Approximating Multisector New Keynesian Models — Extended Version*

Carlos Carvalho  
Central Bank of Brazil  
PUC-Rio

Fernanda Nechio  
Federal Reserve Bank of San Francisco

June, 2017

Abstract

We show that a calibrated three-sector model with a suitably chosen distribution of price stickiness can closely approximate the dynamic properties of New Keynesian models with a much larger number of sectors. The parameters of the approximate three-sector distribution are such that both the approximate and the original distributions share the same (i) average frequency of price changes, (ii) cross-sectional average of durations of price spells, (iii) cross-sectional standard deviation of durations of price spells, (iv) cross-sectional skewness of durations of price spells, and (v) cross-sectional kurtosis of durations of price spells. These results provide the tools for a growing literature that tries to estimate empirically-relevant multisector models with much reduced computational costs.

JEL classification codes: E22, J6, E12

Keywords: multisector approximation, heterogeneity, factor specificity

*In this supplement to our main paper we provide several extensions of the model and results. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of San Francisco, the Federal Reserve System or the Central Bank of Brazil. E-mails: cvianac@econ.puc-rio.br and fernanda.nechio@sf.frb.org.
1 Introduction

The macroeconomics literature has provided substantial evidence of significant heterogeneity in the average frequencies with which firms in different sectors of the economy change their prices, both in the U.S. and in other countries (e.g., Blinder et al., 1998, Bils and Klenow, 2004, Nakamura and Steinsson, 2008, and Klenow and Malin, 2011 for the U.S. economy; Dhyne et al., 2006, and references cited therein for the euro area).

The literature has also shown that accounting for this heterogeneity matters for the dynamic properties of New Keynesian economies (e.g., Carvalho, 2006, Nakamura and Steinsson, 2010, Carvalho and Nechio, 2011 and Pasten, Schoenle and Weber, 2016). Models that allow for heterogeneity in frequency of price adjustments yield substantially different dynamics for aggregate variables than models that assume firms adjusting with a common (average) frequency. In response to monetary shocks, for example, heterogeneous multisector models feature much larger and more persistent real effects than one-sector economies with similar average degrees of nominal rigidities. These different dynamics obtained in multisector economies are quantitatively important and have been shown to better match the empirical properties of aggregate data. Heterogeneity in price stickiness also has implications for optimal monetary policy (e.g., Aoki, 2001 and Eusepi, Hobijn and Tambalotti, 2011).

Working with large-scale multisector models, however, is computationally costly, as models with many sectors frequently result in a sizable state-space. As a consequence, while one can find several calibrated multisector models in the literature, the set of papers that entertain estimating such large-scale models is much more limited (exceptions include, for example, Carvalho and Lee, 2011 and Bouakez, Cardia and Ruge-Murcia, 2009).

In this paper we show how a large-dimension multisector economy can be approximated by a three-sector model that, aside from the number of sectors, features the same model ingredients as the original economy. In particular, we show that for a suitably chosen distribution of price stickiness, aggregate variables in the three-sector model feature similar dynamics than the corresponding variables in the original model. A comparison of the impulse response functions of the main aggregate variables in response to monetary and technology shocks show that the three-sector economy follows extremely closely the dynamic properties of the original (larger-scale) model.

The empirical evidence shows that the distribution of price stickiness is highly asymmetric, and with a much longer right tail. Therefore, there is a larger mass of sectors adjusting more frequently and a smaller, but positive, mass adjusting very infrequently. These features are well summarized by five moments of the distribution, namely; (i) average frequency of price
changes, (ii) cross-sectional average of durations of price spells, (iii) cross-sectional standard deviation of durations of price spells, (iv) the cross-sectional skewness of durations of price spells, and (v) cross-sectional kurtosis of durations of price spells.

Our approximation approach entails constructing an artificial small-scale distribution of price rigidity that matches the same shape features of the original one. More specifically, given these five moments of the original distribution of price stickiness, we build a three-sector distribution by choosing sectoral weights and price adjustment frequencies such that both the approximate and the original distributions share the same aforementioned five moments.¹

We apply our approximation strategy to the U.S. distribution of price stickiness derived from Nakamura and Steinsson (2008), as well as to artificially-generated arbitrary distributions of price rigidity. We test the performance of our approximation using the three dynamic stochastic general equilibrium models of heterogeneity presented in Carvalho and Nechio (2016). All three models feature multisector economies where heterogeneity in frequency of price adjustment is the only source of heterogeneity. The models, however, differ in their assumptions about the mobility of the production inputs across sectors and firms within each sector. One model assumes that capital and labor are free to move across firms and sectors, i.e., an economy-wide multisector model. The other model assumes, instead, that capital and labor can freely move across firms, but cannot move across sectors, and hence, factors are sector-specific. The third model assumes that capital and labor are specific at the firm level. Carvalho and Nechio (2016) show that the dynamics of these three economies can substantially differ depending on the assumptions for the distribution of price stickiness.

The results show that, for all variations of DSGE economies considered, for either the U.S. distribution or the arbitrary ones, our approximation approach performs extremely well. In particular, three-sector economies parameterized with distributions of price stickiness that match the five aforementioned moments yield impulse response functions to monetary and technology shocks that are markedly close to the ones obtained in their corresponding original economies.

The rest of the paper proceeds as follows. Section 2 describes our approximation approach and data. Section 3 presents the DSGE models used to compare variables dynamics under the approximated and the original distributions of price stickiness. In Section 4 we present results using the approximation based on the U.S. empirical distribution of price stickiness and in Section 5 we show results using arbitrary distributions of price stickiness. Section 6 revisits the analysis while considering alternative shock specifications. The last section concludes.

¹ We have 5 degrees of freedom (2 weights and 3 frequencies of price change) to match 5 moments from the original distribution of price stickiness. Hence, we match it with a three-sector distribution.
2 A three-sector approximation

The vast literature that highlights the properties of firms’ price adjustments agrees on some key features of the distribution of price changes. The literature points to a sizable heterogeneity in frequency of price adjustment across sectors of the economy, with some of those sectors adjusting prices very frequently, while some others taking much longer. It also shows that the distribution of price adjustments is highly asymmetric and right-skewed, with a right tail that is much longer than the left one (see, for example Figures 1 of both Bils and Klenow, 2004 and Nakamura and Steinsson, 2010). Therefore, the shape of the distribution is such that the sectors that adjust more often account for a larger share of overall consumer expenditures, but there are some (lower-weighted) sectors that adjust very infrequently.

These features of the distributions of price stickiness are important in determining the dynamics of aggregate variables in New Keynesian models (e.g., Carvalho, 2006, Nakamura and Steinsson, 2010, Carvalho and Nechio, 2011). Hence, in order to construct an artificial distribution that mimics the properties of the original distribution of price stickiness, one should account for higher order moments of the latter. This would allow one to also capture the skewness and the long tail of the original distribution.

To obtain an artificial distribution that accounts for the above properties, we first calculate the following moments of the original distribution of price stickiness:

(i) Average frequency of price changes:
\[ \bar{\alpha} = \sum_{s=1}^{S} f_s \alpha_s, \]

(ii) Cross-sectional average of the expected durations of price spells:
\[ \bar{d} = \sum_{s=1}^{S} f_s \alpha_s^{-1}, \]

(iii) Cross-sectional standard deviation of the expected durations of price spells:
\[ \sigma_d = \sqrt{\sum_{s=1}^{S} f_s (\alpha_s^{-1} - \bar{d})^2}, \]

(iv) Skewness of the cross-sectional distribution of expected durations of price spells:
\[ S_d = \frac{1}{\sigma_d^3} \sum_{s=1}^{S} f_s (\alpha_s^{-1} - \bar{d})^3, \]

(v) Kurtosis of the cross-sectional distribution of expected durations of price spells:
\[ K_d = \frac{1}{\sigma_d^4} \sum_{s=1}^{S} f_s (\alpha_s^{-1} - \bar{d})^4, \]

where, for each sector \( s \in \{1, \ldots, S\} \), \( f_s \) stands for sector-s expenditure share and \( \alpha_s \) stands for sector-s frequency of price adjustment.

For any given price distribution, we can construct a three-sector approximate distribution that matches the above five moments of the original one. More specifically, we consider an
approximation in which we choose two sectoral weights and three frequencies of price changes for the artificial three-sector distribution such that both the approximate and the original distributions share the same the five moments above.²

In what follows, we construct approximate distributions based on the U.S. empirical distribution and on artificially-generated arbitrary distributions of price stickiness. In particular, in Section 4 we rely on the U.S. empirical evidence and build a three-sector approximate price stickiness distribution that matches the empirical moments of the price distribution reported in Nakamura and Steinsson (2008). In Section 5 we construct arbitrary (large-scale) distributions and obtain their corresponding approximate three-sector distributions. To assess our approximation based on this moment-matching approach, we use the dynamic stochastic general equilibrium (DSGE) multisector models of Carvalho and Nechio (2016), which we summarize in Section 3, and compare the dynamics of aggregate variables in the original (large-scale) multisector economy to the dynamics of the corresponding variables in the three-sector economies which feature the approximate distribution of price stickiness.

3 Three New Keynesian models of factor specificity

Carvalho and Nechio (2016) present three versions of multisector DSGE models. In all three models, identical infinitely-lived consumers supply labor and capital to intermediate firms that they own, invest in a complete set of state-contingent financial claims, and consume a final good. The latter is produced by competitive firms that bundle varieties of intermediate goods. The monopolistically competitive intermediate firms that produce these varieties are divided into sectors that only differ in their frequency of price changes. Labor and capital are the only inputs in the production of intermediate goods.

The models differ, however, on their assumptions about the mobility of factor inputs across sectors. In their benchmark model, they assume that capital and labor can be reallocated freely across firms in the same sector but cannot move across sectors, i.e., factors are sector-specific. The authors also entertain versions of their model in which factors are firm-specific, i.e., cannot move across sectors nor firms within sectors; and factors are economy-wide and can move freely across firms and sectors.

In the following, we provide some key ingredients of all three versions of Carvalho and Nechio (2016) models, and refer the reader to the paper for detailed descriptions.

² That is, we choose a set of \( f_1, f_2, \alpha_1, \alpha_2 \) and \( \alpha_3 \) such that the resulting moments of this three-sector distribution match the moments of the original one. Since we have a three-sector distribution, once \( f_1 \) and \( f_2 \) are set, by construction, \( f_3 = 1 - (f_1 + f_2) \).
3.1 Sector-specific factors

The representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{s=1}^{S} \omega_s N_{s,t}^{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_tC_t + P_tI_t + E_t [\Theta_{t,t+1} B_{t+1}] \leq \sum_{s=1}^{S} W_{s,t} N_{s,t} + B_t + T_t + \sum_{s=1}^{S} Z_{s,t} K_{s,t},$$

the law of motion for the stocks of sector-specific capital:

$$K_{s,t+1} = (1-\delta) K_{s,t} + \Phi (I_{s,t}, K_{s,t}) I_{s,t}, \forall s$$

$$\Phi (I_{s,t}, K_{s,t}) = \Phi \left( \frac{I_{s,t}}{K_{s,t}} \right) = 1 - \frac{1}{2} \kappa \left( \frac{I_{s,t}}{K_{s,t}} - \frac{\delta}{K_{s,t}} \right)^2,$$

$$I_{s,t} \geq 0, \forall s$$

and a standard “no-Ponzi” condition. \(E_t\) denotes the time-\(t\) expectations operator, \(C_t\) is consumption of the final good, \(N_{s,t}\) denotes total labor supplied to firms in sector \(s\), \(W_{s,t}\) is the associated nominal wage rate, and \(\omega_s\) is the relative disutility of supplying labor to sector \(s\). \(I_{s,t}\) denotes investment in sector-\(s\) capital, \(I_t \equiv \sum_{s=1}^{S} I_{s,t}\) is capital supplied to firms in sector \(s\), and \(Z_{s,t}\) is the associated nominal return on capital.

The final good can be used for either investment or consumption, and sells at the nominal price \(P_t\). \(B_{t+1}\) accounts for the state-contingent value of the portfolio of financial securities held by the consumer at the beginning of \(t+1\). Under complete financial markets, agents can choose the value of \(B_{t+1}\) for each possible state of the world at all times, subject to the no-Ponzi condition and the budget constraint. \(T_t\) stands for profits received from intermediate firms. The absence of arbitrage implies the existence of a nominal stochastic discount factor \(\Theta_{t,t+1}\) that prices in period \(t\) any financial asset portfolio with state-contingent payoff \(B_{t+1}\) at the beginning of period \(t+1\).\(^3\) \(\beta\) is the time-discount factor, \(\sigma^{-1}\) denotes the intertemporal elasticity of substitution, \(\gamma^{-1}\) is the Frisch elasticity of labor supply, \(\delta\) is the rate of depreciation. Finally, the adjustment-cost function, \(\Phi (\cdot)\), is specified as in Chari, Kehoe and McGrattan (2000), where \(\Phi\) is convex and satisfies \(\Phi (\delta) = 1, \Phi' (\delta) = 0,\) and \(\Phi'' (\delta) = -\frac{\kappa}{\delta}\).

A representative competitive firm produces the final good, which is a composite of varieties

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\(^3\) To avoid cluttering the notation, we omit explicit reference to the different states of nature.
of intermediate goods. Monopolistically competitive firms produce such varieties. These firms are divided into sectors indexed by \( s \in \{1, ..., S\} \), each featuring a continuum of firms. Sectors differ in the degree of price rigidity, as we detail below. Overall, firms are indexed by their sector \( s \), and are further indexed by \( j \in [0, 1] \). The distribution of firms across sectors is given by sectoral weights \( f_s > 0 \), with \( \sum_{s=1}^{S} f_s = 1 \).

The final good is used for both consumption and investment by combining the intermediate varieties. More specifically, the representative final-good-producing firm solves:

\[
\text{max } P_t Y_t - \sum_{s=1}^{S} f_s \int_0^1 P_{s,j,t} Y_{s,j,t} \, dj
\]

s.t.

\[
Y_t = \left( \sum_{s=1}^{S} f_s Y_{s,t} \right)^{\eta \over \eta - 1} \nabla_{Y_t}
\]

\[
Y_{s,t} = \left( f_s \int_0^1 P_{s,j,t} Y_{s,j,t} \, dj \right)^{\theta \over \theta - 1},
\]

where \( Y_t \) is the final good, \( Y_{s,t} \) is the aggregation of sector-\( s \) intermediate goods, and \( Y_{s,j,t} \) is the variety produced by firm \( j \) in sector \( s \). The parameters \( \eta \geq 0 \), and \( \theta > 1 \) are, respectively, the elasticity of substitution across sectors and the elasticity of substitution within sectors. \( P_t \) is the price of the final good, \( P_{s,t} \) is the price index of sector-\( s \) intermediate goods, and \( P_{s,j,t} \) is the price charged by firm \( j \) from sector \( s \).

Monopolistically competitive firms produce varieties of the intermediate good by employing capital and labor. For analytical tractability, we assume that intermediate firms set prices as in Calvo (1983). The frequency of price changes varies across sectors and it is the only source of (ex-ante) heterogeneity.

In each period, each firm \( j \) in sector \( s \) changes its price independently with probability \( \alpha_s \), and when it does, it chooses \( X_{s,j,t} \) to solve:

\[
\text{max } E_t \sum_{l=0}^{\infty} \Theta_{t,t+l} (1 - \alpha_s)^l \left[ X_{s,j,t} Y_{s,j,t+l} - W_{s,t+l} N_{s,j,t+l} - Z_{s,t+l} K_{s,j,t+l} \right]
\]

s.t.

\[
Y_{s,j,t} = \left( \frac{P_{s,j,t}}{P_{s,t}} \right) ^{-\theta} \left( \frac{P_{s,t}}{P_t} \right) ^{-\eta} Y_t
\]

\[
Y_{s,j,t} = A_t \left( K_{s,j,t} \right) ^{1-\chi} \left( N_{s,j,t} \right) ^{\chi},
\]

\[
\log A_t = \rho_A \log A_{t-1} + \sigma \varepsilon_{a,t} \]

where \( \chi \) is the elasticity of output with respect to labor, \( A_t \) is a technology shock, \( \rho_A \in [0, 1) \) is the shock persistence, and \( \varepsilon_{a,t} \) is a zero-mean, unit-variance \( i.i.d. \) shock.

For brevity, we refrain from showing all equilibrium conditions and refer the reader to
Carvalho and Nechio (2016) for details. We note that as a direct implication of the assumption of sector specificity at capital and labor markets, returns on both inputs, and hence, firms’ real marginal costs are equalized at the sectoral level.

### 3.2 Firm-specific and economy-wide factors

To consider the firm-specific and the economy-wide multisector economy, one needs to reformulate the consumers’ and intermediate firms’ problems. In the firm-specific case, in equilibrium, returns to capital and labor are specific at the firm level. In contrast, in the economy-wise case, capital and labor returns are equalized across firms and sectors. Therefore, differently from the model of Section 3.1, in the firm-specific model, firms’ real marginal costs are equalized at the firm level, while in the economy-wide model, firms’ real marginal costs are equalized across firms and sectors. For additional details about all three models, see Carvalho and Nechio (2016).

### 3.3 Monetary policy

To close the model, as in the benchmark version of the models presented in Carvalho and Nechio (2016), we assume that the growth rate of nominal aggregate demand follows a first-order autoregressive, $AR(1)$, process, thus, leaving monetary policy implicit.\(^4\) Denoting nominal aggregate demand by $M_t \equiv P_t Y_t$:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \sigma_{m,t} \epsilon_{m,t},$$

where $m_t \equiv \log(M_t)$, $\rho_m$ determines the autocorrelation in nominal aggregate demand growth, and $\epsilon_{m,t}$ is a purely monetary, uncorrelated, zero-mean, unit-variance $i.i.d.$ shock. In Section 6 we consider an alternative specification for monetary policy.

All model versions are solved and analyzed using a loglinear approximation around a zero-inflation steady state.

### 3.4 Parameterization

Aside from the distribution of price stickiness, which we detail in the next two sections, we parameterize the three models as in Carvalho and Nechio (2016) and, unless otherwise stated, keep those parameters unchanged in the different variants of the models entertained below.

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\(^4\) This is a common assumption in the literature (e.g. Mankiw and Reis, 2002).
In particular, we set the intertemporal elasticity of substitution $\sigma^{-1}$ to 1/2, the (inverse) labor supply elasticity $\gamma$ to 0.5, the elasticity of output with respect to labor to $\chi = 2/3$, and the consumer discount factor $\beta$ such that it implies a time-discount rate of 4% per year.

We set the elasticity of substitution between varieties of the same sector to $\theta = 7$. Relying on the estimates provided by Hobijn and Nechio (2017), we set the elasticity of substitution across sectors to unity, $\eta = 1$ (i.e. the aggregator that converts sectoral into final output is Cobb-Douglas).

To specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of $\rho_m$ typically fall in the range of 0.4 to 0.7, which maps into a range of roughly 0.75 – 0.90 at a monthly frequency.\(^5\) We set $\rho_m = 0.8$, and the standard deviation of the shocks $\sigma_{\varepsilon_m} = 0.6\%$ (roughly 1% at a quarterly frequency), in line with the same estimation results.\(^6\) For technology shocks, we set $\rho_a = 0.965$ and the standard deviation of the shocks $\sigma_{\varepsilon_m} = 0.6\%$ (roughly 1% at a quarterly frequency).

Finally, we calibrate investment adjustment costs, $\kappa$, such that each model roughly matches the standard deviation of investment relative to the standard deviation of GDP in the data. For that, we consider monetary and technology shocks one at a time. When analyzing the response to monetary shocks, we set $\kappa$ to 40, while when analyzing the response to technology shocks, we set $\kappa$ to 15. The remaining parameter values are unchanged from the baseline parameterization when considering one or the other shock.

To be able to assess our approximation approach, it remains to specify the original distributions of price stickiness to be approximated. In what follows, we construct approximate distributions based on the U.S. empirical distribution in Sections 4, and based on artificially-generated arbitrary distributions of price stickiness in Section 5.

### 4 Approximating the U.S. empirical distribution

In this section we resort to the available microeconomic evidence on price rigidity in the U.S. to pin down the distribution of price rigidity across sectors.

In particular, we use the statistics on the frequency of regular price changes — those that are not due to sales or product substitutions — reported by Nakamura and Steinsson (2008). More specifically, the U.S. distribution of price stickiness we consider consists of weights and

\(^5\) See, for instance, Mankiw and Reis (2002).

\(^6\) All results for volatilities scale-up proportionately with $\sigma_{\varepsilon_m}$. 
frequencies of price changes for 67 expenditure classes of goods and services. In the model, each class is identified with a sector.

Following the approach described in Section 2, we construct a three-sector approximate distribution. The original and the artificial distributions are such that the average frequency of price changes equals 0.2 (which implies prices changing, on average, once every 4.7 months), the cross-sectional average of durations of price spells equals 13.6 months, the cross-sectional standard deviation of durations of price spells equals 11.7 months, the cross-sectional skewness of durations of price spells equals 1.3 months, and the cross-sectional kurtosis of durations of price spells equals 5.2 months.

Figure 1 shows the impulse response functions (IRFs) to a monetary shock for the three models; economy-wide, sector-, and firm-specific factor markets. For brevity, the figure only reports the IRFs for real GDP and inflation. The approximation for other variables are provided in the Appendix Figures A1 to A3.

The panels show that for all three models, the three-sector model calibrated with the approximate distribution does a great job in replicating the impulse response functions of output and inflation (and similarly for the other aggregate variables in the model). The largest approximation error is attained at the firm-specific model and equals 0.0006.

Figure 2 shows the impulse response functions of GDP and inflation to a technology shock for the same three models. Results are qualitatively similar. For all model versions, the three-sector model calibrated with the approximate distribution tracks very closely the dynamics of the impulse response functions of output and inflation in the original 67-sector models. The approximation for other variables are provided in the Appendix Figures A4 to A6.

5 Approximating arbitrary distributions

The results above are based on an approximate three-sector distribution obtained from the U.S. empirical evidence. In this section we consider, instead, departing from artificially-generated

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7 We discard the category “Girls’ Outerwear,” for which the reported frequency of regular price changes is zero. We renormalize the expenditure weights to sum to unity. The frequency of price changes for each expenditure class is obtained as the weighted average of the frequencies for the underlying categories, using the expenditure weights provided by Nakamura and Steinsson (2008). Expenditure-class weights are given by the sum of the expenditure weights for those categories. As an example of what this aggregation entails, the resulting “New and Used Motor Vehicles” class consists of the categories “Subcompact Cars”, “New Motorcycles”, “Used Cars”, “Vehicle Leasing” and “Automobile Rental”; the “Fresh Fruits” class comprises four categories: “Apples”, “Bananas”, “Oranges, Mandarins etc.” and “Other Fresh Fruits.”

8 Note that we aggregate the original 271 categories from Nakamura and Steinsson (2008) up into their corresponding 67 expenditure classes of goods and services. Approximating directly from the 271-sector distribution from Nakamura and Steinsson (2008) yields similar results, as reported in the Appendix Figures A7 and A8.
Figure 1: Impulse response functions in the 67-sector and the approximated 3-sector economies to a monetary shock.
Figure 2: Impulse response functions in the 67-sector and the approximated 3-sector economies to a technology shock.
distributions of price stickiness and comparing the impulse response functions obtained when the models are parameterized with these distributions to those obtained when the models are parameterized with their corresponding three-sector approximate distributions.

Turning to the details, we construct arbitrary 67-sector distributions in which both the frequency of price adjustment and the sectoral weight are arbitrarily chosen. In particular, we start by constructing a vector of frequencies of price changes that ranges from 0.999 to 0.008, declining in equal steps. Therefore, in this artificial 67-sector distribution, price changes range from, on average, once a month, in the most flexible sector, to once every 120 months, in the stickiest sector. Next, we associate a set of arbitrary sectoral weights to this vector of frequencies. More specifically, we generate sectoral weights from a $Beta$ distribution in which both shape parameters range from 0.5 to 10, in incremental steps of 0.25. This combination of shape parameters yields a set of 1521 arbitrary vectors of sectoral weights. Finally, each vector of weights is paired with the arbitrary vector of frequencies of price changes. This pairing yields 1521 sets of 67-sector price stickiness distributions.\footnote{Note that all distributions have the same vector of price stickiness but vary in the set of sectoral weights.}

We, then, turn to the approximation for each of these arbitrary distributions. For that, we follow the same steps as in Section 4. More specifically, for each arbitrary (67-sector) distribution, we calculate the five empirical moments described in Section 2 and obtain its corresponding approximate three-sector distribution. Next, we parameterize the models of Section 3 under the original and its approximated arbitrary distribution, and compare the responses to monetary and technology shocks under the two parameterizations.

The exercise described above yields 1521 original and approximated parameterizations of the three DSGE economies of Section 3. While we could, in principle, compare how each approximation performs for each economic variable over time (in analogy to Figures 1 and 2), we, instead, report a summary of our results. In particular, for each model, variable and approximation, we calculate the largest difference between the response of the 67-sector and the approximated three-sector models within the first five years after the shock. This yields a distribution of the largest approximation errors for each aggregate variable in the different models.\footnote{After each model converges to the equilibrium, any comparison between the original and the approximate impulse response functions yields perfect matching. Therefore, we opt to show results considering the time series up to five years after the shock hits the economy, in analogy to the results reported in Figures 1 and 2. Versions of Figures 3 and 4 that consider a longer time series yield histograms that are further concentrated near zero, and are available upon request.}

Figures 3 and 4 report the main results. They show the histograms of the approximation errors for real GDP and inflation for the three models in response to a monetary and technology shock, respectively. Both Figures show that, for all three models of factor specificity, the three-

\section*{References}

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sector approximation performs extremely well. In all cases, the largest approximation error across all arbitrary price stickiness distributions is within the order of $10^{-4}$. The Appendix Figures A9 to A14 confirm that these results also hold for the other main aggregate variables in the models.

**Figure 3:** Distribution of largest approximation errors in response to a monetary shock.

### 6 Robustness

In this section we consider an alternative specification for monetary policy as a robustness exercise. More specifically, we assume that monetary policy is conducted according to an
Figure 4: Distribution of largest approximation errors in response to a technology shock.
interest rate rule subject to persistent shocks:

\[ IR_t = \beta^{-1} \left( \frac{P_t}{P_{t-1}} \right)^{\phi_{\pi}} \left( \frac{GDP_t}{GDP^n_t} \right)^{\phi_y} \epsilon_{v,t}, \]

where \( IR_t \) is the short-term nominal interest rate, \( GDP_t \) is gross domestic product, \( GDP^n_t \) denotes gross domestic product when all prices are flexible, \( \phi_{\pi} \) and \( \phi_y \) are the parameters associated with Taylor-type interest-rate rules. \( \epsilon_{v,t} \) is a persistent shock with process \( \epsilon_{v,t} = \rho_v \epsilon_{v,t-1} + \sigma_{\epsilon_v} \epsilon_{v,t} \) where \( \epsilon_{v,t} \) is a zero-mean, unit-variance i.i.d. shock, and \( \rho_v \in [0,1) \).

To calibrate the version of the model with the above interest rate rule, we set \( \phi_{\pi} = 1.5 \), \( \phi_y = 0.5/12 \), and \( \rho_v = 0.965 \), and set the investment cost parameter \( \kappa \) to 45, which roughly implies that the model matches the standard deviation of investment in the data relative to the standard deviation of GDP. The remaining parameter values are unchanged from the baseline specification.

We, then, follow the same steps as described in Sections 4 and 5, and assess our approximation using the U.S. empirical distribution of price stickiness and the set of arbitrary distributions, respectively.

Figure 5 shows the impulse response functions of GDP and inflation to an expansionary monetary shock for the three models of Section 3. Figure 6 reports the distribution of the largest approximation errors obtained when the three models are calibrated using the arbitrary distributions described in Section 5.11 The approximation in the DSGE models assuming an interest rule is quite accurate, with the three-sector approximated economy follows closely the dynamics of the original one. Figure 6 reinforces our conclusions.

Finally, we also entertain a version of the three models in which we exclude capital accumulation, assuming a labor-only economy, and a version in which we modify the Taylor rule to allow for policy inertia. In all cases, our results are qualitatively unchanged and are available upon request.

7 Conclusion

Our results show that a calibrated model with a suitably chosen three-sector distribution of price stickiness can closely approximate the dynamic properties of New Keynesian models calibrated with a much larger distribution. In particular, the set of sectoral weights and frequencies of price adjustment in the approximated distribution are such that the latter and the original distributions share the same (i) average frequency of price changes, (ii) cross-sectional average

11 The approximation for all other variables are provided in the Appendix Figures A15 to A20.
Figure 5: Impulse response functions in the 67-sector and the approximated 3-sector economies: interest rate rule.
Figure 6: Distribution of largest approximation errors: interest rate rule.
of durations of price spells, \((iii)\) cross-sectional standard deviation of durations of price spells, \((iv)\) cross-sectional skewness of durations of price spells, and \((v)\) cross-sectional kurtosis of durations of price spells of the original distribution of price stickiness. Under those conditions, we show that, for alternative multisector models with different assumptions for factor inputs mobility, type of shocks and monetary policy specifications, the original and the approximate distributions yield impulse response functions that are markedly close. These results provide the tools for a growing literature that tries to estimate empirically relevant multisector models with much reduced computational costs.

References


Carvalho, Carlos, and Jae Won Lee. 2011. “Sectoral price facts in a sticky-price model.” Federal Reserve Bank of New York Staff Reports 495.


A Appendix

A.1 Additional evidence

Figure A1: Impulse response functions in the 67-sector and the approximated 3-sector economies – economy-wide factor markets: monetary shock.
Figure A2: Impulse response functions in the 67-sector and the approximated 3-sector economies – sector-specific factor markets: monetary shock.
Figure A3: Impulse response functions in the 67-sector and the approximated 3-sector economies – firm-specific factor markets: monetary shock.
Figure A4: Impulse response functions in the 67-sector and the approximated 3-sector economies – economy-wide factor markets: technology shock.
Figure A5: Impulse response functions in the 67-sector and the approximated 3-sector economies – sector-specific factor markets: technology shock.
Figure A6: Impulse response functions in the 67-sector and the approximated 3-sector economies – firm-specific factor markets: technology shock.
Figure A7: Impulse response functions in the 271-sector and the approximated 3-sector economies: monetary shock.
Figure A8: Impulse response functions in the 271-sector and the approximated 3-sector economies: technology shock.
Figure A9: Distribution of approximation errors – economy-wide factor markets: monetary shock.
Figure A10: Distribution of approximation errors – sector-specific factor markets: monetary shock.
Figure A11: Distribution of approximation errors – firm-specific factor markets: monetary shock.
Figure A12: Distribution of approximation errors – economy-wide factor markets: technology shock.
Figure A13: Distribution of approximation errors – sector-specific factor markets: technology shock.
Figure A14: Distribution of approximation errors – firm-specific factor markets: technology shock.
Figure A15: Impulse response functions in the 67-sector and the approximated 3-sector economies – economy-wide factor markets: interest rate rule.
Figure A16: Impulse response functions in the 67-sector and the approximated 3-sector economies – sector-specific factor markets: interest rate rule.
Figure A17: Impulse response functions in the 67-sector and the approximated 3-sector economies – firm-specific factor markets: interest rate rule.
Figure A18: Distribution of approximation errors – economy-wide factor markets: interest rate rule.
Figure A19: Distribution of approximation errors – sector-specific factor markets: interest rate rule.
Figure A20: Distribution of approximation errors – firm-specific factor markets: interest rate rule.