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Abstract

In an economy with frictional goods and labor markets there exist a price and a wage that implement the constrained efficient allocation. This price maximizes the marginal revenue of labor, balancing a price and a trading effect on firm revenue, and this wage trades off the benefits of job creation against the cost of turnover in the labor market. We show under bargaining over prices and wages that a double Hosios condition: (i) implements the constrained efficient allocation; (ii) also minimizes the elasticity of labor market tightness and job creation to a demand shock, and; (iii) that the relative response of wages to that of unemployment to changes in demand flattens as workers lose bargaining power, and it is steepest when there is efficient rent sharing in the goods market between consumers and producers, thereby relating changes in the slope of a wage Phillips curve to the constrained efficiency of allocations.

JEL Classification: E24, E32, J63, J64.
Keywords: Aggregate demand, unemployment, search and matching frictions, market power, constrained efficiency, wage Phillips curve.

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1 Introduction

Product markets have become more concentrated and workers have seen their bargaining power erode over the last several decades. At the same time the wage Phillips curve, the original trade-off between unemployment and wage growth in Phillips 1958, has flattened.[1] This paper provides a model where such a trade-off between wages and unemployment naturally arises and depends, in particular, on structural parameters related to the market power of firms, workers and consumers in goods and labor markets. Further, the paper identifies four regimes characterizing excess or deficit of demand for labor or goods relative to efficiency (in a second-best sense). Depending on the regime the Phillips relation is more or less steep, connecting changes in the slope of the Phillips curve to departures form efficiency in the labor and goods markets.

In the economy studied here participants — consumers, producers, and labor — all have some market power and interact in frictional markets. The trading frictions take the form of search in labor and goods markets, which leads to the existence of positive surplus to transactions. Market power in dividing the rents from trade determine whether a good is sold near its marginal cost or closer to its marginal utility and whether labor is paid its marginal product or nearer its reservation wage. The model is therefore convenient to study the real effects of changes in demand, and how prices, wages, and markups relate to market power, the slope of a wage Phillips curve, and the efficiency of allocations.

The environment considers the marginal decisions of firms to hire and to produce, and that of consumers to expand their consumption set in a one worker — one firm — one consumer environment.[2] Production requires hiring a worker, and sales require establishing a customer relationship. In the labor market, firms search for workers and expect a certain profit from selling goods and paying wages. Relative to a standard model of equilibrium unemployment, the payoff to entry in the labor market — the demand for labor — is now a function of the equilibrium in the goods market. In the goods market, consumers decide how much to actively search for products to add to their consumption basket depending on the ease of finding a product and the consumption surplus after paying the price of the good. The implication for firms is a downward-sloping demand curve in price, and goods market power has two opposing effects on the marginal revenue product of labor. A greater market power allows firms to obtain a higher price but conversely the higher price generates a negative trading effect on revenue: firms face less consumer demand in the goods market relative to the aggregate supply of goods, increasing the difficulty for all firms to find customers.

A central result is first that the double Hosios condition on prices and wages (equalizing workers and consumers bargaining power to their respective elasticity in the matching of labor and goods market) leads to constrained efficiency. Second, this condition is also the condition that minimizes the elasticity of labor market tightness, and hence job creation, to a demand shock. The demand shocks is modeled here as a sudden change in consumer preferences for entering the

[1] Evidence for a flattening was Phillips curve appears at least since the work of Katz and Krueger (1999), and more recently Galí and Gambetti (2018) and Leduc and Wilson (2017). The has been a recent debate regarding changes in the slope of the price Phillips curve, e.g., O. Blanchard, Cerutti, and Summers 2015; O. J. Blanchard 2016, McLeay and Tenreyro 2019; Hazell et al. (2020).

[2] This is formally equivalent to a large representative consumer and firm.

market for goods. The socially efficient allocation in the goods market is obtained for a degree of firm's goods market power that balances the positive price and negative trading effects in the goods market, thereby maximizing the marginal revenue product of labor and the demand for labor for a given division of rents in the labor market. At this so-called Hosios-efficiency in the goods market, the offsetting price and trading effects result in a minimized elasticity of aggregate demand, and of the marginal revenue product of labor, to changes in consumer preferences. Similarly, a Hosios-condition also exists in the labor market balancing the cost of turnover against the value-added of additional production. This double efficiency is a reference point for our analysis.[3]

The third result is that the relative response of wages and unemployment to changes in demand — the slope of the wage Phillips curve — is explicitly related to market power in setting wages and prices and the constrained efficiency of allocations. An increase in firm market power in setting wages will increase the markup of prices over wages and flatten the wage Phillips curve, and this flattening is inefficient only if firms already extract too much surplus from a match with labor. Rising firm market power in the goods market always increases the markup of prices over wages. It flattens the wage Phillips curve only if firms are extracting too much surplus from consumers almost always.[4] Otherwise it actually steepens the response of wages relative to that of unemployment following a change in aggregate demand. The results hold in an extension in which we look at the dynamics response of wages and unemployment to a demand shock. To sum up, the slope of the Phillips curve depends non-monotonically on the Hosios-efficiency of the goods market and thus non monotonically to firms' markup power vis-a-vis consumers. In contrast, in the labor market, the slope depends monotonically in workers bargaining power, and a change in slope reflects a more efficient allocation if the change in worker market power is in the direction of labor market efficiency.

Our work continues a recent strand of research has revived the original ideas of the search literature where consumers needed to search in the goods market in order to consume (e.g., Diamond 1971, 1982). An early attempt to integrate goods and labor frictions with money is in Shi (1998), and Gourio and Rudanko (2014) present a model in which firms develop a customer base on a search frictional product market to study the implication for firm dynamics. Trading frictions in product markets studied in new monetarist models (see Lagos, Rocheteau, and Wright 2017 for a review), usually in conjunction with credit market frictions, establish a similar mechanism through which product market frictions affect the marginal revenue of firms and the equilibrium in the labor market. In Berentsen, Menzio, and Wright (2011) it is inflation, reducing the real income of consumers, that affects the real demand faced by firms, their markups, and demand for labor.[5] Our results can also be related to the recent work of Mangin and Julien (2018), who de-

[3] In addition, we show that efficient prices and wages follow simple linear rules in market tightness. If a price setting mechanism in a decentralization can conform to these rules under restrictions placed on parameters, then the decentralized allocation can be constrained efficient.

[4] The result, as we show in the appendix, hold for a wide range of model parameter values but has not been establish unambiguously everywhere. In extreme parameterizations of the model that result in a rate of unemployment close to 0 the result can be reserved.

[5] See also Branch, Petrosky-Nadeau, and Rocheteau (2014). Petrosky-Nadeau and Wasmer (2015) study the business cycle implication of extending the search model of equilibrium unemployment to a search frictional product market. In earlier work in the money search literature Lehmann and Van der Linden (2010) obtain of a level of product market tightness result in the highest level of labor demand through a similar trade off as in our environment. Bai, Rios-Rull,

scribe a generalized Hosios condition in a variety of environments.[6] The efficiency trade off in the goods market is similar to underlying the efficiency properties of competitive search environments (Moen 1997). After maximizing the social surplus within a consumption match by setting the tightness of the goods market, a planner then chooses the efficient level of employment (the scale of the economy) by setting the tightness of the labor market, or relative demand for labor, so as to balance the cost of turnover against the value-added of additional production as in the original work of Hosios (1990).

The model clarifies how market power in setting wages and prices underlie the slope of the wage Phillips curve as an equilibrium outcome, providing a framework for understanding the implications of changes in the structure of labor and goods markets that have tilted power towards firms in setting prices and wages. In the labor market, the literature focuses on the decline in unions (Schanbel 2013), the rise of large employers (Azar, Marinescu, and Steinbaum 2017, Benmelech, Bergman, and Kim 2018), and the increasing use of contracts that limit workers' bargaining power such as non-compete clauses and binding arbitration (Krueger and Posner 2018, Starr, Prescott, and Bishara 2019). In the goods market, research points to the rise in concentration in sales, with a few large firms dominating in a majority of economic sectors (Grullon, Larkin, and Michaely 2017). Markups of price over marginal cost have been trending up at the aggregate (Nekarda and Ramey 2013) and sectoral (Hall 2018) levels since the late 1980s. Most recently, market power of firms has been shown to have widely changed in both the US and in Europe over the last decades (see e.g. Gutiérrez and Philippon 2017).[7] Strictly speaking, however, markups can capture firm market power in either the labor or the product market. This will also be the case in our environment where the observation of changing markups and slope of the Phillips curve are not independent, and can be related to departures from an efficient sharing of rents in the goods and labor markets.

The environment studied reinforces central aspects of the New-Keynesian research agenda without relying on price and wage rigidities. As in standard New Keynesian theory (e.g., Galí 2011), greater firm market power in the labor market flattens the wage Phillips curve. In contrast, greater firm market power in the goods market increases the markup of price over marginal cost but has no effect on the slope of the wage Phillips curve in standard New Keynesian theory. We also relate to O. J. Blanchard and Giavazzi (2003) who present a model in which the rents available to workers when bargaining wages with firms depend on the size of the rents in the goods market. In their environment, goods market rents arise from monopolistic competition, and the effect of moving towards competitive pricing is a monotonic decrease in unemployment. This is in contrast with our environment in which the efficient price in the goods market is between these

and Storesletten (2011), in a directed search environment, explore the implications for the measurement of aggregate productivity. Similar findings arise in Michaillat and Saez (2014) who argue that changes in goods market frictions must account for a large share of fluctuations in the labor market. More recently, Bethune, Choi, and Wright (2016) study retail trade markets with trading frictions in a mixture of directed and random search in a New Monetarist framework.

[6]They derive efficiency conditions for matching markets that include the classical congestion externality and what they term an output externality which occurs when the surplus of a match depends on entry by at least one of the agents. In our environment, a related externality arises from the interaction of multiple markets in general equilibrium: the labor match surplus is not independent of equilibrium in the goods market.

[7]Firm-level data suggest that a change in market composition toward high markup firms drives the aggregate trends (Autor et al. 2017, Kehrig and Vincent 2018, De Loecker, Eeckhout, and Unger 2018).

two extremes, and leads to the lowest rate of unemployment.

The rest of the paper is organized as follows. Section 2 sets out the environment and the trading frictions agents face in the labor and goods markets and describes the set of prices and wages that can sustain equilibria with positive entry in the goods and labor markets. Section 3 derives the constrained efficient allocation and discusses the separation into price and wage regimes, while a decentralization under bargaining is studied in Section 4. Section 5 studies the effects of demand shocks away from the constrained efficient allocation, and the relation between market power and the slope of the wage Phillips curve, both at and out of steady state, is the subject of Section 5.3.[8] Section 6 concludes.

2 Economies with frictional goods and labor markets

2.1 Environment - matching and separation

Matching, separation, and market tightness

In the labor market, workers are either unemployed or employed. Let 1 be the total labor force, and $\mathcal{U}$ is the number of unemployed workers. Firms post a number $\mathcal{V}$ of vacancies. These two inputs combine through an increasing and constant returns-to-scale function $M_L(\mathcal{V}, \mathcal{U})$ into a number of matches in the labor market. Workers separate from these employment relations at constant rate s^L. The law of motion of unemployment is therefore (time is continuous):

$$\dot{\mathcal{U}} = s^L (1 - \mathcal{U}) - M_L (\mathcal{U}, \mathcal{V}) \tag{1}$$

In the goods market, consumers derive utility from consuming a bundle of goods $\mathcal{C}$. This bundle is a stock of goods and services to which new goods are added by exerting D_U units of search effort to find products supplied by one-worker-firms not currently serving another consumer $\mathcal{N}_G$. Inflows of new goods into the consumption basket are governed by an increasing and constant returns-to-scale function $M_G (D_U, \mathcal{N}_G)$. Goods exit the consumption basket when one of two events occurs. First, a consumer can quit a particular consumption relationship. This arrives at a constant rate s^G. Second, consumer-firm relationships are also terminated when the producer, i.e., the firm, loses its worker. This occurs at rate s^L, and we denote the sum both separation rates by $s = s^G + s^L$. Therefore, the laws of motion in the goods market are:

$$\begin{aligned} \dot{\mathcal{C}} &= M_G (\mathcal{D}_U, \mathcal{N}_G) - s\mathcal{C} && (2) \\ \dot{\mathcal{N}_G} &= M_L (\mathcal{U}, \mathcal{V}) + s^G \mathcal{N}_\pi - M_G (\mathcal{D}_U, \mathcal{N}_G) - s^L \mathcal{N}_G && (3) \end{aligned}$$

where the number of firms matched with a consumer $\mathcal{N}_\pi$ follows the same law of motion as the bundle of goods $\mathcal{C}$. The measure D_U can be thought of as unmatched consumption demand. The inflows into the mass of firms searching in the goods market, in equation (3), stem from two sources: the inflow of new hires in the labor market, and the inflow from firms that have lost a

[8]The role of alternative bargaining assumptions is studied in appendix section F.1.

consumer, $s^G \mathcal{N}_\pi$. Likewise, there are two sources of outflows: new matches with consumers in the goods market, and separations of workers $\left(s^L \mathcal{N}_G\right)$.

These laws of motion imply identities that are helpful for analyzing the social planner's problem. First, the total number of filled jobs corresponds to the total number of employed workers. That is, $1 - \mathcal{U} = \mathcal{N}_G + \mathcal{N}_\pi$. Second, the number of firms supplying to consumers equals the consumption bundle, namely $\mathcal{N}_\pi = \mathcal{C}$.

Tightness in goods and labor markets

Tightness in both markets is defined from the perspective of the buyer: $\theta = \mathcal{V}/\mathcal{U}$ in the labor market and $\xi = D_U/\mathcal{N}_G$ in the goods market. Transitions are quicker in a tight market for the seller. The job finding rate $f(\theta) = M_L/\mathcal{U}$ is increasing in labor market tightness, and the job filling rate $q(\theta) = M_L/\mathcal{V}$ is decreasing in labor market tightness. Greater tightness ξ in the goods market reflects more consumer search for a good relative to the number of firms searching for consumers. The firm's meeting rate in the goods market $\lambda(\xi) = M_G/\mathcal{N}_G$ is increasing in ξ, while the consumer's meeting rate $\psi(\xi) = M_G/D_U$ is decreasing in ξ. The elasticity of matching in the labor market with respect to the unemployed is denoted $\eta_L(\theta)$, while the elasticity of matching in the goods market with respect to consumer demand D_U is denoted by $\eta_G(\xi)$. This is summarized below:

Labor market matching frictions:	Goods market matching frictions:
$\theta = \mathcal{V}/\mathcal{U}$	$\xi = D_U/\mathcal{N}_G$
$q(\theta) = M_L/\mathcal{V}$ with $q' < 0$	$\psi(\xi) = M_G/D_U$ with $\psi' < 0$
$f(\theta) = M_L/\mathcal{U} = \theta q(\theta)$ with $f' > 0$	$\lambda(\xi) = M_G/\mathcal{N}_G = \psi(\xi)\xi$ with $\lambda' > 0$
$\eta_L(\theta) = -\theta q'(\theta)/q(\theta) \in (0,1)$	$\eta_G(\xi) = \xi\lambda'(\xi)/\lambda(\xi) \in (0,1)$

2.2 Entry and the set of sustainable decentralized equilibria

Firms are allowed to freely enter the labor market, and consumers the goods market. These entry conditions allow us to define the set of wages and prices that could sustain equilibria with positive tightness of the goods and labor markets. In the following sections we determine the efficiency of a decentralized allocation relative to the planner's allocation, and then show how different price and wage setting mechanisms could implement the constrained efficient allocation.

Value functions of agents: workers, consumers, and producers

In principle, the value of employment at a firm searching in the goods market and paying a wage w_g, could be distinct from that at a firm selling its product and paying a wage w_π. In our baseline workers earn a wage ω regardless of the status of the firm, an assumption we relax in the extensions of Section F.1 of the appendix. The asset values of employment and unemployment to a worker, W_e and W_u respectively, are given by:

$$rW_e = \omega + s^L\left(W_u - W_e\right) \tag{4}$$

$$rW_u = z + f(\theta)\left(W_e - W_u\right) \tag{5}$$

while z is the flow utility from non-employment, $f(\theta)$ the job finding rate, and r is the rate of time discounting.

The problems facing consumers and workers are separated by the assumption of risk neutrality and a pooling of resources among worker-consumers. Implicitly, any resource not spent on the search good is used as a numeraire with a constant marginal utility. We do not report the income of the consumer because it scales up both consumer value functions in the same way, dropping out of the consumption surplus.

In the goods market, consumers determine privately the optimal level of search effort D_U at marginal utility cost σ to find goods to add to their consumption bundle. Each good provides a marginal benefit to the consumer Φ in terms of numeraire, at a cost in numeraire $\mathcal{P}$. The consumer asset values in a steady-state are:

$$rW_{D_U} = -\sigma + \psi(\xi)\left(W_C - W_{D_U}\right) \tag{6}$$

$$rW_C = (\Phi - \mathcal{P}) + s^G\left(W_{D_U} - W_C\right) + s^L\left(W_{D_U} - W_C\right) \tag{7}$$

where W_{D_U} and W_C denote, respectively, the value functions of searching for a good and of being matched with a seller . Consumers find goods at rate ψ and enjoy the consumption surplus $(W_C - W_{D_U})$ in equation (6). Consumption of the search good ends following either a change in tastes, which arrives at rate s^G, or following a labor turnover event at the firm selling the good. Indeed, the last term $s^L\left(W_{D_U} - W_C\right)$ captures the fact that, if the firm from which the good is purchased loses its worker, there is an interruption in the flow of surplus as the good is no longer produced.

The value of searching, J_g, and being matched, J_π, in the goods market to the firm are given by:

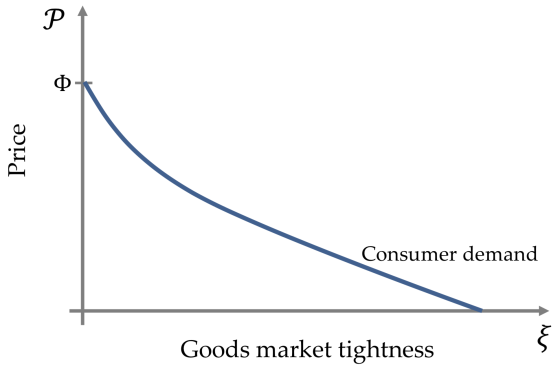
$$rJ_g = -\omega + \lambda\left(J_\pi - J_g\right) + s^L\left(J_v - J_g\right) \tag{8}$$

$$rJ_\pi = \mathcal{P} - \omega + s^G\left(J_g - J_\pi\right) + s^L\left(J_v - J_\pi\right) \tag{9}$$

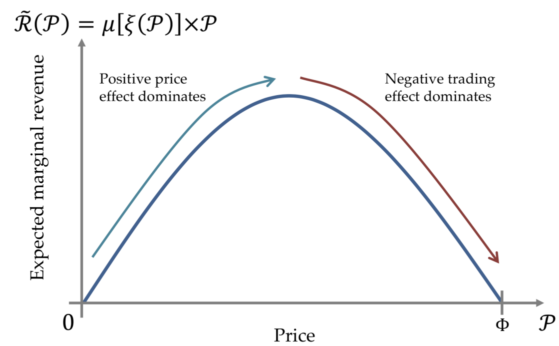
The firm matches with a consumer at rate λ, and gains the goods market surplus $\left(J_\pi - J_g\right)$. However, it faces the risk of a labor turnover shock in both stages g and π, and a loss $\left(J_v - J_g\right)$ and $(J_v - J_\pi)$, respectively. The profit flow $\mathcal{P} - \omega$ from selling the good may also be interrupted following a loss of the consumer, at rate s^G. In this event the worker remains with the firm and the capital loss is $\left(J_g - J_\pi\right)$.

Finally, in the labor market, the value of a vacancy to a firm is the sum of an outflow cost γ and a capital gain $J_g - J_v$ following a match with a worker at rate $q(\theta)$:

$$rJ_v = -\gamma + q(\theta)\left(J_g - J_v\right) \tag{10}$$



(a) Consumer goods market entry condition



(b) Price and the expected marginal revenue of labor

Figure 1: Consumer demand and the expected marginal revenue of labor

2.3 Entry in the markets for goods and labor

We are now in a position to define the set of wages and prices that could sustain equilibria with positive tightness of the goods and labor market.

Goods market entry

Free entry of consumer search in the goods market leads to $W_{D_U} = 0$ in equilibrium. Applying this to equations (6) and (7) we have a goods market entry condition:

$$\frac{\sigma}{\psi(\xi)} = \frac{\Phi - \mathcal{P}}{r+s} \tag{11}$$

under which entry occurs until the average cost of finding a search good, σ/ψ, equals the consumer's consumption surplus $W_{D_M} - W_{D_U} = (\Phi - \mathcal{P})\,/\,(r+s)$. The latter is simply the discounted present value of the net benefit flow in units of numeraire from consumption $(\Phi - \mathcal{P})$.[9] The entry condition describes a negative relation between the price and tightness of the goods market, $\xi(\mathcal{P})$ with $\xi'(\mathcal{P}) < 0$ (Figure 1a), and an equilibrium with positive goods market tightness can exist as long as the price is below the marginal utility Φ. In effect this is a downward-sloping demand curve, and the price elasticity of demand $\frac{d\mathcal{C}}{d\mathcal{P}}\frac{\mathcal{P}}{\mathcal{C}} = -\frac{1-\eta_G}{\eta_G}\frac{\Phi}{\Phi-\mathcal{P}}$ depends on the elasticity of the goods matching function, η_G, and the flow consumption surplus $\Phi - \mathcal{P}$. Firms face a high elasticity of demand when consumer entry has less effect on the product finding rate ψ (low η_G), and when goods are priced near their marginal utility, giving consumers a small surplus.

A humped curve of expected marginal revenue product of labor in price

From the perspective of a firm entering the labor market, the marginal revenue expected from hiring labor and selling in the goods market depends on the price $\mathcal{P}$ and the relative ease of finding and retaining a buyer. The amount of time a unit of production generates sales is, therefore

[9] As separate issue not explored in this work which we only note here is that the environment retains monetary neutrality. All relevant net payoffs are measured in units of numeraire and scale with the introduction of a nominal price level, having no effect on the allocations.

a function $\mu(\xi)$ the tightness of the goods market ξ:

$$\mu(\xi) = \frac{\lambda(\xi)}{r + s + \lambda(\xi)} \in (0, 1) \tag{12}$$

The factor $\mu(\xi)$ discounts for the firm's likelihood of finding ($\lambda(\xi)$) and retaining (s) its demand in the goods market. This factor may also be interpreted as a discounted rate of capacity utilization. In the extreme where there is no demand (when ξ tends to 0), meeting a consumer is infinitely slow and μ tends to 0. In the other extreme, as the goods market is infinitely tight, matching is instantaneous and μ tends to 1.[10]

The annuity value of revenue generated by a worker for the firm at entry, denoted by $\mathcal{R}(\xi, \mathcal{P})$, can thus be defined as:

$$\mathcal{R}(\xi, \mathcal{P}) = \mu(\xi) \times \mathcal{P}$$

Since goods market tightness, and hence $\lambda(\xi)$, is declining in $\mathcal{P}$, the trading factor μ may also be expressed as a declining function of the price $\tilde{\mu}(\mathcal{P}) \equiv \mu[\xi(\mathcal{P})]$, with $\tilde{\mu}'(\mathcal{P}) < 0$. It follows that the expected revenue from selling the good as a function of the price is:

$$\tilde{\mathcal{R}}(\mathcal{P}) \quad = \quad \tilde{\mu}(\mathcal{P}) \times \mathcal{P} > 0 \text{ for } \mathcal{P} \in (0, \Phi)$$

$\tilde{\mathcal{R}}(\mathcal{P})$ is positive for a price within the range of 0 (no revenue from sales) and the marginal utility Φ (no buyers for the good), with $\frac{\partial \tilde{\mathcal{R}}(\mathcal{P})}{\partial \mathcal{P}} = \frac{\partial \tilde{\mu}(\mathcal{P})}{\partial \mathcal{P}}\mathcal{P} + \tilde{\mu}(\mathcal{P}) \lesseqgtr 0$ and $\frac{\partial^2 \tilde{\mathcal{R}}(\mathcal{P})}{\partial \mathcal{P}^2} < 0$. The revenue function $\tilde{\mathcal{R}}(\mathcal{P})$ is concave through a trading effect — an increase in the price makes it less likely to find a consumer — and a price effect — the added revenue of a higher price keeping the demand for the good constant (i.e., no change in goods market tightness).

The combination of these forces produces a hump-shaped curve for expected marginal revenue of labor, represented in Figure 1b. An increase in price leads to greater expected revenue when the price effect dominates, that is, when $\tilde{\mu}(\mathcal{P})$ is greater than the absolute value of the negative trading effect $\frac{\partial \tilde{\mu}(\mathcal{P})}{\partial \mathcal{P}}\mathcal{P}$. This is a situation when the price elasticity of expected trading $\left|\frac{\partial \tilde{\mu}(\mathcal{P})}{\partial \mathcal{P}} \frac{\mathcal{P}}{\tilde{\mu}(\mathcal{P})}\right|$ is less than 1. When $\left|\frac{\partial \tilde{\mu}(\mathcal{P})}{\partial \mathcal{P}} \frac{\mathcal{P}}{\tilde{\mu}(\mathcal{P})}\right| > 1$ the trading effect dominates and the expected revenue is declining in the price. These effects are similar to what arises in competitive search (Moen 1997) where firms take into account the effects of a wage offer on the queue in their labor market.

Labor market entry and job creation

Firms are assumed to freely enter the labor market, resulting in an equilibrium with $J_v = 0$ and, combining (8), (9), and (10), we have:

$$\frac{\gamma}{q(\theta)} = \frac{\tilde{\mathcal{R}}(\mathcal{P}) - \omega}{r + s^L} \tag{13}$$

This equation is a generalization of the classical entry equation in the labor market, the job creation condition, now that firms anticipate the effects of goods market frictions on the marginal revenue

[10] This interpretation also found in Petrosky-Nadeau and Wasmer (2017). It converges to 1 as goods market frictions disappear.

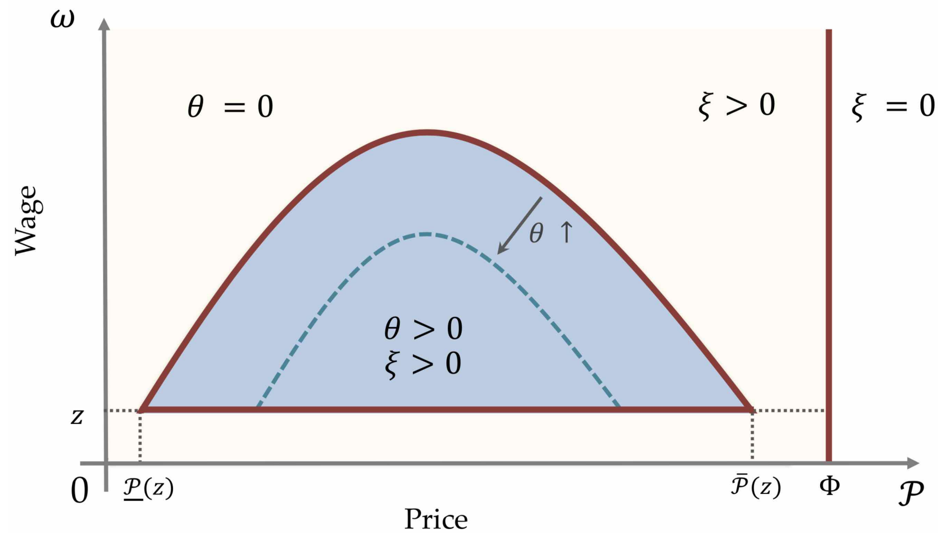


Figure 2: Set of equilibria with positive goods and labor market tightness in wage and price space

$\tilde{\mathcal{R}}\left(\mathcal{P}\right)$. An equilibrium with positive labor market tightness can exist if $\tilde{\mathcal{R}}(\mathcal{P}) > \omega$. As seen above, $\tilde{\mathcal{R}}(\mathcal{P})$ is positive and and concave in the price for $\mathcal{P} \in (0, \Phi)$ through the trading and price effects. It follows that, for a given wage, labor market tightness is positive and concave in the price. That is, equation (13) implicitly defines a function $\theta(\mathcal{P};\omega)$ with $\partial\theta(\mathcal{P};\omega)/\partial\mathcal{P} \lesseqgtr 0$ and $\partial^2\theta(\mathcal{P};\omega)/\partial\mathcal{P}^2 < 0$. Labor market tightness is positive as long as $\tilde{\mathcal{R}}(\mathcal{P}) > \omega$, placing two bounds on the price for entry of firms in the labor market: the two prices that solve $\omega = \tilde{\mu}(\mathcal{P})\mathcal{P}$. This occurs for a low price level $\underline{\mathcal{P}}(\omega)$ with a high likelihood of finding a consumer, and a high price level $\overline{\mathcal{P}}(\omega)$ with a slack goods market in which it is difficult to find a consumer.

Set of decentralized equilibria with positive goods and labor market tightness

The shaded area of Figure 2 represents all the pairs of price and wage $(\mathcal{P}, \omega)$ for which $\theta > 0$ and $\xi > 0$. As long as the price $\mathcal{P}$ is below the marginal utility Φ, there can exist an equilibrium with positive goods market tightness ξ (see equation (11)). In the labor market, the job creation condition defines iso-labor market tightness loci in price and wage space: $\omega = \tilde{\mathcal{R}}(\mathcal{P}) - \left(r + s^L\right)\gamma/q(\theta)$. These iso-tightness loci inherit the shape of the expected marginal revenue function $\tilde{\mathcal{R}}(\mathcal{P})$ and represent all the pairs of prices and wages that could sustain a given equilibrium tightness of the labor market. The solid curve in Figure 2 corresponds to the boundary for prices and wages for an equilibrium with strictly positive firm entry in the labor market. All loci below correspond to equilibria with increasing labor market tightness. Finally, the worker's reservation wage z places a lower bound on ω for an equilibrium with positive labor market tightness.

3 Efficiency in an economy with frictional goods and labor markets

We now characterize the constrained efficient allocation chosen by a social planner, before discussing how this constrained efficient allocation can be decentralized.

3.1 Social planner's problem

Each job produces 1 unit of the search good, and the good is valued by the consumer at the margin by $\Phi > 0$. There is a flow cost to searching in the goods market $\sigma > 0$. The non-employed have flow utility $z > 0$, and there is a flow cost to job vacancies $\gamma > 0$. The social planner balances the utility of consumption and these costs of production. It seeks to maximize the value of consumption of the search good and non-employment utility, net of labor and goods market search costs:

$$\Omega = \int_0^\infty e^{-rt} \left[\Phi \mathcal{C} + z\mathcal{U} - \gamma \mathcal{V} - \sigma D_U\right] dt$$

Equations (1) to (3) are dynamic frictional constraints. They apply to both the decentralized equilibrium and the social planner's program. Using these constraints and the identities in the goods and labor markets, we have the following Hamiltonian for the planner's problem:[11]

$$\begin{aligned} \Omega = \max_{D_U, \mathcal{V}, \mathcal{U}, \mathcal{C}} \quad & e^{-rt} \left[\Phi \mathcal{C} + z\mathcal{U} - \gamma \mathcal{V} - \sigma D_U\right] \\ & + \Psi_U \left[M_L(\mathcal{U}, \mathcal{V}) - s^L (1 - \mathcal{U})\right] \\ & + \Psi_{\mathcal{C}} \left[s\mathcal{C} - M_G(D_U, 1 - \mathcal{U} - \mathcal{C})\right] \end{aligned}$$

where the planner chooses consumer search units D_U and job vacancies $\mathcal{V}$, subject to the laws of motion of the co-states of unemployment $\mathcal{U}$ and matched pairs of buyers and sellers $\mathcal{C}$. The associated multipliers are Ψ_U and $\Psi_{\mathcal{C}}$, respectively.

Proposition 1. *The social planner's allocation is unique, and is a pair of constrained efficient goods and labor market tightness $\left(\xi^{opt}, \theta^{opt}\right)$ that solve the pair of conditions for optimal consumer entry and optimal labor market entry, denoted by CE^{OPT} and LE^{OPT} respectively:*

$$CE^{OPT}: \quad (r+s)\frac{\sigma}{\psi(\xi)} = \eta_G \Phi - (1-\eta_G)\sigma\xi \tag{14}$$

$$LE^{OPT}: \quad \left(r+s^L\right)\frac{\gamma}{q(\theta)} = (1-\eta_L)\left[\Phi - z - \frac{\sigma(r+s)}{\psi(\xi)}\frac{1}{\eta_G}\right] - \eta_L \gamma\theta \tag{15}$$

The constrained efficient level of tightness in the goods market ξ^{opt} is uniquely determined by equation (11). It is independent of equilibrium labor market tightness. It is set at a level equating the costs of forming a consumption match to the social benefit of the marginal consumption match. The latter allows for an optimal balancing of the benefits of consumption (with weight η_G) and the opportunity cost for the consumer of searching for goods (with the complement weight). The condition is represented as the vertical line in (ξ, θ) space of Figure 3.

The equilibrium condition (15) describes a decreasing relation between labor and goods market tightness. It is represented as the decreasing solid curve in Figure 3. The equation is similar to that obtained in models of equilibrium unemployment with perfect product markets, e.g. in Pissarides (2000). It is identical when goods market frictions disappear, when either $\sigma = 0$ or $\psi(\xi)$ tends to infinity. The tightness in the labor market achieved under perfect goods markets $\bar{\theta}$ (when σ/ψ

[11] Appendix section A details the planner's problem and its solution.

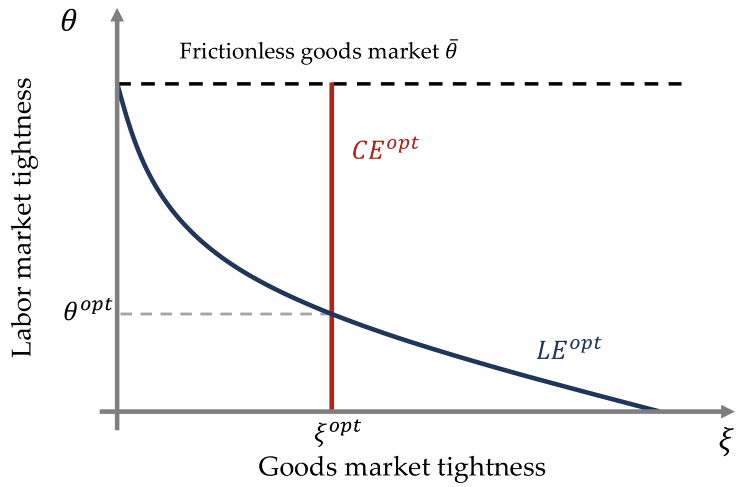


Figure 3: Constrained efficient equilibrium goods and labor market tightness ξ^{opt} and θ^{opt}.

tends toward zero) is represented as the horizontal dashed line in Figure 3. Goods market frictions unambiguously reduce labor market tightness relative to what the social planner could achieve in their absence. Forming a match in the goods market is costly, reducing the net surplus to creating employment.

Proposition 2. *The constrained efficient tightness of the goods market ξ^{opt} maximizes the marginal revenue of labor $\mathcal{R}(\xi, \mathcal{P})$.*

Proof: Combining the definition of $\mathcal{R}(\xi, \mathcal{P})$ with the consumer's entry condition 1a to eliminate the price $\mathcal{P}$, we can express the marginal revenue of labor as an increasing and concave function of goods market tightness ξ:

$$\hat{\mathcal{R}}(\xi) = \frac{\lambda}{r+s+\lambda}\left(\Phi - \frac{\sigma(r+s)}{\psi}\right)$$

The solution to $\frac{\partial \hat{\mathcal{R}}(\xi)}{\partial \xi} = 0$ is $(r+s)\frac{\sigma}{\psi(\xi)} = \eta_G \Phi - (1-\eta_G)\sigma\xi$, which corresponds to the planner's equilibrium condition (14) for goods market tightness. □

Writing the planner's problem at a steady state, with $r = 0$, provides much of the intuition for the choices made by the planner in deciding on the constrained efficient allocation. Indeed the planner's objective becomes:

$$\Omega^s = \hat{\mathcal{R}}(\xi)(1-\mathcal{U}) + z\mathcal{U} - \gamma\mathcal{V}$$

Maximizing Ω^s by choosing goods market tightness is achieved by maximizing $\hat{\mathcal{R}}(\xi)$. The planner's allocation seeks to maximize the value-added produced by labor in each consumer-seller pair by choosing ξ^{opt}. The planner then chooses the scale of the economy with the efficient tightness of the labor market θ^{opt} given an efficient social value $\hat{\mathcal{R}}(\xi)$.

3.2 Slackness regimes and the efficiency of prices and wages

Proposition 3. *There exists a unique price $\mathcal{P}^{opt}$ for which the decentralized tightness of the goods market is constrained efficient. Moreover:*

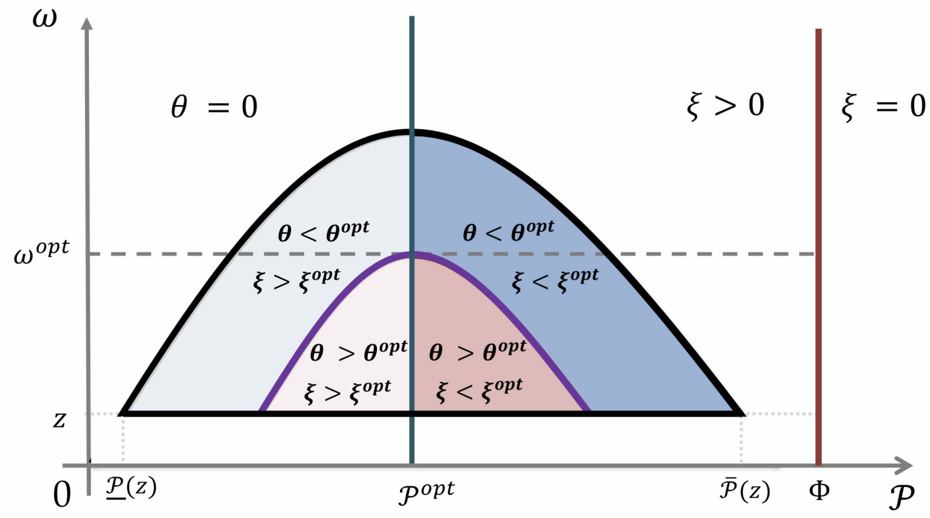


Figure 4: Possible equilibria and tightness regimes in price and wage space
(1) slack labor and tight goods market; (2) slack labor and slack goods market; (3) tight labor and slack goods market; (4) tight labor and tight goods market. The purple iso-labor market tightness condition $\omega = \tilde{\mathcal{R}}(\mathcal{P}) - (r + s^L)\,\gamma/q(\theta^{opt})$ evaluated at the planner's θ^{opt} separates tight and slack labor markets, while goods market tightness regimes are separated by the constrained efficient price $\mathcal{P}^{opt} = (1 - \eta_G)\,(\Phi + \sigma\xi^{opt})$.

for any $\mathcal{P} > \mathcal{P}^{opt}$, the decentralized $\xi^* < \xi^{opt}$ and firm expected revenue $\mathcal{R} < \mathcal{R}^{opt}$;
for any $\mathcal{P} < \mathcal{P}^{opt}$, the decentralized $\xi^* > \xi^{opt}$ and firm expected revenue $\mathcal{R} < \mathcal{R}^{opt}$.

The decentralized equilibrium consumer entry condition (11), and its unique mapping of price to goods market tightness, implies that a single price $\mathcal{P}^{opt}$ implements the constrained efficient tightness of the goods market ξ^{opt}. It is the price at which the consumer's consumption surplus $(\Phi - \mathcal{P})\,/\,(r + s)$ equals the marginal social benefit of a goods market match, the right-hand-side of the planner's goods market condition $(\eta_G\Phi - (1 - \eta_G)\,\sigma\xi)\,/\,(r + s)$. It also results in a maximized annuity value of the marginal revenue of labor at entry $\mathcal{R}\,(\xi, \mathcal{P})$, exactly balancing the price and trading effects in the goods market. Any decentralized equilibrium with a price below $\mathcal{P}^{opt}$ induces excess consumer entry and a goods market that is too tight, or $\xi^* > \xi^{opt}$. This corresponds to all regions with the left of the vertical solid line in Figure 4 at $\mathcal{P}^{opt}$. Symmetrically, any decentralized equilibrium with a price $\mathcal{P} > \mathcal{P}^{opt}$ has insufficient demand for goods or $\xi^* < \xi^{opt}$. Whether there is excess or insufficient demand for goods, however, it is always the case that marginal revenue of labor is inefficiently low, $\mathcal{R} < \mathcal{R}^{opt}$, when the goods market deviates from the constrained efficient allocation.

Proposition 4. *There exists a continuum of prices and wages $(\mathcal{P}, \omega)$ on the iso-labor market tightness locus for θ^{opt}: $\omega = \tilde{\mathcal{R}}(\mathcal{P}) - (r + s^L)\,\gamma/q(\theta^{opt})$, that implement the constrained efficient tightness of the labor market. Moreover:*

for any pair $(\mathcal{P}, \omega)$ above the iso-labor market tightness locus for θ^{opt}, the decentralized $\theta < \theta^{opt}$;
for any pair $(\mathcal{P}, \omega)$ below the iso-labor market tightness locus for θ^{opt}, the decentralized $\theta > \theta^{opt}$.

The labor market entry condition (13) describes pairs of prices and wages that implement a given equilibrium labor market tightness. The planner's solution determines a unique constrained efficient θ^{opt} which can be implemented in a decentralization by the price-wage pairs on the iso-labor market tightness for θ^{opt}. This locus of prices and wages is represented as the solid (purple) curve in Figure 4. Any pair of prices and wages below this curve results in excessive demand for labor. Any pair above that curve results in inefficiently low demand for labor.[12]

There thus arises, naturally, four slackness regimes in terms of the efficiency of the resulting allocations in the goods and labor markets. In all regimes, movements in the price towards the efficient allocation in the goods market increases expected marginal revenue of labor, thereby increasing firm entry into the labor market. In the regimes with excessive demand for labor, wages must rise in order to curb the already tight labor market and the additional pressure from rising firm revenue as the price approaches the efficient price. In regimes with insufficient demand for labor, however, achieving efficiency can require either a decrease or an increase in the wage. If the starting point is a decentralized equilibrium in which the wage is above ω^{opt}, efficiency is achieved by lowering the wage. If the starting point is a decentralized equilibrium in which the wage is below ω^{opt}, the interaction between frictional labor and goods markets generate a situation in which moving towards greater tightness in the labor market (and hence, lower unemployment) occurs with higher wages (through a greater marginal revenue of labor as the price converges to the efficient level).

3.3 Implementing the constrained efficient allocation with price and wage rules

Even before considering particular wage and price mechanisms that decentralize the economy we can derive simple price and wage rules that would implement the constrained efficient allocation. Starting in the goods market, the combination of the entry condition (11) with the social planner's condition for goods market tightness (14) yields a simple rule for the price:

$$\mathcal{P} \quad = \quad (1-\eta_G)\,(\Phi+\sigma\xi) \tag{16}$$

that leads the decentralized equilibrium tightness of the goods market ξ^* to be the constrained efficient ξ^{opt}. Likewise, in the labor market, combining the labor market entry condition (13) and the social planner's condition for labor market tightness (15) yields a simple rule for the wage:

$$\omega \quad = \quad (1-\eta_L)\,z+\eta_L\left[\gamma\theta+\tilde{\mathcal{R}}\left(\mathcal{P}^{opt}\right)\right] \tag{17}$$

that leads the decentralized equilibrium in the labor market to be constrained efficient.[13]

[12] For a given price $\mathcal{P}$ there exists a wage $\omega^{opt}(\mathcal{P})$ such that the decentralized tightness of the labor market equals the constrained efficient θ^{opt}. From the labor market entry condition (13) this wage follows $\omega^{opt}(\mathcal{P}) = \mathcal{R}(\xi,\mathcal{P}) - (r+s^L)\,\gamma/q(\theta^{opt})$. If the prevailing wage firms expect to pay $\omega < \omega^{opt}$, there will be excess firm entry and tightness in the labor market. If $\omega > \omega^{opt}$ there is too little firm entry and the labor market is too slack.

[13] These price and wage rules implementing constrained efficient allocations in the goods and labor markets are plotted in $(\xi,\mathcal{P})$ space in Figures 11a and 11b of the appendix.

4 A decentralization under price and wage bargaining

We now study allocations where prices in the goods and labor market are determined through bargaining, and whether restrictions can be placed on bargaining power to achieve a constrained efficient allocation. In this section each consumer-producer and worker-firm pair engage in bilateral Nash bargaining. The share of the match surplus accruing to a particular party in bargaining is its market power, and we relate market power to the efficiency of the resulting decentralized allocations. Several alternative assumptions on price and wage setting, such as allowing for strategic behavior, are studied in Section F.1.[14]

4.1 Price and wage bargaining and equilibrium

Price setting in the goods market

The price in the goods market $\mathcal{P}$ divides the total consumption surplus, $(W_{\mathcal{C}} - W_{D_U}) + (J_\pi - J_g)$, by solving the Nash problem:

$$\mathcal{P} = \text{argmax}\,(W_{\mathcal{C}} - W_{D_U})^{\alpha_G}\left(J_\pi - J_g\right)^{1-\alpha_G} \tag{18}$$

where $\alpha_G \in (0,1)$ is the bargaining strength of a consumer. The slope of $W_{\mathcal{C}}$ in equation (7) and the slope of J_π in equation (9) are opposite but identical in absolute value. Therefore the price must satisfy the sharing rule $(1-\alpha_G)(W_{\mathcal{C}} - W_{D_U}) = \alpha_G\left(J_\pi - J_g\right)$, dividing the surplus into shares $(1-\alpha_G)$ for the producer and its complement α_G for the consumer. As such, α_G is the consumer's market power, and $(1-\alpha_G)$ the producer's market power. The sharing condition leads to a simple rule for determining prices:

$$\mathcal{P} = (1-\alpha_G)(\Phi + \xi\sigma) \tag{19}$$

The price increases in the market power of the firm, $(1-\alpha_G)$, the marginal utility of consumers Φ and their relative threat points, captured by $\xi\sigma$. Higher search costs and a tighter search market for consumers (greater consumer demand relative to the supply of goods) push the negotiated price up as the seller has a relatively better outside option.

Wage setting in the labor market

The wage splits the labor match surplus $(W_e - W_u) + \left(J_g - J_v\right)$ by solving the Nash problem

$$w = \text{argmax}\,(W_e - W_u)^{\alpha_L}\left(J_g - J_v\right)^{1-\alpha_L} \tag{20}$$

where $\alpha_L \in (0,1)$ is the bargaining strength of a worker. As each side's respective match surplus is of the same absolute slope with respect to the wage, the solution is a wage that satisfies the surplus sharing rule $(1-\alpha_L)(W_e - W_u) = \alpha_L\left(J_g - J_v\right)$, dividing the total match surplus into shares α_L for

[14] The detailed derivations for this section are available in Appendix B.1.

the worker, and its complement $(1-\alpha_L)$ for the firm. The resulting wage rule

$$w = (1-\alpha_L)\, z + \alpha_L \left[\gamma\theta + \mathcal{R}(\xi,\mathcal{P})\right] \tag{21}$$

increases with non-employment utility z and labor market tightness. It also reflects the outcome of the equilibrium in the goods market along the two dimensions determining the expected marginal revenue $\mathcal{R}(\xi,\mathcal{P})$, the level of relative demand ξ and the price $\mathcal{P}$.

Equilibrium

Eliminating prices and wages in the consumer entry condition (11) and in the firm entry condition (13), the decentralized equilibrium is a pair (ξ,θ) that solves the two entry conditions in the goods and labor markets, denote by CE* and LE*, respectively:

$$\text{CE}^*: \quad (r+s)\,\frac{\sigma}{\psi(\xi)} \;=\; \alpha_G\Phi - (1-\alpha_G)\,\xi\sigma \tag{22}$$

$$\text{LE}^*: \quad \left(r+s^L\right)\frac{\gamma}{q(\theta)} \;=\; (1-\alpha_L)\left[\Phi - z - \frac{(r+s)\,\sigma}{\psi\,(\xi)}\frac{1}{\alpha_G}\right] - \alpha_L\gamma\theta \tag{23}$$

As long as consumer market power α_G is strictly positive there is a unique equilibrium with positive goods market tightness. The equilibrium price in the goods market is independent of labor market tightness and, as long as the expected marginal revenue of labor $\mathcal{R}(\xi,\mathcal{P})$ is greater than the flow value of non-employment z, there exists a unique equilibrium with positive labor market tightness. Taking the limit where product market frictions are eliminated (as $(r+s)\,\sigma/\psi\,(\xi)$ tends to 0) recovers the standard Diamond-Mortensen-Pissarides labor market entry condition.

4.2 Constrained efficiency

Comparing the equilibrium conditions (22) in the decentralized economy to the social planner's allocation in (14), tightness in the goods market is constrained efficient if $\alpha_G = \eta_G$. Turning to the labor market, comparing equations (23) and (15), the decentralized equilibrium in the labor market corresponds to the social planner's allocation if $\alpha_L = \eta_L$ *and* $\alpha_G = \eta_G$. At this point, the search externalities in the goods and labor markets are internalized and the decentralized agents reach an optimal allocation. This leads to a double Hosios condition:

Proposition 5. *The decentralized allocation in an economy with search and bargaining in goods and labor markets, and in which wages are not bargained conditional on whether the firm is selling its product, is constrained efficient if and only if* $\alpha_L = \eta_L$ *and* $\alpha_G = \eta_G$.

The Hosios conditions can also be seen through the bargained wage and price rules. The goods market price (18) corresponds to the optimal price in (16) if the firm's share of the surplus $\alpha_G = \eta_G$. The wage in (21) coincides with the wage delivering the constrained efficient tightness of the labor market ω^{opt} in (17) if the worker's bargaining weight $\alpha_L = \eta_L$ *and* the goods market is at the constrained efficient allocation with $\mathcal{P}^{opt}$. [15]

[15] This result is a generalization of our finding in Petrosky-Nadeau and Wasmer (2017) (chapter 7) where the social

Market power and the efficiency of allocations

Whether there is excess demand or supply in either the goods or the labor market relative to the planner's allocation can be represented in market power space (α_G, α_L). This is similar to the separation in price and wage space of Section 3.2.

Begin with the goods market. Goods market tightness ξ^* is strictly increasing in the consumer's bargaining power α_G. Thus if $\alpha_G < \eta_G$, there is insufficient entry of consumers and demand, i.e. $\xi^* < \xi^{opt}$. If $\alpha_G > \eta_G$, there is excessive demand and $\xi^* > \xi^{opt}$. The condition for constrained efficiency in the goods market, $\alpha_G = \eta_G$, is plotted as a vertical solid line in Figure 5. To the left of the curve, the goods market is too slack. To the right of the curve, the goods market is too tight.

The effect of goods market bargaining power on tightness in the labor market, in contrast, is non-monotonic and operates through the hump-shaped property of the expected marginal revenue of labor. As with iso-labor market curves in price and wage space, iso-labor market tightness curves also exist in market power space (α_G, α_L). Indeed, the marginal revenue of labor $\mathcal{R}(\xi, \mathcal{P})$ is strictly lower than at the planner's allocation for any $\alpha_G \neq \eta_G$. Thus, moving away from the Hosios condition in the goods market it is possible to maintain $\theta^* = \theta^{opt}$ by giving more of the labor match surplus to the firm, i.e., with an $\alpha_L < \eta_L$, compensating either for a negative trading effect (when $\alpha_G < \eta_G$) or for a lower price (when $\alpha_G > \eta_G$). The solid (red) curve of Figure 5 represents all the pairs of market power in the goods and labor markets (α_G, α_L) that implement the constrained efficient θ^{opt}.

Proposition 6. *The decentralized equilibrium goods market tightness ξ^* is strictly decreasing in the seller's bargaining power α_G. The decentralized equilibrium tightness of the labor market θ^* is a concave function of α_G, maximized when the seller's share of the goods market surplus $\alpha_G = \eta_G$. The decentralized equilibrium goods market tightness reaches its optimal value $\xi^* = \xi^{opt}$ if $\alpha_G = \eta_G$, while labor market tightness $\theta^* = \theta^{opt}$ if $\alpha_L = \tilde{\alpha}_L(\alpha_G)$, where $\tilde{\alpha}_L(\alpha_G)$ is given by the iso-labor market tightness condition (24).*

$$\tilde{\alpha}_L: \quad \left(r + s^L\right) \frac{\gamma}{q(\theta^{opt})} = (1 - \tilde{\alpha}_L) \left[\Phi - z - \frac{\sigma(r+s)}{\psi(\xi^*)} \frac{1}{\alpha_G}\right] - \tilde{\alpha}_L \gamma \theta^{opt} \tag{24}$$

All locations below the curve correspond to a decentralized labor market allocation with $\theta^* > \theta^{opt}$. All locations above the curve correspond to decentralized allocations with $\theta^* < \theta^{opt}$. The intersection of both curves, which corresponds to $\alpha_L = \eta_L$ and $\alpha_G = \eta_G$, is the constrained efficient allocation in which $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$. Thus all possible deviations of allocations away from the constrained efficient can be represented in (α_G, α_L) space. We thus have the four regimes, depending on which market is either too tight or too slack relative to the socially efficient, as a function of market power in each market.

planner was prevented from affecting goods market tightness because consumer search effort was inelastic. Therefore, only the labor market could be constrained efficient, with the condition $\alpha_L = \eta_L$, for constant value of goods market tightness. Here, the endogenous entry of consumers leads an entry condition in the goods market. Thus two conditions instead of one are required to reach efficiency, in order to the internalize the trading externalities on both the labor and the goods markets.

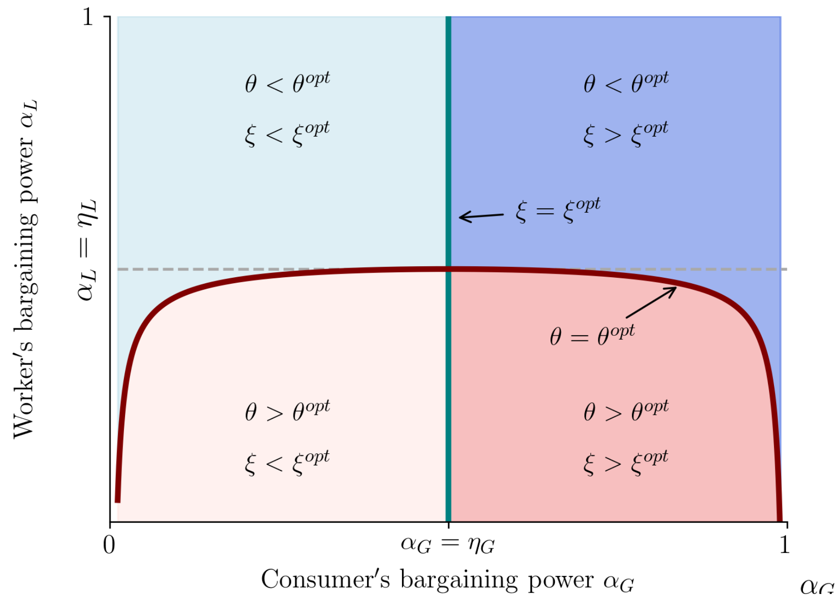


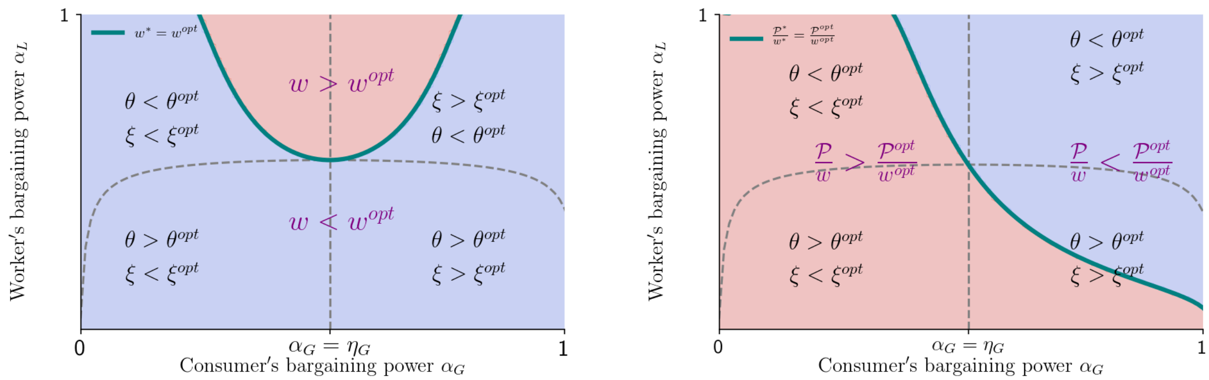
Figure 5: Market power and the efficiency of tightness in the goods and labor markets
The tight and slack labor market regimes are separated by the iso-labor market tightness condition in labor and goods market bargaining space (24) evaluated at θ^{opt}. The goods market efficiency condition $\alpha_G = \eta_G$ separates tight and slack goods markets.

Market power, wages, and markups

In all four tightness regimes, the price in the goods market is either too high or too low but never both. This is not the case for wages (Figure 6a). Allocations with insufficient job creation ($\theta^* < \theta^{opt}$) can arise when the wage is either too high or too low. In the standard model of equilibrium unemployment this only occurs when the wage is too high. In the presence of product market frictions this can occur even with a low wage the further the equilibrium in the goods market departs from the constrained efficient allocation. This is because the expected marginal revenue of labor $\mathcal{R}(\xi, \mathcal{P})$ is maximized at $\xi^* = \xi^{opt}$. Allocations with excessive job creation, $\theta^* > \theta^{opt}$, only arise with wages that are too low.

The markup of price over wage is strictly decreasing in labor's market power α_L in setting the wage, and in the consumer's market power α_G in setting the price in the goods market. A greater worker bargaining weight α_L allows labor to obtain a higher wage and the is no change in the price, resulting in a lower markup. Greater consumer bargaining power results in a lower price. If α_G increases towards η_G the wage increases as well. If α_G increases beyond η_G the wage declines, but less than the fall in the price.

The efficiency of the markup of price over wage is presented in Figure 6b. A tight labor market in which firms have excess market power setting the price in the goods market always has a markup that is inefficiently high. The markup is always inefficiently low in the mirror regime of a slack labor market with a tight goods market and too much market power for consumers. The situation is mixed in regimes where the efficiency of tightness in both the labor and goods markets go in the same direction.



(a) Wage and market power

(b) Mark up and market power

Figure 6: Market power and the efficiency of markups and wages.
Regions in which the decentralized equilibrium features an inefficiently high outcome of interest are in red. Inefficiently low decentralized values are in blue. The loci of bargaining strengths for which $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$ are represented by the dashed lines.

5 Constrained efficiency and stability in response to changes in demand

We begin by deriving the effect of a demand shock on aggregate activity, and establish that the elasticity of the marginal revenue of labor increases as market power deviates from the efficient division of rents in the goods market, and the double Hosios condition minimizes the response of labor market tightness to changes in demand.

5.1 Shocks to demand and the marginal revenue of labor

We consider a shock to preferences that changes the demand for goods. In particular, a drop in the disutility σ leads consumers to demand more goods at the prevailing price, tilting the demand curve up. The same shock tilts the price curve down because consumers are in a better position bargaining on the price since they are more willing to shop for other sellers.[16] Thus a drop in σ in our notion of a positive demand shock because it results in an increased relative demand ξ in the goods market.

Proposition 7. *The elasticity of goods market tightness ξ to a change in consumer demand is minimized at the good market Hosios condition $\alpha_G = \eta_G$.*

Proof: The elasticity of ξ to a decline in the cost of searching in the goods market $-\mathrm{d}\sigma$,

$$\frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi} = \frac{\Phi}{(1-\eta_G)\,\Phi + \eta_G \mathcal{R}} > 1 \tag{25}$$

[16] This is illustrated in Figure 12 of the appendix.

is positive and strictly greater than 1 as $\Phi > \mathcal{R}$. As $\alpha_G = \eta_G$ maximizes the marginal revenue $\mathcal{R}$, it also minimizes the elasticity of goods market tightness to a demand shock.[17] □

Proposition 8. *A positive demand shock increasing raises marginal revenue $\mathcal{R}(\xi, \mathcal{P})$ through both the trading and price effects, and the elasticity of the marginal revenue to changes in demand is minimized at the good market Hosios condition $\alpha_G = \eta_G$.*

Proof: Recall the definition of the expected marginal revenue $\mathcal{R}(\xi, \mathcal{P}) = \mu(\xi)\mathcal{P}$. A positive demand shock makes it simultaneously easier to find a customer base ($\mu(\xi)$ increases following an increase in ξ), and the price paid for the good $\mathcal{P}$ increases. Formally, the elasticity of the marginal revenue is given by:

$$\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{R}} \quad = \quad \frac{\mathrm{d}\mu}{-\mathrm{d}\sigma}\frac{\sigma}{\mu} + \frac{\mathrm{d}\mathcal{P}}{-\mathrm{d}\sigma}\frac{\sigma}{\mathcal{P}} = \alpha_G \frac{\eta_G\left(\Phi - \mathcal{R}\right)}{\left(1-\eta_G\right)\Phi + \eta_G\mathcal{R}} > 0 \tag{26}$$

where $\frac{\mathrm{d}\mu}{-\mathrm{d}\sigma}\frac{\sigma}{\mu} = \eta_G\left(\frac{r+s}{r+s+\lambda}\right)\frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi} > 0$ is the trading effect, and $\frac{\mathrm{d}\mathcal{P}}{-\mathrm{d}\sigma}\frac{\sigma}{\mathcal{P}} = \left(\frac{\sigma\xi}{\Phi+\sigma\xi}\right)\left(1 + \frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi}\right) > 0$ is the price effect. A positive demand shock thus always increases the marginal revenue of labor. Moreover, the elasticity of the marginal revenue to a demand shock, which is decreasing in the $\mathcal{R}$ in (26), increases away from the goods market efficiency condition $\alpha_G = \eta_G$. □

When firms have a high degree of market power in price setting ($\alpha_G < \eta_G$), consumer demand and the rate of capacity utilization $\mu(\xi)$ are very elastic to a demand shock, leading to a high elasticity of the marginal product $\mathcal{R}$. When firms have little market power ($\alpha_G > \eta_G$), the response of the price is highly elastic to a demand shock, driving a larger elasticity of the marginal product $\mathcal{R}$ to a change in demand.

5.2 Labor market responses to changes in demand

Proposition 9. *The elasticity of labor market tightness to a change in demand is minimized at the double Hosios condition in the goods and labor markets, $\alpha_G = \eta_G$ and $\alpha_L = \eta_L$. Deviations from constrained efficiency in either the goods or labor market result in a greater elasticity of labor market tightness to a change in demand.*

Proof: The elasticity of labor market tightness to a demand shock is the product of the elasticity of θ to a change in the marginal revenue of labor and the elasticity of $\mathcal{R}$ to the change in demand:

$$\frac{\mathrm{d}\theta}{-\mathrm{d}\sigma}\frac{\sigma}{\theta} \quad = \quad \left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right) \times \left(\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{R}}\right) = \frac{\mathcal{R}}{\eta_L\left(\mathcal{R}-z\right) + \left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta} \times \left(\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{R}}\right) \tag{27}$$

The first term, the elasticity of labor market tightness to the change in $\mathcal{R}$, has a familiar form (see Shimer 2005, for instance), and is minimized at the labor market constrained efficiency condition $\alpha_L = \eta_L$ for a given goods market power α_G. This arises from the fact that the term $\frac{\alpha_L}{1-\alpha_L}\theta$ is maximized at the labor market Hosios condition, as shown in section C.3 of the appendix. Moreover,

[17] Appendix section D provides the detailed derivations this section. The results of this section go through with a change in the marginal utility Φ as the source of changes in demand.

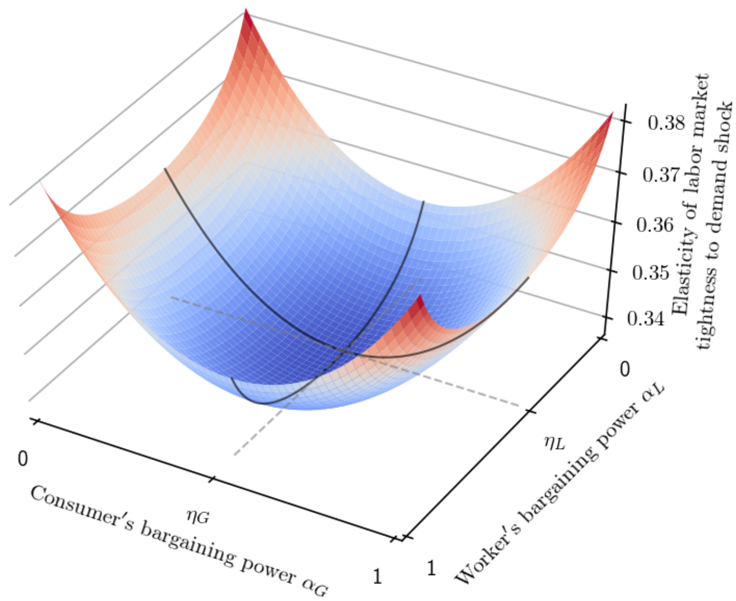


Figure 7: Market power and the elasticity of labor market tightness to changes in demand

deviations from the Hosios condition in the goods market reduce the labor match surplus, effectively the gap between $\mathcal{R}$ and the flow value of non-employment z in the denominator of (27). This in itself increases the elasticity of labor market tightness to marginal revenue, as does the lower θ resulting from deviations away from $\alpha_G = \eta_G$. $\square$

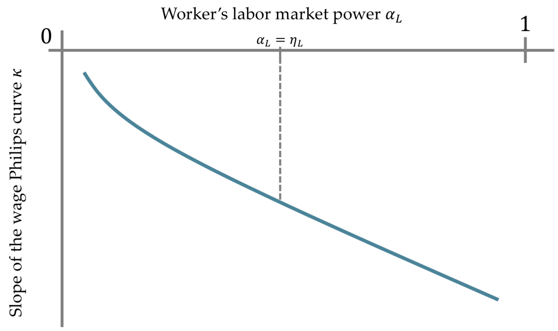
The sensitivity of the labor market tightness to demand shocks is illustrated in Figure 7 (for a numerical solution of the model at a steady state). The solid black curves indicate the loci for either $\alpha_G = \eta_G$ or $\alpha_L = \eta_L$. The figure illustrates how deviating from either the labor or the goods market constrained efficiency conditions leads to a greater elasticity of labor market tightness and hence job creation relative to the elasticity at the social planner's allocation.

Finally, the elasticities of the wage and unemployment to a demand shock, the responses the underly a wage Phillips curved discussed in the next section, are:

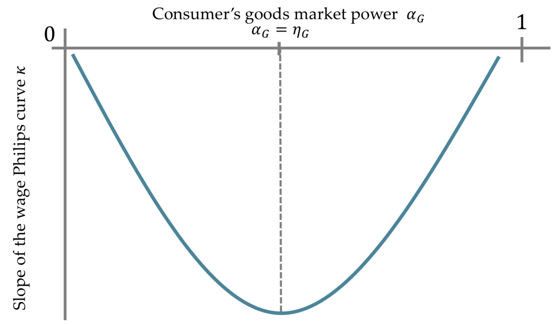
$$\frac{\mathrm{d}w}{\text{-d}\sigma}\frac{\sigma}{w} = \frac{\alpha_L}{w}\left[\mathcal{R} + \gamma\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right)\right]\left(\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{R}}\right) > 0 \tag{28}$$

$$\frac{\mathrm{d}\mathcal{U}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{U}} = -(1-\eta_L)\mathcal{N}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right)\left(\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma}\frac{\sigma}{\mathcal{R}}\right) < 0 \tag{29}$$

The response of the wage is proportional to the worker's wage setting market power α_L. With a lower α_L, workers receive a smaller fraction of the increase in their marginal revenue $\mathcal{R}$ and obtain less wage increases from a tightening labor market θ. Changes in goods market power α_G affect the wage response through its impact on the response of the marginal revenue $\mathcal{R}$ and labor market



(a) Labor market power



(b) Goods market power

Figure 8: Market power and the slope of the wage Phillips curve

tightness, as discussed above. As a result it is lowest at the constrained efficient allocation in the goods market, or when $\alpha_G = \eta_G$. The response of the rate of unemployment operates through changes in the demand for labor θ and its impact on the job finding rate out of unemployment. As a result, the response of the unemployment rate increases as workers' wage bargaining power α_L weakens, given a degree of goods market power α_G. For a given level of labor market power α_L, in contrast, the response of unemployment to the demand shock is weakest at the constrained efficiency condition $\alpha_G = \eta_G$.

5.3 A wage Phillips curve when goods and labor markets are frictional

The wage Phillips curve (Phillips 1958), a negative correlation between wage and unemployment growth, is at the heart of the inflation-output trade off faced by policymakers to the extent that wage growth underpins inflationary pressures. A flat curve implies less inflationary pressure from wages to a given expansion in economic activity. We study the relation of the slope of the wage Phillips curve to market power in the goods and labor markets and show the properties derived here also hold along the dynamic path of the economy, out of steady state.

The slope of the Phillips curve and the role of market power

The slope of the wage Phillips curve may be defined as the relative response of wages and the rate of unemployment to a demand shock. That is, it is defined as $\kappa \equiv \hbar_{w,\sigma}/\hbar_{\mathcal{U},\sigma}$, where $\hbar_{w,\sigma} \equiv \frac{\mathrm{d}w}{\text{-d}\sigma}$ and $\hbar_{\mathcal{U},\sigma} \equiv \frac{\mathrm{d}\mathcal{U}}{\text{-d}\sigma}$. The model's slope of a wage Phillips curve is thus given by:

$$\kappa \quad \equiv \quad -\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{\left[\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\right)^{-1}+\gamma\theta\right]}{\mathcal{U}\mathcal{N}}<0 \tag{30}$$

Proposition 10. *Given a level of consumer market power in price bargaining a reduction of worker's market power in wage bargaining always flattens the wage Phillips curve. In turn, under the sufficient condition that $\mathcal{U} \leq \mathcal{N}$, the slope of the wage Phillips curve (30) is steepest at the constrained efficient allocation in the goods market, and then flattens away from $\alpha_G = \eta_G$, given a level of worker market power in the wage bargaining.*

Proof: The effect of the labor market power is the known result in models of equilibrium unemployment that a weakening of worker's wage bargaining power results in a wage that is less responsive, and an unemployment rate that is more responsive, to economic shocks. The effect of the consumer's market power on the slope of the wage Phillips curve comes from the analysis of $\left[\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\right)^{-1}+\gamma\theta\right]/(\mathcal{U}\mathcal{N})$. Let $A=\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\right)^{-1}+\gamma\theta=\eta_L(\mathcal{R}-z)+\left(1+(1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta>0$, and $B=\mathcal{U}\mathcal{N}>0$. Given previous propositions on the elasticity of labor market tightness and the marginal revenue $\mathcal{R}$, it can be shown that at the good market Hosios condition $\alpha_G=\eta_G$ we have (see appendix section D for additional details):

$$
\begin{aligned}
\frac{\partial \frac{A}{B}}{\partial \alpha_G} &= \frac{A'B-AB'}{B^2}=0\\
\frac{\partial^2 \frac{A}{B}}{\partial \alpha_G^2} &= \frac{A''B-AB''}{B^2}=\frac{\left[\eta_L\frac{\partial^2\mathcal{R}}{\partial\alpha_G^2}+\left(1+(1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\frac{\partial^2\theta}{\partial\alpha_G^2}\right]B-A(\mathcal{N}-\mathcal{U})\frac{\partial^2\mathcal{U}}{\partial\alpha_G^2}}{B^2}
\end{aligned}
$$

As established earlier, both the marginal revenue $\mathcal{R}$ and labor market tightness θ are increasing and concave in α_G, and are maximized at $\alpha_G=\eta_G$. In addition, from the fact that $\frac{\partial^2\mathcal{R}}{\partial\alpha_G^2},\frac{\partial^2\theta}{\partial\alpha_G^2}<0$ it follows that $\frac{\partial^2\mathcal{U}}{\partial\alpha_G^2}<0$. Therefore $(\mathcal{N}-\mathcal{U})>0$ is a sufficient condition for $\frac{\partial^2\frac{A}{B}}{\partial\alpha_G^2}<0$ and the slope of the wage Phillips curve being most pronounced with respect to goods market power when $\alpha_G=\eta_G$. □

Changes in labor market power in favor of the firm, a reduction in α_L, always flatten the wage Phillips curve by reducing the wage response to an increase in demand and, at the same time, increasing the response of the rate of unemployment. Workers receive a smaller fraction of the increase in their marginal revenue $\mathcal{R}$ and obtain fewer wage increases from a tightening labor market θ, thereby creating a larger response in the demand for labor and reduction in the rate of unemployment. If the change in α_L is towards the socially efficient wage (α_L is declining from above η_L) then a flattening curve is welfare improving. This is not the case when the worker's bargaining power is already below the socially efficient division of rents in the labor market (see Figure 8a).[18]

Changes in goods market power α_G away from the socially efficient division of rents flatten the wage Phillips curve. Therefore, in contrast to the markup of prices over wages that increases monotonically with firms' market power in goods price setting, a flattening of wage Phillips curve indicates a departure from efficient rent sharing in either the consumer's or the producer's favor. There are two sources of the increase in the unemployment response $\mathrm{d}\mathcal{U}/\mathrm{d}\sigma$ away from $\alpha_G=\eta_G$. The negative response of unemployment to the change in demand increases more than the rise in the response of a wage, which is now closer to the worker's (fixed) reservation wage (see Figure 8b).

The joint relation between goods and labor market power to the slope of the wage Phillips curve is presented in Figure 9. The conditions $\alpha_G=\eta_G$, minimizing the slope of the wage Phillips curve κ for a given degree of labor market power α_L is plotted as the solid teal curve. As second

[18] Proposition 10 could be extended to the slope of the Phillips curve defined in terms of ratios of elasticities. In that case, the Phillips curve slopes as $\kappa^{elasticity}\equiv\varphi_{w,\sigma}/\varphi_{\mathcal{U},\sigma}=-\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{\left[\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\right)^{-1}+\gamma\theta\right]}{w\mathcal{N}}<0$, where $\varphi_{w,\sigma}\equiv\frac{\mathrm{d}w}{\text{-}\mathrm{d}\sigma}\frac{\sigma}{w}$ and $\varphi_{\mathcal{U},\sigma}\equiv\frac{\mathrm{d}\mathcal{U}}{\text{-}\mathrm{d}\sigma}\frac{\sigma}{\mathcal{U}}$, still defined in equations (28) and (29). The results are similar to Proposition 7: the Phillips curves always reaches an optimum in Hosios in the goods market. This corresponds to a minimum in our parameter values and more generally in a large portion of the parameter space. However, we could not find a simple sufficient condition like the one in Proposition 7 and provide details in Online Appendix G for a less simple sufficient condition and a discussion.

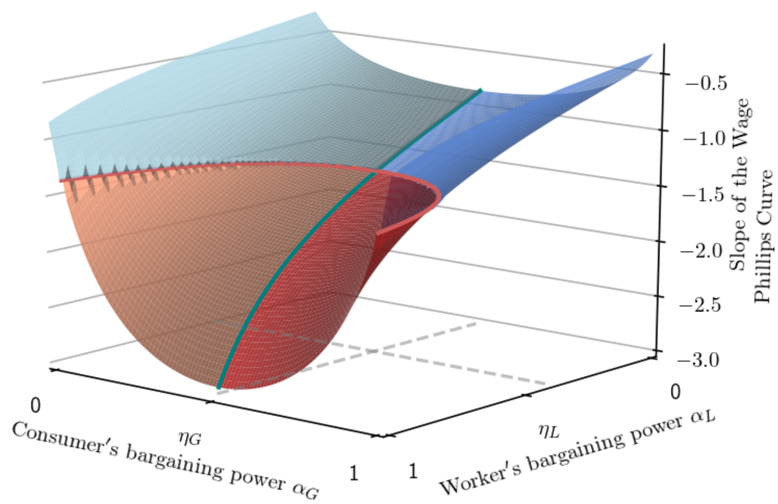


Figure 9: Goods and Labor market power and the slope of the wage Phillips curve
Notes: numerical solution, see appendix for parameter values

(red) solid curve represent an iso-slope of the wage Phillips curve in market power space and in particular, the two solid curves intersect at the level corresponding to the constrained efficient allocation achieved at $\alpha_G = \eta_G$ and $\alpha_L = \eta_L$. This iso-κ curve also corresponds to the combination of α_L and α_G that divide of the labor market slack regimes relative to θ^{opt} discuss in section 4.2. As such, in regimes with excess slack in the labor market, the slope of the Phillips curve is flatter than is socially efficient, while in regimes with excess tightness in the labor market the Phillips curve is too steep.

In contrast to New Keynesian wage Phillips curve there is a clear separation between the cost of adjusting employment at the firm and labor's market power (see Galí 2011, for example). In a New Keynesian environment, where market power is defined as the markup of the wage over the marginal rate of substitution between leisure and consumption, it is entirely determined by the firm's production technology and, in particular, the substitutability of differentiated labor services. A decline in the substitutability of labor services results in a greater gap between the wage and the marginal rate of substitution of the worker, interpreted as an increase in the worker's bargaining power. In addition, a New Keynesian wage Phillips curve requires a friction in resetting the wage without which the slope is infinite. Lastly, the elasticity of matching in the labor market η_L affects the slope of the wage Phillips curve in a manner similar to a decline in the substitutability of labor inputs in a New Keynesian environment.

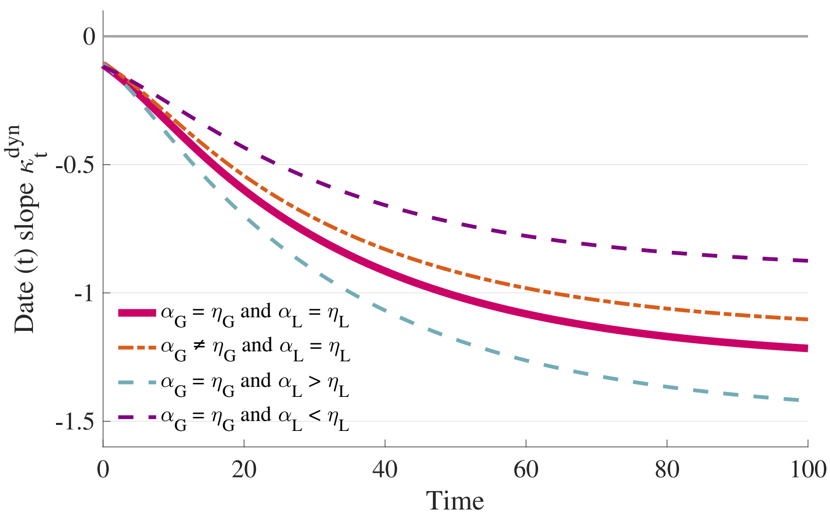


Figure 10: Dynamic wage Phillips curve κ_t

Dynamics

We now analyze the dynamics of the model in continuous time, extending the Bellman equations to include time derivatives. We further assume that wages and prices are fixed at the time of creation of the relevant match and therefore do not change for 'older' pairs after the shock. The adjustment of market tightness θ and ξ following a one-time shock on σ is instantaneous due to the free-entry of consumers and firms, and the fact that price-setters take into account the new value of σ when they bargain with consumers. We denote by a star the instantaneous values of labor and goods market tightness as a function of the current value of σ_t: $\theta^*(\sigma_t)$ and $\xi^*(\sigma_t)$.[19]

The dynamics of adjustment to a shock is characterized by the gradual evolution of the state variables of the system. Their law of motions are:

$$\dot{\mathcal{U}} = s^L\left(1 - \mathcal{U}_t\right) - \theta^* q(\theta^*)\mathcal{U}_t \tag{31}$$

$$\dot{\mathcal{N}_\pi} = 1 - \dot{\mathcal{N}_G} - \dot{\mathcal{U}} = \mathcal{N}_{Gt}\lambda(\xi^*) - (s_L + s_G)\mathcal{N}_\pi \tag{32}$$

while at any time the identities $\mathcal{C}_t = \mathcal{N}_{\pi t}$ and $\mathcal{N}_{gt} = 1 - \mathcal{U}_t - \mathcal{N}_{\pi t}$ hold. The number of newly job opened and filled jobs after a permanent shock at time 0 is a_t, and evolves according to:

$$\dot{a}_t = \theta^* q(\theta^*)\mathcal{U}_t - s^L a_t$$

which we track in order to calculate the fraction of firms having adjusted their wage $\aleph_t = a_t/(\mathcal{N}_{gt} + \mathcal{N}_{\pi t})$. This fraction evolves from 0 at the time of the shock to 1 in the new-steady-state, and allows us to calculate the average wage in the economy.

The dynamics of the Phillips curve, therefore, can be characterized as follows. First, the slope of new wages w_t^{new} to a one-time change in σ is the value of $\varphi_{w^{new},\sigma} = \frac{\partial w^*}{\partial \sigma}$ previously calculated where w^* is the steady-state value of the wage. Second, the elasticity of average wages $w^e =$

[19] Appendix E contains the derivation of the entire dynamic system and demonstrates the instantaneous adjustment of θ_t and ξ_t to $\theta^*(\sigma_t)$ and $\xi^*(\sigma_t)$ respectively.

$\aleph_t w^{new} + (1 - \aleph_t) w^{old}$ to a one-time change in σ is

$$\frac{\partial w^e}{\partial \sigma} = \left(\aleph_t \frac{\partial w^{new}}{\partial \sigma} + (1 - \aleph_t) \times 0 \right) = \aleph_t \frac{\partial w^{new}}{\partial \sigma}$$

Finally, the instantaneous slope of the Phillips curve, at a point in time, is given by:

$$\kappa_t^{dyn} = \left. \frac{dw^e}{d\mathcal{U}_t} \right|_{\sigma_t} = \frac{\frac{\partial w^e}{\partial \sigma}}{\partial \mathcal{U}^* / d\sigma + (\mathcal{U}_t - \mathcal{U}_t^*)} = \frac{\aleph_t \frac{\partial w^{new}}{\partial \sigma}}{d\mathcal{U}^* / d\sigma + (\mathcal{U}_t - \mathcal{U}_t^*)} \tag{33}$$

using that the variation of $\mathcal{U}_t$ arises from both the variation of the final steady-state value and the convergence to it: $d\mathcal{U}_t = \partial \mathcal{U}^* / d\sigma + (\mathcal{U}_t - \mathcal{U}_t^*)$. The instantaneous Phillips wage curve is will be rather flat close the to initial shock as very few firms have adjusted their wage in the first periods, and will slowly converges to its maximum value as wages adjust over time with job turnover.[20]

The results confirm the steady state analytical results of the previous section carry through to the transitional dynamics. For a given worker's bargaining weight in setting wages α_L, the goods market Hosios condition $\alpha_G = \eta_G$ minimizes the slope of the wage Phillips curve. This can be seen comparing the red (for $\alpha_G = \eta_G$) and dashed orange ($\alpha_G \neq \eta_G$) paths in Figure 10. For a given degree of rent sharing in the goods market, the instantaneous slope of the Phillips curve for $\alpha_L < \eta_L$ (purple line) is flatter than $\alpha_L > \eta_L$ (blue line).

6 Conclusion

When frictions affect several markets, inefficiency in one market may affect the efficiency of outcomes in the other markets. In our case, the optimal tightness of the goods market maximizes the marginal revenue of firms and its demand for labor conditional on a wage. The efficient wage then balances the social benefits of an additional job against the cost of turnover in the labor market. This double Hosios condition also minimizes the economy's response to a demand shock. Moreover, the relative response of wages and unemployment to changes in demand — the wage Phillips curve — can be explicitly related to market power in setting wages and prices. Rising firm market power in the labor market flattens the wage Phillips curve. This flattening is inefficient only if firms already extract too much surplus from a match with labor. Rising firm market power in the goods market flattens the wage Phillips curve only if firms are extracting too much surplus from consumers. Otherwise it actually steepens the response of wages relative to that of unemployment following a change in aggregate demand. This departs from the standard New Keynesian framework in which changes in markups are not related to a changing output-wage inflation trade-off. As such, we present a mechanism by which changes in market power in either goods or labor market markets would affect inflation pressures from wage growth, and the trade off faced by a monetary authority trying to respond to demand shocks.

[20]The dynamics are solved for and simulated using a simple ODE system for a given set of parameters described in appendix E.3. The time to convergence of the state-variables is of the order of magnitude of 1 year (time period is monthly), and the key parameter for the speed of the dynamics and adjustments to wages and of the Phillips curve is therefore the rate of turnover s_L.

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Appendix

A Social planner's problem

This section provides the detailed derivations for Section 3. Recall the social planner's program:

$$
\begin{aligned}
H = \max_{D_U,\mathcal{V},\mathcal{U},\mathcal{C}} \quad & e^{-rt}\left[\Phi\mathcal{C} + z\mathcal{U} - \gamma\mathcal{V} - \sigma D_U\right] \\
& +\Psi_{\mathcal{C}}\left[\left(s^G + s^L\right)\mathcal{C} - M_G\left(D_U, 1-\mathcal{U}-\mathcal{C}\right)\right] \\
& +\Psi_U\left[M_L\left(\mathcal{U},\mathcal{V}\right) - s^L\left(1-\mathcal{U}\right)\right]
\end{aligned}
$$

using $\frac{\partial M_G}{\partial \mathcal{U}} = \frac{\partial M_G}{\partial \mathcal{N}_G}\frac{\partial \mathcal{N}_G}{\partial \mathcal{U}}$, $\frac{\partial M_G}{\partial \mathcal{C}} = \frac{\partial M_G}{\partial \mathcal{N}_G}\frac{\partial \mathcal{N}_G}{\partial \mathcal{C}}$, and the following properties of the matching functions: $\frac{\partial M_L}{\partial \mathcal{U}} = \eta_L f(\theta)$; $\frac{\partial M_G}{\partial D_U} = \eta_G \psi(\xi)$; $\frac{\partial M_L}{\partial \mathcal{V}} = (1-\eta_L)q(\theta)$; $\frac{\partial M_G}{\partial \mathcal{N}_G} = (1-\eta_G)\lambda(\xi)$. The first order conditions are:

$$\frac{\partial H}{\partial D_U} = 0 \quad \rightarrow \quad e^{-rt}(-\sigma) - \Psi_{\mathcal{C}}\eta_G\psi = 0 \tag{A.1}$$

$$\frac{\partial H}{\partial \mathcal{V}} = 0 \quad \rightarrow \quad e^{-rt}(-\gamma) + \Psi_U\left(1-\eta_L\right)q = 0 \tag{A.2}$$

$$\frac{\partial H}{\partial \mathcal{C}} = -\dot{\Psi}_{\mathcal{C}} \quad \rightarrow \quad e^{-rt}\Phi + \Psi_{\mathcal{C}}\left[\left(s^G+s^L\right) + \left(1-\eta_G\right)\lambda\right] = -\dot{\Psi}_{\mathcal{C}} \tag{A.3}$$

$$\frac{\partial H}{\partial \mathcal{U}} = -\dot{\Psi}_{\mathcal{U}} \quad \rightarrow \quad e^{-rt}z + \Psi_{\mathcal{U}}\left(\eta_L f + s^L\right) + \Psi_{\mathcal{C}}\left(1-\eta_G\right)\lambda = -\dot{\Psi}_{\mathcal{U}} \tag{A.4}$$

A.1 Constrained efficient goods market tightness

To obtain the efficient goods market tightness, begin with (A.1):

$$\Psi_{\mathcal{C}} = -\frac{e^{-rt}\sigma}{\eta_G\psi} \quad \text{and} \quad \dot{\Psi}_{\mathcal{C}} = \frac{re^{-rt}\sigma}{\eta_G\psi}$$

Combining with (A.3) we have

$$
\begin{aligned}
e^{-rt}\Phi + \Psi_{\mathcal{C}}\left[(s^G+s^L) + (1-\eta_G)\lambda(\theta)\right] &= -\dot{\Psi}_{\mathcal{C}} \\
e^{-rt}\Phi - \frac{e^{-rt}\sigma}{\eta_G\psi}\left[(s^G+s^L) + \left(1-\eta_G\right)\lambda\right] &= \frac{re^{-rt}\sigma}{\eta_G\psi} \\
\eta_G\Phi - \sigma\xi\left(1-\eta_G\right) &= \frac{\sigma}{\psi}\left(r+s\right)
\end{aligned}
$$

A.2 Constrained efficient labor market tightness

Solving for Ψ_U in (A.2) we obtain:

$$\Psi_U = \frac{e^{-rt}\gamma}{q\left(1-\eta_L\right)} \quad \text{and} \quad \dot{\Psi}_U = \frac{-re^{-rt}\gamma}{q\left(1-\eta_L\right)}$$

and in combination with (A.4), we have:

$$\begin{aligned}
-\dot{\Psi}_{\mathcal{U}} &= e^{-rt}z + \Psi_{\mathcal{U}}\left(\eta_L f + s^L\right) + \Psi_{\mathcal{C}}\left(1-\eta_G\right)\lambda \\
\frac{-re^{-rt}\gamma}{q\left(1-\eta_L\right)} &= e^{-rt}z + \frac{e^{-rt}\gamma}{q\left(1-\eta_L\right)}\left(\eta_L f + s^L\right) - \frac{e^{-rt}\sigma}{\eta_G\psi}\left(1-\eta_G\right)\lambda \\
\frac{-r\gamma}{q\left(1-\eta_L\right)} &= z + \frac{\gamma}{q\left(1-\eta_L\right)}\left(\eta_L f + s^L\right) - \frac{\sigma}{\eta_G\psi}\left(1-\eta_G\right)\lambda \\
\frac{\left(r+s^L\right)\gamma}{q\left(1-\eta_L\right)} &= \frac{\sigma}{\eta_G\psi}\left(1-\eta_G\right)\lambda - z - \frac{\eta_L\gamma f}{q\left(1-\eta_L\right)} \\
\frac{\left(r+s^L\right)\gamma}{q} &= \left(1-\eta_L\right)\left[\frac{\sigma}{\eta_G\psi}\left(1-\eta_G\right)\lambda - z\right] - \eta_L\gamma\theta \\
\frac{\left(r+s^L\right)\gamma}{q} &= \left(1-\eta_L\right)\left[\sigma\xi\left(\frac{1-\eta_G}{\eta_G}\right) - z\right] - \eta_L\gamma\theta
\end{aligned}$$

Note that from the equilibrium condition for goods market tightness we have that $\sigma\xi\left(\frac{1-\eta_G}{\eta_G}\right) = \Phi - \frac{\sigma}{\eta_G\psi(\xi)}\left(r+s\right)$. This delivers the social planner's job creation condition (15) in the text.

A.3 Planner's goods market allocation and the expected marginal revenue of labor

We are looking to establish that marginal revenue of labor $\mathcal{R}(\xi,\mathcal{P})$ is maximized at the planner's choice for goods market tightness ξ^{opt}. Recall the marginal revenue function

$$\mathcal{R}(\xi,\mathcal{P}) = \frac{\lambda}{r+s+\lambda}\mathcal{P}$$

It can be expressed as a function of the decentralized tightness of the goods market alone by using the consumer entry condition (11):

$$\frac{\sigma\left(r+s\right)}{\psi} = \Phi - \mathcal{P} \quad \Rightarrow \quad \mathcal{P} = \Phi - \frac{\sigma\left(r+s\right)}{\psi}$$

and hence

$$\hat{\mathcal{R}}(\xi) = \frac{\lambda}{r+s+\lambda}\left(\Phi - \frac{\sigma\left(r+s\right)}{\psi}\right)$$

Taking the first derivative of $\hat{\mathcal{R}}(\xi)$ and setting it to zero, and making use of $\psi = \lambda/\xi$; $\frac{\partial\lambda}{\partial\xi} = \eta_G\frac{\lambda}{\xi}$; $\frac{\partial\psi}{\partial\xi} = \left(\eta_G - 1\right)\frac{\psi}{\xi}$, we have:

$$\frac{\partial\hat{\mathcal{R}}(\xi)}{\partial\xi} = \frac{\frac{\partial\lambda}{\partial\xi}\left(\Phi - \frac{\sigma(r+s)}{\psi}\right)}{r+s+\lambda} + \frac{\lambda\frac{\partial\psi}{\partial\xi}}{r+s+\lambda}\frac{\sigma(r+s)}{\psi^2} - \frac{\lambda\frac{\partial\lambda}{\partial\xi}\left(\Phi - \frac{\sigma(r+s)}{\psi}\right)}{(r+s+\lambda)^2}$$

and

$$\begin{aligned}\frac{\partial \hat{\mathcal{R}}(\xi)}{\partial \xi} = 0 &= \eta_G \left(\Phi - \frac{\sigma(r+s)}{\psi} \right) \left(\frac{r+s}{r+s+\lambda} \right) + (\eta_G - 1) \frac{\sigma(r+s)}{\psi} \\ 0 &= \eta_G \Phi - \frac{\sigma(r+s)}{\psi} + (\eta_G - 1) \frac{\sigma}{\psi} \lambda\end{aligned}$$

which yields

$$\frac{\sigma(r+s)}{\psi} = \eta_G \Phi - (1 - \eta_G) \sigma \xi$$

This corresponds to the planner's equilibrium condition for goods market tightness and hence ξ^{opt} maximizes the marginal revenue $\mathcal{R}$.

A.4 Steady state planner's problem

Consider the planner's objective $\Phi\mathcal{C} + z\mathcal{U} - \gamma\mathcal{V} - \sigma D_U$ at a steady state, and setting $r = 0$. From the laws of motion we have directly that $\mathcal{C} = \mathcal{N}_\pi$. Next, from the law of motion for $\mathcal{C}$ at a steady state:

$$M_G = s\mathcal{C} \to \lambda \mathcal{N}_G = s\mathcal{N}_\pi \to \mathcal{N}_G = \frac{s}{\lambda}\mathcal{N}_\pi$$

and then from the identity $\mathcal{N}_G + \mathcal{N}_\pi = 1 - \mathcal{U}$ we have

$$\mathcal{N}_\pi = \frac{\lambda}{\lambda + s}(1 - \mathcal{U})$$

Recall the definition of goods market tightness $\xi = D_U/\mathcal{N}_G$. Since $\mathcal{N}_G = \frac{s}{\lambda}\mathcal{N}_\pi$ and $\psi = \lambda/\xi$, we have:

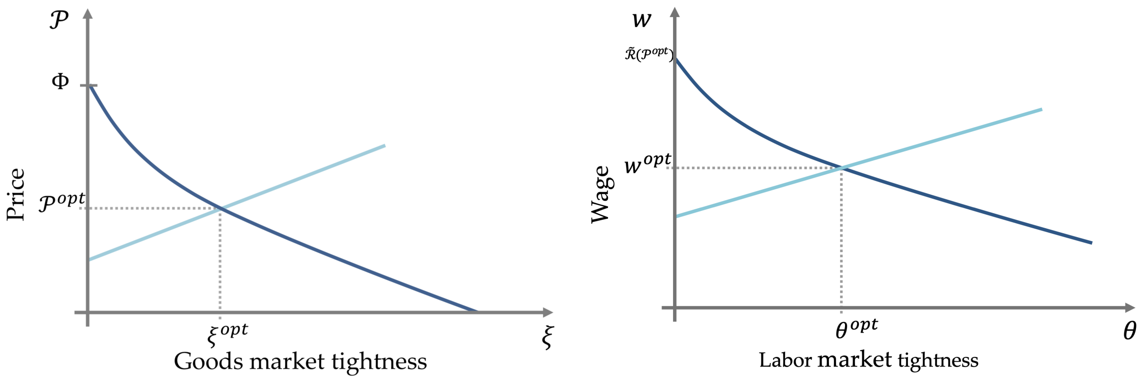
$$D_U = \xi \mathcal{N}_G = \xi \frac{s}{\lambda} \mathcal{N}_\pi = \frac{s}{\psi} \mathcal{N}_\pi$$

We can now restate the planner's problem as

$$(1 - U)\frac{\lambda}{\lambda + s}\left(\Phi - \frac{\sigma}{\psi}s\right) + z\mathcal{U} - \gamma\mathcal{V}$$

Notice that $\frac{\lambda}{\lambda+s}\left(\Phi - \frac{\sigma}{\psi}s\right) \equiv \hat{\mathcal{R}}(\xi)$ is the firm's revenue $\hat{\mathcal{R}}(\xi)$ we derived for the earlier proof. The planner is indeed maximizing the firm's revenue.

A.5 Additional charts



(a) Goods market equilibrium with the consumer entry condition (11) and price rule (16)

(b) Labor market equilibrium with job creation condition (13) and wage rule (17).

Figure 11: Price and wage rules implementing the constrained efficient allocation in goods and labor market

B Decentralized equilibrium

In what follows we allow the wage paid to workers differ when the firm is searching in the goods market and selling to and denote them, respectively, w_g and w_π. In this general setting the asset values for employment distinguish the two states for the firm are denoted W_{eg} and $W_{e\pi}$.

The consumer, worker, and firm Bellman equations are reproduced here for convenience:

$$\begin{aligned}
rW_{D_U} &= -\sigma + \psi\left(W_{\mathcal{C}} - W_{D_U}\right) && \text{(A.5)}\\
rW_{\mathcal{C}} &= (\Phi - \mathcal{P}) + s^G\left(W_{D_U} - W_{\mathcal{C}}\right) + s^L\left(W_{D_U} - W_{\mathcal{C}}\right) && \text{(A.6)}\\
rW_{e\pi} &= w_\pi + s^L\left(W_u - W_{e\pi}\right) + s^G\left(W_{eg} - W_{e\pi}\right) && \text{(A.7)}\\
rW_{eg} &= w_g + s^L\left(W_u - W_{eg}\right) + \lambda\left(W_{e\pi} - W_{eg}\right) && \text{(A.8)}\\
rW_u &= z + f(\theta)\left(W_{eg} - W_u\right) && \text{(A.9)}\\
rJ_v &= -\gamma + q(\theta)\left(J_g - J_v\right) && \text{(A.10)}\\
rJ_g &= -w_g + \lambda\left(J_\pi - J_g\right) + s^L\left(J_v - J_g\right) && \text{(A.11)}\\
rJ_\pi &= \mathcal{P} - w_\pi + s^G\left(J_g - J_\pi\right) + s^L\left(J_v - J_\pi\right) && \text{(A.12)}
\end{aligned}$$

The above Bellman equations lead to expressions for the consumer and firm private surpluses in a goods market match:

$$\begin{aligned}
W_{\mathcal{C}} - W_{D_U} &= \frac{(\Phi - \mathcal{P}) - rW_{D_U}}{r+s} && \text{(A.13)}\\
J_\pi - J_g &= \frac{\mathcal{P} - w_\pi - \left(r + s^L\right)J_g + s^L J_v}{r+s} && \text{(A.14)}
\end{aligned}$$

A worker's and the firm's surplus generated from forming a match in the labor market are, respectively:

$$
\begin{aligned}
W_{eg} - W_u &= \frac{w_g + \lambda\left(W_{e\pi} - W_{eg}\right) - rW_u}{r + s^L} \\
J_g - J_v &= \frac{-w_g + \lambda\left(J_\pi - J_g\right)}{r + s^L}
\end{aligned}
$$

The worker's surplus is the present discounted value of the wage w_g, plus the change in match surplus $\left(W_{e\pi} - W_{eg}\right)$ when a consumer is found, net of the worker's outside option rW_u. The firm's surplus is the present discounted value of the difference between the wage paid to the worker and the expected gain once a customer has been found, $\left(J_\pi - J_g\right)$.

The match surpluses between a worker and firm, once a match in the goods market is formed and the firm moves to the profit making state, gain respectively:

$$
W_{e\pi} - W_{eg} = \frac{w_\pi - w_g}{r + s + \lambda} \tag{A.15}
$$

$$
J_\pi - J_g = \frac{\mathcal{P} - \left(w_\pi - w_g\right)}{r + s + \lambda} \tag{A.16}
$$

A worker gains the present value of any change in the bargained wage. A firm gains the present value of the revenue from selling its good net of any change in the cost of labor.

B.1 Price and wage determination: benchmark assumptions

Our benchmark assumption is $w_g = w_\pi = \omega$, in which case $rW_{eg} = rW_{e\pi} = rW_e = w + s^L\left(W_u - W_e\right)$.

Price bargaining

Starting from the sharing rule

$$
\left(1 - \alpha_G\right)\left(W_C - W_{D_U}\right) = \alpha_G\left(J_\pi - J_g\right) \tag{A.17}
$$

the left-hand side can be replaced with equation (A.13), and the right-hand side with (A.14):

$$
\left(1 - \alpha_G\right)\left(\Phi - \mathcal{P}\right) = \alpha_G\left[\mathcal{P} - \omega + s^L\left(J_v - J_g\right)\right]
$$

Re-arranging, we obtain an expression for the price:

$$
\mathcal{P} = \left(1 - \alpha_G\right)\Phi + \alpha_G\left[\omega + s^L\left(J_g - J_v\right)\right]
$$

We further have, using (8),

$$
\begin{aligned}
s^L(J_g - J_v) &= -\omega + \lambda\left(J_\pi - J_g\right) = -\omega + \lambda\frac{1 - \alpha_G}{\alpha_G}\left(W_C - W_{D_U}\right) \\
&= -\omega + \lambda\frac{1 - \alpha_G}{\alpha_G}\frac{\sigma}{\psi(\xi)} = -\omega + \frac{1 - \alpha_G}{\alpha_G}\sigma\xi
\end{aligned}
$$

such that one obtains:

$$\mathcal{P} \quad = \quad (1-\alpha_G)\left[\Phi+\sigma\xi\right]$$

Wage bargaining

The first order condition to the wage setting problem is:

$$\frac{\partial\left(W_e-W_u\right)}{\partial\omega}=\frac{1}{r+s^L}=-\frac{\partial\left(J_g-J_v\right)}{\partial\omega}$$

and the simple Nash-bargaining rule applies:

$$(1-\alpha_L)\left(W_e-W_u\right)=\alpha_L\left(J_g-J_v\right) \tag{A.18}$$

The wage is therefore the solution to

$$\begin{aligned}(1-\alpha_L)\left[\omega-rW_u\right] &= \alpha_L\left[-\omega+\lambda\left(J_\pi-J_g\right)\right]\\ \omega &= (1-\alpha_L)\,rW_u+\alpha_L\lambda\left(J_\pi-J_g\right)\end{aligned}$$

From the wage bargaining sharing rule and the firm's free-entry we have $rW_u=z+f(\theta)\left(W_e-W_u\right)=z+\gamma\theta\alpha_L/\left(1-\alpha_L\right)$. Moreover, the firm's surplus can be expressed as $J_\pi-J_g=\mathcal{P}/\left(r+s+\lambda\right)$, such that we have the wage rule (with $\mu=\lambda/\left(r+s+\lambda\right)$):

$$\omega=(1-\alpha_L)\,z+\alpha_L\left[\gamma\theta+\mu\mathcal{P}\right]$$

B.2 Equilibrium conditions

Starting from the labor market entry condition:

$$\begin{aligned}\left(r+s^L\right)\frac{\gamma}{q} &= -w_g+\lambda\left(J_\pi-J_g\right)\\ \left(r+s^L\right)\frac{\gamma}{q} &= -w_g+\lambda\left(\frac{\mathcal{P}-\left(w_\pi-w_g\right)}{r+s+\lambda}\right)\\ \left(r+s^L\right)\frac{\gamma}{q} &= \frac{\lambda\mathcal{P}-\lambda\left(w_\pi-w_g\right)-\left(r+s+\lambda\right)w_g}{r+s+\lambda}\\ \left(r+s^L\right)\frac{\gamma}{q} &= \frac{\lambda\left(\mathcal{P}-w_\pi\right)-\left(r+s\right)w_g}{r+s+\lambda}\\ \left(r+s^L\right)\frac{\gamma}{q} &= \frac{\lambda\mathcal{P}-\left(\lambda w_\pi+\left(r+s\right)w_g\right)}{r+s+\lambda}\\ \left(r+s^L\right)\frac{\gamma}{q} &= \mu\mathcal{P}-\omega\end{aligned}$$

where $\omega=\frac{\lambda w_\pi+(r+s)w_g}{r+s+\lambda}$, and simply $\omega=w$ in the constant wage profile case. Inserting wage rule:

$$\left(r+s^L\right)\frac{\gamma}{q(\theta)} \quad = \quad (1-\alpha_L)\left[\mu\mathcal{P}-z\right]-\alpha_L\gamma\theta$$

Next we show that the expected marginal revenue of labor $\mathcal{R}(\xi,\mathcal{P}) = \mu\mathcal{P}$ can be re-expressed in two ways.[21] First, from the consumer entry condition, we have:

$$\begin{aligned}
\frac{\sigma}{\psi} &= \frac{\alpha_G}{1-\alpha_G}\left(J_\pi - J_g\right) \\
\frac{\sigma}{\lambda}\xi &= \frac{\alpha_G}{1-\alpha_G}\frac{\mathcal{P}}{r+s+\lambda} \\
\mu\mathcal{P} &= \frac{1-\alpha_G}{\alpha_G}\sigma\xi
\end{aligned}$$

and second, again from the consumer entry condition, we have:

$$\begin{aligned}
\frac{\sigma}{\psi} &= \frac{\alpha_G\Phi - (1-\alpha_G)\,\sigma\xi}{r+s} \\
(r+s)\,\frac{\sigma}{\alpha_G\psi} &= \Phi - \frac{1-\alpha_G}{\alpha_G}\sigma\xi \\
\frac{1-\alpha_G}{\alpha_G}\sigma\xi &= \Phi - \frac{\sigma\,(r+s)}{\psi}\frac{1}{\alpha_G} = \mu\mathcal{P}
\end{aligned}$$

C Elasticities to changes in Demand

C.1 Response of goods market tightness to a demand shock

Starting from the good market entry condition $\frac{\sigma(r+s)}{\psi} = \alpha_G\Phi - (1-\alpha_G)\,\xi\sigma$ we have:

$$\begin{aligned}
\frac{r+s}{\psi}d\sigma - \frac{\sigma\,(r+s)}{\psi^2}\frac{\partial\psi}{\xi}d\xi &= -\,(1-\alpha_G)\,(\sigma d\xi + \xi d\sigma) \\
\left(\frac{\sigma\,(r+s)}{\psi} + (1-\alpha_G)\,\sigma\xi\right)\frac{d\sigma}{\sigma} &= -\left((1-\eta_G)\,\frac{\sigma\,(r+s)}{\psi} + (1-\alpha_G)\,\sigma\xi\right)\frac{d\xi}{\xi} \\
\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} &= -\left(\frac{(1-\alpha_G)\,\sigma\xi + \frac{\sigma(r+s)}{\psi}}{(1-\alpha_G)\,\sigma\xi + (1-\eta_G)\,\frac{\sigma(r+s)}{\psi}}\right) < -1 \qquad \text{(A.19)}
\end{aligned}$$

which, with a little more substitutions yields:

$$\begin{aligned}
\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} &= -\left(\frac{(1-\alpha_G)\,\sigma\xi + \alpha_G\Phi - (1-\alpha_G)\,\sigma\xi}{(1-\alpha_G)\,\sigma\xi - (1-\eta_G)\,[\alpha_G\Phi - (1-\alpha_G)\,\sigma\xi]}\right) \\
\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} &= -\left(\frac{\alpha_G\Phi}{\alpha_G\,(1-\eta_G)\,\Phi + \eta_G\,(1-\alpha_G)\,\sigma\xi}\right) \\
\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} &= -\left(\frac{\Phi}{(1-\eta_G)\,\Phi + \eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right)\sigma\xi}\right) \\
\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} &= -\left(\frac{\Phi}{(1-\eta_G)\,\Phi + \eta_G\mathcal{R}}\right) < -1
\end{aligned}$$

[21] The following derivations make use of $\frac{\sigma}{\psi} = W_{\mathcal{C}} - W_{D_U}$; $W_{\mathcal{C}} - W_{D_U} = \frac{\alpha_G}{1-\alpha_G}\left(J_\pi - J_g\right)$; $\left(J_\pi - J_g\right) = \frac{\mathcal{P}-\left(w_\pi - w_g\right)}{r+s+\lambda}$; $\frac{\sigma}{\psi} = \frac{(\Phi-\mathcal{P})}{r+s}$ $w_\pi = \alpha_L\mathcal{P} + w_g$; $\mathcal{P} = (1-\alpha_G)\,(\Phi+\sigma\xi) + \alpha_G\left(w_\pi - w_g\right) = (1-\alpha_G)\,(\Phi+\sigma\xi) + \alpha_G\alpha_L\mathcal{P}$.

$\mathcal{R}$ is maximized at the efficiency condition $\alpha_G = \eta_G$. It follows that elasticity of goods market tightness to a demand shock is minimized at $\alpha_G = \eta_G$ and is equal to $\frac{d\xi}{d\sigma}\frac{\sigma}{\xi} = -\left(\frac{\Phi}{(1-\eta_G)(\Phi+\sigma\xi^{opt})}\right) < -1$.

C.2 Response of the marginal revenue to a demand shock

Begin with the elasticity of the price

$$\begin{aligned}
\mathcal{P} &= (1-\alpha_G)(\Phi+\sigma\xi) \\
\mathrm{d}\mathcal{P} &= (1-\alpha_G)(\sigma\mathrm{d}\xi + \xi(-\mathrm{d}\sigma)) \\
\mathrm{d}\mathcal{P} &= (1-\alpha_G)\,\sigma\xi\left(\frac{\mathrm{d}\xi}{\xi} + \frac{(-\mathrm{d}\sigma)}{\sigma}\right) \\
\frac{\mathrm{d}\mathcal{P}}{-\mathrm{d}\sigma}\frac{\sigma}{\mathcal{P}} &= \frac{(1-\alpha_G)\,\sigma\xi}{\mathcal{P}}\left(\frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi}+1\right) \\
\frac{\mathrm{d}\mathcal{P}}{-\mathrm{d}\sigma}\frac{\sigma}{\mathcal{P}} &= \frac{\sigma\xi}{(\Phi+\sigma\xi)}\left(1+\frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi}\right) > 0
\end{aligned}$$

and the elasticity of μ:

$$\begin{aligned}
\mu &= \frac{\lambda}{r+s+\lambda} \\
\mathrm{d}\mu &= \left[\frac{\frac{\partial\lambda}{\partial\xi}}{r+s+\lambda} - \frac{\lambda\frac{\partial\lambda}{\partial\xi}}{(r+s+\lambda)^2}\right]\frac{\partial\xi}{\partial\sigma}(-d\sigma) \\
&= \left[\frac{\frac{\eta_G}{\xi}\lambda}{r+s+\lambda} - \frac{\frac{\eta_G}{\xi}\lambda^2}{(r+s+\lambda)^2}\right]\frac{\partial\xi}{\partial\sigma}(-d\sigma) \\
\frac{\mathrm{d}\mu}{-\mathrm{d}\sigma} &= \eta_G\left(\frac{\lambda}{r+s+\lambda}\right)\left(1-\frac{\lambda}{r+s+\lambda}\right)\frac{\partial\xi}{\partial\sigma}\frac{1}{\xi} \\
\frac{\mathrm{d}\mu}{-\mathrm{d}\sigma}\frac{\sigma}{\mu} &= \eta_G\left(\frac{r+s}{r+s+\lambda}\right)\frac{\mathrm{d}\xi}{-\mathrm{d}\sigma}\frac{\sigma}{\xi} > 0
\end{aligned}$$

Using $\mathcal{R} = \mu\mathcal{P}$ and $\mathcal{P} = \left(\Phi - \frac{(r+s)}{\psi}\sigma\right)$, we first have:

$$\begin{aligned}
\mathrm{d}\mathcal{R} &= \mathcal{P}\frac{\partial\mu}{\partial\xi}\frac{\partial\xi}{\partial\sigma}\mathrm{d}\sigma - \mu\frac{(r+s)}{\psi}\mathrm{d}\sigma + \mu\frac{(r+s)}{\psi^2}\sigma\frac{\partial\psi}{\partial\xi}\frac{\partial\xi}{\partial\sigma}\mathrm{d}\sigma \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\sigma &= \mathcal{P}\frac{\partial\mu}{\partial\xi}\frac{\partial\xi}{\partial\sigma}\sigma + \mu\frac{(r+s)}{\psi^2}\sigma(\eta_G-1)\frac{\psi}{\xi}\frac{\partial\xi}{\partial\sigma}\sigma - \mu\frac{(r+s)}{\psi}\sigma \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \mathcal{P}\frac{\partial\mu}{\partial\xi}\frac{\partial\xi}{\partial\sigma}\frac{\sigma}{\mu\mathcal{P}} + \frac{\mu}{\mu\mathcal{P}}\frac{(r+s)}{\psi^2}\sigma(\eta_G-1)\frac{\psi}{\xi}\frac{\partial\xi}{\partial\sigma}\sigma - \frac{\mu}{\mu\mathcal{P}}\frac{(r+s)}{\psi}\sigma \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\partial\mu}{\partial\xi}\frac{\xi}{\mu}\right)\left(\frac{\partial\xi}{\partial\sigma}\frac{\sigma}{\xi}\right) - \frac{(1-\eta_G)}{\mathcal{P}}\frac{(r+s)\,\sigma}{\psi}\left(\frac{\partial\xi}{\partial\sigma}\frac{\sigma}{\xi}\right) - \frac{(r+s)}{\psi}\frac{\sigma}{\mathcal{P}} \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\partial\xi}{\partial\sigma}\frac{\sigma}{\xi}\right)\left[\left(\frac{\partial\mu}{\partial\xi}\frac{\xi}{\mu}\right) - (1-\eta_G)\frac{(r+s)\,\sigma}{\psi\mathcal{P}}\right] - \frac{(r+s)}{\psi}\frac{\sigma}{\mathcal{P}}
\end{aligned}$$

Recall that $\frac{\partial \mu}{\partial \xi}\frac{\xi}{\mu} = \eta_G\left(\frac{r+s}{r+s+\lambda}\right)$ such that:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left[\eta_G\left(\frac{r+s}{r+s+\lambda}\right) - (1-\eta_G)\frac{(r+s)\,\sigma}{\psi\mathcal{P}}\right] - \frac{(r+s)}{\psi}\frac{\sigma}{\mathcal{P}} \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= (r+s)\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left[\left(\frac{\eta_G}{r+s+\lambda}\right) - (1-\eta_G)\frac{\sigma}{\psi\mathcal{P}}\right] - \frac{\sigma}{\psi\mathcal{P}}\right]
\end{aligned}$$

Now use the fact that $\left(\frac{1}{r+s+\lambda}\right) = \frac{\mathcal{R}}{\lambda\mathcal{P}}$ and $\frac{\sigma}{\psi} = \frac{\alpha_G}{1-\alpha_G}\frac{\mathcal{R}}{\lambda}$:[22]

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= (r+s)\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left[\eta_G\left(\frac{\mathcal{R}}{\lambda\mathcal{P}}\right) - (1-\eta_G)\left(\frac{\alpha_G}{1-\alpha_G}\right)\frac{\mathcal{R}}{\lambda\mathcal{P}}\right] - \left(\frac{\alpha_G}{1-\alpha_G}\right)\frac{\mathcal{R}}{\lambda\mathcal{P}}\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \frac{(r+s)\,\mathcal{R}}{\lambda\mathcal{P}}\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left[\eta_G - (1-\eta_G)\left(\frac{\alpha_G}{1-\alpha_G}\right)\right] - \left(\frac{\alpha_G}{1-\alpha_G}\right)\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \frac{(r+s)\,\mathcal{R}}{\lambda\mathcal{P}}\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left(\frac{\eta_G-\alpha_G}{1-\alpha_G}\right) - \left(\frac{\alpha_G}{1-\alpha_G}\right)\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \frac{(r+s)\,\mathcal{R}}{\lambda\mathcal{P}}\left(\frac{\alpha_G}{1-\alpha_G}\right)\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left(\frac{\eta_G-\alpha_G}{\alpha_G}\right) - 1\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \frac{\Phi-\mathcal{P}}{\mathcal{P}}\left[\left(\frac{\partial \xi}{\partial \sigma}\frac{\sigma}{\xi}\right)\left(\frac{\eta_G-\alpha_G}{\alpha_G}\right) - 1\right]
\end{aligned}$$

where we used $\frac{(r+s)}{\mathcal{P}}\left(\frac{\alpha_G}{1-\alpha_G}\right)\frac{\mathcal{R}}{\lambda} = \frac{(r+s)}{\mathcal{P}}\frac{\sigma}{\psi} = \frac{(r+s)}{\mathcal{P}}\frac{\Phi-\mathcal{P}}{r+s} = \frac{\Phi-\mathcal{P}}{\mathcal{P}}$. Recall that $\mathcal{P} = (1-\alpha_G)(\Phi+\sigma\xi)$ to obtain:

$$\begin{aligned}
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\alpha_G\Phi - (1-\alpha_G)\,\sigma\xi}{(1-\alpha_G)(\Phi+\sigma\xi)}\right)\left[\left(\frac{-\Phi}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right)\left(\frac{\eta_G-\alpha_G}{\alpha_G}\right) - 1\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\Phi - \left(\frac{1-\alpha_G}{\alpha_G}\right)\sigma\xi}{\left(\frac{1-\alpha_G}{\alpha_G}\right)(\Phi+\sigma\xi)}\right)\left[\left(\frac{\Phi}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right)\left(\frac{\alpha_G-\eta_G}{\alpha_G}\right) - 1\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\Phi-\mathcal{R}}{\left(\frac{1-\alpha_G}{\alpha_G}\right)\Phi+\mathcal{R}}\right)\left[\left(\frac{\Phi}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right)\left(\frac{\alpha_G-\eta_G}{\alpha_G}\right) - 1\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\alpha_G(\Phi-\mathcal{R})}{(1-\alpha_G)\,\Phi+\alpha_G\mathcal{R}}\right)\left[\left(\frac{(\alpha_G-\eta_G)\,\Phi}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right) - \alpha_G\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\alpha_G(\Phi-\mathcal{R})}{(1-\alpha_G)\,\Phi+\alpha_G\mathcal{R}}\right)\left[\frac{(\alpha_G-\eta_G)\,\Phi - \alpha_G\left[(1-\eta_G)\,\Phi+\eta_G\mathcal{R}\right]}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\alpha_G(\Phi-\mathcal{R})}{(1-\alpha_G)\,\Phi+\alpha_G\mathcal{R}}\right)\left[\frac{-\eta_G\Phi+\alpha_G\eta_G\Phi+\alpha_G\eta_G\mathcal{R}}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= \left(\frac{\alpha_G(\Phi-\mathcal{R})}{(1-\alpha_G)\,\Phi+\alpha_G\mathcal{R}}\right)\left[\frac{-\eta_G\left[(1-\alpha_G)\,\Phi-\alpha_G\mathcal{R}\right]}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}\right] \\
\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}} &= -\alpha_G\frac{\eta_G(\Phi-\mathcal{R})}{(1-\eta_G)\,\Phi+\eta_G\mathcal{R}}
\end{aligned}$$

[22] $\frac{\sigma}{\psi} = \frac{\alpha_G}{1-\alpha_G}\left(J_\pi - J_g\right) = \frac{\alpha_G}{1-\alpha_G}\frac{\mathcal{P}}{r+s+\lambda} = \frac{\alpha_G}{1-\alpha_G}\frac{\mathcal{R}}{\lambda}$

C.3 Response of labor market tightness to a demand shock

Starting from the job creation condition:

$$
\begin{aligned}
\frac{\gamma\left(r+s^L\right)}{q} &= \left(1-\alpha_L\right)\left(\mathcal{R}-z\right)-\alpha_L\gamma\theta \\
-\frac{\gamma\left(r+s^L\right)}{q^2}\frac{\partial q}{\partial\theta}\mathrm{d}\theta &= \left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}\mathrm{d}\sigma-\alpha_L\gamma\mathrm{d}\theta \\
\left[\frac{\eta_L}{\theta}\frac{\gamma\left(r+s^L\right)}{q}+\alpha_L\gamma\right]\mathrm{d}\theta &= \left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}\mathrm{d}\sigma \\
\left[\frac{\eta_L}{\theta}\left[\left(1-\alpha_L\right)\left(\mathcal{R}-z\right)-\alpha_L\gamma\theta\right]+\alpha_L\gamma\right]\mathrm{d}\theta &= \left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}\mathrm{d}\sigma \\
\left[\frac{\eta_L}{\theta}\left(1-\alpha_L\right)\left(\mathcal{R}-z\right)-\eta_L\alpha_L\gamma+\alpha_L\gamma\right]\mathrm{d}\theta &= \left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}\mathrm{d}\sigma \\
\left[\frac{\eta_L}{\theta}\left(1-\alpha_L\right)\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\alpha_L\gamma\right]\mathrm{d}\theta &= \left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}\mathrm{d}\sigma \\
\frac{\mathrm{d}\theta}{\mathrm{d}\sigma} &= \frac{\left(1-\alpha_L\right)\frac{\partial\mathcal{R}}{\partial\sigma}}{\frac{\eta_L}{\theta}\left(1-\alpha_L\right)\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\alpha_L\gamma} \\
\frac{\mathrm{d}\theta}{\mathrm{d}\sigma} &= \frac{\theta}{\eta_L\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta}\frac{\partial\mathcal{R}}{\partial\sigma}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}} &= \frac{\theta}{\eta_L\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta} \\
\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta} &= \frac{\mathcal{R}}{\eta_L\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta}
\end{aligned}
$$

Minimization with respect to labor market power α_L

One has the job creation condition and elasticity of labor market tightness:

$$
\begin{aligned}
\left(r+s^L\right)\frac{\gamma}{q(\theta)} &= \left(1-\alpha_L\right)\left[\mathcal{R}-z\right]-\alpha_L\gamma\theta \\
\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right) &= \frac{\mathcal{R}}{\eta_L\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta}
\end{aligned}
$$

Establishing that the elasticity in the last equation is minimized at the labor market Hosios condition amounts to establishing the term in the denominator:

$$M(\alpha_L)=\frac{\alpha_L}{1-\alpha_L}\theta$$

is minimized at at $\alpha_L=\eta_L$.

The first is to show it is at an optimum w.r.t. α_L at $\alpha_L=\eta_L$. From here on let $M'=\partial M/\partial\alpha_L$. Begin by writing the first derivative as

$$M' \quad = \quad \frac{1}{1-\alpha_L}\left[\alpha_L\theta'+\frac{\theta}{1-\alpha_L}\right] \tag{A.20}$$

where $\theta' = \partial\theta/\partial\alpha_L$ and note that if $\alpha_L = \eta_L$ is an optimum, then at that point $M' = 0$ and from (A.20)

$$\frac{\theta'}{\theta} = -\frac{1}{\eta_{L(1-\eta_L)}} \quad \text{at} \quad \alpha_L = \eta_L$$

Now, working from the job creation condition we can obtain:

$$\begin{aligned}
\theta' &= \frac{-\left[\mathcal{R} - z + \gamma\theta\right]}{\left(1-\alpha_L\right)\left[\mathcal{R} - z\right]\frac{\eta_L}{\theta} + \alpha_L\left(1-\eta_L\right)\gamma} \\
\frac{\theta'}{\theta} &= \frac{-\left[\mathcal{R} - z + \gamma\theta\right]}{\eta_L\left(1-\alpha_L\right)\left[\mathcal{R} - z\right] + \alpha_L\left(1-\eta_L\right)\gamma\theta}
\end{aligned}$$

which at $\alpha_L = \eta_L$ is equal to:

$$\frac{\theta'}{\theta} = \frac{-1}{\eta_L\left(1-\eta_L\right)}$$

As a result $M' = 0$ at $\alpha_L = \eta_L$.

The second step is to show that $M'' < 0$ at at $\alpha_L = \eta_L$. Begin with the second derivative:

$$\begin{aligned}
M'' &= \frac{\alpha_L}{(1-\alpha_L)}\theta'' + \frac{2\theta'}{(1-\alpha_L)^2} + \frac{\theta}{(1-\alpha_L)^3} \\
M'' &= \frac{\alpha_L}{(1-\alpha_L)}\theta'' + \frac{\theta}{(1-\alpha_L)^2}\left(\frac{2\theta'}{\theta} + \frac{1}{(1-\alpha_L)}\right)
\end{aligned}$$

where $\theta'' < 0$, $\theta' < 0$ and $\theta > 0$. Evaluating M'' at $\alpha_L = \eta_L$, and making use of $\frac{\theta'}{\theta} = \frac{-1}{\eta_L(1-\eta_L)}$, we have

$$M''(\alpha_L = \eta_L) = \frac{\eta_L}{(1-\eta_L)}\theta'' + \frac{\theta}{(1-\eta_L)^3}\left(\frac{-2}{\eta_L} + 1\right) < 0$$

C.4 Wage response of a demand shock

Recall and take the first derivative of the wage equation with respect to a change in σ:

$$\begin{aligned}
w &= \alpha_L\left[\mathcal{R} + \gamma\theta\right] + \left(1-\alpha_L\right)z \\
\frac{\mathrm{d}w}{\mathrm{d}\sigma} &= \alpha_L\left[\frac{\partial\mathcal{R}}{\partial\sigma} + \gamma\frac{\partial\theta}{\partial\sigma}\right] \\
&= \alpha_L\left[1 + \gamma\frac{\partial\theta}{\partial\mathcal{R}}\right]\frac{\partial\mathcal{R}}{\partial\sigma} \\
\frac{\mathrm{d}w}{\text{-}\mathrm{d}\sigma}\frac{\sigma}{w} &= \frac{\alpha_L}{w}\left[\mathcal{R} + \gamma\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right)\right]\left(\frac{\mathrm{d}\mathcal{R}}{\text{-}\mathrm{d}\sigma}\frac{\sigma}{\mathcal{R}}\right) > 0
\end{aligned}$$

C.5 Response of unemployment to a demand shock

Take the first derivative of the steady state rate of unemployment with respect to a change in σ:

$$
\begin{aligned}
\mathcal{U} &= \frac{s^L}{s^L + f(\theta)} \\
\frac{\mathrm{d}\mathcal{U}}{\mathrm{d}\sigma} &= -\frac{s^L}{\left[s^L + f(\theta)\right]^2} \frac{\partial f}{\partial \theta} \frac{\partial \theta}{\partial \mathcal{R}} \frac{\partial \mathcal{R}}{\partial \sigma} \\
&= -\frac{\mathcal{U} f}{\left[s^L + f(\theta)\right] \theta} \left(\frac{\partial f}{\partial \theta} \frac{\theta}{f} \right) \frac{\partial \theta}{\partial \mathcal{R}} \frac{\partial \mathcal{R}}{\partial \sigma} \\
&= -\frac{\mathcal{U}\mathcal{N}}{\theta} (1 - \eta_L) \frac{\partial \theta}{\partial \mathcal{R}} \frac{\partial \mathcal{R}}{\partial \sigma} \\
\frac{\mathrm{d}\mathcal{U}}{\mathrm{d}\sigma} &= -(1 - \eta_L) \Gamma(\theta) \frac{\partial \theta}{\partial \mathcal{R}} \frac{\partial \mathcal{R}}{\partial \sigma} \\
\frac{\mathrm{d}\mathcal{U}}{\text{-d}\sigma} \frac{\sigma}{\mathcal{U}} &= -(1 - \eta_L) \mathcal{N} \left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}} \frac{\mathcal{R}}{\theta} \right) \left(\frac{\mathrm{d}\mathcal{R}}{\text{-d}\sigma} \frac{\sigma}{\mathcal{R}} \right) < 0
\end{aligned}
$$

where $\Gamma(\theta) = \frac{\mathcal{U}\mathcal{N}}{\theta}$ with $\Gamma' < 0$, $\Gamma(\theta = 0) \to \infty$ and $\Gamma(\theta \to \infty) = 0$.

C.6 Additional charts

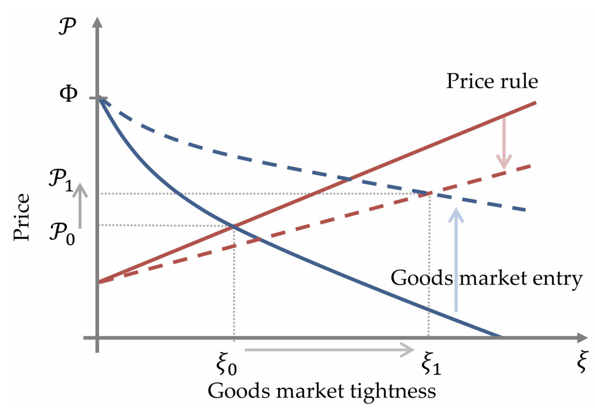


Figure 12: Goods market equilibrium following a positive demand shock

D Deriving the slope of a wage Phillips curve in steady-state

The slope of the wage Phillips curve is the value κ^{slope} can be defined as:

$$
\kappa^{slope} \equiv -\left(\frac{\alpha_L}{1 - \eta_L} \right) \frac{\left[\theta \left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}} \right)^{-1} + \gamma\theta \right]}{\mathcal{U}\mathcal{N}} < 0 \tag{A.21}
$$

The building blocks for this section are:

$$
\begin{aligned}
\mathcal{R} &= \mathcal{P}\mu \\
\frac{\sigma(r+s)}{\psi} &= \alpha_G \Phi - (1-\alpha_G)\xi\sigma \\
\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}} &= \frac{\theta}{\eta_L(\mathcal{R}-z) + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\gamma\theta} \\
w &= (1-\alpha_L)z + \alpha_L[\gamma\theta + \mathcal{R}(\xi,\mathcal{P})] \\
\mathcal{N} &= \frac{f(\theta)}{s^L + f(\theta)}
\end{aligned}
$$

and we proceed to establish whether

$$
\begin{aligned}
\left.\frac{\partial\kappa^{slope}}{\partial\alpha_G}\right|_{\alpha_G=\eta_G} &= 0 \\
\left.\frac{\partial^2\kappa^{slope}}{\partial\alpha_G^2}\right|_{\alpha_G=\eta_G} &> 0
\end{aligned}
$$

D.1 Establishing $\left.\frac{\partial\kappa^{slope}}{\partial\alpha_G}\right|_{\alpha_G=\eta_G} = 0$

From now on in this section, we denote by X' the first order derivative of the equilibrium variable X with respect to α_G and by $''$ the second derivative with respect to the same variable. This can be partial or total, and we use these notation if there is no confusion. We first establish that if $\alpha_G = \eta_G$, then $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}} = 0$.

Recall that $\mu = \frac{\lambda}{r+s+\lambda}$ and

$$
\frac{\partial\mu}{\partial\xi}\frac{\xi}{\mu} = \eta_G\left(\frac{r+s}{r+s+\lambda}\right) = \eta_G\frac{(r+s)\mathcal{R}}{\lambda\mathcal{P}} = \eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)
$$

using $\frac{(r+s)\mathcal{R}}{\lambda\mathcal{P}} = \left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{(r+s)\sigma}{\mathcal{P}\psi}\right) = \left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)$. Next, using the consumer entry condition to obtain

$$
\begin{aligned}
\mathcal{P}' &= \frac{(r+s)}{\psi^2}\frac{\partial\psi}{\partial\xi}\sigma\xi' = \frac{(r+s)}{\psi}\left(\frac{\partial\psi}{\partial\xi}\frac{\xi}{\psi}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\sigma}{\alpha_G} \\
&= \left(\frac{\eta_G-1}{\alpha_G}\right)\frac{(r+s)\sigma}{\psi}\left(\xi'\frac{\alpha_G}{\xi}\right) \\
&= \left(\frac{\eta_G-1}{\alpha_G}\right)(\Phi-\mathcal{P})\left(\xi'\frac{\alpha_G}{\xi}\right)
\end{aligned}
$$

Turning to $\mathcal{R}' = \frac{\partial\mathcal{R}}{\partial\alpha_G}$:

$$
\begin{aligned}
\mathcal{R} &= \mu\mathcal{P} \\
\mathcal{R}' &= \mu'\mathcal{P} + \mathcal{P}'\mu \\
\mathcal{R}' &= \left(\frac{\partial\mu}{\partial\xi}\frac{\xi}{\mu}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} + \mathcal{P}'\mu \\
\mathcal{R}' &= \eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} + (\eta_G - 1)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} \\
\mathcal{R}' &= \left[\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G}\right]\left[\frac{\eta_G-\alpha_G}{\alpha_G}\right] \\
&= \left[\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mathcal{R}}{\alpha_G}\right]\left[\frac{\eta_G-\alpha_G}{\alpha_G}\right] \\
\Leftrightarrow \mathcal{R}'\frac{\alpha_G}{\mathcal{R}} &= \frac{\eta_G-\alpha_G}{\alpha_G}\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)
\end{aligned}
$$

Result G-1. Optimum of revenues at Hosios in the goods market
If $\alpha_G = \eta_G$, then $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}} = 0$.

We can now turn to establishing κ^{slope} is also at an optimum at $\alpha_G = \eta_G$. For this, let $\theta\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\right)^{-1} + \gamma\theta = A$ and let $\mathcal{UN} = B$. Then,

$$\kappa^{slope} = -\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{A}{B}$$

Since $A = \eta_L(\mathcal{R} - z) + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta$, $A' = 0$ at $\alpha_G = \eta_G$ as both $\mathcal{R}$ and θ are at an optimum with respect to α_G. Likewise $B' = 0$ as $\mathcal{U}$ is at an optimum with respect to α_G. Therefore

$$\left.\frac{\partial A/B}{\partial\alpha_G}\right|_{\alpha_G=\eta_G} = \frac{A'B - AB'}{B^2} = 0$$

Result G-2. Optimum of κ^{slope} at Hosios in the goods market
$\left(\frac{A}{B}\right)' = \frac{A'B-AB'}{B^2} = 0$ at $\alpha_G = \eta_G$.
As supplementary results, we have the following equalities.
Result G-3. Optimum in (θ, N) at Hosios in the goods market
First, we have that, at $\alpha_G = \eta_G$,

$$\theta' = 0 \text{ and } \theta'' \leq 0$$
$$\mathcal{N}' = 0 \text{ and } \mathcal{N}'' \leq 0$$

The first result comes from the fact that θ is increasing in an concave in $\mathcal{R}$ from the job creation condition. The second one is obtained from the fact that $\mathcal{N}$ is monotonically increasing in θ so that

$$
\begin{aligned}
\mathcal{N}' &= \frac{\partial \mathcal{N}}{\partial \theta}\theta' \\
\mathcal{N}'' &= \frac{\partial^2 \mathcal{N}}{\partial \theta^2}\left(\theta'\right)^2 + \frac{\partial \mathcal{N}}{\partial \theta}\theta'' + \frac{\partial}{\partial \alpha_G}\left(\frac{\partial \mathcal{N}}{\partial \theta}\right)\theta' \\
&= 0 + \frac{\partial \mathcal{N}}{\partial \theta}\theta'' + 0
\end{aligned}
$$

D.2 Establishing $\left.\frac{\partial^2 \kappa^{slope}}{\partial \alpha_G^2}\right|_{\alpha_G=\eta_G} \geq 0$

The second derivative of the slope is studied at Hosios in the goods market, and therefore, given that first order derivatives are zero in that point,

$$
\begin{aligned}
(A/B)'' &= \frac{A''}{B} - \frac{AB''}{B^2} \\
&= \frac{BA'' - AB''}{B^2}
\end{aligned}
$$

We have

$$
\begin{aligned}
A'' &= \eta_L \mathcal{R} + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta'' \\
B'' &= \mathcal{U}''\mathcal{N} + \mathcal{U}\mathcal{N}'' = \mathcal{U}''\left(\mathcal{N} - \mathcal{U}\right)
\end{aligned}
$$

where the second line uses $\mathcal{N}'' = -\mathcal{U}''$. We then obtain

$$
BA'' - AB'' = \left[\eta_L \mathcal{R} + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta''\right]\mathcal{U}\mathcal{N} - A\mathcal{U}''\left(\mathcal{N} - \mathcal{U}\right)
$$

It is easy to verify that, since $\theta'' \leq 0$ and $\mathcal{U}'' \geq 0$, that the sign of the expression above is non-positive as soon as $\mathcal{N} \geq \mathcal{U}$. Hence, remembering that $\kappa^{slope} = -\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{A}{B}$ the second derivative of κ^{slope} with respect to α_G is positive and it corresponds to a minimum of the slope.

E Dynamics

E.1 New Bellman equations

To prove that the two tightness variables evolve immediately to their final value, let us start from the most general formulation in which prices and wages may vary over time within each firm. We will then assume that they are rigid and the dynamics will be greatly simplified. The Bellman equations have time-varying terms described below, beginning with workers:

$$rW_e = \omega_t + s^L (W_u - W_e) + \frac{\partial W_e}{\partial t} \tag{A.22}$$

$$rW_u = z + f(\theta_t) (W_e - W_u) + \frac{\partial W_u}{\partial t} \tag{A.23}$$

hence the surplus of workers with respect to the firm is:

$$\Sigma_W = W_e - W_u = \frac{\omega_t - z + \frac{\partial \Sigma_W}{\partial t}}{r + s^L + f(\theta_t)}$$

Turning to consumers:

$$rW_{D_U} = -\sigma_t + \psi(\xi_t) (W_C - W_{D_U}) + \frac{\partial W_{D_U}}{\partial t} = 0 \tag{A.24}$$

$$rW_C = (\Phi - \mathcal{P}_t) + s^G (W_{D_U} - W_C) + s^L (W_{D_U} - W_C) + \frac{\partial W_C}{\partial t} \tag{A.25}$$

such that the surplus of the consumer is:

$$\Sigma_C = W_C - W_{D_U} = W_C = \frac{\Phi - \mathcal{P}_t + \frac{\partial W_C}{\partial t}}{r + s + \psi(\xi_t)}$$

This also leads, with free entry in the goods market, to

$$\frac{\sigma_t}{\psi(\xi_t)} = \frac{\Phi - \mathcal{P}_t + \frac{\partial W_C}{\partial t}}{r + s} \tag{A.26}$$

Now, in this equation, the term $\frac{\partial W_C}{\partial t}$ arises from equation (A.25). Hence, absent any bubbles (e.g deviation from fundamentals), W_C only varies to the extent that existing prices do vary over time. So, if we assume that prices within a firm are set at a constant level, it follows that $\frac{\partial W_C}{\partial t} = 0$, and therefore ξ_t and σ_t are linked by the free-entry equation value of the price

$$\frac{\sigma_t}{\psi(\xi_t)} = \frac{\Phi - \mathcal{P}(\sigma_t)}{r + s}$$

where $\mathcal{P}(\sigma_t)$ is value of the price at the calendar time of the creation of the match between the consumer and the firm. Hence, $\xi_t = \xi^*(\sigma_t)$.

Next, looking at firms:

$$rJ_v = -\gamma + q_t (J_g - J_v) + \frac{\partial J_v}{\partial t} = 0 \tag{A.27}$$

$$rJ_g = -\omega_t + \lambda_t (J_\pi - J_g) + s^L (J_v - J_g) + \frac{\partial J_g}{\partial t} \tag{A.28}$$

$$rJ_\pi = \mathcal{P}_t - \omega_t + s^G (J_g - J_\pi) + s^L (J_v - J_\pi) + \frac{\partial J_\pi}{\partial t} \tag{A.29}$$

or deriving the firm's surplus with respect to consumers $\Sigma_{F,C} = J_\pi - J_g$:

$$\Sigma_{F,C} = \frac{\mathcal{P}_t + \frac{\partial \Sigma_{F,C}}{\partial t}}{r + s + \lambda_t}$$

while the firm's entry surplus with respect to workers is $\Sigma_{F,W} = J_g - J_v$ and then

$$\Sigma_{F,W} = \frac{-\omega_t + \gamma + \lambda_t \Sigma_{F,C} + \frac{\partial \Sigma_{F,W}}{\partial t}}{r + s^L + q_t}$$

E.2 Dynamic equilibrium

With

$$\mu(\xi_t) = \frac{\lambda(\xi_t)}{r + s + \lambda(\xi_t)} \tag{A.30}$$

and

$$\tilde{\mathcal{R}}_t(\mathcal{P}) = \mu(\xi_t)\left(\mathcal{P}_t + \frac{\partial J_\pi}{\partial t}\right)$$

$$\frac{\gamma}{q(\theta_t)} = \frac{\tilde{\mathcal{R}}_t(\mathcal{P}_t) - \omega_t}{r + s^L} \tag{A.31}$$

With a similar reasoning, $\frac{\partial J_\pi}{\partial t}$ varies only to the extent that wages and prices vary over the match. Assuming that wages are rigid for an existing firm, one has again that $\frac{\partial J_\pi}{\partial t} = 0$. Hence,

$$\frac{\gamma}{q(\theta_t)} = \frac{\mathcal{R}_t \mathcal{P}(\sigma_t) - \omega(\sigma_t)}{r + s^L}$$

and therefore, one has $\theta_t = \theta^*(\sigma_t)$.

Finally, given that time derivatives will all cancel out in bargaining, one still has for price and wages:

$$\mathcal{P}_t^{new} = (1 - \alpha_G)(\Phi + \sigma \xi^*(\sigma_t)) \tag{A.32}$$

and

$$\omega_t^{new} = (1 - \alpha_L) z + \alpha_L \left[\gamma \theta^*(\sigma_t) + \tilde{\mathcal{R}}_t(\mathcal{P}_t^{new})\right] \tag{A.33}$$

E.3 Solving and simulating the dynamic system

The dynamics are solved and simulated using Matlab's differential equations functions (@ode45) using the following parameter values: $r = 0.0038$, $\alpha_L = 0.5$, $\eta_L = 0.5$, $\chi_L = 0.5$, $s^L = 0.032$; $\gamma = 0.16$, $z = 0.9$; $\alpha_G = 0.5$, $\eta_G = 0.5$, $\chi_G = 0.5$, $s^G = 0.01$, $\Phi = 1$ and an initial value of σ of 0.05. For the purposes of the simulations we shock σ, increasing its value to 0.15.

Online Appendix

F Online Appendices

F.1 Extensions to alternative assumptions on price and wage setting

In our baseline, prices and wages are bargained independently, and the wage is bargained to apply both when the firm is searching and matched in the goods market. There are, however, several alternatives that take into account the sequential nature of meetings and strategic interactions. One could allow, in particular, for different outside options in the negotiation, and one could consider various strategic interactions in bargaining across different matches in the labor and the goods markets. That is, wages could be negotiated in each stage separately, with corresponding wage w_g and w_π, and could take into account future effects on prices in the goods market.

Our baseline assumption was a case in which wages are not bargained separately in stages g and π, an assumption we relax in Section F.1.[23] The first extension does not allow for strategic effects in wage bargaining. Agents do not anticipate how wages may affect the goods market match surplus and prices. This is explored separately in Section F.1.

In all cases considered the outside option in negotiating the wage in stage g is assumed to be the dissolution of the labor match. That is,

$$w_g \quad = \quad \text{argmax} \left(W_{eg} - W_u\right)^{\alpha_L} \left(J_g - J_v\right)^{1-\alpha_L} \tag{A.1}$$

Disagreement when bargaining over wage w_π leads each side to revert to its value in stage g instead of a full dissolution of the match. That is, the wage solves the bargaining problem

$$w_\pi \quad = \quad \text{argmax} \left(W_{e\pi} - W_{eg}\right)^{\alpha_L} \left(J_\pi - J_g\right)^{1-\alpha_L} \tag{A.2}$$

Wages bargained sequentially: no strategic interactions

This first extension does not allow for strategic interactions in bargaining and assumes that the wage in the entry stage w_g does not affect the subsequent wage w_π. Let $\tilde{\alpha}_G$ be given by the expression:

$$\tilde{\alpha}_G(\alpha_L, \alpha_G) = \alpha_G \times \left(\frac{1-\alpha_L}{1-\alpha_L\alpha_G}\right) \tag{A.3}$$

This is an effective consumer share of the goods market match surplus, after the worker has been paid the bargained wage. If $\alpha_L = 0$, meaning the firm in stage π receives the entire surplus of the match with the worker, none of the economic surplus from production is dissipated or transferred to labor. The consumer receives a share corresponding to its bargaining power, $\tilde{\alpha}_G = \alpha_G$. If instead $\alpha_L = 1$, the firm in stage π had no surplus from the match with labor. The effective consumer bargaining weight is then $\tilde{\alpha}_G = 0$ as the consumer receives no consumption surplus irrespective of its goods market power. All of the surplus from production has accrued to labor.

[23] In addition, we explicitly allow for a different bargaining power of workers in stage π, and denote it by α'_L in the appendix. Assuming $\alpha'_L = \alpha_L$ the wage rules are identical whether the outside option is a break up of the labor match in wage bargaining during stage π, giving the value J_v for the firm and W_U to the worker.

The effective consumer share $\tilde{\alpha}_G$ plays an important role in the equilibrium allocation, a pair (ξ, θ) that solves:[24]

$$(r+s)\,\frac{\sigma}{\psi(\xi)} \;=\; \tilde{\alpha}_G\Phi - (1-\tilde{\alpha}_G)\,\xi\sigma \tag{A.4}$$

$$\left(r+s^L\right)\frac{\gamma}{q(\theta)} \;=\; \left(1-\alpha_L'\right)\left[\Phi - \frac{(r+s)\,\sigma}{\psi(\xi)}\left(\frac{1}{\tilde{\alpha}_G}\right)\right] - (1-\alpha_L)\,z - \alpha_L\gamma\theta \tag{A.5}$$

It is then easy to derive that:

Proposition 11. *When wages are set separately at firms searching in the goods market and matched in the goods market, the decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if $\alpha_L = \eta_L$ and $\tilde{\alpha}_G(\alpha_L, \alpha_G) = \eta_G$, where the second equality is equivalent to $\alpha_G = \eta_G \times \frac{1}{1-\alpha_L(1-\eta_G)} \geq \eta_G$.*

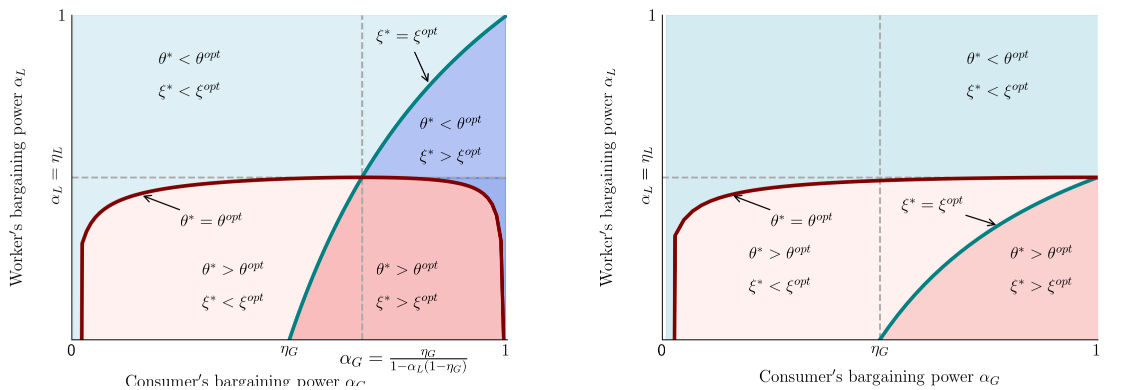
As in Section 4.2, the possible deviations of allocations away from the constrained efficient can be represented in (α_G, α_L) space. The condition for constrained efficiency in the goods market, $\alpha_G = \frac{\eta_G}{1-\alpha_L(1-\eta_G)}$, is plotted as the upward slope curve in the range $\alpha_G \in [\eta_G, 1]$. The curve represents all the pairs of bargaining weights in the goods and labor market that deliver $\xi^* = \xi^{opt}$. To the right of the curve the goods market is too tight. To the left of the curve the goods market is too slack. In particular, it states that for greater worker bargaining strength α_L consumers need to receive a greater share of the surplus in the goods market to ensure the efficient tightness of the goods market. If not, if effective consumer market power is too weak, there is an insufficient entry of consumer in the goods market relative to goods as the decentralized price $\mathcal{P}$ is too high. In the limit, as the worker share α_L tends to 1, the consumer needs to receive a share of the goods market surplus α_G that also tends to 1. At the other end of rent sharing, when the worker receives its reservation wage ($\alpha_L = 0$), efficient goods market tightness is achieved for $\alpha_G = \eta_G$. Below $\alpha_G = \eta_G$ it is not possible to restore constrained efficiency in the goods market by varying market power in the labor market.[25]

The second (red) curve separating the bargaining weight space is defined by the combinations of bargaining weights α_L and α_G which deliver a decentralized labor market tightness θ^* exactly equal to the constrained efficient θ^{opt}, as earlier.[26] All locations below the curve correspond to a decentralized labor market allocation with $\theta^* > \theta^{opt}$. All locations above the curve correspond to decentralized allocations with $\theta^* < \theta^{opt}$. The intersection of both curves, which corresponds to $\alpha_L = \eta_L$ and $\alpha_G = \eta_G / \left[1 - \alpha_L\left(1-\eta_G\right)\right]$, is the constrained efficient allocation in which $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$.

[24] The wage rules for this extension are derived in the appendix and presented as equations (A.7) and (A.9), and lead to the price equation (A.13).

[25] In addition, the α_L for constrained efficiency is always strictly less than α_G when $\eta_G \geq 0.5$. For any $0 < \eta_G < 0.5$, there a crossing at $\tilde{\alpha}_G$ with $\alpha_L < \alpha_G$ when $\alpha_G < \tilde{\alpha}_G$, and $\alpha_L > \alpha_G$ when $\alpha_G > \tilde{\alpha}_G$.

[26] The curve is defined by the function $\left(r+s^L\right)\frac{\gamma}{q(\theta^{opt})} = (1-\tilde{\alpha}_L)\left[\frac{\lambda(\xi^*)}{r+s+\lambda(\xi^*)}\left(\frac{1-\alpha_G}{1-\tilde{\alpha}_L\alpha_G}\right)(\Phi+\xi^*\sigma) - z\right] - \tilde{\alpha}_L\gamma\theta^{opt}$ in which θ^{opt} is taken as given, and ξ^* is the decentralized goods market tightness that solves (A.4).



(a) No strategic interaction

(b) Strategic effect of wages on price bargaining for $\eta_G = 1 - \eta_L$

Figure 13: Efficiency of tightness in the goods and labor markets with sequential wage bargaining

Wages bargained sequentially: workers and firms in stage π strategically anticipate the effect on prices

Suppose now that the worker and the firm in negotiating the wage w_π internalize the effect on the consumption match surplus in the goods market. That is, they know the bargained price will follow $\mathcal{P} = (1-\alpha_G)(\Phi + \xi\sigma) + \alpha_G (w_\pi - w_g)$. In this case the sharing rule derived from the Nash maximand must take into account that $\frac{\partial W_{e\pi}}{\partial w_\pi} = \frac{1}{r+s}$ and that $\frac{\partial J_\pi}{\partial w_\pi} = \frac{-(1-\alpha_G)}{r+s}$. The wage must now satisfy a more complex sharing rule, $\alpha_L (J_\pi - J_g) = (1-\alpha_L)(1-\alpha_G)(W_{e\pi} - W_{eg})$, giving a larger share to the worker. This share, $\alpha_L / [1 - \alpha_G (1-\alpha_L)]$ tends to α_L when the consumer has no bargaining power ($\alpha_G = 0$), and tends to 1 when $\alpha_G = 1$.

The implication of this strategic context for the goods market is a new effective bargaining share of the consumer:

$$\hat{\alpha}_G = \alpha_G (1 - \alpha_L)$$

More bargaining power to labor reduces the effective share of the consumption surplus for the consumer.

The equilibrium decentralized tightness of the goods and labor markets (ξ, θ) solve:

$$\begin{aligned} \frac{\sigma(r+s)}{\psi(\xi)} &= \hat{\alpha}_G \Phi - (1-\hat{\alpha}_G)\xi\sigma \\ \left(r+s^L\right)\frac{\gamma}{q(\theta)} &= (1-\alpha_L)\left[\Phi - \frac{\sigma(r+s)}{\psi(\xi)}\left(\frac{1}{\hat{\alpha}_G}\right)\right] - (1-\alpha_L) z - \alpha_L \gamma\theta \end{aligned}$$

from which we have the following proposition:

Proposition 12. *When wages are set separately at firms searching in the goods market and matched in the goods market, and wage bargaining at selling firms internalizes the effect on the goods market match surplus, the decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if $\alpha_L = \eta_L$ and $\hat{\alpha}_G(\alpha_L, \alpha_G) = \eta_G$.*

The conditions for constrained efficiency remain similar in nature in this extension as well. The worker's bargaining weight must equal the elasticity of the labor matching function with respect to unemployment, and the effective bargaining weight of the consumer in the goods market $\hat{\alpha}_G$ must equal the elasticity of the goods matching function with respect to consumer entry η_G. However, the second equality, which is equivalent to $\alpha_G = \eta_G \times \frac{1}{1-\alpha_L} \geq \eta_G$ has the following additional implication. If $\eta_G < 1 - \eta_L$, then the α_G that implements the constrained efficient allocation is less than 1. In this case, deviations away from the constrained efficient fall once again into one of four tightness regimes. If $\eta_G = 1 - \eta_L$, then the α_G that implements the constrained efficient allocation equals 1. In this case the regime with $\theta^* < \theta^{opt}$ and $\xi^* > \xi^{opt}$ does not exist. It is plotted in Figure 13b. Finally, If $\eta_G > 1 - \eta_L$, then there exists no α_G that can implement the constrained efficient allocation.

Detailed derivations: Sequential wage bargaining, no strategic interaction

The outside option in negotiating the wage in stage g is the dissolution of the labor match:

$$w_g \quad = \quad \text{argmax} \left(W_{eg} - W_u\right)^{\alpha_L} \left(J_g - J_v\right)^{1-\alpha_L} \tag{A.6}$$

The solution for the stage g wage must satisfy the sharing rule $(1-\alpha_L)\left(W_{eg} - W_u\right) = \alpha_L \left(J_g - J_v\right)$, such that we have:

$$\begin{aligned}
(1-\alpha_L)\left(w_g - rW_u + \lambda(W_{e\pi} - W_{eg})\right) &= \alpha_L \left(-w_g + \lambda\left(J_\pi - J_g\right)\right) \\
w_g &= (1-\alpha_L)\, rW_u \\
w_g &= (1-\alpha_L)\, z + \alpha_L \gamma\theta
\end{aligned} \tag{A.7}$$

There are two alternatives for the outside option when bargaining over the wage in stage π. In the first case the outside option is a loss of the consumer, but not the dissolution of the labor match. In the second case the outside option is the dissolution of the labor match. Both assumptions result in the same wage rule w_π if the worker's bargaining strength α_L is the same in both goods market stages g and π (see below).

(i) Assumption 1: loss of the consumer as outside option

In the first case the wage solves the bargaining problem

$$w_\pi \quad = \quad \text{argmax} \left(W_{e\pi} - W_{eg}\right)^{\alpha'_L} \left(J_\pi - J_g\right)^{1-\alpha'_L} \tag{A.8}$$

From equations describing the private surpluses, it is immediate that each has the same absolute slope (see in particular equations (A.15) and (A.16)). As such, the wage must satisfy the sharing rule $(1-\alpha'_L)\left(W_{e\pi} - W_{eg}\right) = \alpha'_L \left(J_\pi - J_g\right)$. This leads to a few interesting results. First, replacing the surpluses by their corresponding expressions, the wage increment is:

$$\begin{aligned}
\left(1-\alpha_L'\right)\left(W_{e\pi}-W_{eg}\right) &= \alpha_L'\left(J_\pi - J_g\right) \\
\left(1-\alpha_L'\right)\left(\frac{w_\pi - w_g}{r+s+\lambda}\right) &= \alpha_L'\left(\frac{\mathcal{P}-\left(w_\pi - w_g\right)}{r+s+\lambda}\right) \\
\left(1-\alpha_L'\right)\left(w_\pi - w_g\right) &= \alpha_L'\left(\mathcal{P}-\left(w_\pi - w_g\right)\right) \\
w_\pi - w_g &= \alpha_L'\mathcal{P}
\end{aligned}$$

Thus the wage w_π reflects the value of the sale of the good:

$$w_\pi = \left(1-\alpha_L\right)z + \alpha_L\gamma\theta + \alpha_L'\mathcal{P} \tag{A.9}$$

The expected wage payment to labor from the perspective of a firm entering the labor market, that which affects the entry incentives, is $\omega = \mu w_\pi + (1-\mu)w_g$. Given the results above, one has that:

$$\omega = \left(1-\alpha_L\right)z + \alpha_L\gamma\theta + \alpha_L'\mu\mathcal{P} \tag{A.10}$$

(ii) Assumption 2: Dissolution of labor match as outside option with $\alpha_L = \alpha_L'$

When the outside option is the dissolution of the labor match, the wage bargained while the firm is in a match with a consumer solves:

$$w_\pi = \text{argmax}\left(W_{\pi g} - W_u\right)^{\alpha_L}\left(J_\pi - J_v\right)^{1-\alpha_L} \tag{A.11}$$

The resulting wage must satisfy the sharing rule $\left(1-\alpha_L\right)\left(W_{e\pi}-W_u\right) = \alpha_L\left(J_\pi - J_v\right)$. Since the wage bargained in stage g must still satisfy the sharing rule $\left(1-\alpha_L\right)\left(W_{eg}-W_u\right) = \alpha_L\left(J_g - J_v\right)$, it remains the case that we have $\left(1-\alpha_L\right)\left(W_{e\pi}-W_{eg}\right) = \alpha_L\left(J_\pi - J_g\right)$, as in the previous assumption on the outside option during stage π wage bargaining. The wage rule in π is thus once again:

$$w_\pi = \left(1-\alpha_L\right)z + \alpha_L\left(\gamma\theta + \mathcal{P}\right)$$

Labor market equilibrium condition Define $\tilde{\alpha}_G = \alpha_G\left(\frac{1-\alpha_L'}{1-\alpha_L'\alpha_G}\right)$. Recall the general firm entry condition, and insert the wage expression ω to obtain:

$$\begin{aligned}
\left(r+s^L\right)\frac{\gamma}{q} &= \mu\mathcal{P} - \omega \\
\left(r+s^L\right)\frac{\gamma}{q(\theta)} &= \left(1-\alpha_L'\right)\mu(\xi)\mathcal{P} - \left(1-\alpha_L\right)z - \alpha_L\gamma\theta
\end{aligned} \tag{A.12}$$

The price equation is now:

$$\begin{aligned}
\mathcal{P} &= \left(1-\alpha_G\right)\left[\Phi + \sigma\xi\right] + \alpha_G\alpha_L'\mathcal{P} \\
\mathcal{P} &= \left(1-\tilde{\alpha}_G\right)\left(\Phi + \xi\sigma\right)
\end{aligned} \tag{A.13}$$

and the goods market entry equation remains: $\frac{\sigma}{\psi(\xi)} = \frac{(\Phi-\mathcal{P})}{r+s}$. To obtain the final equilibrium conditions, note that

$$\begin{aligned}\frac{\sigma}{\psi s} &= \frac{\alpha_G}{1-\alpha_G}\left(J_\pi - J_g\right)\\ \frac{\sigma}{\lambda}\xi &= \frac{\left(1-\alpha_L'\right)\alpha_G}{1-\alpha_G}\frac{\mathcal{P}}{r+s+\lambda} \quad \Rightarrow \quad \mu\mathcal{P} = \frac{1-\tilde{\alpha}_G}{\tilde{\alpha}_G}\sigma\xi\end{aligned}$$

and that

$$\begin{aligned}\frac{\sigma\left(r+s\right)}{\psi} &= \tilde{\alpha}_G\Phi - \left(1-\tilde{\alpha}_G\right)\sigma\xi\\ \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\tilde{\alpha}_G}\right) &= \Phi - \frac{1-\tilde{\alpha}_G}{\tilde{\alpha}_G}\sigma\xi \quad \Rightarrow \quad \frac{1-\tilde{\alpha}_G}{\tilde{\alpha}_G}\sigma\xi = \Phi - \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\tilde{\alpha}_G}\right)\end{aligned}$$

such that

$$\mu\mathcal{P} = \Phi - \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\tilde{\alpha}_G}\right)$$

and we have the job creation condition and consumer entry equations derived in the text:

$$\left(r+s^L\right)\frac{\gamma}{q(\theta)} = \left(1-\alpha_L\right)\left[\Phi x - z - \frac{\left(r+s\right)\sigma}{\psi\left(\xi\right)}\left(\frac{1}{\tilde{\alpha}_G}\right)\right] - \alpha_L\gamma\theta \tag{A.14}$$

$$\left(r+s\right)\frac{\sigma}{\psi(\xi)} = \tilde{\alpha}_G\Phi x - \left(1-\tilde{\alpha}_G\right)\xi\sigma \tag{A.15}$$

Constrained efficiency Comparing with the planner's allocation, the decentralized economy is constraint efficient if $\alpha_L = \alpha_L' = \eta_L$ and $\tilde{\alpha}_G = \eta_G$. The last condition leads to

$$\alpha_G\frac{\left(1-\alpha_L'\right)}{1-\alpha_G\alpha_L'} = \eta_G \quad \Rightarrow \quad \alpha_G = \frac{\eta_G}{1-\alpha_L'\left(1-\eta_G\right)}$$

Detailed derivation: Sequential wage bargaining with strategic interactions

Suppose that the worker and the firm in negotiating the wage w_π internalize the effect on the consumption match surplus in the goods market. That is, they now take into account $\mathcal{P} = \left(1-\alpha_G\right)\left(\Phi+\xi\sigma\right) + \alpha_G\left(w_\pi - w_g\right)$ when solving

$$w_\pi = \text{argmax}\left(W_{e\pi} - W_{eg}\right)^{\alpha_L'}\left(J_\pi - J_g\right)^{1-\alpha_L'}$$

The slopes of the match surpluses with respect to the wage are $\frac{\partial W_{e\pi}}{\partial w_\pi} = \frac{1}{r+s}$ and $\frac{\partial J_\pi}{\partial w_\pi} = \frac{-(1-\alpha_G)}{r+s}$ for the worker and the firm, respectively. As such, the first order condition

$$\frac{\alpha_L'}{W_{e\pi}-W_{eg}}\frac{\partial W_{e\pi}}{\partial w_\pi} + \frac{1-\alpha_L'}{J_\pi - J_g}\frac{\partial J_\pi}{\partial w_\pi} = 0$$

leads to the sharing rule:

$$\alpha_L'\left(J_\pi - J_g\right) = \left(1-\alpha_L'\right)\left(1-\alpha_G\right)\left(W_{e\pi} - W_{eg}\right)$$

We can now derive the wage:

$$\begin{aligned}
\alpha_L' \left(J_\pi - J_g\right) &= \left(1-\alpha_L'\right)\left(1-\alpha_G\right)\left(W_{e\pi} - W_{eg}\right) \\
\alpha_L' \left[\mathcal{P} - \left(w_\pi - w_g\right)\right] &= \left(1-\alpha_L'\right)\left(1-\alpha_G\right)\left(w_\pi - w_g\right) \\
\alpha_L' \mathcal{P} &= \left[1 - \alpha_G\left(1-\alpha_L'\right)\right]\left(w_\pi - w_g\right) \\
w_\pi &= \frac{\alpha_L'}{1-\alpha_G\left(1-\alpha_L'\right)}\mathcal{P} + w_g
\end{aligned}$$

Plugging into the price equation:

$$\begin{aligned}
\mathcal{P} &= \left(1-\alpha_G\right)\left(\Phi + \xi\sigma\right) + \frac{\alpha_G \alpha_L'}{1-\alpha_G\left(1-\alpha_L'\right)}\mathcal{P} \\
\mathcal{P}\left[\frac{1-\alpha_G}{1-\alpha_G\left(1-\alpha_L'\right)}\right] &= \left(1-\alpha_G\right)\left(\Phi + \xi\sigma\right) \\
\mathcal{P} &= \left[1-\alpha_G\left(1-\alpha_L'\right)\right]\left(\Phi + \xi\sigma\right)
\end{aligned}$$

Denote $\hat{\alpha}_G = \alpha_G\left(1-\alpha_L'\right)$. To obtain the equilibrium in (ξ, θ) start with:

$$\begin{aligned}
\frac{\sigma}{\psi} &= \frac{\alpha_G}{1-\alpha_G}\left(J_\pi - J_g\right) \\
\frac{\sigma}{\lambda}\xi &= \frac{\alpha_G}{1-\alpha_G}\frac{\left(1-\frac{\alpha_L'}{1-\alpha_G\left(1-\alpha_L'\right)}\right)\mathcal{P}}{r+s+\lambda} \\
\mu\mathcal{P} &= \frac{1-\alpha_G}{\alpha_G}\left(\frac{1-\alpha_G\left(1-\alpha_L'\right)}{1-\alpha_G+\alpha_G\alpha_L'-\alpha_L'}\right)\sigma\xi \\
\mu\mathcal{P} &= \left(\frac{1-\alpha_G\left(1-\alpha_L'\right)}{\alpha_G\left(1-\alpha_L'\right)}\right)\sigma\xi \Rightarrow \mu\mathcal{P} = \left(\frac{1-\hat{\alpha}_G}{\hat{\alpha}_G}\right)\sigma\xi
\end{aligned}$$

and

$$\begin{aligned}
\frac{\sigma\left(r+s\right)}{\psi} &= \hat{\alpha}_G\Phi - \left(1-\hat{\alpha}_G\right)\sigma\xi \\
\frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\hat{\alpha}_G}\right) &= \Phi - \frac{1-\hat{\alpha}_G}{\hat{\alpha}_G}\sigma\xi \Rightarrow \frac{1-\hat{\alpha}_G}{\hat{\alpha}_G}\sigma\xi = \Phi - \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\hat{\alpha}_G}\right)
\end{aligned}$$

such that

$$\mu\mathcal{P} = \Phi - \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\hat{\alpha}_G}\right)$$

The consumer entry and job creation conditions become:

$$\begin{aligned}
\frac{\sigma\left(r+s\right)}{\psi(\xi)} &= \hat{\alpha}_G\Phi - \left(1-\hat{\alpha}_G\right)\xi\sigma \\
\left(r+s^L\right)\frac{\gamma}{q(\theta)} &= \left(1-\alpha_L'\right)\left[\Phi - \frac{\sigma\left(r+s\right)}{\psi}\left(\frac{1}{\hat{\alpha}_G}\right)\right] - \left(1-\alpha_L\right)z - \alpha_L\gamma\theta
\end{aligned}$$

G Hosios and the slope of the wage Philipps curve defined as a ratio of elasticities (footnote 18)

If we instead define the slope of the wage Phillips curve is in terms of relative elasticities we have:

$$\kappa^{elast} \equiv -\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{\left[\mathcal{R}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right)^{-1}+\gamma\theta\right]}{w\mathcal{N}}<0 \tag{A.16}$$

The main equations of the model are:

$$\begin{aligned}
\mathcal{R} &= \mathcal{P}\mu \\
\frac{\sigma\left(r+s\right)}{\psi} &= \alpha_G\Phi-\left(1-\alpha_G\right)\xi\sigma \\
\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta} &= \frac{\mathcal{R}}{\eta_L\left(\mathcal{R}-z\right)+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\gamma\theta} \\
w &= \left(1-\alpha_L\right)z+\alpha_L\left[\gamma\theta+\mathcal{R}(\xi,\mathcal{P})\right] \\
\mathcal{N} &= \frac{f(\theta)}{s^L+f(\theta)}
\end{aligned}$$

We need to verify whether

$$\begin{aligned}
\left.\frac{\partial\kappa^{elast}}{\partial\alpha_G}\right|_{\alpha_G=\eta_G} &= 0 \\
\left.\frac{\partial^2\kappa^{elast}}{\partial\alpha_G^2}\right|_{\alpha_G=\eta_G} &> 0
\end{aligned}$$

so that the curve has a minimum in its slope in Hosios in the goods market.

G.1 Establishing $\left.\frac{\partial\kappa^{elast}}{\partial\alpha_G}\right|_{\alpha_G=\eta_G}=0$

Let $\mathcal{R}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\mathcal{R}}\frac{\mathcal{R}}{\theta}\right)^{-1}+\gamma\theta=A$ and let $w\mathcal{N}=B$. Then,

$$\kappa^{elast}=-\left(\frac{\alpha_L}{1-\eta_L}\right)\frac{A}{B}$$

with

$$\begin{aligned}
A &= \eta_L\left(\mathcal{R}-z\right)+\left(1+\left(1-\eta_L\right)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta \\
B &= \left[\left(1-\eta_L\right)z+\eta_L\left(\gamma\theta+\mathcal{R}\right)\right]\mathcal{N} \\
&= \left[\eta_L\left(\mathcal{R}-z\right)+\eta_L\gamma\theta-z\right]\mathcal{N}
\end{aligned}$$

Recall that $\mu = \frac{\lambda}{r+s+\lambda}$ and

$$\frac{\partial \mu}{\partial \xi}\frac{\xi}{\mu} = \eta_G\left(\frac{r+s}{r+s+\lambda}\right) = \eta_G\frac{(r+s)\mathcal{R}}{\lambda\mathcal{P}} = \eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)$$

using $\frac{(r+s)\mathcal{R}}{\lambda\mathcal{P}} = \left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{(r+s)\sigma}{\mathcal{P}\psi}\right) = \left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)$.

From now on in this section, we denote by X' the first order derivative of the equilibrium variable X with respect to α_G and by $''$ the second derivative with respect to the same variable. This can be partial or total, and we use these notation if there is no confusion. Using $\mathcal{P} = (1-\alpha_G)(\Phi+\sigma\xi)$, $\mathcal{R}(\xi,\mathcal{P}) = \frac{\lambda}{r+s+\lambda}\mathcal{P}$; $\mathcal{R}(\xi,\mathcal{P}) = \frac{1-\alpha_G}{\alpha_G}\sigma\xi$ and using the consumer entry condition, one obtains:

$$\begin{aligned}
\mathcal{P}' &= \frac{(r+s)}{\psi^2}\frac{\partial\psi}{\partial\xi}\sigma\xi' = \frac{(r+s)}{\psi}\left(\frac{\partial\psi}{\partial\xi}\frac{\xi}{\psi}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\sigma}{\alpha_G} \\
&= \left(\frac{\eta_G-1}{\alpha_G}\right)\frac{(r+s)\sigma}{\psi}\left(\xi'\frac{\alpha_G}{\xi}\right) \\
&= \left(\frac{\eta_G-1}{\alpha_G}\right)(\Phi-\mathcal{P})\left(\xi'\frac{\alpha_G}{\xi}\right)
\end{aligned}$$

Turning to $\mathcal{R}' = \frac{\partial\mathcal{R}}{\partial\alpha_G}$:

$$\begin{aligned}
\mathcal{R} &= \mu\mathcal{P} \\
\mathcal{R}' &= \mu'\mathcal{P} + \mathcal{P}'\mu \\
\mathcal{R}' &= \left(\frac{\partial\mu}{\partial\xi}\frac{\xi}{\mu}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} + \mathcal{P}'\mu \\
\mathcal{R}' &= \eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} + (\eta_G-1)\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G} \\
\mathcal{R}' &= \left[\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mu\mathcal{P}}{\alpha_G}\right]\left[\frac{\eta_G-\alpha_G}{\alpha_G}\right] \\
&= \left[\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)\frac{\mathcal{R}}{\alpha_G}\right]\left[\frac{\eta_G-\alpha_G}{\alpha_G}\right] \\
\Leftrightarrow \mathcal{R}'\frac{\alpha_G}{\mathcal{R}} &= \frac{\eta_G-\alpha_G}{\alpha_G}\left(\frac{\Phi-\mathcal{P}}{\mathcal{P}}\right)\left(\xi'\frac{\alpha_G}{\xi}\right)
\end{aligned}$$

Result F-1. Optimum of revenues at Hosios in the goods market

If $\alpha_G = \eta_G$, then $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}} = 0$.

Moreover, the sign of the slope $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}}$ is of the sign of $\left[\eta_G\left(\frac{1-\alpha_G}{\alpha_G}\right) + (\eta_G-1)\right]$ from which we have that $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}} > 0$ if $\alpha_G < \eta_G$, and $\frac{\partial\mathcal{R}}{\partial\alpha_G}\frac{\alpha_G}{\mathcal{R}} < 0$ otherwise. Therefore the marginal revenue $\mathcal{R}$ is maximized for $\alpha_G = \eta_G$.

Turning to the wage Philipps curve, we can now establish that it is at an optimum with respect to α_G at $\alpha_G = \eta_G$:

Result F-2. Optimum of κ^{elast} at Hosios in the goods market

$\left(\frac{A}{B}\right)' = \frac{A'B-AB'}{B^2} = 0$ at $\alpha_G = \eta_G$.

G.2 Intermediate results

As supplementary results, we have the following equalities.

Result F-3. Optimum in (θ, w, N) at Hosios in the goods market

First, we have that, at $\alpha_G = \eta_G$,

$$\theta' = 0 \text{ and } \theta'' \leq 0$$
$$\mathcal{N}' = 0 \text{ and } \mathcal{N}'' \leq 0$$
$$w' = 0 \text{ and w}'' \leq 0$$

The second line is obtained from the fact that $\mathcal{N}$ is monotonically increasing in θ so that

$$\begin{aligned}
\mathcal{N}' &= \frac{\partial \mathcal{N}}{\partial \theta}\theta' \\
\mathcal{N}'' &= \frac{\partial^2 \mathcal{N}}{\partial \theta^2}\left(\theta'\right)^2 + \frac{\partial \mathcal{N}}{\partial \theta}\theta'' + \frac{\partial}{\partial \alpha_G}\left(\frac{\partial \mathcal{N}}{\partial \theta}\right)\theta' \\
&= 0 + \frac{\partial \mathcal{N}}{\partial \theta}\theta'' + 0
\end{aligned}$$

since $\frac{\partial \mathcal{N}}{\partial \theta} = 0$.

The last lines is due to the fact that

$$\begin{aligned}
w' &= \frac{\partial w}{\partial \theta}\theta' + \frac{\partial w}{\partial \mathcal{R}}\mathcal{R}' = \alpha_L \gamma \theta' + \alpha_L \mathcal{R}' \\
w'' &= \frac{\partial^2 w}{\partial \theta^2}\left(\theta'\right)^2 + \frac{\partial w}{\partial \theta}\theta'' + \frac{\partial}{\partial \alpha_G}\left(\frac{\partial w}{\partial \theta}\right)\theta' \\
&= 0 + \alpha_L \gamma \theta'' + 0
\end{aligned}$$

In addition, one can differentiate the free entry-condition in the goods market to get an expression for the slope of ξ which is not zero at $\alpha_G = \eta_G$.

Result F-4. First derivative of good market tightness

$$\frac{\partial \xi}{\partial \alpha_G} = \left(\Phi + \xi\sigma\right)\frac{\xi}{\alpha_G \Phi}$$

Result F-5. Second derivatives of wages and employment

Given that

$$\begin{aligned}
w &= \alpha_L \left[\mathcal{R} + \gamma\theta\right] + (1-\alpha_L)\, z \\
w'' &= \alpha_L \left[\mathcal{R}'' + \gamma\theta''\right] \\
\mathcal{N} &= f/(s+f) \\
\mathcal{N}' &= \frac{f'(s+f) - ff'}{\left(s^L+f\right)^2} = \frac{sdf}{\left(s^L+f\right)^2} \\
\mathcal{N}'' &= \frac{f^2\left(sf''\right) - 2ff'.sf'}{f^4} \\
&= \frac{sf'' - 2(f')^2.s/f}{f^2} = \left(\frac{s}{f}\right)\frac{f'' - 2(f')^2/f}{f} \\
&= \mathcal{U}\mathcal{N}\frac{f'' - 2(f')^2/f}{f}
\end{aligned}$$

Result F-6: second derivative of f

$$\begin{aligned}
f' &= f/\theta\,(1-\eta_L)\,\theta' = \chi_l\theta^{-\eta_L}\,(1-\eta_L)\,\theta' \\
f'' &= (-\eta_L)\,\chi_l\theta^{-\eta_L-1}\,(1-\eta_L)\,\theta'' \\
&= (-\eta_L)\,(1-\eta_L)\,(q(\theta)/\theta)\,\theta''
\end{aligned}$$

Finally, a last result that will be useful is to determine the link between θ'' and $\mathcal{R}''$. Use the free-entry equation in the labor market:

$$\frac{\gamma\left(r+s^L\right)}{q} = (1-\alpha_L)\,(\mathcal{R}-z) - \alpha_L\gamma\theta$$

Denote by $G(\theta, \mathcal{R}, \alpha_G)$ the relation linking the three variables. Note that α_G does not enter the equation, but yet θ and $\mathcal{R}$ are function of α_G. Thus will simplify our analysis. One has first:

$$G'_\theta\theta' + G'_\mathcal{R}\mathcal{R}' = 0 \tag{A.17}$$

which gives the theorem of implicit functions, that is:

$$\frac{\partial\theta}{\partial\mathcal{R}} = \frac{-G'_\mathcal{R}}{G'_\theta}$$

and we already know that

$$\frac{\partial\theta}{\partial\mathcal{R}}\frac{\mathcal{R}}{\theta} = \frac{\mathcal{R}}{\eta_L\,(\mathcal{R}-z) + (1-\eta_L)\,\frac{\alpha_L}{1-\alpha_L}\gamma\theta}$$

Second, expanding on A.17 and using a differentiation again, one has that:

$$G'_\theta\theta'' + G''_{\theta\theta}\left(\theta'\right)^2 + 2G''_{\theta\mathcal{R}}\mathcal{R}'\theta' + G''_{\mathcal{R}\mathcal{R}}\left(\mathcal{R}'\right)^2 + G'_\mathcal{R}\mathcal{R}'' = 0$$

Therefore, given that all first order derivatives are zero in $\alpha_G = \eta_G$, one has

$$\theta'' = \frac{-G'_{\mathcal{R}}}{G'_{\theta}} \mathcal{R}''$$
$$\theta'' = \frac{\theta}{\eta_L (\mathcal{R} - z) + (1-\eta_L) \frac{\alpha_L}{1-\alpha_L} \gamma\theta} \mathcal{R}''$$

Note that $A = \eta_L (\mathcal{R} - z) + \left(1 + (1-\eta_L) \frac{\alpha_L}{1-\alpha_L}\right) \gamma\theta$ so that:
Result F-7. Link between second derivatives

$$\theta'' = \frac{\theta}{A - \gamma\theta} \mathcal{R}''$$

G.3 Establishing $\left.\frac{\partial^2 \kappa^{elast}}{\partial \alpha_G^2}\right|_{\alpha_G = \eta_G} > 0$

$$\begin{aligned}
A' &= \frac{dA}{d\alpha_G} = \eta \frac{\partial R}{\partial \alpha_G} + (1+\eta)\gamma \frac{\partial \theta}{\partial \alpha_G} = \eta R' + (1+\eta)\gamma\theta' \\
B' &= \frac{dw}{d\alpha_G} \mathcal{N} + w \frac{d\mathcal{N}}{d\alpha_G} = w'\mathcal{N} + \mathcal{N}'w \\
(A/B)' &= \frac{A'}{B} - \frac{AB'}{B^2} \\
(A/B)'' &= \frac{BA'' - A'B'}{B^2} - \frac{B^2 (A'B' + AB'') - 2BB'(AB')}{B^4}
\end{aligned}$$

This second derivative of the slope is studied at Hosios in the goods market, and therefore, given that first order derivatives are zero in that point,

$$\begin{aligned}
(A/B)'' &= \frac{A''}{B} - \frac{AB''}{B^2} \\
&= \frac{BA'' - AB''}{B^2} \\
&= \frac{1}{B} \left(\eta_L \mathcal{R}'' + \left(1 + (1-\eta_L) \frac{\alpha_L}{1-\alpha_L}\right) \gamma\theta'' - \frac{A}{B} \left[\mathcal{N} w'' + w \mathcal{N}''\right] \right)
\end{aligned}$$

Pose $$Z = \eta_L \mathcal{R}'' + \left(1 + (1-\eta_L) \frac{\alpha_L}{1-\alpha_L}\right) \gamma\theta'' - \frac{A}{B} \left[\mathcal{N} w'' + w \mathcal{N}''\right]$$

We now want to study the sign of Z. The first two terms are negative given the negative second derivatives, while the last term is negative, so that the sign is not immediate. Now, replacing w'' and $\mathcal{N}''$ from Result F-5, then using Result F-6 to transform into functions of θ'' and $\mathcal{R}''$, one gets, in $\alpha_G = \eta_G$:

$$
\begin{aligned}
Z &= \eta_L \mathcal{R}'' + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta'' - \frac{A}{B}\left[\mathcal{N}w'' + w\mathcal{N}''\right] \\
&= \left(\eta_L \mathcal{R}'' + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta''\right) - \frac{A}{B}\left[\mathcal{N}\left(\alpha_L\left[\mathcal{R}'' + \gamma\theta''\right]\right) + w\mathcal{U}\mathcal{N}\frac{f''}{f}\right] \\
&= \left(\eta_L \mathcal{R}'' + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta''\right) - \frac{A}{B}\left[\mathcal{N}\left(\alpha_L\left[\mathcal{R}'' + \gamma\theta''\right]\right) + w\mathcal{U}\mathcal{N}\frac{(-\eta_L)(1-\eta_L)(q(\theta)/\theta)\,\theta''}{f}\right] \\
&= \mathcal{R}''\left[\eta_L - \frac{A}{w}\alpha_L\right] + \theta''\left[\left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma - \frac{A}{w}\alpha_L\gamma + \frac{A}{w\mathcal{N}}w\mathcal{U}\mathcal{N}\frac{(-\eta_L)(1-\eta_L)\left(f(\theta)/\theta^2\right)}{f}\right] \\
Z &= \mathcal{R}''\left[\eta_L - \frac{A}{w}\alpha_L\right] + \theta''\left[\left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma - \frac{A}{w}\alpha_L\gamma + A\mathcal{U}\frac{\eta_L(1-\eta_L)}{\theta^2}\right]
\end{aligned}
$$

finally using Result F-7:

$$
Z = \mathcal{R}''\left\{\eta_L - \frac{A}{w}\alpha_L + \frac{\theta}{A-\gamma\theta}\left[(1+\eta_L)\gamma - \frac{A}{w}\alpha_L\gamma + A\mathcal{U}\frac{\eta_L(1-\eta_L)}{\theta^2}\right]\right\}
$$

and so

$$
\left.\frac{\partial^2\kappa^{elast}}{\partial\alpha_G^2}\right|_{\alpha_G=\eta_G} = -\left(\frac{\alpha_L}{1-\eta_L}\right)\mathcal{R}''M
$$

where

$$
M = \eta_L - \frac{A}{w}\alpha_L + \frac{\theta}{A-\gamma\theta}\left[(1+\eta_L)\gamma - \frac{A}{w}\alpha_L\gamma + A\mathcal{U}\frac{\eta_L(1-\eta_L)}{\theta^2}\right]
$$

So the sufficient condition is now that $M > 0$. The condition generally holds for simpler cases. First, let us prove a last set of results:

Result F-8. Limits

The limit in $\alpha_L = 1$ of $\frac{\alpha_L}{1-\alpha_L}\gamma\theta$ is zero.

$$
\frac{\gamma\left(r+s^L\right)}{q(\theta)} = (1-\alpha_L)(\mathcal{R}-z) - \alpha_L\gamma\theta
$$

When $\alpha_L \to 1$, workers get all the surplus, meaning θ asymptotically tends to 0, as all three terms in the above equation. Note that both $1/q(\theta)$ and θ converge to zero, but that $\theta/(1/q(\theta)) = f(\theta) \sim \theta^{1-\eta_L}$ tends to zero, so θ tends faster to zero. Multiply by $q(\theta)$, one gets:

$$
\gamma\left(r+s^L\right) = (1-\alpha_L)(\mathcal{R}-z)\,q(\theta) - \alpha_L\gamma\theta q(\theta)
$$

so we know that

$$
(1-\alpha_L)(\mathcal{R}-z)\,q(\theta) \to \gamma\left(r+s^L\right)
$$

Now, the quantity:

$$
\frac{\alpha_L}{1-\alpha_L}\gamma\theta = \left[\frac{\alpha_L}{(1-\alpha_L)\,q(\theta)}\right]\left[\gamma\theta q(\theta)\right] \to 0
$$

since the first bracketed term tends to a finite value.

One also has, off limit:

$$\frac{\gamma\left(r+s^{L}\right)}{q(\theta)}=\left(1-\alpha_{L}\right)(\mathcal{R}-z)-\alpha_{L}\gamma\theta$$

$$\frac{\alpha_{L}\gamma\theta}{1-\alpha_{L}}=(\mathcal{R}-z)-\frac{\gamma\left(r+s^{L}\right)}{q(\theta)(1-\alpha_{L})}$$

which is the total labor surplus times $(r+s^{L})$.

Similarly,

$$\frac{\gamma\left(r+s^{L}\right)}{q(\theta)}=\left(1-\alpha_{L}\right)(\mathcal{R}-z)-\alpha_{L}\gamma\theta$$

implies

$$\frac{\alpha_{L}\gamma\theta}{\mathcal{R}-z}=-\frac{\gamma\left(r+s^{L}\right)}{q(\theta)(\mathcal{R}-z)}+\left(1-\alpha_{L}\right)\rightarrow-\frac{\gamma\left(r+s^{L}\right)\left(1-\alpha_{L}\right)}{\gamma\left(r+s^{L}\right)}+\left(1-\alpha_{L}\right)=0$$

We can now return to the sign of M in different cases.

G.4 Polar case 1: if $\alpha_L \to 0$ then $A \to \eta_L(\mathcal{R}-z)+\gamma\theta$

$$M=\eta_{L}+\frac{\theta}{\eta_{L}\left(\mathcal{R}-z\right)}(1+\eta_{L})\gamma>0$$

and by continuity, $M>0$ in a range around $\alpha_L=0$.

G.5 Polar case 2: if $\alpha_L \to 1$

From Result F-8, $\theta \to 0$ and $\frac{\alpha_L}{1-\alpha_L}\gamma\theta \to 0$; while $A=\eta_L(\mathcal{R}-z)+\left(1+(1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta \to \eta_L(\mathcal{R}-z)$ and $w=(1-\alpha_L)z+\alpha_L[\gamma\theta+\mathcal{R}(\xi,\mathcal{P})] \to \mathcal{R}(\xi,\mathcal{P})$ therefore

$$\begin{aligned}M\sim&\left[\eta_{L}-\frac{\eta_{L}\left(\mathcal{R}-z\right)}{\mathcal{R}}\right]+\\&\frac{\theta}{\eta_{L}\left(\mathcal{R}-z\right)}\left[(1+\eta_{L})\gamma-\frac{\eta_{L}\left(\mathcal{R}-z\right)}{\mathcal{R}(\xi,\mathcal{P})}\gamma+\frac{\eta_{L}\left(\mathcal{R}-z\right)}{\mathcal{N}}\alpha_{L}\frac{s\left(\eta_{L}\right)\left(1-\eta_{L}\right)\left(1/\theta^{2}\right)}{f}\right] &\text{(A.18)}\\\sim&\;\eta_{L}\frac{z}{\mathcal{R}}+\frac{1}{\theta}\left[\frac{1}{\mathcal{N}}\alpha_{L}\frac{s\left(\eta_{L}\right)\left(1-\eta_{L}\right)}{f(\theta)}\right] &\text{(A.19)}\end{aligned}$$

and the second term in brackets tends to $+\infty$ and so by continuity $M>0$ in a range around $\alpha_L=1$.

Polar case 4: let have $z \to \mathcal{R}$

This implying $\theta=0$: then $A=\eta_L(\mathcal{R}-z)+\left(1+(1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta=0$, $w=z$. We also have that

$$\frac{\alpha_{L}\gamma\theta}{\mathcal{R}-z}\rightarrow0$$

and

$$(\mathcal{R}-z)\sim\theta^{\eta_{L}}$$

and $A = \eta_L (\mathcal{R} - z) + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\gamma\theta - \gamma\theta = \eta_L(\mathcal{R}-z) + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\gamma\theta \sim \eta_L\theta^{\eta_L} + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\gamma\theta$:

$$M \sim [\eta_L] + \frac{\theta^{1-\eta_L}}{\eta_L}\left[(1+\eta_L)\gamma + \frac{A}{\mathcal{N}}\alpha_L \frac{s(\eta_L)(1-\eta_L)(1/\theta^2)}{f}\right]$$
$$M = [\eta_L] + \frac{1}{\eta_L}\left[(1+\eta_L)\gamma + \frac{A}{\mathcal{N}}\alpha_L \frac{s(\eta_L)(1-\eta_L)(1/\theta^2)}{f}\right] \to \infty \tag{A.20}$$

so again $M > 0$ by continuity in a range of z around $\mathcal{R}$.

G.6 Polar case 5: let have $\mathcal{R} \to +\infty$

In that case, $\theta \to +\infty$ and $\mathcal{R} \sim \frac{\alpha_L}{1-\alpha_L}\gamma\theta$ according to Result F-8. It follows that

$$A \sim \eta_L\mathcal{R} + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\mathcal{R}(1-\alpha_L) \sim 1 + \eta_L(1-\alpha_L)\mathcal{R} > 0$$

$$A/w \sim \frac{\eta_L\mathcal{R} + \left(1 + (1-\eta_L)\frac{\alpha_L}{1-\alpha_L}\right)\mathcal{R}(1-\alpha_L)}{\alpha_L\mathcal{R} + \mathcal{R}(1-\alpha_L)} \sim 1 + \eta_L(1-\alpha_L) > 0$$

Now, we have

$$M = \left[\eta_L - \frac{A}{w}\alpha_L\right] + \frac{\theta}{A - \gamma\theta}\left[(1+\eta_L)\gamma - \frac{A}{w}\alpha_L\gamma + \frac{A}{\mathcal{N}}\alpha_L\frac{s(\eta_L)(1-\eta_L)(1/\theta^2)}{f}\right]$$

$$M \sim [\eta_L - \alpha_L - \eta_L(1-\alpha_L)\alpha_L] + \frac{(1-\alpha_L)}{\eta_L + ((1-\eta_L)\alpha_L)}[(1+\eta_L) - (1+\eta_L(1-\alpha_L))]$$

The first bracketed term is

$$\eta_L - \alpha_L - \eta_L(1-\alpha_L)\alpha_L = \eta_L(1-(1-\alpha_L)\alpha_L) - \alpha_L$$

could be negative. But the second term is,

$$(1+\eta_L) - (1+\eta_L(1-\alpha_L)) = \eta_L\alpha_L > 0$$

Combine:

$$M \sim \frac{[\eta_L(1-(1-\alpha_L)\alpha_L) - \alpha_L][\eta_L + ((1-\eta_L)\alpha_L)] + (1-\alpha_L)\eta_L\alpha_L}{\eta_L + ((1-\eta_L)\alpha_L)}$$

Compute

$$
\begin{aligned}
\mathcal{D} &= [\eta_L - \alpha_L - \eta_L(1-\alpha_L)\alpha_L]\,[\eta_L + ((1-\eta_L)\,\alpha_L)] + (1-\alpha_L)\eta_L\alpha_L \\
&= [\eta_L - \alpha_L - \eta_L(1-\alpha_L)\alpha_L]\,[\eta_L] \\
&+ [\eta_L - \alpha_L - \eta_L(1-\alpha_L)\alpha_L]\,[((1-\eta_L)\,\alpha_L)] \\
&+ (1-\alpha_L)\eta_L\alpha_L \\
&= \eta_L^2 - \alpha_L\eta_L - \eta_L^2(1-\alpha_L)\alpha_L \\
&+ \eta_L\,(1-\eta_L)\,\alpha_L - \alpha_L^2\,(1-\eta_L) - \eta_L(1-\alpha_L)\alpha_L^2\,(1-\eta_L) \\
&+ (1-\alpha_L)\eta_L\alpha_L
\end{aligned}
$$

In the specific case $\eta_L - \alpha_L$ one has

$$
\begin{aligned}
\mathcal{D} &= -\alpha_L^3(1-\alpha_L) - (1-\alpha_L)^2\alpha_L^3 + (1-\alpha_L)\alpha_L^2 \\
&= \alpha_L^3\left[-1 + \alpha_L - (1-\alpha_L)^2\right] + (1-\alpha_L)\alpha_L^2 \\
&= \alpha_L^3\left[-1 + \alpha_L - 1 - \alpha_L^2 + 2\alpha_L\right] + (1-\alpha_L)\alpha_L^2 \\
&= \alpha_L^2\left[-2\alpha_L - \alpha_L^3 + 3\alpha_L^2\right] + \alpha_L^2 - \alpha_L^3 \\
&= \alpha_L^2\left[1 - 3\alpha_L + 3\alpha_L^2 - \alpha_L^3\right]
\end{aligned}
$$

Over (0,1) the function $\mathcal{D}(x) = 1 - 3x + 3x^2 - x^3$ is positive. Indeed, $\mathcal{D}'(x) = -3 + 6x - 3x^2 = 0$. It's $\Delta = 0$ and so $\mathcal{D}'(x) = 0$ if and only if

$$
x = \frac{-6}{2 * -3} = 1
$$

so it has a double maximum in 1. So $\mathcal{D}'(1) = 0$ and $\mathcal{D}'(x) < 0$ so the function continuously decreases from 1 to 0 over the interval and thus is always positive.