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Abstract

We assess the impact of news concerning the reforms associated with "Abenomics" using an arbitrage-free term structure model of nominal and real yields. Our model explicitly accounts for the deflation protection enhancement embedded in Japanese inflation-indexed bonds issued since 2013, which pay their original nominal principal when deflation has occurred from issue to maturity. The value of this enhancement is sizable and time-varying, with substantive impacts on estimates of expected inflation compensation. After properly accounting for deflation protection, our results suggest that Japanese inflation risk premia were mostly negative during this period. Moreover, long-term inflation expectations remained positive throughout, despite extensive spells of realized deflation. Finally, initial market responses to policy changes associated with Abenomics and afterwards were not as inflationary as they appear under standard modeling procedures, implying that the program was less "disappointing" than many perceive.

JEL Classification: C32, E43, E52, G12, G17

Keywords: Japan, affine arbitrage-free term structure model, unconventional monetary policy, deflation, inflation expectations

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1 Introduction

After the election of Prime Minister Shinzo Abe in December 2012, Japan embarked on a program of extensive policy reforms, popularly known as "Abenomics." This program contained three components, designated by the Prime Minister as the "three arrows," consisting of monetary, fiscal, and structural reforms. The monetary reforms included the adoption of an explicit two-percent inflation target by the Bank of Japan (BoJ) and the launch of an asset-purchase program that was planned to double the monetary base, commonly referred to as "quantitative and qualitative monetary easing" (QQE).[1] Fiscal reforms would include a "flexible" policy consisting of short-run fiscal expansion followed by fiscal consolidation over the medium term, see Hoshi (2013). Finally, the structural reforms were broadly characterized as including labor market reforms and reduced subsidies in a variety of industries, particularly agriculture, as stressed by Hausman and Wieland (2014).

The launch of Abenomics was received with substantial enthusiasm, with the initial monetary policy balance sheet expansion exceeding expectations.[2] While some expressed skepticism, particularly about the potential for extensive structural reforms,[3] most studies indicate that movements in financial variables associated with the monetary policy changes under Abenomics were initially optimistic. The introduction of QQE appears to have pushed up inflation expectations, with those reported by the BoJ increasing from less than one percent to close to two percent.[4] By the fall of 2013, analysts were quite optimistic about a fundamental shift in monetary policy under Abe. Hoshi (2013) concluded that "Abenomics's first arrow seems to be moving in the right direction. At least in the financial market, the inflation expectation has been increasing." These results were also robust to alternative measures of inflation expectations, including those based on PPP,[5] which indicated about a 200 basis point initial increase in inflation expectations. de Michelis and Iacoviello (2016) report that after the launch of Abenomics 6-10 year inflation expectations from five-year forward inflation swap rates five years ahead increased from 0 to 1.2 percent, while consensus forecasts of inflation at the same horizon increased by 80 basis points.

Corroborating reactions were seen in other markets as well. The yen initially depreciated against the dollar by about 20 percent, while the Nikkei stock price index rose by about 60 percent, see Ito (2014). Hausman and Wieland (2014) estimate that Abenomics raised overall output growth in 2013 by between 0.9 and 1.8 percentage points.

Still, the overall impact of the monetary stimulus alone is left uncertain by the simultaneous efforts in fiscal and structural reforms. In particular, efforts to isolate the impact of the first "monetary" arrow through VAR-based methods suggest a more moderate, but still

[1]For an early review and assessment of these programs, see Hausman and Wieland (2014, 2015).

[2]For example, see Ito (2014).

[3]For example, Hoshi (2013) noted that the framework for structural reforms lacked "focus," and accurately predicted that implementation in all of the 52 areas of reform would prove to be difficult.

[4]See Bank of Japan (2013).

[5]See Krugman (2013) and Mandel and Barnes (2013).

substantive positive impact, see Dell-Arriccia et al. (2018). Hattori and Yetman (2017) combine forecasts and also document an increase in inflation expectations following the launch of Abenomics. However, they also find an increase in the dispersion of those expectations, which they interpret as an indication of the lack of credibility in the BoJ inflation targeting regime.

Most analyses conclude that Abenomics disappointed relative to initial expectations. Katz (2014) notes that real wages to date of his writing had fallen by two percent under Abe. Hausman and Wieland (2015) note that realized Japanese output in 2015 was substantially lower than forecasts at the launch of the program in 2012. They also note that, as of 2015, inflation expectations remained 50-100 basis points below the BoJ two-percent inflation target. They associate this sluggish movement in inflation expectations with imperfect credibility of the inflation target. The BoJ Tankan survey indicated similar shortfalls.

Many associated the disappointment with the Abenomics program to a failure to carry out the initially-promised reforms. The October 30, 2012 BoJ increase in the size of the Asset Purchase Program and launch of the Bank Lending Facility was accompanied by a joint statement recognizing the need for the government and the BoJ to "work together" to overcome deflation.[6] However, the anticipated need for medium-term fiscal consolidation in Japan mitigated the impact of short-term expansionary measures. Moreover, implemented structural reforms were modest at best.

In this paper, we revisit the implications of the Abenomics reforms in a high-frequency event study framework. Specifically, we analyze the information reflected in the prices of inflation-indexed Japanese government bonds. To do so, we construct an arbitrage-free term structure model of Japanese nominal and real yields, using the methods of, e.g., Abrahams et al. (2016) and D'Amico et al. (2018). We jointly model Japanese nominal and real government bond yields, accounting for the value of the deflation protection option embedded in the contract of Japanese inflation-indexed bonds issued since 2013 using the approach of Christensen et al. (2012). As in the case of that study of inflation-protected U.S. treasuries, these bonds also implicitly offer "deflation protection" in the form of paying off the original nominal principal at maturity when deflation has occurred since issuance. As we demonstrate below, these enhancements are particularly important over our sample period containing low and often negative Japanese inflation.

Finally, to obtain the appropriate persistence of the dynamic factors in the model, we incorporate long-term forecasts of inflation from surveys of professional forecasters, as advocated by Kim and Orphanides (2012).[7] Our model allows us to account for inflation risk premia, as in Christensen et al. (2010), and hence to identify bond investors' underlying inflation expectations.

[6]Cited in Hoshi (2013).

[7]These are the ten-year projections of CPI inflation ex fresh food that can be constructed semi-annually from the Consensus Forecasts survey.

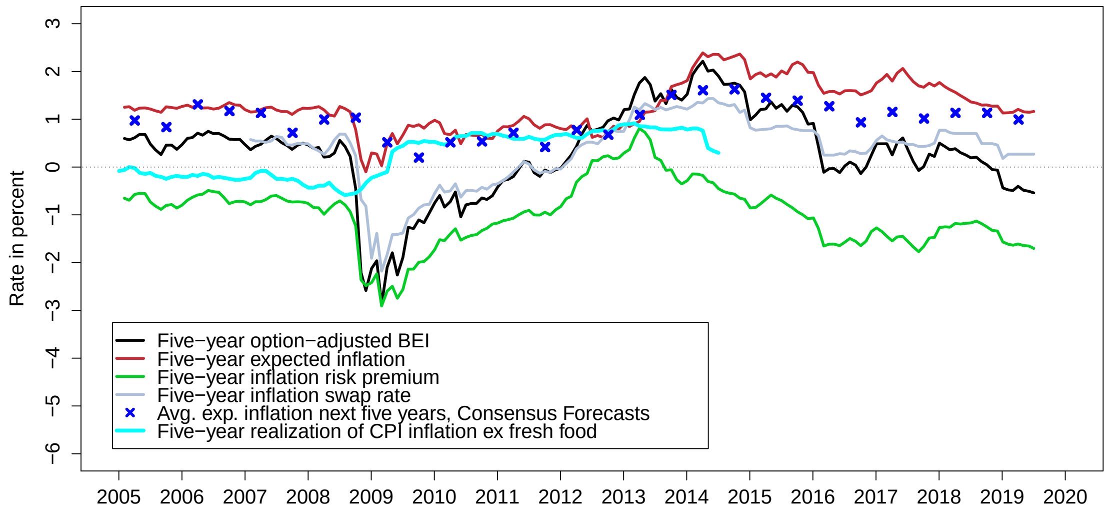


Figure 1: **Five-Year Option-Adjusted Japan BEI Decomposition**
Illustration of (i) five-year fitted option-adjusted break-even inflation (BEI) calculated as the difference between the fitted five-year nominal yield and the fitted five-year option-adjusted real yield from joint model of Japanese nominal and real government bond yields, (ii) estimated five-year expected inflation, and (iii) residual five-year inflation risk premium. Also shown are five-year inflation swap rates (Source: Bloomberg), mean five-year expected inflation (Source: Consensus Forecasts survey), and subsequent five-year realization of CPI inflation ex-fresh-food.

Our model is Gaussian and does not respect any lower bounds on nominal yields. This could potentially bias our results over the portion of our sample when Japanese yields appeared to be constrained by the zero lower bound, but we expect that any bias would be modest. Moreover, our sample also includes the period of negative nominal Japanese rates since the beginning of 2016, over which the existence of a lower bound on nominal yields is unclear. Our Gaussian dynamics are required under our methodology to account for the value of the deflation protection enhancement in Japanese inflation-indexed bonds, which we consider more crucial for generating accurate estimates, given the large size of this enhancement for Japan. We therefore leave alternative dynamic formulations for future research.

To preview our results, Figure 1 shows our model's decomposition of the five-year option-adjusted breakeven inflation (BEI) rate into the underlying expected inflation component and the residual inflation risk premium since 2005.[8] Over this period, the five-year option-adjusted Japanese BEI has averaged 0.30 percent. However, inflation expectations averaged 1.28 percent, implying that inflation risk premia were significantly negative on average. Negative pricing of inflation risk suggests that investors in the Japanese government bond market view

[8]In online appendix F, we provide a similar decomposition of the five-year forward option-adjusted BEI starting five years ahead, frequently referred to as the 5yr5yr BEI.

exposure to Japanese inflation risk as desirable, perhaps due to diversification opportunities relative to other assets. This somewhat unusual pattern can be rationalized in other ways as well. For example, if monetary policy is expected to be constrained by the effective lower bound in a recession, low inflation or outright deflation would coincide with high marginal utility giving rise to negative inflation risk premia since nominal bonds would serve as a good hedge under those circumstances.

A number of alternative series verify the accuracy of our model. First, Figure 1 also shows the five-year inflation swap rate. Assuming no financial frictions, theory would predict that inflation swap rates and matching option-adjusted BEI rates should be identical. We take the closeness of these two series as evidence that our model is performing well. We also discipline the model estimation with survey measures of Japanese ten-year inflation expectations. This leaves our estimates of expected inflation at the five-year horizon very similar to the survey data.[9] Finally, we compare our estimated five-year forecast of expected inflation to the subsequent realization of average CPI inflation ex fresh food. While there is some discrepancy between these series in the early years of our sample, they are relatively close for the available period since 2009. Overall, these results all suggest that our model generates realistic inflation dynamics.

Another important product of our model is an estimate of the price of deflation risk in the Japanese government bond market. We measure the deflation risk premium by calculating the spread between the par yield of a synthetic newly issued inflation-indexed bond without deflation protection and that of a similar bond with the same maturity that includes deflation protection. Figure 2 shows this series constructed at the ten-year maturity. Our estimate of the deflation risk premium is large, averaging 74 basis points, and it exhibits notable time variation with a standard deviation of 50 basis points. Furthermore, it spikes during the global financial crisis when Japanese CPI inflation fell sharply. Thus, our model suggests that deflation protection would be quite valuable during the financial crisis, as would be expected, providing another desirable check on the validity of our analysis.

More importantly, the deflation risk premium bottomed out in early 2013, immediately after Shinzo Abe reassumed power in December 2012 and optimism about the prospects for Abenomics to eliminate deflation was at its peak. However, the value of the deflation risk premium has since trended up, as Abenomics failed to provide its expected boost to inflation and economic performance. Note that our model indicates that it is the *priced* long-term deflation risk that has trended up since 2014, while both actual and priced near-term deflation risk have been negligible since the spring of 2013.

To study the impact of the Bank of Japan's key monetary policy changes since 2013, we use our dynamic term structure model, combined with an event study approach similar to Christensen and Rudebusch (2012), who investigate the response of U.S. and U.K. government

[9] As we do not include the five-year survey inflation forecasts in the model estimation, this comparison serves as a real model validation.

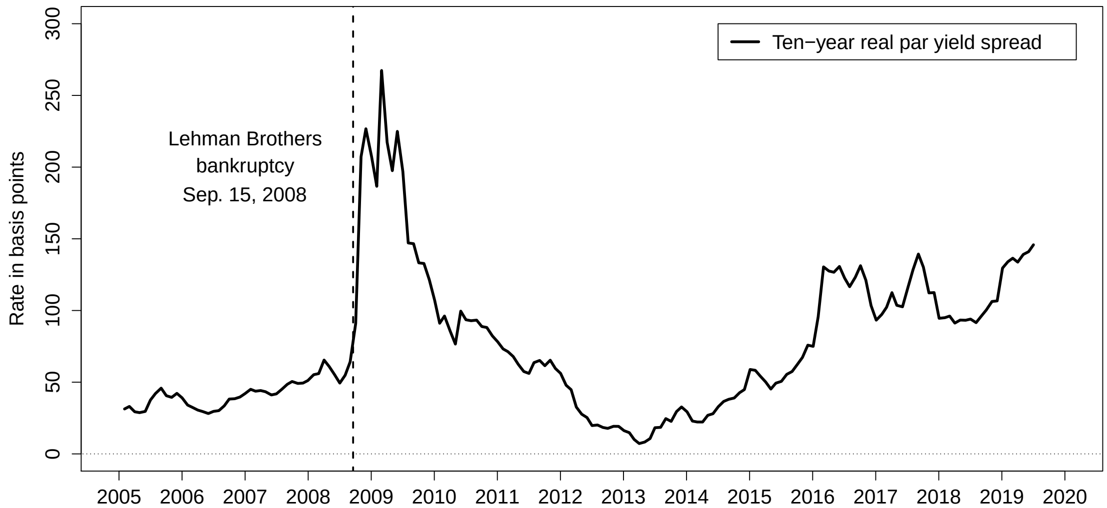


Figure 2: **Value of Ten-Year Deflation Protection Options**
Shown is the "deflation risk premium" defined as the spread between the par yield of a synthetic newly issued ten-year inflation-indexed bond lacking deflation protection and that of a deflation-protected bond with the same maturity.

bond yields to announcements regarding their respective unconventional monetary policy initiatives. We estimate our model with daily data, which allows for decomposition of one-day long-term yield changes into changes to expected inflation and associated risk premia.[10]

We identify six primary announcements associated with monetary policy changes under Abenomics. Our event dates include the introduction of an explicit two-percent inflation target and open-ended expansion of the asset purchase program on January 22, 2013; the introduction of the BoJ quantitative and qualitative easing program (QQE), under which the BoJ committed to double its monetary base and its holdings of JGBs over the following two years on April 3, 2013, see Hausman and Wieland (2014); the expansion of the BoJ QQE program, in which it raised its targeted monetary base expansion from 60-70 trillion yen to 80 trillion yen on October 31, 2014, see Hausman and Wieland (2015); the movement by the BoJ into negative policy rates on January 29, 2016; the introduction of "yield curve control" by the BoJ on September 21, 2016, under which the BoJ committed to keeping short-term rates on reserves at -0.1 percent and to continuing to purchase long-term JGBs to keep the ten-year rate close to 0, see Dell'Ariccia et al. (2018). Furthermore, it committed to overshooting its two-percent inflation target. Finally, we include the strengthening of the

[10]Gagnon et al. (2011), Christensen and Rudebusch (2012), and Bauer and Rudebusch (2014) provide term structure model decompositions of the U.S. experience with unconventional monetary policies, while Christensen and Krogstrup (2019) use a similar approach to evaluate the Swiss experience with unconventional reserve expansions.

framework for continuous powerful monetary easing announced on July 31, 2018, in which the BoJ committed to maintain the existing extremely low levels of short- and long-term interest rates for an extended period of time.

Our results demonstrate that the value of the deflation protection enhancement is generally decreasing with policy announcements signaling enhancement or implementation of the Abenomics program, as would be expected. As a result, our estimated changes in inflation expectations on these announcement dates are smaller than those that one would obtain without the deflation protection adjustment. For example, without adjusting for changes in deflation protection our benchmark model yields fitted estimates of changes in the five- and ten-year BEI rates of 11.1 and 7.7 basis points, respectively, over our event window for the announcement of the BoJ adoption of an explicit two-percent inflation target. However, after adjusting for the deflation protection option, the changes are more modest, at 9.1 and 6.5 basis points, respectively. Similarly, without adjusting we would conclude that the adoption of yield curve control had pushed up the five- and ten-year yields by 0.5 and 1.1 basis points respectively, while after adjustment we estimate that both yields actually fell.

Other events yielded similar results. Overall then, our results suggest that the initial market response to policies under Abenomics were not as optimistic as one would perceive without properly accounting for changes in the value of deflation protection. As such, our analysis suggests that the program was less "disappointing" than it appears. Rather than thwarted by the failure to implement announced policies, the reason that is often given for the disappointing outcomes under the Abe regime, our analysis suggests that market participants never were optimistic about the prospects for the program from the start.

The remainder of the paper is structured as follows. Section 2 contains the data description, while Section 3 details the no-arbitrage term structure model we use and presents the empirical results. Section 4 analyzes the deflation risk premium and its impact on our results, while Section 5 describes our event study of the impact of key monetary policy changes since 2013. Section 6 concludes. The appendix explains the bond decomposition we use, while additional appendices available online contain details on bond price formulas, model estimation, and various robustness checks.

2 Japanese Government Bond Data

The Japanese government bond market is large by international standards. As of December 2018, the total outstanding notional amount of marketable bonds issued by the government of Japan was 1,100.5 trillion yen, of which close to 1 percent represented inflation-indexed bonds.[11] In total, Japanese government debt equaled 238% of Japanese nominal national

[11]Source: https://www.mof.go.jp/english/jgbs/publication/newsletter/jgb2019_02e.pdf

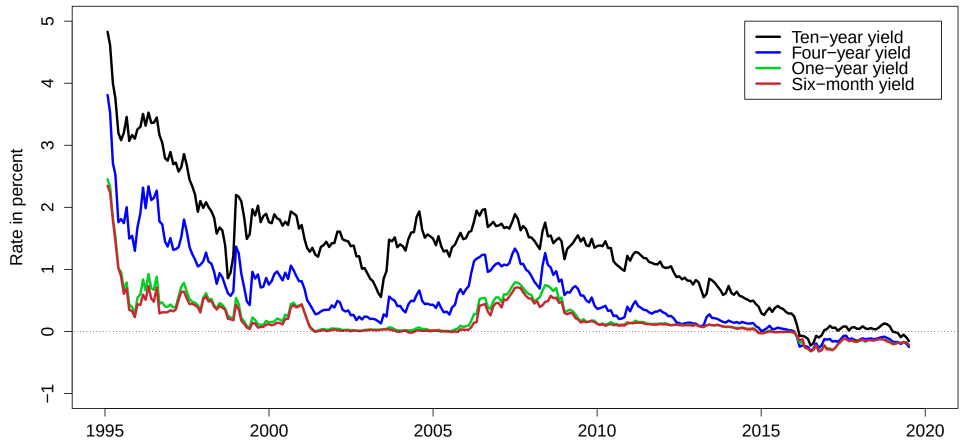


Figure 3: **Japanese Nominal Government Bond Yields**
Illustration of the Japanese nominal government zero-coupon bond yields with maturities of six months, one year, four years, and ten years. The data series are monthly covering the period from January 31, 1995 to June 29, 2018.

GDP at the end of April 2019, far above the level of any other major industrialized country.[12]

2.1 Nominal Bonds

We extend the Japanese nominal government bond yield series in Kim and Singleton (2012), which originally ended in March 2008, with Japanese nominal government zero-coupon yields to June 2019 downloaded from Bloomberg as in Christensen and Rudebusch (2015). This data set contains six maturities: six-month yields and one-, two-, four-, seven-, and ten-year yields, with all yields being continuously compounded and available at daily frequency.

Our extension allows us to assess the effects of the most recent unconventional policies pursued by the Japanese government and the Bank of Japan since Shinzo Abe reassumed power in December 2012, as well as the negative interest rate policy introduced in January 2016 and the yield curve control announced in September 2016. To facilitate empirical implementation, we use the data at the end of each month from January 31, 1995 to June 28, 2019, a total of 294 monthly observations, although we also perform a set of estimations with daily data.

Figure 3 shows the variation over time for four of our nominal yields. All maturities display a persistent drop in yields since the mid-1990s. There is also a persistent decline in the yield spreads, particularly in the neighborhood of the zero lower bound. The yield spread between the ten- and one-year yield was larger than 200 basis points at the start of the sample

[12]Source: https://www.mof.go.jp/english/jgbs/publication/debt_management_report/2019/esaimu2019-3-ho.pdf

JGBi (coupon, maturity)	No. obs.	Issuance Date	Issuance amount	Number of auctions	Total notional amount
(1) 1.2% 3/10/2014	86	3/10/2004	100	1	100
(2) 1.1% 6/10/2014	88	6/10/2004	300	1	300
(3) 0.5% 12/10/2014	98	12/10/2004	500	1	500
(4) 0.5% 6/10/2015	100	6/10/2005	500	1	500
(5) 0.8% 9/10/2015	96	9/12/2005	500	1	500
(6) 0.8% 12/10/2015	90	12/12/2005	500	1	500
(7) 0.8% 3/10/2016	92	3/10/2006	500	1	500
(8) 1% 6/10/2016	87	6/12/2006	500	2	1000
(9) 1.1% 9/10/2016	89	10/11/2006	500	1	500
(10) 1.1% 12/10/2016	88	12/12/2006	500	2	1000
(11) 1.2% 3/10/2017	84	4/10/2007	500	1	500
(12) 1.2% 6/10/2017	91	6/12/2007	500	2	1000
(13) 1.3% 9/10/2017	81	10/10/2007	500	1	500
(14) 1.2% 12/10/2017	84	12/11/2007	500	2	1000
(15) 1.4% 3/10/2018	80	4/10/2008	500	1	500
(16) 1.4% 6/10/2018	80	6/10/2008	500	2	1000
(17) 0.1% 9/10/2023	69	10/10/2013	300	2	600
(18) 0.1% 3/10/2024	61	4/10/2014	400	2	800
(19) 0.1% 9/10/2024	57	10/10/2014	500	2	1000
(20) 0.1% 3/10/2025	50	5/12/2015	500	4	2000
(21) 0.1% 3/10/2026	37	4/14/2016	400	4	1600
(22) 0.1% 3/10/2027	27	4/13/2017	400	4	1600
(23) 0.1% 3/10/2028	14	5/15/2018	400	2	800
(24) 0.1% 3/10/2029	2	5/10/2019	400	1	400

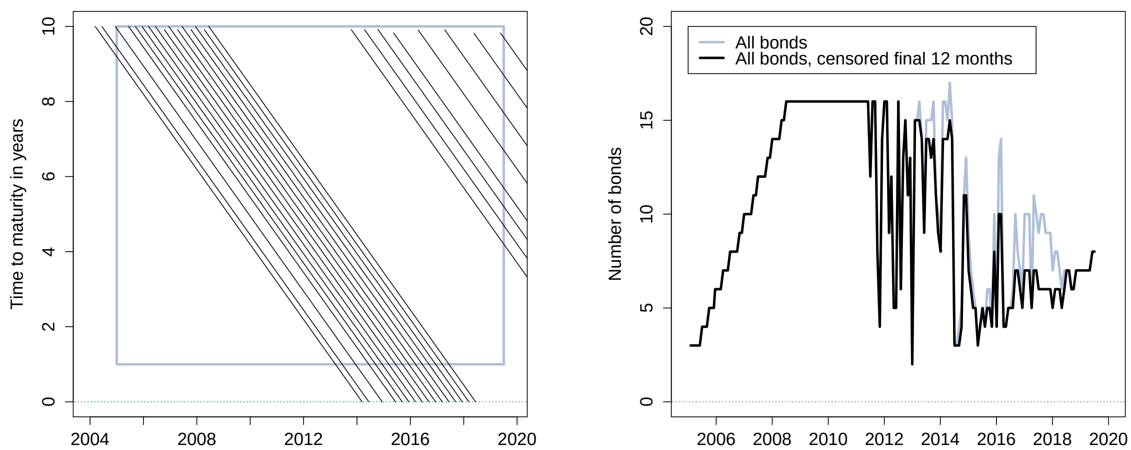
Table 1: **Sample of Japanese Real Government Bonds**
The table reports the characteristics, first issuance date and amount, the total number of auctions, and total amount issued in billions of Japanese yen for the sample of Japanese inflation-indexed government bonds (JGBi). Also reported are the number of monthly observation dates for each bond during the sample period from January 31, 2005 to June 28, 2019.

and less than 25 basis points at the end of the sample. Kim and Singleton (2012) find that a two-factor model is adequate to fit their data and we therefore choose to use a two-factor model for the nominal yields to capture both of these stylized facts.[13]

2.2 Real Bonds

The Japanese government has issued inflation-indexed bonds—known as JGBi—since the spring of 2004. These are all ten-year bonds, which were issued in two separate periods. From March 2004 until June 2008, a total of 16 bonds were issued on a nearly quarterly frequency. The program was then temporarily halted in the aftermath of the global financial

[13]One might be concerned about JGB market liquidity, given the BoJ's purchases of close to 45 percent of all outstanding JGBs by the end of our sample. However, Kurosaki et al. (2015) and Sakiyama and Kobayashi (2018) provide detailed statistical evidence on the liquidity and trading patterns in the JGB cash market and find that the market does not appear to have been impaired during our sample period.



(a) Distribution of inflation-indexed bonds

(b) Number of inflation-indexed bonds

Figure 4: **Real Japanese Government Bond Sample**
Panel (a) shows the maturity distribution of available Japanese inflation-indexed government bonds (JGBi) on any given date. The solid grey rectangle indicates the sample used in our empirical analysis. The sample is restricted to start on January 31, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity. Panel (b) reports the number of outstanding inflation-indexed bonds available at a given point in time for various samples.

crisis. However, shortly after Shinzo Abe reassumed power, the program was resumed. New inflation-indexed bonds have been issued roughly once a year since then. These are government bonds whose principal amount fluctuates in proportion with the consumer price index (CPI) excluding fresh food.

This latter period of issuance included the deflation protection enhancement noted in the introduction. These bonds are guaranteed to pay off at par at maturity, even if there was net deflation between the issuance and maturity dates. This effectively placed a deflation protection option into the bond contract.[14] Table 1 contains the contractual details of all 24 JGBi's in our sample as well as their individual number of monthly observations.

The distribution of individual JGBi's for every date in our sample is illustrated in Figure 4(a). Each bond's trajectory over time in terms of remaining years to maturity is represented by a diagonal solid black line that starts at its date of issuance with a value equal to its original maturity and ends at zero on its maturity date. The two waves of JGBi issuances are clearly visible.

The solid grey rectangle in Figure 4(a) indicates the sub-sample of bonds used in our empirical analysis. The sample is restricted to start on January 31, 2005, and limited to inflation-indexed bond prices with more than one year remaining to maturity.

Figure 4(b) shows the distribution across time of the number of JGBi's included in the

[14]See https://www.mof.go.jp/english/jgbs/topics/bond/10year_inflation/index.htm

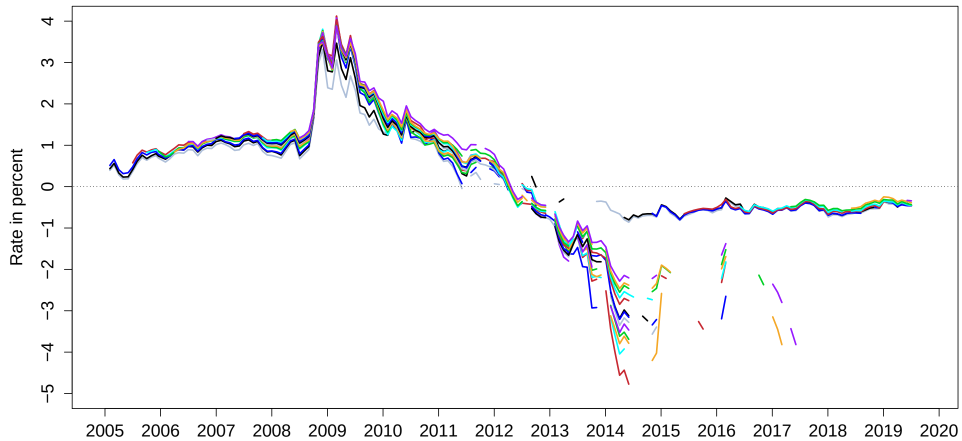


Figure 5: **Yield to Maturity of Japanese Real Government Bonds**
Illustration of the yield to maturity of the Japanese inflation-indexed bonds considered in this paper, which are subject to two sample choices: (1) sample limited to the period from January 31, 2005, to June 28, 2019; (2) censoring of a bond's price when it has less than one year to maturity.

sample. Our sample starts with three bonds and increases to sixteen bonds by 2008. The number of bonds available then gradually declines beginning in 2011, as bonds from the first wave of issuances start to mature. At the end of our sample there are seven bonds. The number of inflation-indexed bonds $n_R(t)$ combined with the time variation in the cross-sectional dispersion in the maturity dimension observed in Figure 4(a) provides the identification of the real factors in our model.[15]

Figure 5 shows the yields to maturity for all 24 Japanese inflation-indexed bonds. We see notable changes in the level and slope of the Japanese real yield curve, which motivates our choice to model the inflation-indexed data with two real yield factors. Note also that the series for individual bonds show gaps as the bonds approach maturity. Our use of all available bond price information in combination with the Kalman filter is designed to handle such data gaps.

3 Model Estimation and Results

In this section, we first detail the joint model of nominal and real yields that serves as the benchmark in our analysis. We then describe the decomposition of nominal and real bond yields into underlying expectations and residual risk premium components before we explain how to evaluate the value of the deflation enhancement embedded in individual inflation-indexed bonds. This is followed by a description of the restrictions imposed to achieve econo-

[15]Finlay and Wende (2012) represent an early example of analysis like ours based on prices from a limited number of Australian inflation-indexed bonds.

metric identification of our model and its estimation. We end the section with a brief summary of our estimation results.

3.1 An Arbitrage-Free Model of Nominal and Real Yields

Our joint model of nominal and real yields has a state vector denoted by $X_t = (L_t^N, S_t^N, L_t^R, S_t^R)$, where (L_t^N, S_t^N) represent level and slope factors in the nominal yield curve, while (L_t^R, S_t^R) represent separate level and slope factors in the real yield curve.[16] The instantaneous nominal and real risk-free rates are defined as

$$r_t^j = L_t^j + S_t^j, \quad j = N, R.$$

To obtain a Nelson and Siegel (1987) factor loading structure in the yield functions, the risk-neutral, or $\mathbb{Q}$, dynamics of the state variables must be assumed to be given by the following system of stochastic differential equations:

$$\begin{pmatrix} dL_t^N \\ dS_t^N \\ dL_t^R \\ dS_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\lambda^N & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\lambda^R \end{pmatrix} \begin{pmatrix} L_t^N \\ S_t^N \\ L_t^R \\ S_t^R \end{pmatrix} dt + \begin{pmatrix} \sigma_{11} & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} \begin{pmatrix} dW_t^{L^N,\mathbb{Q}} \\ dW_t^{S^N,\mathbb{Q}} \\ dW_t^{L^R,\mathbb{Q}} \\ dW_t^{S^R,\mathbb{Q}} \end{pmatrix}.$$

Based on this specification of the $\mathbb{Q}$-dynamics, nominal and real zero-coupon bond yields preserve a simplified Nelson and Siegel (1987) factor loading structure:[17]

$$y_t^j(\tau) = L_t^j + \left(\frac{1 - e^{-\lambda^j \tau}}{\lambda^j \tau}\right) S_t^j - \frac{A^j(\tau)}{\tau}, \quad j = N, R, \tag{1}$$

where the nominal and real yield-adjustment terms are given by

$$\begin{aligned}
\frac{A^N(\tau)}{\tau} &= \frac{\sigma_{11}^2}{6}\tau^2 + (\sigma_{21}^2 + \sigma_{22}^2)\Big[\frac{1}{2(\lambda^N)^2} - \frac{1}{(\lambda^N)^3}\frac{1-e^{-\lambda^N\tau}}{\tau} + \frac{1}{4(\lambda^N)^3}\frac{1-e^{-2\lambda^N\tau}}{\tau}\Big] \\
&\quad +\sigma_{11}\sigma_{21}\Big[\frac{1}{2\lambda^N}\tau + \frac{1}{(\lambda^N)^2}e^{-\lambda^N\tau} - \frac{1}{(\lambda^N)^3}\frac{1-e^{-\lambda^N\tau}}{\tau}\Big]; \\
\frac{A^R(\tau)}{\tau} &= \frac{\sigma_{31}^2 + \sigma_{32}^2 + \sigma_{33}^2}{6}\tau^2 \\
&\quad +(\sigma_{41}^2 + \sigma_{42}^2 + \sigma_{43}^2 + \sigma_{44}^2)\Big[\frac{1}{2(\lambda^R)^2} - \frac{1}{(\lambda^R)^3}\frac{1-e^{-\lambda^R\tau}}{\tau} + \frac{1}{4(\lambda^R)^3}\frac{1-e^{-2\lambda^R\tau}}{\tau}\Big] \\
&\quad +(\sigma_{31}\sigma_{41} + \sigma_{32}\sigma_{42} + \sigma_{33}\sigma_{43})\Big[\frac{1}{2\lambda^R}\tau + \frac{1}{(\lambda^R)^2}e^{-\lambda^R\tau} - \frac{1}{(\lambda^R)^3}\frac{1-e^{-\lambda^R\tau}}{\tau}\Big].
\end{aligned}$$

[16] Chernov and Mueller (2012) provide evidence of a hidden factor in the U.S. nominal yield curve that is observable from real yields and inflation expectations. Our joint model accommodates this stylized fact via the (L_t^R, S_t^R) factors.

[17] See the online supplementary appendix for the derivation of the bond yield formulas.

To implement our model empirically, we need to specify the risk premia that connect these factor dynamics under the $\mathbb{Q}$-measure to the dynamics under the real-world $\mathbb{P}$-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical $\mathbb{P}$-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002). Under the Gaussian framework, this specification implies that the risk premia Γ_t depend on the state variables; that is,

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^4$ and $\gamma^1 \in \mathbf{R}^{4\times 4}$ contain unrestricted parameters. Thus, the resulting unrestricted four-factor joint model of nominal and real yields has $\mathbb{P}$-dynamics given by

$$dX_t = K^P(\theta^P - X_t) + \Sigma dW_t^P,$$

where K^P is an unrestricted 4×4 mean-reversion matrix, θ^P is a 4×1 vector of mean levels, and Σ is a 4×4 lower triangular volatility matrix.

This is the transition equation in the Kalman filter estimation. Going forward, we refer to this Gaussian joint four-factor model of nominal and real yields as the $G^J(4)$ model and use it as our base model for estimation.

3.2 Decomposing Bond Yields

As explained in the appendix, the price of a nominal zero-coupon bond with maturity in τ years can be written as

$$P_t^N(\tau) = P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right),$$

where $P_t^R(\tau)$ is the price of a real zero-coupon bond that pays one consumption unit in τ years, M_t^R is the real stochastic discount factor, and Π_t is the price level.

By taking logs, this can be converted into

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where $y_t^N(\tau)$ and $y_t^R(\tau)$ are nominal and real zero-coupon yields as described in the previous section, while the market-implied average rate of inflation expected at time t for the period from t to $t + \tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right] \tag{2}$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences, as in, for example, Rudebusch and Swanson (2011).

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

Finally, we define the nominal and real term premia as

$$TP_t^j(\tau) = y_t^j(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^j]ds, \quad j = N, R.$$

That is, the nominal term premium is the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal rate r_t^N. The interpretation for the real term premium is similar. The model thus allows us to decompose nominal and real yields into their respective term premia and short-rate expectations components.

3.3 Deflation Protection Option Values

We next evaluate the value of the deflation protection enhancement that has been embedded in Japanese inflation-indexed bonds issued since 2013. As inflation in Japan has averaged close to zero since the inception of deflation protection in 2013, the potential for net deflation over the life of bonds issued after that date has been non-trivial, leaving the deflation protection enhancement likely to be of significant value. It follows that the failure to account for the deflation protection enhancement would likely reduce the quality of estimates of BEI from JGB yields.

Consider an inflation-indexed bond issued at time t_0 with a reference price index value equal to Π_{t_0}. By time t, its accrued inflation compensation is $\frac{\Pi_t}{\Pi_{t_0}}$, which we define as the "inflation index ratio." There are then two mutually exclusive scenarios: First, the net price index change to maturity T could be sufficiently positive that the inflation index ratio is greater than one. Given this outcome, the bond will pay off its inflation-adjusted principal $\frac{\Pi_T}{\Pi_{t_0}}$ at maturity.

Alternatively, the net price index change between t and T may be insufficient, leaving the net change less than one. Given that outcome, the deflation protection option will be in the money, as the inflation-indexed bond returns its original principal. We show in online Appendix A that the value of the deflation protection option, DOV_t, is then given by

$$DOV_t\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = \Bigg[E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^N ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big] - E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq\frac{\Pi_{t_0}}{\Pi_t}\}}\Big]\Bigg].$$

The option value will be lower when accrued inflation compensation is larger, as it is less likely that the net price index change over the bond's remaining life will be sufficiently low (or negative) to bring the option back into the money. Moreover, when accrued inflation is larger, the option value is lower the shorter is the remaining time to maturity, as the probability of bringing the option back into the money at maturity is reduced.

3.4 Model Estimation and Econometric Identification

We estimate the $G^J(4)$ model using a conventional likelihood-based approach, where we extract the latent pricing factors from the observed data, which in our case are nominal zero-coupon yields and inflation-indexed mid-market yields to maturity. The functional form for nominal yields is specified as affine and provided in equation (1), whereas the expression for the yield to maturity $\hat{y}_t^R$ of an inflation-indexed bond with maturity at T that pays an annual coupon C semi-annually is given by the solution to the following fixed-point problem

$$\hat{P}_t^R = C(t_1-t)\exp\left\{-(t_1-t)\hat{y}_t^R\right\} + \sum_{k=2}^{n}\frac{C}{2}\exp\left\{-(t_k-t)\hat{y}_t^R\right\} + \exp\left\{-(T-t)\hat{y}_t^R\right\}, \quad (3)$$

where $\hat{P}_t^R$ is the model-implied inflation-indexed bond price

$$\begin{aligned}\hat{P}_t^R = {} & C(t_1-t)\exp\left\{-(t_1-t)y_t^R(t_1-t)\right\} \\ & +\sum_{k=2}^{n}\frac{C}{2}\exp\left\{-(t_k-t)y_t^R(t_k-t,)\right\} \\ & +\exp\left\{-(T-t)y_t^R(T-t)\right\} + DOV_t\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big)\end{aligned} \quad (4)$$

and Π_t/Π_{t_0} is the accrued inflation compensation since issuance. That is, at time t we use the real yields $y_t^R(\tau)$ in equation (1) to discount the coupon payments.

The last term in equation (4) accounts for the deflation option value (DOV). The principal at maturity is only adjusted for inflation if accumulated inflation since issuance of the bond is positive. We only include the value of this option for the inflation-indexed bonds that have that contractual feature and compute it using an approach similar to the one outlined in Christensen et al. (2012), see the online supplementary appendix for details.[18] Following Joslin et al. (2011), all nominal yields have independent Gaussian measurement errors $\varepsilon_t^{N,i}$ with zero mean and a common standard deviation σ_ε^N, denoted $\varepsilon_{y,t}^i \sim \mathcal{NID}\big(0,(\sigma_\varepsilon^N)^2\big)$ for $i = 1, 2, \ldots, n_N$. We also account for measurement errors in the yields to maturity of the inflation-indexed bonds through $\varepsilon_t^{R,i}$, where $\varepsilon_t^{R,i} \sim \mathcal{NID}\big(0,(\sigma_\varepsilon^R)^2\big)$ for $i = 1, 2, \ldots, n_R(t)$.

3.4.1 Survey Forecasts

We also incorporate long-term forecasts of inflation from surveys of professional forecasters in our model estimation. These are the projected ten-year CPI inflation ex fresh food that can be constructed semi-annually from the Consensus Forecasts survey.

As demonstrated by Kim and Orphanides (2012), the inclusion of long-term survey forecasts can help the model better capture the appropriate persistence of the factors under the objective $\mathbb{P}$-dynamics, which can otherwise suffer from significant finite-sample bias.[19] Indeed, as reported in online appendix C, we find that our estimation results are considerably less accurate in terms of the model's implied inflation expectations when we omit the survey inflation forecasts from our model estimation.

The measurement equation for the survey expectations incorporating these long-term forecasts takes the form

$$\pi_t^{CF}(10) = \pi_t^e(10) + \varepsilon_t^{CF},$$

where $\pi_t^e(10)$ is the model-implied ten-year expected inflation calculated using equation (2), which is affine in the state variables, while the measurement error is $\varepsilon_t^{CF} \sim \mathcal{NID}\big(0,(\sigma_\varepsilon^{CF})^2\big)$.

To improve the tractability of our model estimation, we impose the parameter restriction $\kappa_{44}^{\mathbb{P}} = \lambda^R$. This creates a direct connection between the $\mathbb{P}$- and $\mathbb{Q}$-dynamics of the real yield slope factor S_t^R that facilitates identification.

Regarding the empirical identification of the parameters in the volatility matrix Σ, note that since $\frac{A^N(\tau)}{\tau}$ contains three unique elements that are functions of τ, the three volatility parameters σ_{11}, σ_{21}, and σ_{22} can be empirically identified from solely observing nominal yields. In turn, this implies that the remaining seven volatility parameters $(\sigma_{31}, \sigma_{32}, \sigma_{33}, \sigma_{41}, \sigma_{42}, \sigma_{43}, \sigma_{44})$ must be identified from real yields. However, it is clear from the real yield-adjustment term $\frac{A^R(\tau)}{\tau}$ that only three of these parameters can be econometrically identified as long as the information set is limited to nominal and real yields. Thus, in reality, only $(\sigma_{33}, \sigma_{43}, \sigma_{44})$ can be identified. As a result, we can not estimate the volatility correlations between the nominal and real yield curve risk factors. We therefore restrict the volatility matrix Σ to a diagonal

[18] We do not account for the approximately 2.5 month lag in the inflation indexation. Grishchenko and Huang (2013) and D'Amico et al. (2018) find that this adjustment normally is within a few basis points for the implied yield on U.S. TIPS and hence it is likely to be very small for our Japanese data as well.

[19] Also see Bauer et al. (2012).

Maturity in months	$G^J(4)$ model	
	Mean	RMSE
6	6.09	9.91
12	1.15	5.96
24	-4.60	7.46
48	-6.19	10.85
84	-0.39	11.88
120	0.00	0.00
All maturities	-0.66	8.64

Table 2: **Pricing Errors of Nominal Yields**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese nominal yields in our benchmark $G^J(4)$ model. All errors are reported in basis points.

$K^{\mathbb{P}}$	$K^{\mathbb{P}}_{\cdot,1}$	$K^{\mathbb{P}}_{\cdot,2}$	$K^{\mathbb{P}}_{\cdot,3}$	$K^{\mathbb{P}}_{\cdot,4}$	$\theta^{\mathbb{P}}$		Σ
$K^{\mathbb{P}}_{1,\cdot}$	3.7170	3.9473	-0.3675	-0.09157	0.0049	$\Sigma_{1,1}$	0.0039
	(0.2555)	(0.2850)	(0.1518)	(0.1172)	(0.0131)		(0.0003)
$K^{\mathbb{P}}_{2,\cdot}$	-0.0405	0.0667	0.1035	0.0897	-0.0084	$\Sigma_{2,2}$	0.0041
	(0.2770)	(0.3129)	(0.1167)	(0.1241)	(0.0124)		(0.0004)
$K^{\mathbb{P}}_{3,\cdot}$	-2.4089	-2.7092	0.3386	0.2051	-0.0084	$\Sigma_{3,3}$	0.0068
	(0.3443)	(0.3569)	(0.1240)	(0.0841)	(0.0150)		(0.0002)
$K^{\mathbb{P}}_{4,\cdot}$	3.0266	3.3619	0.1757	0.4314	-0.0054	$\Sigma_{4,4}$	0.0148
	(0.3128)	(0.3522)	(0.1226)	(0.0111)	(0.0144)		(0.0012)

Table 3: **Estimated Benchmark Model Parameters**
The estimated parameters for the mean-reversion matrix $K^{\mathbb{P}}$, the mean vector $\theta^{\mathbb{P}}$, and the volatility matrix Σ in our benchmark $G^J(4)$ model. The $\mathbb{Q}$-related parameters are estimated at $\lambda^N = 0.1088$ (0.0050) and $\lambda^R = \kappa^{\mathbb{P}}_{44} = 0.4314$. The numbers in parentheses are the estimated standard deviations.

matrix, as recommended by Christensen et al. (2011).[20]

Finally, we note that the model is estimated with the standard extended Kalman filter due to the nonlinear measurement equations for the inflation-indexed bond yields, see online Appendix B for details and Andreasen et al. (2019) for evidence of the robustness of this approach.

3.5 Estimation Results

We next present the estimation results for our benchmark $G^J(4)$ model. First, Table 2 documents that the model fits all of the nominal yields well, as the overall root mean-squared error (RMSE) is only 8.64 basis points.

The summary statistics of the fitted errors for each JGBi calculated as described in equa-

[20]One could in theory identify the remaining volatility parameters from the value of the deflation protection options embedded in the Japanese inflation-indexed bonds issued since 2013. However, these bonds are quite limited in number and sample period, and we are doubtful that the identification of these parameters in this manner would be successful.

JGBi (coupon, maturity)	Pricing errors	
	Mean	RMSE
(1) 1.2% 3/10/2014	-6.13	15.29
(2) 1.1% 6/10/2014	6.84	14.59
(3) 0.5% 12/10/2014	-1.38	9.39
(4) 0.5% 6/10/2015	6.84	11.48
(5) 0.8% 9/10/2015	2.96	7.69
(6) 0.8% 12/10/2015	-0.40	9.93
(7) 0.8% 3/10/2016	-1.52	8.25
(8) 1% 6/10/2016	1.21	10.29
(9) 1.1% 9/10/2016	-4.62	8.20
(10) 1.1% 12/10/2016	-4.64	7.28
(11) 1.2% 3/10/2017	-5.98	10.70
(12) 1.2% 6/10/2017	0.83	5.75
(13) 1.3% 9/10/2017	-1.68	5.05
(14) 1.2% 12/10/2017	0.12	7.11
(15) 1.4% 3/10/2018	-3.18	11.26
(16) 1.4% 6/10/2018	7.44	13.62
(17) 0.1% 9/10/2023	5.87	11.43
(18) 0.1% 3/10/2024	2.31	4.27
(19) 0.1% 9/10/2024	-1.21	3.84
(20) 0.1% 3/10/2025	-1.47	3.18
(21) 0.1% 3/10/2026	-3.28	3.70
(22) 0.1% 3/10/2027	-3.65	4.70
(23) 0.1% 3/10/2028	0.96	2.58
(24) 0.1% 3/10/2029	2.10	2.46
All yields	0.00	9.56
Max $\mathcal{L}^{EKF}$	18,361.56	

Table 4: **Pricing Errors of Japanese Real Government Bond Yields to Maturity**
This table reports the mean pricing errors (Mean) and the root mean-squared pricing errors (RMSE) of Japanese inflation-indexed bond (JGBi) yields to maturity in our benchmark $G^J(4)$ model. The errors are computed as the difference between the observed yield to maturity downloaded from Bloomberg and the corresponding model-implied yield. All errors are reported in basis points.

tion (3) are reported in Table 4. The RMSE for all yield errors combined is 9.56 basis points, which is only slightly above the corresponding statistic for the nominal yields. As such, we consider the model's fit to the real yield data to be satisfactory as well.

We also find that the estimated measurement error standard deviations within our benchmark model are $\sigma_\varepsilon^N = 0.0011$, $\sigma_\varepsilon^R = 0.0010$, and $\sigma_\varepsilon^{CF} = 0.0013$, which also match well with the properties of the corresponding fitted error series.

Second, we report the estimated dynamic parameters of our benchmark $G^J(4)$ model in Table 3. The volatility parameters in the Σ matrix are estimated with precision. For the mean-reversion parameters in the $K^{\mathbb{P}}$ matrix and the mean parameters in the $\theta^{\mathbb{P}}$ vector, the results are more mixed in that some of them are highly statistically significant, while others are clearly insignificant.

4 Deflation Risk Analysis

In this section, we first assess how the actual and priced probability of deflation has evolved since 2005. We then analyze the value of the deflation protection enhancement offered by JGBi's issued since 2013 and study its impact on our estimates of BEI rates.

4.1 Calculation of Deflation Probabilities

Using the estimated benchmark $G^J(4)$ model, we can examine whether the change in the price index (i.e., the inflation rate) from time t to $t+\tau$ will fall below a certain critical level q. This event is denoted as

$$\frac{\Pi_{t+\tau}}{\Pi_t} = e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds} \leq (1+q).$$

Taking logs, we get

$$Y_{t,t+\tau} = \ln\Big(\frac{\Pi_{t+\tau}}{\Pi_t}\Big) = \int_t^{t+\tau}(r_s^N - r_s^R)ds \leq \ln(1+q).$$

As shown in Christensen et al. (2012), the conditional distribution of this integral term is

$$Y_{t,t+\tau} \sim N\Big(m_Y^{\mathbb{P}}(t,\tau), \sigma_Y^{\mathbb{P}}(\tau)^2\Big),$$

where $m_Y^{\mathbb{P}}(t,\tau)$ and $\sigma_Y^{\mathbb{P}}(\tau)^2$ are the distribution's conditional mean and variance, respectively, under the real-world $\mathbb{P}$ probability measure.[21] The probability of the change in the price index being below the critical level q is therefore equivalent to

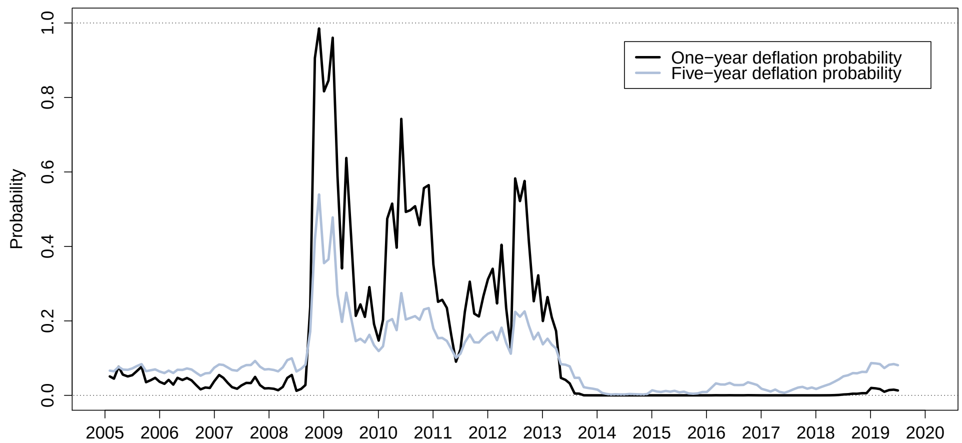
$$Prob_t\big(Y_{t,t+\tau} \leq \ln(1+q)\big) = Prob_t\left(\frac{Y_{t,t+\tau} - m_Y^{\mathbb{P}}(t,\tau)}{\sigma_Y^{\mathbb{P}}(\tau)} \leq \frac{\ln(1+q) - m_Y^{\mathbb{P}}(t,\tau)}{\sigma_Y^{\mathbb{P}}(\tau)}\right) = \Phi\Big(\frac{\ln(1+q) - m_Y^{\mathbb{P}}(t,\tau)}{\sigma_Y^{\mathbb{P}}(\tau)}\Big).$$

In particular, to assess deflationary outcomes we fix $q = 0$ to obtain

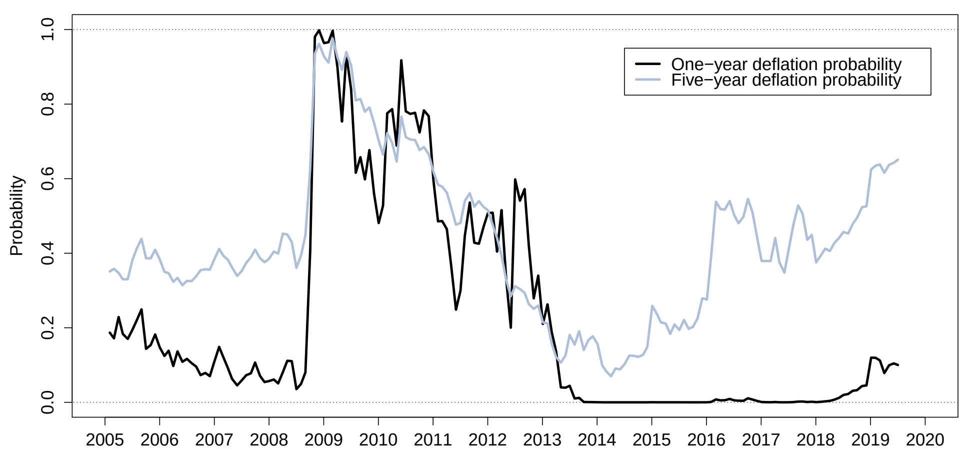
$$Prob_t\big(Y_{t,t+\tau} \leq 0\big) = \Phi\Big(\frac{-m_Y^{\mathbb{P}}(t,\tau)}{\sigma_Y^{\mathbb{P}}(\tau)}\Big).$$

Figure 6 illustrates the actual and priced probabilities of deflation according to the $G^J(4)$ model. The objective deflation probabilities have been negligible since the spring of 2013. However, priced long-term deflation probabilities have trended up since 2014 and are elevated

[21] Risk-neutral inflation probabilities are readily obtained by replacing the real-world dynamics of the state variables with their risk-neutral dynamics.



(a) Objective $\mathbb{P}$-measure



(b) Risk-neutral $\mathbb{Q}$-measure

Figure 6: **Estimated Deflation Probabilities**
Panel (a) shows the estimated probability of net deflation over the next one- and five-year period under the objective $\mathbb{P}$ probability measure. Panel (b) shows the corresponding probabilities estimated under the risk-neutral $\mathbb{Q}$-measure.

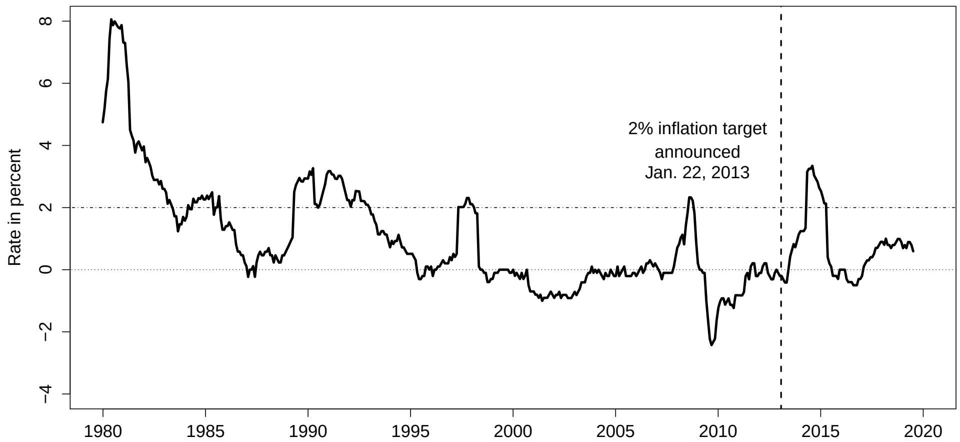


Figure 7: **Japanese CPI Inflation ex Fresh Food**

at the end of our sample period. In the following, we therefore explore how these developments have affected the value of deflation protection offered by recently issued JGBi's.

4.2 Deflation Option Values

In this section, we examine the estimated values of the deflation protection options embedded in the JGBi's issued since 2013.

For perspective, Figure 7 shows the year-over-year change in the Japanese Consumer Price Index (CPI) excluding fresh food since 1980.[22] Consumer price inflation in Japan has been persistently low since the mid-1990s, with extended spells of deflation interrupted by brief short-lived upticks in inflation. As a result, many of the inflation index ratios ($\frac{\Pi_t}{\Pi_{t_0}}$) for the JGBi's in our sample issued before 2013 have extended periods with inflation index ratios below one, as shown in Figure 8. The deflation protection option is likely to be of significant value for these bonds.

Figure 9 shows the estimated option value of deflation protection for each JBGi as implied by our benchmark $G^J(4)$ model. We measure this option value as a yield spread between the model-implied yield to maturity based on the fitted price of a bond *without* deflation protection and the model-implied yield to maturity based on the price of a fitted bond *with* the deflation protection enhancement included. As expected, the deflation protection option values are typically between 50 and 100 basis points since their launch in 2013. Neglecting this enhancement would therefore result in substantive errors in estimating expected inflation compensation from Japanese bond yields.[23]

[22]This is the price index targeted by the BoJ.

[23]Figure 9 also demonstrates that as inflation was also low or negative during the global financial crisis, such options also would have been of considerable value at that time, had they been included in JGBi's.

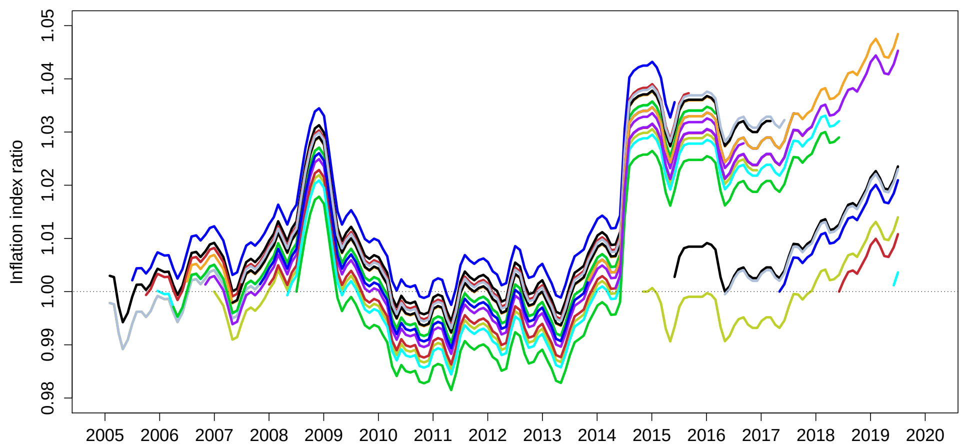


Figure 8: **Inflation Index Ratios of Japanese Inflation-Indexed Bonds**
Shown are the inflation index ratios ($\frac{\Pi_t}{\Pi_{t_0}}$) for all 24 JGBi's in our sample.

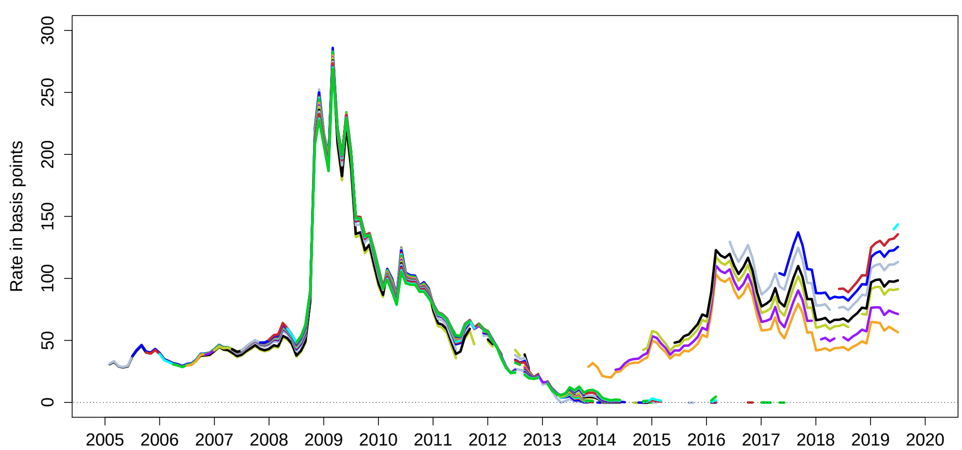


Figure 9: **Value of Deflation Protection Options in Japanese Inflation-Indexed Bonds**
Estimated values of deflation protection options implied by our benchmark $G^J(4)$ model for all 24 JGBi's in our sample. Note that only JGBi's issued since 2013 offer the deflation protection.

4.2.1 Deflation Option Values Measured as Par Yield Spreads

To have a consistent measure of deflation protection values across time, which is not affected by variation in inflation index ratios, coupon differences, and maturity mismatches, we follow Christensen et al. (2012) and construct synthetic ten-year real par-coupon yield spreads.

We calculate the deflation option values by comparing the prices of a newly issued JGBi without any accrued inflation compensation, but with deflation protection and a similar JGBi that does not offer this protection. First, consider the latter hypothetical JGBi with T years remaining to maturity that pays an annual coupon C semi-annually. As this bond does not offer any deflation protection, its par coupon is determined by the equation

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q[e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q[e^{-\int_t^T r_s^R ds}] = 1.$$

The first term is the sum of the present value of the $2T$ coupon payments using the model's fitted real yield curve at day t. The second term is the discounted value of the principal payment. We denote the coupon rate that solves this equation as C_{NO}.

Next, consider the corresponding JGBi with deflation protection, but no accrued inflation compensation. Since its coupon payments are not protected against deflation, the difference is in accounting for the deflation protection on the principal payment as explained in Section 3.3. Therefore, the par coupon for this bond is given by the solution to the following equation

$$\sum_{i=1}^{2T} \frac{C}{2} E_t^Q[e^{-\int_t^{t_i} r_s^R ds}] + E_t^Q\left[e^{-\int_t^T r_s^R ds}\right] + \left[E_t^Q\left[e^{-\int_t^T r_s^N ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq 1\}}\right] - E_t^Q\left[e^{-\int_t^T r_s^R ds}\mathbf{1}_{\{\frac{\Pi_T}{\Pi_t}\leq 1\}}\right]\right] = 1,$$

where the last term on the left-hand side represents the net present value of the deflation protection of the principal in the JGBi contract.[24] We denote as C_O the par-coupon yield of the new hypothetical JGBi that solves this equation.

The difference between C_{NO} and C_O is a measure of the advantage of holding a newly issued JGBi at the inflation adjustment floor. Figure 2 in the introduction shows the difference between the C_{NO} and C_O values that solve the pricing equations at the ten-year maturity using our estimated benchmark $G^J(4)$ model.[25] Prior to the financial crisis, the differences between the two synthetic JGBi yields were averaging less than 50 basis points. However, the yield differences then spiked wth the onset of the crisis. After the crisis ended, the yield difference gradually declined and reached a bottom in the spring of 2013 when hopes for the success of Abenomics were at their peak. Since then, the yield difference has trended higher again, reaching a plateau near 100 basis points in early 2016 where it has remained until the end of our sample.[26]

[24] The online supplementary appendix explains how these contingent conditional expectations are calculated within the $G^J(4)$ model using the contingent claim pricing results of Duffie et al. (2000).

[25] In online Appendix D, we document that the reported results are insensitive to the inclusion of the survey inflation forecasts in the model estimation, while they are sensitive to including the option adjustment for obvious reasons.

[26] The sizable yield spread suggests that seasoned pre-2013 and more recently post-2013 JGBi's should not be pooled to construct real yield curves without correcting for the value of the deflation protection.

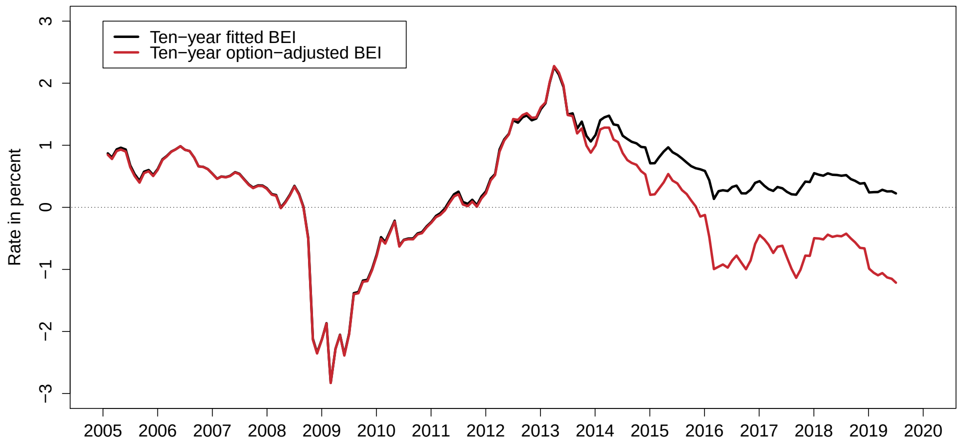


Figure 10: **Effect of Option Adjustment on Ten-Year BEI**

4.3 Deflation Option-Adjusted Breakeven Inflation

To illustrate the impact of the deflation protection enhancement on our estimate of breakeven inflation (BEI), we take fitted BEI from our benchmark $G^J(4)$ model estimated without either option adjustment or survey information and compare it to the option-adjusted estimate of BEI from the same model estimated with option adjustment, but without the survey information. While the former represents a flexible fit to the raw bond price data, the latter provides the cleanest direct estimate of the option-adjusted BEI rates.

Figure 10 plots the ten-year BEI estimated in these two ways. Since the launch of the option-enhanced bonds in late 2013, there is a wide and sustained wedge between the estimates of BEI, with an average slightly above 100 basis points since 2016. Importantly, the option-adjusted BEI is below the fitted BEI from a standard smoothing of the observed JGBi prices. This is because the JGBi prices adjusted for the deflation protection are below the observed prices, which converts into a higher option-adjusted real yield or, equivalently, a lower option-adjusted BEI. Thus, it is crucial to account for the deflation option values in estimating BEI. Indeed, failure to account for the deflation protection enhancement results in substantive overestimation of BEI rates.[27]

5 Monetary Policy under Shinzo Abe

In this section, we use our estimated benchmark model to evaluate the immediate market reactions to policy actions undertaken by the Bank of Japan since Shinzo Abe reassumed

[27]While our study is the first to our knowledge to account for the deflation protection enhancement in Japanese bonds, Grishchenko et al. (2016) analyze the deflation option values embedded in U.S. TIPS prices, while Fleckenstein et al. (2017) study the price of deflation risk in the U.S. inflation swaption market.

No.	Date	Announcement description
I	Jan. 22, 2013	Introduction of price stability target defined as a 2 percent year-over-year change in the all items consumer price index (CPI) excluding fresh food to be achieved at the earliest possible time. Also, the existing asset purchase program will be completed by January 2014 as planned and followed by open-ended asset purchases.
II	Apr. 4, 2013	Introduction of quantitative and qualitative easing (QQE) of monetary policy. ¥60-70 trillion annual increase in the monetary base. Aim to achieve price stability target within about two years.
III	Oct. 31, 2014	Expansion of the QQE policy to ¥80 trillion annual increase in the monetary base.
IV	Jan. 29, 2016	Introduction of negative interest rates.
V	Sep. 21, 2016	Introduction of yield curve control and commitment to overshoot the 2 percent price stability target.
VI	Jul. 31, 2018	Strengthening of the framework for continuous powerful monetary easing and commitment to maintain existing extremely low levels of short- and long-term interest rates for an extended period of time.

Table 5: **Key Policy Announcements by the Bank of Japan**

power in December 2012.

5.1 Key Monetary Policy Changes

We consider six key policy announcements, which are listed in Table 5. These include the introduction of an explicit inflation target and open-ended expansion of the asset purchase program on January 22, 2013; the introduction of quantitative and qualitative easing (QQE) policy on April 4, 2013; the expansion of the QQE program on October 31, 2014; the movement by the BoJ into negative policy rates on January 29, 2016; the introduction of "yield curve control" by the BoJ on September 21, 2016 in addition to a commitment to overshoot its two-percent inflation target; and the strengthening of the framework for continuous powerful monetary easing announced on July 31, 2018.[28]

5.2 Bond Market Results

Since bond prices, like other asset prices, are the result of transactions between forward-looking investors, any effects of policy changes should be reflected in bond prices on announcement, rather than implementation. For there to be a price response to an announcement, it must contain new information about future policy, or about the relative demand

[28] Arai (2017) also performs a high-frequency event study of BoJ policy announcements, but his data ends in July 2013 and therefore only offers an early assessment of BoJ policies under Abe. Furthermore, his main focus is on the pass-through of monetary policy shocks to corporate bonds, stocks, and the exchange rate.

for and supply of assets going forward. We think that this characterization is reasonable for the announcements we study here. While policy action was expected, the exact timing and content are likely to have been at least partially a surprise. Still, the interpretation of market movements around these announcements should be interpreted as the impact of their surprise components, and in particular not of the anticipated policy changes.

We use a one-day window as the baseline for the event study, in line with the literature on the U.S. experience with unconventional monetary policy (see, e.g., Krishnamurthy and Vissing-Jorgensen, 2011). A narrow window is sufficient thanks to the size and depth of the Japanese government bond market, which allows adequate time for market participants to digest and trade on the new information. Furthermore, a narrow window minimizes the risk of confounding factors polluting the measurement of the announcement effects. By investigating the change between the day before the announcements and the day of the announcements, we allow for enough time for the response to materialize after each announcement.

Table 6 reports the one-day changes in five key BEI rates in response to the six considered announcements. We report daily changes for fitted BEI rates from the benchmark $G^J(4)$ model incorporating survey data without (top panel) and with (bottom panel) adjustments for the deflation protection option values.[29]

Our results demonstrate that the value of the deflation protection option is generally decreasing with policy announcements signaling enhancement or implementation of the Abenomics program. As a result, estimated changes in inflation expectations on these announcement dates are smaller than would be obtained without adjusting for changes in deflation expectations. For example, fitting our benchmark model without adjusting for changes in deflation protection yields estimates of changes in the five-year and ten-year BEI rates of 11.1 and 7.7 basis points, respectively, over our event window for the announcement of the BoJ adoption of an explicit two-percent inflation target (event I). However, after adjusting for the deflation protection option, the changes are more modest, at 9.1 and 6.5 basis points, respectively. Similarly, without adjusting for the change in the value of the deflation protection option, we would conclude that the adoption of yield curve control had pushed up the five- and ten-year yields by 0.5 and 1.1 basis points, respectively. However, after adjusting for the deflation protection option we estimate that both yields actually fell.

Other events yielded similar results. The lone exception is the April 3, 2013 event, which announced the launch of QQE. For that event, we obtain a surprising estimate of a 4.9 basis point decline over our event window without the inflation protection option adjustment. This estimated change is attenuated to a decline of 2.9 basis points with the deflation protection adjustment included. Nevertheless, five out of our six events (and all of the ones with an estimated positive change in the ten-year yield without the deflation protection option adjustment) find a lower change in the ten-year yield after controlling for deflation protection.

[29] In online Appendix E, we report the one-day changes in observable nominal yields and matching fitted real yields across five maturities.

Event		Fitted BEI				
		1-year	2-year	5-year	7-year	10-year
I	Jan. 21, 2013	28.7	49.4	93.8	115.0	140.6
	Jan. 22, 2013	48.7	66.1	104.9	124.3	148.2
	Change	**20.0**	**16.6**	**11.1**	**9.2**	**7.7**
II	Apr. 3, 2013	113.9	132.2	170.9	189.0	210.8
	Apr. 4, 2013	121.7	136.6	169.6	185.8	205.8
	Change	**7.8**	**4.4**	**-1.3**	**-3.2**	**-4.9**
III	Oct. 30, 2014	452.6	346.6	178.4	128.0	90.3
	Oct. 31, 2014	457.5	350.9	181.8	131.1	93.1
	Change	**4.9**	**4.4**	**3.4**	**3.0**	**2.7**
IV	Jan. 28, 2016	280.3	209.3	96.6	62.6	37.2
	Jan. 29, 2016	269.0	201.3	94.1	61.9	38.1
	Change	**-11.3**	**-8.0**	**-2.5**	**-0.7**	**0.9**
V	Sep. 20, 2016	205.8	148.6	57.9	30.6	10.3
	Sep. 21, 2016	204.9	148.3	58.4	31.4	11.4
	Change	**-0.9**	**-0.4**	**0.5**	**0.8**	**1.1**
VI	Jul. 30, 2018	98.2	82.0	58.8	53.6	52.2
	Jul. 31, 2018	101.1	84.4	60.5	55.1	53.4
	Change	**2.9**	**2.4**	**1.7**	**1.4**	**1.3**

Event		Option-Adjusted BEI				
		1-year	2-year	5-year	7-year	10-year
I	Jan. 21, 2013	31.3	50.8	94.8	116.7	143.1
	Jan. 22, 2013	46.1	63.6	104.0	124.4	149.6
	Change	**14.8**	**12.8**	**9.1**	**7.8**	**6.5**
II	Apr. 3, 2013	110.5	129.7	171.8	192.0	215.8
	Apr. 4, 2013	114.4	132.0	171.1	190.2	212.9
	Change	**3.9**	**2.3**	**-0.6**	**-1.8**	**-2.9**
III	Oct. 30, 2014	443.8	346.2	172.7	112.9	63.0
	Oct. 31, 2014	447.1	349.1	174.7	114.5	64.2
	Change	**3.4**	**2.9**	**2.0**	**1.6**	**1.3**
IV	Jan. 28, 2016	299.4	216.1	66.7	14.1	-30.5
	Jan. 29, 2016	280.7	200.2	55.8	5.1	-38.0
	Change	**-18.7**	**-15.9**	**-11.0**	**-9.1**	**-7.4**
V	Sep. 20, 2016	212.6	136.2	-1.2	-49.7	-91.2
	Sep. 21, 2016	212.7	136.2	-1.3	-49.8	-91.3
	Change	**0.1**	**0.0**	**-0.1**	**-0.1**	**-0.1**
VI	Jul. 30, 2018	150.3	106.7	29.5	3.2	-18.3
	Jul. 31, 2018	151.8	108.1	30.7	4.3	-17.2
	Change	**1.6**	**1.4**	**1.2**	**1.1**	**1.1**

Table 6: **One-Day Responses of Japanese BEI**

The table reports the one-day response of Japanese BEI at five different maturities around the BoJ announcement dates. All numbers are measured in basis points.

Note that long-term inflation expectations appear to remain well-anchored throughout our estimation period. Only the first event that provides a meaningful upward push to five- and ten-year BEI rates, while movements around the others fail to meaningfully raise inflation expectations. Indeed, we find that the introduction of negative interest rates in January 2016 resulted in a sizable drop across all maturities of BEI rates once one accounts for the value of the deflation protection option.

5.3 Yield Decompositions with Term Structure Models

We next use the benchmark $G^J(4)$ model to decompose the one-day bond yield reactions with and without option adjustment, both estimated with daily data. We focus on ten-year yields in our analysis. Ten-year yields are commonly used as the benchmark long-term yield in most government bond markets around the world, including Japan. They are also key long-term rates of interest for monetary policy, and have served as the most popular maturity for studies of financial market reactions to unconventional monetary policies.[30] On a practical note, ten-year yields are the longest maturity represented in our data for both nominal and real bonds, and Japanese short- and medium-term nominal yields were constrained near the zero lower bound for most of our sample (see Figure 3).

5.3.1 Nominal Yield Decompositions

Recall that nominal term premia are defined as the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal short rate r_t^N

$$TP_t^N(\tau) = y_t^N(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^N]ds.$$

Figure 11 shows the nominal yield decomposition at the ten-year maturity since 2005.[31] We note that the average expected nominal short rates over the next ten-years have fluctuated around zero during our sample period. As a consequence, our model attributes the 2 percentage point declines in the ten-year nominal yield since 2006 almost entirely to declines in the ten-year nominal term premium. However, we do note a softening in the nominal short rate expectations component during the 2012 Abe campaign.

We can map these results to the event study by looking at the daily change in the ten-year nominal yield decomposition around the six key BoJ announcements analyzed in the paper. Specifically, the models are used to decompose the observed nominal zero-coupon yields into three components:

[30] For example, see Gagnon et al. (2011), Christensen and Rudebusch (2012), and Christensen and Krogstrup (2019).

[31] The decomposition starts in 2005 because the expectations appearing in the definition of the term premium are functions of the real yield factors, which are not identified prior to 2005.

Decomposition from $G^J(4)$ model without option adjustment					
Event		Avg. short rate next ten years	Ten-year term premium	Residual	Ten-year yield
I	Jan. 21, 2013	-35	99	15	78
	Jan. 22, 2013	-35	99	15	78
	Change	**0**	**0**	**0**	**0**
II	Apr. 3, 2013	-41	91	4	54
	Apr. 4, 2013	-40	88	-5	42
	Change	**1**	**-3**	**-10**	**-11**
III	Oct. 30, 2014	2	37	6	44
	Oct. 31, 2014	2	37	5	44
	Change	**0**	**0**	**-1**	**0**
IV	Jan. 28, 2016	-4	21	4	20
	Jan. 29, 2016	-6	19	-2	11
	Change	**-2**	**-1**	**-6**	**-9**
V	Sep. 20, 2016	-11	-2	7	-6
	Sep. 21, 2016	-11	-2	8	-5
	Change	**0**	**1**	**0**	**1**
VI	Jul. 30, 2018	-22	24	7	9
	Jul. 31, 2018	-22	24	4	6
	Change	**0**	**0**	**-3**	**-3**

Decomposition from $G^J(4)$ model with option adjustment					
Event		Avg. short rate next ten years	Ten-year term premium	Residual	Ten-year yield
I	Jan. 21, 2013	-38	102	15	78
	Jan. 22, 2013	-38	102	15	78
	Change	**0**	**0**	**0**	**0**
II	Apr. 3, 2013	-43	93	4	54
	Apr. 4, 2013	-43	91	-5	42
	Change	**1**	**-2**	**-10**	**-11**
III	Oct. 30, 2014	-3	41	6	44
	Oct. 31, 2014	-3	41	6	44
	Change	**0**	**0**	**0**	**0**
IV	Jan. 28, 2016	-2	17	5	20
	Jan. 29, 2016	-3	15	-1	11
	Change	**-1**	**-3**	**-6**	**-9**
V	Sep. 20, 2016	-4	-8	7	-6
	Sep. 21, 2016	-4	-8	7	-5
	Change	**0**	**0**	**0**	**1**
VI	Jul. 30, 2018	-15	17	7	9
	Jul. 31, 2018	-15	17	4	6
	Change	**0**	**0**	**-3**	**-3**

Table 7: **Decomposition of One-Day Responses of Nominal Ten-Year Yield**
The decomposition of one-day responses of the Japanese ten-year nominal government bond yield on six BoJ announcement dates into changes in (i) the average expected nominal short rate over the next ten years, (ii) the ten-year term premium, and (iii) the unexplained residual based on the $G^J(4)$ model estimated with daily data and including the Consensus Forecasts of ten-year CPI inflation. All numbers are measured in basis points.

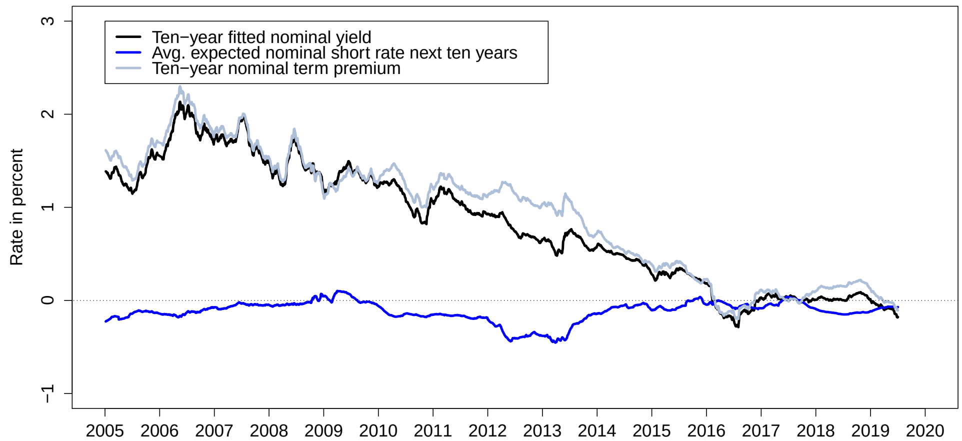


Figure 11: **Ten-Year Nominal Yield Decomposition**

(i) the estimated average expected nominal short rate until maturity;

(ii) the term premium defined as the difference between the model-fitted nominal yield and the average expected nominal short rate; and

(iii) a residual that reflects variation not accounted for by the model.

The results of these daily decompositions are reported in Table 7. In light of the relatively stable nominal short-rate expectations component in Figure 11, it is not surprising that most of the reaction of the ten-year nominal yield to the six key BoJ announcements are ascribed to either the ten-year nominal term premium or the unexplained residual. Indeed, for the two largest reactions on April 4, 2013 and January 29, 2016, most of the decline in the nominal ten-year yield is accounted for by the unexplained residuals, and this holds independent of the option adjustment, which matters little for the model fit of nominal yields.

5.3.2 Real Yield Decompositions

Similarly, real term premia are defined as the difference in expected real return between a buy and hold strategy for a τ-year real bond and an instantaneous rollover strategy at the risk-free real rate r_t^R

$$TP_t^R(\tau) = y_t^R(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^R]ds.$$

Figure 12 shows the real yield decomposition at the ten-year maturity since 2005. We note that the average expected real short rates over ten-year periods have fluctuated systematically in negative territory around a level slightly below negative one percent during the shown period. As a consequence, practically all the variation in the ten-year option-adjusted real

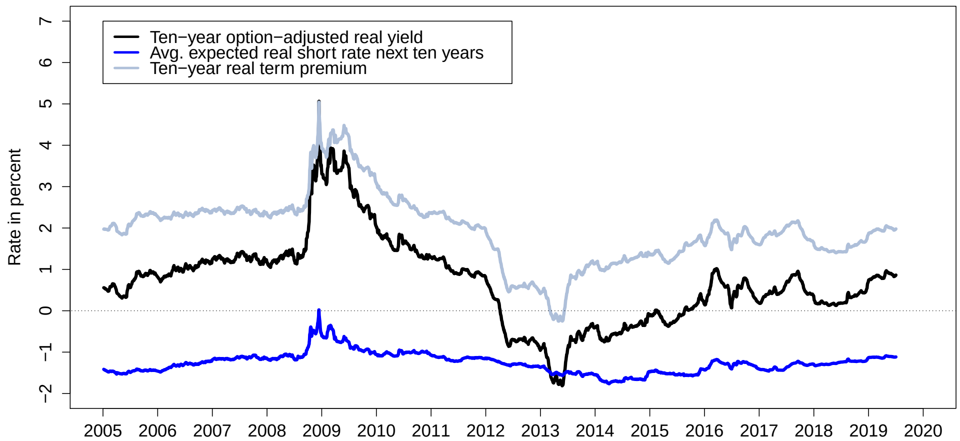


Figure 12: **Ten-Year Option-Adjusted Real Yield Decomposition**

yield is driven by changes in the ten-year real term premium, which has remained positive throughout our sample period except for a short-lived drop below zero in the spring of 2013.

For our event study, we again use the daily change in the ten-year real yield decomposition around the six key BoJ announcements. However, we do not observe the ten-year real yield directly. This leaves us with no residual analogous to that which we used in the analysis of the ten-year nominal yield. Instead, we use our model to decompose the fitted real zero-coupon yields into two components:

(i) the estimated average expected real short rate until maturity and

(ii) the term premium defined as the difference between the model-fitted real yield and the average expected real short rate.

The result of these daily decompositions are reported in Table 8. As with the nominal yield decompositions, it is again not too surprising that the model indicates that most of the real yield response came about through changes in the real term premium, rather than through expectations about future real short rates.

As for the real yield reaction overall, we note that the option adjustment tends to temper the estimated reaction. For the January 29, 2016 announcement, this effect is so large that the negative real yield response estimated without option adjustment turns positive after its inclusion. Hence, the introduction of negative nominal short rates *pushed up* real yields.

5.3.3 Breakeven Inflation Decompositions

Recall that the decomposition of the BEI rates is given by

Decomposition from $G^J(4)$ model without option adjustment				
Event		Avg. short rate next ten years	Ten-year term premium	Ten-year yield
I	Jan. 21, 2013	-138	61	-77
	Jan. 22, 2013	-141	56	-84
	Change	**-3**	**-5**	**-8**
II	Apr. 3, 2013	-159	-3	-161
	Apr. 4, 2013	-159	1	-158
	Change	**0**	**3**	**3**
III	Oct. 30, 2014	-163	111	-52
	Oct. 31, 2014	-164	110	-55
	Change	**-1**	**-2**	**-2**
IV	Jan. 28, 2016	-140	119	-21
	Jan. 29, 2016	-141	115	-25
	Change	**-1**	**-3**	**-4**
V	Sep. 20, 2016	-138	115	-23
	Sep. 21, 2016	-138	114	-24
	Change	**0**	**-1**	**-1**
VI	Jul. 30, 2018	-131	81	-50
	Jul. 31, 2018	-131	80	-51
	Change	**-1**	**-1**	**-1**

Decomposition from $G^J(4)$ model with option adjustment				
Event		Avg. short rate next ten years	Ten-year term premium	Ten-year yield
I	Jan. 21, 2013	-134	54	-79
	Jan. 22, 2013	-136	50	-86
	Change	**-2**	**-4**	**-6**
II	Apr. 3, 2013	-151	-15	-166
	Apr. 4, 2013	-151	-14	-165
	Change	**0**	**1**	**1**
III	Oct. 30, 2014	-165	140	-25
	Oct. 31, 2014	-166	140	-26
	Change	**0**	**-1**	**-1**
IV	Jan. 28, 2016	-135	181	46
	Jan. 29, 2016	-134	183	50
	Change	**2**	**2**	**4**
V	Sep. 20, 2016	-124	203	79
	Sep. 21, 2016	-124	204	79
	Change	**0**	**1**	**1**
VI	Jul. 30, 2018	-123	143	20
	Jul. 31, 2018	-123	142	19
	Change	**0**	**-1**	**-1**

Table 8: **Decomposition of One-Day Responses of the Real Ten-Year Yield**
The decomposition of one-day responses of the Japanese ten-year real government bond yield on six BoJ announcement dates into changes in (i) the average expected real short rate over the next ten years and (ii) the ten-year term premium based on the $G^J(4)$ model estimated with daily data and including the Consensus Forecasts of ten-year CPI inflation. All numbers are measured in basis points.

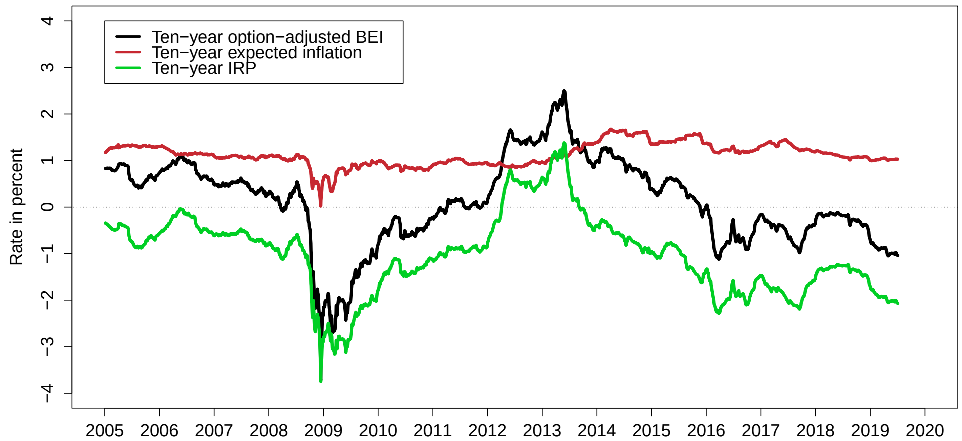


Figure 13: **Ten-Year Option-Adjusted BEI Decomposition**

$$BEI_t(\tau) = y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

where $\pi_t^e(\tau)$ is the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$, while $\phi_t(\tau)$ is the associated inflation risk premium.

Figure 13 shows the decomposition of the ten-year fitted option-adjusted BEI. As ten-year expected inflation has remained stable at a level slightly above one percent since 2005, the large variation in the fitted ten-year BEI is almost entirely driven by changes in the inflation risk premium, which has been negative most of this period.

The inflation risk premium did turn positive during 2012, coinciding with increasing optimism about the Abe reforms. However, it has been on a downward trajectory since the spring of 2013. The negative inflation risk premium that prevailed since that date implies that bond investors view future economic downturns as likely to coincide with low inflation.

Finally, Table 9 reports the daily changes in the ten-year BEI decomposition around the six BoJ announcements. The six events in our study do not appear to have played a major role in the persistent changes in BEI in Figure 13. While the first event, the introduction of the 2-percent inflation target and the expansion of the asset purchase program, helped push up both inflation expectations and inflation risk premia, this was almost offset by the second event announcing the launch of the QQE program.

More importantly, adjusting for the deflation option turns out to be critical for the assessment of the financial market reaction to these announcements. The most notable case is the January 29, 2016 introduction of negative interest rates. When we exclude the deflation protection option in the valuation of JGBi, the $G^J(4)$ model indicates that the announcement resulted in a slight firming in the ten-year BEI, driven by an increase in the inflation risk

Decomposition from $G^J(4)$ model without option adjustment				
Event		Ten-year exp. inflation	Ten-year IRP	Ten-year BEI
I	Jan. 21, 2013	100	41	141
	Jan. 22, 2013	103	45	148
	Change	**3**	**4**	**8**
II	Apr. 3, 2013	115	96	211
	Apr. 4, 2013	116	90	206
	Change	**1**	**-6**	**-5**
III	Oct. 30, 2014	163	-73	90
	Oct. 31, 2014	164	-71	93
	Change	**1**	**2**	**3**
IV	Jan. 28, 2016	133	-96	37
	Jan. 29, 2016	132	-94	38
	Change	**-1**	**2**	**1**
V	Sep. 20, 2016	125	-115	10
	Sep. 21, 2016	125	-114	11
	Change	**0**	**1**	**1**
VI	Jul. 30, 2018	106	-54	52
	Jul. 31, 2018	107	-53	53
	Change	**1**	**1**	**1**

Decomposition from $G^J(4)$ model with option adjustment				
Event		Ten-year exp. inflation	Ten-year IRP	Ten-year BEI
I	Jan. 21, 2013	94	49	143
	Jan. 22, 2013	97	53	150
	Change	**3**	**4**	**7**
II	Apr. 3, 2013	106	110	216
	Apr. 4, 2013	107	106	213
	Change	**1**	**-4**	**-3**
III	Oct. 30, 2014	161	-98	63
	Oct. 31, 2014	162	-98	64
	Change	**0**	**1**	**1**
IV	Jan. 28, 2016	132	-163	-31
	Jan. 29, 2016	129	-167	-38
	Change	**-3**	**-5**	**-7**
V	Sep. 20, 2016	119	-210	-91
	Sep. 21, 2016	119	-210	-91
	Change	**0**	**0**	**0**
VI	Jul. 30, 2018	106	-125	-18
	Jul. 31, 2018	107	-124	-17
	Change	**0**	**1**	**1**

Table 9: **Decomposition of One-Day Responses of the Ten-Year BEI**
The decomposition of one-day responses of the Japanese ten-year BEI on six BoJ announcement dates into changes in (i) the ten-year expected inflation and (ii) the ten-year inflation risk premium (IRP) based on the $G^J(4)$ model estimated with daily data and including the Consensus Forecasts of ten-year CPI inflation. All numbers are measured in basis points.

premium. However, once we account for the option value, the model shows a large drop in the ten-year option-adjusted BEI of 7 basis points driven by declines in both the ten-year expected inflation rate and the ten-year inflation risk premium.[32]

6 Conclusion

This paper uses an arbitrage-free term structure model of nominal and real yields on Japanese government bonds to evaluate the impact of news associated with policy announcements under "Abenomics," the extensive reform program adopted under Japanese Prime Minister Shinzo Abe. To our knowledge, our analysis is the first to assess the impact of these announcements with proper adjustment for the deflation protection enhancements embedded in the inflation-indexed bonds issued under Abe. We demonstrate that due to Japan's long experience with low inflation and pessimistic outlook, the value of these enhancements were typically large, ranging from 50-100 basis points since they were included in 2013. Moreover, they are volatile, suggesting that their incorporation would also be influential in the determination of high-frequency analyses of the impacts of policy announcements under Abenomics.

We confirm this conjecture in an event study of six important policy announcements under Abenomics from January 2013 through July 31, 2018. Our results demonstrate that changes in inflation expectations on our announcement dates were generally smaller and less optimistic than one would obtain without the deflation protection adjustment. As such, one might conclude that the Abenomics program was not as "disappointing" as early analysis indicated. Our results suggest that market participants were skeptical about the prospects for Abenomics to engineer an escape from Japan's low inflation environment from the beginning. However, one caveat is that the market's skepticism may have been driven by pessimism about the government's implementation of its announced policy reform.

[32]As shown in online appendix C, we get qualitatively similar results in the event study when we drop the survey inflation forecasts from the model estimation.

Appendix: Bond Yield Decomposition

In this appendix, we describe the decomposition of nominal and real bond yields into underlying expectations and residual risk premium components using arbitrage-free term structure models.

We follow Merton (1974) and assume the existence of a continuously-traded continuum of nominal and real zero-coupon bonds. This implies that inflation risk is spanned by the nominal and real yields. This allows us to decompose the nominal and real yields into the sum of the corresponding short-rate expectations and associated term premia using our arbitrage-free term structure model.

To begin, define the nominal and real stochastic discount factors as M_t^N and M_t^R, respectively. Their dynamics are standard and given by

$$\begin{aligned} dM_t^N/M_t^N &= -r_t^N dt - \Gamma_t' dW_t^{\mathbb{P}}, \\ dM_t^R/M_t^R &= -r_t^R dt - \Gamma_t' dW_t^{\mathbb{P}}, \end{aligned}$$

where Γ_t contains the risk premia.

Under our no-arbitrage condition, the price of a nominal bond that pays one unit of currency in τ years and the price of a real bond that pays one consumption unit in τ years must satisfy

$$P_t^N(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] \quad \text{and} \quad P_t^R(\tau) = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right],$$

where $P_t^N(\tau)$ and $P_t^R(\tau)$ are the prices of the zero-coupon, nominal and real bonds for maturity τ at time t and $E_t^{\mathbb{P}}[.]$ is the conditional expectations operator under the real-world (or $\mathbb{P}$-) probability measure.

The no-arbitrage condition also requires that the price of a consumption unit, denoted as the overall price level Π_t, is the ratio of the real and nominal stochastic discount factors:

$$\Pi_t = \frac{M_t^R}{M_t^N}.$$

By Ito's lemma, the dynamic evolution of Π_t is given by

$$d\Pi_t = (r_t^N - r_t^R)\Pi_t dt.$$

Thus, in the absence of arbitrage, the instantaneous growth rate of the price level is equal to the difference between the instantaneous nominal and real risk-free rates.[33] Correspondingly, we can express the stochastic price level at time t+τ as

$$\Pi_{t+\tau} = \Pi_t e^{\int_t^{t+\tau}(r_s^N - r_s^R)ds}.$$

The relationship between the yields and inflation expectations can be obtained by decomposing the price of the nominal bond as follows

$$\begin{aligned} P_t^N(\tau) &= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^N}{M_t^N}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R/\Pi_{t+\tau}}{M_t^R/\Pi_t}\right] = E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] + cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] \\ &= P_t^R(\tau) \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] \times \left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right). \end{aligned}$$

[33]Note that the price level Π_t is a stochastic process as long as r_t^N and r_t^R are stochastic processes.

Converting this price into yield to maturity using

$$y_t^N(\tau) = -\frac{1}{\tau}\ln P_t^N(\tau) \quad \text{and} \quad y_t^R(\tau) = -\frac{1}{\tau}\ln P_t^R(\tau),$$

we obtain

$$y_t^N(\tau) = y_t^R(\tau) + \pi_t^e(\tau) + \phi_t(\tau),$$

where the market-implied average rate of inflation expected at time t for the period from t to $t+\tau$ is

$$\pi_t^e(\tau) = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right] = -\frac{1}{\tau}\ln E_t^{\mathbb{P}}\left[e^{-\int_t^{t+\tau}(r_s^N - r_s^R)ds}\right]$$

and the associated inflation risk premium for the same time period is

$$\phi_t(\tau) = -\frac{1}{\tau}\ln\left(1 + \frac{cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right]}{E_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}\right] \times E_t^{\mathbb{P}}\left[\frac{\Pi_t}{\Pi_{t+\tau}}\right]}\right).$$

This last equation demonstrates that the inflation risk premium can be positive or negative. It is positive if and only if

$$cov_t^{\mathbb{P}}\left[\frac{M_{t+\tau}^R}{M_t^R}, \frac{\Pi_t}{\Pi_{t+\tau}}\right] < 0.$$

That is, the riskiness of nominal bonds relative to real bonds depends on the covariance between the real stochastic discount factor and inflation, and is ultimately determined by investor preferences.

Now, the BEI rate is defined as

$$BEI_t(\tau) \equiv y_t^N(\tau) - y_t^R(\tau) = \pi_t^e(\tau) + \phi_t(\tau),$$

that is, the difference between nominal and real yields of the same maturity. Note that it can be decomposed into the sum of expected inflation and the inflation risk premium.

Finally, we define the nominal and real term premia as

$$\begin{aligned} TP_t^N(\tau) &= y_t^N(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^N]ds, \\ TP_t^R(\tau) &= y_t^R(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s^R]ds. \end{aligned}$$

That is, the nominal term premium is the difference in expected nominal return between a buy and hold strategy for a τ-year nominal bond and an instantaneous rollover strategy at the risk-free nominal rate r_t^N. The interpretation for the real term premium is similar. The model thus allows us to decompose nominal and real yields into their respective term premia and short-rate expectations components.

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Online Appendix

Assessing Abenomics: Evidence from Inflation-Indexed Japanese Government Bonds

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The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

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Contents

A Deflation Protection Option Values

In this appendix, we explain how we calculate the value of the deflation protection enhancement that has been embedded in Japanese inflation-indexed bonds issued since 2013.

Consider an inflation-indexed bond issued at time t_0 with a reference price index value equal to Π_{t_0}. By time t, its accrued inflation compensation is $\frac{\Pi_t}{\Pi_{t_0}}$, which we define as the "inflation index ratio." There are then two mutually exclusive scenarios to consider. First, the net price index change to maturity T could be sufficiently positive that the net change from issuance to maturity is greater than one. This would imply:

$$\frac{\Pi_t}{\Pi_{t_0}} \times \frac{\Pi_T}{\Pi_t} > 1 \quad \Longleftrightarrow \quad \frac{\Pi_T}{\Pi_t} > \frac{\Pi_{t_0}}{\Pi_t}.$$

Given this outcome, the bond will pay off its inflation-adjusted principal $\frac{\Pi_T}{\Pi_{t_0}}$ at maturity.

Alternatively, the net price index change between t and T may be insufficient, leaving the net change less than one

$$\frac{\Pi_t}{\Pi_{t_0}} \times \frac{\Pi_T}{\Pi_t} \leq 1 \quad \Longleftrightarrow \quad \frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}.$$

Given that outcome, the deflation protection option will be in the money, as the inflation-indexed bond returns its original principal.

The net present value of the principal payment per yen invested at time t is therefore

$$NPV_t^{principal}\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = E_t^{\mathbb{Q}}\left[\frac{\Pi_T}{\Pi_t} \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} > \frac{\Pi_{t_0}}{\Pi_t}\}}\right] + E_t^{\mathbb{Q}}\left[1 \cdot e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}\}}\right].$$

Moreover, since

$$\frac{\Pi_T}{\Pi_t} = e^{\int_t^T (r_s^N - r_s^R) ds},$$

the equation can be rewritten as

$$NPV_t^{principal}\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds}\Big] + \left[E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}\}}\Big] - E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}\}}\Big]\right].$$

It then follows that the value of the deflation protection option, DOV_t, is given by

$$DOV_t\Big(\frac{\Pi_t}{\Pi_{t_0}}\Big) = \left[E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^N ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}\}}\Big] - E_t^{\mathbb{Q}}\Big[e^{-\int_t^T r_s^R ds} \mathbf{1}_{\{\frac{\Pi_T}{\Pi_t} \leq \frac{\Pi_{t_0}}{\Pi_t}\}}\Big]\right].$$

B The Extended Kalman Filter Estimation

In this appendix, we describe the estimation of the $G^J(4)$ model, which is based on the extended Kalman filter. For affine Gaussian models, in general, the conditional mean vector and the conditional covariance matrix are[1]

$$\begin{aligned} E^{\mathbb{P}}[X_T|\mathcal{F}_t] &= (I - \exp(-K^{\mathbb{P}}\Delta t))\theta^{\mathbb{P}} + \exp(-K^{\mathbb{P}}\Delta t)X_t, \\ V^{\mathbb{P}}[X_T|\mathcal{F}_t] &= \int_0^{\Delta t} e^{-K^{\mathbb{P}}s}\Sigma\Sigma' e^{-(K^{\mathbb{P}})'s}ds, \end{aligned}$$

where $\Delta t = T - t$. Conditional moments of discrete observations are computed and the state transition equation is obtained as

$$X_t = (I - \exp(-K^{\mathbb{P}}\Delta t))\theta^{\mathbb{P}} + \exp(-K^{\mathbb{P}}\Delta t)X_{t-1} + \xi_t,$$

where Δt is the time between observations.

In the standard Kalman filter, the measurement equation is linear

$$y_t = A + BX_t + \varepsilon_t$$

and the assumed error structure is

$$\begin{pmatrix} \xi_t \\ \varepsilon_t \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right],$$

where the matrix H is assumed to be diagonal, while the matrix Q has the following structure

$$Q = \int_0^{\Delta t} e^{-K^{\mathbb{P}}s}\Sigma\Sigma' e^{-(K^{\mathbb{P}})'s}ds.$$

In addition, the transition and measurement errors are assumed to be orthogonal to the initial state.

Now consider Kalman filtering, which is used to evaluate the likelihood function. Due to the assumed stationarity, the filter is initialized at the unconditional mean and variance of the state variables under the $\mathbb{P}$-measure: $X_0 = \theta^{\mathbb{P}}$ and $\Sigma_0 = \int_0^{\infty} e^{-K^{\mathbb{P}}s}\Sigma\Sigma' e^{-(K^{\mathbb{P}})'s}ds$. Denote the information available at time t by $Y_t = (y_1, y_2, \ldots, y_t)$, and denote model parameters by ψ. Consider period $t-1$ and suppose that the state update X_{t-1} and its mean square error

[1] Throughout conditional and unconditional covariance matrices are calculated using the analytical solutions provided in Fisher and Gilles (1996).

matrix Σ_{t-1} have been obtained. The prediction step is

$$X_{t|t-1} = E^{\mathbb{P}}[X_t|Y_{t-1}] = \Phi_t^{X,0}(\psi) + \Phi_t^{X,1}(\psi)X_{t-1},$$

$$\Sigma_{t|t-1} = \Phi_t^{X,1}(\psi)\Sigma_{t-1}\Phi_t^{X,1}(\psi)' + Q_t(\psi),$$

where $\Phi_t^{X,0} = (I - \exp(-K^{\mathbb{P}}\Delta t))\theta^{\mathbb{P}}$, $\Phi_t^{X,1} = \exp(-K^{\mathbb{P}}\Delta t)$, and $Q_t = \int_0^{\Delta t} e^{-K^{\mathbb{P}}s}\Sigma\Sigma' e^{-(K^{\mathbb{P}})'s}ds$, while Δt is the time between observations.

In the time-t update step, $X_{t|t-1}$ is improved by using the additional information contained in Y_t:

$$X_t = E^{\mathbb{P}}[X_t|Y_t] = X_{t|t-1} + \Sigma_{t|t-1}B(\psi)'F_t^{-1}v_t,$$

$$\Sigma_t = \Sigma_{t|t-1} - \Sigma_{t|t-1}B(\psi)'F_t^{-1}B(\psi)\Sigma_{t|t-1},$$

where

$$v_t = y_t - E^{\mathbb{P}}[y_t|Y_{t-1}] = y_t - A(\psi) - B(\psi)X_{t|t-1},$$

$$F_t = cov(v_t) = B(\psi)\Sigma_{t|t-1}B(\psi)' + H(\psi),$$

$$H(\psi) = diag(\sigma_\varepsilon^2(\tau_1), \dots, \sigma_\varepsilon^2(\tau_N)).$$

At this point, the Kalman filter has delivered all ingredients needed to evaluate the Gaussian log likelihood, the prediction-error decomposition of which is

$$\log l(y_1, \dots, y_T; \psi) = \sum_{t=1}^{T}\Big(-\frac{N}{2}\log(2\pi) - \frac{1}{2}\log|F_t| - \frac{1}{2}v_t'F_t^{-1}v_t\Big),$$

where N is the number of observed yields. Now, the likelihood is numerically maximized with respect to ψ using the Nelder-Mead simplex algorithm. Upon convergence, the standard errors are obtained from the estimated covariance matrix,

$$\widehat{\Omega}(\widehat{\psi}) = \frac{1}{T}\Big[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial \log l_t(\widehat{\psi})}{\partial \psi}\frac{\partial \log l_t(\widehat{\psi})}{\partial \psi}'\Big]^{-1},$$

where $\widehat{\psi}$ denotes the estimated model parameters.

In the $G^J(4)$ model, the extended Kalman filter is needed because the measurement equations of the inflation-indexed yields are no longer affine functions of the state variables. Instead, the measurement equation takes the general form

$$\overline{y}_t^R(\tau^i) = z(X_t; \tau^i, C^i, \psi) + \varepsilon_t^{R,i}, \tag{1}$$

where $\overline{y}_t^R(\tau^i)$ is the observed yield to maturity implied by the mid-market clean price (i.e., without accrued interest) of the inflation-indexed bond i at time t, while $z(X_t; \tau^i, C^i, \psi)$ is the corresponding model-implied yield to maturity.

In the extended Kalman filter, equation (1) is linearized using a first-order Taylor expansion around the best guess of X_t in the prediction step of the Kalman filter algorithm. Thus, in the notation introduced above, this best guess is denoted $X_{t|t-1}$ and the approximation is given by

$$z(X_t; \tau^i, C^i, \psi) \approx z(X_{t|t-1}; \tau^i, C^i, \psi) + \frac{\partial z(X_t; \tau^i, C^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}} (X_t - X_{t|t-1}).$$

Thus, by defining

$$\begin{aligned} A_t(\psi) &\equiv z(X_{t|t-1}; \tau^i, C^i, \psi) - \frac{\partial z(X_t; \tau^i, C^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}} X_{t|t-1}, \\ B_t(\psi) &\equiv \frac{\partial z(X_t; \tau^i, C^i, \psi)}{\partial X_t}\Big|_{X_t = X_{t|t-1}}, \end{aligned}$$

the measurement equation can be given on an affine form as

$$\overline{y}_t^R(\tau^i) = A_t(\psi) + B_t(\psi) X_t + \varepsilon_t^{R,i}$$

and the steps in the algorithm proceed as previously described. Andreasen et al. (2019) document that this estimation method is robust and reliable.

C $G^J(4)$ Model Results without Survey Information

In this appendix, we assess the sensitivity of our estimation results to the inclusion of the survey inflation forecasts.

Figure 1 shows the ten-year expected inflation implied by the $G^J(4)$ model when estimated with and without the ten-year inflation expectations from the Consensus Forecasts surveys of professional forecasters, which are also shown in the figure. We note that, with survey information included, the $G^J(4)$ model is able to provide a very close fit to the survey inflation forecasts. On the other hand, when we estimate the $G^J(4)$ model without the survey inflation

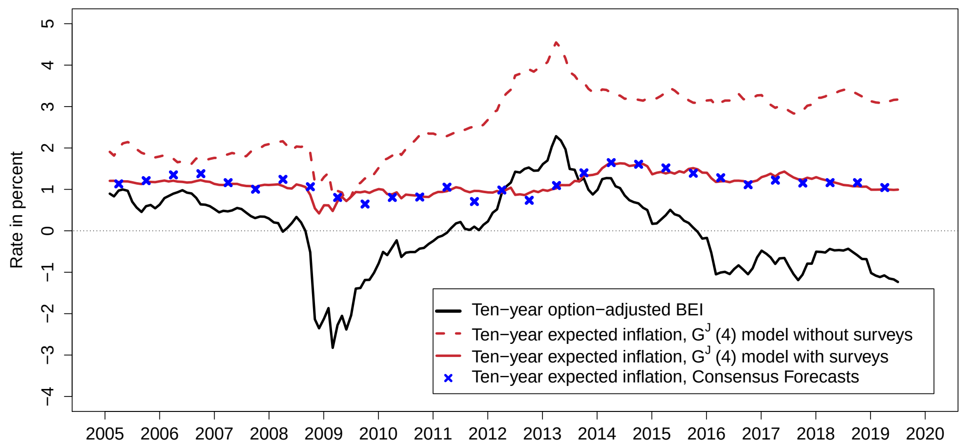


Figure 1: **Sensitivity of Ten-Year Expected Inflation to Including Surveys**

forecasts, the model-implied inflation expectations appear to be unreasonably high. This supports our choice to focus on the $G^J(4)$ model estimated with the survey inflation forecasts. Equally important, the estimated ten-year option-adjusted BEI rates from the two estimations are practically indistinguishable and therefore only shown with a single solid black line in the figure.

In light of the sensitivity of the model-implied inflation expectations to the inclusion of the survey forecasts in the model estimation, we repeat our high-frequency analysis by estimating the $G^J(4)$ model with and without the deflation option adjustment, but *without* the survey forecasts in both cases. We then use the estimated model to decompose the change in the daily ten-year BEI around the six key BoJ announcement events.

The results are reported in Table 1, where we note that now we get more variation in the inflation expectations component, which seems reasonable given that the model's expectations generator is not informed by—and therefore tied to—the expectations reflected in the Consensus Forecasts surveys. However, as noted earlier, the model's estimate of ten-year BEI is not affected by excluding the survey forecasts from the model estimation. Therefore, qualitatively, the decompositions are very similar to those reported in the main text where the survey forecast were included in the model estimation. Specifically, it remains the case that the first two events in 2013 appear to have provided a boost to inflation expectations, while the introduction of negative rates in January 2016 appear to have depressed long-term inflation expectations quite notably.

Decomposition from $G^J(4)$ model without option adjustment				
Event		Ten-year exp. inflation	Ten-year IRP	Ten-year BEI
I	Jan. 21, 2013	289	-149	140
	Jan. 22, 2013	293	-145	148
	Change	**4**	**4**	**8**
II	Apr. 3, 2013	334	-123	210
	Apr. 4, 2013	336	-131	206
	Change	**2**	**-7**	**-5**
III	Oct. 30, 2014	388	-297	91
	Oct. 31, 2014	389	-295	93
	Change	**1**	**1**	**3**
IV	Jan. 28, 2016	398	-360	37
	Jan. 29, 2016	397	-359	38
	Change	**-1**	**2**	**1**
V	Sep. 20, 2016	405	-394	10
	Sep. 21, 2016	404	-393	11
	Change	**-1**	**2**	**1**
VI	Jul. 30, 2018	390	-338	52
	Jul. 31, 2018	390	-337	53
	Change	**0**	**1**	**1**

Decomposition from $G^J(4)$ model with option adjustment				
Event		Ten-year exp. inflation	Ten-year IRP	Ten-year BEI
I	Jan. 21, 2013	40	104	143
	Jan. 22, 2013	42	108	150
	Change	**2**	**4**	**7**
II	Apr. 3, 2013	41	175	216
	Apr. 4, 2013	47	166	213
	Change	**6**	**-9**	**-3**
III	Oct. 30, 2014	378	-315	63
	Oct. 31, 2014	380	-315	64
	Change	**2**	**0**	**1**
IV	Jan. 28, 2016	426	-456	-30
	Jan. 29, 2016	420	-457	-38
	Change	**-7**	**-1**	**-8**
V	Sep. 20, 2016	427	-518	-91
	Sep. 21, 2016	427	-518	-91
	Change	**-1**	**0**	**0**
VI	Jul. 30, 2018	345	-363	-18
	Jul. 31, 2018	344	-362	-17
	Change	**-1**	**1**	**1**

Table 1: **Decomposition of One-Day Responses of the Ten-Year BEI**
The decomposition of one-day responses of the Japanese ten-year BEI on six BoJ announcement dates into changes in (i) the ten-year expected inflation and (ii) the ten-year inflation risk premium (IRP) based on the $G^J(4)$ model estimated with daily data, but *without* the Consensus Forecasts of ten-year CPI inflation. All numbers are measured in basis points.

D Sensitivity of the Deflation Risk Premium

In this appendix, we explore the sensitivity of our estimated deflation risk premium series to various model estimation choices.

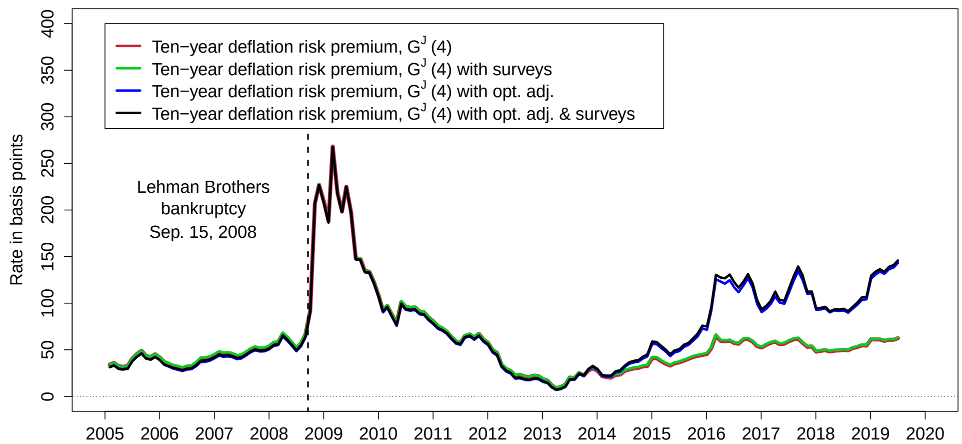


Figure 2: **Ten-Year Deflation Risk Premiums**

Figure 2 shows the ten-year deflation risk premium calculated from four different specifications of the $G^J(4)$ model: without either option adjustment or survey information, with either option adjustment or survey information, and with both option adjustment and survey information.

The results show that the estimated deflation risk premiums are nearly identical from 2005 to mid-2014. For the remaining part of the sample there is a wedge between the premiums from the two specifications that adjust for the deflation protection option values on one side and those from the two specifications that do not adjust for the option values.

This underscores the importance of accounting for the values of the deflation protection options in the model estimation. It also demonstrates that the calculated deflation risk premiums are entirely unaffected whether or not the survey information is included in the model estimation as their value is determined by the models' risk-neutral $\mathbb{Q}$-dynamics.

E Bond Market Reaction to BoJ Announcements

In this appendix, we report the bond market reaction to the six events included in our event study analysis. Specifically, we measure the one-day reaction in our observed nominal yields at five of the six maturities in our data. These are reported in the top panel of Table 2. As for real yields, we take the fitted real yields from our benchmark $G^J(4)$ model estimated using daily data without either option adjustment or survey information, which represents a flexible fit to the raw bond price data and offers the cleanest direct read of the changes in real yields without any adjustments whatsoever. These results are reported in the bottom panel of Table 2.

F Long-Term BEI Decomposition

In this appendix, we decompose our estimates of the option-adjusted BEI over a five-year period starting five years ahead (a.k.a. the 5yr5yr BEI) into its expectations and risk premium components. The 5yr5yr BEI is a market-based measure of inflation compensation, which is frequently used to monitor bond investors' long-term inflation expectations.

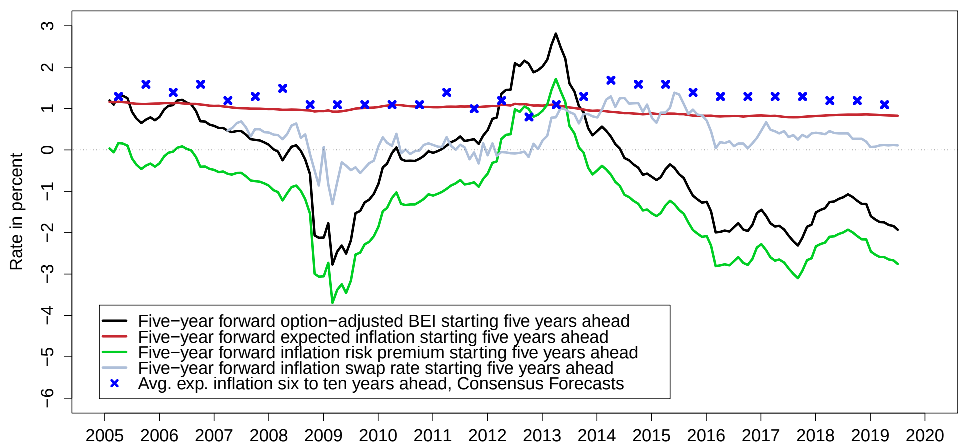


Figure 3: **5yr5yr Option-Adjusted BEI Decomposition**

Figure 3 shows the result of its decomposition based on our estimated benchmark $G^J(4)$ model. First, note that the option-adjusted 5yr5yr BEI has varied quite notably since 2005. It dropped deep into negative territory during the financial crisis. However, it turned highly positive as Shinzo Abe assumed power in late 2012. Since then it has experienced a persistent decline as enthusiasm concerning the prospects for Abenomics diminished. By the end of our sample, 5yr5yr BEI for Japan stood at negative 1.93 percent.

Importantly, though, the model decomposition shows that bond investors' long-term inflation expectations have remained positive and relatively stable at around 1 percent throughout

Event		Observed nominal yields				
		1-year	2-year	4-year	7-year	10-year
I	Jan. 21, 2013	7.7	6.5	9.9	38.0	78.4
	Jan. 22, 2013	8.4	7.1	10.8	38.4	78.5
	Change	**0.7**	**0.6**	**0.9**	**0.4**	**0.1**
II	Apr. 3, 2013	6.8	5.9	10.1	29.2	53.8
	Apr. 4, 2013	6.8	6.4	9.7	23.7	42.5
	Change	**0**	**0.5**	**-0.4**	**-5.5**	**-11.3**
III	Oct. 30, 2014	1.5	2.8	8.3	23.3	44.3
	Oct. 31, 2014	2.2	3.2	8.3	23.0	44.0
	Change	**0.7**	**0.4**	**0**	**-0.3**	**-0.3**
IV	Jan. 29, 2016	-3.5	-2.4	-0.8	5.1	20.0
	Jan. 29, 2016	-8.2	-8.9	-8.8	-3.5	10.9
	Change	**-4.7**	**-6.5**	**-8.0**	**-8.6**	**-9.1**
V	Sep. 20, 2016	-31.1	-28.4	-23.8	-17.7	-5.8
	Sep. 21, 2016	-30.1	-27.9	-22.7	-16.8	-5.1
	Change	**1.0**	**0.5**	**1.1**	**0.9**	**0.7**
VI	Jul. 30, 2018	-12.8	-12.0	-10.5	-2.6	9.3
	Jul. 31, 2018	-12.7	-11.9	-10.4	-3.5	6.5
	Change	**0.1**	**0.1**	**0.1**	**-0.9**	**-2.8**

Event		Fitted real yields				
		1-year	2-year	4-year	7-year	10-year
I	Jan. 21, 2013	-26.5	-38.4	-54.2	-67.8	-76.8
	Jan. 22, 2013	-46.2	-54.8	-66.4	-76.9	-84.3
	Change	**-19.7**	**-16.4**	**-12.2**	**-9.1**	**-7.6**
II	Apr. 3, 2013	-113.4	-124.6	-139.6	-152.5	-161.2
	Apr. 4, 2013	-122.0	-130.0	-140.8	-150.7	-157.9
	Change	**-8.6**	**-5.4**	**-1.3**	**1.8**	**3.3**
III	Oct. 30, 2014	-454.5	-342.6	-203.1	-100.4	-52.2
	Oct. 31, 2014	-458.9	-346.5	-206.4	-103.1	-54.7
	Change	**-4.5**	**-3.9**	**-3.2**	**-2.7**	**-2.5**
IV	Jan. 29, 2016	-286.5	-212.1	-119.6	-52.1	-21.2
	Jan. 29, 2016	-277.9	-206.9	-118.8	-54.5	-25.1
	Change	**8.6**	**5.2**	**0.8**	**-2.4**	**-4.0**
V	Sep. 20, 2016	-235.0	-175.3	-101.3	-47.6	-23.5
	Sep. 21, 2016	-233.9	-174.6	-101.2	-48.0	-24.1
	Change	**1.1**	**0.7**	**0.1**	**-0.3**	**-0.6**
VI	Jul. 30, 2018	-112.7	-93.6	-70.6	-55.1	-49.7
	Jul. 31, 2018	-115.7	-96.2	-72.5	-56.6	-51.1
	Change	**-3.0**	**-2.5**	**-2.0**	**-1.5**	**-1.3**

Table 2: **One-Day Responses of Japanese Government Bond Yields**
The table reports the one-day response of five Japanese government bond yields around the BoJ announcement dates. All numbers are measured in basis points.

our sample period, while it is the 5yr5yr inflation risk premium that is the primary source of the variation in the 5yr5yr BEI. This result is consistent with the long-term inflation forecasts for the period six to ten years ahead reported for Japan in the Consensus Forecasts surveys and shown with blue crosses in the figure, which also remain positive and vary relatively little over the course of the sample. As such, while the initial enthusiasm and ultimate disappointment in the Abenomics program resulted in notable movements in the inflation risk premium, we find little change over the episode in investors' long-term expected inflation.

We also include the 5yr5yr inflation swap rates.[2] While this series exhibits a greater discrepancy with our fitted option-adjusted BEI series, some part of this difference is likely due to low liquidity in the inflation swap market.[3]

G Yield Data on the BoJ Announcement Dates

Figure 4 shows the available nominal and real yields on the day before and on the day of the six BoJ announcements we consider.

First and most importantly, we note that we have a full term structure of JGBi yield observations with the exception of January 21, 2013, when we only observe a narrow range of JGBi yields. Still, given our full panel of daily observations for the entire sample combined with the Kalman filter, which significantly narrows the admissible range of the estimated state variables, the model decomposition even on that date is likely to be about as accurate as it is on any other day in the sample.

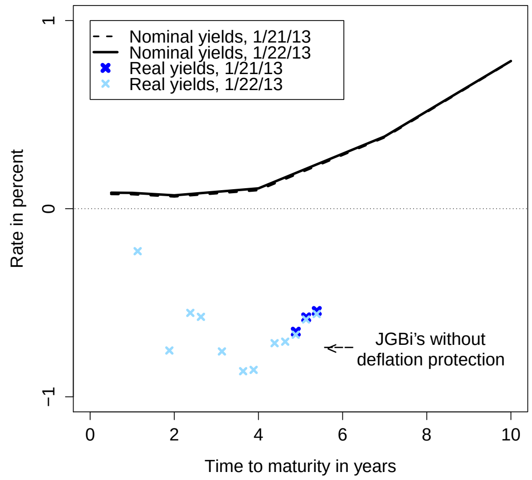
Second, the available JGBi yields for the events in 2014 and 2016 represent a mix of bonds with and without deflation protection underscoring the importance of adjusting for price effects tied to this compositional heterogeneity for our assessment.

H Sensitivity to Eliminating Individual JGBi's

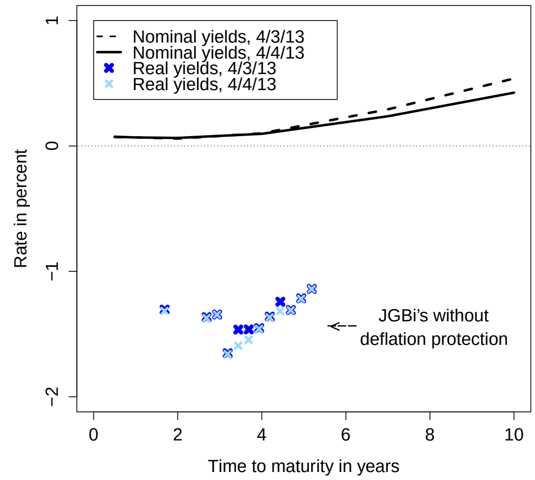
In light of the somewhat unusual universe of available JGBi's in terms of their cross sectional distribution, which at times is sparse and narrow, one could rightly be concerned about the overall robustness of our results. To address such concerns, we undertake the following exercise. To begin, we start from the full sample, drop the first JGBi from it and re-estimate

[2] Source: Bloomberg.

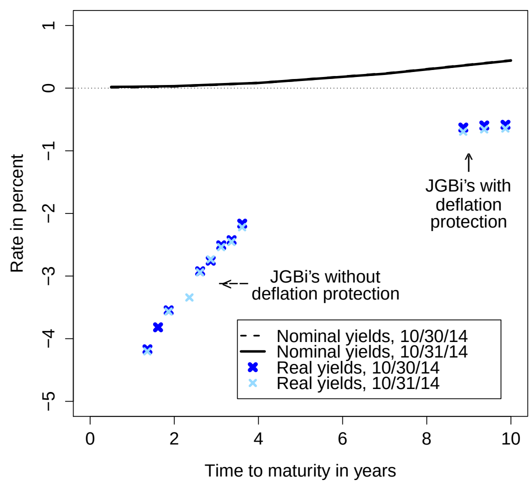
[3] We also used our benchmark model to construct market-based estimates of the natural real rate r_t^* as in Christensen and Rudebusch (2019). Our estimate suggests that the natural rate in Japan has been close to minus one percent since 2005. However, we do find that both the 5yr5yr option-adjusted real yield and our r_t^* estimate have trended up since 2013. These results are available from the authors on request.



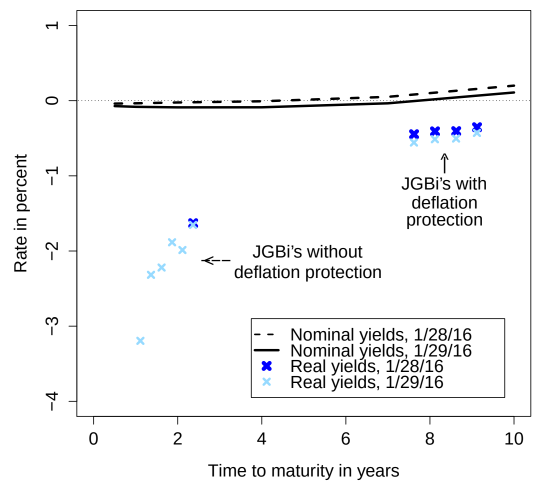
(a) Jan. 22, 2013



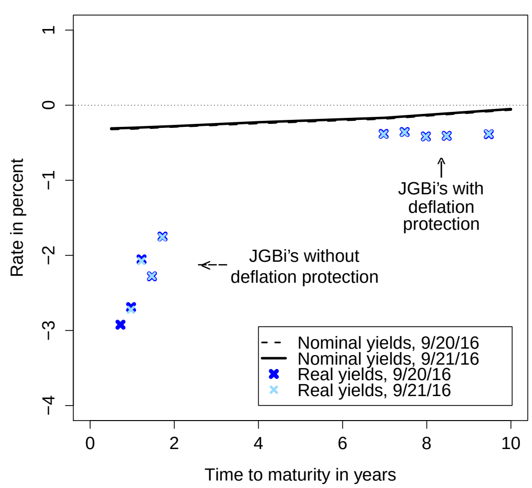
(b) Apr. 4, 2013



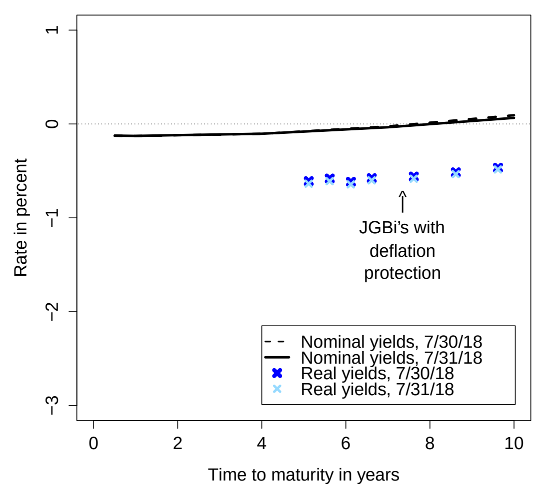
(c) Oct. 31, 2014



(d) Jan. 29, 2016



(e) Sep. 21, 2016



(f) Jul. 31, 2018

Figure 4: **Available Bond Yields around BoJ Announcement Dates**

the model. Next, we start from the full sample, drop the second JGBi from it and re-estimate the model. This is repeated down to the elimination of the last JGBi from our full sample, a total of 24 estimations.

It turns out that eliminating individual JGBi's from our sample has very little impact on our estimation results. To demonstrate this, we compare the five-year expected inflation and the ten-year deflation risk premium from these 24 estimations (all shown with thin grey lines in the following) to the corresponding results from our original estimation based on the full sample (shown with a thick black line in the following). In Figure 5, the top panel provides the comparison of the five-year expected inflation, while the bottom panel shows the comparison of the ten-year deflation risk premium. In both panels, we note that the thick black line based on the full-sample results is hardly distinguishable from any of the 24 grey lines in each panel. This leads us to conclude that our results are not driven by the price variation from any individual JGBi, but rather reflect the collective variation of the entire real yield curve as measured through our JGBi data.

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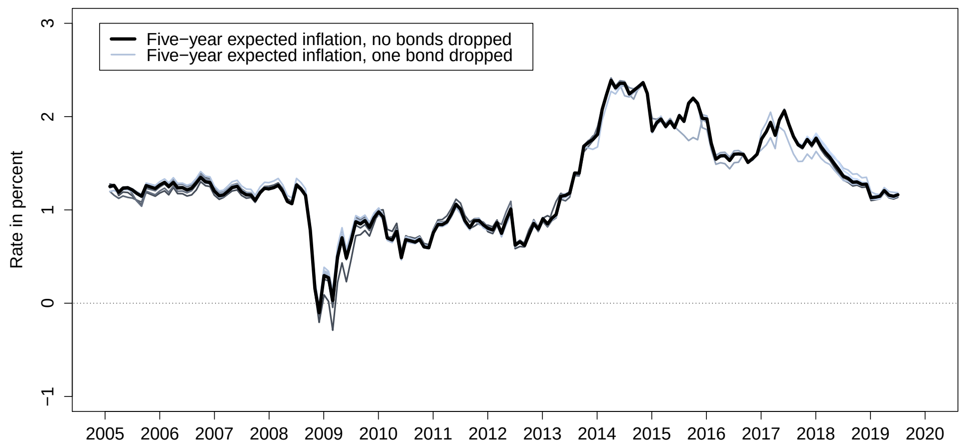
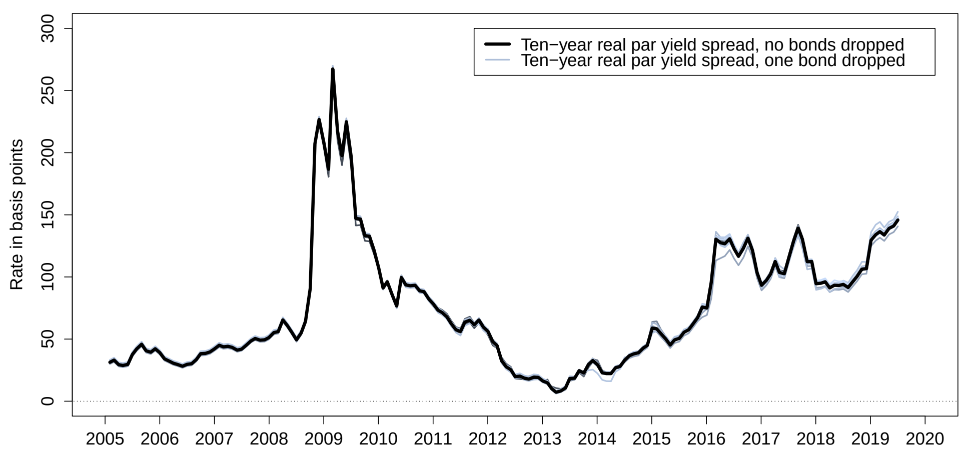


(a) Five-year expected inflation



(b) Ten-year deflation risk premium

Figure 5: **Sensitivity to Eliminating Individual JGBi's**
Panel (a) shows the estimated five-year expected inflation from the full sample and from model estimations where a single JGBi is dropped from the full sample each time (a total of 24 different estimations). Panel (b) shows the corresponding estimates of the ten-year deflation risk premium as defined in Section 4.2.1 of the paper.