Swiss Unconventional Monetary Policy: Lessons for the Transmission of Quantitative Easing

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Abstract

We analyze the reaction of long-term government bond yields to announcements by the Swiss National Bank (SNB) to implement unconventional monetary policy initiatives during the summer of 2011. Since these policies included an expansion of central bank reserves without any purchases of long-term securities, they provide novel insights into the transmission mechanism of quantitative easing. Using dynamic term structure models, we decompose the response of Swiss government bond yields into changes to expectations about future short-term interest rates and term premiums. We find that the declines in yields following the announcements of the reserve expansions reflected reduced term premiums, whereas expectations about future short-term rates changed little. We interpret this as evidence that expansions of reserves by themselves can give rise to portfolio balance effects.

JEL Classification: G12, E43, E52, E58

Keywords: Term structure modeling, monetary policy, quantitative easing
1 Introduction

After having lowered conventional policy rates to their effective zero lower bound by early 2009, a number of major central banks have engaged in large-scale asset purchases—frequently referred to as quantitative easing (QE)—to provide further monetary stimulus through unconventional means. The stated aims of such purchases differ slightly across countries, but usually involve reducing long-term interest rates, either broadly or in specific markets. Whether QE programs have reduced long-term interest rates—and through what channels—has become the topic of a large and growing literature.

This literature has focused on two main channels. One is a signaling channel, which works through changing market expectations about future monetary policy (see, e.g., Christensen and Rudebusch 2012 and Bauer and Rudebusch 2013); another is a portfolio balance channel arising from changes in the supply available in the market for the assets that the central bank has purchased (see, e.g., Gagnon et al. 2011 and Krishnamurthy and Vissing-Jorgensen 2011).1,2 Bernanke and Reinhart (2004), however, point out that portfolio balance effects of QE programs can arise through an additional reserve channel. Namely, the increase in the supply of reserves may put upward pressure on asset prices more broadly. When the central bank buys specific securities in large quantities and pays for these by issuing central bank reserves, both channels can work simultaneously. This is the case for all three QE programs conducted by the Federal Reserve since 2008, and for the Bank of England’s asset purchase programs. Both central banks conducted QE by buying large quantities of safe and liquid long-term bonds in exchange for newly issued reserves.3 The implication is that the effects of QE programs on long-term yields documented in the previous empirical literature may represent portfolio balance effects derived either from reductions in the relative supply of long-term bonds or from the increased supply of central bank reserves. Critically, the effects of these two different channels cannot be separately identified in those cases.

This paper addresses the empirical relevance of the reserve channel by investigating the unconventional monetary policies conducted by the Swiss National Bank (SNB) in August 2011. To counter increasing deflationary concerns and a rapid appreciation of the Swiss franc at the time, the SNB announced and carried out three consecutive expansions of reserves held as sight deposits at the SNB. The expansions were large and carried out within the span of

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1 See also Joyce et al. (2011), Hamilton and Wu (2012), Thornton (2012), and Neely (2013) for discussions.
2 There is also a third potential channel for QE to work, namely through its effect on liquidity and market functioning; see Christensen and Gillan (2014) and Kandrac (2014) for discussions and analysis in the context of U.S. QE programs.
3 There is one exception, namely the Federal Reserve’s “Maturity Extension Program” (MEP) that operated from September 2011 through 2012. This program involved purchases of more than $600 billion of long-term Treasury securities (defined as bonds with more than six years to maturity) financed by selling an equal amount of shorter-term Treasuries (defined as bonds with less than three years to maturity). Thus, the MEP represents a case of sizable purchases and sales of securities without any change in the amount of reserves. This contrasts with the Swiss National Bank program we study, which features the opposite combination. See Cahill et al. (2013) and Li and Wei (2013) for analysis of the Fed’s MEP.
a few weeks, making the program unprecedented in terms of both its size and how quickly it was implemented.\(^4\) While the expansions were achieved through purchases of a combination of assets, they were announced as, and centered around, an expansion of reserves. Equally important, the expansions were achieved without any purchase of long-term debt securities. Thus, these actions left the market supply of long-term government bonds—as well as the supply of close substitutes—unchanged. This makes for a very interesting case study of the transmission of quantitative easing to long-term yields.

The question we are interested in is whether the SNB’s expansion of reserves in August 2011 affected long-term Swiss government bond yields, and through which channels.\(^5\) We document that yields did respond in the immediate aftermath of the announcements. Long-term Swiss Confederation bond yields dropped by a cumulative total of 28 basis points following the three SNB announcements of reserve injections. Relative to the yield on the ten-year Swiss Confederation bond of 1.33 percent on the eve of the first announcement, 28 basis points represent a substantial and significant drop.

Such yield declines could primarily happen through three channels. The first is the portfolio balance effect derived from an expanded supply of reserves held by banks, as emphasized by Bernanke and Reinhart (2004). The second is a possible portfolio balance effect related to the assets that the SNB purchased to achieve the reserve expansions. Given the short maturity of these assets, we argue that direct substitution effects are most likely negligible. As a consequence, this channel is unlikely to have been important for long-term yields. Third, as with other QE programs, the SNB announcements could have produced signaling effects.

To separately identify these channels in the data, we follow the literature and use term structure models combined with an event study approach similar to Christensen and Rudebusch (2012, henceforth CR), who investigate the response of U.K. and U.S. government bond yields to their respective unconventional policy initiatives. Performing rolling daily re-estimations of dynamic term structure models of Swiss Confederation bond yields, allows us to decompose, in real time, long-term yield changes into changes to expected short-rate and term premium components.\(^6\) The expected short-rate component is then associated with monetary policy expectations, while portfolio balance effects are associated with the term premium.

With estimated changes in term premiums and monetary policy expectations in hand, we evaluate and compare the responses around the SNB announcements. We find that the

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\(^4\)By early September 2011, the SNB’s balance sheet had expanded by an amount equal to 30 percent of Swiss GDP.

\(^5\)Our focus on Swiss government bond yields is, in part, motivated by the findings of Ranaldo and Rossi (2010), who study the response of various Swiss financial assets to SNB monetary policy announcements. They find that the bond market shows strong reactions to such events. This suggests that a focus on Swiss Confederation bonds will provide the clearest reading of investors’ reactions to the SNB announcements.

\(^6\)Gagnon et al. (2011), CR, and Bauer and Rudebusch (2013) are among the previous studies that provide term structure model decompositions of the U.S. experience with unconventional monetary policies. Mirkov and Sutter (2013) also use term structure models to analyze both the U.S. and Swiss experience with such policies, but they do not make a real-time event study like ours.
drop in long-term Swiss Confederation bond yields was predominantly in the term premium, suggesting portfolio balance effects. By contrast, we find signaling effects to have been less important in driving the response of long-term yields to the SNB’s announcements. Given the nature of the SNB reserve expansions, we conclude that the most likely driver of the identified portfolio balance effects were the reserve expansions themselves, rather than the reduced supply of the assets that the SNB bought. To our knowledge, this is the first paper, using data on unconventional monetary policies in the aftermath of the global financial crisis, to show that an expansion of reserves can have significant portfolio balance effects on long-term bond yields in the absence of long-term bond purchases.

Regarding the relative importance of signaling versus portfolio balance effects, our findings are similar to those reported by CR in their analysis of the U.K. QE program. We speculate that this could be linked to the fact that neither the U.K. QE program nor the SNB announcements studied here were accompanied by any type of forward guidance that could have affected bond investors’ expectations about future monetary policy. This contrasts with findings for the U.S. QE program, where both CR and Bauer and Rudebusch (2013) report evidence of significant signaling effects consistent with the forward guidance provided by the FOMC.7

These findings have a number of policy implications. First, they suggest that it is possible to design effective quantitative easing programs with the aim of influencing long-term interest rates in economies where institutional or market factors preclude large-scale central bank purchases of long-lived securities. Second, when exiting large-scale asset purchase programs, the management of reserves could warrant as much attention as the wind-down of the purchased assets. And finally, the effect of central bank unconventional policies may depend crucially on central bank communication policies, as also emphasized by CR.

The remainder of the paper is structured as follows. The next section describes the context and details of the SNB’s three expansions of reserves in August 2011. In Section 3, we discuss in greater detail how we expect the expansion of reserves to have affected interest rates. Section 4 contains the model-based event analysis of the market reaction around the SNB announcements. It introduces the event study approach, the data, and our empirical term structure models, and it describes how we use the models to extract short-term interest rate expectations and term premiums from bond yields. Furthermore, it contains our main empirical results. In Section 5, we analyze the identified drop in the term premium and carry out related robustness checks, while Section 6 concludes and discusses the policy implications. Appendices contain additional event information, empirical results, and technical formulas.

7At first, in December 2008, the Federal Reserve introduced the formulation that its target rate would be exceptionally low for “some time.” In March 2009, the language in FOMC statements was changed to state that an exceptionally low target rate would be warranted for “an extended period of time” before explicit forward guidance was given starting with the FOMC statement in August 2011.
Figure 1: The Exchange Rate between the Swiss Franc and the Euro.
Panel (a) shows the daily movements in the exchange rate between the Swiss franc and the euro since 1999. Panel (b) shows the daily movements around the four 2011 SNB unconventional policy announcements, indicated with vertical lines. In both panels, the minimum exchange rate level of 1.20 announced on September 6, 2011, is shown with a dotted black horizontal line. Source: SNB.

2 The SNB’s Expansion of Reserves in August 2011

In normal times, the SNB ensures price stability by setting a target range for a representative short-term money market interest rate, the three-month CHF LIBOR, and by steering market rates toward this target through short-term repo operations. The exchange rate is floating under normal circumstances. This policy framework reached its limit in March 2009 when, in response to developments related to the financial crisis, the SNB reduced its target rate to what was considered its effective lower bound. Further monetary policy easing continued to be desirable, but a complicating factor was the persistent strengthening of the Swiss franc due to sustained safe-haven pressures starting in late 2008, see Figure 1(a). The appreciation added considerable downward pressure on Swiss consumer prices despite the low interest rate level.

As a response, the SNB adopted a number of unconventional policies. In March 2009, these included foreign exchange interventions to prevent further appreciation, extension of the maturity for repo operations, and a relatively small, targeted, and short-lived bond purchase program.8

When economic prospects temporarily improved, the bond purchase program was discontinued by the end of 2009, and exited in 2010, and foreign exchange interventions were officially discontinued in the summer of 2010. By that time, however, the foreign exchange

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8See Kettemann and Krogstrup (2014) for an overview and analysis of the impact of this program.
Interventions had resulted in a substantial expansion of the SNB’s balance sheet and central bank reserves. A large part of these reserves were gradually absorbed starting in 2010, through reverse repo operations and the sale of short-term SNB bills.\textsuperscript{9} Still, the exchange rate continued to appreciate. In 2011, the intensification of the European debt crisis compounded woes and resulted in increasing risk of severe deflation in Switzerland.

Against this background, the SNB introduced new unconventional policy measures in August and September 2011. First, on August 3, the SNB announced that it would further lower the top of the target range for the three-month CHF LIBOR from 75 to 25 basis points (the lower end of the range was already at zero), and that it would aim at the lower end of the range. At the same time, it announced that it would significantly increase its supply of liquidity to Swiss money markets.\textsuperscript{10} Specifically, the SNB would expand banks’ sight deposits (i.e., central bank reserves) from CHF 30 billion to CHF 80 billion.\textsuperscript{11} The stated intention was to push down money market interest rates, thereby making the Swiss franc less attractive against other currencies. No intentions of affecting long-term yields or risk premiums were stated.

The reserve expansion was to be achieved by buying back SNB bills from the markets, by not rolling over maturing SNB bills, and by allowing reverse repos with banks to expire. The intended mix of these operations could only be observed ex post. Figure 1(b) shows that the exchange rate appreciation briefly paused, but quickly resumed following this first announcement.

One week later, on August 10, the SNB announced that it would again expand reserves, this time by a further CHF 40 billion.\textsuperscript{12} To achieve the second expansion quickly, the SNB would, in addition to the previous types of operations, also conduct short-term foreign exchange swaps (primarily of one week maturity). The exchange rate reversed course and briefly depreciated following this announcement. The depreciation was not considered sufficient, however, and on August 17, the SNB announced it would raise reserves further, this time by CHF 80 billion. This final expansion would take the total level of reserves to roughly CHF 200 billion.\textsuperscript{13}

The exchange rate response was again muted. In the weeks that followed, the appreciation resumed. Therefore, on September 6, the SNB adopted a minimum exchange rate for the Swiss franc of 1.20 francs per euro, and stated its willingness to buy foreign currency in unlimited

\textsuperscript{9}SNB bills are short-term debt securities with maturities up to one year issued by the SNB.
\textsuperscript{11}Banks’ sight deposits are equivalent to central bank reserves. Approximately 300 banks hold sight deposits at the SNB. Sight deposits are non-interest bearing and readily available for payment transactions and represent legal payment instruments. Banks also hold sight deposits as a liquidity reserve and in order to fulfill the statutory minimum reserve requirements. The SNB directly influences the aggregate amount of sight deposits, and hence the liquidity in the Swiss franc money market, through its money market operations. Total SNB sight deposits also include deposits held by the Swiss government and a smaller number of nonbank financial institutions.
<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Announcement description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Aug. 3, 2011, 8:55 a.m.</td>
<td>Target range for three-month CHF LIBOR lowered to 0 to 25 basis points. In addition, banks’ sight deposits at the SNB will be expanded from CHF 30 billion to CHF 80 billion.</td>
</tr>
<tr>
<td>II</td>
<td>Aug. 10, 2011, 9:05 a.m.</td>
<td>Banks’ sight deposits at the SNB will rapidly be expanded from CHF 80 billion to CHF 120 billion.</td>
</tr>
<tr>
<td>III</td>
<td>Aug. 17, 2011, 8:55 a.m.</td>
<td>Banks’ sight deposits at the SNB will immediately be expanded from CHF 120 billion to CHF 200 billion.</td>
</tr>
<tr>
<td></td>
<td>Sep. 6, 2011, 10:00 a.m.</td>
<td>The SNB announces a minimum exchange rate for the Swiss franc to the euro of 1.20 francs per euro and is prepared to buy foreign currency in unlimited quantities to defend it.</td>
</tr>
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Table 1: SNB Policy Announcements in August and September 2011.

quantities to defend it.\textsuperscript{14} The exchange rate immediately moved to 1.20 and has remained at or above this threshold since.

Our focus is on the three expansions of reserves announced in August 2011 (events I-III in Table 1). The sum of these reserve expansions amounted to CHF 170 billion, or about 30 percent of Swiss GDP in 2011. In comparison, the U.S. aggregate QE programs have yet to reach such a magnitude.\textsuperscript{15} Figure 2 shows the reserve expansions and their main counterparts on the SNB balance sheet. A large part was achieved by repurchasing SNB bills and allowing bills to mature without new issuance. The total volume of outstanding bills was reduced by CHF 66 billion in August alone. By the end of 2011, outstanding bills had been reduced by nearly CHF 100 billion. Expiration of reverse repos amounted to CHF 26 billion in August, after which all reverse repo operations had expired. Liquidity-increasing repos were subsequently carried out, but these contributed only a small part of the overall reserve expansion. The largest part of the expansions in August was achieved through other measures, most notably foreign exchange swaps. As SNB bills were increasingly bought back during the rest of 2011, a corresponding part of the foreign exchange swaps were allowed to expire.

To be able to learn something from the market response to these announcements using an event study, at least part of these measures must have been unexpected when they were announced. We therefore briefly address this issue here. Clearly, the public was expecting a monetary policy reaction to the worsening situation in August 2011. There was plenty of discussion in the Swiss media and a certain level of pressure from political and interest


\textsuperscript{15}As of the end of 2013, the Federal Reserve’s balance sheet totaled $4.1 trillion, or about 25 percent of U.S. GDP.
groups to enact exchange rate measures to counter what was seen as an unsustainable and unacceptable appreciation in the spring and early summer of 2011. The public called for a floor or peg for exchange rates, or for interventions to reverse the exchange rate trend. There was also speculation about the SNB introducing negative interest rates, and for good reason. The SNB had responded to a strongly appreciating exchange rate in the 1970s by introducing negative interest rates on foreign bank deposits, before finally introducing an exchange rate floor to the German mark in 1979.

However, the timing and specific nature and content of the announcements were very likely to have been unexpected. The three announcements were not pre-announced, and followed unscheduled meetings of the SNB’s Governing Board.\textsuperscript{16} The public debate prior to the announcements did not include any discussion of possible liquidity expansions. Reserve expansions had not been used before as a policy tool by the SNB, nor had it ever been publicly discussed as a possible means to counter exchange rate appreciation pressures during this episode. Moreover, the sheer size of the expansions seems to have been a complete surprise. Thus, the announcements appear to satisfy the requirements for a classic event study of the type we perform later in the paper.

\textsuperscript{16}The SNB normally releases its monetary policy statements on a scheduled quarterly basis in mid-March, mid-June, mid-September, and mid-December.
3 Transmission to Long-Term Interest Rates

In this section, we show that the announcements of the SNB’s reserve expansions were associated with drops in Swiss long-term interest rates, and we offer some theories about possible transmission channels.

Figure 3 shows the movements of the daily ten-year Swiss Confederation bond yield during the summer and early fall of 2011, and the dates of the three announcements of reserve expansions as well as the date the exchange rate floor was introduced. The yield was already on a downward trend due to strong global safe-haven pressures and high risk aversion when the first announcement was made. During the weeks of the three announcements, however, the drop in the yield seems to have accelerated. Moreover, yields invariably fell following all three announcements. In Section 4, we show that yields dropped by a collective 28 basis points following the three announcements. The drop in response to the last and most forceful announcement was the strongest, and most significant.

Through which channels could these announcements have reduced long-term yields? Below, we outline some theories about possible transmission channels to long-term yields. To structure the discussion, note that the yield of a bond can be written as consisting of a risk-neutral part that represents the expected future short interest rates until maturity, and a
term premium which compensates investors for the added risk they take when investing in a fixed-income bond of a given maturity instead of investing the same amount in the short-term money market:

\[ y_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} E_t^P[r_s] ds + TP_t(\tau), \]

where \( t \) is time and \( \tau \) is time until maturity. \( RN_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} E_t^P[r_s] ds \) is the risk-neutral component of the yield that is identical for all bonds independent of the issuer. The term \( TP_t(\tau) \) captures macro risks such as uncertainty regarding the growth and inflation outlook, changes in overall risk aversion, issuer-specific risks such as the credit risk of the issuer in question and liquidity risk of the bond. Finally, it also captures a premium due to supply and demand factors in the presence of market imperfections.

The effect of the expansion of reserves can be divided into two broad categories, namely policy signaling effects and portfolio balance effects. The former affect the risk-neutral component of the yield, while portfolio balance effects are specific to the security, and hence affect the term premium. We discuss each of these types of effects below and their relevance in the context of the Swiss reserve expansions in August 2011.

### 3.1 Policy Signaling Effects

Policy signaling affects the risk-neutral part of the yield, \( RN_t(\tau) \). Thus, the policy announcements could have changed the market view of how the SNB intended to set short-term interest rates in the future, that is, for how long the SNB intended to keep the short-term policy rate at the zero lower bound, and how quickly it would increase that rate after exiting the zero lower bound. If the announcements in August 2011 indicated that the SNB was more concerned about the subdued outlook for inflation than previously perceived, we should expect measures of average expected future short-term policy interest rates to fall in response to the announcements.

In the empirical analysis in Section 4, we find that such signaling effects were small in connection with the announcements of reserve expansions. However, the strong reaction of short money market rates to the announcements was generally interpreted as a form of signalling effect at the time. In the following, we reconcile these two views by taking a quick look at Swiss money market rates around the August 2011 announcements.

Figure 4 plots the development in selected short-maturity Swiss franc term overnight indexed swap (TOIS) rates. Changes in TOIS rates are usually taken as good proxies for changes in expected future short-term interest rates. The depicted rates dropped by 30

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17 This is of course a simplification. See Bauer and Rudebusch (2013) for a thorough discussion.

18 TOIS quotes are collected around 11 a.m. on each business day. We would ideally want to investigate long-term Swiss franc TOIS rates, which would reflect the expected policy path over a longer horizon. However, traded TOIS contracts with long maturities are few and the market for such contracts developed only recently and is not liquid. For this reason, we consider TOIS rates of the more liquid part of the market with maturities up to six months.
to 70 basis points and turned negative in the weeks following the first announcement. The strongest reaction came after the third announcement, when the three-month TOIS rate fell 17 basis points to -0.24 percent within a few hours of the announcement and a further 22 basis points on the following day, reaching its lowest point ever of -0.46 percent. To put this reaction into perspective, a change of 22 basis points in the three-month TOIS rate amounts to seven standard deviations of its daily variation since records began in 2000. The SNB’s intermediate aim of pushing down money market rates through reserve expansions clearly was very successful.

A negative three-month TOIS rate means that the counterparty paying the floating rate is willing to pay a fixed rate (for example 0.46 percent) for a three-month period for the right to also pay the floating overnight rate to the counterparty. This only makes sense if there is a possibility that the overnight rate could turn negative during the next three months. As already discussed, the financial press at the time indeed speculated that the SNB might introduce negative interest rates. It is therefore likely that investors placed a much higher probability on the SNB introducing negative interest rates after having observed that the SNB was prepared to take steps like those announced in August 2011.

We consider these strong dips into negative territory to represent a short-term expectation,
that is, market participants may have increased the probability they attached to the SNB imposing negative interest rates, but if negative interest rates were imposed, they did not expect those rates to stay negative for long. We hence do not consider the drops in rates to imply signaling effects for long-term yields. Our reasons for this interpretation are provided in the following.

First, market participants were expecting the SNB to take crisis measures, rather than seek to loosen the overall monetary policy stance. A crisis measure such as negative interest rates, if effective, should only affect expected short rates in the very near term (during the crisis), making any effect on longer-term interest rates very small. One parallel would be the market reaction around the approaching debt ceiling deadline for the U.S. federal government in October 2013. Unlike the Swiss case, where we can only speculate about what type of scenarios investors were fearing, the U.S. debt ceiling episode presented a tangible risk of default at a specific, known time. This makes it useful for drawing comparisons. Figure 5 shows yields on outstanding U.S. Treasury bills on two days, one several weeks before the official deadline and the other just days before it. Bills that would mature immediately after the debt ceiling deadline were seriously affected, while bills with maturities further in the future barely responded. Apparently, investors expected that, even if a technical default were to happen, it would be short-lived—measures would be taken to solve the problem. The key

Figure 5: U.S. Treasury Bill Curve ahead of the U.S. Debt Ceiling Deadline.
Illustration of the U.S. Treasury bill curve on October 8, 2013, a few days before the official debt ceiling for the U.S. federal government would be breached. For comparison the Treasury bill curve on September 18, 2013, is shown. Source: Bloomberg.
takeaway is that rather extreme priced expectations for near-term events can exist with no material implications for medium- and long-term expectations. We suspect that the Swiss money market reaction following the SNB announcements in August 2011 is an example of this case.

Second, the rapid reversal in the rates after August 17, 2011, implies that the net decline from the end of July 2011 through September 2011 is much smaller and more consistent with the variation observed in the Swiss Confederation bond market that our empirical model-based analysis in Section 4 relies upon.

Third, changes in expected future short rates are not confirmed by the monthly Consensus Forecasts survey of professional forecasters. This survey suggests that the biggest decline in short-rate expectations occurred between the surveys dated July 11, 2011, and August 8, 2011, that is, in response to the first announcement that also included a lowering of the target range for the three-month CHF LIBOR. The September and October 2011 surveys show more muted responses.

To summarize, we find the dramatic declines in short-term money market rates around the SNB announcements to be exaggerated and reflect expectations about crisis measures rather than revisions to medium- and long-term expectations about future monetary policy.

3.2 Portfolio Balance Effects

Portfolio balance effects are related to the relative supplies of different assets in the market. Theory suggests that when assets with otherwise near-identical risk and return characteristics are considered imperfect substitutes by some market participants (e.g., due to preferred habitats) and markets are segmented, a change in the relative market supply of an asset may affect its relative price (see Tobin 1969 and Vayanos and Vila 2009). According to such theories, for market participants to be willing to hold more of an asset that has increased in relative supply, the relative price of this asset will have to fall, or its expected return relative to those of other assets will have to increase.

If the SNB reserve expansions had portfolio balance effects on long-term interest rates, this can only have been through substitution effects due to the reduced supply of the assets bought by the SNB, or through portfolio reallocations by banks in response to the increase in central bank reserves.

First, we consider whether the assets purchased by the SNB were likely to have caused a reduction in term premiums of long-term bonds through portfolio balance effects. To begin, take the example of short-term bills. Whether a reduction in short-term bills would spill over into higher demand for long-term bonds, would in theory depend on the substitutability between short- and long-term safe assets. Very little is known about this substitutability. However, we find it unlikely that the difference in maturity of the two assets would not play a strongly differentiating role. While we cannot exclude an effect, it would likely be of second
Now, we consider whether the changes in reserves per se might have had portfolio balance effects on asset prices. To keep the exposition simple, Figure 6 shows simplified versions of the aggregate balance sheets of three types of market players in asset markets, namely the central bank, reserve holding banks and non-bank financial institutions. Below, we give an example of how a portfolio balance effect of reserves could work through the balance sheet and asset allocation choices of banks.

Regarding repo collateral, the relatively small reductions in repo collateral associated with the reserve expansions are unlikely to have increased demand for long-term Swiss franc bonds in August 2011. The reason is that the general collateral basket for Swiss franc repos comprises foreign currency collateral, making the pool from which to draw such collateral much larger than Swiss franc bonds. The Swiss secured lending market usually functions in an environment with scarcity of Swiss franc-denominated collateral. Partly as a result of this scarcity, there is broad availability and acceptance of collateral denominated in foreign currency for Swiss repo operations. Regarding FX swaps, short-term changes in foreign exchange is unlikely to have affected the price of government bonds, as the foreign exchange changes related to the SNB operations were minuscule relative to the foreign exchange market.

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Figure 6: Balance Sheets of an Asset Market with Three Actors.
Stylized balance sheets of three actors in a financial market, the central bank, reserve holding banks and non-bank financial institutions. The central bank can transact with both types of institutions.
Take the example of SNB purchases of SNB bills from the market. SNB bills are short-term liquid assets, which are arguably very close, if not perfect, substitutes for reserves. Assume for the sake of argument that the two are in fact perfect substitutes and hence carry the same zero yield at the zero lower bound. The red arrows in Figure 6 show what happens to the central bank balance sheet when it purchases short-term government bonds from the private financial sector; its assets increase with the amount of short-term bonds purchased and its liabilities increase with the same amount of reserves. Assume now that the counterparty to the central bank is a bank. In this case, the matching arrows in the private sector are the green ones. The aggregate bank balance sheet size remains unchanged, but the composition of short-term assets shifts from short-term bonds toward reserves. If the two types of assets were in fact perfect substitutes at the zero lower bound, then banks might not consider this asset swap to change their portfolio composition, and would hence see no need to take further actions to change its portfolio. No asset prices would necessarily change. This is the usual argument against portfolio balance effects of reserves at the zero lower bound. However, we see at least one reason why there could still be a portfolio balance effects of central bank short-term bond purchases at the zero lower bound, as explained in the following.

When the central bank purchases its short-term bonds from non-bank financial firms on the other hand, we have the case of the blue arrows in Figure 6. Carpenter et al (2013) find that the ultimate sellers of assets to the Federal Reserve in connection with its QE programs were non-bank financial institutions. The U.K. asset purchase program was, at least initially, mainly conducted in assets which were held by non-bank financial institutions as well. So this case is of practical relevance for understanding the transmission of QE. As before, we assume that short-term bonds and reserves are perfect substitutes for banks. In addition, we assume that deposits held by non-bank financial institutions at their correspondent banks are also perfect substitutes to short-term government bonds. Since non-bank financial firms cannot accept and hold reserves, the central bank in this case credits the reserves with the correspondent banks, which then in turn credit the deposits to their customers, the non-bank financial firms. Under our asset substitutability assumptions, the balance sheet and composition of the non-bank financial firms would be largely unchanged, and hence would be unlikely to induce any further asset price effects. However, the banks’ aggregate balance sheet has now grown by the amount of reserves issued on the asset side, and by the new deposits on the liability side. Critically, the banks had no say in these transactions, which they are obliged to carry out on behalf of their customers. The bank balance sheet impact, in other words, happens endogenously when the counterparty of the central bank is not a bank. Assuming the banks considered their asset allocation optimal before the balance sheet expansion, and provided the newly issued deposits are considered a stable source of funding

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20 Other types of reserve expanding operations conducted by the SNB would have had very similar effects.
21 This is obviously far fetched as the risk profile of the two clearly differ, but if we relax the assumption, the case for portfolio balance effects would only be stronger.
at the aggregate banking sector level, then it is unlikely that banks would view their new asset allocation as optimal since it has become more heavily tilted toward reserves than before. Banks may individually try to diversify out of excess reserves and into other assets. In the aggregate, however, banks have to hold the reserves created by the central bank's open market operations, and they can only sell reserves to each other. They might seek to purchase assets from each other using reserves, and this process will continue until relative asset prices have adjusted sufficiently for individual banks to be content holding the increased amount of reserves. Finally, with reserves being the numeraire, their price cannot change, instead the prices of other assets in banks' portfolios have to increase.

In principle, all securities held by banks in their financial asset portfolios could be affected according to this logic. To limit our focus and make our study comparable to the existing literature, however, we consider only Swiss long-term Confederation bonds in our empirical examination. One reason why banks are likely initially to have increased their demand for Swiss Confederation bonds in response to reserve injections is that such bonds are liquid, safe, and benefit from a zero risk weight for calculating regulatory risk-weighted assets. Risk weights arguably have represented an important balance sheet constraint for bank portfolio choices in recent years. Another reason why this effect could be important for Swiss Confederation bonds is that the size of the reserve expansions in 2011 was large relative to both the size of the entire Swiss Confederation bond market (around CHF 100 billion in recent years) and banks' holdings of these (about CHF 11 billion of these were held by banks in Switzerland in 2011).22 If only a small proportion of the reserve injections in 2011 resulted in higher bank demand for Confederation bonds, the effect on the relatively small Confederation bond market could likely be substantial.

The empirical literature on portfolio balance effects from changes in reserves is scarce. We are not aware of any event studies of QE programs focusing on the effect of reserves. Two related papers, Krogstrup et al. (2012) and Mirkov and Sutter (2013), empirically investigate the association between reserves and long-term yields in connection with post-financial crisis unconventional monetary policies in the United States and Switzerland, and find tentative evidence of reserve effects.

To conclude, portfolio balance effects could have affected the yields of Swiss long-term Confederation bonds through the increase in central bank reserves. Whether or not such effects are relevant is an empirical question. We next turn to the data.

22Foreign banks with sight deposits at the SNB could have held additional Confederation bonds. Data on Confederation bond supply and bank holdings are available in the annual Swiss National Bank publications “Banks in Switzerland” and “Swiss Financial Accounts.”
4 Empirical Analysis

In this section, we first describe the event study method we use to analyze the effects of the SNB announcements. Second, we detail the Swiss government bond yield data set used in the analysis and its reaction to the announcements. Third, we describe how bond yields can be decomposed into a short-rate expectations component and an associated term premium component, and we introduce the specific class of Gaussian term structure models we use for that purpose before we proceed to finding a preferred specification and documenting its performance. We end the section by using the term structure models to perform a real-time decomposition of the yield responses to the SNB announcements into separate short-rate expectations and term premium components.

4.1 Event Study Methodology

As bond prices are forward looking asset prices, any potential portfolio balance effects will be reflected in bond prices at the time markets become aware of a future change to relative asset supplies. The price impact thus occurs not when a policy is implemented, but when it becomes known to the market. Remaining unexpected effects might occur at or around implementation of a policy, but as long as we do not know what was expected in the first place, we also do not know how to interpret a potential implementation effect. Assuming that the policy announcements in August 2011 contained new information for financial market participants about the relative supply of assets, we therefore limit our study to an event analysis of these announcements.

We use a two-day window as the baseline for the event study, in line with the literature (see, e.g., Joyce and Tong 2012). A broad window is necessary because we do not know exactly when during the morning the yield data we investigate are collected (further details about the data are provided below). The bond data could have been collected at the same time, around 09:00 a.m., or several hours after the announcements were made. Moreover, we need to allow market participants sufficient time to digest and factor in the new information contained in the unusual announcements. In fact, results reported in Appendix B show that, for all three announcements, the responses that materialize between the morning before the announcements and the recording of the data on the morning of the announcements are rather small.

Ranaldo and Rossi (2010) find that, in the past, Swiss bond markets have taken up to 30 minutes to respond to conventional, and hence familiar, types of SNB policy announcements. The event window should allow for at least this amount of time for markets to digest and react. By investigating the change between the morning of the day before the announcements and the morning of the day after the announcements, we allow for a minimum of 24 hours, but no more than 26 hours, for the response to materialize after each announcement.
Figure 7: Statistics for Bonds Used in Yield Curve Construction.
Illustration of the number of bonds used in the construction of the daily Swiss government zero-coupon bond yield curve as well as their minimum and maximum maturity. The sample covers the period from January 6, 1998, to December 30, 2011.

The drawback of a broad event window is a higher risk of including news not related to the event. In the robustness section, we therefore carefully consider whether other events could be driving our results. Moreover, the event study technique suffers from the fact that we cannot accurately assess what was expected before each announcement. The discussion in Section 2 of expectations around the time of the announcements suggests that some action was likely expected by market participants prior to the announcements, although the specific nature of the announcements was likely to have been a surprise. This could result in some degree of underestimation of the interest rate response. At the very least, however, our results provide a benchmark to compare with alternative ways of extracting market reactions.

4.2 Daily Data on Confederation Bond Yields

We now describe the yield data derived from Swiss Confederation bonds and used in the empirical analysis, and take a second look at how yields behaved in the event windows around the three policy announcements.

The specific Swiss bond yields analyzed in this paper are zero-coupon yields constructed
using a smooth discount function based on the Svensson (1995) yield curve: \( y(\tau) = \beta_0 + \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \beta_1 + \left[ \frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right] \beta_2 + \left[ \frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right] \beta_3. \)

For each business day, this function is used to price a set of observed Swiss Confederation bond prices. Figure 7 shows the number as well as the shortest and longest maturity of the bonds used in the daily estimation of the discount function over the sample period. The zero-coupon yields derived from this approach should constitute a very good approximation to the true underlying Swiss government zero-coupon yield curve over the maturity range covered by the underlying pool of bonds.\(^{24}\) Using the fitted values of the four coefficients, \((\beta_0(t), \beta_1(t), \beta_2(t), \beta_3(t))\), and the two parameters, \((\lambda_1(t), \lambda_2(t))\), we obtain zero-coupon bond yields with six maturities: one, two, three, five, seven, and ten years to maturity. The summary statistics are provided in Table 2, while Figure 8 illustrates the constructed time series of the one-, two-, five-, and ten-year Swiss government zero-coupon bond yields. The figure shows that the term structure is upward sloping on average, and that short- and

\(^{23}\)These are computed daily by SNB staff.

<table>
<thead>
<tr>
<th>Maturity (months)</th>
<th>Mean (percent)</th>
<th>Std. dev. (percent)</th>
<th>Skewness</th>
<th>Kurtosis</th>
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</thead>
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<td>-0.32</td>
<td>2.82</td>
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</table>

Table 2: Summary Statistics for the Swiss Government Bond Yields.
Summary statistics for the sample of daily Swiss government zero-coupon bond yields covering the period from January 6, 1998, to December 30, 2011, a total of 3,475 observations.

<table>
<thead>
<tr>
<th>Event</th>
<th>Maturity 1-year</th>
<th>Maturity 2-year</th>
<th>Maturity 3-year</th>
<th>Maturity 5-year</th>
<th>Maturity 7-year</th>
<th>Maturity 10-year</th>
</tr>
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<td>17</td>
<td>24</td>
<td>65</td>
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<td>Aug. 4, 2011</td>
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<td>61</td>
<td>98</td>
<td>131</td>
</tr>
<tr>
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<td>-5</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
</tr>
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<td>Aug. 9, 2011</td>
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<td>47</td>
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<td>119</td>
</tr>
<tr>
<td>Aug. 11, 2011</td>
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<td>Change</td>
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<td>0</td>
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<td>-17</td>
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<td>-20</td>
</tr>
<tr>
<td>Total net change</td>
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<td>-10</td>
<td>-15</td>
<td>-25</td>
<td>-28</td>
<td>-28</td>
</tr>
</tbody>
</table>

Table 3: Two-Day Responses of Swiss Government Bond Yields.
The table reports the two-day response of the six Swiss government bond yields used in model estimation around the SNB announcement dates. All numbers are measured in basis points.

medium-term yields are more volatile than long-term yields. These are stylized facts shared by both U.S. Treasury and U.K. gilt yield data.

Table 3 shows the two-day response of the Swiss government bond yields to the SNB announcements. As noted in Section 3, there is a clear negative yield response, on net, to the announcements with long-term yields declining about twice as much as their shorter-term counterparts.25

Focusing on the ten-year yield, the drop of a few basis points following the first announcement was within the standard deviation of two-day yield changes during the sample period (about 5 basis points). However, the change in the yield following the second announcement was slightly above. The yield drop was particularly strong in connection with the final and

25Daily Bloomberg data for the mid-market yield to maturity of the 2% Swiss Confederation bond with maturity on May 25, 2022, confirm the magnitude of the declines in the long-term yields.
most forceful announcement. The ten-year yield fell by 20 basis points between the morning of the day before and the morning of the day after that announcement, amounting to four standard deviations of two-day changes in that yield over our sample period. By contrast, the exchange rate barely reacted, making it unlikely that the movements in yields were driven by exchange rate changes.

We now address the question of whether these drops reflected expected future policy rates or term premiums. For this, we need to decompose yields into term premiums and expected future short rates.

4.3 Empirical Term Structure Models

In order to accurately decompose the two-day bond yield reactions, we need a term structure model that performs well in forecasting short-term policy interest rates. With such a forecast as a proxy for market expectations of future policy rates, we can then define and compute the term premium by rewriting equation 1:

\[ TP_t(\tau) = y_t(\tau) - \frac{1}{\tau} \int_t^{t+\tau} E_t^P[r_s] ds. \]  

That is, the term premium is the difference in expected returns between a buy-and-hold strategy for a \( \tau \)-year Treasury bond and an instantaneous rollover strategy based on the risk-free rate \( r_t \).

We use the arbitrage-free Nelson-Siegel (AFNS) model class developed in Christensen et al. (2011, henceforth CDR). This model class, the model selection procedure we use as well as the performance of our preferred specification of these term structure models are described in the remainder of this section.

4.3.1 The Empirical Affine Model

First, we note that the Nelson-Siegel factors embedded in AFNS models fit the data on Swiss yields well. It is typically found that three factors are sufficient to model the time variation in the cross section of U.S. Treasury bond yields (e.g., Litterman and Scheinkman, 1991). We observe a similar phenomenon for our sample of Swiss government bond yields. Indeed, 99.89 percent of the total variation is accounted for by three factors. Table 4 reports the

\[ TP_t(\tau) = y_t(\tau) - \frac{1}{\tau} \int_t^{t+\tau} E_t^P[r_s] ds. \]  

For the entire sample period since 1998, only one two-day change was larger than that observed on August 17. That extreme event took place on November 20, 2008, in connection with the global financial market turmoil following the Lehman Brothers bankruptcy. At that time, the ten-year yield fell 29 basis points over two days.

Note that, if the measure announced on August 17, 2011, led market participants to believe more strongly that the SNB would take measures to induce the exchange rate to depreciate in the future, we should have expected to see an increase in the yield to compensate for the expected depreciation risk according to interest rate parity conditions.

Note that a Jensen’s inequality term has been left out for the rollover strategy in this definition.

Mirkov and Sutter (2013) and Söderlind (2010) are among the previous studies to analyze Swiss yields using Gaussian term structure models.
Table 4: Eigenvectors of Principal Components in Swiss Government Bond Yields.
The loadings of yields of various maturities on the three first principal components are shown. The final row shows the proportion of all bond yield variability accounted for by each principal component. The data consist of daily Swiss government zero-coupon bond yields covering the period from January 6, 1998, to December 30, 2011, a total of 3,475 observations.

eigenvectors that correspond to the three first principal components of the sample. The first principal component accounts for 95.0 percent of the variation in the Swiss government bond yields, and its loading across maturities is uniformly positive. Thus, like a level factor, a shock to this component changes all yields in the same direction irrespective of maturity. The second principal component accounts for 4.4 percent of the variation in these data and has sizable positive loadings for the shorter maturities and sizable negative loadings for the long maturities. Thus, like a slope factor, a shock to this component steepens or flattens the yield curve. Finally, the third component, which accounts for 0.5 percent of the variation, has a hump-shaped factor loading as a function of maturity, which is naturally interpreted as a curvature factor. This motivates our use of the Nelson and Siegel (1987) model with its level, slope, and curvature factors for modeling this sample of Swiss government bond yields, even though the state variables that we estimate are not identical to the principal component factors discussed here.  

The AFNS model class that we consider has three state variables, \( X_t = (L_t, S_t, C_t) \), characterized by the following system of stochastic differential equations under the risk-neutral \( Q \)-measure used for pricing:

\[
\begin{bmatrix}
    dL_t \\
    dS_t \\
    dC_t
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 0 \\
    0 & \lambda & -\lambda \\
    0 & 0 & \lambda
\end{bmatrix}
\begin{bmatrix}
    \theta_1^Q \\
    \theta_2^Q \\
    \theta_3^Q
\end{bmatrix} -
\begin{bmatrix}
    L_t \\
    S_t \\
    C_t
\end{bmatrix}
\]

\[dt + \Sigma \begin{bmatrix}
    dW_{t}^{L,Q} \\
    dW_{t}^{S,Q} \\
    dW_{t}^{C,Q}
\end{bmatrix}, \quad \lambda > 0.
\]

30A number of recent papers use principal components as state variables, Joslin et al. (2011) is an example.
In addition, the instantaneous risk-free rate is defined by

\[ r_t = L_t + S_t. \]

This specification implies that zero-coupon bond yields are given by

\[ y_t(\tau) = L_t + \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) S_t + \left( \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right) C_t - \frac{A(\tau)}{\tau}, \tag{3} \]

where the factor loadings in the yield function match the level, slope, and curvature loadings introduced in Nelson and Siegel (1987). \( A(\tau)/\tau \) is a yield-adjustment term, which captures convexity effects due to Jensen’s inequality and ensures absence of arbitrage.

The factor dynamics of the maximally flexible specification of the AFNS model are then given by

\[
\begin{pmatrix}
  dL_t \\
  dS_t \\
  dC_t
\end{pmatrix} = \begin{pmatrix}
  \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P \\
  \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\
  \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P
\end{pmatrix} \begin{pmatrix}
  \theta_1^P \\
  \theta_2^P \\
  \theta_3^P
\end{pmatrix} - \begin{pmatrix}
  L_t \\
  S_t \\
  C_t
\end{pmatrix} dt + \begin{pmatrix}
  \sigma_{11} & 0 & 0 \\
  \sigma_{21} & \sigma_{22} & 0 \\
  \sigma_{31} & \sigma_{32} & \sigma_{33}
\end{pmatrix} \begin{pmatrix}
  dW_{t,L,P}^P \\
  dW_{t,S,P}^P \\
  dW_{t,C,P}^P
\end{pmatrix}. \tag{4}
\]

We estimate this model for Swiss zero-coupon yields using a standard Kalman filter. Equation (3) is the measurement equation, and equation (4) is the transition equation in the Kalman filter; see CDR for technical details.

### 4.3.2 Model Selection and Performance Evaluation

We start out by using only the pre-crisis part of our sample, that is, the period from January 1998 to January 2008, to identify appropriate specifications of the AFNS model framework described above. This avoids the shocks and noise from the crisis. Furthermore, to ease the computational burden, these exercises are performed with data at weekly frequency, unlike the daily data used in the real-time model estimations in the event study itself.

First, we build on the findings in CDR and limit the \( \Sigma \) volatility matrix to be diagonal. To determine the appropriate specification of the mean-reversion matrix \( K^P \), a general-to-specific modeling strategy is applied. Thus, after each estimation, we restrict the least significant parameter estimates to zero and then re-estimate the model. This strategy is continued down to the most parsimonious specification, which has a diagonal \( K^P \) matrix. The final specification choice is based on the values of the Akaike and Bayes information criteria as per Christensen et al. (2010, 2014) and CR.\(^{32}\) The summary statistics of the model selection process are reported in Table 5. The Akaike information criterion is minimized by specification

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\(^{31}\)The model is completed with a risk premium specification that connects the \( Q \)-dynamics to the dynamics under the real-world \( P \)-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical \( P \)-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium introduced in Duffee (2002).

\(^{32}\)See Harvey (1989) for further details.
Table 5: Evaluation of Alternative Specifications of the AFNS Model.

There are seven alternative estimated specifications of the AFNS model of Swiss government bond yields with the unrestricted 3-by-3 $K^P$ matrix being the most flexible. Each specification is listed with its maximum log likelihood value ($\log L$), number of parameters ($k$), the $p$-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the information criteria (AIC and BIC). The sample is weekly from January 9, 1998, to January 4, 2008, a total of 522 observations.

(2), while the Bayes information criterion is minimized by specification (6), that is, by the specifications of the mean-reversion matrix $K^P$ given by

$$K^P_{AIC} = \begin{pmatrix} \kappa^P_{11} & 0 & \kappa^P_{13} \\ \kappa^P_{21} & \kappa^P_{22} & \kappa^P_{23} \\ \kappa^P_{31} & \kappa^P_{32} & \kappa^P_{33} \end{pmatrix}$$

and

$$K^P_{BIC} = \begin{pmatrix} \kappa^P_{11} & 0 & 0 \\ 0 & \kappa^P_{22} & 0 \\ 0 & 0 & \kappa^P_{33} \end{pmatrix}.$$

Due to the lack of any established benchmark model for Swiss government bond yields, we choose to compare the selected AFNS models to relevant alternative AFNS models. Specifically, we include the unconstrained AFNS model in equation (4), which is the AFNS model closest to the canonical $A_0(3)$ model of Dai and Singleton (2000), as well as the most parsimonious independent-factor AFNS model favored by CDR. Also, we consider the AFNS model with diagonal $\Sigma$ volatility matrix, but unrestricted $K^P$ mean-reversion matrix, which is the starting point for our model selection procedure, in addition to the specification favored by CR for U.S. data with the $K^P$ matrix given by

$$K^P_{CR} = \begin{pmatrix} \kappa^P_{11} & 0 & 0 \\ \kappa^P_{21} & \kappa^P_{22} & \kappa^P_{23} \\ 0 & 0 & \kappa^P_{33} \end{pmatrix}.$$  

We now re-estimate the specific AFNS models identified above on a weekly basis for the period after January 2008, adding one week of data at the time, in order to fully reflect the data available to market participants in real-time during this period. We then use these...
models to forecast the three-month CHF LIBOR six months, one year, and two years ahead on a weekly basis over the period from January 4, 2008, until December 30, 2011. As the three-month CHF LIBOR has been the target policy rate of the SNB since 1998, this exercise sheds light on the ability of the various AFNS models to deliver reasonable policy expectations. The summary statistics for the forecast errors relative to the subsequent realizations of the three-month CHF LIBOR are reported in Table 6, which also contains the forecast errors obtained using a random walk assumption. We note the strong forecast performance of the preferred AFNS model according to the BIC. In the remainder of the paper, we will refer to this specification as the preferred AFNS model.\footnote{Unreported results for Diebold and Mariano (1995) tests of forecast accuracy show that the preferred AFNS model’s short rate forecasts are statistically significantly more accurate than the random walk at all three forecast horizons. However, among the AFNS models, the preferred model’s performance is not, in general, statistically superior to that of the other models.}

Table 6 reports the estimated parameters for the preferred AFNS model. The usual pattern from applications of AFNS models to U.S. data is also present for Swiss data. The level factor is the most persistent factor and has the lowest volatility, while the curvature

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|c|c|c|c|}
\hline
Forecasting method & \multicolumn{2}{c|}{Six-month forecast} & \multicolumn{2}{c|}{One-year forecast} & \multicolumn{2}{c|}{Two-year forecast} \\
\hline & Mean & RMSE & Mean & RMSE & Mean & RMSE \\
\hline Random walk & 33.55 & 79.33 & 62.37 & 113.65 & 71.26 & 123.57 \\
Unconstrained AFNS model & -1.54 & 60.86 & 44.62 & 73.67 & 80.63 & 93.56 \\
Unrestricted $K^P$ AFNS model & 22.28 & 63.40 & 72.80 & 85.43 & 110.27 & 116.27 \\
Indep.-factor AFNS model & 11.80 & 60.18 & 51.96 & 72.89 & 78.46 & 90.14 \\
CR AFNS model & 29.35 & 67.88 & 80.94 & 93.03 & 118.82 & 124.77 \\
Preferred AIC AFNS model & 21.60 & 62.83 & 71.48 & 83.23 & 107.97 & 113.27 \\
Preferred BIC AFNS model & 13.82 & 61.00 & 54.31 & 73.97 & 80.22 & 91.15 \\
\hline
\end{tabular}
\caption{Summary Statistics for Policy Target Rate Forecast Errors.}
\end{table}

Summary statistics of the forecast errors—mean and root mean squared errors (RMSEs)—of the three-month CHF LIBOR six months, one year, and two years ahead. The forecasts are weekly starting on January 4, 2008, and running until December 30, 2011, a total of 209 forecasts for all three forecast horizons. All measurements are expressed in basis points.

Figure 9 compares the forecasts at the one-year horizon from the preferred AFNS model to the corresponding mean from the Consensus Forecasts survey and to the subsequent realizations of the three-month CHF LIBOR. Since mid-2009 the model has generated one-year-ahead short rate forecasts that have systematically been a bit above the subsequent realizations. This suggests that bond investors never anticipated the period of the zero interest rate policy to be as long as it turned out to be.\footnote{In unreported results, we analyze whether this outcome could be caused by finite-sample bias in the estimation of the model’s mean-reversion $P$-dynamics, that is, the $K^P$ matrix. Specifically, we followed the approach described in CR of imposing a near-unit-root property on the AFNS level factor. However, we did not obtain better forecast performance for any of the specifications studied. As a consequence, we proceed without any adjustments to the original model estimates.} This is also consistent with the evidence from the Consensus Forecasts survey of professional forecasters.

Table 7 reports the estimated parameters for the preferred AFNS model. The usual pattern from applications of AFNS models to U.S. data is also present for Swiss data. The level factor is the most persistent factor and has the lowest volatility, while the curvature

\footnote{The model output used in the forecast exercise is the model-implied three-month yield.}
Figure 9: Forecasts of the Three-Month CHF LIBOR.
Illustration of the forecasts of the three-month CHF LIBOR one year ahead from the preferred AFNS model. For comparison the mean one-year-ahead forecast from the Consensus Forecasts survey is also shown. Subsequent realizations of the three-month CHF LIBOR are included, so at date $t$, the figure shows forecasts as of time $t$ and the realization from $t$ plus one year. The model forecast data and the realizations are weekly covering the period from January 4, 2008, to December 30, 2011, while the Consensus Forecast survey is monthly covering the period from January 14, 2008, to December 12, 2011.

Table 7: Parameter Estimates of the Preferred AFNS Model.
The estimated parameters of the $K^P$ matrix, the $\theta^P$ vector, and the $\Sigma$ matrix for the AFNS model preferred according to BIC. The associated estimated $\lambda$ is 0.2726 (0.0045) with maturity measured in years. Estimated standard deviations of the parameter estimates are given in parentheses. The maximum log likelihood value is 19,456.32.

factor is the least persistent and most volatile factor. The slope factor has dynamic qualities in between the two other factors in terms of persistence and volatility. Furthermore, the estimated mean parameters in $\theta^P$ accord well with the estimated paths of the three state variables. In particular, the negative values for $\theta_2^P$ and $\theta_3^P$ are consistent with the average
<table>
<thead>
<tr>
<th>Maturity in months</th>
<th>Preferred AFNS model</th>
<th>Mean</th>
<th>RMSE</th>
<th>$\bar{\sigma}_\varepsilon(\tau_i)$</th>
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<tr>
<td>84</td>
<td>-0.39</td>
<td>1.14</td>
<td>1.44</td>
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<td>0.00</td>
<td>0.00</td>
<td>2.24</td>
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Table 8: Summary Statistics for the Fitted Errors.
The mean and root mean squared error of the fitted errors of the Swiss government bond yields across six different maturities are shown. Also reported are the estimated measurement error standard deviations for each maturity. All numbers are measured in basis points. The data are daily covering the period from January 6, 1998, to January 3, 2008.

Table 8 contains the summary statistics for the fitted yield errors. The fit is good, as indicated by the low root mean squared fitted errors (RMSE), and almost without bias, on average, as the mean errors are very close to zero across the entire maturity range.

We also test the significance of the parameter restrictions imposed on the $K^P$ mean-reversion matrix in the preferred AFNS model relative to the corresponding AFNS model with an unrestricted $K^P$ matrix, using standard likelihood ratio tests. Figure 10 shows that the parameter restrictions have been statistically insignificant for most sample cutoff dates. Thus, the preferred AFNS model is flexible enough to capture the relevant information in the data throughout this period. The figure also shows the LR tests of the parameter restrictions in the more parsimonious independent-factor AFNS model, which appears to be too tightly parameterized as its parameter restrictions are never supported by the data at conventional significance levels independent of the sample cutoff date.

### 4.4 Decomposition of the Yield Responses to the SNB Announcements

We now use the empirical term structure models to assess the different channels of transmission of the SNB announcements to Swiss long-term yields. We decompose Swiss zero-coupon yields into three components: 37

(i). the estimated average expected short rate until maturity;

(ii). the term premium defined as the difference between the model fitted yield and the average expected short rate; and

(iii). a residual that reflects variation not accounted for by the model.
Figure 10: Likelihood Ratio Tests of Parameter Restrictions in AFNS Models.
Illustration of the value of likelihood ratio tests of the restrictions imposed in the independent-factor and preferred AFNS models relative to the AFNS model with unrestricted $K^P$ matrix and diagonal $\Sigma$ matrix. The analysis covers weekly re-estimations from January 4, 2008, to December 30, 2011, a total of 209 observations, while the full data set used in the estimation covers the period from January 9, 1998, to December 30, 2011.

Figure 11 shows the result of the decomposition of ten-year Swiss government bond yields since 2008 (excluding the residual, which is sufficiently small to be irrelevant). Over the period from August 1, 2011, to the date of the introduction of the exchange rate floor on September 6, 2011, the observed ten-year government bond yield declined 35 basis points. According to our preferred AFNS model, policy expectations as reflected in the estimated average expected short rate over the next ten years only declined 4 basis points, while the ten-year term premium accounts for 31 of the 35 basis point yield decline, or 89 percent. However, the key question is to what extent the announced reserve expansions by the SNB are the driver of these yield changes. This would be more likely if they take place in the immediate aftermath of the announcements. With the yield decomposition in hand, we hence resume the event analysis. We use our AFNS models, now estimated in real time at daily frequency, to decompose the response of the Swiss government bond yield to the SNB announcements into the three components described above.

Table 9 contains the results of decomposing the two-day ten-year yield responses based on the empirical AFNS models.\footnote{Appendix D provides the analytical formulas required for the decomposition in the preferred AFNS model.} Despite the differences in statistical fit and forecast perfor-
Figure 11: Decomposition of Ten-Year Yield.

Panel (a) shows the daily real-time decomposition of the variation of the ten-year Swiss government bond yield into (i) estimated average expected short rate forecasted until maturity and (ii) the term premium defined as the difference between the observed government bond yield and the average expected short rate based on the preferred AFNS model of Swiss government bond yields. Panel (b) shows the daily movements around the four 2011 SNB unconventional policy announcements, indicated with solid black vertical lines. The data are daily covering the period from January 3, 2008, to December 30, 2011.

...mance documented earlier, the models agree on what drove yield changes on the announcement dates. Three of the four models, including our preferred specification, indicate that policy expectations were only revised marginally lower in response to the announcements, so that most of the yield declines are associated with declines in term premiums. We also note that all four models indicate that short rate expectations declined around the first announcement on August 3, 2011, when the target range for the three-month CHF LIBOR was lowered. This is the only evidence of any notable signaling effect that we detect in our analysis.

Still, changes in term premiums could reflect factors other than portfolio balance effects, not least because the episode took place during a turbulent phase of the European sovereign debt crisis. We address this next.

5 Robustness

In this section, we argue that, while other factors could have contributed to pushing term premiums lower in the wake of the SNB’s announcements of reserve expansions, the reserve expansions remain the most likely direct drivers of the identified term premium declines on the event dates.

The SNB policy announcements in August 2011 came at a time of substantial market
Table 9: Decompositions of Two-Day Responses of Ten-Year Yield.
The decomposition of two-day responses of the ten-year Swiss government bond yield on three SNB announcement dates into changes in (i) the average expected target rate over the next ten years, (ii) the ten-year term premium, and (iii) the unexplained residual based on empirical AFNS models of Swiss government bond yields. All changes are measured in basis points.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>Avg. target rate next ten years</th>
<th>Ten-year term premium</th>
<th>Residual</th>
<th>Ten-year yield</th>
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</thead>
<tbody>
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<td>-5</td>
<td>2</td>
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<td>-1</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
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<td>1</td>
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<tr>
<td></td>
<td>Preferred AFNS</td>
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<td>-1</td>
<td>1</td>
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<tr>
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<td>-20</td>
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<td></td>
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<td>Preferred AFNS</td>
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<td>-25</td>
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</table>

The decomposition of two-day responses of the ten-year Swiss government bond yield on three SNB announcement dates into changes in (i) the average expected target rate over the next ten years, (ii) the ten-year term premium, and (iii) the unexplained residual based on empirical AFNS models of Swiss government bond yields. All changes are measured in basis points.

upheaval, high volatility, and flights to safety. This was an intense part of the European sovereign debt crisis. Hence, elevated risk perceptions could have increased the flight to safety from risky assets into Swiss Confederation bonds during this period, which would have induced a fall in term premiums. We cannot control for safe haven or risk aversion effects directly, as the term premium contains all portfolio balance and risk premium effects and does not allow us to discriminate between their different sources. Instead, we indirectly control for such effects through measures of foreign market developments, bond market liquidity, and financial market uncertainty and include them in regressions of Swiss term premiums. We also consider whether other events that could have moved the Swiss term premium happened in the announcement windows. Finally, in the absence of intraday data on Swiss Confederation bond yields, we investigate intraday data on ten-year swap rates.

### 5.1 Measures of Foreign Developments, Liquidity, and Uncertainty

We first want to eliminate foreign term premiums as a key driver of the declines in Swiss term premiums following the SNB announcements. For this purpose, we make the simplifying assumption that the Swiss policy actions had a minimal, if any, effect on foreign market activity. This allows us to treat the yield changes abroad, and decompositions thereof, as purely exogenous variables. Furthermore, for simplicity, we limit our focus to the U.S. and
euro-area government bond markets, widely regarded as the two most liquid fixed-income markets in the world.

For the United States, we choose to rely on the shadow-rate model analyzed in Christensen and Rudebusch (2013). The shadow-rate modeling approach allows us to preserve the Gaussian factor dynamics, while we obtain bond yields that respect the zero lower bound. This aspect matters for modeling U.S. Treasury yields in the most recent period.

Without an established euro-area benchmark term structure model, we went through a model selection analysis similar to the one described for the Swiss yields using German government bond yields. The resulting preferred AFNS model for German yields has $P$-dynamics given by

$$
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t
\end{pmatrix} = \begin{pmatrix}
    10^{-7} & 0 & 0 \\
    0 & \kappa_{22}^P & \kappa_{23}^P \\
    0 & 0 & \kappa_{33}^P
\end{pmatrix} \begin{pmatrix}
    0 \\
    \theta_2^P \\
    \theta_3^P
\end{pmatrix} - \begin{pmatrix}
    L_t \\
    S_t \\
    C_t
\end{pmatrix} \, dt + \begin{pmatrix}
    \sigma_{11} & 0 & 0 \\
    0 & \sigma_{22} & 0 \\
    0 & 0 & \sigma_{33}
\end{pmatrix} \begin{pmatrix}
    dW_t^{L,P} \\
    dW_t^{S,P} \\
    dW_t^{C,P}
\end{pmatrix}.
$$

This specification is identical to the AFNS specification CR preferred for their analysis of U.K. gilt yield responses to the Bank of England’s QE programs.

Figure 12 shows the ten-year term premium from our preferred AFNS model for Swiss yields and compares it to the estimates of the corresponding foreign term premiums. To establish the dynamic relationship between these various term premium estimates, we focus on the period from January 3, 2008, to June 30, 2011, that lies before the SNB announcements, but after the outbreak of the global financial crisis. As reported in Table 10, the correlations between the Swiss and euro-area ten-year term premiums and the Swiss and U.S. ten-year term premiums were both -10.9 percent for this period. Using five-year term premiums gives similar results. Thus, in general, the connection between Swiss and foreign term premiums was relatively weak in this period.

Since part of the term premium could be a premium investors require for assuming the liquidity risk of Swiss Confederation bonds, we also want to control for changes in liquidity. To capture variation in the liquidity of the Swiss Confederation bond market, we use the average bid-ask spread of all available Confederation bonds weighted by the outstanding notional of each bond.

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39. As Christensen and Rudebusch (2013) only consider weekly data, we estimate their shadow-rate model on an updated sample of the daily Treasury yields used in CR.
40. It is not obvious that we would want to enforce a lower yield bound for either the Swiss or the German bond yields also analyzed, mainly because in both of these samples yields have actually been well below zero for intermediate maturities for extended periods in recent years. Hence, in these cases, a standard Gaussian modeling approach appears to be fully warranted.
41. The data are publicly available on the website of the German Bundesbank.
42. The results leading to this conclusion are available from the authors upon request.
43. Note that this is not inconsistent with significant foreign influence on Swiss fixed-income developments. The effects would just have to come through the expectations channel for future domestic and foreign monetary policy.
44. The data are based on SNB staff’s own calculations.
Figure 12: Swiss and Foreign Ten-Year Term Premiums.
Panel (a) shows the daily Swiss, euro-area, and U.S. ten-year term premiums. Panel (b) shows the daily movements around the four 2011 SNB unconventional policy announcements, indicated with solid black vertical lines. The data cover the period from January 3, 2008, to December 30, 2011. The source of each is detailed in the main text.

<table>
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<td>Euro</td>
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<th>Ten-year term premiums</th>
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<td>Euro</td>
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<td>U.S.</td>
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Table 10: Pairwise Correlations of Term Premiums.
The top table contains the pairwise correlations between the estimated Swiss, euro-area, and U.S. five-year term premiums. The bottom table reports the corresponding pairwise correlations between the estimated ten-year term premiums. The sample is daily from January 3, 2008, to June 30, 2011, a total of 854 observations.

Confederation bonds from April 3, 2000, to December 12, 2011.\(^{46}\) Since the raw daily series is rather volatile, we also show the smoothed two-week moving average. However, we emphasize that we only rely on the raw daily series in the subsequent regression analysis. For the period from January 4, 2008, to August 2, 2011, that is, up until the day before the first SNB

\(^{46}\)This was the sample made available to us.
Figure 13: Bid-Ask Spreads in the Swiss Confederation Bond Market.
Panel (a) shows the daily weighted average bid-ask spread in the Swiss Confederation bond market over the period from April 3, 2000, to December 12, 2011. Also shown is the smoothed two-week moving average. Panel (b) shows the daily movements around the four 2011 SNB unconventional policy announcements, indicated with solid black vertical lines. The data is based on SNB staff’s own calculations.

announcement, the correlation between the term premium and the smoothed bid-ask spread was 43.3 percent. This suggests a positive connection between the two series. Thus, both on economic and statistical grounds, we want to control for the impact of this measure of market liquidity.

Our final control variable is a measure of priced economic uncertainty, namely the VIX options-implied volatility index. It represents near-term uncertainty about the general stock market as reflected in one-month options on the S&P 500 stock price index and is widely used as a gauge of investor fear and risk aversion. When the price of uncertainty goes up as reflected in higher values of the VIX, risk premiums tend to go up. However, in the case of Switzerland during the European sovereign debt crisis, spikes in the VIX could also be a trigger for safe-haven demand for Swiss assets. Thus, the role of the VIX for Swiss term premiums is not clear beforehand, but like the other measures we want to control for changes in market uncertainty in our analysis.

5.2 Regression Analysis

To account for how Swiss term premiums ordinarily respond to market and foreign developments, we run simple ordinary least squares regressions, and lag the euro-area and U.S. term premiums as well as the VIX by one day. Lagging is important because the euro-area and U.S. financial market data are recorded at market close, whereas the Swiss data are recorded
The results of our regressions are reported in Table 11. Overall, the $R^2$s are somewhat low, but all coefficients are statistically significant with the exception of the coefficient on the VIX in the five-year regressions. Furthermore, there is a difference in the dynamic interaction with the foreign risk premiums. Increases in U.S. term premiums tend to put downward pressure on Swiss term premiums. By contrast, Swiss and euro-area term premiums tend to move in the same direction, possibly due to the geographical proximity and close economic ties between Switzerland and the core euro-area countries. Finally, the coefficients of the bid-ask spread that serves as a proxy for market liquidity has the expected positive sign and high statistical significance. This indicates that Swiss term premiums tend to go up when market liquidity deteriorates and bid-ask spreads widen.

Table 12 summarizes the decompositions of the one-day responses of foreign ten-year
Decomposition from models

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>Avg. target rate next ten years</th>
<th>Ten-year term premium</th>
<th>Residual</th>
<th>Ten-year yield</th>
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<td>U.S. model</td>
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<td>-18</td>
<td>-3</td>
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Table 12: Decompositions of One-Day Responses of Foreign Ten-Year Yields.
The decomposition of one-day responses of ten-year foreign government bond yields on three SNB announcement dates into changes in (i) the average expected target rate over the next ten years, (ii) the ten-year term premium, and (iii) the unexplained residual based on the empirical foreign term structure models described in the main text. All changes are measured in basis points.

government bond yields around the time of the three SNB announcements.\textsuperscript{47} Based on the statistical relationship reported in Table 11, the adjustment for the foreign one-day net response and the two-day net response in the bid-ask spread is \(0.4901 \cdot 5.54 - 0.1084 \cdot (-17.76) + 1.2185 \cdot 11.09 - 0.0058 \cdot 0.0525 = 18.15\) basis points, of which 4.64 basis points arise from changes in foreign term premiums and 13.51 basis points derive from the spikes in bid-ask spreads and the VIX. This suggests that the effect from the controlling factors would actually have pushed up the Swiss ten-year term premium around the SNB announcements. For robustness, we repeat the analysis using two-day responses for all variables instead. This produces an adjustment of \(0.4901 \cdot 2.76 - 0.1084 \cdot (-36.68) + 1.2185 \cdot 11.09 - 0.0058 \cdot 0.0206 = 18.84\) basis points with 5.32 basis points derived from foreign term premium changes and the residual 13.51 basis points coming mainly from the deterioration in Swiss bond market liquidity. The conclusion remains that foreign and market liquidity influences have tended to offset some of the declines in the Swiss term premiums that we estimate.\textsuperscript{48} Overall, this suggests that, if anything, we may be underestimating the term premium effects of the SNB policy announcements.

5.3 Other Events

As a final robustness check, we investigate whether there could have been other major news events occurring around the time of the SNB announcements that could potentially explain our findings. Appendix A provides a log of the most notable economic events in August and September 2011. The first important event that comes to mind is the FOMC meeting on August 9, 2011, when U.S. monetary policy makers introduced explicit monetary policy

\textsuperscript{47} For the foreign series, we only use a one-day event window because they are recorded at the close of the market local time.

\textsuperscript{48} In Appendix C, we repeat this analysis for the five-year maturity and obtain very similar results.
forward guidance for the first time. Even though this event may corrupt the Swiss financial market response to the second SNB announcement on August 10, 2011, we note that this is not the dominating event on either the Swiss or foreign side in our controlling exercises. Thus, our conclusion is unlikely to be biased to any notable extent by this particular event.

We did not find any important events that could have been driving financial markets on the day of the strongest drop in the Swiss term premium, August 17, 2011. European financial markets were relatively calm that day. This changed the following day with an event that was symptomatic for the general state of stress relating to the European sovereign debt crisis in the fall of 2011. Rumors circulated on August 18, 2011, that the ECB’s U.S. dollar facility was tapped by a European bank, signalling funding stress. This triggered a bank equity sell-off. Between market close on August 17, 2011, and August 18, 2011, European stock prices (bank stock prices in particular) plunged and volatility measures rose sharply, suggesting a strong flight to safety. Importantly, though, this flight to safety in European equity markets did not overlap in time with the drop in the Swiss term premium on August 17, 2011.

5.4 Intraday Evidence

Unfortunately, we do not have access to intraday prices for Swiss Confederation bonds that could help us shed light on the exact timing of the market reaction to the three SNB announcements. Instead, we look at a close substitute for Confederation bonds for which we do have intraday data, namely quoted rates on ten-year interest rate swap contracts in Swiss francs.

Figure 14 shows the intraday variation of the Swiss ten-year interest rate swap rate on the day before, and on the day of, the three SNB announcements. First, we note that there is little overall variation on the day before each SNB announcement. It hence does not appear as if there is anything material going on in the run up to any of the three SNB announcements. Second, the market reaction differs across the three announcements. In Figure 14(b), we see evidence of the signaling effect around the first announcement as the ten-year swap rate drops immediately following the announcement. In contrast, Figures 14(d) and 14(f) indicate that the responses to the last two announcements were rather gradual, and do not suggest that investors took much of a signal about future policy rates from those measures—again consistent with our model decompositions. In contrast, the market reactions follow a pattern of gradual yield declines throughout the remainder of the announcement day. We interpret this as evidence that it took some time for investors to digest and assess the impact on risk premiums from the extraordinary measures. This also supports the use of two-day event windows. Overall, the intraday evidence is consistent with the findings from the daily data.
Figure 14: Intraday Movement of the Swiss Ten-Year Interest Rate Swap Rate.

Intraday variation in the Swiss ten-year interest rate swap rate on the day before, and on the day of, the three SNB announcements. Source: Bloomberg.
6 Conclusion

In the rapidly growing literature on the effects of QE on financial markets, two channels have received the most attention, the signaling channel and the portfolio balance effect of changes in the market supply of the purchased assets. In this paper, we emphasize that another source of portfolio balance effects may be important, notably the potential portfolio reallocation effect arising from the mere expansion of excess reserves that is a defining part of any QE program.

To obtain evidence on the latter channel, we study the unconventional monetary policy measures undertaken by the SNB in the late summer of 2011, prior to introducing the exchange rate floor in early September 2011. The design of these policies provides a unique case study for identifying portfolio balance effects of reserves. The reason is that, in contrast to the QE programs conducted in the United Kingdom and the United States, the SNB policy measures involved an unprecedented expansion of excess reserves without any outright purchases of domestic long-term securities. Thus, this represents a pure QE program in the terminology of Bernanke and Reinhart (2004).

Long-term Swiss Confederation bond yields dropped by a cumulative total of 28 basis points, or one fifth of the yield, following the three SNB announcements of reserve injections. To understand the transmission channels through which the expansion of reserves affected long-term yields, we apply standard event study techniques and the estimation of dynamic term structure models to data on Swiss Confederation bond yields. The modeling approach allows us to decompose the observed yield changes into a component that represents expectations of future short policy rates, and a residual term premium component. We find that the main part of the drop in yields in response to the SNB announcements reflect declines in term premiums. Only the first announcement on August 3, 2011, which included a lowering of the upper bound for the three-month CHF LIBOR, is associated with any notable signaling effect. These findings are robust across model specifications. We obtain similar results when we focus on five-year yields. Also, the conclusions are robust to controlling for variations in foreign term premiums, market liquidity, and market uncertainty. Intraday data supports this interpretation as well. To our knowledge, this paper is the first to document that part of the transmission channel of QE programs to long-term interest rates may derive, at least partly, from a portfolio balance effect through the expansion of reserves.

Our findings are likely to reflect specific market and institutional features of Switzerland. It remains an open question whether they extend to other countries. It is an important question, however. The presence of a bank portfolio balance effect resulting from quantitative easing through an expansion of reserves has implications for the design of future QE programs, for the design of the exit from such programs, and for central bank communication. Thus, more research is needed to better understand the financial market impact of changes in central bank reserves, and the factors that underlie it.
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<td>Aug. 2, 2011</td>
<td>President Obama signs bill to raise the U.S. federal government debt ceiling.</td>
</tr>
<tr>
<td>Aug. 3, 2011</td>
<td>First SNB announcement.</td>
</tr>
<tr>
<td>Aug. 4, 2011</td>
<td>Bank of Japan intervenes in currency markets to prop up the yen. ECB Monetary Policy Decision: No changes to the main refinancing rate.</td>
</tr>
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<td>Aug. 6, 2011</td>
<td>S&amp;P lowers U.S. Treasury debt rating from AAA to AA+ for the first time.</td>
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<tr>
<td>Aug. 7, 2011</td>
<td>ECB Statement: ECB will actively implement its securities purchase program.</td>
</tr>
<tr>
<td>Aug. 9, 2011</td>
<td>FOMC Statement: Federal funds target rate to remain exceptionally low at least through mid-2013.</td>
</tr>
<tr>
<td>Aug. 11, 2011</td>
<td>Financial market authorities of Belgium, Italy, France, and Spain as well as the European financial regulator, ESMA, announce a ban on all forms of short selling of the shares of banks and other financial firms.</td>
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<tr>
<td>Aug. 18, 2011</td>
<td>Market rumors that the ECB dollar facility was tapped for the first time since February 23, 2011. This led to a major sell-off in European bank stocks and an increase in market volatility.</td>
</tr>
<tr>
<td>Aug. 26, 2011</td>
<td>Federal Reserve Chairman Ben Bernanke’s speech at Jackson Hole. No explicit QE news.</td>
</tr>
<tr>
<td>Sep. 6, 2011</td>
<td>SNB introduces exchange rate floor.</td>
</tr>
<tr>
<td>Sep. 21, 2011</td>
<td>FOMC statement: Announcement of the Maturity Extension Program (MEP). The program involves Treasury purchases of $400 billion with maturities longer than 6 years, financed by sales of Treasuries with less than 3 years to maturity.</td>
</tr>
</tbody>
</table>

Table 13: News and Market Events in August and September 2011.

Appendix A: Important Events in August and September 2011
Table 14: One-Day Responses of Swiss Government Bond Yields.
The table reports the one-day response of the six Swiss government bond yields used in model estimation around three SNB announcement dates. All numbers are measured in basis points.

| Event     | Maturity | 1-year | 2-year | 3-year | 5-year | 7-year | 10-year | Change  | 1-year | 2-year | 3-year | 5-year | 7-year | 10-year | Change  |
|-----------|----------|--------|--------|--------|--------|--------|--------|---------|--------|--------|--------|--------|--------|--------|---------|---------|
| Aug. 2, 2011 | 30       | 17     | 24     | 65     | 100    | 133    |        | -2      | -4     | -4     | -2     | -1     | -1     |         |         |
| Aug. 9, 2011 | 26       | 13     | 14     | 47     | 83     | 119    |        | -1      | -1     | -1     | -1     | 0      | 1      | -3      |         |
| Aug. 10, 2011| 21       | 4      | 10     | 50     | 88     | 122    |        | -5      | -9     | -5     | 3      | 4      | 3      |         |         |
| Aug. 16, 2011| 19       | 8      | 13     | 49     | 84     | 119    |        | -1      | -1     | 0      | -1     | -2     | -4     |         |         |
| Aug. 17, 2011| 17       | 7      | 12     | 48     | 82     | 115    |        | -1      | -1     | 0      | -1     | -2     | -4     |         |         |

Total net change: -9, -14, -9, 0, 1, -3

Table 15: Decompositions of One-Day Responses of Ten-Year Yield.
The decomposition of one-day responses of the ten-year Swiss government bond yield on three SNB announcement dates into changes in (i) the average expected target rate over the next ten years, (ii) the ten-year term premium, and (iii) the unexplained residual based on empirical AFNS models of Swiss government bond yields. All changes are measured in basis points.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>Decomposition from models</th>
<th>Ten-year yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Avg. target rate next ten years</td>
<td>Ten-year term premium</td>
</tr>
<tr>
<td>I Aug. 3, 2011</td>
<td>Unconstrained AFNS</td>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K^p$ AFNS</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>II Aug. 10, 2011</td>
<td>Unconstrained AFNS</td>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K^p$ AFNS</td>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>III Aug. 17, 2011</td>
<td>Unconstrained AFNS</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K^p$ AFNS</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Total net change</td>
<td>Unconstrained AFNS</td>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K^p$ AFNS</td>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>-4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-4</td>
<td>2</td>
</tr>
</tbody>
</table>

Appendix B: Decomposition of One-Day Yield Responses

In this appendix, we provide the decomposition of the one-day responses based on our empirical models of Swiss government bond yields. Table 14 contains the one-day changes in the six bond yields used in model estimation, while Table 15 reports the results from the model decompositions of the one-day response of the ten-year yield around the SNB announcements.
Appendix C: Decomposition of Five-Year Yield Responses

In this appendix, we provide the decomposition of the one- and two-day responses of the five-year Swiss government bond yield based on our empirical models of the Swiss government bond yield curve. Table 16 contains the decompositions of the one-day changes, while Table 17 reports the results from the model decompositions of the two-day changes. Furthermore, Table 18 summarizes the decompositions of the two-day responses of foreign five-year government bond yields around the time of the three SNB announcements.

Based on the regression results reported in Table 11, the adjustment at the five-year maturity for the one-day net response of the foreign term premiums and the VIX and the two-day net response of the market liquidity measure is 0.7799 · 5.45 − 0.2727 · (−10.76) + 1.0644 · 11.09 − 0.0027 · 0.0525 = 18.99 basis points with 7.18 basis points derived from changes in foreign term premiums, while 11.80 basis points is associated with the spike in bid-ask spreads in the Swiss Confederation bond market and in the VIX. Thus, the conclusion remains that the effect from foreign term premiums and the deterioration in market liquidity would actually have pushed up the Swiss five-year term premium around the SNB announcements. For robustness, we repeat the analysis using two-day responses for all variables instead. This produces an adjustment of 0.7799 · 1.97 − 0.2727 · (−21.11) + 1.0644 · 11.09 − 0.0027 · 0.0206 = 19.10 basis points with 7.29 basis points associated with foreign developments, while 11.80 basis points derive mainly from the weakening in Swiss bond market liquidity conditions. Thus, the conclusion remains the same, namely that accounting for spillovers from foreign term premiums, market uncertainty, and market liquidity measures would actually push in the direction of observing smaller term premium declines, if not outright term premium increases.
### Table 17: Decompositions of Two-Day Responses of Five-Year Yield.

The decomposition of two-day responses of the five-year Swiss government bond yield on three SNB announcement dates into changes in (i) the average expected target rate over the next five years, (ii) the five-year term premium, and (iii) the unexplained residual based on empirical AFNS models of Swiss government bond yields. All changes are measured in basis points.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>Decomposition from models</th>
<th>Five-year yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. target rate</td>
<td>Five-year term premium</td>
<td>Residual</td>
</tr>
<tr>
<td></td>
<td>next five years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I Aug. 3, 2011</td>
<td>Unconstrained AFNS</td>
<td>-6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K_p$ AFNS</td>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td>II Aug. 10, 2011</td>
<td>Unconstrained AFNS</td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K_p$ AFNS</td>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>1</td>
<td>-5</td>
</tr>
<tr>
<td>III Aug. 17, 2011</td>
<td>Unconstrained AFNS</td>
<td>-1</td>
<td>-16</td>
</tr>
<tr>
<td></td>
<td>Unrestricted $K_p$ AFNS</td>
<td>5</td>
<td>-22</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
<td>0</td>
<td>-17</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>1</td>
<td>-18</td>
</tr>
<tr>
<td>Total net change</td>
<td>Unconstrained AFNS</td>
<td>-10</td>
<td>-14</td>
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<td>Unrestricted $K_p$ AFNS</td>
<td>2</td>
<td>-27</td>
</tr>
<tr>
<td></td>
<td>Indep.-factor AFNS</td>
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<td>-23</td>
</tr>
<tr>
<td></td>
<td>Preferred AFNS</td>
<td>-1</td>
<td>-24</td>
</tr>
</tbody>
</table>

### Table 18: Decompositions of One-Day Responses of Foreign Five-Year Yields.

The decomposition of one-day responses of five-year foreign government bond yields on three SNB announcement dates into changes in (i) the average expected target rate over the next five years, (ii) the five-year term premium, and (iii) the unexplained residual based on the empirical foreign term structure models described in the main text. All changes are measured in basis points.

<table>
<thead>
<tr>
<th>Event</th>
<th>Model</th>
<th>Decomposition from models</th>
<th>Five-year yield</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Avg. target rate</td>
<td>Five-year term premium</td>
<td>Residual</td>
</tr>
<tr>
<td></td>
<td>next five years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I Aug. 3, 2011</td>
<td>Euro-area model</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>U.S. model</td>
<td>5</td>
<td>-4</td>
</tr>
<tr>
<td>II Aug. 10, 2011</td>
<td>Euro-area model</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>U.S. model</td>
<td>2</td>
<td>-4</td>
</tr>
<tr>
<td>III Aug. 17, 2011</td>
<td>Euro-area model</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td></td>
<td>U.S. model</td>
<td>1</td>
<td>-3</td>
</tr>
<tr>
<td>Total net change</td>
<td>Euro-area model</td>
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<td>5</td>
</tr>
<tr>
<td></td>
<td>U.S. model</td>
<td>8</td>
<td>-11</td>
</tr>
</tbody>
</table>

### Appendix D: Analytical Formulas for Policy Expectations and Term Premiums in the Preferred AFNS Model

In this appendix, we derive the analytical formulas for policy expectations and term premiums within the preferred AFNS model of Swiss Confederation yields.
For a start, we note that the term premium is defined as

\[ TP_t(\tau) = y_t(\tau) = \frac{1}{\tau} \int_t^{t+\tau} E_t^P[r_s]ds. \]

In the preferred AFNS model, as in any AFNS model, the instantaneous short rate is defined as

\[ r_t = L_t + S_t, \]

while the specification of the \( P \)-dynamics is given by

\[
\begin{pmatrix}
    dL_t \\
    dS_t \\
    dC_t
\end{pmatrix} = \begin{pmatrix}
    \kappa_{11}^P & 0 & 0 \\
    0 & \kappa_{22}^P & 0 \\
    \kappa_{31}^P & 0 & \kappa_{33}^P
\end{pmatrix}\begin{pmatrix}
    \theta_1^P \\
    \theta_2^P \\
    \theta_3^P
\end{pmatrix} + \begin{pmatrix}
    L_t \\
    S_t \\
    C_t
\end{pmatrix} dt + \begin{pmatrix}
    \sigma_{11} & 0 & 0 \\
    0 & \sigma_{22} & 0 \\
    0 & 0 & \sigma_{33}
\end{pmatrix} \begin{pmatrix}
    dW_t^L, P \\
    dW_t^S, P \\
    dW_t^C, P
\end{pmatrix}.
\]

Thus, the mean-reversion matrix is given by

\[ K^P = \begin{pmatrix}
    \kappa_{11}^P & 0 & 0 \\
    0 & \kappa_{22}^P & 0 \\
    \kappa_{31}^P & 0 & \kappa_{33}^P
\end{pmatrix}. \]

Its matrix exponential can be calculated analytically and is given by

\[ \exp(-K^P \tau) = \begin{pmatrix}
    \text{e}^{-\kappa_{11}^P \tau} & 0 & 0 \\
    0 & \text{e}^{-\kappa_{22}^P \tau} & 0 \\
    -\kappa_{31}^P \frac{\text{e}^{-\kappa_{11}^P \tau} - \text{e}^{-\kappa_{33}^P \tau}}{\kappa_{33} - \kappa_{11}} & 0 & \text{e}^{-\kappa_{33}^P \tau}
\end{pmatrix}. \]

Thus, the conditional mean of the state variables is

\[ E_t^P[X_{t+\tau}] = \theta^P + \begin{pmatrix}
    \text{e}^{-\kappa_{11}^P \tau} & 0 & 0 \\
    0 & \text{e}^{-\kappa_{22}^P \tau} & 0 \\
    -\kappa_{31}^P \frac{\text{e}^{-\kappa_{11}^P \tau} - \text{e}^{-\kappa_{33}^P \tau}}{\kappa_{33} - \kappa_{11}} & 0 & \text{e}^{-\kappa_{33}^P \tau}
\end{pmatrix} \begin{pmatrix}
    L_t - \theta_1^P \\
    S_t - \theta_2^P \\
    C_t - \theta_3^P
\end{pmatrix} = \begin{pmatrix}
    \theta_1^P + \text{e}^{-\kappa_{11}^P \tau}(L_t - \theta_1^P) \\
    \theta_2^P + \text{e}^{-\kappa_{22}^P \tau}(S_t - \theta_2^P) \\
    \theta_3^P - \kappa_{31}^P \frac{\text{e}^{-\kappa_{11}^P \tau} - \text{e}^{-\kappa_{33}^P \tau}}{\kappa_{33} - \kappa_{11}}(L_t - \theta_1^P) + \text{e}^{-\kappa_{33}^P \tau}(C_t - \theta_3^P)
\end{pmatrix}. \]

In order to get back to the term premium formula, we note that the conditional expectation of the instantaneous short rate process is:

\[ E_t^P[r_s] = E_t^P[L_s + S_s] = \theta_1^P + \text{e}^{-\kappa_{11}^P (s-t)}(L_t - \theta_1^P) + \theta_2^P + \text{e}^{-\kappa_{22}^P (s-t)}(S_t - \theta_2^P). \]

Now, we integrate the expected short rate over the time interval from \( t \) to \( t + \tau \) as in the definition of the term premium:

\[
\begin{align*}
\int_t^{t+\tau} E_t^P[r_s]ds &= \int_t^{t+\tau} \left( \theta_1^P + e^{-\kappa_{11}^P (s-t)}(L_t - \theta_1^P) + \theta_2^P + e^{-\kappa_{22}^P (s-t)}(S_t - \theta_2^P) \right)ds \\
&= (\theta_1^P + \theta_2^P)\tau + (L_t - \theta_1^P) \int_t^{t+\tau} e^{-\kappa_{11}^P (s-t)}ds + (S_t - \theta_2^P) \int_t^{t+\tau} e^{-\kappa_{22}^P (s-t)}ds \\
&= (\theta_1^P + \theta_2^P)\tau + (L_t - \theta_1^P) \left[ \frac{-1}{\kappa_{11}^P} e^{-\kappa_{11}^P (s-t)} \right]_t^{t+\tau} + (S_t - \theta_2^P) \left[ \frac{-1}{\kappa_{22}^P} e^{-\kappa_{22}^P (s-t)} \right]_t^{t+\tau} \\
&= (\theta_1^P + \theta_2^P)\tau + (L_t - \theta_1^P) \frac{1 - e^{-\kappa_{11}^P \tau}}{\kappa_{11}^P} + (S_t - \theta_2^P) \frac{1 - e^{-\kappa_{22}^P \tau}}{\kappa_{22}^P}.
\end{align*}
\]
The relevant term to go into the term premium formula is the average expected short rate
\[
\frac{1}{\tau} \int_{t}^{t+\tau} E_t^P[r_s] ds = \theta_1^P + \theta_2^P + (L_t - \theta_1^P) \frac{1 - e^{-\kappa_{11}^P \tau}}{\kappa_{11}^P} + (S_t - \theta_2^P) \frac{1 - e^{-\kappa_{22}^P \tau}}{\kappa_{22}^P}.
\]

The final expression for the term premium is then given by
\[
TP_t(\tau) = y_t(\tau) - \frac{1}{\tau} \int_{t}^{t+\tau} E_t^P[r_s] ds
= L_t + \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} S_t + \left( \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} - e^{-\lambda^P \tau} \right) C_t - \frac{A(\tau)}{\tau}
- \theta_1^P - \theta_2^P - (L_t - \theta_1^P) \frac{1 - e^{-\kappa_{11}^P \tau}}{\kappa_{11}^P} - (S_t - \theta_2^P) \frac{1 - e^{-\kappa_{22}^P \tau}}{\kappa_{22}^P}
= \left( 1 - \frac{1 - e^{-\kappa_{11}^P \tau}}{\kappa_{11}^P} \right) L_t + \left( \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} - \frac{1 - e^{-\kappa_{22}^P \tau}}{\kappa_{22}^P} \right) S_t + \left( \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} - e^{-\lambda^P \tau} \right) C_t
- \left( 1 - \frac{1 - e^{-\kappa_{11}^P \tau}}{\kappa_{11}^P} \right) \theta_1^P - \left( 1 - \frac{1 - e^{-\kappa_{22}^P \tau}}{\kappa_{22}^P} \right) \theta_2^P - \frac{A(\tau)}{\tau}.
\]

We now provide the formulas for the decomposition of forward rates in the preferred AFNS model. In AFNS models, in general, the instantaneous forward rate is given by
\[
f_t(\tau) = L_t + e^{-\lambda^P \tau} S_t + \lambda^P e^{-\lambda^P \tau} C_t + A^I(\tau),
\]
where the yield-adjustment term in the instantaneous forward rate function is:
\[
A^I(\tau) = \frac{\partial A(\tau)}{\partial \tau}
= \frac{1}{2} \sigma_{11}^2 \tau^2 - \frac{1}{2} \left( \sigma_{21}^2 + \sigma_{22}^2 \right) \left( \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} \right)^2
- \frac{1}{2} \left( \sigma_{31}^2 + \sigma_{32}^2 + \sigma_{33}^2 \right) \left[ \frac{1}{\lambda^P} - \frac{2}{\lambda^P} e^{-\lambda^P \tau} + \frac{2}{\lambda^P} e^{-2\lambda^P \tau} + \frac{2}{\lambda^P} e^{-2\lambda^P \tau} + \tau e^{-2\lambda^P \tau} \right]
- \sigma_{11} \sigma_{21} \left[ \frac{1 - e^{-\lambda^P \tau}}{\lambda^P} \right] - \sigma_{11} \sigma_{31} \left[ \frac{1}{\lambda^P} - \frac{1}{\lambda^P} e^{-\lambda^P \tau} - \tau e^{-\lambda^P \tau} \right]
- \left( \sigma_{21} \sigma_{31} + \sigma_{22} \sigma_{32} \right) \left[ \frac{1}{\lambda^P} - \frac{2}{\lambda^P} e^{-\lambda^P \tau} - \frac{1}{\lambda^P} e^{-2\lambda^P \tau} + \frac{1}{\lambda^P} e^{-2\lambda^P \tau} + \frac{1}{\lambda^P} e^{-2\lambda^P \tau} \right].
\]

The instantaneous forward rate term premium in the preferred AFNS model is then given by
\[
TP^I_t(\tau) = f_t(\tau) - E_t^P[r_{t+\tau}]
= L_t + e^{-\lambda^P \tau} S_t + \lambda^P e^{-\lambda^P \tau} C_t + A^I(\tau) - \left( \theta_1^P + e^{-\kappa_{11}^P \tau} (L_t - \theta_1^P) + \theta_2^P + e^{-\kappa_{22}^P \tau} (S_t - \theta_2^P) \right)
= \left( 1 - e^{-\kappa_{11}^P \tau} \right) L_t + \left( e^{-\lambda^P \tau} - e^{-\kappa_{22}^P \tau} \right) S_t + \lambda^P e^{-\lambda^P \tau} C_t + A^I(\tau) - \left( 1 - e^{-\kappa_{11}^P \tau} \right) \theta_1 - \left( 1 - e^{-\kappa_{22}^P \tau} \right) \theta_2.
\]
References


