can be nonzero. This is because even a risk-neutral household can benefit from an asset that pays off well when the price of the household's total consumption bundle is low. An asset that pays off well when current and future wages are low (and hence leisure is cheap) or current and future interest rates are high (and hence future consumption is cheap) is preferable to an asset that pays off poorly in those situations. Even a risk-neutral household would be willing to pay a premium for such an asset—implying a lower ψ_t^i—and this effect is captured by the second term in (A29).

The fact that households in the present paper face a consumption-leisure tradeoff as well as a current-vs.-future consumption tradeoff implies that the second term in (A29) is more general than just changes in the household's investment opportunities. Indeed, the second term in (A29) is better described as being due to "changes in purchasing opportunities." The decomposition in (A29) also suggests that ψ_t^i is more accurately described as an "expected excess return" rather than a "risk premium" because only the first term in (A29) represents compensation to the household for bearing risk; the second term is not compensation for risk but rather reflects changes in the household's purchasing opportunities over time.

Finally, the decomposition (A29) can be written in terms of relative rather than absolute risk aversion using Definition 2:

Corollary A1. *In terms of relative risk aversion, the risk premium in* (A29) *can be written as:*

$$\psi_t^i \;=\; R^c(a,l;\Theta)\,\mathrm{Cov}_t\!\left(dr_{t+1}^i,\, \frac{d\hat{A}_{t+1}}{A}\right) \;-\; \mathrm{Cov}_t(dr_{t+1}^i, d\Phi_{t+1}) \tag{A30}$$

where A is as in Definition 2.

Proof of Lemma A1

Differentiating the household's Euler equation (13) and evaluating at steady state yields

$$U''(c)(dc_t^* - E_t dc_{t+1}^*) = \beta U'(c) E_t dr_{t+1}. \tag{A31}$$

It follows that, for each $k = 1, 2, \ldots$,

$$E_t dc_{t+k}^* \;=\; dc_t^* + \frac{\beta c}{\gamma}\, E_t \sum_{i=1}^{k} dr_{t+i}. \tag{A32}$$

Combining (3)–(4), differentiating, and evaluating at steady state yields

$$E_t \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} (dc_{t+k}^* - w dl_{t+k}^* - l dw_{t+k} - dd_{t+k} - a dr_{t+k}) \;=\; (1+r)\, da_t. \tag{A33}$$

Differentiating the household's unemployment Euler equation (17) and evaluating at steady state yields

$$\begin{aligned} \frac{V''(l{+}u)}{f(\Theta)}(dl_\tau^* + du_\tau^*) \;-\; \frac{V'(l{+}u)}{f(\Theta)^2}\, df_\tau \;=\; \beta E_\tau \Bigg[& wU''(c) dc_{\tau+1}^* \;+\; U'(c) dw_{\tau+1} \\ & + \; V''(l{+}u)\, \frac{1-s-f(\Theta)}{f(\Theta)}\, (dl_{\tau+1}^* + du_{\tau+1}^*) \;-\; V'(l+u) \frac{1-s}{f(\Theta)^2}\, df_{\tau+1} \Bigg]. \end{aligned} \tag{A34}$$

Rearranging (A34) gives

$$\begin{aligned} dl_\tau^* + du_\tau^* \;=\; & \beta(1{-}s{-}f(\Theta))\, E_\tau\big(dl_{\tau+1}^* + du_{\tau+1}^*\big) \;-\; \frac{\gamma}{\chi}\frac{l+u}{c}\, (1 - \beta(1{-}s{-}f(\Theta)))\, E_\tau dc_{\tau+1}^* \\ & + \; \frac{l+u}{\chi w}\, (1 - \beta(1{-}s{-}f(\Theta)))\, E_\tau dw_{\tau+1}^* \;+\; \frac{l+u}{\chi f(\Theta)}\, df_\tau \;-\; \beta \frac{l+u}{\chi} \frac{1-s}{f(\Theta)}\, E_\tau df_{\tau+1}, \end{aligned} \tag{A35}$$

which, assuming $|s + f(\Theta)| < 1$, can be solved forward to yield

$$\begin{aligned} dl_\tau^* + du_\tau^* \;=\; & -\frac{\gamma}{\chi}\frac{l+u}{c}\, dc_\tau^* \;-\; \beta \frac{l+u}{\chi}\, E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} dr_{\tau+k} \\ & + \; \frac{l+u}{\chi}\, (1 - \beta(1{-}s{-}f(\Theta)))\, E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} dw_{\tau+k} \\ & + \; \frac{l+u}{\chi f(\Theta)}\, df_\tau \;-\; \beta \frac{l+u}{\chi}\, E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} df_{\tau+k}. \end{aligned} \tag{A36}$$

Note that, when current and future r, w, and $f(\Theta)$ do not change, (A36) reduces to (21). Define

$$\begin{aligned} d\omega_\tau \;\equiv\; & -\beta E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} dr_{\tau+k} \;+\; \big(1 - \beta(1{-}s{-}f(\Theta))\big)\, E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} dw_{\tau+k} \\ & + \; \frac{1}{f(\Theta)}\, df_\tau \;-\; \beta E_\tau \sum_{k=1}^{\infty} \beta^{k-1} (1{-}s{-}f(\Theta))^{k-1} df_{\tau+k}. \end{aligned} \tag{A37}$$

Then

$$dl^*_\tau + du^*_\tau \;=\; -\frac{\gamma}{\chi}\frac{l+u}{c}\,dc^*_\tau \;-\; \frac{l+u}{\chi}\,d\omega_\tau, \tag{A38}$$

so that (A38) corresponds to a standard intratemporal optimality condition between $l+u$ and c, and $d\omega_\tau$ represents the change in the present discounted value of the gains from searching for a job, analogous to dw_τ in a frictionless model.

Differentiating the transition equation (2) for labor and evaluating at steady state yields

$$dl^*_{t+k} \;=\; (1-s-f(\Theta))\,dl^*_{t+k-1} \;+\; f(\Theta_{t+k-1})\left(dl^*_{t+k-1} + du^*_{t+k-1}\right) \;+\; udf_{t+k-1}. \tag{A39}$$

Solving backward to time t gives

$$dl^*_{t+k} \;=\; (1-s-f(\Theta))^k dl_t \;+\; \sum_{i=1}^{\infty}(1-s-f(\Theta))^{k-1-i}\left[f(\Theta)\left(dl^*_{t+i} + du^*_{t+i}\right) \;+\; udf_{t+i}\right]. \tag{A40}$$

Substituting (A38) into (A40) and taking expectations gives

$$\begin{aligned}
E_t dl^*_{t+k} \;=\; & (1-s-f(\Theta))^k dl_t \;-\; \frac{\gamma}{\chi}\frac{l+u}{c}f(\Theta)\frac{1-(1-s-f(\Theta))^k}{s+f(\Theta)}\,dc^*_t \\
& + \frac{l+u}{\chi}f(\Theta)E_t\sum_{i=0}^{k-1}(1-s-f(\Theta))^{k-1-i}d\omega_{t+i} \;+\; uE_t\sum_{i=0}^{k-1}(1-s-f(\Theta))^{k-1-i}df_{t+i} \\
& - \beta\,\frac{l+u}{\chi}\,\frac{f(\Theta)}{s+f(\Theta)}\,E_t\sum_{i=1}^{k-1}\left[1-(1-s-f(\Theta))^{k-i}\right]dr_{t+i}.
\end{aligned} \tag{A41}$$

Inserting (A41) into the budget constraint (A16) gives

$$\begin{aligned}
& \frac{1+r}{r}\left[1+\frac{\gamma}{\chi}\frac{wl}{c}\frac{s+f(\Theta)}{r+s+f(\Theta)}\right]dc^*_t \;-\; \frac{1+r}{r+s+f(\Theta)}\,wdl_t \;-\; \frac{1}{r+s+f(\Theta)}\,E_t\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\,wudf_{t+k} \\
& \quad - \frac{1}{\chi}\frac{s+f(\Theta)}{r+s+f(\Theta)}\,E_t\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\,ld\omega_{t+k} \;+\; \frac{1+r}{r}\frac{\beta c}{\gamma}\left[1+\frac{\gamma}{\chi}\frac{wl}{c}\frac{s+f(\Theta)}{r+s+f(\Theta)}\right]E_t\sum_{k=1}^{\infty}\frac{1}{(1+r)^k}\,dr_{t+k} \\
& \quad - E_t\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\left[ldw_{t+k} + dd_{t+k} + adr_{t+k}\right] \;=\; (1+r)da_t.
\end{aligned} \tag{A42}$$

Solving for dc^*_t, using the definitions of λ and φ, yields

$$\begin{aligned}
dc^*_t \;=\; & \frac{r}{1+r}\,\frac{1}{1+w\lambda\varphi}\Bigg[(1+r)da_t + (1+r)\frac{w}{r+s+f(\Theta)}\,dl_t \\
& \qquad + E_t\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\left(l\,dw_{t+k} + dd_{t+k} + adr_{t+k} + \frac{wu}{r+s+f(\Theta)}\,df_{t+k}\right)\Bigg] \\
& - \frac{\beta c}{\gamma}\,E_t\sum_{k=1}^{\infty}\frac{1}{(1+r)^k}\,dr_{t+k} \;+\; \frac{r}{1+r}\frac{c}{\gamma}\frac{w\lambda\varphi}{1+w\lambda\varphi}\,E_t\sum_{k=0}^{\infty}\frac{1}{(1+r)^k}\,d\omega_{t+k}.
\end{aligned} \tag{A43}$$

Expressions for Risk Aversion with Balanced Growth

The results in the main text carry through essentially unchanged to the case of balanced growth. The corresponding expressions are collected and proved here in Lemma A2, Proposition A2, and Corollary A2.

Along a balanced growth path, $x \in \{l, r, u\}$ satisfies $x_{t+k} = x_t$ for $k = 1, 2, \ldots$, and we drop the time subscript to denote the constant value. For $x \in \{a, c, w, d\}$, we have $x_{t+k} = G^k x_t$ for $k = 1, 2, \ldots$, for some $G \in (0, 1+r)$, and we let x_t^{bg} denote the balanced growth path value. We denote the balanced growth path value of Θ_t by Θ_t^{bg}, although the elements of Θ may grow at different constant rates over time (or remain constant). The job-finding rate $f(\Theta_t^{bg})$ must be constant along the balanced growth path because $f(\Theta_t^{bg}) = sl/u$, and we write this value as $f(\Theta)$. Additional details regarding balanced growth are provided in King, Plosser, and Rebelo (1988, 2002).

Lemma A2. *Given Assumptions 1–6 and 7′, along the balanced growth path, $\partial c_{t+k}^*/\partial a_t = G^k \partial c_t^*/\partial a_t$, $k = 1, 2, \ldots$, and*

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{1 + r - G}{1 + \dfrac{\gamma}{\chi}\, \dfrac{w_t^{bg} l}{c_t^{bg}}\, \dfrac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}}\,. \tag{A44}$$

Proof: The household's Euler equation (13), evaluated along the (nonstochastic) balanced growth path, implies

$$U''(c_t^{bg}) \;=\; \beta(1+r)U''(c_{t+1}^{bg}) \;=\; \beta(1+r)U''(Gc_t^{bg}). \tag{A45}$$

As in King, Plosser, and Rebelo (2002), assume that preferences U are consistent with balanced growth for all initial asset stocks and wages in a neighborhood of a_t^{bg} and w_t^{bg}, and hence for all initial values of (c_t, l_t) in a neighborhood of (c_t^{bg}, l). Thus, (A45) can be differentiated to yield

$$U''(c_t^{bg}) \;=\; \beta(1+r)G\, U''(Gc_t^{bg}). \tag{A46}$$

Differentiating (A45) with respect to a_t yields

$$U''(c_t^{bg})\frac{\partial c_t^*}{\partial a_t} \;=\; \beta(1+r)\, U''(c_{t+1}^{bg})\frac{\partial c_{t+1}^*}{\partial a_t}\,. \tag{A47}$$

Solving for $\partial c_{t+1}^*/\partial a_t$ and using (A46) yields $\partial c_{t+1}^*/\partial a_t = G\, \partial c_t^*/\partial a_t$.

Next, use the household's budget constraint (1)–(2) and the above to solve for $\partial c_t^*/\partial a_t$, following along the lines of (16)–(25).

The larger is G, the smaller is $\partial c_t^*/\partial a_t$, since the household chooses to absorb a greater fraction of asset shocks in future periods.

Proposition A2. *Given Assumptions 1–6 and 7′, absolute risk aversion satisfies*

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) \;=\; \frac{-\mathbb{V}_{11}(a_{t+1}^{bg}, l_{t+1}^{bg}; \Theta_{t+1}^{bg})}{\mathbb{V}_1(a_{t+1}^{bg}, l_{t+1}^{bg}; \Theta_{t+1}^{bg})} \tag{A48}$$

and

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) \;=\; \frac{-U''(c_t^{bg})}{U'(c_t^{bg})}\, \frac{\frac{1+r}{G} - 1}{1 + \dfrac{\gamma}{\chi}\, \dfrac{w_t^{bg} l}{c_t^{bg}}\, \dfrac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}}. \tag{A49}$$

Proof: Proposition 1 implies (A48). Assumptions 1–6 imply (11)–(12). Substituting (11)–(12) and Lemma A2 into (A49) gives

$$R^a(a_t^{bg}, l; \Theta_t^{bg}) \;=\; \frac{-U''(c_{t+1}^{bg})}{U'(c_{t+1}^{bg})}\, \frac{1 + r - G}{1 + \dfrac{\gamma}{\chi}\, \dfrac{w_t^{bg} l}{c_t^{bg}}\, \dfrac{G\big(s + f(\Theta)\big)}{(1 + r - G) + G\big(s + f(\Theta)\big)}}. \tag{A50}$$

Using (A34)–(A39) completes the proof.

Note that (A50) agrees with Proposition 2 when $G = 1$. The larger is G, the smaller is R^a, since larger G implies greater household wealth and ability to absorb asset shocks.

Corollary A2. *Given Assumptions 1–6 and 7′, relative risk aversion satisfies*

$$R^c(a_t^{bg}, l; \Theta_t^{bg}) \;=\; \frac{\gamma}{1 + \dfrac{\gamma}{\chi}\,\dfrac{w_t^{bg} l}{c_t^{bg}}\,\dfrac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}}. \tag{A51}$$

Proof: As in Definition 2, define wealth A_t^{bg} in beginning- rather than end-of-period-t units; this requires multiplying by $((1+r)/G)^{-1}$ rather than just $(1+r)^{-1}$. Then the present discounted value of consumption along the balanced growth path is given by $A_t^{bg} = c_t^{bg}/(\frac{1+r}{G} - 1)$. Substituting these into Proposition A2 completes the proof.

Thus, the expressions for relative risk aversion are largely unchanged by balanced growth. The labor market flexibility index takes the form $\dfrac{s + f(\Theta)}{(\frac{1+r}{G} - 1) + s + f(\Theta)}$ rather than $\dfrac{s + f(\Theta)}{r + s + f(\Theta)}$, which has the effect of decreasing the importance of r and amplifying the importance of $s + f(\Theta)$ in the index, but other than that the expression is unchanged.

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