Optimal Fiscal and Monetary Policy Under Sticky Prices*

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Abstract

This paper studies optimal fiscal and monetary policy under sticky product prices. The theoretical framework is a stochastic production economy without capital. The government finances an exogenous stream of purchases by levying distortionary income taxes, printing money, and issuing one-period nominally risk free bonds. The main findings of the paper are: First, for a miniscule degree of price stickiness (i.e., many times below available empirical estimates), the optimal volatility of inflation is near zero. This result stands in stark contrast with the high volatility of inflation implied by the Ramsey allocation when prices are flexible. The finding is in line with a recent body of work on optimal monetary policy under nominal rigidities that ignores the role of optimal fiscal policy. Second, even small deviations from full price flexibility induce near random walk behavior in government debt and tax rates. Third, sluggish price adjustment raises the average nominal interest rate above the one called for by the Friedman rule. Finally, an interest-rate feedback rule whereby the nominal interest rate depends on inflation and output fits well the data emanating from the Ramsey economy. However, the fitted rule is not of the Taylor type, for the implied inflation coefficient is less than one and close to zero and the output coefficient is negative. *JEL Classification:* E52, E61, E63.

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1 Introduction

Two distinct branches of the existing literature on optimal monetary policy deliver diametrically opposed policy recommendations concerning the long-run and cyclical behavior of prices and interest rates. One branch follows the theoretical framework laid out in Lucas and Stokey (1983). It studies the joint determination of optimal fiscal and monetary policy in flexible-price environments with perfect competition in product and factor markets. In this group of papers, the government's problem consists in financing an exogenous stream of public spending by choosing the least disruptive combination of inflation and distortionary income taxes. The criterion under which policies are evaluated is the welfare of the representative private agent. A basic result of this literature is the optimality of the Friedman rule. A zero opportunity cost of money has been shown to be optimal under perfect-foresight in a variety of monetary models, including cash-in-advance, money-in-the-utility function, and shopping-time models.[1]

In a significant contribution to the literature, Chari et al. (1991) characterize optimal monetary and fiscal policy in stochastic environments with nominal non-state-contingent public debt. They prove that the Friedman rule is also optimal under uncertainty: the government finds it optimal to set the nominal interest rate to zero at all dates and all states of the world. In addition, Chari et al. show that income tax rates are remarkably stable over the business cycle, and that the inflation rate is highly volatile and serially uncorrelated. Under the Ramsey policy, the government uses unanticipated inflation as a lump-sum tax on financial wealth. The government is able to do this because public debt is nominal and non-state-contingent. Thus, inflation plays the role of a shock absorber of unexpected innovations in the fiscal deficit.

On the other hand, a more recent literature focuses on characterizing optimal monetary policy in environments with nominal rigidities and imperfect competition.[2] Besides its emphasis on the role of price rigidities and market power, this literature differs from the earlier one described above in two important ways. First, it assumes, either explicitly or implicitly, that the government has access to (endogenous) lump-sum taxes to finance its budget. An important implication of this assumption is that there is no need to use unanticipated inflation as a lump-sum tax; regular lump-sum taxes take up this role. Second, the government is assumed to be able to implement a production (or employment) subsidy so as to eliminate the distortion introduced by the presence of monopoly power in product and factor markets.

A key result of this literature is that the optimal monetary policy features an inflation rate that is zero or close to zero at all dates and all states.[3] In addition, the nominal interest rate is not only different from zero, but also varies significantly over the business cycle. The reason why price stability turns out to be optimal in environments of the type described here is straightforward: the government keeps the price level constant in order to minimize (or completely eliminate) the costs introduced by inflation under nominal rigidities.

This paper aims to incorporate in a unified framework the essential elements of the two approaches to optimal policy described above. Specifically, in this paper we build a model that shares two elements with the earlier literature: (a) The only source of regular taxation available to the government is distortionary income taxes. In particular, the fiscal authority cannot adjust lump-

[1] See, for example, Chari et al. (1991), Correia and Teles (1996), Guidotti and Végh (1993), and Kimbrough (1986).

[2] See, for example, Erceg, Henderson, and Levin (1999), Galí and Monacelli (2000), Khan, King, and Wolman (2000), Rotemberg and Woodford (1999), Woodford (1999), and Woodford (2000).

[3] In models where money is used exclusively as a medium of account or when money enters in an additively separable way in the utility function, the optimal inflation rate is typically strictly zero. Khan, King, and Wolman (2000) show that when a nontrivial transaction role for money is introduced, the optimal inflation rate lies between zero and the one called for by the Friedman rule. However, in calibrated model economies they find that the optimal rate of inflation is in fact very close to zero and smooth.

sum taxes endogenously in financing its outlays. (b) The government cannot implement production subsidies to undo distortions created by the presence of imperfect competition. At the same time, our model shares two important assumptions with the more recent body of work on optimal monetary policy. First, product markets are not perfectly competitive. In particular, we assume that each firm in the economy is the monopolistic producer of a differentiated good. Second, nominal prices are assumed to be sticky. We introduce price stickiness à la Rotemberg (1982) by assuming that firms face a convex cost of adjusting the price of the good they produce. An assumption maintained throughout this paper that is common to all of the papers cited above (except for Lucas and Stokey, 1983) is that the government has the ability to fully commit to the implementation of announced fiscal and monetary policies.

In an earlier study, Schmitt-Grohé and Uribe (2001a), we characterize optimal fiscal and monetary policy in a flexible price economy under imperfect competition. Under such environment, we find that if the government is unable to tax monopoly profits at a hundred percent rate or if the government can set income and profit taxes separately, the Friedman rule ceases to be optimal. When product markets are imperfectly competitive, the Ramsey planner resorts to a positive nominal interest rate as an indirect way to tax profits. The nominal interest rate represents an indirect tax on profits because households must hold (non-interest-bearing) fiat money in order to convert income into consumption. Another result of the aforementioned study that is central to the current analysis is that while the first moments of inflation, the nominal interest rate, and tax rates are sensitive to the degree of market power in the Ramsey allocation, the cyclical properties of these variables are similar to those arising in perfectly competitive environments. In particular, it is optimal for the government to smooth tax rates and to make the inflation rate highly volatile. Thus, as in the case of perfect competition, the government uses variations in the price level as a state-contingent tax on financial wealth.

When prices are sticky, the government faces a tradeoff in choosing the path of inflation. On the one hand, the government would like to use unexpected inflation as a non-distorting tax on nominal wealth. In this way the fiscal authority can minimize the need to vary distortionary income taxes over the business cycle. On the other hand, changes in the rate of inflation come at a cost, for firms face nominal rigidities.[4] The main result of this paper is that under plausible calibrations of the degree of price stickiness, this trade off is overwhelmingly resolved in favor of price stability. The optimal fiscal/monetary regime features relatively low inflation volatility. Thus, the Ramsey allocation delivers an inflation process that is more in line with the predictions of the more recent body of literature on optimal monetary policy referred to above, which ignores fiscal constraints by assuming that the government can resort to lump-sum taxation.

The remainder of the paper is organized in 6 sections. Section 2 describes the economic environment and defines a competitive equilibrium. Section 3 presents the Ramsey problem and shows that the allocations and prices that satisfy the constraints of the Ramsey problem are equivalent to the allocations and prices that constitute a competitive equilibrium. Section 4 analyzes the business-cycle properties of Ramsey allocations. It first describes the calibration of the model, then presents the quantitative results, and finally discusses the accuracy of the numerical solution method. Sectionr̃efsec:interest-rate-feedback-rules investigates whether the time series process for the nominal interest rate implied by the Ramsey policy can be represented as a Taylor-type interest rate feedback rule. Section 8 presents concluding remarks.

[4]Christiano and Fitzgerald (2000) and Sims (2001) also remark on the desirability of quantitatively investigating the costs and benefits of price volatility in environments with sluggish price adjustment.

2 The Model

In this section we develop a simple infinite-horizon production economy with imperfectly competitive product markets and sticky prices. A demand for money is motivated by assuming that money facilitates transactions. The government finances an exogenous stream of purchases by levying distortionary income taxes, printing money, and issuing one-period nominally risk-free bonds.

2.1 The Private Sector

Consider an economy populated by a large number of identical households. Each household has preferences defined over processes of consumption and leisure and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \tag{1}$$

where c_t denotes consumption, h_t denotes labor effort, $\beta \in (0,1)$ denotes the subjective discount factor, and E_0 denotes the mathematical expectation operator conditional on information available in period 0. The single period utility function U is assumed to be increasing in consumption, decreasing in effort, strictly concave, and twice continuously differentiable.

In each period $t \geq 0$, households can acquire two types of financial assets: fiat money, M_t, and one-period, state-contingent, nominal assets, D_{t+1}, that pay the random amount D_{t+1} of currency in a particular state of period $t+1$. Money facilitates consumption purchases. Specifically, consumption purchases are subject to a proportional transaction cost $s(v_t)$ that depends on the household's money-to-consumption ratio, or consumption-based money velocity,

$$v_t = \frac{P_t c_t}{M_t}, \tag{2}$$

where P_t denotes the price of the consumption good in period t. The transaction cost function satisfies the following assumption:

Assumption 1 *The function $s(v)$ satisfies: (a) $s(v)$ is nonnegative and twice continuously differentiable; (b) There exists a level of velocity $\underline{v} > 0$, to which we refer as the satiation level of money, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (c) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$; and (d) $2s'(v) + vs''(v) > 0$ for all $v \geq \underline{v}$.*

Assumption 1(a) states that the transaction cost is non-negative and smooth. Assumption 1(b) ensures that the Friedman rule, i.e., a zero nominal interest rate, need not be associated with an infinite demand for money. It also implies that both the transaction cost and the distortion it introduces vanish when the nominal interest rate is zero. Assumption 1(c) guarantees that in equilibrium money velocity is always greater than or equal to the satiation level. As will become clear shortly, assumption 1(d) ensures that the demand for money is decreasing in the nominal interest rate. (Note that assumption 1(d) is weaker than the more common assumption of strict convexity of the transaction cost function.)

The consumption good c_t is assumed to be a composite good made of a continuum of intermediate differentiated goods. The aggregator function is of the Dixit-Stiglitz type. Each household is the monopolistic producer of one variety of intermediate goods. The intermediate goods are produced using a linear technology, $z_t \tilde{h}_t$, that takes labor, $\tilde{h}_t$, as the sole input and is subject to an exogenous productivity shock, z_t. The household hires labor from a perfectly competitive market. The demand for the intermediate input is of the form $Y_t d(p_t)$, where Y_t denotes the level of aggregate demand and p_t denotes the relative price of the intermediate good in terms of the composite consumption good. The relative price p_t equals $\tilde{P}_t/P_t$, where $\tilde{P}_t$ is the nominal price of the intermediate good produced by the household and P_t is the price of the composite consumption good. The demand function $d(\cdot)$ is assumed to be decreasing and to satisfy $d(1) = 1$ and $d'(1) < -1$.[5] The monopolist sets the price of the good it supplies taking the level of aggregate demand as given, and is constrained to satisfy demand at that price, that is,

$$z_t\tilde{h}_t \geq Y_t d(p_t). \tag{3}$$

We follow Rotemberg (1982) and introduce sluggish price adjustment by assuming that the firm faces a resource cost that is quadratic in the inflation rate of the good it produces

$$\text{Price adjustment cost } = \frac{\theta}{2}\left(\frac{\tilde{P}_t}{\tilde{P}_{t-1}} - 1\right)^2.$$

The parameter θ measures the degree of price stickiness. The higher is θ the more sluggish is the adjustment of nominal prices. If $\theta = 0$, then prices are flexible.

Each period the household is assumed to receive profits from financial institutions in the amount Π_t. The household takes Π_t as exogenous. We introduce the variable Π_t because we want to allow for the possibility that in equilibrium only a fraction of the transaction and price adjustment costs be true resource costs. We do so by assuming that part of these costs are rebated to the public in a lump-sum fashion.[6]

The flow budget constraint of the household in period t is then given by:

$$\begin{aligned} P_t c_t[1 + s(v_t)] + M_t + E_t r_{t+1} D_{t+1} \quad \leq \quad & M_{t-1} + D_t + P_t\left[p_t Y_t d(p_t) - w_t\tilde{h}_t - \frac{\theta}{2}\left(\frac{P_t}{P_{t-1}}\frac{p_t}{p_{t-1}} - 1\right)^2\right] \\ & +(1 - \tau_t)P_t w_t h_t + \Pi_t, \end{aligned} \tag{4}$$

where w_t is the real wage rate and τ_t is the labor income tax rate. The variable r_{t+1} denotes the period-t price of a claim to one unit of currency in a particular state of period $t + 1$ divided by the probability of occurrence of that state conditional on information available in period t. The left-hand side of the budget constraint represents the uses of wealth: consumption spending, including transactions costs, money holdings, and purchases of interest bearing assets. The right-hand side shows the sources of wealth: money, the payoff of contingent claims acquired in the previous period, profits from the sale of the differentiated good net of the price-adjustment cost, after-tax labor income, and profits received from financial institutions.

In addition, the household is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j\to\infty} E_t q_{t+j+1}(D_{t+j+1} + M_{t+j}) \geq 0, \tag{5}$$

at all dates and under all contingencies. The variable q_t represents the period-zero price of one unit of currency to be delivered in a particular state of period t divided by the probability of occurrence of that state given information available at time 0 and is given by

$$q_t = r_1 r_2 \ldots r_t,$$

[5] The restrictions on $d(1)$ and $d'(1)$ are necessary for the existence of a symmetric equilibrium.

[6] We motivate the rebate of the transaction cost by assuming that the amount rebated represents pure profits of financial institutions owned (in equal shares) by the public.

with $q_0 \equiv 1$.

The household chooses the set of processes $\{c_t, h_t, \tilde{h}_t, p_t, v_t, M_t, D_{t+1}\}_{t=0}^{\infty}$, so as to maximize (1) subject to (2)-(5), taking as given the set of processes $\{Y_t, P_t, \Pi_t, w_t, r_{t+1}, \tau_t, z_t\}_{t=0}^{\infty}$ and the initial condition $M_{-1} + D_0$.

Let the multiplier on the flow budget constraint be λ_t/P_t and the one on the production constraint be $mc_t\lambda_t/P_t$. Then the first-order conditions of the household's maximization problem are (2)-(5) holding with equality and

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)] \tag{6}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1-\tau_t)w_t}{1 + s(v_t) + v_t s'(v_t)} \tag{7}$$

$$v_t^2 s'(v_t) = 1 - E_t r_{t+1} \tag{8}$$

$$\frac{\lambda_t}{P_t} r_{t+1} = \beta \frac{\lambda_{t+1}}{P_{t+1}} \tag{9}$$

$$mc_t = \frac{w_t}{z_t} \tag{10}$$

$$\begin{aligned} 0 \quad = \quad & \lambda_t[Y_t d(p_t) + p_t Y_t d'(p_t) - \theta\pi_t(\pi_t p_t/p_{t-1} - 1) - mc_t Y_t d'(p_t)] \\ & + \beta\theta E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1}p_{t+1}/p_t - 1)p_{t+1}/{p_t}^2, \end{aligned} \tag{11}$$

where $\pi_t = P_t/P_{t-1}$ denotes gross consumer price inflation. The interpretation of these optimality conditions is straightforward. The first-order condition (6) states that the transaction cost introduces a wedge between the marginal utility of consumption and the marginal utility of wealth. The assumed form of the transaction cost function ensures that this wedge is zero at the satiation point $\underline{v}$ and increasing in money velocity for $v > \underline{v}$. Equation (7) shows that both the labor income tax rate and the transaction cost distort the consumption/leisure margin. Given the wage rate, households will tend to work less and consume less the higher is τ or the smaller is v_t. Equation (8) implicitly defines the household's money demand function. Note that $E_t r_{t+1}$ is the period-t price of an asset that pays one unit of currency in every state in period $t+1$. Thus $E_t r_{t+1}$ represents the inverse of the risk-free gross nominal interest rate. Formally, letting R_t denote the gross risk-free nominal interest rate, we have

$$R_t = \frac{1}{E_t r_{t+1}}$$

Our assumptions about the form of the transactions cost function imply that the demand for money is strictly decreasing in the nominal interest rate and unit elastic in consumption. Equation (9) represents a standard pricing equation for one-step-ahead nominal contingent claims. Equation (10) states that marginal cost equals the ratio of wages to the marginal product of labor. Finally, equation (11) states that the presence of price-adjustment costs prevent firms in the short run from setting their prices so as to equate marginal revenue, $p_t + d(p_t)/d'(p_t)$, to marginal cost, mc_t.

2.2 The Government

The government faces a stream of public consumption, denoted by g_t, that is exogenous, stochastic, and unproductive. These expenditures are financed by levying labor income taxes at the rate τ_t, by printing money, and by issuing one-period, risk-free (non-contingent), nominal obligations, which we denote by B_t. The government's sequential budget constraint is then given by

$$M_t + B_t = M_{t-1} + R_{t-1}B_{t-1} + P_t g_t - \tau_t P_t w_t h_t$$

for $t \geq 0$. The monetary/fiscal regime consists in the announcement of state-contingent plans for the nominal interest rate and the tax rate, $\{R_t, \tau_t\}$.

2.3 Equilibrium

We restrict attention to symmetric equilibria where all households charge the same price for the good they produce. As a result, we have that $p_t = 1$ for all t. It then follows from the fact that all firms face the same wage rate, the same technology shock, and the same production technology, that they all hire the same amount of labor. That is, $\tilde{h}_t = h_t$. Also, because all firms charge the same price, we have that the marginal revenue of the individual monopolist is constant and equal to $1 + 1/d'(1)$. Let

$$\eta = d'(1)$$

denote the equilibrium value of the elasticity of demand faced by the monopolist. Then in equilibrium equation (11) gives rise to the following expectations augmented Phillips curve

$$\lambda_t \pi_t(\pi_t - 1) = \beta E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_t \eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - \frac{w_t}{z_t}\right], \tag{12}$$

which has become a standard element in much of the recent related literature on optimal monetary policy.

Because all households are identical, in equilibrium there is no borrowing or lending among them. Thus, all interest-bearing asset holdings by private agents are in the form of government securities. That is,

$$D_t = R_{t-1}B_{t-1}$$

at all dates and all contingencies. In equilibrium, it must be the case that the nominal interest rate is non-negative,

$$R_t \geq 1.$$

Otherwise pure arbitrage opportunities would exist and households' demand for consumption would not be well defined.

Finally, as explained earlier, we assume that only a fraction of the transaction and price-adjustment costs represents a true resource costs. The remainder of these costs are assumed to be rebated to the household in a lump-sum fashion. Thus, in equilibrium

$$\Pi_t = (1-\alpha_1)c_t s(v_t) + (1-\alpha_2)\frac{\theta}{2}(\pi_t - 1)^2,$$

where $\alpha_1, \alpha_2 \in [0, 1]$.

We are now ready to define an equilibrium. A competitive equilibrium is a set of plans $\{c_t,\ h_t,\ M_t,\ B_t,\ v_t,\ mc_t,\ \lambda_t,\ P_t,\ q_t,\ r_{t+1}\}$ satisfying the following conditions:

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)] \tag{13}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1-\tau_t)z_t mc_t}{1 + s(v_t) + v_t s'(v_t)} \tag{14}$$

$$v_t^2 s'(v_t) = \frac{R_t - 1}{R_t} \tag{15}$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{16}$$

$$R_t = \frac{1}{E_t r_{t+1}} \geq 1 \tag{17}$$

$$\lambda_t \pi_t(\pi_t - 1) = \beta E_t \lambda_{t+1} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_t \eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - mc_t\right] \tag{18}$$

$$M_t + B_t + \tau_t P_t z_t mc_t h_t = R_{t-1}B_{t-1} + M_{t-1} + P_t g_t \tag{19}$$

$$\lim_{j\to\infty} E_t q_{t+j+1}(R_{t+j}B_{t+j} + M_{t+j}) = 0 \tag{20}$$

$$q_t = r_1 r_2 \ldots r_t \quad \text{with } q_0 = 1 \tag{21}$$

$$[1 + \alpha_1 s(v_t)]c_t + g_t + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 = z_t h_t \tag{22}$$

$$v_t = P_t c_t / M_t, \tag{23}$$

given policies $\{R_t, \tau_t\}$, exogenous processes $\{z_t, g_t\}$, and the initial condition $R_{-1}B_{-1} + M_{-1} > 0$. The optimal fiscal and monetary policy is the process $\{R_t, \tau_t\}$ associated with the competitive equilibrium that yields the highest level of utility to the representative household, that is, that maximizes (1). As is well known, in the absence of price stickiness, the Ramsey planner will always find it optimal to confiscate the entire initial nominal wealth of the household by choosing a policy that results in an infinite initial price level, $P_0 = \infty$. This is because such a confiscation amounts to a nondistortionary lump-sum tax. To avoid this unrealistic feature of optimal policy, it is typically assumed in the flexible price literature that the initial price level is given. We follow this tradition here to make our results comparable to this literature. However, we note that in the presence of price adjustment costs it may not be optimal for the Ramsey planner to choose $P_0 = \infty$. The reason is twofold. First, such policy would be distortionary as it would introduce a large deviation of marginal cost from marginal revenue. Second, an infinitely large initial inflation would absorb a large number of resources in the case that the implementation of price changes requires the use of real resources, which is the case if $\alpha_2 > 0$.

3 The Ramsey Problem

A key difference between our sticky-price model with non-state-contingent nominal government debt and models with flexible prices (such as Chari et al., 1991, and Schmitt-Grohé and Uribe, 2001a) or models with sticky prices but state-contingent debt (like the model considered by Correia et al., 2001) is that in our model the primal form of the competitive equilibrium cannot any longer be reduced to a single intertemporal implementability (budget) constraint in period 0 and a feasibility constraint holding in every period. This feature of the Ramsey problem is akin to the one identified by Marcet, Sargent and Seppala (2000) in their analysis of optimal policy in a real economy without state-contingent debt.

The following proposition presents a simpler form of the competitive equilibrium and establishes that it is equivalent to the definition of competitive equilibrium given in section 2.3.

Proposition 1 *Plans* $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}_{t=0}^{\infty}$ *satisfying (13), (18), (22),*

$$\lambda_t = \beta\rho(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}} \tag{24}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t,h_t)\gamma(v_t)}{U_c(c_t,h_t)}\right)h_t = \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t; \qquad t > 0 \tag{25}$$

$$\frac{c_0}{v_0} + b_0 + \left(mc_0 z_0 + \frac{U_h(c_0,h_0)\gamma(v_0)}{U_c(c_0,h_0)}\right)h_0 = \frac{R_{-1}B_{-1} + M_{-1}}{P_{-1}\pi_0} + g_0$$

$$\lim_{j\to\infty} E_t\left\{\beta^j \frac{\lambda_{t+j+1}}{\pi_{t+j+1}}\left(\rho(v_{t+j})b_{t+j} + \frac{c_{t+j}}{v_{t+j}}\right)\right\} = 0 \tag{26}$$

$$v_t \geq \underline{v} \qquad \text{and} \qquad v_t^2 s'(v_t) < 1,$$

for all dates and under all contingencies given $(R_{-1}B_{-1} + M_{-1})/P_{-1}$*, are the same as those satisfying (13)-(23), where*

$$\gamma(v_t) \equiv 1 + s(v_t) + v_t s'(v_t)$$

and

$$\rho(v_t) \equiv 1/[1 - v_t^2 s'(v_t)].$$

Proof: See the appendix.■

We will assume that the government has the ability to commit to the contingent policy rules it announces at period 0. It then follows from proposition 1 that the Ramsey problem can be stated as choosing contingent plans $c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t$ so as to maximize (1) subject to (13), (18), (22), (24)-(26), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$, taking as given $(M_{-1} + R_{-1}B_{-1})/P_0$ and the exogenous stochastic processes g_t and z_t.

The Lagrangian of the Ramsey planner's problem as well as the associated first-order conditions are shown in the appendix.

3.0.1 Alternative representation of the Ramsey constraints

While it is not possible to reduce the constraints of the Ramsey problem to a single intertemporal budget constraint in period 0 and one feasibility constraint holding at every date and at every state, as is the case under price flexibility, it is possible to express the set of constraints the Ramsey planner faces in terms of a sequence of intertemporal budget constraints rather than in terms of the sequence of transversality conditions given in (26). The next proposition presents such a representation.

Proposition 2 *Plans $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}_{t=0}^{\infty}$ satisfying the feasibility constraint (22), the expectations augmented Phillips curve*

$$\pi_t(\pi_t - 1) = \beta E_t \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \frac{\gamma(v_t)}{\gamma(v_{t+1})} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta} \left[\frac{1+\eta}{\eta} - mc_t \right] \tag{27}$$

the sequential budget constraint of the government,

$$\frac{c_t}{v_t} + b_t + \left[mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)} \right] h_t = \frac{\rho(v_{t-1}) b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t \quad \forall t \geq 1 \tag{28}$$

$$\frac{c_0}{v_0} + b_0 + \left[mc_0 z_0 + \frac{U_h(c_0, h_0)\gamma(v_0)}{U_c(c_0, h_0)} \right] h_0 = \frac{R_{-1}B_{-1} + M_{t-1}}{P_{-1}\pi_0} + g_0,$$

the sequence of intertemporal budget constraints

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ U_c(c_{t+j}, h_{t+j}) c_{t+j} \phi(v_{t+j}) + U_h(c_{t+j}, h_{t+j}) h_{t+j} + z_{t+j} h_{t+j} (mc_{t+j} - 1) \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right. \tag{29}$$

$$\left. + \alpha_2 \frac{\theta}{2} (\pi_{t+j} - 1)^2 \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \right\} = \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1}) b_{t-1}}{\pi_t} \right]$$

and the boundary conditions on v_t

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

for all dates and under all contingencies given $(R_{-1}B_{-1} + M_{-1})/P_{-1}$, are the same as those satisfying the definition of a competitive equilibrium, that is, (13)-(23).

Proof: See the appendix.■

4 Dynamic Properties of Ramsey Allocations

In this section we characterize numerically the dynamic properties of Ramsey allocations. We compute dynamics by solving first- and second-order logarithmic approximations to the Ramsey planner's policy functions around a non-stochastic Ramsey steady state. In what follows, we first describe the calibration of the model. Then we present the quantitative results. Finally, we discuss the accuracy of the solution method.

4.1 Calibration

We calibrate our model to the U.S. economy. The time unit is meant to be a year. We assume that up to period 0, the economy is in the non-stochastic steady state of a competitive equilibrium with constant paths for consumption, hours, nominal interest rates, inflation, tax rates, etc. To facilitate comparison to the case of price flexibility we adopt, where possible, the calibration of Schmitt-Grohé and Uribe (2001a). Specifically, we assume that in the steady state the inflation rate is 4.2 percent per year, which is consistent with the average growth rate of the U.S. GDP deflator over the period 1960:Q1 to 1998:Q3, that the debt-to-GDP ratio is 0.44 percent, which corresponds to the figure observed in the United States in 1995 (see the 1997 Economic Report of the President, table B-79), and that government expenditures are equal to 20 percent of GDP, a figure that is in line with postwar U.S. data. We follow Prescott (1986) and set the subjective discount rate β to 0.96 to be consistent with a steady-state real rate of return of 4 percent per year.

We assume that the single-period utility index is of the form

$$U(c,h) = \ln(c) + \delta \ln(1-h)$$

We set the preference parameter δ so that in the flexible-price steady state households allocate 20 percent of their time to work. The resulting parameter value is $\delta = 2.9$.[7]

To calibrate the price elasticity of demand η, we use the fact that in a flexible price equilibrium the markup of prices over marginal costs is related to the price elasticity of demand as $1+\mu = \eta/(1+\eta)$. Drawing from the empirical study of Basu and Fernald (1997), we assign a value of 0.2 to the value added markup of prices over marginal cost, μ. Basu and Fernald estimate gross output production functions and obtain estimates for the gross output markup of about 1.1. They show that their estimates are consistent with values for the value added markup of up to 25 percent.

To calibrate the degree of price stickiness, we use Sbordone's (1998) estimate of a linear new-Keynesian Phillips curve. Such a Phillips curve arises in our model from a log-linearization of equilibrium condition (18) around a zero-inflation steady state:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{h}{\theta\mu}\hat{mc}_t,$$

where a circumflex denotes log-deviations from the steady state. Using quarterly postwar U.S. data, Sbordone estimates the coefficient $\theta\mu/h$ to be 17.5. Given our calibration $h = 0.2$ and $\mu = 0.2$, we have that the price-stickiness coefficient θ is 17.5. As pointed out by Sbordone, in a Calvo-Yun staggered price setting model, this value of θ implies that firms change their price on average every 9 months. Because in our model the time unit is a year, we set θ equal to 17.5/4.

We use the following specification for the transactions cost technology:

$$s(v) = Av + B/v - 2\sqrt{AB} \tag{30}$$

This functional form implies a satiation point for consumption-based money velocity, $\underline{v}$, equal to $\sqrt{B/A}$. The money demand function implied by the above transaction technology is of the form

$$v_t^2 = \frac{B}{A} + \frac{1}{A}\frac{R_t - 1}{R_t},$$

To identify the parameters A and B, we estimate this equation using quarterly U.S. data from 1960:1 to 1999:3. We measure v as the ratio of non-durable consumption and services expenditures

[7]See Schmitt-Grohé and Uribe (2001a) for a derivation of the exact relations used to identify δ.

to M1. The nominal interest rate is taken to be the three-month Treasury Bill rate. The OLS estimate implies that $A = 0.0111$ and $B = 0.07524$.[8] At the calibrated steady-state interest rate of 8.2 percent per year, the implied semi-elasticity of money demand with respect to the nominal interest rate ($\partial \ln m / \partial R$) is equal to -2.82. Wwhen the nominal interest rate is zero, our money demand specification also implies a finite semi-elasticity equal to -6.6.

In our baseline calibration we assume that both transaction costs and price-adjustment costs represent true resource costs, that is, we set $\alpha_1 = \alpha_2 = 1$.

Government spending, g_t, and labor productivity, z_t, are assumed to follow independent AR(1) processes in their logarithms,

$$\ln g_t = (1 - \lambda^g) \ln g + \lambda^g \ln g_{t-1} + \epsilon_t^g; \quad \epsilon_t^g \text{ distributes } N(0, \sigma_{\epsilon^g})$$

and

$$\ln z_t = \lambda^z \ln z_{t-1} + \epsilon_t^z; \quad \epsilon_t^z \text{ distributes } N(0, \sigma_{\epsilon^z})$$

We assume that $(\lambda^g, \sigma_{\epsilon^g}) = (0.9, 0.03)$ and that $(\lambda^z, \sigma_{\epsilon^z}) = (0.82, 0.02)$. This specification is in line with the calibration of the stochastic processes for g_t and z_t given in Chari et al. (1995). Table 1 summarizes the calibration of the economy.

4.2 Numerical Results

In Schmitt-Grohé and Uribe (2001a) we show that under flexible prices it is possible to find an exact numerical solution to the Ramsey problem. The reason is that in that case the constraints of the Ramsey problem reduce to a feasibility constraint and a single intertemporal implementability constraint. On the other hand, when price adjustment is sluggish and the government issues only nominal state non-contingent debt, the Ramsey problem contains a sequence of intertemporal implementability constraints, one for each date and state. This complication renders impossible the task of finding an exact numerical solution. One is thus forced to resort to approximation techniques. In this section we limit attention to results based on log-linear approximations to the Ramsey planner's optimality conditions. In section 4.4, we present results based on a second-order approximation to the Ramsey planner's decision rules. We show there that the results of this section are robust to higher-order approximations.

Table 2 displays a number of sample moments of key macroeconomic variables under the Ramsey policy. The moments are computed as follows. We first generate simulated time series of length T for the variables of interest and compute first and second moments. We repeat this procedure J times and then compute the average of the moments. In the table, T equals 100 years and J equals 500. In section 4.4 we explain the criterion for choosing these two parameter values.

4.3 Preliminaries: Ramsey Allocations in Flexible-Price Economies

The top panel of table 2 corresponds to a flexible-price economy with perfect competition ($\theta = 0$ and $\eta = -\infty$), the middle panel to a flexible-price economy with imperfect competition ($\theta = 0$, $\eta = -6$), and the bottom panel to an economy with sluggish price adjustment and imperfect competition ($\theta = 17.5/4$ and $\eta = -6$).

Under flexible prices and perfect competition, the nominal interest rate is constant and equal to zero. That is, the Friedman rule is optimal. Because under perfect competition the nominal

[8] The estimated equation is $v_t^2 = 6.77 + 90.03(R_t - 1)/R_t$. The t-statistics for the constant and slope of the regression are, respectively, 6.81 and 5.64; The $\bar{R}^2$ of the regression is 0.16. Instrumental variable estimates using three lagged values of the dependent and independent variables yield similar estimates for A and B.

Table 1: Calibration

Symbol	Definition	Value	Description
Calibrated Parameters:			
β		0.96	Subjective discount factor
π		1.042	Gross inflation rate
h		0.2	Fraction of time allocated to work
s_g	g/y	0.2	Government consumption to GDP ratio
s_b	$B/(Py)$	0.44	Public debt to GDP ratio
$1+\mu$	$\eta/(1+\eta)$	1.2	Gross value-added markup
θ		17.5/4	Degree of price stickiness
A		0.01	Parameter of transaction cost function
			$s(v) = Av + B/v - 2\sqrt{AB}$
B		0.08	Parameter of transaction cost function
α_1		1	Fraction of transaction cost not rebated
α_2		1	Fraction of price-adjustment cost not rebated
λ^g		0.9	Serial correlation of $\log g_t$
σ_{ϵ^g}		0.0302	Standard deviation of innovation to $\ln g_t$
λ^z		0.82	Serial correlation of technology shock
σ_{ϵ^z}		0.0229	Standard deviation of innovation to $\ln z_t$

Note. The time unit is a year. The variable $y \equiv zh$ denotes steady-state output.

interest rate is zero at all times, the distortion introduced by the transaction cost is driven to zero in the Ramsey allocation ($s(v) = s'(v) = 0$). On the other hand, distortionary income taxes are far from zero. The average value of the labor income tax rate is 18.7 percent. The Ramsey planner keeps this distortion smooth over the business cycle; the standard deviation of τ is 0.04 percentage points.

In the Ramsey allocation with perfect competition and flexible prices, inflation is on average negative (-3.7 percent per year). The most striking feature of the Ramsey allocation is the high volatility of inflation. A two-standard deviation band on each side of the mean features a deflation rate of 15.7 percent at the lower end and inflation of 8.3 percent at the upper end. The Ramsey planner uses the inflation rate as a state-contingent lump-sum tax/transfer on households' financial wealth. This lump-sum tax/transfer appears to be used mainly in response to unanticipated changes in the state of the economy. This is reflected in the fact that inflation displays a near zero serial correlation.[9]

The high volatility and low persistence of the inflation rate stands in sharp contrast to the smooth and highly persistent behavior of the labor income tax rate. Our results on the dynamic properties of the Ramsey economy under perfect competition and flexible prices are consistent with those obtained by Chari et al. (1991).

Under imperfect competition and flexible prices, the volatility and correlation properties of

[9]The result that in the Ramsey equilibrium inflation acts as a lump-sum tax on wealth is due to Chari et al. (1991) and has recently been stressed by Sims (2001).

Table 2: Dynamic Properties of the Ramsey Allocation (Linear Approximation)

Variable	Mean	Std. Dev.	Auto. corr.	Corr(x, y)	Corr(x, g)	Corr(x, z)
Flexible Prices and Perfect Competition ($\theta = 0$ and $\eta = -\infty$)						
τ	18.7	0.044	0.834	-0.322	0.844	-0.516
π	-3.66	6.04	-0.0393	-0.245	0.313	-0.321
R	0	0	–	–	–	–
y	0.25	0.00843	0.782	1	0.203	0.975
h	0.25	0.00217	0.834	-0.322	0.846	-0.516
c	0.21	0.00827	0.778	0.955	-0.0797	0.997
Flexible Prices and Imperfect Competition ($\theta = 0$)						
τ	25.8	0.0447	0.616	0.236	-0.845	0.511
π	-1.82	6.8	-0.0411	-0.207	0.329	-0.321
R	1.83	0.0313	0.797	-0.237	0.845	-0.513
y	0.208	0.00675	0.783	1	0.289	0.951
h	0.208	0.0024	0.833	-0.237	0.845	-0.513
c	0.168	0.00645	0.777	0.93	-0.0624	0.998
Baseline Sticky-Price Economy						
τ	25.1	0.998	0.743	-0.283	0.476	-0.238
π	-0.16	0.171	0.0372	-0.123	0.385	-0.289
R	3.85	0.562	0.865	-0.949	-0.0372	-0.969
y	0.209	0.00713	0.815	1	0.199	0.943
h	0.208	0.00253	0.813	-0.124	0.611	-0.424
c	0.168	0.00707	0.819	0.938	-0.131	0.958

Note. τ, π, and R are expressed in percentage points and y, h, and c in levels. Unless indicated otherwise, the parameter values are: $\alpha_1 = 1$, $\alpha_2 = 1$, $\beta = 1/1.04$, $\delta = 2.9$, $g = 0.04$, $b_{-1} = 0.088$, $\eta = -6$, $\theta = 17.5/4$, $A = 0.0111$, $B = 0.07524$,

$T = 100$, and $J = 500$.

inflation, income tax rates, and other real variables are virtually unchanged. The main effect of imperfect competition is that the Friedman rule ceases to be optimal. The average nominal interest rate rises to 1.8 percent. The reason for this departure from the Friedman rule is the presence of monopoly profits. These profits represent pure rents for the owners of the monopoly rights, which the Ramsey planner would like to tax at a hundred percent rate. If profit taxes are either unavailable or restricted to be less than one hundred percent, then social planner uses inflation as an indirect tax on profits. Inflation acts as an indirect tax on profits because when consumers transform profits into consumption, they must hold money to perform the required transaction. The Friedman rule reemerges if (a) monopoly profits are completely confiscated; (b) profit tax rates are constrained to be equal to income tax rates; (c) monopolistically competitive firms make no profits (as could be the case in the presence of fixed costs); and (d) the Ramsey planner has access to consumption taxes.[10] Another difference between the perfectly and imperfectly competitive economies is that in the latter the average income tax rate is 7 percentage points higher than in the former, even though initial public debt and the process for government purchases are the same in both economies. The reason for this difference is that under imperfect competition, the labor income tax base is smaller due to the presence of market power.

4.3.1 Price Stickiness and Optimal Inflation Volatility

The bottom panel of table 2 displays some dynamic properties of the Ramsey allocation in an economy featuring market power and sticky prices. The Ramsey allocation is characterized by a near-zero average inflation rate of -0.16 percent per year. A key finding of this paper is the the dramatic drop in the standard deviation of inflation from about 7 percent per year under flexible prices to a mere 0.17 percent per year when prices adjust sluggishly.[11] This implication of the Ramsey allocation under sticky prices is more in accord with the recent literature on optimal monetary policy that ignores fiscal considerations (see the references cited in footnote 2).

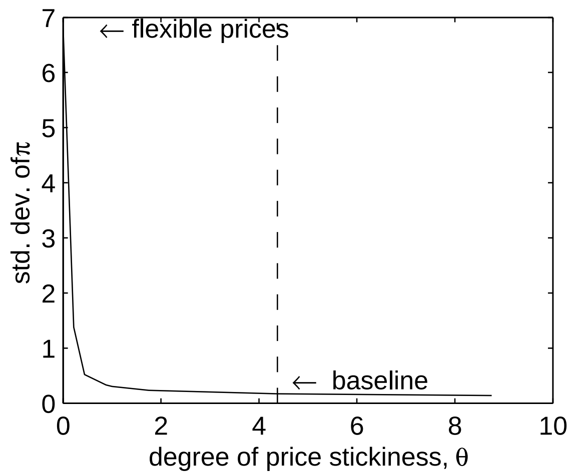
It is intuitively easy to see why allowing for price stickiness must induce a less volatile rate of inflation. If price changes are brought about at a cost, then it is natural to expect that a benevolent government will try to implement policies consistent with a more stable behavior of prices. However, the quantitative effect of an empirically plausible degree of price rigidity on inflation volatility is not clear a priori. When price adjustment is costly, the social planner faces a tradeoff. On the one hand, the planner would like to use unexpected changes in the price level as a state-contingent lump-sum tax or transfer on nominal wealth. In this way, the benevolent government avoids the need to resort to changes in distortionary taxes and interest rates over the business cycle. The use of inflation for this purpose would imply a relatively large volatility in prices. On the other hand, the Ramsey planner has incentives to stabilize the price level in order to minimize the costs associated with nominal price changes. Table 2 shows that for the degree of stickiness that has been estimated for the U.S. economy, the tradeoff is to a large extent resolved in favor of price stability.

Indeed the impact of price stickiness on the optimal degree of inflation volatility turns out to be much stronger than suggested by the numbers in table 2. Figure 1 shows that a minimum amount of price stickiness suffices to make price stability the central goal of optimal policy. Specifically, when the degree of price stickiness, embodied in the parameter θ, is assumed to be 10 times smaller than the estimated value for the US economy used in the bottom panel of table 2, the volatility of

[10] For a formal derivation of these results and a more detailed discussion, see Schmitt-Grohé and Uribe (2001a).

[11] Siu (2001) obtains similar results in a cash-credit economy where nominal rigidities are introduced by assuming that a fraction of firms must set their price one period in advance and the only source of uncertainty are government purchases shocks.

Figure 1: Degree of Price Stickiness and Optimal Inflation Volatility



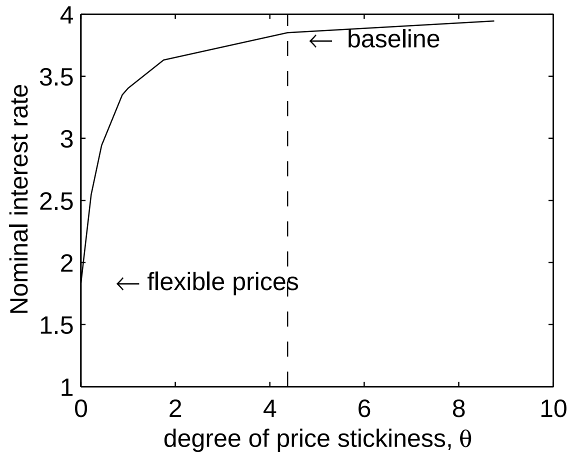
The baseline value of θ is 4.4. The standard deviation of inflation is measured in percent per year.

inflation is below 0.52 percent per year, 13 times smaller than under full price flexibility.

Therefore, the question arises as to why even a marginal degree of price stickiness can turn undesirable the use of a seemingly powerful fiscal instrument, such as large re- or devaluations of private real financial wealth through surprise inflation. Our conjecture is that in the flexible-price economy, the welfare gains of surprise inflations or deflations are very small. Our intuition is as follows. Under flexible prices it is optimal for the central bank to keep the nominal interest rate constant over the business cycle. This means that large surprise inflations must be as likely as large deflations, as variations in real interest rates are small. In other words, inflation must have a near-i.i.d. behavior. As a result, high inflation volatility cannot be used by the Ramsey planner to reduce the average amount of resources to be collected via distortionary income taxes, which would be a first-order effect. The volatility of inflation serves primarily the purpose of smoothing the process of income tax distortions—a second-order source of welfare losses—without affecting their average level.

Another way to gain intuition for the dramatic decline in optimal inflation volatility that takes place even at very modest levels of price stickiness is to interpret price volatility as a way for the government to introduce real state-contingent public debt. Under flexible prices the government uses state-contingent changes in the price level as a non-distorting tax or transfer on private holdings of government assets. In this way, non-state contingent nominal public debt becomes state-contingent in real terms. So, for example, in response to an unexpected increase in government spending (a war, say) the Ramsey planner does not need to increase tax rates by much because by inflating away part of the public debt he can ensure intertemporal budget balance. It is therefore clear that introducing costly price adjustment is as if the government was limited in its ability to issue real state-contingent debt. It follows that the larger is the welfare gain associated with the ability to issue real state-contingent public debt—as opposed to non-state contingent debt—the larger is the amount of price stickiness required to reduce the optimal degree of inflation volatility. Recent work by Marcet, Sargent, and Seppala (2000) shows that indeed the level of welfare under the Ramsey policy in an economy without state-contingent public debt is virtually the same as in an economy

Figure 2: Degree of Price Stickiness and Deviations from the Friedman Rule



The baseline value of θ is 4.4. The nominal interest rate is measured as percent per year.

with state-contingent debt. Our finding that a small amount of price stickiness is all it takes to bring the optimal volatility of inflation from a very large level to near zero is thus perfectly in line with the finding of Marcet, Sargent, and Seppala.[12]

4.3.2 Price Stickiness and Deviations from the Friedman Rule

In our baseline sticky-price economy the Friedman rule fails to hold. The average nominal interest rate is 3.8 percent per year. This significant deviation from the Friedman rule can be decomposed in two parts. First, as shown by Schmitt-Grohé and Uribe (2001a), the presence of monopolistic competition induces the social planner to tax money balances as an indirect way to tax monopoly profits. Comparing the top and middle panels of table 2, it follows that imperfect competition induces a deviation from the Friedman rule of 1.8 percentage points per year. Comparing the middle and bottom panels, it then follows that in our baseline economy price stickiness explains half of the 3.8 percentage points by which the nominal interest rate deviates from the Friedman rule. Indeed, as figure 2 illustrates, there exists a strong increasing relationship between the degree of price stickiness and the average nominal interest rate associated with the Ramsey allocation. The intuition behind this result is simple. The more costly it is for firms to alter nominal prices, the closer to zero is the inflation rate chosen by the benevolent government.

4.4 Accuracy of Solution

The quantitative results presented above are based on a log-linear approximation to the first-order conditions of the Ramsey problem. In Schmitt-Grohé and Uribe (2001a) we show how to compute exact numerical solutions to the Ramsey problem in the flexible-price economies (with perfectly and imperfectly competitive product markets). The availability of exact solutions allows us to evaluate the accuracy of the log-linear solution for the flexible-price economies considered above. The calibrations of the flexible-price economies considered in this paper and in Schmitt-Grohé and

[12]In section 5 we present further evidence that an economy with (even a small amount of) price stickiness behaves like one without real state contingent debt.

Table 3: Accuracy of the approximate numerical solution

Variable	Mean	Std. Dev.	Auto. corr.	Mean	Std. Dev.	Auto. corr.
Flexible Prices and Perfect Competition						
	Exact Solution			Log-Linear Approximation		
τ	18.8	0.0491	0.88	18.7	0.044	0.834
π	-3.39	7.47	-0.0279	-3.66	6.04	-0.0393
R	0	0	–	0	0	–
Flexible Prices and Imperfect Competition						
	Exact Solution			Log-Linear Approximation		
τ	26.6	0.042	0.88	25.8	0.0447	0.616
π	-1.46	7.92	-0.0239	-1.82	6.8	-0.0411
R	1.95	0.0369	0.88	1.83	0.0313	0.797
Baseline Sticky-Price Economy						
	Log-Quadratic Approximation			Log-Linear Approximation		
τ	25.3	0.908	0.719	25.1	0.998	0.743
π	-0.148	0.206	0.237	-0.16	0.171	0.0372
R	3.82	0.689	0.892	3.85	0.562	0.865

Note. τ, π, and R are expressed in percentage points.

Uribe (2001a) are identical. The top and middle panels of table 3 shows that the quantitative results obtained using the exact numerical solution and a log-linear approximation are remarkably close. The most noticeable differences concerns the unconditional expectation of the inflation rate and the nominal interest rate as well as the standard deviation of inflation. As expected, under the log-linear approximation the sum of the mean of inflation and the nominal interest rate are much closer to the real interest rate that prevails in the nonstochastic steady state than under the exact solution. In addition, the log-linear approximation seems to underpredict the optimal volatility of inflation somewhat.

Here we make an effort to gauge the accuracy of such approximation by comparing it to results based on a log-quadratic approximation. The details of the quadratic approximation technique are described in a technical appendix to this paper (Schmitt-Grohé and Uribe, 2001b). The results shown in the bottom panel of table 3 suggest that the log-linear and log-quadratic approximation deliver similar quantitative results. In particular the dramatic decline in inflation volatility vis-a-vis the flexible-price economy also arises under the higher-order approximation.

We close our discussion of numerical accuracy by pointing out that in both the flexible- and sticky-price economies the first-order approximation to the Ramsey allocation features a unit root. As a result, the local approximation techniques employed here become more inaccurate the longer is the simulated time series used to compute sample moments. The reason is that in the long run the log-linearized equilibrium system is bound to wander far away from the point around which the approximation is taken. We choose to restrict attention to time series of length 100 years because for this sample size the log-linear model of the flexible-price economy performs well in comparison to the exact solution. The need to keep the length of the time series relatively short also applies when a log-quadratic approximation is used. If the system deviates far from the point of approximation, then the quadratic terms might introduce large errors. These discrepancies can render the quadratic approximation even more imprecise than the lower-order one. The quadratic approximation is guaranteed to perform better than the linear one only if the system's dynamics are close enough to the point around which the model is approximated.

5 Near Random Walk Property of Taxes and Public Debt under Sticky Prices

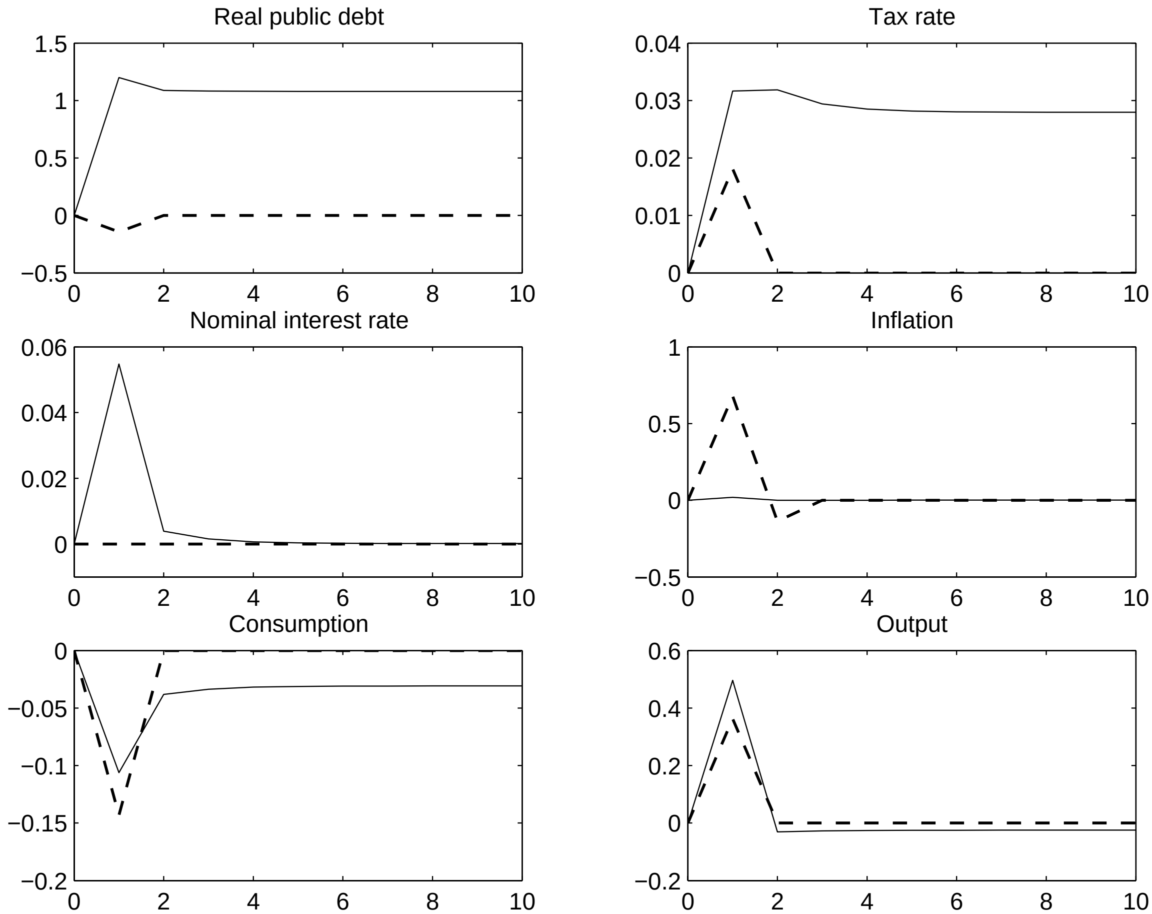
Lucas and Stokey (1983) show that under state contingent government debt tax rates and public debt inherit the stochastic process of the underlying exogenous shocks. This, implies, for example, that if the shocks driving business cycles are serially uncorrelated, so are bonds and tax rates. The work of Barro (1979) and more recently Marcet, Sargent, and Seppala (2000) suggests that the Lucas and Stokey result hinges on the assumption that the government can issue state-contingent debt. These authors show that independently of the assumed process for the shocks generating aggregate fluctuations, tax rates and public debt exhibit near random walk behavior. It is well-known from the work of Chari, Christiano, and Kehoe (1995) that the Ramsey allocation of a flexible price economy with nominally non-state-contingent debt behaves like one with real state-contingent debt. This is because state-contingent variations in prices effectively turn nominally risk-free debt into real state-contingent debt.

In this section we investigate the extent to which the introduction of nominal rigidities brings the Ramsey allocation closer to the one arising in an economy without real state contingent debt. In other words, we wish to find out whether the Barro-Marcet-Sargent-Seppala result can be obtained, not by ruling out complete markets for real public debt, but instead by introducing sticky prices in an economy in which the government issues only non-state-contingent nominal debt.

To this end, we consider the response of the flexible- and sticky-price economies under optimal fiscal and monetary policy to a serially uncorrelated government purchases shock. The result is displayed in figure 3. The response of the flexible price economy is shown with a dashed line and the response of the sticky price economy with a solid line. Government purchases are assumed to increase by 3 percent (one standard deviation) in period 1. Under flexible prices and perfect competition (the Chari, et al. economy), taxes and bonds, like the shock itself, return after one period to their pre-shock values. By contrast, under sticky prices both variables are permanently affected by the shock. Specifically, when prices are sticky, bonds and taxes jump up on impact and then converge to values above their pre-shock levels. The difference in behavior under the two model specifications can be explained entirely by the behavior of the price level. Under flexible prices, the Ramsey planner inflates away part of the real value of outstanding nominal debt, bringing real public debt to its pre-shock level in just one period. Under sticky prices, the government finds it optimal not to increase the price level much. This is because price increases are costly. Instead, the planner finances the increase in government spending partly by increasing public debt and partly by increasing taxes. In order to avoid a large distortion at the time of the shock, the planner smoothes the tax increase over time. As a consequence, the stock of public debt displays a persistent increase.

Thus, our sticky price model appears to replicate the near random walk behavior of bonds and tax rates found under the Ramsey allocation in real models without state-contingent debt, the Barro-Marcet-Sargent-Seppala result. Indeed, the Barro-Marcet-Sargent-Seppala result obtains not only under the baseline calibration of the degree of price stickiness (i.e., $\theta = 17.5/4$, or firms change prices once every 9 months), but for a minimal degree of nominal rigidities. Specifically, if we reduce θ by a factor of 10, bonds and tax rates maintain their near-random-walk behavior. This result is consistent with figure 1, which documents that a small amount of price rigidity suffices to bring the volatility of inflation close to zero.

Figure 3: Impulse response to an iid government purchases shock



— sticky-price model - - - flexible prices and perfect competition

Note: The size of the innovation in government purchases is one standard deviation (3 percent increase in g). The shock takes place in period 1. Public debt, consumption, and output are measured in percent deviations from their pre-shock levels. The tax rate, the nominal interest rate, and the inflation rate are measured in percentage points.

6 The Role of Market Incompleteness

In a recent paper, Correia et al. (2001) show that sticky prices are irrelevant for the real allocation under the Ramsey policy. In their model price stickiness takes the form of prices being set one period in advance, the demand for money stems from a cash-in-advance constraint, and the government issues a complete array of contingent claims. The results of Correia et al. might at first seem at odds with those presented in previous sections of this paper. For we show that under sticky prices consumption and hours behave quite differently than under flexible prices. In particular, we find that in the sticky-price model these two variables display near-random-walk dynamics, whereas under flexible prices they inherit the stochastic process of the exogenous shocks.

In this section we reconcile these seemingly contradictory results. We show that what leads to the difference in results is neither the assumed source of nominal rigidities nor the way in which the demand for money is motivated. Rather, what drives the difference in results is the assumed structure of government-issued assets. The following proposition presents this result formally. It shows that under complete contingent claims markets the real allocation implied by the Ramsey problem in our economy is identical under sticky and flexible prices.

Proposition 3 *(Irrelevance of price stickiness with state-contingent government debt) Suppose that* $\alpha_2 = 0$ *and that in any competitive equilibrium* $\lim_{j\to\infty} E_t\beta^j\lambda_{t+j}\pi_{t+j}(\pi_{t+j}-1) = 0$. *Then if the government can issue state-contingent nominal debt, the real allocation* $\{c_t, h_t, v_t\}$ *is the same under sticky prices* ($\theta > 0$) *as under flexible prices* ($\theta = 0$).

Proof: See the appendix.■

The assumptions under which the irrelevance of price stickiness under complete markets is proved are that sticky prices involve no direct resource costs ($\alpha_2 = 0$) and that the equilibrium inflation rate and marginal utility of wealth do not grow at a rate greater than the discount factor. The assumption that α_2 must be zero is obviously necessary. One can show that if α_2 is greater than zero, then under complete markets the Ramsey constraints take the form of a feasibility equation like (22) and a single intertemporal implementability constraint in c_t, v_t, h_t, and π_t. Our conjecture is that in this case, while the real allocation will differ from the one arising under full price flexibility, it will be similar to it in the sense that real variables will not display near random walk behavior.

7 Interest-Rate Feedback Rules

In this section we address the question of whether the time series arising from the Ramsey allocation imply a relation between the nominal interest rate, inflation, and output consistent with available estimates of such relationship for U.S. data. In recent years there has been a revival of empirical and theoretical research aimed at understanding the macroeconomic consequences of monetary policy regimes that take the form of interest-rate feedback rules. One driving force of this renewed interest can be found in empirical studies showing that in the past two decades monetary policy in the United States is well described as following such a rule. In particular, an influential paper by Taylor (1993) characterizes the Federal Reserve as following a simple rule whereby the federal funds rate is set as a linear function of inflation and the output gap with coefficients of 1.5 and 0.5, respectively. Taylor emphasizes the stabilizing role of an inflation coefficient greater than unity, which loosely speaking implies that the central bank raises real interest rates in response to increases in the rate of inflation. After his seminal paper, interest-rate feedback rules with this feature have become known as Taylor rules. Taylor rules have also been shown to represent an adequate description of monetary policy in other industrialized economies (see, for example, Clarida, Galí, and Gertler, 1998).

To see whether the nominal interest rate process associated with the Ramsey allocation can be well represented by a linear combination of inflation and output, we estimate the following regression using artificial time series from the sticky-price model.

$$R_t = \beta_0 + \beta_1 \pi_t + \beta_2 y_t + u_t.$$

Here the nominal interest rate, R_t, and inflation, π_t, are measured in percent per year, and output, y_t, is measured as percent deviation from its mean value. To generate time series for R_t, π_t, and y_t, we draw artificial time series of size 100 for the two shocks driving business cycles in our model, government consumption and productivity shocks. We use these realizations to compute the implied time series of the endogenous variables of interest using the baseline calibration of the sticky-price model. We then proceed to estimate the above equation. We repeat this procedure 500 times and take the median of the estimated regression coefficients. The OLS estimate of the interest rate feedback rule is

$$R_t = 0.04 - 0.14\,\pi_t - 0.16\,y_t + u_t; \quad R^2 = 0.92.$$

Clearly, an interest rate feedback rule fits quite well the optimal interest rate process. The R^2 coefficient of the regression is above 90 percent. However, the estimated interest-rate feedback rule does not resemble a Taylor rule. First, the coefficient on inflation is less than unity, and indeed insignificantly different from zero with a negative point estimate. Second, the output coefficient is negative. The results are essentially unchanged if one estimates the feedback rule by instrumental variables using lagged values of π, y, and R as instruments. Thus, an econometrician working with a data sampled from the Ramsey economy would conclude that monetary policy is passive, in the sense that the interest rate does not seem to react to changes in the rate of inflation.

The results are also insensitive to the introduction of a smoothing term à la Sack (1998) in the above interest-rate rule. Specifically, adding the nominal interest rate with one lag to the set of explanatory variables yields

$$R_t = 0.03 + 0.15\,\pi_t - 0.11\,y_t + 0.34\,R_{t-1} + u_t; \quad R^2 = 0.96.$$

One issue that has attracted the attention of both empirical and theoretical studies on interest-rate feedback rules is whether the central bank looks at contemporaneous or past measures of inflation. It turns out that in our Ramsey economy, a backward-looking rule also features an inflation coefficient significantly less than one. Specifically, replacing π_t with π_{t-1} in our original specification of the interest rate rule we obtain

$$R_t = 0.04 + 0.21\,\pi_{t-1} - 0.16\,y_t + u_t; \quad R^2 = 0.92.$$

We close this section by pointing out that the results should not be interpreted as suggesting that optimal monetary policy can be implemented by passive interest-rate feedback rules like the ones estimated above. In order to arrive at such conclusion, one would have, in addition, to identify the underlying fiscal regime. Then, one would have to check whether in a competitive equilibrium where the government follows the resulting monetary/fiscal regime, welfare of the representative household is close enough to that obtained under the Ramsey allocation. An obvious problem that one might encounter in performing this exercise is that the competitive equilibrium fails to be unique at the estimated policy regime. This is a matter that deserves further investigation.

8 Conclusion

The focus of this paper is the implications of price stickiness for the optimal degree of price volatility in economies where the government does not have access to lump-sum taxation and public debt is state-non-contingent. The central finding is that for plausible calibrations of the degree of nominal rigidity the volatility of inflation associated with the Ramsey allocation is near zero. Indeed, a very small amount of price stickiness suffices to make the optimal inflation volatility many times lower than the one arising under full price flexibility.

One can interpret this result as indicating that in the U.S. economy the degree of sluggish price adjustment is high enough to induce a benevolent government to refrain from generating surprises in the price level as a way to tax nominal wealth in response to adverse shocks. (This is the primary role of inflation under flexible prices.) Instead, the government concentrates on keeping a relatively stable path for prices.

An important assumption driving the result that significantly less inflation volatility is desirable in the presence of sticky prices is that government debt is state-noncontingent. When government debt is state contingent, the presence of sticky prices may introduce no difference in the in the Ramsey real allocation (see also Correia et al., 2001). The reason for this result is that, as shown in Lucas and Stokey (1983), when government debt is state contingent and prices are fully flexible, the Ramsey allocation does not pin down the price level uniquely. In this case there is an infinite number of price level processes (and thus of money supply processes) that can be supported as Ramsey outcomes. Loosely speaking, the introduction of price stickiness simply "uses this degree of freedom" without altering other aspects of the Ramsey solution. This is not possible under state-noncontingent debt. For in this case the price level is uniquely determined in the flexible-price economy. Thus, the presence of nominal rigidities modifies the optimal real allocation in fundamental ways.

Appendix

Proof of Proposition 1

We first show that plans $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}$ satisfying (13)-(23) also satisfy (24) (25), (26) $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$. It follows from the definition of $\rho(v_t)$ and (15) that $\rho(v_t) = R_t$. It is easy to see then that (15), (17), and assumption 1 together imply that $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$. Taking expectations conditional on information available at time t of (16), using the definition of $\rho(v_t)$, and combining it with (17) one obtains (24). To obtain (25) divide (19) by P_t. Solve (14) for τ_t and use the resulting expression to eliminate τ_t from (19). Use (23) to replace M_t/P_t and let $b_t = B_t/P_t$. Finally, multiply and divide (20) by P_{t+j} and replace q_{t+j+1} with (21) and (16). Multiply by $\lambda_t/(q_t P_t)$ to get(26).

Next, we must show that for any plan $\{c_t, h_t, v_t, \pi_t, \lambda_t, b_t, mc_t\}$ satisfying (13), (18), (22), (24) (25), (26) and $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$ one can construct plans $\{M_t,\ B_t,\ q_t,\ r_{t+1}, \tau_t, R_t\}$ so that (14)-(17), (19)-(21), and (23) hold at all dates and under all contingencies. Set τ_t such that (14) holds. Set $R_t = \rho(v_t)$. It follows from the definition of $\rho(v_t)$ that (15) holds. Assumption 1, the constraints $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$ ensure that $R_t \geq 1$. Let r_{t+1} be given by (16). Taking expected value and comparing the resulting expression to (24) shows that (17) is satisfied. With r_t in hand, let q_t be given by (21). Using $B_t = b_t P_t$ and (23) to write $M_t/P_t = c_t/v_t$, and the definition of τ_t we recover (19). Let $P_t = \pi_t P_{t-1}$ and recall that P_{-1} is given. Multiply (26) by $q_t P_t/\lambda_t$. Note that $q_t P_t \lambda_t \beta^j \lambda_{t+j+1}/\pi_{t+j+1}$ using (16) and (21) can be expressed as $q_{t+j+1}P_{t+j}$. Finally, replace c_{t+j}/v_{t+j} with (23) to obtain (20).$\blacksquare$

Proof of Proposition 2

We first show that plans $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}$ satisfying (13)-(23) also satisfy (27)-(29), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$. It follows from the definition of $\rho(v_t)$ and (15) that $\rho(v_t) = R_t$. It is easy to see then that (15), (17), and assumption 1 together imply that $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$. To obtain (27) divide (18) by λ_t and then use (13) to eliminate λ_t. Next divide (19) by P_t. Solve (14) for τ_t and use the resulting expression to eliminate τ_t from (19). Use (23) to replace M_t/P_t and let $b_t = B_t/P_t$. This yields (28). For any $t, j \geq 0$, (19) can be written as

$$M_{t+j} + B_{t+j} + \tau_{t+j}P_{t+j}z_{t+j}mc_{t+j}h_{t+j} = R_{t+j-1}B_{t+j-1} + M_{t+j-1} + P_{t+j}g_{t+j}$$

Let $W_{t+j+1} = R_{t+j}B_{t+j} + M_{t+j}$ and note that W_{t+j+1} is in the information set of time $t+j$. Use this expression to eliminate B_{t+j} from (19) and multiply by q_{t+j} to obtain

$$q_{t+j}M_{t+j}(1 - R_{t+j}^{-1}) + q_{t+j}E_{t+j}r_{t+j+1}W_{t+j+1} - q_{t+j}W_{t+j} = q_{t+j}[P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}mc_{t+j}z_{t+j}h_{t+j}],$$

where we use (17) to write R_{t+j} in terms of r_{t+j+1}. Take expectations conditional on information available at time t and sum for $j = 0$ to $j = J$

$$E_t \sum_{j=0}^{J} \left[q_{t+j}M_{t+j}(1 - R_{t+j}^{-1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}mc_{t+j}z_{t+j}h_{t+j}) \right] = -E_t q_{t+J+1}W_{t+J+1} + q_t W_t.$$

Take limits for $J \to \infty$. By (20) the limit of the right hand side is well defined and equal to $q_t W_t$. Thus, the limit of the left-hand side exists. This yields:

$$E_t \sum_{j=0}^{\infty} \left[q_{t+j}M_{t+j}(1 - R_{t+j}^{-1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}mc_{t+j}z_{t+j}h_{t+j}) \right] = q_t W_t$$

By (16) we have that $P_{t+j}q_{t+j}/q_t = \beta^j \lambda_{t+j} P_t/\lambda_t$. Use (13) to eliminate λ_{t+j}, (23) to eliminate M_{t+j}/P_{t+j} to obtain

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \left[\frac{c_{t+j}}{v_{t+j}}(1 - R_{t+j}^{-1}) - (g_{t+j} - \tau_{t+j} mc_{t+j} z_{t+j} h_{t+j}) \right] = \frac{W_t}{p_t} \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$

Solve (14) for τ_{t+j}. Then $\tau_{t+j} mc_{t+j} z_{t+j} h_{t+j} = mc_{t+j} z_{t+j} h_{t+j} + \gamma(v_{t+j})/U_c(c_{t+j}, h_{t+j}) U_h(c_{t+j}, h_{t+j}) h_{t+j}$. Use this in the above expression and replace g_{t+j} with (22). This yields

$$E_t \sum_{j=0}^{\infty} \beta^j \Bigg[U_c(c_{t+j}, h_{t+j}) c_{t+j} \frac{1 + \alpha_1 s(v_{t+j}) + \frac{1 - R_{t+j}^{-1}}{v_{t+j}}}{\gamma(v_{t+j})} + U_h(c_{t+j}, h_{t+j}) h_{t+j} +$$
$$\frac{z_{t+j} h_{t+j} U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})}(mc_{t+j} - 1) + \alpha_2 \frac{\theta}{2}(\pi_{t+j} - 1)^2 \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \Bigg] = \frac{W_t}{P_t} \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$

Finally, use (15) to replace $(1 - R_{t+j}^{-1})/v_{t+j}$ with $v_{t+j} s'(v_{t+j})$ and use the definitions of $\phi(v_t)$ and W_t to get (29).

We next show that that plans $\{c_t, h_t, v_t, \pi_t, b_t, mc_t\}$ satisfying (22), (27)-(29), and $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$ also satisfy (13)-(23). Construct λ_t so that it satisfies (13). Let τ_t be given by (14). Let R_t be given by (15). Let r_{t+1} be given by (16). Let q_t be given by (21) and M_t/P_t by (23). By the same arguments given in the proof of Proposition 2 on can show that (18) and (19) then hold. Thus, what remains to be shown is that (17) and (20) are satisfied. Note that $R_t = \rho(v_t) = 1/[1 - v_t^2 s'(v_t)]$, then the restriction $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$ and assumption 1 imply that $R_t \geq 1$. Write (29) as

$$U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + z_t h_t (mc_t - 1) \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$
$$+ E_t \sum_{j=1}^{\infty} \beta^j \Bigg\{ U_c(c_{t+j}, h_{t+j}) c_{t+j} \phi(v_{t+j}) + U_h(c_{t+j}, h_{t+j}) h_{t+j} + z_{t+j} h_{t+j} (mc_{t+j} - 1) \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \quad (31)$$
$$+ \alpha_2 \frac{\theta}{2}(\pi_{t+j} - 1)^2 \frac{U_c(c_{t+j}, h_{t+j})}{\gamma(v_{t+j})} \Bigg\} = \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1}) b_{t-1}}{\pi_t} \right]$$

Make a change of index $h = j - 1$.

$$U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + z_t h_t (mc_t - 1) \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$
$$+ \beta E_t \sum_{h=0}^{\infty} \beta^h \{ U_c(c_{t+h+1}, h_{t+h+1}) c_{t+h+1} \phi(v_{t+h+1}) + U_h(c_{t+h+1}, h_{t+h+1}) h_{t+h+1}$$
$$+ z_{t+h+1} h_{t+h+1} (mc_{t+h+1} - 1) \frac{U_c(c_{t+h+1}, h_{t+h+1})}{\gamma(v_{t+h+1})} \quad (32)$$
$$+ \alpha_2 \frac{\theta}{2}(\pi_{t+h+1} - 1)^2 \frac{U_c(c_{t+h+1}, h_{t+h+1})}{\gamma(v_{t+h+1})} \Bigg\} = \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1}) b_{t-1}}{\pi_t} \right]$$

Using (29) this expression can be simplified to read:

$$U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + z_t h_t (mc_t - 1) \frac{U_c(c_t, h_t)}{\gamma(v_t)} + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 \frac{U_c(c_t, h_t)}{\gamma(v_t)}$$
$$+ \beta E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{\gamma(v_{t+1})} \left[\frac{c_t/v_t + \rho(v_t) b_t}{\pi_{t+1}} \right] \right\} = \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1}) b_{t-1}}{\pi_t} \right] \quad (33)$$

Take expectations of (16) and use the resulting expression to eliminate $\beta E_t \left\{ \frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})\pi_{t+1}} \right\}$. This yields

$$
\begin{aligned}
U_c(c_t,h_t)c_t\phi(v_t) + U_h(c_t,h_t)h_t + z_t h_t(mc_t - 1)\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \alpha_2\frac{\theta}{2}(\pi_t-1)^2\frac{U_c(c_t,h_t)}{\gamma(v_t)} \\
+E_t r_{t+1}\frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[c_t/v_t + \rho(v_t)b_t\right] = \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left[\frac{c_{t-1}/v_{t-1} + \rho(v_{t-1})b_{t-1}}{\pi_t}\right]
\end{aligned}
\tag{34}
$$

Multiply by $P_t\gamma(v_t)/U_c(c_t,h_t)$ and replace $\alpha_2\theta/2(\pi_t-1)^2$ with (22). Combine (15) with (23) to express , $c_t/v_t(v_t^2 s'(v_t))$ as $M_t/P_t(1-R_t^{-1})$. Finally, use (14) to replace $U_h/U_c\gamma(v_t)h_t$. The resulting expression is

$$
M_t(1-R_t^{-1}) + \tau P_t mc_t z_t h_t - P_t g_t + E_t r_{t+1}(M_t + R_t B_t) = M_{t-1} + R_{t-1}B_{t-1} \tag{35}
$$

Subtracting (19) from this expression it follows that (17) must hold. Finally, we must show that (20) holds. Multiply (19) in period $t+j$ by q_{t+j} and take information conditional on information available at time t to get

$$
E_t[q_{t+j}M_{t+j}(1-r_{t+j+1}) + q_{t+j+1}W_{t+j+1}] = E_t[q_{t+j}W_{t+j} + q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}w_{t+j}h_{t+j})]
$$

Now sum for $j=0$ to $j=J$.

$$
E_t\sum_{j=0}^{J}[q_{t+j}M_{t+j}(1-r_{t+j+1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}w_{t+j}h_{t+j})] = -E_t q_{t+J+1}W_{t+J+1} + q_t W_t
$$

Divide by $q_t P_t$

$$
E_t\sum_{j=0}^{J}\frac{q_{t+j}P_{t+j}}{q_t P_t}[(c_{t+j}/v_{t+j})(1-r_{t+j+1}) - (g_{t+j} - \tau_{t+j}w_{t+j}h_{t+j})] = -E_t q_{t+J+1}W_{t+J+1}/(q_t P_t) + \frac{W_t}{P_t}
$$

It follows from (29) that the limit of the left-hand side of the above expression as $J \to \infty$ is W_t/P_t. Hence the limit of the right-hand side is well defined. It then follows that

$$
\lim_{J\to\infty} E_t q_{t+J+1}W_{t+J+1} = 0
$$

for every date t. Using the definition of W_t, one obtains immediately (20). ■

The Lagrangian of the Ramsey Problem

$$
\begin{aligned}
\mathcal{L} \;=\; & E_0\sum_{t=0}^{\infty}\beta^t\Big\{U(c_t,h_t) + \lambda_t^f\left[z_t h_t - [1+\alpha_1 s(v_t)]c_t - g_t - \alpha_2\frac{\theta}{2}(\pi_t-1)^2\right] \\
& +\lambda_t^b\left[\lambda_t - \beta\rho(v_t)E_t\frac{\lambda_{t+1}}{\pi_{t+1}}\right] \\
& +\lambda_t^s\left[\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t,h_t)\gamma(v_t)}{U_c(c_t,h_t)}\right)h_t - \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} - \frac{c_{t-1}}{v_{t-1}\pi_t} - g_t\right] \\
& +\lambda_t^p\left[\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}(\pi_{t+1}-1) + \frac{\eta z_t h_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right) - \pi_t(\pi_t-1)\right] \\
& + \lambda_t^c[U_c(c_t,h_t) - \lambda_t\gamma(v_t)]\Big\}
\end{aligned}
\tag{36}
$$

First-Order Conditions of the Ramsey problem for $t \geq 1$

$$z_t h_t = [1 + \alpha_1 s(v_t)]c_t + g_t + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 \tag{37}$$

$$\lambda_t = \beta \rho(v_t) E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{38}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)} \right) h_t = \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t \tag{39}$$

$$\pi_t(\pi_t - 1) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta} \left(\frac{1+\eta}{\eta} - mc_t \right) \tag{40}$$

$$U_c(c_t, h_t) = \lambda_t \gamma(v_t) \tag{41}$$

$$U_c(t) - \lambda_t^f [1 + \alpha_1 s(v_t)] + \frac{\lambda_t^s}{v_t} + \lambda_t^s h_t \gamma(v_t) M_c(t) - \beta E_t \frac{\lambda_{t+1}^s}{v_t \pi_{t+1}} + \lambda_t^c U_{cc}(t) = 0 \tag{42}$$

$$U_h(t) + \lambda_t^f z_t + \lambda_t^s(mc_t z_t + M_t \gamma(v_t)) + \lambda_t^s h_t M_h(t)\gamma(v_t) + \frac{\lambda_t^p \eta z_t}{\theta} \left(\frac{1+\eta}{\eta} - mc_t \right) + \lambda_t^c U_{ch}(t) = 0 \tag{43}$$

$$\lambda_t^b - \frac{\rho(v_{t-1})\lambda_{t-1}^b}{\pi_t} - \beta \frac{\lambda_t^p}{\lambda_t^2} E_t \lambda_{t+1} \pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_{t-1}^p}{\lambda_{t-1}} \pi_t(\pi_t - 1) - \lambda_t^c \gamma(v_t) = 0 \tag{44}$$

$$-\lambda_t^f \alpha_1 s'(v_t) c_t - \beta \lambda_t^b \rho'(v_t) E_t \frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t^s c_t}{v_t^2} + \lambda_t^s M_t h_t \gamma'(v_t) - \beta b_t \rho'(v_t) E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} + \beta \frac{c_t}{v_t^2} E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} - \lambda_t^c \lambda_t \gamma'(v_t) = 0 \tag{45}$$

$$-\lambda_t^f \alpha_2 \theta(\pi_t - 1) + \lambda_{t-1}^b \rho(v_{t-1}) \frac{\lambda_t}{\pi_t^2} + \lambda_t^s \frac{\rho(v_{t-1})b_{t-1} + c_{t-1}/v_{t-1}}{\pi_t^2} + \lambda_{t-1}^p \frac{\lambda_t}{\lambda_{t-1}}(2\pi_t - 1) - \lambda_t^p(2\pi_t - 1) = 0 \tag{46}$$

$$\lambda_t^s = \beta \rho(v_t) E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} \tag{47}$$

$$\lambda_t^s = \frac{\eta}{\theta} \lambda_t^p \tag{48}$$

$$\lim_{j \to \infty} E_t \left\{ \beta^j \frac{\lambda_{t+j+1}}{\pi_{t+j+1}} \left(\rho(v_{t+j}) b_{t+j} + \frac{c_{t+j}}{v_{t+j}} \right) \right\} = 0 \tag{49}$$

First-Order Conditions of the Ramsey Problem at time 0

$$z_t h_t = [1 + \alpha_1 s(v_t)]c_t + g_t + \alpha_2 \frac{\theta}{2}(\pi_t - 1)^2 \tag{50}$$

$$\lambda_t = \beta\rho(v_t)E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{51}$$

$$\frac{c_t}{v_t} + b_t + \left(mc_t z_t + \frac{U_h(c_t, h_t)\gamma(v_t)}{U_c(c_t, h_t)} \right) h_t = \frac{\rho(v_{t-1})b_{t-1}}{\pi_t} + \frac{c_{t-1}}{v_{t-1}\pi_t} + g_t \tag{52}$$

$$\pi_t(\pi_t - 1) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}(\pi_{t+1} - 1) + \frac{\eta z_t h_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right) \tag{53}$$

$$U_c(c_t, h_t) = \lambda_t \gamma(v_t) \tag{54}$$

$$U_c(t) - \lambda_t^f[1 + \alpha_1 s(v_t)] + \frac{\lambda_t^s}{v_t} + \lambda_t^s h_t \gamma(v_t) M_c(t) - \beta E_t \frac{\lambda_{t+1}^s}{v_t \pi_{t+1}} + \lambda_t^c U_{cc}(t) = 0 \tag{55}$$

$$U_h(t) + \lambda_t^f z_t + \lambda_t^s(mc_t z_t + M_t \gamma(v_t)) + \lambda_t^s h_t M_h(t)\gamma(v_t) + \frac{\lambda_t^p \eta z_t}{\theta}\left(\frac{1+\eta}{\eta} - mc_t\right) + \lambda_t^c U_{ch}(t) = 0 \tag{56}$$

$$\lambda_t^b - \beta \frac{\lambda_t^p}{\lambda_t^2} E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1} - 1) - \lambda_t^c \gamma(v_t) = 0 \tag{57}$$

$$-\lambda_t^f \alpha_1 s'(v_t)c_t - \beta\lambda_t^b \rho'(v_t)E_t \frac{\lambda_{t+1}}{\pi_{t+1}} - \frac{\lambda_t^s c_t}{v_t^2} + \lambda_t^s M_t h_t \gamma'(v_t) - \beta b_t \rho'(v_t) E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} + \beta \frac{c_t}{v_t^2} E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} - \lambda_t^c \lambda_t \gamma'(v_t) = 0 \tag{58}$$

$$\lambda_t^s = \beta\rho(v_t)E_t \frac{\lambda_{t+1}^s}{\pi_{t+1}} \tag{59}$$

$$\lambda_t^s = \frac{\eta}{\theta}\lambda_t^p \tag{60}$$

$$\lim_{j\to\infty} E_t \left\{ \beta^j \frac{\lambda_{t+j+1}}{\pi_{t+j+1}} \left(\rho(v_{t+j})b_{t+j} + \frac{c_{t+j}}{v_{t+j}} \right) \right\} = 0 \tag{61}$$

Steady State of the Ramsey Economy

Assume that $b_t = b_{-1}$ for all t and that $x_t = x_{t-1} = x_{t+1} = x$ for all endogenous and exogenous variables. Also, $z = 1$. Note that the steady-state value of the marginal cost $mc_t = w_t/z_t$ is simply w.

$$h = [1 + \alpha_1 s(v)]c + g + \alpha_2 \frac{\theta}{2}(\pi - 1)^2 \tag{62}$$

$$1 = \beta\rho(v)\frac{1}{\pi} \tag{63}$$

$$\frac{c}{v} + b + \left(w + \frac{U_h(c,h)\gamma(v)}{U_c(c,h)}\right) h = \frac{\rho(v)b}{\pi} + \frac{c}{v\pi} + g \tag{64}$$

$$\pi(\pi - 1) = \frac{\eta h}{\theta(1-\beta)}\left(\frac{1+\eta}{\eta} - w\right) \tag{65}$$

$$U_c(c,h) = \lambda\gamma(v) \tag{66}$$

$$U_c - \lambda^f[1 + \alpha_1 s(v)] + \frac{\lambda^s}{v} + \lambda^s h\gamma(v)M_c - \beta\frac{\lambda^s}{v\pi} + \lambda^c U_{cc} = 0 \tag{67}$$

$$U_h + \lambda^f + \lambda^s(w + M\gamma(v)) + \lambda^s h M_h \gamma(v) + \frac{\lambda^p \eta}{\theta}\left(\frac{1+\eta}{\eta} - w\right) + \lambda^c U_{ch} = 0 \tag{68}$$

$$\lambda^b - \frac{\rho(v)\lambda^b}{\pi} - \beta\frac{\lambda^p}{\lambda^2}\lambda\pi(\pi - 1) + \frac{\lambda^p}{\lambda}\pi(\pi - 1) - \lambda^c\gamma(v) = 0 \tag{69}$$

$$-\lambda^f\alpha_1 s'(v)c - \beta\lambda^b\rho'(v)\frac{\lambda}{\pi} - \frac{\lambda^s c}{v^2} + \lambda^s M h\gamma'(v) - \beta b\rho'(v)\frac{\lambda^s}{\pi} + \beta\frac{c}{v^2}\frac{\lambda^s}{\pi} - \lambda^c\lambda\gamma'(v) = 0 \tag{70}$$

$$-\lambda^f\alpha_2\theta(\pi - 1) + \lambda^b\rho(v)\frac{\lambda}{\pi^2} + \lambda^s\frac{\rho(v)b + c/v}{\pi^2} + \lambda^p(2\pi - 1) - \lambda^p(2\pi - 1) = 0 \tag{71}$$

$$\pi = \beta\rho(v) \tag{72}$$

$$\lambda^s = \frac{\eta}{\theta}\lambda^p \tag{73}$$

Proof of Proposition 3

We begin by defining a competitive equilibrium in the complete markets sticky price economy. A competitive equilibrium is a set of plans $\{c_t,\, h_t,\, M_t,\, v_t,\, mc_t,\, \lambda_t,\, P_t,\, q_t,\, r_{t+1}\}$ satisfying the following conditions:

$$U_c(c_t, h_t) = \lambda_t[1 + s(v_t) + v_t s'(v_t)] \tag{74}$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = \frac{(1-\tau_t)z_t mc_t}{1 + s(v_t) + v_t s'(v_t)} \tag{75}$$

$$v_t^2 s'(v_t) = 1 - E_t r_{t+1} \geq 0 \tag{76}$$

$$\lambda_t r_{t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{77}$$

$$\lambda_t \pi_t(\pi_t - 1) = \beta E_t \lambda_{t+1}\pi_{t+1}(\pi_{t+1} - 1) + \frac{\lambda_t \eta z_t h_t}{\theta}\left[\frac{1+\eta}{\eta} - mc_t\right] \tag{78}$$

$$M_t + E_t r_{t+1} D_{t+1} + \tau_t P_t z_t mc_t h_t = D_t + M_{t-1} + P_t g_t \tag{79}$$

$$\lim_{j\to\infty} E_t q_{t+j+1}(D_{t+j+1} + M_{t+j}) = 0 \tag{80}$$

$$q_t = r_1 r_2 \ldots r_t \quad \text{with } q_0 = 1 \tag{81}$$

$$[1 + \alpha_1 s(v_t)]c_t + g_t = z_t h_t \tag{82}$$

$$v_t = P_t c_t / M_t, \tag{83}$$

given policies $\{D_t, \tau_t\}$, exogenous processes $\{z_t, g_t\}$, and the initial condition $D_0 + M_{-1} > 0$.

Claim 1 *Plans* $\{c_t, h_t, v_t\}_{t=0}^{\infty}$ *satisfying*

$$z_t h_t = [1 + \alpha s(v_t)]c_t + g_t, \tag{84}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{U_c(c_t, h_t)}{\gamma(v_t)} \frac{z_t h_t}{\eta} \right\} = \frac{U_c(c_0, h_0)}{\gamma(v_0)} \left(\frac{D_0 + M_{-1}}{P_0} + \frac{\theta}{\eta}\pi_0(\pi_0 - 1) \right), \tag{85}$$

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2 s'(v_t) < 1,$$

given $D_0 + M_{-1}$ *and* P_0*, where*

$$\gamma(v_t) \equiv 1 + s(v_t) + v_t s'(v_t)$$

and

$$\phi(v_t) \equiv [1 + \alpha_1 s(v_t) + v_t s'(v_t)]/\gamma(v_t),$$

are the same as those satisfying (74)- (83).

Proof: We first show that plans $\{c_t, h_t, v_t\}$ satisfying (74)-(83) also satisfy (84), (85), $v_t \geq \underline{v}$, and $v_t^2 s'(v_t) < 1$.

Obviously, (82) implies (84). Furthermore, (76) and the fact that $r_{t+1} > 0$ because $\lambda_t, P_t > 0$ in the competitive equilibrium imply that $v_t^2 s'(v_t) < 1$. Assumption 1 together with (76) implies that $v_t \geq \underline{v}$. Let $W_{t+1} = D_{t+1} + M_t$ and note that W_{t+1} is in the information set of time $t+1$. Use this expression to eliminate D_t from (79) and multiply by q_t to obtain

$$q_t M_t (1 - E_t r_{t+1}) + q_t E_t r_{t+1} W_{t+1} - q_t W_t = q_t [P_t g_t - \tau_t P_t z_t mc_t h_t].$$

Take expectations conditional on information available at time zero and sum for $t = 0$ to $t = T$ to obtain

$$E_0 \sum_{t=0}^{T} [q_t M_t (1 - E_t r_{r+1}) - q_t (P_t g_t - \tau_t P_t z_t mc_t h_t)] = -E_0 q_{T+1} W_{T+1} + W_0.$$

In writing this expression, we use the fact that $q_0 = 1$. Take limits for $T \to \infty$. By (80) the limit of the right hand side is well defined and equal to W_0. Thus, the limit of the left-hand side exists. This yields:

$$E_0 \sum_{t=0}^{\infty} [q_t M_t (1 - E_t r_{t+1}) - q_t (P_t g_t - \tau_t P_t z_t mc_t h_t)] = W_0$$

By (77) we have that $P_t q_t = \beta^t \lambda_t P_0 / \lambda_0$. Use (74) to eliminate λ_t, (83) to eliminate M_t/P_t, and (76) to replace $(1 - E_t r_{t+1})$ to obtain

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_c(c_t, h_t)}{\gamma(v_t)} \left[c_t v_t s'(v_t) - (g_t - \tau_t z_t mc_t h_t) \right] = \frac{W_0}{P_0} \frac{U_c(c_0, h_0)}{\gamma(v_0)}$$

Solve (14) for τ_t and (78) for mc_t. Then $\tau_t z_t mc_t h_t = (1+\eta)/\eta z_t h_t + \gamma(v_t)/U_c(c_t, h_t) U_h(c_t, h_t) h_t + \frac{\theta}{\eta \lambda_t} [\beta E_t \lambda_{t+1} \pi_{t+1} (\pi_{t+1} - 1) - \lambda_t \pi_t (\pi_t - 1)]$. Use this in the above expression and replace g_t with (82). This yields

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t \left[U_c(c_t, h_t) c_t \phi(v_t) + U_h(c_t, h_t) h_t + \frac{z_t h_t}{\eta} \frac{U_c(c_t, h_t)}{\gamma(v_t)} \right] \\ & + \frac{\theta}{\eta} \left[\lim_{j \to \infty} E_0 \beta^{j+1} \lambda_{j+1} \pi_{j+1} (\pi_{j+1}) - \frac{U_c(c_0, h_0)}{\gamma(v_0)} \pi_0 (\pi_0 - 1) \right] \\ & \qquad = \frac{W_0}{P_0} \frac{U_c(c_0, h_0)}{\gamma(v_0)}, \end{aligned}$$

where we used the definitions $\phi(v_t) = \frac{1 + \alpha s(v_t) + v_t s'(v_t)}{\gamma(v_t)}$. If we now make the additional assumption that in any competitive equilibrium

$$\lim_{j \to \infty} E_0 \beta^{j+1} \lambda_{j+1} \pi_{j+1} (\pi_{j+1}) = 0$$

and use $W_0 = D_0 + M_{-1}$, then we obtain (85).

Now we show that plans $\{c_t, h_t, v_t\}$ that satisfy $v_t \geq \underline{v}$, $v_t^2 s'(v_t) < 1$, (84), and (85) also satisfy (74)-(83) at all dates and all contingencies.

Clearly, (84) implies (82). Given a plan $\{c_t, h_t, v_t\}$ proceed as follows. Use (76) to construct a process for r_{t+1}. Note that given assumption 1, the constraints $v_t \geq \underline{v}$ and $v_t^2 s'(v_t) < 1$ ensure that $1 - E_t r_{t+1} \geq 0$ and that $E_t r_{t+1} > 0$. Let λ_t be given by (74) and P_{t+1} be given by (77). In this way, we obtain for a given choice of the process for r_{t+1} a unique process for π_t. Hence we can use (78) to construct a process for mc_t. Of course, only if the resulting marginal cost process is positive can the plan $\{c_t, h_t, v_t\}$ be supported as a competitive equilibrium, that is, we would like to ensure that $mc_t > 0$. We assume that there exists at least one choice of the r_{t+1} process such that $mc_t > 0$ all dates all contingencies. Then let τ_t be given by (75) and M_t by (83). Construct D_{t+1} (one contingent claim pay-off for each state of the world in $t+1$) as the solution to:

$$E_{t+1}\sum_{j=0}^{\infty}\beta^j\left[U_c(c_{t+j+1},h_{t+j+1})c_{t+j+1}\phi(v_{t+j+1}) + U_h(c_{t+j+1},h_{t+j+1})h_{t+j+1}\right.$$
$$\left.+\frac{z_{t+j+1}h_{t+j+1}}{\eta}\frac{U_c(c_{t+j+1},h_{t+j+1})}{\gamma(v_{t+j+1})}\right] = \left(\frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})}\right)\left(\frac{D_{t+1}+M_t}{P_{t+1}} + \frac{\theta}{\eta}\pi_{t+1}(\pi_{t+1}-1)\right) \quad (86)$$

We wish to show that if D_t is defined in this way, then(79), (80) and (81) also hold. Use the definition of D_t and (85) to get for any $t \geq 0$:

$$\begin{aligned}
& U_c(c_t,h_t)c_t\phi(v_t) + U_h(c_t,h_t)h_t + \frac{z_t h_t}{\eta}\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \\
& E_t\sum_{j=1}^{\infty}\beta^j\left[U_c(c_{t+j},h_{t+j})c_{t+j}\phi(v_{t+j}) + U_h(c_{t+j},h_{t+j})h_{t+j} + \frac{z_{t+j}h_{t+j}}{\eta}\frac{U_c(c_{t+j},h_{t+j})}{\gamma(v_{t+j})}\right] \\
= \; & \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left(\frac{D_t+M_{t-1}}{P_t} + \frac{\theta}{\eta}\pi_t(\pi_t-1)\right)
\end{aligned}$$

Make a change of index. Let $k = j-1$ and use the definition of D_{t+1}. Then the above expression can be written as

$$\begin{aligned}
& U_c(c_t,h_t)c_t\phi(v_t) + U_h(c_t,h_t)h_t + \frac{z_t h_t}{\eta}\frac{U_c(c_t,h_t)}{\gamma(v_t)} + \\
& \beta E_t\left[\frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})}\left(\frac{D_{t+1}+M_t}{P_{t+1}} + \frac{\theta}{\eta}\pi_{t+1}(\pi_{t+1}-1)\right)\right] = \frac{U_c(c_t,h_t)}{\gamma(v_t)}\left(\frac{D_t+M_{t-1}}{P_t} + \frac{\theta}{\eta}\pi_t(\pi_t-1)\right)
\end{aligned}$$

Multiplying by $\gamma(v_t)P_t/U_c(c_t,h_t)$ yields

$$\begin{aligned}
& P_t c_t(1+\alpha_1 s(v_t) + v_t s'v(t)) + U_h(c_t,h_t)/U_c(c_t,h_t)\gamma(v_t)P_t h_t + \frac{P_t z_t h_t}{\eta} \\
& +\beta E_t\left[P_t\frac{U_c(c_{t+1},h_{t+1})}{\gamma(v_{t+1})}\frac{\gamma(v_t)}{U_c(c_t,h_t)}\left(\frac{D_{t+1}+M_t}{P_{t+1}} + \frac{\theta}{\eta}\pi_{t+1}(\pi_{t+1}-1)\right)\right] = \left(D_t + M_{t-1} + P_t\frac{\theta}{\eta}\pi_t(\pi_t-1)\right)
\end{aligned}$$

Then use (74) and (77) to simplify the expression to

$$\begin{aligned}
& P_t c_t(1+\alpha_1 s(v_t) + v_t s'v(t)) + U_h(c_t,h_t)/U_c(c_t,h_t)\gamma(v_t)P_t h_t + \frac{P_t z_t h_t}{\eta} \\
& +E_t\left[r_{t+1}(D_{t+1}+M_t) + \beta\frac{\theta}{\eta}\frac{P_t\lambda_{t+1}}{\lambda_t}\pi_{t+1}(\pi_{t+1}-1)\right] = \left(D_t + M_{t-1} + P_t\frac{\theta}{\eta}\pi_t(\pi_t-1)\right)
\end{aligned}$$

Using (75), (78), (76), and (82) this expression can be written as:

$$\tau_t z_t mc_t P_t h_t + E_t r_{t+1} D_{t+1} + M_t = D_t + M_{t-1} + P_t g_t,$$

which is (79).

Finally, we must show that (80) holds. Let q_t be given by (81). We just established that (79) holds at every date and under every contingency. Multiply (79) in period $t + j$ by q_{t+j} and take expectations conditional on information available at time t to get

$$E_t[q_{t+j}M_{t+j}(1 - r_{t+j+1}) + q_{t+j+1}W_{t+j+1}] = E_t[q_{t+j}W_{t+j} + q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}z_{t+j}mc_{t+j}h_{t+j})]$$

Now sum for $j = 0$ to $j = J$.

$$E_t\sum_{j=0}^{J}[q_{t+j}M_{t+j}(1 - r_{t+j+1}) - q_{t+j}(P_{t+j}g_{t+j} - \tau_{t+j}P_{t+j}z_{t+j}mc_{t+j}h_{t+j})] = -E_tq_{t+J+1}W_{t+J+1} + q_tW_t$$

Divide by q_tP_t

$$E_t\sum_{j=0}^{J}\frac{q_{t+j}P_{t+j}}{q_tP_t}[(c_{t+j}/v_{t+j})(1 - r_{t+j+1}) - (g_{t+j} - \tau_{t+j}z_{t+j}mc_{t+j}h_{t+j})] = -E_tq_{t+J+1}W_{t+J+1}/(q_tP_t) + \frac{W_t}{P_t}$$

Using the definition of D_t given by (86) and the assumption that

$$\lim_{J\to\infty} E_t\beta^{J+1}\lambda_{J+1}\pi_{J+1}(\pi_{J+1}) = 0$$

at every date and every contingency, it follows that the limit of the left-hand side of the above expression as $J \to \infty$ is W_t/P_t. Hence the limit of the right-hand side is well defined. It then follows that

$$\lim_{J\to\infty} E_tq_{t+J+1}W_{t+J+1} = 0$$

for every date t. Using the definition of W_t, one obtains (80). ■

Finally, Schmitt-Grohé and Uribe (2001a) show that the constraints of the Ramsey problem under flexible prices are (84),

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{U_c(c_t,h_t)c_t\phi(v_t) + U_h(c_t,h_t)h_t + \frac{U_c(c_t,h_t)}{\gamma(v_t)}\frac{z_th_t}{\eta}\right\} = \frac{U_c(c_0,h_0)}{\gamma(v_0)}\left(\frac{D_0 + M_{-1}}{P_0}\right),$$

$$v_t \geq \underline{v} \quad \text{and} \quad v_t^2s'(v_t) < 1,$$

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