APPENDIX TO “RESERVE REQUIREMENTS AND OPTIMAL CHINESE STABILIZATION POLICY”

CHUN CHANG, ZHENG LIU, MARK M. SPIEGEL, JINGYI ZHANG

ABSTRACT. This appendix shows some additional details of the model and equilibrium conditions in Chang, Liu, Spiegel, and Zhang (2017).
I. The model

The economy is populated by a continuum of infinitely lived households. The representative household consumes a basket of differentiated goods purchased from retailers. Retailers produce differentiated goods using a homogeneous wholesale good as the only input. The wholesale good is itself a composite of intermediate goods produced by two types of firms: SOEs and POEs. The two types of firms have identical production technologies ex-ante except that the average productivity of SOEs is assumed to be lower than that of POEs.

Firms face working capital constraints. Each firm finances wages and rental payments using both internal net worth and external debt. Following Bernanke, Gertler, and Gilchrist (1999), we assume that external financing is subject to a costly state verification problem. In particular, each firm can observe its own idiosyncratic productivity shocks. Firms with sufficiently low productivity relative to their nominal debt obligations will default and be liquidated. The lender suffers a liquidation cost when taking over the project to seize available revenue.

We generalize the BGG framework to a two-sector environment with SOEs and POEs that have access to different sources of external financing. We assume that SOEs only borrow through formal on-balance-sheet loans. As is effectively the case in China, we also assume that these loans are backed by government guarantees. In contrast, POEs only borrow through off-balance-sheet loans, which are neither regulated nor backed by the government. While banks face no default risk on the guaranteed loans to SOEs, they face expected default costs for off-balance sheet loans extended to POEs, as in the BGG framework.\footnote{Our framework is by necessity a simplification of reality. The off-balance sheet lending in our model is a stand-in for the more diverse and complex set of nonbank financing activity in China, including private loans and corporate bonds. Our framework could be extended to allow separate non-banks to borrow from commercial banks off-balance sheet and then extend loans to POEs. Finally, large and profitable Chinese private firms typically have no difficulties accessing bank loans, but they rely more on non-bank channels for finance, such as equity and bond markets. This observation does not contradict our model’s reallocation mechanism associated with changes in RR policy.}

I.1. Households. There is a continuum of infinitely lived and identical households with unit mass. The representative household has preferences represented by the expected utility function

\[
U = E \sum_{t=0}^{\infty} \beta_t \left[ \ln(C_t) - \Psi \frac{H_t^{1+\eta}}{1+\eta} \right],
\]

where \( E \) is an expectation operator, \( C_t \) denotes consumption, and \( H_t \) denotes labor hours. The parameter \( \beta \in (0, 1) \) is a subjective discount factor, \( \eta > 0 \) is the inverse Frisch elasticity of labor supply, and \( \Psi > 0 \) reflects labor disutility.
The household faces the sequence of budget constraints
\[ C_t + I_t + \frac{D_t}{P_t} = w_t H_t + r^k_t K_{t-1} + R_{t-1} \frac{D_{t-1}}{P_{t-1}} + T_t, \tag{2} \]
where \( I_t \) denotes capital investment, \( D_t \) denotes deposits in banks, \( w_t \) denotes the real wage rate, \( r^k_t \) denotes the real rent rate on capital, \( K_{t-1} \) denotes the level of the capital stock at the beginning of period \( t \), \( R_{t-1} \) is the gross nominal interest rate on household savings determined from information available in period \( t - 1 \), \( P_t \) denotes the price level, and \( T_t \) denotes the lump-sum transfers from the government and earnings received from firms based on the household’s ownership share.

The capital stock evolves according to the law of motion
\[ K_t = (1 - \delta) K_{t-1} + \left[1 - \frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_t \right) \right] I_t, \tag{3} \]
where we have assumed that changes in investment incur an adjustment cost reflected by parameter \( \Omega_k \). The constant \( g_t \) denotes the steady-state growth rate of investment.

The household chooses \( C_t, H_t, D_t, I_t, \) and \( K_t \) to maximize (1), subject to the constraints (2) and (3). The optimizing conditions are summarized by the following equations:
\[ \Lambda_t = \frac{1}{C_t}, \tag{4} \]
\[ w_t = \frac{\Psi H_t}{\Lambda_t}, \tag{5} \]
\[ 1 = E_t \beta R_t \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}, \tag{6} \]
\[ 1 = q^k_t \left[ 1 - \frac{\Omega_k}{2} \left( \frac{I_t}{I_{t-1}} - g_t \right)^2 - \Omega_k \left( \frac{I_t}{I_{t-1}} - g_t \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t q^k_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_k \left( \frac{I_{t+1}}{I_t} - g_t \right) \left( \frac{I_{t+1}}{I_t} \right)^2, \tag{7} \]
\[ q^k_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} [q^k_{t+1} (1 - \delta) + r^k_{t+1}],. \tag{8} \]

where \( \Lambda_t \) denotes the Lagrangian multiplier for the budget constraint (2), \( \pi_t = \frac{P_t}{P_{t-1}} \) denotes the inflation rate from period \( t - 1 \) to period \( t \), and \( q^k_t \equiv \frac{\Lambda^k_k}{\Lambda_t} \) is Tobin’s \( q \), with \( \Lambda^k_t \) being the Lagrangian multiplier for the capital accumulation equation (3).

I.2. Retail sector and price setting. There is a continuum of retailers, each producing a differentiated retail product indexed by \( z \in [0, 1] \). The retail goods are produced using a homogeneous wholesale good, with a constant-returns technology. Retailers are price takers in the input market and face monopolistic competition in their product markets. Retail price adjustments are subject to a quadratic cost, as in ?.

The production function of retail good of type \( z \) is given by
\[ Y_t(z) = M_t(z), \tag{9} \]
where \( Y_t(z) \) denotes the output of the retail good and \( M_t(z) \) the intermediate input.

The final good for consumption and investment (denoted by \( Y^f_t \)) is a Dixit-Stiglitz composite of all retail products given by

\[
Y^f_t = \left[ \int_0^1 Y_t(z)^{(\epsilon-1)/\epsilon} \, dz \right]^{\epsilon/(\epsilon-1)},
\]

where \( \epsilon > 1 \) denotes the elasticity of substitution between retail goods.

The optimizing decisions of the final good producer lead to a downward-sloping demand schedule for each retail product \( z \):

\[
Y^d_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\epsilon} Y^f_t,
\]

where \( P_t(z) \) denotes the price of retail product \( z \).

The zero-profit condition for the final good producer implies that the price level \( P_t \) is related to retail prices by

\[
P_t = \left[ \int_0^1 P_t(z)^{(1-\epsilon)/\epsilon} \, dz \right]^{1/(1-\epsilon)}.
\]

Each retailer takes as given the demand schedule (11) and the price level \( P_t \), and sets a price \( P_t(z) \) to maximize profit. Price adjustments are costly, with the cost function given by

\[
\frac{\Omega_p}{2} \left( \frac{P_t(z)}{\pi P_{t-1}(z)} - 1 \right)^2 C_t,
\]

where \( \Omega_p \) measures the size of the adjustment cost and \( \pi \) is the steady-state inflation rate. Retailer \( z \) chooses \( P_t(z) \) to maximize its expected discounted profit

\[
E_t \sum_{i=0}^{\infty} \beta^i \Lambda_{t+i} \left[ \left( \frac{P_{t+i}(z)}{P_{t+i}} - p_{w,t+i} \right) Y^d_{t+i}(z) - \frac{\Omega_p}{2} \left( \frac{P_{t+i}(z)}{\pi P_{t+i-1}(z)} - 1 \right)^2 C_{t+i} \right],
\]

where \( p_{w,t} \) is the relative price of the wholesale good (expressed in consumption units) and \( Y^d_{t+i}(z) \) is given by the demand schedule (11).

We focus on a symmetric equilibrium in which \( P_t(z) = P_t \) for all \( z \). The optimal price-setting decision implies that

\[
p_{w,t} = \frac{\epsilon - 1}{\epsilon} + \frac{\Omega_p}{\epsilon} \frac{1}{Y_t} \left[ \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} C_t - \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} C_{t+1} \right].
\]

I.3. The wholesale goods sector. The wholesale goods used by retailers as inputs are a composite of intermediate goods produced by firms in the SOE sector and the POE sector. Denote by \( Y_{st} \) and \( Y_{pt} \) the products produced by SOE firms and POE firms, respectively. The quantity of the wholesale good \( M_t \) is given by

\[
M_t = \left( \phi Y_{st}^\sigma_{m-1} + (1 - \phi) Y_{pt}^\sigma_{m-1} \right)^{\sigma_{m-1}/\sigma_m},
\]
where \( \phi \in (0, 1) \) measures the share of SOE goods and \( \sigma_m > 0 \) is the elasticity of substitution between goods produced by the two sectors.

Denote by \( p_{st} \) and \( p_{pt} \) the relative price of SOE products and POE products, respectively, both expressed in final consumption good units. Cost-minimizing by the wholesale good producer implies that

\[
Y_{st} = \phi^\sigma_m \left( \frac{p_{st}}{p_{wt}} \right)^{-\sigma_m} M_t, \quad Y_{pt} = (1 - \phi)^\sigma_m \left( \frac{p_{pt}}{p_{wt}} \right)^{-\sigma_m} M_t. \tag{16}
\]

The zero-profit condition in the wholesale sector implies that the wholesale price is related to the sectoral prices through

\[
p_{wt} = \left( \phi^\sigma_m p_{st}^{1-\sigma_m} + (1 - \phi)^\sigma_m p_{pt}^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}}. \tag{17}
\]

I.4. The intermediate goods sectors. We now present the environment in the SOE and POE intermediate goods sectors. We focus on a representative firm in each sector \( j \in \{s, p\} \).

A firm in sector \( j \) produces a homogeneous intermediate good \( Y_{jt} \) using capital \( K_{jt} \) and two types of labor inputs— household labor \( H_{jt} \) and entrepreneurial labor \( H_{jt}^e \), with the production function

\[
Y_{jt} = A_{jt} \omega_{jt} (K_{jt})^{1-\alpha} \left[ (H_{jt}^e)^{1-\theta} H_{jt}^{\theta} \right]^{\alpha}, \tag{18}
\]

where \( A_{jt} \) denotes productivity of firms in sector \( j \), and the parameters \( \alpha \in (0, 1) \) and \( \theta \in (0, 1) \) are input elasticities in the production technology. The term \( \omega_{jt} \) is an idiosyncratic productivity shock that is i.i.d. across firms and time, and is drawn from the distribution \( F(\cdot) \) with a nonnegative support. We assume that the idiosyncratic productivity shocks are drawn from a Pareto distribution with the cumulative density function \( F(\omega) = 1 - \left( \frac{\omega}{\omega_m} \right)^k \) over the range \([\omega_m, \infty)\), where \( \omega_m > 0 \) is the scale parameter and \( k \) is the shape parameter.

Productivity \( A_{jt} \) contains a common deterministic trend \( g^t \) and a sector-specific stationary component \( A_{jt}^m \) so that \( A_{jt} = g^t A_{jt}^m \). The stationary component \( A_{jt}^m \) follows the stochastic process

\[
\ln A_{jt}^m = (1 - \rho_j) \ln \bar{A}_j + \rho_j \ln A_{jt-1}^m + \epsilon_{jt}, \tag{19}
\]

where \( \bar{A}_j \) is the steady-state level of \( A_{jt}^m \), \( \rho_j \in (-1, 1) \) is a persistence parameter, and the term \( \epsilon_{jt} \) is an i.i.d. innovation drawn from a log-normal distribution \( N(0, \sigma_j) \).

Firms face working capital constraints. In particular, they need to pay wage bills and capital rents before production takes place. Firms finance their working capital payments through their beginning-of-period net worth \( N_{jt, t-1} \) and through borrowing, \( B_{jt} \). The working capital constraint for a firm in sector \( j \in \{s, p\} \) is given by

\[
\frac{N_{jt, t-1} + B_{jt}}{p_t} = w_t H_{jt} + w_{jt}^e H_{jt}^e + r_t^k K_{jt}. \tag{20}
\]

where \( w_{jt}^e \) denotes the real wage rate of managerial labor in sector \( j \).
Given the working capital constraints in Eq. (20), cost-minimization implies that factor demand satisfies

\[ w_t H_{jt} = \alpha \theta \frac{N_{j,t-1} + B_{jt}}{P_t}, \]

\[ w^e_t H^e_{jt} = \alpha (1 - \theta) \frac{N_{j,t-1} + B_{jt}}{P_t}, \]

\[ r_t K_{jt} = (1 - \alpha) \frac{N_{j,t-1} + B_{jt}}{P_t}. \]

(21)  
(22)  
(23)

Substituting these optimal choices of input factors in the production function (18), we obtain the firm’s revenue (in final good units)

\[ p_{jt} Y_{jt} = \tilde{A}_{jt} \omega_{jt} \frac{N_{j,t-1} + B_{jt}}{P_t}, \]

(24)

where the term \( \tilde{A}_{jt} \) is given by

\[ \tilde{A}_{jt} = p_{jt} A_{jt} \left( \frac{1 - \alpha}{\tau_t} \right)^{1-\alpha} \left[ \left( \frac{\alpha (1 - \theta)}{w^e_{jt}} \right)^{1-\theta} \left( \frac{\alpha \theta}{w_t} \right)^{\theta} \right]^\alpha. \]

(25)

We interpret \( \tilde{A}_{jt} \) as the rate of return on the firm’s investment financed by external debt and internal funds.

I.5. Financial intermediaries and debt contracts. Financial intermediation takes place through a continuum of competitive representative commercial banks, which we model in terms of single representative bank. At the beginning of each period \( t \), the bank obtains household deposits \( D_t \) at interest rate \( R_t \). It lends \( B_{st} \) on-balance sheet to the SOE sector, and \( B_{pt} \) off-balance sheet to the private sector. On-balance-sheet (SOE) loans are subject to reserve requirements, but off-balance-sheet (POE) loans are not. In addition, SOE loans are guaranteed by the government and the bank does not face default risk on these loans. In contrast, the interest rate charged to POEs contains a credit spread that reflects the bank’s expected losses due to default, as in the BGG framework.

Since the government guarantees repayments of SOE loans, there is no default risk on bank loans and the bank charges a risk-free loan rate of \( R_{st} \). The bank earns zero profits on SOE loans in equilibrium. However, the reserve requirements drive a wedge between the loan rate and the deposit rate such that

\[ (R_{st} - 1)(1 - \tau_t) = (R_t - 1), \]

(26)

where \( R_{st} \) represents the interest rate on SOE loans.

The bank is also competitive in off-balance sheet lending, with funding costs given by \( R_{pt} = R_t \) as this activity is not subject to reserve requirements.

Since the lender can only observe a borrower’s realized returns at a cost, it charges a state-contingent gross interest rate \( Z_{jt} (j = s, p) \) on all loans to cover monitoring and liquidation
costs. Under this financial arrangement, firms with sufficiently low levels of realized productivity will not be able to make repayments. There is therefore a cut-off level of productivity \( \bar{\omega}_{jt} \) such that firms with \( \omega_{jt} < \bar{\omega}_{jt} \) choose to default, where \( \bar{\omega}_{jt} \) satisfies

\[
\bar{\omega}_{jt} \equiv \frac{Z_{jt}B_{jt}}{A_{jt}(N_{j,t-1} + B_{jt})}.
\]  

(27)

If the firm defaults, the lender pays a liquidation cost and obtains the revenue. In the process of liquidating, a fraction \( m_j \) of output is lost. Furthermore, the government is assumed to cover a fraction \( l_j \) of the loan losses financed by lump-sum taxes collected from the households, where \( l_s = 1 \) and \( l_p = 0 \) such that the government covers the entire loss to banks for SOE defaults but nothing for POE defaults.

We now describe the optimal contract. Under the loan contract characterized by \( \bar{\omega}_{jt} \) and \( B_{jt} \), the expected nominal income for a firm in sector \( j \) is given by

\[
\int_{\bar{\omega}_{jt}}^{\infty} \tilde{A}_{jt} \omega_{jt}(N_{j,t-1} + B_{jt}) dF(\omega) - (1 - F(\bar{\omega}_{jt}))Z_{jt}B_{jt}
\]

\[
= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})[\int_{\bar{\omega}_{jt}}^{\infty} \omega dF(\omega) - (1 - F(\bar{\omega}_{jt}))\bar{\omega}_{jt}]
\]

\[
= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\bar{\omega}_{jt}),
\]  

(28)

where \( f(\bar{\omega}_{jt}) \) is the share of production revenue going to the firm under the loan contract.

The expected nominal income for the lender is given by,

\[
(1 - F(\bar{\omega}_{jt}))Z_{jt}B_{jt} + \int_{0}^{\bar{\omega}_{jt}} \{(1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})
\]

\[
+ l_j[Z_{jt}B_{jt} - (1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})]\} dF(\omega)
\]

\[
= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})\{[1 - (1 - l_j)F(\bar{\omega}_{jt})]\bar{\omega}_{jt} + (1 - m_j)(1 - l_j)\int_{0}^{\bar{\omega}_{jt}} \omega dF(\omega)\}
\]

\[
= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_j(\bar{\omega}_{jt}),
\]  

(29)

where \( g_j(\bar{\omega}_{jt}) \) is the share of production revenue going to the lender. Note that

\[
f(\bar{\omega}_{jt}) + g_j(\bar{\omega}_{jt}) = 1 - m_j \int_{0}^{\bar{\omega}_{jt}} \omega dF(\omega) + l_j \int_{0}^{\bar{\omega}_{jt}} [\bar{\omega}_{jt} - (1 - m_j)\omega] dF(\omega).
\]  

(29)

The optimal contract is a pair \( (\bar{\omega}_{jt}, B_{jt}) \) chosen at the beginning of period \( t \) to maximize the borrower’s expected period \( t \) income,

\[
\max \tilde{A}_{jt}(N_{j,t-1} + B_{jt})f(\bar{\omega}_{jt})
\]  

subject to the lender’s participation constraint

\[
\tilde{A}_{jt}(N_{j,t-1} + B_{jt})g_j(\bar{\omega}_{jt}) \geq R_{jt}B_{jt}.
\]  

(31)
The optimizing conditions for the contract characterize the relation between the leverage ratio and the productivity cut-off

$$\frac{N_{jt-1}}{B_{jt} + N_{jt-1}} = -\frac{g_j(\varpi_{jt})}{f'(\varpi_{jt})} \frac{\tilde{A}_{jt}f(\varpi_{jt})}{R_{jt}}. \quad (32)$$

Following Bernanke et al. (1999), we assume that a manager in sector $j \in \{s,p\}$ survives at the end of each period with probability $\xi_j$, so that the average lifespan for the firm is $\frac{1}{\xi_j}$. The $1-\xi_j$ fraction of exiting managers is assumed to be replaced by an equal mass of new managers, so that the population size of managers stays constant. New managers have start-up funds equal to their managerial labor income $w_{jt}^e H_{jt}^e$. For simplicity, we follow the literature and assume that each manager supplies one unit of labor inelastically and the managerial labor is sector specific (so that $H_{jt}^e = 1$ for $j \in \{s,p\}$).

The end-of-period aggregate net worth of all firms in sector $j$ consists of profits earned by surviving firms plus managerial income

$$N_{jt} = \xi_j \tilde{A}_{jt}(N_{jt-1} + B_{jt})f(\varpi_{jt}) + P_t w_{jt}^e H_{jt}^e. \quad (33)$$

I.6. Government policy. The government conducts monetary policy by following a Taylor-type rule, under which the nominal deposit rate responds to deviations of inflation from target and changes in the output gap. The government’s interest-rate rule is given by

$$R_t = \bar{R} \left( \frac{\bar{\pi}_t}{\bar{\pi}} \right)^{\psi_{rp}} \left( \frac{GDP_t}{\bar{GDP}} \right)^{\psi_{ry}}, \quad (34)$$

where $\bar{R}$ and $\bar{\pi}$ denote the steady-state interest rate and inflation rate, respectively, and the parameters $\psi_{rp}$ and $\psi_{ry}$ are the response coefficients. The term $GDP_t$ denotes the output gap, defined as the deviation of real GDP from its trend.

In the benchmark economy, we assume that the government fixes the required reserve ratio at $\tau_t = \bar{\tau}$.

Government spending consists of an autonomous component, $G_t$, which is a constant fraction of real GDP and the SOE bailout costs.

I.7. Market clearing and equilibrium. The final good is used for consumption, investment, government spending, paying price adjustment costs, and covering bankruptcy costs. Final-good market clearing implies that

$$Y_t = C_t + I_t + G_t + \frac{\Omega_p}{2} (\bar{\pi}_t - \bar{\pi})^2 C_t + \tilde{A}_{mt} N_{st-1} + B_{st} \frac{N_{st-1}}{P_t} m_s \int_0^{\bar{\omega}_{st}} \omega dF(\omega)$$

$$+ \tilde{A}_{pt} N_{pt-1} + B_{pt} \frac{N_{pt-1}}{P_t} m_p \int_0^{\bar{\omega}_{pt}} \omega dF(\omega). \quad (35)$$
Intermediate goods market clearing implies that
\[ M_t = \left( \phi Y_{st}^{\sigma_m-1} + (1 - \phi) Y_{pt}^{\sigma_m-1} \right)^{\sigma_m-1}. \] (36)

Capital market clearing implies that
\[ K_{t-1} = K_{st} + K_{pt}. \] (37)

Labor market clearing implies that
\[ H_t = H_{st} + H_{pt}. \] (38)

Bond market clearing implies that
\[ B_{st} + B_{pt} = (1 - \tau_t) D_t. \] (39)

For convenience of discussion, we define real GDP as the final output net of the costs of firm bankruptcies and price adjustments. In particular, real GDP is defined as
\[ GDP_t = C_t + I_t + G_t. \] (40)

We also define two measures of aggregate TFP, one based on gross output and the other based on value added (i.e., GDP). Output-based TFP is defined as
\[ A_{Y,t} = \frac{Y_t^{\gamma_f}}{(K_{st} + K_{pt})^{1-\alpha} H_t^{\alpha \theta}}, \] (41)
while value-added based TFP is defined as
\[ A_{GDP,t} = \frac{GDP_t}{(K_{st} + K_{pt})^{1-\alpha} H_t^{\alpha \theta}}. \] (42)

II. SUMMARY OF EQUILIBRIUM CONDITIONS

On a balanced growth path, output, consumption, investment, real bank loans and real wage rates all grow at a constant rate \( g \). To obtain balanced growth, we make the stationary transformations
\[
\begin{align*}
    y_t^f &= \frac{Y_t^{\gamma_f}}{g_t}, m_t = \frac{M_t}{g_t}, c_t = \frac{C_t}{g_t}, i_t = \frac{I_t}{g_t}, k_t = \frac{K_t}{g_t}, gd_{pt} = \frac{GDP_t}{g_t}, 
    
    y_{st} &= \frac{Y_{st}}{g_t}, y_{pt} = \frac{Y_{pt}}{g_t}, k_{st} = \frac{K_{st}}{g_t}, k_{pt} = \frac{K_{pt}}{g_t}, \bar{w}_t = \frac{w_t}{g_t}, \bar{w}_{s,e,t} = \frac{w_{s,e,t}}{g_t}, \bar{w}_{p,e,t} = \frac{w_{p,e,t}}{g_t}, 
    
    n_{st} &= \frac{N_{st}}{P_t g_t}, n_{pt} = \frac{N_{pt}}{P_t g_t}, b_{st} = \frac{B_{st}}{P_t g_t}, b_{pt} = \frac{B_{pt}}{P_t g_t}. 
\end{align*}
\]

On the balanced growth path, the transformed variables, the interest rate and the inflation rate are all constants.

The balanced growth equilibrium is summarized by the following equations:
1) Households.

\[ k_t = \frac{1-\delta}{g} k_{t-1} + i_t \left[ 1 - \frac{\Omega_k}{2} \left( \frac{i_t g}{i_{t-1}} - g \right)^2 \right], \quad (A1) \]

\[ \lambda_t = \frac{1}{c_t}, \quad (A2) \]

\[ \tilde{\omega}_t = \Psi H^0_t \tilde{f}, \quad (A3) \]

\[ 1 = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1} g}, \quad (A4) \]

\[ 1 = q^k_t \left[ 1 - \frac{\Omega_k}{2} \left( \frac{i_t g}{i_{t-1}} - g \right)^2 - \Omega_k \left( \frac{g_i t}{i_{t-1}} - g \right) \frac{i_t g}{i_{t-1}} \right] + \beta E_t q^k_{t+1} \frac{\lambda_{t+1}}{\lambda_t g} \Omega_k \left( \frac{i_t g}{i_{t-1}} - g \right) \left( \frac{i_t g}{i_{t-1}} + 1 \right) \quad (A5) \]

\[ q^k_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t g} \left[ (1 - \delta) q^k_{t+1} + r^k_{t+1} \right]. \quad (A6) \]

2) Firms and banks.

\[ y_{st} = A^m_t \tilde{A}_s \lambda_{st}^{1-\alpha} (H^\theta_{st})^\alpha, \quad (A7) \]

\[ \tilde{\omega}_t H_{st} = \alpha \theta \left( \frac{n_{s,t-1}}{\pi_t g} + b_{st} \right), \quad (A8) \]

\[ \tilde{w}_{s,c,t} = \left( \frac{n_{s,t-1}}{\pi_t g} + b_{st} \right) \alpha (1 - \theta), \quad (A9) \]

\[ k_{st} r^k_t = (1 - \alpha) \left( \frac{n_{s,t-1}}{\pi_t g} + b_{st} \right), \quad (A10) \]

\[ \tilde{A}_{st} = \frac{p_{st} y_{st}}{n_{s,t-1} + b_{st}}, \quad (A11) \]

\[ \tilde{A}_{st} \left( \frac{n_{s,t-1}}{\pi_t g} + b_{st} \right) g_s (\tilde{\omega}_{st}) = b_{st} R_{st}, \quad (A12) \]

\[ \frac{n_{s,t-1}}{\pi_t g} + b_{st} = \frac{-g_s' (\tilde{\omega}_{st}) f(\tilde{\omega}_{st}) \tilde{A}_{st}}{R_{st}}, \quad (A13) \]

\[ n_{st} = \tilde{w}_{s,c,t} + \xi_s \tilde{A}_{st} \left( \frac{n_{s,t-1}}{\pi_t g} + b_{st} \right) f(\tilde{\omega}_{st}), \quad (A14) \]

\[ y_{pt} = A^m_t \lambda_{p}^{1-\alpha} (H^\theta_{pt})^\alpha, \quad (A15) \]

\[ \tilde{w}_t H_{pt} = \alpha \theta \left( \frac{n_{p,t-1}}{\pi_t g} + b_{pt} \right), \quad (A16) \]

\[ \tilde{w}_{p,c,t} = \alpha (1 - \theta) \left( \frac{n_{p,t-1}}{\pi_t g} + b_{pt} \right), \quad (A17) \]

\[ r^k_t k_{pt} = (1 - \alpha) \left( \frac{n_{p,t-1}}{\pi_t g} + b_{pt} \right), \quad (A18) \]

\[ \tilde{A}_{pt} = \frac{p_{pt} y_{pt}}{n_{p,t-1} + b_{pt}}, \quad (A19) \]

\[ \tilde{A}_{pt} \left( \frac{n_{p,t-1}}{\pi_t g} + b_{pt} \right) g_p (\tilde{\omega}_{pt}) = b_{pt} R_{pt}, \quad (A20) \]

\[ \frac{n_{p,t-1}}{\pi_t g} + b_{pt} = \frac{-g_p' (\tilde{\omega}_{pt}) f(\tilde{\omega}_{pt}) \tilde{A}_{pt}}{R_{pt}}, \quad (A21) \]
\begin{equation}
\tilde{w}_{p,e,t} + \xi_p \tilde{A}_{pt} \left( \frac{n_{p,t-1}}{\pi_{tg}} + b_{pt} \right) f(\bar{\omega}_{pt}), \quad (A22)
\end{equation}
\begin{equation}
(R_{st} - 1)(1 - \tau_t) = R_t - 1, \quad (A23)
\end{equation}
\begin{equation}
R_{pt} = R_t, \quad (A24)
\end{equation}

3) Pricing, market clearing and monetary policy.

\begin{equation}
p_{wt} = \frac{c - 1}{c} + \frac{\Omega_p}{c} \frac{1}{y_t} [\left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} c_t - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} c_{t+1}], \quad (A25)
\end{equation}

\begin{equation}
\ln \left( \frac{R_{st}}{R_t} \right) = \psi_{ry} \ln \left( \frac{gdp_{st}}{gdp_t} \right) + \psi_{rp} \ln \left( \frac{\pi_{st}}{\pi_t} \right), \quad (A26)
\end{equation}

\begin{equation}
\ln \left( \frac{\tau_{st}}{\tau_t} \right) = \psi_{ty} \ln \left( \frac{gdp_{st}}{gdp_t} \right) + \psi_{tp} \ln \left( \frac{\pi_{st}}{\pi_t} \right), \quad (A27)
\end{equation}

\begin{equation}
g_t = gdp_t f^c, \quad (A28)
\end{equation}

\begin{equation}
y_f = i_t + c_t + g_t + c_t \frac{\Omega_p}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 + \tilde{A}_{st} \left( \frac{n_{s,t-1}}{\pi_{tg}} + b_{st} \right) m_s \int_0^{\bar{\omega}_{st}} \omega dF(\omega)
\end{equation}

\begin{equation}
+ \tilde{A}_{pt} \left( \frac{n_{p,t-1}}{\pi_{tg}} + b_{pt} \right) m_p \int_0^{\bar{\omega}_{pt}} \omega dF(\omega), \quad (A29)
\end{equation}

\begin{equation}
y_f = m_t, \quad (A30)
\end{equation}

\begin{equation}
m_t = \left( \phi y_{st}^{\sigma_m^{-1}} + (1 - \phi) y_{pt}^{\sigma_m^{-1}} \right)^{\frac{\sigma_m}{\sigma_m - 1}}, \quad (A31)
\end{equation}

\begin{equation}
y_{st} = \phi^{\sigma_m} \left( \frac{p_{st}}{p_{wt}} \right)^{-\sigma_m} y_t, \quad (A32)
\end{equation}

\begin{equation}
y_{pt} = (1 - \phi)^{\sigma_m} \left( \frac{p_{pt}}{p_{wt}} \right)^{-\sigma_m} y_t, \quad (A33)
\end{equation}

\begin{equation}
k_{t-1} = k_{st} + k_{pt}, \quad (A34)
\end{equation}

\begin{equation}
H_t = H_{st} + H_{pt}, \quad (A35)
\end{equation}

\begin{equation}
gdp_t = g_t + i_t + c_t. \quad (A36)
\end{equation}

where

\begin{equation}
f(\bar{\omega}_{st}) = \frac{1}{k-1} \omega_m^k \bar{\omega}_{st}^{1-k}, \quad (A37)
\end{equation}

\begin{equation}
f'(\bar{\omega}_{st}) = -\omega_m^k \bar{\omega}_{st}^{-k}, \quad (A38)
\end{equation}

\begin{equation}
g_s(\bar{\omega}_{st}) = \omega_m^{-k \frac{k}{K-1}} (1 - l_s) (1 - m_s) + l_s \bar{\omega}_{st} + (1 - l_s) \left[ 1 - \frac{(1 - m_s) k}{k-1} \right] \omega_m^k \bar{\omega}_{st}^{1-k}, \quad (A39)
\end{equation}

\begin{equation}
g'_s(\bar{\omega}_{st}) = l_s + (1 - l_s) (1 - m_s k) \omega_m^k \bar{\omega}_{st}^{-k}, \quad (A40)
\end{equation}
\[ f(\bar{\omega}_{pt}) = \frac{1}{k-1} \omega_m^{\frac{k}{k-1}} \bar{\omega}_{pt}^{1-k}, \quad (A41) \]
\[ f'(\bar{\omega}_{pt}) = -\omega_m^{\frac{k}{k-1}} \bar{\omega}_{pt}^{-k}, \quad (A42) \]
\[ g_p(\bar{\omega}_{pt}) = \omega_m^{\frac{k}{k-1}} (1 - l_p)(1 - m_p) + l_p \bar{\omega}_{pt} + (1 - l_p)[1 - \frac{(1-m_p)k}{k-1}]\omega_m^{\frac{k}{k-1}} \bar{\omega}_{pt}^{1-k}, \quad (A43) \]
\[ g'_p(\bar{\omega}_{pt}) = l_p + (1 - l_p)(1 - m_p)k \omega_m^{\frac{k}{k-1}} \bar{\omega}_{pt}^{-k}, \quad (A44) \]
\[ \int_{0}^{\omega_{st}} \omega dF(\omega) = \frac{k}{k-1} (\omega_m - \omega_m^{\frac{k}{k-1}} \bar{\omega}_{st}^{1-k}), \quad (A45) \]
\[ \int_{0}^{\omega_{pt}} \omega dF(\omega) = \frac{k}{k-1} (\omega_m - \omega_m^{\frac{k}{k-1}} \bar{\omega}_{pt}^{1-k}). \quad (A46) \]

The system of 36 equations from (A1) to (A36) determine the equilibrium solution for the 36 endogenous variables summarized in the vector,
\[ \begin{bmatrix} y^f_t, m_t, c_t, i_t, g_t, gd_p_t, k_t, \lambda_t, q^k_t, H_t, H_{st}, H_{pt,} \\
y_{st}, y_{pt}, k_{st}, k_{pt}, n_{st}, n_{pt}, b_{st}, b_{pt}, A_{st}, A_{pt}, \bar{\omega}_{st}, \bar{\omega}_{pt,} \\
\bar{\omega}_t, \bar{\omega}_{s,e,t}, \bar{\omega}_{p,e,t}, r^k_t, R_t, R_{st}, R_{pt}, \pi_t, p_{wt}, p_{st}, p_{pt}, \tau \end{bmatrix} \]
REFERENCES

