

# Online Appendix to Factor Specificity and Real Rigidities\*

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## Abstract

We develop a multisector model in which capital and labor are free to move across firms within each sector, but cannot move across sectors. To isolate the role of sectoral specificity, we compare our model with otherwise identical multisector economies with either economy-wide or firm-specific factor markets. Sectoral factor specificity generates within-sector strategic substitutability and tends to induce across-sector strategic complementarity in price setting. Our model can produce either more or less monetary non-neutrality than those other two models, depending on parameterization and the distribution of price rigidity across sectors. Under the empirical distribution for the U.S., our model behaves similarly to an economy with firm-specific factors in the short-run, and later on approaches the dynamics of the model with economy-wide factor markets. This is consistent with the idea that factor price equalization might take place gradually over time, so that firm-specificity may serve as a reasonable short-run approximation, whereas economy-wide markets are likely a better description of how factors of production are allocated in the longer run.

*JEL classification codes:* E22, J6, E12

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# A Online Appendix

## A.1 First-order conditions for model with sectoral factor markets

The first-order conditions for consumption and labor are:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l P_t}{\Theta_{t,l} P_{t+l}},$$

$$\frac{W_{s,t}}{P_t} = \omega_s N_{s,t}^\gamma C_t^\sigma, \quad \forall s.$$

Consumers' allocation of sectoral investment  $I_{s,t}$  and capital  $K_{s,t+1}$  yields:

$$Q_{s,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left( \frac{Z_{s,t+1}}{P_{t+1}} + Q_{s,t+1} \left[ (1 - \delta) - \Phi' \left( \frac{I_{s,t+1}}{K_{s,t+1}} \right) \left( \frac{I_{s,t+1}}{K_{s,t+1}} \right)^2 \right] \right) \right\},$$

$$Q_{s,t} \left( \Phi' \left( \frac{I_{s,t}}{K_{s,t}} \right) \frac{I_{s,t}}{K_{s,t}} + \Phi \left( \frac{I_{s,t}}{K_{s,t}} \right) \right) = 1,$$

where  $Q_{s,t}$  denotes Tobin's  $q$  for sector  $s$ .

In all models, the solution must also satisfy a transversality condition:

$$\lim_{l \rightarrow \infty} E_t [\Theta_{t,l} B_l] = 0.$$

## A.2 First-order conditions for model with firm-specific factors

The first-order conditions for consumption and labor are now:

$$\frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} = \frac{\beta^l P_t}{\Theta_{t,l} P_{t+l}},$$

$$\frac{W_{s,j,t}}{P_t} = N_{s,j,t}^\gamma C_t^\sigma, \quad \forall s, j.$$

Consumers' allocation of investment and capital to firm  $j$  in sector  $s$  is such that:

$$Q_{s,j,t} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left( \frac{Z_{s,j,t+1}}{P_{t+1}} + Q_{s,j,t+1} \left[ (1 - \delta) - \Phi' \left( \frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right) \left( \frac{I_{s,j,t+1}}{K_{s,j,t+1}} \right)^2 \right] \right) \right\},$$

$$Q_{s,j,t} \left( \Phi' \left( \frac{I_{s,j,t}}{K_{s,j,t}} \right) \frac{I_{s,j,t}}{K_{s,j,t}} + \Phi \left( \frac{I_{s,j,t}}{K_{s,j,t}} \right) \right) = 1,$$

where  $Q_{s,j,t}$  denotes Tobin's  $q$  of firm  $j$  in sector  $s$ . Optimal price setting implies:

$$X_{s,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l} (\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1})^{-1} W_{s,j,t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l}},$$

where:

$$\Lambda_{s,t} = \left( \frac{1}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t. \quad (\text{A.1})$$

From cost-minimization, real marginal costs can be expressed as:

$$MC_{s,j,t} = \frac{1}{\chi^\chi (1 - \chi)^{1-\chi}} \left( \frac{W_{s,j,t}}{P_t} \right)^\chi \left( \frac{Z_{s,j,t}}{P_t} \right)^{(1-\chi)}. \quad (\text{A.2})$$

Note that marginal costs are now firm-specific. This is a direct implication of the assumption of firm-specific capital and labor markets.

### A.3 First-order conditions for model with economy-wide factor markets

The first-order conditions for consumption and labor are:

$$\begin{aligned} \frac{C_t^{-\sigma}}{C_{t+l}^{-\sigma}} &= \frac{\beta^l P_t}{\Theta_{t,l} P_{t+l}}, \\ \frac{W_t}{P_t} &= N_t^\gamma C_t^\sigma. \end{aligned} \quad (\text{A.3})$$

Consumers' allocation of investment  $I_t$  and capital  $K_{t+1}$  yields:

$$\begin{aligned} Q_t \left( \Phi' \left( \frac{I_t}{K_t} \right) \frac{I_t}{K_t} + \Phi \left( \frac{I_t}{K_t} \right) \right) &= 1, \\ Q_t &= \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left( \frac{Z_{t+1}}{P_{t+1}} + Q_{t+1} \left[ (1 - \delta) - \Phi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 \right] \right) \right\}, \end{aligned}$$

where  $Q_t$  denotes Tobin's  $q$ . Optimal price setting implies:

$$X_{s,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l} (\chi K_{s,j,t+l}^{1-\chi} N_{s,j,t+l}^{\chi-1})^{-1} W_{t+l}}{E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \Lambda_{s,t+l}},$$

where:

$$\Lambda_{s,t} = \left( \frac{1}{P_{s,t}} \right)^{-\theta} \left( \frac{P_{s,t}}{P_t} \right)^{-\eta} Y_t. \quad (\text{A.4})$$

From cost-minimization, real marginal costs can be expressed as:

$$MC_{s,j,t} = MC_t \frac{1}{\chi^\chi (1-\chi)^{1-\chi}} \left( \frac{W_t}{P_t} \right)^\chi \left( \frac{Z_t}{P_t} \right)^{(1-\chi)}. \quad (\text{A.5})$$

Note that marginal costs are equalized across firms and sectors. This is a direct implication of the assumption of economy-wide capital and labor markets.

## A.4 Firm-specific model solution

To solve the model, we follow [Woodford \(2005\)](#) and generalize his solution method for a multi-sector economy.

### A.4.1 Rewriting the marginal cost equation

The loglinear versions of the marginal cost, consumption/labor decision, and production function equations are given by, respectively:<sup>1</sup>

$$mc_{s,j,t} = w_{s,j,t} - p_t - (1-\chi)k_{s,j,t}^d - (\chi-1)n_{s,j,t}, \quad (\text{A.6})$$

$$w_{s,j,t} - p_t = \sigma c_t + \gamma n_{s,j,t}, \quad (\text{A.7})$$

$$y_{s,j,t} = (1-\chi)k_{s,j,t} + \chi n_{s,j,t}^d. \quad (\text{A.8})$$

Equations (A.6)-(A.8) yield:

$$k_{s,j,t} = n_{s,j,t} + (w_{s,j,t} - p_t) - (z_{s,j,t} - p_t). \quad (\text{A.9})$$

The marginal cost can be rewritten as a function of capital and labor by replacing (A.9) into (A.8), isolating for  $n_{s,j,t}$ , and replacing for equation (A.7) and (A.8) to obtain:

$$mc_{s,j,t} = \sigma c_t + \gamma n_{s,j,t} - (1-\chi)k_{s,j,t} - (\chi-1)n_{s,j,t}. \quad (\text{A.10})$$

From the production function (A.8):

$$n_{s,j,t} = \frac{1}{\chi} [y_{s,j,t} - a_t - (1-\chi)k_{s,j,t}].$$

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<sup>1</sup>Lower-case letters denote log deviations from steady state.

Substituting for  $n_{s,j,t}$  in equation (A.10) and rearranging yields:

$$mc_{s,j,t} = \sigma c_t + \left( \frac{\gamma - (\chi - 1)}{\chi} \right) y_{s,j,t} - \frac{(1 - \chi)(1 + \gamma)}{\chi} k_{s,j,t}. \quad (\text{A.11})$$

Integrating across all firms in sector  $s$ :

$$\sigma c_t = mc_{s,t} - \left( \frac{\gamma - (\chi - 1)}{\chi} \right) y_{s,t} + \frac{(1 - \chi)(1 + \gamma)}{\chi} k_{s,t}, \quad (\text{A.12})$$

which we can replace at equation (A.11) to obtain:

$$mc_{s,j,t} = mc_{s,t} + \frac{\gamma - (\chi - 1)}{\chi} (y_{s,j,t} - y_{s,t}) - \frac{(1 - \chi)(1 + \gamma)}{\chi} (k_{s,j,t} - k_{s,t}). \quad (\text{A.13})$$

The final firm's problem yields demand for goods produced by firm  $j$  in sector  $s$ :

$$y_{s,j,t} = y_{s,t} - \theta (p_{s,j,t} - p_{s,t}),$$

which can be substituted at equation A.13 to obtain:

$$mc_{s,j,t} = mc_{s,t} - \theta \frac{\gamma - (\chi - 1)}{\chi} (p_{s,j,t} - p_{s,t}) - \frac{(1 - \chi)(1 + \gamma)}{\chi} (k_{s,j,t} - k_{s,t}). \quad (\text{A.14})$$

#### A.4.2 Capital equation

From the consumers' maximization problem, the capital and investment allocations yield:

$$\kappa \delta (i_{s,j,t} - k_{s,j,t}) = E_t \left\{ \begin{array}{l} -\sigma (c_{t+1} - c_t) + [1 - (1 - \delta) \beta] (z_{s,j,t+1} - p_{t+1}) \\ + (1 - \delta) \beta \kappa \delta (i_{s,j,t+1} - k_{s,j,t+1}) + \kappa \delta^2 \beta (i_{s,j,t+1} - k_{s,j,t+1}) \end{array} \right\}. \quad (\text{A.15})$$

Using the law of motion for capital stocks and using equations (A.7)-(A.8) to replace for  $z_{s,j,t+1}$ , one obtains:

$$\kappa (k_{s,j,t+1} - k_{s,j,t}) = E_t \left\{ \begin{array}{l} -\sigma (c_{t+1} - c_t) + [1 - (1 - \delta) \beta] \left[ \begin{array}{l} \frac{1+\gamma}{\chi} \left( \begin{array}{l} y_{s,j,t+1} \\ -(1 - \chi) k_{s,j,t+1} \end{array} \right) \\ + \sigma c_{t+1} - k_{s,j,t+1} \end{array} \right] \\ + \kappa \beta (k_{s,j,t+2} - k_{s,j,t+1}) \end{array} \right\}.$$

Integrating across all firms in sector  $s$  yields:

$$\kappa [k_{s,j,t+1} - k_{s,t+1} - (k_{s,j,t} - k_{s,t})] = E_t \left\{ \left[ 1 - (1 - \delta) \beta \left[ \begin{array}{c} \frac{1+\gamma}{\chi} \left( \begin{array}{c} -\theta (p_{s,j,t+1} - p_{s,t+1}) \\ -(1 - \chi) (k_{s,j,t+1} - k_{s,t+1}) \end{array} \right) \\ -(k_{s,j,t+1} - k_{s,t+1}) \end{array} \right] + \kappa \beta [k_{s,j,t+2} - k_{s,t+2} - (k_{s,j,t+1} - k_{s,t+1})] \right] \right\},$$

where we used again the final-firms demand for goods produced by firm  $j$  in sector  $s$ .

Defining

$$\tilde{p}_{s,j,t} = p_{s,j,t} - p_{s,t}, \quad (\text{A.16})$$

$$\tilde{k}_{s,j,t} = k_{s,j,t} - k_{s,t}, \quad (\text{A.17})$$

we can rewrite the previous equation as:

$$\kappa [\tilde{k}_{s,j,t+1} - \tilde{k}_{s,j,t}] = E_t \left\{ \left[ 1 - (1 - \delta) \beta \left[ \begin{array}{c} \frac{1+\gamma}{\chi} \left( -\theta \tilde{p}_{s,j,t+1} - (1 - \chi) \tilde{k}_{s,j,t+1} \right) - \tilde{k}_{s,j,t+1} \right] + \kappa \beta [\tilde{k}_{s,j,t+2} - \tilde{k}_{s,j,t+1}] \right] \right\}. \quad (\text{A.18})$$

### A.4.3 Pricing rule

The intermediate-firm optimization problem yields the optimal price-setting equation:

$$x_{s,j,t} = (1 - \beta (1 - \alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m (p_{t+m} + mc_{s,j,t+m})$$

Note that because each firm can have a different capital-accumulation history, expectations may vary from one firm to another, and hence,  $E_t$ , the usual expectations at time- $t$  operator, differs from  $E_t^j$ .

Rewriting equation (A.14) at  $t + m$ , and using equations (A.16) and (A.17) yields:

$$mc_{s,j,t+m} = mc_{s,t+m} - \theta \frac{\gamma - (\chi - 1)}{\chi} \tilde{p}_{s,j,t+m} - \frac{(1 - \chi)(1 + \gamma)}{\chi} \tilde{k}_{s,j,t+m},$$

which can be replaced in the optimal pricing equation above to obtain:

$$x_{s,j,t} = (1 - \beta (1 - \alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m \left( p_{t+m} + mc_{s,t+m} - \theta \frac{\gamma - (\chi - 1)}{\chi} (\tilde{x}_{s,j,t} - \sum_{i=1}^m E_t \pi_{s,t+i}) - \frac{(1 - \chi)(1 + \gamma)}{\chi} \tilde{k}_{s,j,t+m} \right),$$

where we used the fact that definitions (A.16) and (A.17) imply:

$$\begin{aligned} E_t^j \tilde{p}_{s,j,t+m} &= \tilde{p}_{s,j,t} - \sum_{i=1}^m E_t \pi_{s,t+i} \\ \tilde{k}_{s,j,t+m} &= k_{s,j,t+m} - k_{s,t+m} \end{aligned}$$

Taking  $p_{s,t}$  from both sides of the price equation above, using the fact that  $p_{s,t} = p_{s,t+m} - \sum_{i=1}^m E_t \pi_{s,t+i}$ , and rewriting yields:

$$\left(1 + \frac{\theta(\gamma - (\chi - 1))}{\chi}\right) \tilde{x}_{s,j,t} = (1 - \beta(1 - \alpha_s)) E_t^j \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m \left( \begin{aligned} &+ \left[1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right] \sum_{i=1}^m E_t \pi_{s,t+i} \\ &- \frac{(1 - \chi)(1 + \gamma)}{\chi} \tilde{k}_{s,j,t+m}. \end{aligned} \right)$$

Note that the only term in the above expression that depends on firm-j expectations is the one associated with capital stocks. Hence, we can rewrite the above expression as:

$$\begin{aligned} \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right) \tilde{x}_{s,j,t} &= (1 - \beta(1 - \alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m \left( \begin{aligned} &+ \left(1 + \theta \frac{\gamma - (\chi - 1)}{\chi}\right) \sum_{i=1}^m E_t \pi_{s,t+i} \\ &- \frac{(1 - \chi)(1 + \gamma)}{\chi} \tilde{k}_{s,j,t+m}. \end{aligned} \right) \quad (\text{A.19}) \end{aligned}$$

#### A.4.4 Guessing a solution

Equation (A.18) can be rewritten as:

$$\begin{aligned} E_t \left\{ Q(L) \tilde{k}_{s,j,t+2} \right\} &= \Xi_1 E_t \tilde{p}_{s,j,t+1} \quad (\text{A.20}) \\ \text{where: } Q(L) &= \beta - AL + L^2 \\ A &= \left[ \beta + [1 - (1 - \delta)\beta] \left[ \frac{\chi + (1 + \gamma)(1 - \chi)}{\kappa\chi} \right] - 1 \right] \\ \Xi &= [1 - (1 - \delta)\beta] \frac{\theta(1 + \gamma)}{\kappa\chi}, \end{aligned}$$

and we could factor the  $Q(L)$  so to obtain two real roots (see [Woodford, 2005](#)).

Because consumer j's decision problem is locally convex, the first-order condition characterizes a locally unique optimal plan, and at the time of price adjustment, the chosen price must depend only on j's relative capital stock and its own sector's state. Hence, j's pricing decision must take the form:

$$\tilde{x}_{s,j,t} = g_{s,t} - \psi_s \tilde{k}_{s,j,t}, \quad (\text{A.21})$$

where  $g_{s,t}$  depends only on the sectoral state and aggregate variables, and the coefficient  $\psi_s$  is to be determined.

Sectoral prices are such that:

$$\int_I (x_{s,i,t} - p_{s,t}) di - (1 - \alpha_s) \pi_{s,t} = 0. \quad (\text{A.22})$$

Given that optimizing firms are chosen at random at each time, and that  $\tilde{k}$  is a gap relative to the sectoral capital:

$$\int_I \tilde{k}_{s,i,t} di = 0. \quad (\text{A.23})$$

Hence, replacing (A.21) and (A.23) into (A.22) yields:

$$g_{s,t} = \frac{1 - \alpha_s}{\alpha_s} \pi_{s,t}.$$

Calvo price setting implies that:

$$E_t \tilde{p}_{s,j,t+1} = (1 - \alpha_s) E_t (\tilde{p}_{s,j,t} - \pi_{s,t+1}) + \alpha_s E_t \tilde{x}_{s,j,t+1}.$$

Using equation (A.21) and  $g_{s,t}$ , we obtain:

$$E_t \tilde{p}_{s,j,t+1} = (1 - \alpha_s) \tilde{p}_{s,j,t} - \alpha_s \psi_s \tilde{k}_{s,j,t+1}. \quad (\text{A.24})$$

Extending [Woodford \(2005\)](#)'s insight for multisector economies, the optimal quantity of investment in any period must depend only on  $j$ 's relative capital stock, its relative price, and the economy's aggregate state. Thus,  $\tilde{k}_{s,j,t+1}$  can be represented as a function of  $\tilde{k}_{s,j,t}$ ,  $\tilde{p}_{s,j,t}$ . Hence, a firm  $j$  individual expectation ( $E_t^j$ ) only involves periods at which the firm is not readjusting. We guess (and verify) that:

$$\tilde{k}_{s,j,t+1} = \kappa_{1,s} \tilde{k}_{s,j,t} - \kappa_{2,s} \tilde{p}_{s,j,t}, \quad (\text{A.25})$$

where the parameters  $\kappa_{1,s}$ ,  $\kappa_{2,s}$ ,  $\psi_s$  and function  $g_{s,t}$  are to be determined.

Using equation (A.24), this guess on capital implies that:

$$E_t \tilde{k}_{s,j,t+2} = (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) \tilde{k}_{s,j,t+1} - \kappa_{2,s} (1 - \alpha_s) \tilde{p}_{s,j,t}.$$

Using this last expression and equation (A.24) to substitute for (A.20) yields:

$$\begin{bmatrix} \beta (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) - A \\ + \alpha_s \psi_s [1 - (1 - \delta) \beta] \frac{\theta(1+\gamma)}{\kappa \chi} \end{bmatrix} \tilde{k}_{s,j,t+1} = -\tilde{k}_{s,j,t} + \begin{bmatrix} [1 - (1 - \delta) \beta] \frac{\theta(1+\gamma)}{\chi \kappa} (1 - \alpha_s) \\ + \kappa_{2,s} (1 - \alpha_s) \beta \end{bmatrix} \tilde{p}_{s,j,t}.$$

Comparing with guess (A.25), we have:

$$\begin{bmatrix} \beta (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) - A \\ + \alpha_s \psi_s [1 - (1 - \delta) \beta] \frac{\theta(1+\gamma)}{\kappa \chi} \end{bmatrix} \kappa_{1,s} = -1 \quad (\text{A.26})$$

$$- \begin{bmatrix} \beta (\kappa_{1,s} + \kappa_{2,s} \alpha_s \psi_s) - A \\ + \alpha_s \psi_s [1 - (1 - \delta) \beta] \frac{\theta(1+\gamma)}{\kappa \chi} \end{bmatrix} \kappa_{2,s} = \begin{bmatrix} [1 - (1 - \delta) \beta] \frac{\theta(1+\gamma)}{\chi \kappa} (1 - \alpha_s) \\ + \kappa_{2,s} (1 - \alpha_s) \beta \end{bmatrix}. \quad (\text{A.27})$$

Note that the system of dynamic equations (A.24) and (A.25) implies:

$$\begin{bmatrix} E_t \tilde{p}_{s,j,t+1} \\ \tilde{k}_{s,j,t+1} \end{bmatrix} = \begin{bmatrix} (1 - \alpha_s) + \alpha_s \psi_s \kappa_{2,s} & -\alpha_s \psi_s \kappa_{1,s} \\ -\kappa_{2,s} & \kappa_{1,s} \end{bmatrix} \begin{bmatrix} E_t \tilde{p}_{s,j,t} \\ \tilde{k}_{s,j,t} \end{bmatrix}.$$

And we have both eigenvalues of the matrix inside the unit circle if and only if:

$$\begin{aligned} \kappa_{1,s} &< (1 - \alpha_s)^{-1} \\ \kappa_{1,s} &< 1 - \kappa_{2,s} \psi_s \\ \kappa_{1,s} &> -1 - \frac{\alpha_s}{2 - \alpha_s} \kappa_{2,s} \psi_s. \end{aligned}$$

#### A.4.5 Optimal pricing rule

The optimal pricing equation (A.19) includes the term  $E_t^j \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m \tilde{k}_{s,j,t+m}$ , which varies according to each firm's capital accumulation history.

Using our solution guesses (A.21) and (A.25), we can rewrite this term as:

$$\begin{aligned} E_t^j \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m \tilde{k}_{s,j,t+m} &= (1 - (1 - \alpha_s) \beta \kappa_{1,s})^{-1} \tilde{k}_{s,j,t} \\ &\quad - \kappa_{2,s} \frac{\beta (1 - \alpha_s)}{(1 - \beta (1 - \alpha_s)) (1 - \beta (1 - \alpha_s) \kappa_{1,s})} \tilde{p}_{s,j,t} \\ &\quad + \kappa_{2,s} \frac{(1 - \alpha_s) \beta}{(1 - \beta (1 - \alpha_s)) (1 - \beta (1 - \alpha_s) \kappa_{1,s})} \sum_{k=1}^{\infty} \beta^k (1 - \alpha_s)^k E_t \pi_{s,t+k}. \end{aligned}$$

where we also used the fact that for any firm that doesn't readjust between times  $t$  and  $t + m$ :

$$\tilde{p}_{s,j,t+m} = \tilde{x}_{s,j,t} - \pi_{s,t+m} - \dots - \pi_{s,t+1}.$$

Hence, the optimal pricing equation (A.19) implies:

$$\begin{aligned} \phi_s \tilde{x}_{s,j,t} &= (1 - \beta(1 - \alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m (p_{t+m} + m c_{s,t+m} - p_{s,t+m}) \\ &+ \left( 1 + \theta \frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s} (1 - \chi) (1 + \gamma) \beta (1 - \alpha_s)}{\chi (1 - \beta(1 - \alpha_s) \kappa_{1,s})} \right) \sum_{k=1}^{\infty} \beta^k (1 - \alpha_s)^k E_t \pi_{s,t+k} \\ &- \frac{(1 - \chi) (1 + \gamma)}{\chi} \frac{(1 - \beta(1 - \alpha_s))}{(1 - (1 - \alpha_s) \beta \kappa_{1,s})} \tilde{k}_{s,j,t}, \end{aligned}$$

where:

$$\phi_s = \left( 1 + \theta \frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s} (1 - \chi) (1 + \gamma) \beta (1 - \alpha_s)}{\chi (1 - \beta(1 - \alpha_s) \kappa_{1,s})} \right).$$

The term  $\phi_s$  is (an important component) of the coefficients of the firm-specific Phillips curve.

Recall that our guesses (A.21) and (A.25) take the form:

$$\tilde{x}_{s,j,t} = g_{s,t} - \psi_s \tilde{k}_{s,j,t}$$

$$\tilde{k}_{s,j,t+1} = \kappa_{1,s} \tilde{k}_{s,j,t} - \kappa_{2,s} \tilde{p}_{s,j,t}.$$

Hence, the solution for the pricing equation above implies that:

$$\begin{aligned} \phi_s g_{s,t} &= (1 - \beta(1 - \alpha_s)) E_t \sum_{m=0}^{\infty} \beta^m (1 - \alpha_s)^m (p_{t+m} + m c_{s,t+m} - p_{s,t+m}) \\ &+ \phi_s \sum_{k=1}^{\infty} \beta^k (1 - \alpha_s)^k E_t \pi_{s,t+k}, \end{aligned}$$

where  $\psi_s$  satisfies:

$$\phi_s \psi_s = \frac{(1 - \chi) (1 + \gamma)}{\chi} \frac{(1 - \beta(1 - \alpha_s))}{(1 - (1 - \alpha_s) \beta \kappa_{1,s})}. \quad (\text{A.28})$$

Note that this last equation can be solved for  $\psi_s$  as a function of  $\kappa_{1,s}$  and  $\kappa_{2,s}$ .

Equation (A.28) along with equations (A.26) and (A.27) form a system of 3 equations and

3 unknowns,  $\psi_s$ ,  $\kappa_{1,s}$  and  $\kappa_{2,s}$ :

$$\begin{aligned} \frac{(1-\chi)(1+\gamma)}{\chi} \frac{(1-\beta(1-\alpha_s))}{(1-(1-\alpha_s)\beta\kappa_{1,s})} &= \phi_s \psi_s \\ [\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi] \kappa_{1,s} &= -1 \\ -[\beta(\kappa_{1,s} + \kappa_{2,s}\alpha_s\psi_s) - A + \alpha_s\psi_s\Xi] \kappa_{2,s} &= [\Xi(1-\alpha_s) + \kappa_{2,s}(1-\alpha_s)\beta], \end{aligned}$$

where:

$$\begin{aligned} \phi_s &= \left( 1 + \theta \frac{\gamma - (\chi - 1)}{\chi} - \frac{\kappa_{2,s}(1-\chi)(1+\gamma)\beta(1-\alpha_s)}{\chi(1-\beta(1-\alpha_s)\kappa_{1,s})} \right), \\ A &= \left[ \beta + [1 - (1-\delta)\beta] \left[ \frac{\chi + (1+\gamma)(1-\chi)}{\kappa\chi} \right] - 1 \right], \\ \Xi &= [1 - (1-\delta)\beta] \frac{\theta(1+\gamma)}{\kappa\chi}. \end{aligned}$$

The coefficients  $\kappa_{1,s}$  and  $\kappa_{2,s}$  are obtained from the solution to the nonlinear system of 3 equations and 3 unknowns  $\psi_s$ ,  $\kappa_{1,s}$  and  $\kappa_{2,s}$ :

Note that in a version of the model without capital accumulation,  $\chi = 1$ ,  $\phi_s$  simplifies to  $\phi = (1 + \theta\gamma)$  – which is familiar from new Keynesian DSGE models. Details of the derivation of these equations are available upon request.

## References

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