APPENDIX TO “THE SLOW JOB RECOVERY IN A MACRO MODEL OF SEARCH AND RECRUITING INTENSITY”

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Abstract. This appendix provides a summary of equilibrium conditions in the DSGE model of Leduc and Liu (2017) and presents some additional results with alternative assumptions on the entry-cost distribution.

I. Summary of equilibrium conditions

A search equilibrium is a system of 17 equations for 17 variables summarized in the vector

\[ [C_t, \Lambda_t, m_t, q_t^u, q_t^v, N_t, u_t, U_t, Y_t, r_t, v_t, J_t^F, w_t^N, w_t, n_t^e, a_t, s_t] . \]

We write the equations in the same order as in the dynare code.

1. Household’s bond Euler equation:

\[ 1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} r_t, \] (B1)

2. Marginal utility of consumption

\[ \Lambda_t = \frac{1}{C_t}, \] (B2)

3. Search intensity

\[ h_1 + h_2(s_t - \bar{s}) = \frac{q_t^u}{s_t} \left\{ \frac{b}{1 - b} (J_t^F - Kn_t) - \frac{h(s_t)}{1 - q_t^u} \right\}, \] (B3)

4. Matching function

\[ m_t = \mu_t (s_t u_t)^\alpha (a_t v_t)^{1-\alpha}, \] (B4)
(5) Job finding rate
\[ q^u_t = \frac{m_t}{u_t}, \tag{B5} \]

(6) Vacancy filling rate
\[ q^v_t = \frac{m_t}{v_t}, \tag{B6} \]

(7) Employment dynamics:
\[ N_t = (1 - \delta_t)N_{t-1} + m_t, \tag{B7} \]

(8) Number of searching workers:
\[ u_t = 1 - (1 - \delta_t)N_{t-1}, \tag{B8} \]

(9) Unemployment:
\[ U_t = 1 - N_t, \tag{B9} \]

(10) Law of motion for vacancies:
\[ v_t = (1 - \rho^o)(1 - q^v_{t-1})v_{t-1} + (\delta_t - \rho^o)N_{t-1} + n_t, \tag{B10} \]

(11) Aggregate production function:
\[ Y_t = Z_tN_t \tag{B11} \]

(12) Aggregate Resource constraint:
\[ C_t + h(s_t)u_t + \kappa(a_t)v_t + \frac{1}{2}Kn_t^2 = Y_t, \tag{B12} \]
where the search cost function and the recruiting cost function are given by
\[ h(s_t) = h_1(s_t - \bar{s}) + \frac{h_2}{2}(s_t - \bar{s})^2 \]
\[ \kappa(a_t) = \kappa_0 + \kappa_1(a_t - \bar{a}) + \frac{\kappa_2}{2}(a_t - \bar{a})^2 \]

(13) Value of vacancy:
\[ Kn_t = -\kappa(a_t) + q^v_t J^F_t + (1 - q^v_t)(1 - \rho^o)E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} Kn_{t+1}, \tag{B13} \]

(14) Recruiting intensity:
\[ \kappa_1 + \kappa_2(a_t - \bar{a}) = \frac{q^v_t}{a_t} \left( J^F_t - (1 - \rho^o)E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} Kn_{t+1} \right), \tag{B14} \]

(15) Match value:
\[ J^F_t = Z_t - w_t + E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ (1 - \delta_{t+1})J^F_{t+1} + \delta_{t+1}Kn_{t+1} \right\}, \tag{B15} \]
(16) Nash bargaining wage:

\[
\frac{b}{1-b} (J_t^F - K_n t) = w_t^N - \phi - \frac{\chi_t}{\Lambda_t} + h(s_t) E_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left[ (1 - \delta_{t+1}) (1 - q_{t+1}^u) \right. \\
\left. \frac{b}{1-b} (J_{t+1}^F - K_{n+1} t) \right].
\]

(B16)

(17) Actual real wage (with real wage rigidity)

\[
w_t = w_{t-1}^N (w_t^N)^\gamma,
\]

(B17)

II. Steady State

(1) Household’s bond Euler equation:

\[1 = \beta r,\]

(C1)

(2) Marginal utility of consumption

\[\Lambda = \frac{1}{C},\]

(C2)

(3) Search intensity

\[h_1 = \frac{q^u}{s} \frac{b}{1-b} (J^F - K_n),\]

(C3)

(4) Matching function

\[m = \mu (\bar{s}u)^\alpha (\bar{a}v)^{1-\alpha},\]

(C4)

(5) Job finding rate

\[q^u = \frac{m}{u},\]

(C5)

(6) Vacancy filling rate

\[q^v = \frac{m}{v},\]

(C6)

(7) Employment dynamics:

\[m = \delta N,\]

(C7)

(8) Number of searching workers:

\[u = U + m,\]

(C8)

(9) Unemployment:

\[U = 1 - N,\]

(C9)

(10) Vacancies:

\[\rho^o + (1 - \rho^o) q^v v = (\delta - \rho^o) N + n,\]

(C10)

(11) Aggregate production function:

\[Y = ZN\]

(C11)
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(12) Aggregate Resource constraint:
\[ C + \kappa_0 v + \frac{1}{2} Kn^2 = Y, \]  \hspace{1cm} (C12)

(13) Value of vacancies:
\[ q^u J^F - \kappa_0 = [1 - \beta (1 - q^u) (1 - \rho^o)] Kn \]  \hspace{1cm} (C13)

(14) Recruiting intensity:
\[ \kappa_1 \hat{a} = q^u [J^F - \beta (1 - \rho^o) Kn] , \]  \hspace{1cm} (C14)

(15) Match value:
\[ [1 - \beta (1 - \delta)] J^F = Z - w + \beta \delta Kn , \]  \hspace{1cm} (C15)

(16) Nash bargaining wage:
\[ w^N = \phi + \frac{\chi}{\Lambda} + \frac{b}{1-b} [1 - \beta (1 - \delta) (1 - q^u)] (J^F - Kn) , \]  \hspace{1cm} (C16)

(17) Actual real wage
\[ w = w^N , \]  \hspace{1cm} (C17)

III. EQUILIBRIUM SYSTEM SCALED BY STEADY STATE (USED IN DYNARE)

Denote by \( \hat{X}_t \equiv \frac{X_t}{X} \) the scaled value of the variable \( X_t \) by its steady-state level. The system of equilibrium conditions can be reduced to the following 17 equations to solve for the 17 endogenous variables summarized in the vector
\[ \begin{align*}
\hat{C}_t, \hat{\Lambda}_t, \hat{\gamma}_t, \hat{m}_t, \hat{\nu}_t, \hat{q}_t^u, \hat{q}_t, \hat{N}_t, \hat{U}_t, \hat{J}_t^F, \hat{\omega}_t^N, \hat{\omega}_t, \hat{n}_t, \hat{a}_t, \hat{s}_t.
\end{align*} \]

(1) Household’s bond Euler equation:
\[ 1 = \frac{\hat{\Lambda}_{t+1}}{E_t} \hat{\gamma}_t , \]  \hspace{1cm} (D1)

(2) Marginal utility of consumption
\[ \hat{\Lambda}_t = \frac{1}{\hat{C}_t} . \]  \hspace{1cm} (D2)

(3) Search intensity
\[ h_1 + h_2 \hat{s}(\hat{s}_t - 1) = \frac{q^u \hat{q}_t^u}{\hat{s}\hat{s}_t} \left\{ \frac{b}{1-b} (J^F \hat{J}_t^F - Kn \hat{n}_t) - \frac{h_1 \hat{s}(\hat{s}_t - 1) + \frac{h_2^2}{2} (\hat{s}_t - 1)^2}{1 - q^u \hat{q}_t^u} \right\} \]  \hspace{1cm} (D3)

(4) Matching function
\[ \hat{m}_t = (\hat{s}_t \hat{\nu}_t)^\alpha (\hat{a}_t \hat{\gamma}_t)^{1-\alpha} , \]  \hspace{1cm} (D4)

(5) Job finding rate
\[ \hat{q}_t^u = \frac{\hat{m}_t}{\hat{\nu}_t} . \]  \hspace{1cm} (D5)
(6) Vacancy filling rate

\[ \hat{q}_t^v = \frac{\hat{m}_t}{\hat{v}_t}, \]  

(D6)

(7) Employment dynamics:

\[ \hat{N}_t = (1 - \delta \exp(\hat{\delta}_t))\hat{N}_{t-1} + \frac{m}{N} \hat{m}_t, \]  

(D7)

(8) Number of searching workers

\[ u\hat{u}_t = 1 - (1 - \delta \exp(\hat{\delta}_t))N\hat{N}_{t-1}, \]  

(D8)

(9) Unemployment:

\[ U\hat{U}_t = 1 - \hat{N}_t, \]  

(D9)

(10) Vacancies:

\[ v\hat{v}_t = (1 - \rho_o)(1 - q^v\hat{q}_t^v)\hat{v}_{t-1} + (\delta \exp(\hat{\delta}_t) - \rho_o)N\hat{N}_{t-1} + \hat{n}_t, \]  

(D10)

(11) Aggregate production function:

\[ \hat{Y}_t = \exp(\hat{z}_t)\hat{N}_t \]  

(D11)

(12) Aggregate Resource constraint:

\[ \hat{Y}_t = \left[ h_1 \hat{s}_t (\hat{\phi}_t - 1) + \frac{h_2 \hat{s}^2}{2} (\hat{\phi}_t - 1)^2 \right] \hat{u}_t + \left[ \kappa_0 + \kappa_1 \hat{a}_t (\hat{\phi}_t - 1) + \frac{\kappa_2 \hat{a}^2}{2} (\hat{\phi}_t - 1)^2 \right] \frac{v}{Y} \hat{v}_t + \frac{C}{Y} \hat{C}_t + \frac{1}{2} \frac{K_n}{Y} \hat{n}_t^2, \]  

(D12)

(13) Value of vacancy:

\[ Kn\hat{n}_t = - \left[ \kappa_0 + \kappa_1 \hat{a}_t (\hat{\phi}_t - 1) + \frac{\kappa_2 \hat{a}^2}{2} (\hat{\phi}_t - 1)^2 \right] + \]  

\[ q^v J^F \hat{q}_t^v \hat{J}_t^F + (1 - q^v\hat{q}_t^v)(1 - \rho_o)E_t \frac{\beta_{t+1}}{\Lambda_t} K_n\hat{n}_{t+1}, \]  

(D13)

(14) Recruiting intensity:

\[ \kappa_1 + \kappa_2 \hat{a}_t (\hat{\phi}_t - 1) = \frac{q^v\hat{q}_t^v}{\hat{a}\hat{a}_t} \left[ J^F \hat{J}_t^F - (1 - \rho_o)E_t \frac{\beta_{t+1}}{\Lambda_t} K_n\hat{n}_{t+1} \right], \]  

(D14)

(15) Match value:

\[ J^F \hat{J}_t^F = \exp(\hat{z}_t) - \hat{w}\hat{w}_t + E_t \frac{\beta_{t+1}}{\Lambda_t} \left\{ (1 - \delta \exp(\hat{\delta}_{t+1}))J^F \hat{J}_{t+1}^F + \delta \exp(\hat{\delta}_{t+1})K_n\hat{n}_{t+1} \right\}, \]  

(D15)
(16) Nash bargaining wage:

\[
\frac{b}{1-b}(J^F j^F_t - Kn \hat{n}_t) = w\hat{w}_t^N - \phi - \chi \exp(\hat{\chi}_t) + \frac{h_1 \hat{s}(\hat{s}_t - 1) + \frac{h_2 \hat{s}^2}{2} (\hat{s}_t - 1)^2}{1 - q^u \hat{q}^u_t} + E_t \frac{\beta \hat{\Lambda}_{t+1}}{\hat{\Lambda}_t} \left[ (1 - \delta \exp(\hat{\delta}_{t+1})) (1 - q^u \hat{q}^u_{t+1}) \frac{b}{1-b}(J^F j^F_{t+1} - Kn \hat{n}_{t+1}) \right].
\]

(D16)

(17) Actual real wage (with real wage rigidity)

\[
\hat{w}_t = \hat{w}_t^\gamma (\hat{w}_t^N)^\gamma,
\]

(D17)

(18) Preference shock process

\[
\hat{\chi}_t = \rho \hat{\chi}_{t-1} + \varepsilon_{\chi t},
\]

(D18)

(19) Technology shock process

\[
\hat{z}_t = \rho \hat{z}_{t-1} + \varepsilon_{zt},
\]

(D19)

(20) Job separation shock process

\[
\hat{\delta}_t = \rho \hat{\delta}_{t-1} + \varepsilon_{\delta t},
\]

(D20)

IV. Robustness to alternative distribution assumptions for entry costs

In the baseline model, we assume that entry costs are drawn from a uniform distribution (i.e., \(\xi = 1\)). This assumption is in line with the literature (Fujita and Ramey, 2007; Coles and Kelishomi, 2011). Here, we examine our model’s quantitative implications under alternative values of \(\xi\). As we discussed in the paper, a larger value of \(\xi\) implies more elastic responses of entry (i.e., vacancy creation) to changes in the value of vacancies. The limiting case with \(\xi = \infty\) approximates an environment with free entry, as in the standard DMP model.

To examine the sensitivity of our results to \(\xi\), we re-estimate the baseline model with two alternative calibrations, one with \(\xi = 2\) and the other with \(\xi = 10\). With a larger value of \(\xi\), the model becomes closer to one with free entry, and firms in the model would rely more on variations in the number of vacancies to respond to macroeconomic shocks and less on varying recruiting intensity. Thus, the exercise here also helps evaluate the quantitative importance of cyclical variations in recruiting intensity for labor market dynamics. As in the baseline model, we evaluate the quantitative performance of the model under these alternative \(\xi\) values by comparing the model’s predictions on the job filling and finding rates and also on the hiring rate to those in the actual data.

Figure 1 shows the job filling rate and the job finding rate in the model estimated under \(\xi = 2\) (the dashed and dotted lines), along with those predicted from the standard matching
function (the dashed lines) as well as those in the data (the solid lines). The figure shows that, similar to the baseline case with $\xi = 1$, our model's predictions on the job filling and finding rates under $\xi = 2$ match the data well. They both outperform the predictions from the standard matching function by a significant margin. Indeed, as we show in Table 1, the model with $\xi = 2$ implies even smaller RMSEs than our baseline model with $\xi = 1$ (0.074 vs. 0.086), both are substantially smaller than those implied by the standard matching function (0.157).

Figure 2 shows the predicted hiring rate in our model with $\xi = 2$ along with that in the data. Again, the model matches the data well. These results suggest that the quantitative results obtained in our baseline model are robust when we increase the value of $\xi$ modestly (from 1 to 2).

It is natural to ask: What if $\xi$ takes an even larger value so that the model gets closer to the standard DMP model with free entry? We provide an answer by re-estimating the DSGE model with $\xi$ fixed at 10. Figure 3 displays the job filling and finding rates implied by the model with $\xi = 10$, along with those from the standard matching function and the data. The figure shows that the model’s predictions appear worse than our baseline model with $\xi = 1$ and become closer to the predictions from the standard matching function. Figure 4 shows the model’s predicted hiring rate when $\xi$ is fixed at 10 versus that in the data. Apparently, the model performs not as well as the baseline model with $\xi = 1$ in matching the hiring data. Table 1 shows that the RMSEs for job filling and finding rates under $\xi = 10$ are significantly larger than those in the baseline case (0.115 vs. 0.086), although they remain smaller than those implied by the standard matching function (0.115 vs. 0.157).
Table 1. Out-of-sample predictions of alternative models: RMSE

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard model</th>
<th>Benchmark model</th>
<th>Alternative estimation</th>
<th>$\xi = 2$</th>
<th>$\xi = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job filling rate</td>
<td>0.1565</td>
<td>0.0864</td>
<td>0.1238</td>
<td>0.0737</td>
<td>0.1151</td>
</tr>
<tr>
<td>Job finding rate</td>
<td>0.1565</td>
<td>0.0862</td>
<td>0.1437</td>
<td>0.0737</td>
<td>0.1151</td>
</tr>
</tbody>
</table>

*Note: The numbers in this table are root mean squared errors calculated based on demeaned data and predictions from each model.*
Figure 1. Job filling rate and job finding rate: Data, standard model, and DSGE model estimated under the calibration $\xi = 2$. 
Figure 2. Hiring rate: Data vs. DSGE model with $\xi = 2$. 
Figure 3. Job filling rate and job finding rate: Data, standard model, and DSGE model estimated under the calibration $\xi = 10$. 
Figure 4. Hiring rate: Data vs. DSGE model with $\xi = 10$. 
REFERENCES

