

FEDERAL RESERVE BANK OF SAN FRANCISCO

WORKING PAPER SERIES

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March 2019

Working Paper 2019-08

<https://www.frbsf.org/economic-research/publications/working-papers/2019/08/>

Suggested citation:

Christensen, Jens H. E., Eric Fischer, Patrick Shultz. 2019. “Bond Flows and Liquidity: Do Foreigners Matter?” Federal Reserve Bank of San Francisco Working Paper 2019-08. <https://doi.org/10.24148/wp2019-08>

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Bond Flows and Liquidity: Do Foreigners Matter?

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Abstract

In their search for yield in the current low interest rate environment, many investors have turned to sovereign debt in emerging economies, which has raised concerns about risks to financial stability from these capital flows. To assess this risk, we study the effects of changes in the foreign-held share of Mexican sovereign bonds on their liquidity premiums. We find that recent increases in foreign holdings of these securities have played a significant role in driving up their liquidity premiums. Provided the higher compensation for bearing liquidity risk is commensurate with the chance of a major foreign-led sell-off in the Mexican government bond market, this development may not pose a material risk to its financial stability.

JEL Classification: E43, E44, F36, G12

Keywords: term structure modeling, liquidity risk, financial market frictions, emerging markets, financial stability, foreign holdings

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This version: March 1, 2019.

1 Introduction

In their search for yield in the current low interest rate environment, many investors have turned to sovereign debt in emerging markets. In light of the history of sovereign credit crises in emerging markets, especially in Latin America, significant debt purchases from investors outside the border could pose risks to financial stability. This is particularly true with monetary policy normalization on the horizon in many advanced economies, which could provide an impetus for foreign investments to move elsewhere. Hence, it seems warranted to study the role of foreigners in these markets.

In general, foreign participation in local currency sovereign bond markets can bring benefits as well as costs, both of which may vary notably across time and complicate assessments of tradeoffs regarding advancing the foreign share. As for potential benefits, foreign investors help develop the domestic fixed-income markets through improvements to technology, trading strategies, and related derivative markets and by diversifying the set of financial market participants, all of which could improve market liquidity. As for potential downside risks, sudden stops in capital inflows are a major concern for emerging market economies integrated into world capital markets. Calvo et al. (2004) define such events as large and unexpected drops in net capital inflows that exceed two standard deviations below prevailing sample means. If aggregate bond spreads are elevated at the same time, they refer to them as *systemic sudden stops*. As described in Calvo et al. (2004), many emerging market countries have experienced sudden stops caused by a drying-up of capital flows from spikes in global risk aversion or rises in global interest rates. Specifically, they find that, for emerging market economies, systemic sudden stops tend to coincide with large, real currency depreciations (exceeding 20 percent) and output collapses averaging about 10 percent from peak to trough. Furthermore, systemic sudden stops in net capital flows by global investors are associated with greater slowdowns in economic activity and higher currency depreciations and are therefore more concerning than sudden flight events triggered by local investors, which tend to merely cause temporary spikes in gross capital outflows with much less negative impact on the domestic economy, as demonstrated by Rothenberg and Warnock (2011). Thus, while systemic sudden stops may be rare, they are a risk that merits careful monitoring by both policymakers and investors alike. In addition, large foreign inflows may lead to domestic asset price and credit bubbles that would further expose local financial markets and the economy to the risk of a sudden reversal in capital flows. Finally, even absent sudden stops, increased foreign participation can make both the local bond markets and the domestic economy more sensitive to shifts in global financial market sentiment.

While research has explored the ties between debt flows and bond prices (see Mitchell et al. 2007 and Beltran et al. 2013 for examples), the connection between debt flows and market functioning and its potential implications for financial stability have received less attention.¹

¹One notable example is Christensen and Gillan (2018), who document that U.S. Treasury Inflation-

To assess the potential financial stability implications of the increased foreign participation in the sovereign debt markets in emerging economies, this paper analyzes the influence of foreign investors on the liquidity premiums of domestic government bond securities in a leading emerging market economy, namely Mexico.

Our focus on Mexican government bonds is motivated by several observations. First, little is known about the magnitudes of liquidity premiums in regular sovereign bond markets. Second, given that Mexico has one of the largest and most important sovereign bond markets among emerging market economies, our analysis can serve as a benchmark for understanding liquidity premiums and financial market frictions in other emerging economies. Finally, the Bank of Mexico maintains a comprehensive database of foreign and domestic holdings for all Mexican government securities that is instrumental to our analysis in order to establish a connection between foreign holdings and bond risk premiums. In short, Mexico offers an ideal setting for studying the question we are interested in.

To estimate the liquidity premiums of Mexican government bonds, we rely on recent research by Andreasen et al. (2018, henceforth ACR), who show how a standard term structure model can be augmented with a liquidity risk factor to accurately measure bond liquidity premiums. Their approach identifies the liquidity risk factor from its unique loading, which mimics the idea that, over time, an increasing fraction of the outstanding notional amount of a given security tends to get locked up in buy-and-hold investors' portfolios. This raises its sensitivity to variation in the market-wide liquidity captured by the liquidity risk factor. By observing a cross section of securities over time, the liquidity risk factor can be separately identified and distinguished from the fundamental risk factors in the model. We model the fundamental frictionless Mexican yields that would prevail in a world without any frictions to trading using a standard Gaussian model, namely the arbitrage-free Nelson-Siegel (AFNS) model introduced in Christensen et al. (2011), which we augment with a liquidity risk factor structured as in ACR. We estimate both AFNS models and liquidity-augmented extensions thereof, denoted AFNS-L models, using price information for individual Mexican bonds.²

Our results can be summarized as follows. First, we find that the liquidity-augmented model improves model fit and delivers robust estimates of the risk factors that drive the variation in the frictionless part of the Mexican government bond yield curve. Second, our results show that liquidity premiums in the Mexican government bond market are of considerable size with an average of 0.57 percent and a standard deviation of 0.19 percent. For comparison, the liquidity premium advantage of newly issued ten-year U.S. Treasuries over comparable seasoned securities has averaged less than 0.15 percent the past two decades, see Christensen et al. (2017). Hence, liquidity risk is an important component in the pricing of Mexican

Protected Securities (TIPS) purchases by the Federal Reserve during its second large-scale asset purchase program lowered the priced frictions in the markets for TIPS and inflation swaps.

²Only a limited number of papers have estimated dynamic term structure models using Mexican government bond yields; Espada and Ramos-Francia (2008b) and Espada et al. (2008) are examples.

government bonds. Furthermore, we see significant variation around a general upward trend during our sample period. The empirical question we are interested in is to what extent this variation can be explained by changes in foreign investor holdings of Mexican government securities. After running regressions with a number of relevant controlling variables, we find a roughly one-to-one relationship in that a one percentage point increase in the foreign-held share of Mexican government bonds raises their liquidity premium by almost one basis point. Given that the foreign market share has increased more than 40 percentage points between 2010 and 2017, our results suggest that the large increase in foreign holdings during this period could have raised the estimated bond liquidity premiums by as much as 0.35 percent.

What are the financial stability implications from these observations? Provided this increase in the compensation demanded by investors for assuming the liquidity risk of Mexican government bonds matches the risk and expected size of any future market sell-off driven by foreigners leaving the Mexican market, there may not be any major threats to the financial stability of the Mexican government bond market at this point. However, if foreigners turn out to represent a less stable and dedicated source of funding than domestic investors, there does appear to be some risk that the Mexican government could face potentially severe funding challenges down the road if foreigners were to decide to move their money elsewhere for whatever reason. Thus, these developments merit careful monitoring. Furthermore, this tilt in the ownership composition of Mexican government bonds could have implications for the monetary policy of the Bank of Mexico as it may be forced to put greater emphasis on foreign developments, which could matter for both macroeconomic outcomes and investors in these markets.

In the current environment characterized by low global interest rates (see Holston et al. 2017) and an ongoing gradual normalization of U.S. monetary policy, a potential trigger for a sudden-stop type of event could be deleveraging by international banks in response to contractionary U.S. monetary policy shocks as described in Bruno and Shin (2015). Such spillover effects are also known as the “risk-taking” transmission channel of monetary policy first highlighted by Borio and Zhu (2012). However, in light of the overall deleveraging of international banks since the global financial crisis, such effects may not be bank-led in the future, but rather materialize directly in the markets for debt securities via asset managers and other “buy side” investors as argued by Shin (2013). He therefore encourages careful examination of the bond yields of emerging market debt securities, and our analysis can be viewed as an attempt to meet that objective.

Specific to the concerns about the normalization of U.S. monetary policy, Iacoviello and Navarro (2018) document that GDP in emerging economies tend to drop in response to U.S. monetary policy tightening, particularly when economic and financial vulnerability is high, see also Ammer et al. (2016). As for the international capital flows that are at the heart of sudden stops, Avdjiev et al. (2017) provide evidence of time variation in the drivers of global

liquidity and suggest that they are likely to have changed since the global financial crisis.

More broadly, a number of papers have studied the drivers of foreign participation in local currency sovereign bonds in emerging market economies and their effects on these assets. For example, Burger and Warnock (2006, 2007) examine bond markets in over 40 countries and find that greater foreign participation in local currency debt markets is explained by countries that have stable inflation rates, strong creditor rights, and greater macroeconomic stability. Other papers attribute some of this greater foreign participation in local currency bond markets since the financial crisis to low U.S. Treasury yields (Miyajima et al. 2015). To examine the effects of this increase in foreign participation, Peiris (2010) uses pre-crisis data to show that foreign investors diversify the investor base and increase liquidity of local currency bond markets. Ebeke and Lu (2015) use post-crisis data for 13 emerging market countries to show that increases in the foreign-held share of local currency sovereign bonds tend to be associated with declines in general yield levels but increases in yield volatility. Xiao (2015) and Zhou et al. (2014) analyze mutual fund portfolio flows in Mexico and find that foreign investors are more responsive to global shocks than local investors. The current paper complements this literature by looking directly at the connection between foreign holdings and sovereign bond liquidity premiums.

The remainder of the paper is organized as follows. Section 2 details our theoretical framework and describes how bond yields can be decomposed into expectations and risk premium components in a world without frictions. The section also describes how the general framework is augmented to account for liquidity risk. Section 3 details our data on bond prices and holdings, while Section 4 describes the model estimation and results. Section 5 analyzes the estimated Mexican government bond liquidity premium and evaluates its connections to foreign holdings of Mexican government bonds. Finally, Section 6 concludes. The appendix contains robustness checks of our regressions.

2 Decomposing Yields with Affine Models

In this section, we first describe how bond yields can be decomposed into the underlying short-rate expectations component and a residual term premium in a world without any frictions to trading. We then describe the wedge between the theoretical frictionless yields and the observed Mexican government bond yields caused by imperfect bond market liquidity. Finally, we augment the frictionless model to adjust the Mexican government bond yields for the liquidity bias.

2.1 Decomposing Yields in a Frictionless World

In this section, we briefly describe how bond prices are modeled in a world without frictions and how such models can be used to decompose yields into a component that reflects investors'

policy expectations and a residual term premium component.

For simplicity, we focus on decomposing $P_t(\tau)$, the price of a zero-coupon bond at time t that has a single payoff, namely one unit of currency, at maturity $t + \tau$. Under standard assumptions (see Cochrane 2001 and the references therein), this price is given by

$$P_t(\tau) = E_t^{\mathbb{P}}\left(\frac{M_{t+\tau}}{M_t}\right),$$

where the stochastic discount factor, M_t , denotes the value at time t_0 of a claim at a future date t , and the superscript \mathbb{P} refers to the actual, or real-world, probability measure underlying the dynamics of M_t .

We follow the usual reduced-form empirical finance approach that models bond prices with unobservable (or latent) factors, here denoted as X_t , and the assumption of no residual arbitrage opportunities.³ We assume that X_t follows an affine Gaussian process with constant volatility, with dynamics in continuous time given by the solution to the following stochastic differential equation (SDE):

$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t)dt + \Sigma dW_t^{\mathbb{P}},$$

where $K^{\mathbb{P}}$ is an $n \times n$ mean-reversion matrix, $\theta^{\mathbb{P}}$ is an $n \times 1$ vector of mean levels, Σ is an $n \times n$ volatility matrix, and $W_t^{\mathbb{P}}$ is an n -dimensional Brownian motion. The dynamics of the stochastic discount function are given by

$$dM_t = r_t M_t dt + \Gamma_t' M_t dW_t^{\mathbb{P}},$$

and the instantaneous risk-free rate, r_t , is assumed affine in the state variables

$$r_t = \delta_0 + \delta_1' X_t,$$

where $\delta_0 \in R$ and $\delta_1 \in R^n$. The risk premiums, Γ_t , are also affine

$$\Gamma_t = \gamma_0 + \gamma_1 X_t,$$

where $\gamma_0 \in R^n$ and $\gamma_1 \in R^{n \times n}$.

Duffie and Kan (1996) show that these assumptions imply that zero-coupon yields are also affine in X_t :

$$y_t(\tau) = -\frac{1}{\tau}A(\tau) - \frac{1}{\tau}B(\tau)'X_t,$$

where $A(\tau)$ and $B(\tau)$ are given as solutions to the following system of ordinary differential

³Ultimately, of course, the behavior of the stochastic discount factor is determined by the preferences of the agents in the economy, as in, for example, Rudebusch and Swanson (2011).

equations

$$\begin{aligned}\frac{dB(\tau)}{d\tau} &= -\delta_1 - (K^{\mathbb{P}} + \Sigma\gamma_1)'B(\tau), \quad B(0) = 0, \\ \frac{dA(\tau)}{d\tau} &= -\delta_0 + B(\tau)'(K^{\mathbb{P}}\theta^{\mathbb{P}} - \Sigma\gamma_0) + \frac{1}{2} \sum_{j=1}^n [\Sigma' B(\tau) B(\tau)' \Sigma]_{j,j}, \quad A(0) = 0.\end{aligned}$$

Thus, the $A(\tau)$ and $B(\tau)$ functions are calculated *as if* the dynamics of the state variables had a constant drift term equal to $K^{\mathbb{P}}\theta^{\mathbb{P}} - \Sigma\gamma_0$ instead of the actual $K^{\mathbb{P}}\theta^{\mathbb{P}}$ and a mean-reversion matrix equal to $K^{\mathbb{P}} + \Sigma\gamma_1$ as opposed to the actual $K^{\mathbb{P}}$.⁴ The difference is determined by the risk premium Γ_t and reflects investors' aversion to the risks embodied in X_t .

Finally, we define the term premium as

$$TP_t(\tau) = y_t(\tau) - \frac{1}{\tau} \int_t^{t+\tau} E_t^{\mathbb{P}}(r_s) ds. \quad (1)$$

That is, the term premium is the difference in expected return between a buy and hold strategy for a τ -year bond and an instantaneous rollover strategy at the risk-free rate r_t .

2.2 A Frictionless Arbitrage-Free Model

To capture the fundamental factors operating the frictionless yield curve described above, we choose to focus on the tractable affine dynamic term structure model introduced in Christensen et al. (2011).⁵ Although the model is not formulated using the canonical form of affine term structure models introduced by Dai and Singleton (2000), it can be viewed as a restricted version of the canonical Gaussian model.⁶

In this arbitrage-free Nelson-Siegel (AFNS) model, the state vector is denoted by $X_t = (L_t, S_t, C_t)$, where L_t is a level factor, S_t is a slope factor, and C_t is a curvature factor. The instantaneous risk-free rate is defined as

$$r_t = L_t + S_t. \quad (2)$$

The risk-neutral (or \mathbb{Q} -) dynamics of the state variables are given by the stochastic differential

⁴The probability measure with these alternative dynamics is frequently referred to as the risk-neutral, or \mathbb{Q} , probability measure since the expected return on any asset under this measure is equal to the risk-free rate r_t that a risk-neutral investor would demand.

⁵To motivate this choice, we note that Espada et al. (2008) show that the first three principal components in their sample of Mexican government bond yields have a level, slope, and curvature pattern in the style of Nelson and Siegel (1987) and account for more than 99 percent of the yield variation.

⁶See Christensen et al. (2011) for details on the derivation of these restrictions.

equations⁷

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & -\lambda \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \end{pmatrix}, \quad (3)$$

where Σ is the constant covariance (or volatility) matrix.⁸ Based on this specification of the \mathbb{Q} -dynamics, zero-coupon bond yields preserve the Nelson and Siegel (1987) factor loading structure as

$$y_t(\tau) = L_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) S_t + \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) C_t - \frac{A(\tau)}{\tau}, \quad (4)$$

where the yield-adjustment term is given by

$$\begin{aligned} \frac{A(\tau)}{\tau} &= \frac{\sigma_{11}^2}{6} \tau^2 + \sigma_{22}^2 \left[\frac{1}{2\lambda^2} - \frac{1}{\lambda^3} \frac{1 - e^{-\lambda\tau}}{\tau} + \frac{1}{4\lambda^3} \frac{1 - e^{-2\lambda\tau}}{\tau} \right] \\ &+ \sigma_{33}^2 \left[\frac{1}{2\lambda^2} + \frac{1}{\lambda^2} e^{-\lambda\tau} - \frac{1}{4\lambda} \tau e^{-2\lambda\tau} - \frac{3}{4\lambda^2} e^{-2\lambda\tau} + \frac{5}{8\lambda^3} \frac{1 - e^{-2\lambda\tau}}{\tau} - \frac{2}{\lambda^3} \frac{1 - e^{-\lambda\tau}}{\tau} \right]. \end{aligned}$$

To complete the description of the model and to implement it empirically, we need to specify the risk premiums that connect these factor dynamics under the \mathbb{Q} -measure to the dynamics under the real-world (or physical) \mathbb{P} -measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical \mathbb{P} -measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). In the Gaussian framework, this specification implies that the risk premiums Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^3$ and $\gamma^1 \in \mathbf{R}^{3 \times 3}$ contain unrestricted parameters.

Thus, the resulting unrestricted three-factor AFNS model has \mathbb{P} -dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{P}} \\ dW_t^{S,\mathbb{P}} \\ dW_t^{C,\mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the Kalman filter estimation.

⁷As discussed in Christensen et al. (2011), with a unit root in the level factor, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon.

⁸As per Christensen et al. (2011), Σ is a diagonal matrix, and $\theta^{\mathbb{Q}}$ is set to zero without loss of generality.

2.3 An Arbitrage-Free Model with Liquidity Risk

Equation (1) highlights that the decomposition of yields into expectations and risk premium components can be distorted if the observed yields are biased by liquidity effects. In this section, we augment the frictionless AFNS model introduced above to account for the liquidity risk of the Mexican bonos we use in the empirical analysis. By adjusting the bond prices for liquidity effects, we obtain estimates of the ideal or frictionless yields that feature in the yield decomposition in equation (1).

To begin, we first introduce the concept of a frictionless price of a security. We think of this as the price that would prevail in a world where there are no obstacles to trading, i.e., there are no bid-ask spreads, no counterparty credit risk, no delay in receiving the proceeds from trades, and it is possible to trade an arbitrary amount of the security without affecting its price. Unfortunately, this hypothetical, clean set of security prices is not observed. Instead, the prices we do observe are of course not free of frictions. They reflect bid-ask spreads, the challenges in finding buyers willing to purchase the amount on offer etc. As a consequence, the prices we observe contain a net present value discount that reflects the sum of frictions current and future holders of the security expect to incur between now and the maturity of the security. Note that, although we use the word discount and think of it as a penalty, the model framework introduced below is flexible enough to allow the discount to be positive. This happens if a security is so desirable that its market value moves above its fundamental value.

Due to the frictions to trading discussed above, the security prices we observe are sensitive to liquidity pressures among investors. As a consequence, the discounting of future cash flows from a security is not performed with the frictionless short rate described in Section 2.1, but rather with a discount function that also accounts for the liquidity risk. In support of this approach we note that recent research by Hu et al. (2013) and others suggest that liquidity is indeed a priced risk factor. Thus, we choose to represent this by a single liquidity risk factor denoted X_t^{liq} . Furthermore, since liquidity risk is security-specific in nature, the function used to discount the cash flow of a given security indexed i is assumed to be unique. Following ACR we let this discount function take the following form

$$\bar{r}_t^i = r_t + \beta^i (1 - e^{-\lambda^{L,i}(t-t_0^i)}) X_t^{liq}, \quad (5)$$

where r_t is the frictionless instantaneous risk-free rate, t_0^i denotes the date of issuance of the security, and β^i is its sensitivity to the variation in the liquidity risk factor. While we could expect the sensitivities to be identical across securities, the results from our subsequent empirical application shows that it is important to allow for the possibility that the sensitivities differ across securities. Furthermore, we allow the decay parameter $\lambda^{L,i}$ to vary across securities as well. Since β^i and $\lambda^{L,i}$ have a nonlinear relationship in the bond pricing formula, it

is possible to identify both empirically. Finally, we stress that equation (5) can be included in any dynamic term structure model to account for security-specific liquidity risks as also emphasized by ACR.

The inclusion of the issuance date t_0^i in the pricing formula is a proxy for the phenomenon that, as time passes, it is typically the case that an increasing fraction of a given security is held by buy-and-hold investors. This limits the amount of the security available for trading and drives up the liquidity premium. Rational and forward-looking investors will take this dynamic pattern into consideration when they determine what they are willing to pay for a security at any given point in time between the date of issuance and the maturity of the bond. This dynamic pattern is built into the model structure we use.

To augment the AFNS model with a liquidity risk factor to account for the liquidity risk embedded in bonos prices, let $X_t = (L_t, S_t, C_t, X_t^{liq})$ denote the state vector of this four-factor AFNS-L model. As before, (L_t, S_t, C_t) represent level, slope, and curvature factors, while X_t^{liq} is the added liquidity factor.

As in the AFNS model, we let the frictionless instantaneous risk-free rate be defined by equation (2), while the risk-neutral dynamics of the state variables used for pricing are given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \kappa_{liq}^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_{liq}^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^{liq} \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{liq,\mathbb{Q}} \end{pmatrix},$$

where Σ continues to be a diagonal matrix. This structure implies that X_t^{liq} is assumed to be an independent Ornstein-Uhlenbeck process under the pricing measure.

Based on the \mathbb{Q} -dynamics above, frictionless Mexican bonos zero-coupon yields preserve the Nelson-Siegel factor loading structure in equation (4). However, due to liquidity risk in the bonos market, bonos prices are sensitive to liquidity pressures and, as detailed above, their pricing is performed with a discount function that accounts for the liquidity risk

$$\bar{r}_t^i = r_t + \beta^i (1 - e^{-\lambda^{L,i}(t-t_0^i)}) X_t^{liq} = L_t + S_t + \beta^i (1 - e^{-\lambda^{L,i}(t-t_0^i)}) X_t^{liq}, \quad (6)$$

where t_0^i denotes the date of issuance of the bonos in question and β^i is its sensitivity to the variation in the liquidity factor with $\lambda^{L,i}$ being the associated decay parameter.

The net present value of one Mexican peso paid by bonos i at time $t + \tau^i$ has the following

exponential-affine form⁹

$$\begin{aligned} P_t^i(t_0^i, \tau^i) &= E_t^{\mathbb{Q}} \left[e^{-\int_t^{t_0^i+\tau^i} \bar{r}^i(s, t_0^i) ds} \right] \\ &= \exp \left(B_1^i(\tau^i) L_t + B_2^i(\tau^i) S_t + B_3^i(\tau^i) C_t + B_4^i(t_0^i, t, \tau^i) X_t^{liq} + A^i(t_0^i, t, \tau^i) \right). \end{aligned}$$

This implies that the model belongs to the class of Gaussian affine term structure models. Note also that, by fixing $\beta^i = 0$ for all i , we recover the AFNS model.

Now, consider the whole value of the bonos issued at time t_0^i with maturity at $t + \tau^i$ that pays a coupon C^i semi-annually. Its price is given by¹⁰

$$P_t^i(t_0^i, \tau^i, C^i) = C^i(t_1 - t) E_t^{\mathbb{Q}} \left[e^{-\int_t^{t_1} \bar{r}^i(s, t_0^i) ds} \right] + \sum_{j=2}^N \frac{C^i}{2} E_t^{\mathbb{Q}} \left[e^{-\int_t^{t_j} \bar{r}^i(s, t_0^i) ds} \right] + E_t^{\mathbb{Q}} \left[e^{-\int_t^{t_0^i+\tau^i} \bar{r}^i(s, t_0^i) ds} \right].$$

Finally, to complete the description of the AFNS-L model, we again specify an essentially affine risk premium structure, which implies that the risk premiums Γ_t take the form

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^4$ and $\gamma^1 \in \mathbf{R}^{4 \times 4}$ contain unrestricted parameters. Thus, the resulting unrestricted four-factor AFNS-L model has \mathbb{P} -dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^{liq} \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^{liq} \end{pmatrix} \right) dt + \Sigma \begin{pmatrix} dW_t^{L, \mathbb{P}} \\ dW_t^{S, \mathbb{P}} \\ dW_t^{C, \mathbb{P}} \\ dW_t^{liq, \mathbb{P}} \end{pmatrix}.$$

This is the transition equation in the extended Kalman filter estimation.

3 Mexican Government Bond Data

This section briefly describes the Mexican government bond data we use in the model estimation.

The set of individual standard Mexican fixed-coupon government bonds, known as bonos, available from Bloomberg at the time of our data pull is illustrated in Figure 1(a). Each bond is represented by a solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. These bonds are all marketable non-callable bonds denominated in Mexican pesos that pay a fixed rate of interest semi-annually. In general, the Mexican government has been issuing five-, ten-, twenty- and thirty-

⁹See Christensen and Rudebusch (2019) for the derivation of this formula.

¹⁰This is the clean price that does not account for any accrued interest and maps to our observed bond prices.

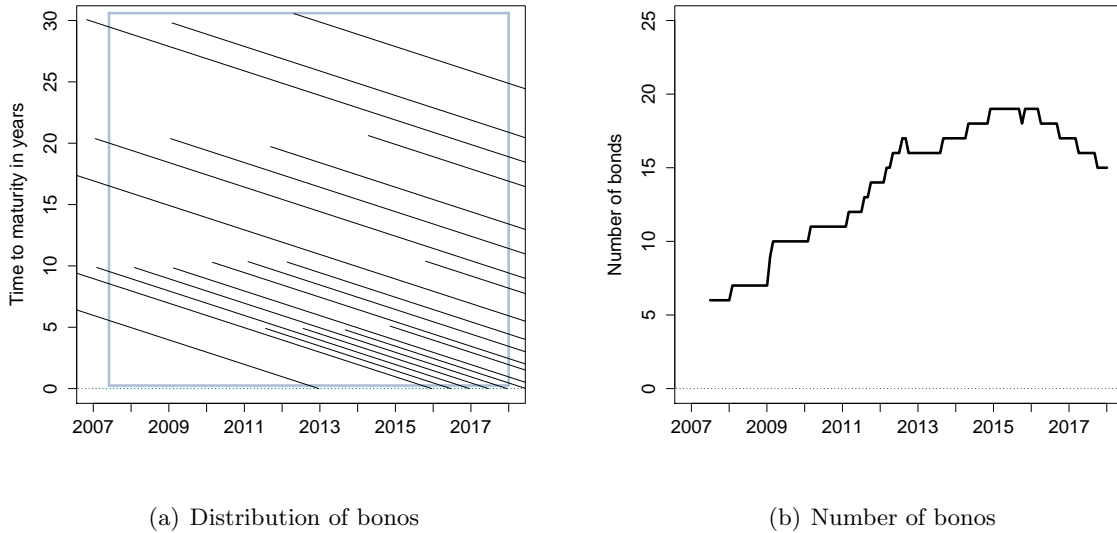


Figure 1: **Overview of the Mexican Bonos Data**

Panel (a) shows the maturity distribution of the Mexican government fixed-coupon bonos considered in the paper. The solid grey rectangle indicates the sample used in the empirical analysis, where the sample is restricted to start on June 30, 2007, and limited to bonos prices with more than three months to maturity after issuance. Panel (b) reports the number of outstanding bonos at each point in time.

year fixed-coupon bonds repeatedly during the shown period. As a consequence, there is a wide variety of bonds with different maturities and coupon rates in the data throughout the considered sample period. It is this variation that provides the foundation for the econometric identification of the factors in the yield curve models we use.

Figure 1(b) shows the distribution across time of the number of bonds included in the sample. We note a gradual increase from six bonds at the start of the sample to fifteen at its end. Combined with the cross sectional dispersion in the maturity dimension observed in Figure 1(a), this implies that we have a very well-balanced panel of Mexican bonos prices.

The contractual characteristics of all 21 bonos securities in our sample are reported in Table 1. The number of monthly observations for each bond using three-month censoring before maturity is also reported in the table. Although each bond is issued at first with a relatively small outstanding notional amount, they grow quickly thanks to a large number of subsequent reopenings that can raise their outstanding amounts to as much as 350 billion pesos, or close to 20 billion U.S. dollars. Thus, these are large bond series even by international standards.

Figure 2 shows the time series of the yields to maturity implied by the observed Mexican bonos prices downloaded from Bloomberg. We note a persistent decline from the peak of the global financial crisis in the fall of 2008 when bonos yields briefly spiked above ten percent to a trough in the spring of 2013 when long-term bonos yields were below six percent. This was followed by a sharp spike in Mexican medium- and long-term yields during the taper

Bonos (coupon, maturity)	Bonos type	No. obs.	Issuance		Number of auctions	Total notional amount
			Date	Amount		
(1) 9% 12/20/2012	10yr	63	1/9/2003	1,500	40	86,618
(2) 8% 12/7/2023	20yr	127	10/30/2003	1,000	36	93,495
(3) 8% 12/17/2015	10yr	99	1/5/2006	3,100	33	106,009
(4) 10% 11/20/2036	30yr	127	10/26/2006	2,000	37	90,065
(5) 7.5% 6/3/2027	20yr	127	1/18/2007	4,650	35	179,629
(6) 7.25% 12/15/2016	10yr	111	2/1/2007	4,800	26	134,687
(7) 7.75% 12/14/2017	10yr	116	1/31/2008	7,650	25	93,220
(8) 8.5% 5/31/2029	20yr	108	1/15/2009	2,000	28	102,349
(9) 8.5% 11/18/2038	30yr	108	1/29/2009	2,000	36	100,072
(10) 8.5% 12/13/2018	10yr	107	2/12/2009	2,500	29	149,197
(11) 8% 6/11/2020	10yr	95	2/25/2010	25,000	20	351,000
(12) 6.5% 6/10/2021	10yr	83	2/3/2011	25,000	29	329,770
(13) 6.25% 6/16/2016	5yr	56	7/22/2011	25,000	17	167,550
(14) 7.75% 5/29/2031	20yr	76	9/9/2011	60,500	22	193,000
(15) 6.5% 6/9/2022	10yr	71	2/15/2012	74,500	22	335,096
(16) 7.75% 11/13/2042	30yr	69	4/20/2012	33,000	38	228,246
(17) 5% 6/15/2017	5yr	56	7/19/2012	30,000	11	145,000
(18) 4.75% 6/14/2018	5yr	53	8/30/2013	211,000	19	339,825
(19) 7.75% 11/23/2034	20yr	45	4/11/2014	15,000	26	104,377
(20) 5% 12/11/2019	5yr	38	11/7/2014	15,000	20	242,067
(21) 5.75% 3/5/2026	10yr	27	10/16/2015	17,000	12	172,711

Table 1: **Sample of Mexican Bonos**

The table reports the bonos name, type, number of monthly observations in the sample period from June 30, 2007, to December 29, 2017, first issuance date and amount, the total number of auctions, and the total amount issued in millions of Mexican pesos for the considered set of Mexican government fixed-coupon bonos based on the information available as of February 2018.

tantrum episode in May 2013. At first, short-term yields remained low, but they started to rapidly move higher as well in the spring of 2016 as the Bank of Mexico tightened the stance of monetary policy to fight elevated levels of price inflation and strengthen the value of the Mexican peso. Thus, there has been a fair amount of variation in the general yield level in Mexico during our sample period. Furthermore, as in U.S. Treasury yield data, there is notable variation in the shape of the yield curve. At times, like early in our sample, yields across maturities are relatively compressed. At other times, the yield curve is steep, with long-term bonos trading at yields that are 3 to 4 percent above those of shorter-term securities like in 2015. It is these characteristics that underlie our choice of using a three-factor model for the frictionless part of the Mexican yield curve similar to what is standard for U.S. and U.K. data; see Christensen and Rudebusch (2012).

Finally, regarding the important question of a lower bound, the Bank of Mexico has never been forced to lower its conventional policy rate even close to zero, and the bond yields in the data have remained well above zero throughout the sample period. Thus, there is no need to

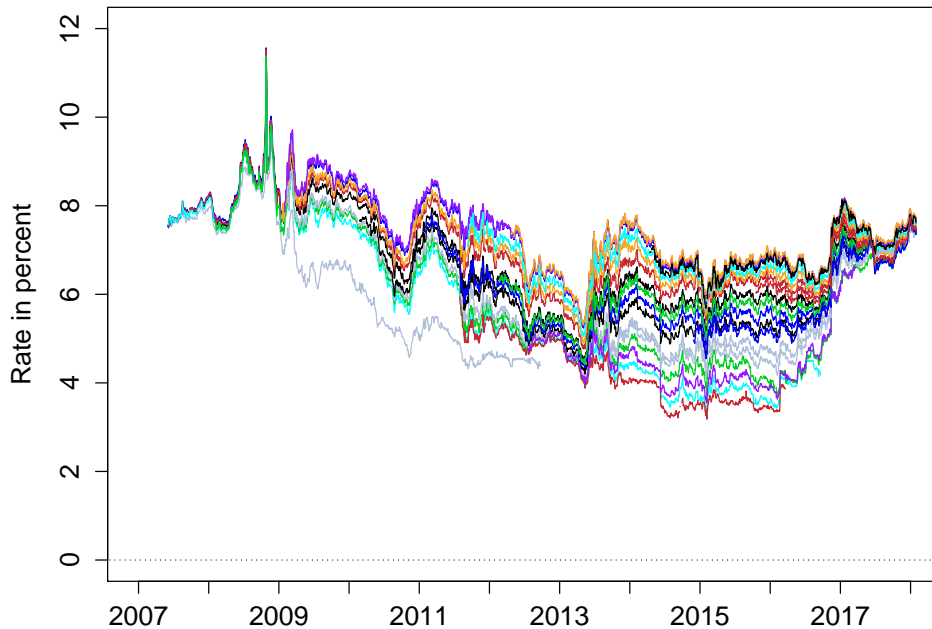


Figure 2: **Mexican Bonos Yields**

Illustration of the yields to maturity implied by the Mexican government fixed-coupon bonos prices downloaded from Bloomberg. The data are monthly covering the period from June 30, 2007, to December 29, 2017, and censor the last three months for each maturing bond.

account for any lower bounds to model these fixed-coupon bond prices, which supports our focus on standard Gaussian yield curve models.

3.1 Mexican Government Debt Holdings

In addition to the bond price data described above, our analysis utilizes data on domestic and foreign holdings of Mexican government debt securities that the Bank of Mexico requires financial intermediaries to report as a way to track market activity in the Mexican sovereign bond markets. These data have been collected since 1978 and are available at daily frequency up to the present. A key strength of the data set is that it covers any change in Mexican government debt holdings by either domestic or foreign investors. For each transaction, the reporting forms also identify the type of Mexican government security. Therefore, we are able to exploit the data reported for holdings of bonos alone and leave other Mexican government securities for future research. Although the data are available at a daily frequency, we use the observations at the end of each month to align them with our bond price data.

Figure 3 shows the monthly level of bonos holdings by domestic residents and foreigners over the period from June 2007 through December 2017. We note that foreigners overtook

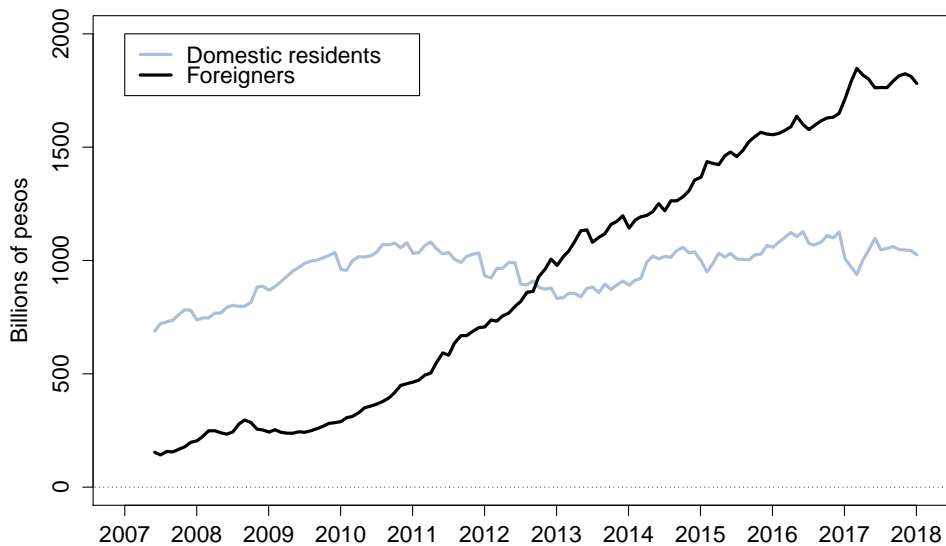


Figure 3: **Net Holdings of Mexican Bonos**

domestic residents in total holdings by late 2012 and have continued to increase their share quite notably and now exceed those of domestic residents by a wide margin. The empirical question we are interested in is whether this dramatic increase in foreign holdings of Mexican government bonds has affected the liquidity risk in this market, but before we can address that question we need to estimate our models.

4 Model Estimation and Results

In this section, we first detail the model estimation and its econometric identification before we proceed to a description of the estimation results.

4.1 Model Estimation and Econometric Identification

Due to the nonlinearity of the bond pricing formulas, the models cannot be estimated with the standard Kalman filter. Instead, we use the extended Kalman filter as in Kim and Singleton (2012), see Christensen and Rudebusch (2019) for details. To make the fitted errors comparable across bonds of various maturities, we follow ACR and scale each bond price by its duration. Thus, the measurement equation for the bond prices takes the following form:

$$\frac{P_t^i(t_0^i, \tau^i, C^i)}{D_t^i(t_0^i, \tau^i, C^i)} = \frac{\widehat{P}_t^i(t_0^i, \tau^i, C^i)}{D_t^i(t_0^i, \tau^i, C^i)} + \varepsilon_t^i,$$

where $\widehat{P}_t(t_0^i, \tau^i, C^i)$ is the model-implied price of bonos i and $D_t(t_0^i, \tau^i, C^i)$ is its duration, which is fixed and calculated before estimation. The robustness and reliability of this approach is documented in Andreasen et al. (2019). In addition, we assume that all bond price measurement equations have *i.i.d.* fitted errors with zero mean and standard deviation σ_ε .

Since the liquidity factor is a latent factor that we do not observe, its level is not identified without additional restrictions. As a consequence, when we include the liquidity factor X_t^{liq} , we let the first thirty-year bonos issued during our sample window have a unit loading on the liquidity factor, that is, bonos number (9) in our sample issued on January 29, 2009, with maturity on November 18, 2038, and a coupon rate of 8.5 percent has $\beta^9 = 1$.

Furthermore, we note that the liquidity decay parameters $\lambda^{L,i}$ can be hard to identify if their values are too large or too small. As a consequence, we impose the restriction that they fall within the range from 0.0001 to 10, which is without practical consequences based on the evidence presented in ACR. Also, for numerical stability during the model optimization, we impose the restrictions that the liquidity sensitivity parameters β^i fall within the range from 0 to 250, which turns out not to be a binding constraint at the optimum.

Finally, we assume that the state variables are stationary and therefore start the Kalman filter at the unconditional mean and covariance matrix. This assumption is supported by the analysis in Chiquiar et al. (2010), who find that Mexican inflation seems to have become stationary at some point in the early 2000s, while De Pooter et al. (2014) document that measures of long-term inflation expectations from both surveys and the Mexican government bond market have remained anchored close to the 3 percent inflation target of the Bank of Mexico at least since 2003. Assuming real rates and bond risk premiums are stationary,¹¹ this evidence would imply that Mexican government bond yields should be stationary as well, as also suggested by visual inspection of the individual yield series depicted in Figure 2.

4.2 Estimation Results

This section presents our benchmark estimation results. In the interest of simplicity, we focus on a version of the AFNS-L model where $K^{\mathbb{P}}$ and Σ are diagonal matrices. As shown in ACR, these restrictions have hardly any effects on the estimated liquidity premium for each bonos, because it is identified from the model's \mathbb{Q} -dynamics, which are independent of $K^{\mathbb{P}}$ and display only a weak link to Σ through the small convexity adjustment in yields. However, as a robustness check, we relax this assumption by considering alternative specifications of the model's \mathbb{P} -dynamics. This exercise reveals that our results are indeed robust to such changes.¹²

The impact of accounting for liquidity risk is apparent in our results. The first two columns in Table 2 show that the bonos pricing errors produced by the AFNS model indicate

¹¹We note that these might be strong assumptions. In the United States, there is evidence of a persistent downward trend in real yields the past two decades; see Christensen and Rudebusch (2019).

¹²These results are available from the authors.

Mexican bonos	Pricing errors				Estimated parameters			
	AFNS		AFNS-L		AFNS-L			
	Mean	RMSE	Mean	RMSE	β^z	SE	$\lambda^{L,z}$	SE
(1) 9% 12/20/2012 [†]	-0.26	6.39	0.34	1.33	1.9637	0.5052	10	8.8736
(2) 8% 12/7/2023 ⁺	-3.72	7.02	0.51	5.33	0.5370	0.2500	0.1035	0.1097
(3) 8% 12/17/2015 [†]	3.35	7.44	0.08	3.85	0.9164	0.2389	10	2.6748
(4) 10% 11/20/2036 [×]	4.83	6.94	0.93	4.15	0.8830	0.0607	10	2.0527
(5) 7.5% 6/3/2027 ⁺	-2.80	6.65	0.82	4.50	0.4527	0.0908	0.7032	1.3433
(6) 7.25% 12/15/2016 [†]	0.93	7.16	0.20	3.53	1.0332	0.3104	0.1757	0.1019
(7) 7.75% 12/14/2017 [†]	-1.44	9.88	0.50	5.19	0.7783	0.1822	0.3435	0.1039
(8) 8.5% 5/31/2029 ⁺	0.59	4.37	0.89	3.54	0.5525	0.0824	10	2.0254
(9) 8.5% 11/18/2038 [×]	3.89	6.83	1.09	3.53	1	n.a.	0.3595	0.2033
(10) 8.5% 12/13/2018 [†]	2.04	5.04	-0.31	4.05	2.0760	1.5795	0.0562	0.0560
(11) 8% 6/11/2020 [†]	1.46	6.21	0.29	4.09	0.6184	0.1180	10	2.0681
(12) 6.5% 6/10/2021 [†]	3.63	6.78	0.78	3.40	0.5650	0.1030	10	2.1463
(13) 6.25% 6/16/2016 [*]	0.42	8.82	-0.29	4.28	0.8591	0.2158	10	2.1143
(14) 7.75% 5/29/2031 ⁺	3.02	5.27	0.77	3.74	0.6470	0.0708	10	2.1449
(15) 6.5% 6/9/2022 [†]	3.70	5.57	0.68	3.35	0.5195	0.0895	10	2.2551
(16) 7.75% 11/13/2042 [×]	-2.31	5.55	0.44	5.03	1.1459	0.0702	10	2.2514
(17) 5% 6/15/2017 [*]	-0.96	8.62	-0.25	5.23	1.1217	0.3066	0.3350	0.1541
(18) 4.75% 6/14/2018 [*]	5.33	8.84	0.22	5.59	0.8294	0.1794	1.3849	0.4361
(19) 7.75% 11/23/2034 ⁺	-0.70	3.95	0.35	2.58	0.7843	0.0833	1.3709	1.1850
(20) 5% 12/11/2019 [*]	2.62	6.82	0.07	3.70	0.6961	0.1381	2.9699	2.0906
(21) 5.75% 3/5/2026 [†]	-2.30	6.43	0.25	3.66	1.106	3.0787	0.0932	0.3557
All yields	1.01	6.86	0.46	4.16	-	-	-	-
Max \mathcal{L}^{EKF}	9,352.62		10,036.44		-	-	-	-

Table 2: **Pricing Errors and Estimated Liquidity Risk Parameters**

This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Mexican government bonds in the AFNS and AFNS-L models estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ . The errors are computed as the difference between the bond market price expressed as yield to maturity and the corresponding model-implied yield. All errors are reported in basis points. Standard errors (SE) are not available (n.a.) for the normalized value of β^z . Asterisk * indicates five-year bonds, dagger † indicates ten-year bonds, plus + indicates twenty-year bonds, and cross × indicates thirty-year bonds.

a reasonable fit, with an overall root mean-squared error (RMSE) of 6.86 basis points. The following two columns reveal a substantial improvement in the pricing errors when correcting for liquidity risk, as the AFNS-L model has a much lower overall RMSE of just 4.16 basis points. Hence, accounting for liquidity risk leads to a notable improvement in the ability of our model to explain bonos market prices. Without exception there is uniform improvement in model fit as measured by RMSE from incorporating the liquidity risk factor into the AFNS model. Note also that neither twenty- nor thirty-year bonds pose any particular challenges for the two models. Thus, both AFNS and AFNS-L models are clearly able to fit those long-term bond yields to a satisfactory accuracy.

The final four columns of Table 2 report the estimates of the specific liquidity parameters

Parameter	AFNS		AFNS-L	
	Est.	SE	Est.	SE
$\kappa_{11}^{\mathbb{P}}$	0.2846	0.1246	0.4789	0.3286
$\kappa_{22}^{\mathbb{P}}$	6.5806	1.2222	0.4156	0.5277
$\kappa_{33}^{\mathbb{P}}$	0.3978	0.2915	0.1564	0.1986
$\kappa_{44}^{\mathbb{P}}$	–	–	0.2016	0.2883
σ_{11}	0.0077	0.0002	0.0120	0.0007
σ_{22}	0.0812	0.0020	0.0199	0.0024
σ_{33}	0.0282	0.0021	0.0226	0.0031
σ_{44}	–	–	0.0175	0.0055
$\theta_1^{\mathbb{P}}$	0.2196	0.0095	0.0933	0.0130
$\theta_2^{\mathbb{P}}$	-0.1803	0.0068	-0.0456	0.0300
$\theta_3^{\mathbb{P}}$	-0.1200	0.0223	-0.0315	0.0609
$\theta_4^{\mathbb{P}}$	–	–	-0.0133	0.0422
λ	0.1770	0.0036	0.2770	0.0097
$\kappa_{liq}^{\mathbb{Q}}$	–	–	2.0004	0.3360
$\theta_{liq}^{\mathbb{Q}}$	–	–	0.0095	0.0020
σ_{ε}	0.0008	1.02×10^{-5}	0.0005	1.12×10^{-5}

Table 3: **Estimated Dynamic Parameters**

The table shows the estimated dynamic parameters for the AFNS and AFNS-L models estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ .

associated with each bonos. Except for the last bonos for which we have few observations, all other bonos in our sample are exposed to liquidity risk as their β^i parameters are significantly different from zero at the conventional 5 percent level.

Table 3 reports the estimated dynamic parameters. We note that the slope factor is the most significantly impacted by adding the liquidity risk factor to the AFNS model. Both its mean-reversion rate and volatility decline significantly, while the changes in these two parameters for the level and curvature factors are much smaller and not statistically significant except for σ_{11} . Furthermore, the mean parameters for all three frictionless factors change quite markedly, but this is due to the latent nature of our models that allows the filtered state variables to have sizable level differences across the two models (not shown), while maintaining a close fit to the data in both estimations. Finally, there is a slight increase in the estimated value of λ as we move from the AFNS model to the AFNS-L model. This implies that in the AFNS-L model the slope factor puts more emphasis on fitting shorter-term bond yields. The fourth factor in the AFNS-L model, the liquidity factor, is a persistent process with a volatility close to that of the level factor. Its mean under the objective \mathbb{P} probability measure is -0.0133, which is close to the average of its filtered path. However, its mean under the risk-neutral \mathbb{Q} probability measure used for pricing is 0.0095, which explains why the estimated bonos liquidity premiums described in the next section are strictly positive.

5 The Bonos Liquidity Premium

In this section, we analyze the bonos liquidity premium implied by the estimated AFNS-L model described in the previous section. First, we formally define the bonos liquidity premium, study its historical evolution, and assess its robustness before we end the section by relating the estimated liquidity premium to foreign holdings of bonos, while controlling for other relevant factors that could affect the liquidity risk of bonos.

5.1 The Estimated Bonos Liquidity Premium

We use the estimated AFNS-L model to extract the liquidity premium in the bonos market. To compute this premium, we first use the estimated parameters and the filtered states $\{X_{t|t}\}_{t=1}^T$ to calculate the fitted bonos prices $\{\hat{P}_t^i\}_{t=1}^T$ for all outstanding securities in our sample. These bond prices are then converted into yields to maturity $\{\hat{y}_t^{c,i}\}_{t=1}^T$ by solving the fixed-point problem

$$\hat{P}_{t=1}^i = C(t_1 - t) \exp\left\{-(t_1 - t)\hat{y}_t^{c,i}\right\} + \sum_{k=2}^n \frac{C}{2} \exp\left\{-(t_k - t)\hat{y}_t^{c,i}\right\} + \exp\left\{-(T - t)\hat{y}_t^{c,i}\right\}, \quad (7)$$

for $i = 1, 2, \dots, N$, meaning that $\{\hat{y}_t^{c,i}\}_{t=1}^T$ is the rate of return on the i th bonos if held until maturity. To obtain the corresponding yields without correcting for liquidity risk, a new set of model-implied bond prices are computed from the estimated AFNS-L model but using only its frictionless part, i.e., using the constraints that $X_{t|t}^{liq} = 0$ for all t as well as $\sigma_{44} = 0$ and $\theta_{liq}^Q = 0$. These prices are denoted $\{\tilde{P}_t^i\}_{t=1}^T$ and converted into yields to maturity $\tilde{y}_t^{c,i}$ using (7). They represent estimates of the prices that would prevail in a world without any financial frictions. The liquidity premium for the i th bonos is then defined as

$$\Psi_t^i \equiv \hat{y}_t^{c,i} - \tilde{y}_t^{c,i}. \quad (8)$$

The average estimated liquidity premium of Mexican bonos implied by the AFNS-L model is shown with a solid black line in Figure 4. We note that the estimated liquidity premium is of considerable size, with an average of 0.57 percent and a standard deviation of 0.19 percent. Hence, liquidity risk is an important component in the pricing of Mexican government bonds. Furthermore, we see significant variation around a general upward trend during our sample period, with notable spikes in the summer of 2011 and spring of 2014 and a persistent decline in the fall of 2016.

Next, we are interested in understanding the determinants of the bonos liquidity premium series and its ties to the increase in foreign holdings of Mexican government bonds described in Section 3.1. Therefore, in Figure 4, we also show the market share of foreigners defined as foreign net holdings divided by total public holdings (solid gray line) where we note a

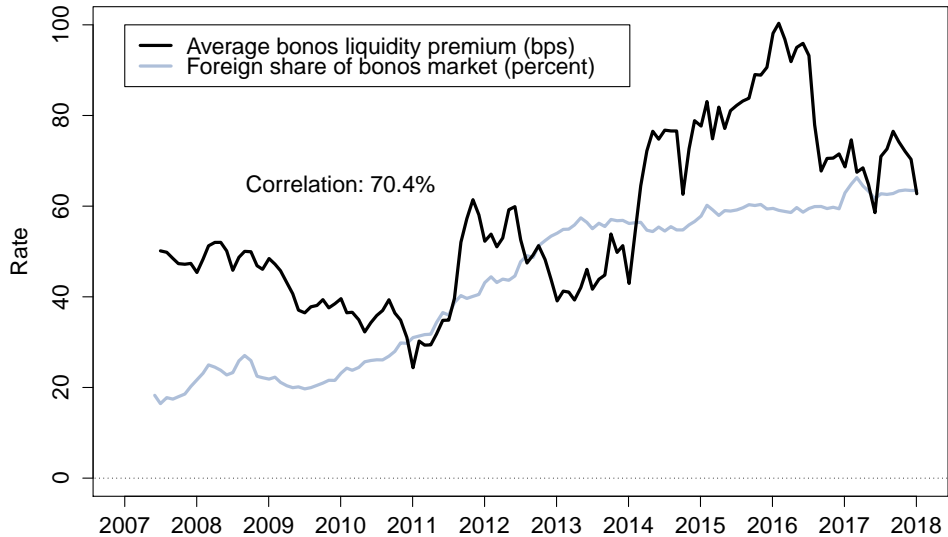


Figure 4: **Estimated Bonos Liquidity Premium and Foreign Share of Bonos Market** Illustration of the average estimated liquidity premium of Mexican bonos for each observation date implied by the AFNS-L model estimated with a diagonal specification of $K^{\mathbb{P}}$ and Σ . The Mexican bonos liquidity premiums are measured as the estimated yield difference between the fitted yield to maturity of individual Mexican bonos and the corresponding frictionless yield to maturity with the liquidity risk factor turned off. Also shown is the share of the bonos market held by foreigners at the end of each month. Both series cover the period from June 30, 2007, to December 29, 2017.

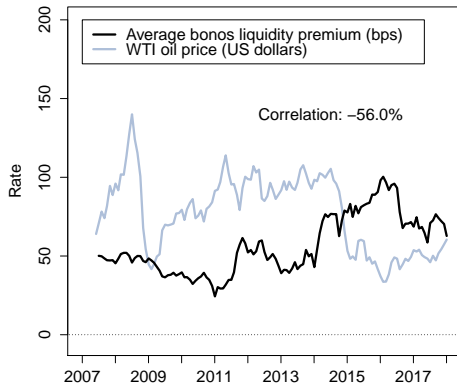
high positive correlation (70 percent) between it and the average estimated bonos liquidity premium. The key empirical question is to what extent variation in the estimated bonos liquidity premium can be explained by changes in the shown foreign-held share of the Mexican bonos market.

5.2 Regression Analysis

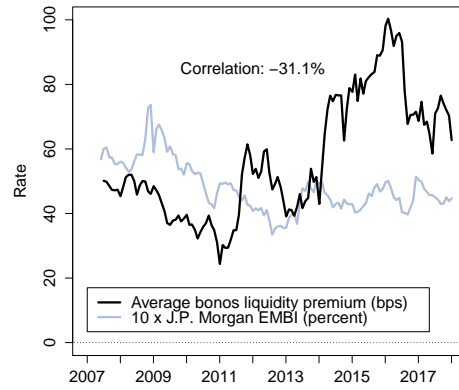
To explain the variation of the bonos liquidity premiums, we run standard regressions with the liquidity premium series as the dependent variable and the share of foreign holdings of bonos as the explanatory factor.¹³ In addition, we include a number of controls that are thought to matter for bonos market liquidity specifically or bond market liquidity more broadly as described in the following.

The first variable is the West Texas Intermediate (WTI) Cushing crude oil price. As a major oil producing country, the revenue and bond issuance of the Mexican government

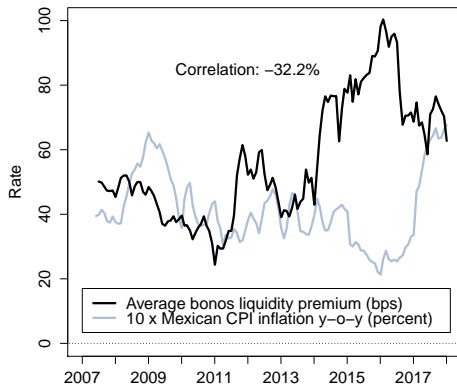
¹³Our analysis is inspired by Hancock and Passmore (2015), who use the U.S. Federal Reserve’s holdings as a share of the U.S. Treasury and mortgage backed securities (MBS) markets as explanatory variables to determine their effect on MBS yields and mortgage rates.



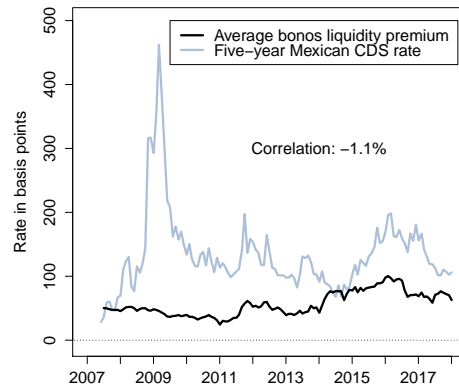
(a) WTI oil price



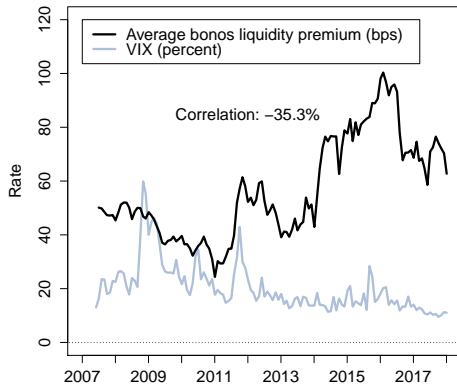
(b) J.P. Morgan emerging market bond index



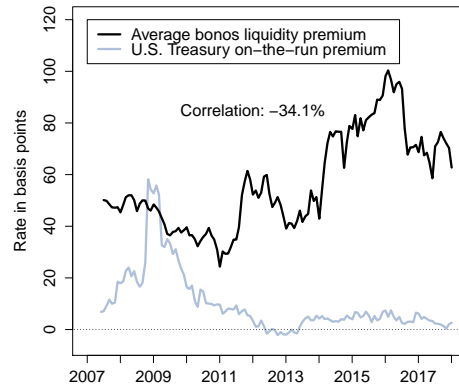
(c) Mexican CPI inflation



(d) Five-year Mexican CDS rate



(e) VIX



(f) U.S. Treasury on-the-run premium

Figure 5: Variables Explaining the Average Bonos Liquidity Premium

In each panel, we compare the average estimated bonos liquidity premium to the indicated explanatory variable. In panel (a) the WTI spot oil price is measured in U.S. dollars. In panel (b) the J.P. Morgan Emerging Market Bond Index (EMBI) is measured in percent. In panel (c) the year-over-year change in the Mexican consumer price index (CPI) is scaled up by a factor of 10. In panel (d) the five-year sovereign CDS rate of Mexico is measured in basis points. In panel (e) the VIX for the S&P 500 is expressed in percent. In panel (f) the U.S. Treasury on-the-run (OTR) premium is measured as the yield difference between the ten-year off-the-run Treasury par yield from Gürkaynak et al. (2007) and the ten-year on-the-run Treasury par yield from the H.15 series at the Board of Governors.

are affected by changes in oil prices, which could play a role for the liquidity in the Mexican government bond market. Second, to control for factors that affect emerging market sovereign bonds more broadly, we include the J.P. Morgan Emerging Market Bond Index (EMBI). Our next set of controls are specific to Mexico, namely the year-over-year change in the Mexican consumer price index (CPI) and the five-year credit default swap (CDS) rate for Mexico, which are both supposed to control for general developments in the Mexican economy of importance to investors in the bonos market. The fifth variable is the VIX, which represents near-term uncertainty about the general stock market as reflected in options on the Standard & Poor's 500 stock price index and is widely used as a gauge of investor fear and risk aversion. The final variable is the yield difference between seasoned (off-the-run) U.S. Treasury securities and the most recently issued (on-the-run) U.S. Treasury security of the same ten-year maturity mentioned earlier. This on-the-run (OTR) premium is a frequently used measure of financial frictions in the U.S. Treasury market.

In Figure 5, we compare each of the control variables to the estimated bonos liquidity premium series. We note that the bonos liquidity premiums are significantly negatively correlated with the WTI oil price as one would expect since high oil prices tend to improve the budget of the Mexican government and thereby the market conditions for bonos. More surprisingly, it is also significantly negatively correlated with the EMBI. However, once we run joint regressions with several explanatory variables, this relationship becomes positive as economic intuition would suggest. Equally surprisingly, it is significantly negatively correlated with Mexican CPI inflation, and this relationship holds up in joint regressions as we discuss below. Furthermore, the bonos liquidity premiums have only a weak correlation to Mexican CDS rates. Finally, neither the VIX nor the U.S. Treasury on-the-run premium turn out to play any significant role for the bonos liquidity premiums, despite being significantly negatively correlated on an individual basis.

We now run standard linear regressions to more formally assess the relative importance of each of these seven variables. First, we run regressions with each explanatory variable in isolation. The results reported in the last two columns of Table 4 confirm the characterizations about the ties between our bonos liquidity premium series and the seven explanatory variables. In particular, the foreign-held share of the bonos market has the largest explanatory power followed by the WTI oil price, while the three financial variables (the CDS rate, VIX, and the on-the-run premium) indeed only have a weak link with the bonos liquidity premium.

The columns labeled (1) and (2) in Table 4 show the results of our preferred joint regression with the four most important variables and the full joint regression with all explanatory variables included, respectively. Three things stand out. First, the two regressions produce essentially identical adjusted R^2 values. Thus, the preferred regression yields about as much explanatory power as possible given our seven variables. Second, the foreign-held share has an estimated coefficient that is statistically insignificant from 1, that is, we find an almost

Explanatory variables	(1)	(2)	(3)	Individual regressions	
				$\hat{\beta}$	adj. R^2
Foreign share	0.82* (0.32)	0.94** (0.30)		0.79* (0.33)	0.49
WTI	-0.24 (0.24)	-0.29 (0.22)	-0.43 (0.23)	-0.44 (0.32)	0.31
EMBI	6.54 (5.47)	5.12 (4.86)	0.36 (5.72)	-7.40 (5.92)	0.09
CPI inflation	-4.14 (2.11)	-4.40* (1.95)	-3.70 (2.40)	-5.31 (16.03)	0.10
CDS rate		-0.08 (0.05)	0.02 (0.05)	0.00 (0.08)	-0.01
VIX		0.16 (0.30)	-0.54 (0.41)	-0.73* (0.30)	0.12
OTR premium		0.40 (0.31)	-0.23 (0.43)	-0.52 (0.39)	0.11
Intercept	25.29 (50.14)	35.69 (39.39)	113.57** (33.94)		
Adjusted R^2	0.67	0.68	0.50		

Table 4: **Regression Results**

The table reports the results of regressions with the average estimated bonos liquidity premium as the dependent variable and seven explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with three lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

one-for-one relationship whereby a 1 percentage point increase in the foreign-held share of Mexican government bonds tends to raise their liquidity premium by close to 1 basis point. Third, the WTI, the EMBI, and Mexican CPI inflation all have a stable relationship with the bonos liquidity premium series in these joint regressions and contribute explanatory power on their own as evidenced by the regression results in the column labeled (3), which omits the foreign-held share as an explanatory variable. However, as this latter regression only generates an adjusted R^2 of 50 percent as compared with the adjusted R^2 of 49 percent from using the foreign-held share in isolation, it is clear that the foreign-held share is the key determining factor in explaining the variation in the bonos liquidity premium series.

As reported in Appendix A, we also run our regressions in first differences without obtaining any significant results. This suggests that our results are driven by stock effects from foreigners holding the bonds rather than by flow effects from their bond purchases.

To summarize, the regression results reveal that the increase in the share of foreign holdings of Mexican bonos is significantly positively correlated with the change in the bonos liquidity premiums, both on its own and after including our control variables. In terms of magnitudes, the results imply that a 1 percentage point increase in the foreign share raises the liquidity premium by 0.01 percent or 1 basis point. Given that the foreign market share

has increased by more than 40 percentage points between 2010 and 2017 as shown in Figure 4, our results suggest that the large increase in foreign holdings during our sample period has played a significant role for the upward trend in the liquidity premiums in the Mexican bonos market since then and raised them by as much as 0.35 percent.

A potential explanation for our findings is tied to the fact that our measure of liquidity premiums is forward-looking and not determined by the current trading conditions in the market, which indeed have improved as measured by transaction costs such as bid-ask spreads (not shown). Hence, investors may expect that, as foreign holdings of Mexican government debt increase, the probability of a large sell-off has increased as well—particularly with the normalization of U.S. monetary policy in progress. As a consequence, investors may demand higher liquidity premiums. If so, our results would imply that investors are indeed being compensated for the added liquidity risk in case these flows were to reverse in coming years. Provided the increased compensation is commensurate with the risk of such events, the expanded role of foreigners in the Mexican government bond market may not pose a material risk to its current financial stability.

An important caveat to any conclusions, though, is that our sample only covers a period of foreign capital inflows into the Mexican sovereign bond market. Thus, we have not been able to model the dynamics of a potential sudden stop in the foreign supply of funds to the Mexican bond market. More broadly, the analysis presented in this paper should be viewed as a first step in connecting capital flows to liquidity premiums and financial stability assessments in emerging markets.

6 Conclusion

In this paper, we analyze the relationship between foreign holdings of Mexican government bonds and the premiums investors demand for assuming their liquidity risk. Our results show that increases in foreign holdings tend to put upward pressure on liquidity premiums in the bonos market. Although foreign holdings of Mexican bonos have increased significantly in recent years and likely contributed to the upward trend in their liquidity premiums, this may not pose a risk to financial stability provided the uptick in liquidity premium compensation is adequate relative to the underlying risk of any major sell-off in coming years. More broadly, this type of research may shed light on the important role that foreign investors play for the stability of financial markets in emerging economies.

Finally, we feel compelled to stress the versatility of our empirical approach. For one, it is straightforward to cast the considered model as a shadow-rate model that respects a lower bound for bond yields using formulas provided in Christensen and Rudebusch (2015) in case that is needed. Also, it is feasible to allow for stochastic volatility using the generalized AFNS models developed in Christensen et al. (2014). Finally, the presented model can be expanded with macroeconomic variables as in Espada and Ramos-Francia (2008a), with expectations

from surveys as in Kim and Orphanides (2012), or with real yields as in ACR. Thus, there are many ways to enrich the analysis. However, we leave it for future research to explore those avenues.

Appendix A: Regressions in First Differences

In this appendix, we repeat the regression exercises in Section 5.2 with all variables measured in monthly changes. The purpose is to analyze whether the dynamic relationship between the average bonos liquidity premium and the foreign-held share is a stock effect as modeled in Section 5.2 or is due to flow effects from monthly changes in the share held by foreigners.

Explanatory variables	(1)	(2)	(3)	Individual regressions	
				$\hat{\beta}$	adj. R^2
Foreign share	0.29 (0.38)	0.25 (0.36)		0.33 (0.40)	0.00
WTI	-0.13** (0.04)	-0.11* (0.04)	-0.11* (0.05)	-0.12* (0.05)	0.03
EMBI	-1.43 (1.25)	-2.50 (1.37)	-2.73 (1.46)	-1.38 (1.23)	0.00
CPI inflation	-1.60 (1.09)	-1.56 (1.03)	-1.52 (1.04)	-1.27 (1.10)	0.00
CDS rate		0.03 (0.02)	0.03 (0.02)	0.02 (0.01)	0.00
VIX		0.03 (0.08)	0.04 (0.08)	0.08 (0.08)	0.00
OTR premium		-0.15 (0.11)	-0.16 (0.11)	-0.03 (0.05)	-0.01
Intercept	0.00 (0.43)	-0.02 (0.41)	0.07 (0.40)		
Adjusted R^2	0.03	0.03	0.03		

Table 5: **Results for Regressions in First Differences**

The table reports the results of regressions in first differences with the average estimated bonos liquidity premium as the dependent variable and seven explanatory variables described in the main text. Standard errors computed by the Newey-West estimator (with 3 lags) are reported in parentheses. Asterisks * and ** indicate significance at the 5 percent and 1 percent levels, respectively.

The results are reported in Table 5. First, all estimated coefficients but one are insignificant. Still, it remains the case that a monthly increase in the foreign-held share of bonos is weakly associated with an increase in the estimated bonos liquidity premium. We take this evidence to suggest that our findings reflect stock effects rather than flow effects. Thus, it is the persistent increase in foreign holdings over time that matter rather than the month-to-month variation.

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