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Inflation Uncertainty and Excess Returns on Stocks and Bonds

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This paper investigates the relation between inflation uncertainty and excess returns on stocks and bonds. It quantifies the effect of inflation uncertainty by comparing actual excess returns with those expected by a hypothetical naive investor who treats inflation forecasts as if they were known with certainty. The evidence suggests that ignoring inflation uncertainty results in only small pricing errors, on average.

It is well known that there is a positive relation between the level and variability of inflation (e.g., see Taylor 1981, Ball and Cecchetti 1990, or Evans 1991). Some economists and policymakers interpret this as evidence that rates of inflation such as those experienced in the United States in the late 1970s and early 1980s impose significant costs on society by increasing the degree of inflation uncertainty. For example, one argument goes as follows: when there is greater uncertainty about inflation, investors may be less eager to hold long-term bonds because their prices are more sensitive to unexpected changes in inflation than are the prices of short-term bills. Thus, when inflation is high, long-term bonds may have to offer a premium in the form of higher expected real returns in order to compensate investors for incurring greater inflation uncertainty. Furthermore, if investment declines when long-term real interest rates rise, inflation uncertainty may reduce the level and possibly the growth rate of output in the long run by reducing the rate of capital accumulation.

While it may be true that there is a positive relation between the level and variability of inflation, this does not necessarily imply that there is a positive relation between the level of inflation and risk premiums on financial assets. The reason is that the risk premium on a security depends not on the variance of its own real return, which may depend on the variance of inflation, but on the covariance between its real return and the stochastic discount factor. For example, in Lucas's (1978) consumption-based capital asset pricing model, the risk premium on a security is related to the covariance between its own real return and the marginal rate of substitution between consumption today and consumption tomorrow. Evidence on the relation between the level and variance of inflation tells us nothing about the relevant covariance term and thus contains little information about the relation between inflation and risk.

This paper investigates one aspect of the relation between inflation uncertainty and risk. It asks whether accounting for inflation uncertainty is important for understanding the equity and term premium puzzles. The equity premium puzzle refers to the fact that the average annual return on stocks is roughly 6 percentage points higher than the average return on Treasury bills. Similarly, the term premium puzzle refers to the fact that the average annual holding return on Treasury notes is roughly 45 to 65 basis points higher than the average holding return on Treasury bills. The equity and term premiums probably arise because stocks and long-term bonds are riskier than short-term bills, but they are regarded as puzzles because it is difficult to find economically appealing, quantitative models of risk premiums (see Mehra and Prescott 1985; Backus, Gregory, and Zin 1989; or Cochrane and Hansen 1992).

This paper follows Labadie (1989) and Giovannini and Labadie (1991) by investigating the quantitative importance of inflation uncertainty for the equity and term premium puzzles. Using a variety of standard parametric models, they found that accounting for inflation uncertainty increases the equity premium but still produces a value that is small relative to the one found in sample. They also found that the inflation risk premium is rather small, on the order of 10 to 15 basis points. In their model, the average real return on nominal bills is only slightly greater than the risk-free real rate, and most of the variation in returns on risky assets is due to variation in the risk-free rate.

Their results are conditional on a discount factor specification that is a bit problematic, however. In particular, Hansen and Singleton (1982) and Hansen and Jagannathan (1991) showed that their discount factor model yields forecastable risk-adjusted excess returns, which suggests either that the model does not price risk correctly or that financial markets are inefficient. Furthermore, Cochrane and Hansen (1992) showed that many other parametric discount factor models also suffer from this problem. Hence, there is little reason to believe that any of the standard parametric models adequately price risk.

This paper complements the research of Giovannini and Labadie by trying a nonparametric or preference-free approach. The basic idea is to conjure up a hypothetical naive investor who ignores inflation uncertainty when pricing nominal securities. Our naive investor makes systematic pricing errors because he implicitly sets inflation risk factors equal to zero. I estimate these pricing errors by comparing actual equity and term premiums with the ones that our naive investor would expect. Large inflation pricing errors imply that inflation uncertainty is important for understanding excess return puzzles.

It turns out that mean inflation pricing errors can be estimated using only minimal assumptions about the discount factor. Specifically, mean pricing errors can be expressed as a function of a number of observable sample moments plus a single unknown parameter, the mean discount factor. Instead of specifying a complete parametric model for the discount factor, I only need to calibrate its mean. I specify a range of reasonable values for this parameter and then estimate inflation pricing errors for a number of values on that range. The advantage of this approach is that it imposes weaker maintained assumptions than those used in prior work.

Using post-war U.S. data, I find that inflation pricing errors are quite *small* relative to observed equity and term premiums. The results are robust with respect to variation in the mean discount factor as well as to various splits of the post-war sample. Inflation pricing errors are also estimated with a reasonable degree of precision, so it is unlikely that the results are due to sampling error. Thus, the paper fails to find evidence that inflation uncertainty is important for excess return puzzles.

The rest of the discussion is organized as follows. Section I explains how inflation pricing errors are calculated. Section II describes the data and reports the empirical results. The paper concludes with a brief summary.

I. Asset Pricing Conditions and Inflation Pricing Errors

It is useful to begin by defining some notation. Let p_t denote the nominal price level, and let b_{jt} be the nominal price of a j-period discount bond. The nominal yield to maturity, y_{jt}, is implicitly defined by the equation

$$b_{jt} = (1 + y_{jt})^{-j} .$$

The nominal one-period holding return on a j-period discount bond is defined as the return earned by buying the security at the beginning of the period and selling it at the end. For a j-period discount bond, the gross one-period holding return is

$$\mathrm{h}_{jt} = b_{j-1,t+1}/b_{jt} .$$ [1]

The gross nominal one-period holding return on an equity is

$$h_{jt} = (v_{jt+1} + d_{jt+1})/v_{jt} ,$$

where v_{jt} is the nominal equity price and d_{jt} is the nominal dividend.

Finally, let m_t denote the one-period stochastic discount factor. For example, in the consumption-based capital asset pricing model (Lucas 1978), m_t is the intertemporal marginal rate of substitution between consumption this period and consumption next. Similarly, under certain conditions,[2] one can use Cochrane's (1991) production-based asset pricing model to express the discount factor as a function of intertemporal marginal rates of transformation.

1. For long-term bonds, which generally pay coupons, I use data on yields to maturity for equivalent discount bonds (see McCulloch 1990 and McCulloch and Kwon 1993).

2. Real and financial returns must each span the underlying state space.

I assume that there are a great many investors who discount future payments at the same rate, that these investors have rational expectations, and that financial markets are complete and efficient. This framework subsumes a wide class of asset pricing models. In fact, Cochrane and Hansen point out that as long as the law of one price holds, one can *always* interpret asset returns using this framework.[3]

These assumptions imply the conditional moment restrictions that follow. First, pick some nominally riskless, short-term discount bond to serve as a benchmark security. The 3-month Treasury bill is a good choice for the benchmark security, as the probability that the U.S. government will default on Treasury bills is essentially zero. The conditional moment restriction for the benchmark security is

$$(1) \qquad E_t m_t (p_t/p_{t+1}) h_{1t} = 1 \,,$$

where h_{1t} is the gross nominal one-period return on the benchmark security and E_t denotes the conditional expectation based on information available at time t. Since 3-month Treasury bills are nominally riskless, this condition can be factored into

$$(2) \qquad h_{1t} E_t m_t (p_t/p_{t+1}) = 1,$$

or

$$E_t m_t (p_t/p_{t+1}) = b_{1t} \,.$$

Using the law of iterated expectations, it follows that this must also hold unconditionally:

$$E m_t (p_t/p_{t+1}) = E b_{1t} \,.$$

Expanding the expectation term on the left-hand side yields

$$(3) \qquad \mu_m \mu_\pi + \sigma_{m\pi} = \mu_b \,,$$

where μ_x denotes the mean of the variable x_t and σ_{xy} denotes the covariance between the variables x_t and y_t. The variable π_t denotes the reciprocal of the gross inflation rate (i.e. $\pi_t \equiv p_t/p_{t+1}$).

Equation (3) is a generalization of the Fisher equation. To a first order approximation, μ_π/μ_b is the mean gross real return on a nominally riskless bond. Similarly, to a first order approximation, $1/\mu_m$ is the mean gross real return on a hypothetical indexed bond. If $\sigma_{m\pi}$ is zero, the price of a nominal bond equals the price of an indexed bond with an adjustment for expected inflation, thus yielding the Fisher equation. If $\sigma_{m\pi}$ is negative, the ex post real return on a nominal bill covaries negatively with the discount factor. In this case, risk averse investors would hedge against unexpected movements in inflation by buying indexed bonds, which would therefore sell at a premium relative to nominally riskless bills.[4] Hence, $\sigma_{m\pi}$ can be interpreted as an inflation risk premium.

For all other securities, the principle of no-arbitrage implies that risk adjusted, real excess returns are unpredictable:

$$(4) \qquad E_t m_t (p_t/p_{t+1}) s_{jt} = 0,$$

where $s_{jt} \equiv (h_{jt} - h_{1t})$ denotes the nominal excess holding return on security j relative to the benchmark security. The law of iterated expectations implies that this must also hold unconditionally. Taking a second-order approximation, one can write the unconditional no-arbitrage restriction as

$$\mu_m \sigma_{\pi s} + \mu_\pi \sigma_{ms} + \mu_s \sigma_{m\pi} + \mu_m \mu_\pi \mu_s = 0 \,.$$[5]

By rearranging the no-arbitrage condition and using equation (3), one can derive the following expression for mean nominal excess returns:

$$(5) \qquad \mu_s = -(\sigma_{ms}/\mu_m + \sigma_{\pi s}/\mu_\pi)(\mu_m \mu_\pi/\mu_b) \,.$$

Nominal excess returns depend on three covariance terms: σ_{ms}, $\sigma_{\pi s}$, and $\sigma_{m\pi}$. First, a security whose nominal excess return covaries positively with the discount factor ($\sigma_{ms} > 0$) provides a hedge against unexpected movements in m_t and thus pays a lower nominal return than the benchmark security. Second, holding the real risk premium constant, a security whose nominal excess return covaries negatively with inflation ($\sigma_{\pi s} > 0$) would pay lower nominal excess returns on average, because the mean real excess return is an increasing function of $\sigma_{\pi s}$ (this follows from Jensen's inequality). Third, as explained above, the ratio $\mu_m \mu_\pi/\mu_b$ can be interpreted as a measure of the degree of inflation risk on nominally riskless bonds.

Now suppose that an investor were to price securities in a naive manner, ignoring inflation uncertainty. Our naive investor would factor the no-arbitrage condition for excess returns into

$$(p_t/p_{t+1}) E_t m_t s_{jt} = 0,$$

which implies that

$$E_t m_t s_{jt} = 0.$$

3. Apart from the law of one price, the model does not have any testable implications if the discount factor is left unspecified, so the general framework cannot be refuted.

4. For example, in simple versions of the consumption-based CAPM, the discount factor is higher than expected when future consumption is lower than expected. A security that tends to pay low returns when consumption is lower than expected amplifies consumption risk. Such securities would sell at a lower price than an indexed bond.

5. This is exact if the joint distribution for m_t, p_t/p_{t+1}, and s_{jt} is symmetric.

Using the law of iterated expectations and expanding the latter condition yields

$$\tilde{\mu}_s = -\sigma_{ms}/\mu_m. \tag{6}$$

If we compare equation (6) with equation (5), we see that the naive investor is making two mistakes. First, he uses the Fisher equation, $\mu_m\mu_\pi = \mu_b$, to price nominal bills. That is, he ignores the inflation risk premium. This error vanishes if $\sigma_{m\pi}=0$. Second, the naive investor ignores Jensen's inequality when computing expected real returns; hence he implicitly sets $\sigma_{\pi s}=0$. This error vanishes if nominal excess returns are uncorrelated with inverse gross inflation. If inflation pricing errors are large, it follows that the naive investor would make big losses by ignoring inflation uncertainty. Hence large pricing errors imply that inflation uncertainty is important for understanding excess return puzzles.

To compute the average pricing error, we need to estimate $-\sigma_{ms}/\mu_m$ and compare it with μ_s. I use a nonparametric approach. Looking back to equation (5), we see that μ_s depends on three observed moments (μ_b, μ_π, and $\sigma_{\pi s}$) and on two unobserved moments (μ_m and σ_{ms}). If we plug in estimates of the observed moments, equation (5) defines a trade-off between admissible values of μ_m and σ_{ms}. I calibrate a variety of plausible values for μ_m and then back the corresponding value for σ_{ms} out of equation (5). This generates a range of plausible values for $\tilde{\mu}_s = -\hat{\sigma}_{ms}/\hat{\mu}_m$. Then I calculate the mean inflation pricing error by subtracting $\tilde{\mu}_s$ from the sample mean, $\hat{\mu}_s$.

II. EMPIRICAL ANALYSIS

This section contains the main empirical analysis. It begins by listing the data sources and by describing how stock and bond returns are adjusted for taxes. Then it explains how the mean discount rate is calibrated. Finally, it reports estimates of inflation pricing errors for excess returns on stocks and bonds.

Data Sources

The data consist of quarterly observations of returns on the S&P 500, returns on various long- and short-term Treasury bonds, and the CPI inflation rate. The sample covers the period 1947 to 1990. Data on the Treasury yield curve are taken from McCulloch (1990) and McCulloch and Kwon (1993), and data on the S&P 500 are published in Ibbotson (1991). The inflation rate was calculated from a CPI series that treats housing on a rental-equivalent basis; see Huizinga and Mishkin (1984) for details.

Adjustments for Taxes

The theory outlined in the previous section ignores one important consideration, viz., the effect of taxes on asset pricing conditions. Since investors care about after-tax returns, pre-tax returns need not satisfy the no-arbitrage conditions even in an efficient capital market. Although this is an important issue, there has been little research on how to modify asset pricing models in order to account for taxes, perhaps because the tax code is so enormously complex. Instead, the asset pricing literature follows one of two standard approaches. The first is to ignore taxes completely and work with pre-tax return data. The second is to make some simple assumptions about the tax code and then to work with approximate after-tax returns.

This paper adopts the latter approach.[6] Both the theory and the data need some modification. First consider the modification to the theory. The pricing condition for the benchmark security (equation 1) and the no-arbitrage condition for excess returns (equation 4) both go through as long as returns are calculated on an after-tax basis. But I need an additional assumption to identify the parameter $\sigma_{m\pi}$. The additional assumption is that the tax rate on Treasury bill returns is known at the beginning of the holding period. If this assumption is satisfied, then the after-tax nominal return on the benchmark security is also riskless, and the conditional moment restriction can then be factored as in equation (2). In this case, it follows that

$$\mu_m\mu_\pi + \sigma_{m\pi} = E(1/\tilde{h}_{1t}),$$

where $\tilde{h}_{1t}$ denotes the gross after-tax nominal return on the benchmark bill.

The assumption that tax rates are known at the beginning of the holding period may be unrealistic. Nonetheless, there are two reasons why I believe that it provides a workable approximation. First, most tax reforms provide advance notice of changes in tax rates, so the assumption is often satisfied. Second, and more importantly, this assumption is used only for identifying $\sigma_{m\pi}$, and the basic conclusions are not sensitive to the value of this parameter.

Now consider the adjustments to the data. McCulloch (1990) and McCulloch and Kwon (1993) adjust the term structure data by assuming that coupons and Treasury bill returns are taxed at the marginal rate on ordinary income and that capital gains on Treasury bonds are taxed at the going capital gains rate when the bonds mature; see McCulloch (1975) for details.

I follow a similar approach when adjusting for taxes on stock returns. I assume that dividends are taxed as ordinary income in the year in which they are paid out. Accounting

6. Pre-tax data yield the same results.

for capital gains taxes is more difficult, since capital gains are taxed when they are realized rather than when they occur. Since capital losses offset prior, unrealized capital gains but do not offset ordinary income, investors have an incentive to defer capital gains in order to offset potential future losses. There is no simple way to account for this deferral option. Instead, I assume that investors smooth capital gains over various arbitrary holding periods, ranging from one to five years, and then pay the capital gains tax rate that is in effect at the end of the averaging period. While this assumption is arbitrary, it goes a long way toward capturing the incentive to smooth capital gains and losses. Fortunately, the precise length of the averaging period does not affect the results.

Calibrating the Mean Discount Factor

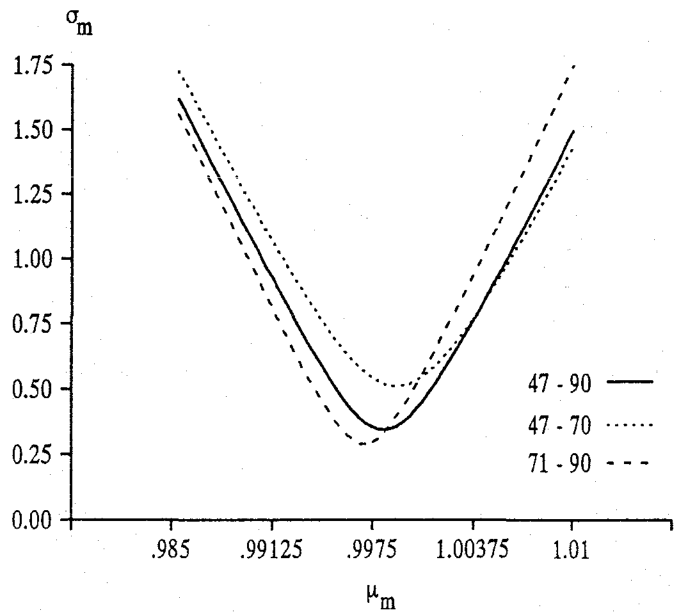
The inflation pricing error is a function of the mean discount factor, which is unobservable. However, this parameter can be restricted to a fairly tight range by appealing to the results of Cochrane and Hansen (1992). Using postwar quarterly data, they show that μ_m must lie between 0.98 and 1.03 in order to ensure that the discount factor is always non-negative. Furthermore, they also show that if μ_m is very far from 0.998, the variance of the discount factor must increase substantially in order for observed returns to be consistent with the unconditional asset pricing conditions.

The latter result is illustrated in Figure 1, which shows Hansen-Jagannathan bounds computed from quarterly real holding returns on 3- and 12-month Treasury bills, 5- and 10-year Treasury notes, and the S&P 500. For various values of μ_m (plotted along the horizontal axis), the Hansen-Jagannathan bound shows the minimum standard deviation of m_t (plotted along the vertical axis) that is consistent with the unconditional moment conditions for these securities.[7]

The solid line shows Hansen-Jagannathan bounds for the period 1947–1990, the dotted line shows the results for the sub-period 1947–1970, and the dashed line shows the results for the sub-period 1971–1990. Over the whole sample, the variance of the discount factor is minimized for $\mu_m = 0.998$, and the minimized value is $\sigma_m = 0.343$. The discount factor is measured in units of inverse gross returns, so the discount factor is considerably more volatile than stock returns. Furthermore, as we move away from $\mu_m = 0.998$, the volatility of the discount factor increases rapidly. Similar results apply to both sub-periods. During the first part of the sample, σ_m is minimized for $\mu_m =$ 0.9988, and the minimized value is 0.511. In the second part of the sample, σ_m is minimized for $\mu_m = 0.9968$, with a minimized value of 0.290. Again, the volatility of the discount factor rises dramatically as we move away from the minimum.

FIGURE 1

HANSEN-JAGANNATHAN BOUNDS



These results indicate either that the mean discount factor is somewhere around 0.997 or 0.999 or that the discount factor is enormously volatile. The former hypothesis seems fairly plausible, since these discount factors imply that the mean riskless real interest rate is somewhere around 0.5 to 1.2 percent per year. The latter hypothesis seems less plausible, since generating this much volatility in discount factors is a major theoretical problem (e.g. see Cochrane and Hansen). Thus, in what follows, I assume that μ_m is close to 0.998.

Inflation Uncertainty and the Equity Premium

Table 1 reports inflation pricing errors for quarterly excess returns on the S&P 500, with empirical 95 percent confidence intervals shown in parentheses.[8] I experimented with

7. Adding additional securities tightens the bound.

8. The Monte Carlo simulations were conducted as follows. First, for each sample period, a vector autoregression was estimated for inverse gross inflation, nominal Treasury bill prices, and excess returns. The estimated models were then used to generate artificial data, and an

alternative capital gains smoothing periods, ranging from 1 to 5 years and found that the results were robust. Thus Table 1 focuses on results for the 1-year smoothing period.

The first column reports results for the period 1947–1990. During this period, the mean equity premium was 116.7 basis points per quarter, and the estimated inflation pricing errors ranged from 0.4 to 1.6 basis points. This suggests that inflation uncertainty accounts for only a very small fraction of the equity premium. Furthermore, the upper 2.5 percent probability bounds for the inflation pricing error range from 2.5 to 3.3 basis points, so there is very little chance that this conclusion is due to sampling error.

The second and third columns report results for the subperiods 1947–1970 and 1971–1990. These dates were chosen more or less arbitrarily to split the sample into low- and high-inflation eras. Although the mean equity premium varies substantially across the two sub-samples, inflation pricing errors do not. For example, the mean quarterly equity premium fell from 187.8 basis points in the low-inflation period to 31.5 basis points in the high-inflation period. On the other hand, estimates of the inflation pricing error changed very little, ranging from –1.3 to 0.6 basis points during the low-inflation period and from 1.4 to 1.7 basis points during the high-inflation period. Thus, even though there was much more inflation uncertainty during the latter half of the sample, inflation pricing errors were not dramatically larger.

The estimates for the split samples are less precise than for the whole sample because there are less data. But the probability bounds still suggest that inflation pricing errors are small. For example, during the low-inflation subperiod, the upper 2.5 percent probability bounds range from 0.9 to 2.7 basis points. In the high-inflation subperiod, the upper probability bounds are a bit higher, ranging from 3.2 to 3.6 basis points. Even so, inflation pricing errors appear to be a relatively small fraction of the total equity premium.

TABLE 1

PRICING ERRORS AND THE EQUITY PREMIUM

	1947–1990	1947–1970	1971–1990
μ_e	116.7	187.8	31.5
$\mu_e - \tilde{\mu}_e$ (.99)	0.4 (-0.8, 2.5)	–1.3 (-3.1, 0.9)	1.4 (-0.1, 3.3)
$\mu_{ee} - \tilde{\mu}_e$ (.995)	1.0 (0.1, 2.6)	-0.4 (-1.6, 1.5)	1.6 (0.3, 3.2)
$\mu_e - \tilde{\mu}_e$ (.998)	1.3 (0.5, 3.0)	0.2 (-0.9, 2.1)	1.7 (0.3, 3.4)
$\mu_e - \tilde{\mu}_e$ (1.0)	1.6 (0.7, 3.3)	0.6 (-0.5, 2.7)	1.7 (0.2, 3.6)
$\sigma_{\pi e}/\mu_\pi$	1.2 (0.5, 2.6)	0.4 (-0.7, 1.5)	1.6 (0.4, 3.0)

NOTE: This table reports mean equity premiums and pricing errors for various time periods and values of the mean discount factor. The entries are measured in basis points per quarter, and empirical 95 percent confidence intervals are shown in parentheses.

Inflation Uncertainty and Term Premiums on Treasury Bonds

Table 2 reports inflation pricing errors for quarterly excess holding returns on various Treasury bills, notes, and bonds. Empirical 95 percent confidence intervals are shown in parentheses.

Panels A through C report results for 6-, 9-, and 12-month Treasury bills, respectively. During the period 1947–1990, mean excess holding returns were roughly 10 to 12 basis points per quarter. The corresponding inflation pricing errors were roughly one-tenth of a basis point or less. Mean excess holding returns were higher during the second half of the sample than during the first half, but the point estimates suggest that inflation pricing errors were still only a fraction of a basis point. The 2.5 percent upper probability bounds are also quite small relative to the mean term premiums.

Panels D through G report results for Treasury notes with maturities of 2, 3, 4, and 5 years, respectively. For the sample as a whole, mean excess holding returns were roughly 13 to 16 basis points per quarter. Inflation pricing errors on these securities were also quite small. For example, for the benchmark discount factor $\mu_m = 0.998$, inflation pricing errors range from 0.3 basis points on 2-year notes to 0.6 basis points on 5-year notes. The upper 2.5 percent probability bounds are also small. For the benchmark value of μ_m they range from 0.5 basis points for 2-year notes to 1.0 basis points for 5-year notes.

Term premiums on medium-term notes were also quite a bit larger in the second half of the sample than in the first. For example, while mean excess returns ranged from 2 to 9 basis points per quarter during the period 1947–1970, they ranged from 25 to 28 basis points per quarter during the period 1971–1990. The point estimates for the inflation pricing errors also increased in magnitude, but they were still small relative to the total term premium. For example,

inflation pricing error was calculated for each artificial data set. The pricing errors were then compiled into an empirical probability distribution, and the upper and lower confidence bounds were read off from that distribution.

TABLE 2

PRICING ERRORS AND TERM PREMIUMS ON TREASURY BONDS

	1947–1990	1947–1970	1971–1990
A. 6-MONTH BILLS			
μ_6	10.1	7.4	13.4
$\mu_6-\tilde{\mu}_6$ (.99)	−0.1	−0.1	0.0
	(−0.2, 0.0)	(−0.1, 0.0)	(−0.1, 0.1)
$\mu_6-\tilde{\mu}_6$ (.995)	0.0	0.0	0.1
	(−0.1, 0.1)	(−0.1, 0.0)	(0.0, 0.2)
$\mu_6-\tilde{\mu}_6$ (.998)	0.0	0.0	0.1
	(0.0, 0.1)	(0.0, 0.1)	(0.0, 0.2)
$\mu_6-\tilde{\mu}_6$ (1.0)	0.0	0.0	0.1
	(0.0, 0.1)	(0.0, 0.1)	(0.0, 0.2)
$\sigma_{\pi 6}/\mu_\pi$	0.0	0.0	0.1
	(0.0, 0.1)	(0.0, 0.0)	(0.0, 0.1)
B. 9-MONTH BILLS			
μ_9	11.7	7.7	16.6
$\mu_9-\tilde{\mu}_9$ (.99)	0.0	−0.1	0.1
	(−0.1, 0.1)	(−0.1, 0.0)	(−0.1, 0.2)
$\mu_9-\tilde{\mu}_9$ (.995)	0.0	0.0	0.2
	(−0.1, 0.1)	(−0.1, 0.0)	(0.0, 0.3)
$\mu_9-\tilde{\mu}_9$ (.998)	−0.1	0.0	0.2
	(0.0, 0.1)	(0.0, 0.1)	(0.1, 0.4)
$\mu_9-\tilde{\mu}_9$ (1.0)	0.1	0.0	0.2
	(0.0, 0.2)	(0.0, 0.1)	(0.1, 0.4)
$\sigma_{\pi 9}/\mu_\pi$	0.0	0.0	0.2
	(0.0, 0.1)	(0.0, 0.0)	(0.1, 0.3)
C. 12-MONTH BILLS			
μ_{12}	12.3	7.3	18.3
$\mu_{12}-\tilde{\mu}_{12}$ (.99)	0.0	−0.1	−0.2
	(−0.1, 0.1)	(−0.1, 0.0)	(0.0, 0.4)
$\mu_{12}-\tilde{\mu}_{12}$ (.995)	0.1	0.0	0.3
	(0.0, 0.2)	(−0.1, 0.0)	(0.1, 0.5)
$\mu_{12}-\tilde{\mu}_{12}$ (.998)	0.1	0.0	0.3
	(0.0, 0.2)	(0.0, 0.1)	(0.1, 0.6)
$\mu_{12}-\tilde{\mu}_{12}$ (1.0)	0.1	0.0	0.4
	(0.0, 0.3)	(0.0, 0.1)	(0.2, 0.6)
$\sigma_{\pi 12}/\mu_\pi$	0.1	0.0	0.3
	(0.0, 0.2)	(−0.1, 0.1)	(0.1, 0.5)
D. 2-YEAR NOTES			
μ_{24}	15.8	8.6	24.5
$\mu_{24}-\tilde{\mu}_{24}$ (.99)	0.1	−0.1	0.5
	(−0.1, 0.3)	(−0.2, 0.0)	(0.1, 0.9)
$\mu_{24}-\tilde{\mu}_{24}$ (.995)	0.2	0.0	0.6
	(0.0, 0.4)	(−0.1, 0.1)	(0.2, 1.1)
$\mu_{24}-\tilde{\mu}_{24}$ (.998)	0.3	0.0	0.6
	(0.0, 0.5)	(−0.1, 0.2)	(0.3, 1.2)
$\mu_{24}-\tilde{\mu}_{24}$ (1.0)	0.3	0.0	0.7
	(0.1, 0.5)	(−0.1, 0.2)	(0.3, 1.3)
$\sigma_{\pi 24}/\mu_\pi$	0.2	0.0	0.6
	(0.1, 0.4)	(−0.1, 0.1)	(0.3, 1.0)
E. 3-YEAR NOTES			
μ_{36}	15.4	7.2	25.4
$\mu_{36}-\tilde{\mu}_{36}$ (.99)	0.3	0.0	0.7
	(0.0, 0.5)	(−0.2, 0.1)	(0.2, 1.4)
$\mu_{36}-\tilde{\mu}_{36}$ (.995)	0.3	0.0	0.8
	(0.1, 0.6)	(−0.1, 0.2)	(0.4, 1.5)
$\mu_{36}-\tilde{\mu}_{36}$ (.998)	0.4	0.0	0.9
	(0.2, 0.7)	(−0.1, 0.2)	(0.4, 1.6)
$\mu_{36}-\tilde{\mu}_{36}$ (1.0)	0.4	0.0	0.1
	(0.2, 0.7)	(−0.1, 0.2)	(0.4, 1.7)
$\sigma_{\pi 36}/\mu_\pi$	0.4	0.0	0.9
	(0.2, 0.6)	(−0.1, 0.2)	(0.4, 1.5)
F. 4-YEAR NOTES			
μ_{48}	14.6	4.5	26.9
$\mu_{48}-\tilde{\mu}_{48}$ (.99)	0.4	0.0	1.0
	(0.1, 0.8)	(−0.2, 0.2)	(0.3, 1.8)
$\mu_{48}-\tilde{\mu}_{48}$ (.995)	0.5	0.0	1.1
	(0.2, 0.8)	(−0.2, 0.2)	(0.5, 1.9)
$\mu_{48}-\tilde{\mu}_{48}$ (.998)	0.5	0.0	1.2
	(0.2, 0.9)	(−0.1, 0.3)	(0.5, 2.0)
$\mu_{48}-\tilde{\mu}_{48}$ (1.0)	0.5	0.1	1.3
	(0.2, 0.9)	(−0.1, 0.3)	(0.6, 2.1)
$\sigma_{\pi 48}/\mu_\pi$	0.5	0.0	1.1
	(0.2, 0.8)	(−0.1, 0.2)	(0.5, 1.9)
G. 5-YEAR NOTES			
μ_{60}	13.3	1.7	27.5
$\mu_{60}-\tilde{\mu}_{60}$ (.99)	0.5	0.1	1.2
	(0.1, 1.0)	(−0.2, 0.3)	(0.4, 2.2)
$\mu_{60}-\tilde{\mu}_{60}$ (.995)	0.6	0.1	1.3
	(0.2, 1.0)	(−0.2, 0.3)	(0.6, 2.3)
$\mu_{60}-\tilde{\mu}_{60}$ (.998)	0.6	0.1	1.4
	(0.3, 1.0)	(−0.1, 0.3)	(0.6, 2.4)
$\mu_{60}-\tilde{\mu}_{60}$ (1.0)	0.6	0.1	1.5
	(0.3, 1.1)	(−0.1, 0.3)	(0.7,2.5)
$\sigma_{\pi 60}/\mu_\pi$	0.6	0.1	1.4
	(0.3, 1.0)	(−0.1, 0.3)	(0.6, 2.2)
H. 10-YEAR NOTES			
μ_{120}	7.8	−13.0	33.0
$\mu_{120}-\tilde{\mu}_{120}$ (.99)	1.0	0.2	2.2
	(0.3, 1.8)	(−0.3, 0.6)	(0.8, 3.9)
$\mu_{120}-\tilde{\mu}_{120}$ (.995)	1.0	0.1	2.3
	(0.4, 1.7)	(−0.3, 0.4)	(1.1, 4.0)
$\mu_{120}-\tilde{\mu}_{120}$ (.998)	1.0	0.1	2.4
	(0.4, 1.8)	(−0.3, 0.4)	(1.2, 4.2)
$\mu_{120}-\tilde{\mu}_{120}$ (1.0)	1.0	0.1	2.5
	(0.4, 1.8)	(−0.3, 0.4)	(1.2, 4.3)
$\sigma_{\pi 120}/\mu_\pi$	1.0	0.1	2.4
	(0.4, 1.7)	(−0.3, 0.4)	(1.1, 3.8)

NOTE: This table reports mean term premiums and pricing errors for various Treasury bonds, time periods, and values of the mean discount factor. The entries are measured in basis points per quarter, and empirical 95 percent confidence intervals are shown in parentheses.

for $\mu_m = 0.998$, the point estimates ranged from 0 to 0.1 basis points during the low-inflation decades and from 0.6 to 1.4 basis points during the high-inflation decades. The upper probability bounds were larger during the second half of the sample, but they were also small relative to the total term premiums. For example, for $\mu_m = 0.998$, the upper probability bounds ranged from 0.2 to 0.3 basis points in the first half of the sample and from 1.2 to 2.5 basis points in the second half.

Finally, Panel H reports results for excess returns on 10-year Treasury bonds. For the sample as a whole, the mean term premium was 7.8 basis points, and the estimated inflation pricing error was 1 basis point, with an upper probability bound of roughly 1.8 basis points. But the mean term premium appears to be an average of two distinct regimes. During the first half of the sample, the mean excess holding return was –13 basis points per quarter, and the inflation pricing error was only a fraction of a basis point. During the second half of the period, the mean excess holding return was 33 basis points per quarter, and the inflation pricing error was roughly 2 or 2.5 basis points.

These results suggest that an investor who paid no attention to inflation uncertainty would have made relatively small pricing errors on Treasury securities. This holds for the sample as a whole and for the high inflation sub-period.

III. CONCLUSION

This paper investigates the relation between inflation uncertainty and excess returns on stocks and bonds. It complements prior research by weakening the maintained assumptions used to isolate the effects of inflation uncertainty. The empirical analysis is non-parametric, so the results cannot be dismissed on the grounds that standard discount factor models have difficulty pricing risk.

The paper quantifies the effects of inflation uncertainty by comparing mean excess returns with the values expected by a hypothetical naive investor who treats inflation forecasts as if they were known with certainty. One way to interpret this exercise is to ask, "How badly would I do if I were to ignore inflation uncertainty completely when pricing financial assets?" The evidence suggests that this would result in only small pricing errors, on average.

Our naive investor implicitly relies on two simplifying assumptions. First, he uses the Fisher real return rather than the exact real return. Second, he assumes that 3-month Treasury bills are riskless. Both are simplifications that macroeconomists often use in applied work. Anyone who resorts to these simplifying assumptions implicitly adopts a prior that inflation uncertainty is unimportant for excess return puzzles. The evidence reported in this paper suggests that this may not be such a bad prior.

Finally, two caveats are in order. First, the empirical analysis is limited to estimates of unconditional pricing errors. It is possible that conditional pricing errors are large and variable but have mean zero. If so, inflation uncertainty could be important for time-varying risk premiums, but my analysis would fail to detect it. Breaking the sample in two represents a small step toward investigating conditional pricing errors, but further use of conditioning information would clearly be desirable.

A second caveat concerns the framework for analysis. This paper interprets failures of parametric discount factor models as failures of the parameterization rather than as a failure of the framework. Other researchers interpret the failure of parametric models as a failure of the framework and have begun to explore models based on incomplete markets and cognitive misperceptions. Those models might deliver different results about the importance of inflation uncertainty.

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