

# Aggregation and the PPP Puzzle in a Sticky-Price Model\*

Carlos Carvalho  
Federal Reserve Bank of New York

Fernanda Nechio  
Princeton University

September, 2008

## Abstract

We study the purchasing power parity (PPP) puzzle in a multi-sector, two-country, sticky-price model. Across sectors, firms differ in the extent of price stickiness, in accordance with recent microeconomic evidence on price setting in various countries. Combined with local currency pricing, this leads sectoral real exchange rates to have heterogeneous dynamics. We show analytically that in this economy, deviations of the real exchange rate from PPP are more volatile and persistent than in a counterfactual one-sector world economy that features the same average frequency of price changes, and is otherwise identical to the multi-sector world economy. When simulated with a sectoral distribution of price stickiness that matches the microeconomic evidence for the U.S. economy, the model produces a half-life of deviations from PPP of 45 months. In contrast, the half-life of such deviations in the counterfactual one-sector economy is only slightly above one year. As a by-product, our model provides a decomposition of this difference in persistence that allows a structural interpretation of the different approaches found in the empirical literature on aggregation and the real exchange rate. In particular, we reconcile the apparently conflicting findings that gave rise to the “PPP Strikes Back debate” (Imbs et al. 2005a,b and Chen and Engel 2005).

*JEL classification codes:* F30, F41, E00

*Keywords:* heterogeneity, aggregation, price stickiness, real exchange rates, PPP puzzle

---

\*We thank seminar participants at the NBER Summer Institute IFM 2008, SOEGW Conference 2008, NY Area Workshop on Monetary Economics, Johns Hopkins, Princeton and NY Fed for helpful comments. We are also grateful to Gianluca Benigno, Roberto Chang, Michael Devereux, Gauti Eggertsson, Charles Engel, Pierre-Olivier Gourinchas, Jean Imbs, Anthony Landry, Paolo Pesenti, Ricardo Reis, H el ene Rey, Chris Sims, and Thomas Wu for comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System. E-mails: carlos.carvalho@ny.frb.org, fnechio@princeton.edu.

# 1 Introduction

Purchasing power parity (PPP) states that, once converted to the same currency, price levels across countries should be equal. As a result, the real exchange rate between any two countries - the ratio of their price levels in a common currency - should be constant and equal to unity. A more flexible version of PPP postulates that real exchange rates should be constant, but not necessarily equal to one. In contrast with the tight predictions of either version of PPP, in the data real exchange rates display large and long-lived fluctuations around their average levels. Rogoff's (1996) survey of the empirical literature on the subject reports a "consensus view" that places estimates of the half-life of deviations from PPP in the range of 3 to 5 years. While he suggests that the high volatility of the real exchange rate could be explained by a model with monetary shocks and nominal rigidities, so far models of this type with plausible nominal frictions have failed to produce the large persistence found in the data; hence, the puzzle.

In this paper, we study the PPP puzzle in a multi-sector, two-country, sticky-price model. We depart from the existing literature by introducing heterogeneity in the frequency of price changes across sectors, in accordance with recent microeconomic evidence on price setting for various countries (e.g. Bils and Klenow 2004; Dhyne et al. 2006 for the Euro area). Combined with local currency pricing, these differences in the extent of price stickiness lead sectoral real exchange rates to have heterogeneous dynamics, which are also evident in the data (Imbs et al. 2005a).

We isolate the role of heterogeneity by comparing the dynamic behavior of the aggregate real exchange rate in such a multi-sector economy with the behavior of the real exchange rate in an otherwise identical one-sector world economy with the same average frequency of price changes. We refer to this auxiliary economy as the *counterfactual one-sector world economy*. We show that, in response to nominal shocks, the aggregate real exchange rate in the heterogeneous economy is more volatile and persistent than in the counterfactual one-sector world economy, and that the difference can be arbitrarily large.

We then investigate whether quantitatively our multi-sector model can solve the PPP puzzle, i.e. produce highly volatile and persistent real exchange rates in response to monetary disturbances, under a plausible parametrization. In particular, to discipline our analysis we use a cross-sectional distribution of the frequency of price changes that matches the recent microeconomic evidence for the U.S. economy. We ask the same question in the counterfactual one-sector world economy. Our multi-sector model produces a half-life of deviations from PPP of 45 months, well within the consensus view of 3 to 5 years. In contrast, such deviations in the one-sector world economy are short-lived, with a half-life only slightly above one year. In order to produce deviations from PPP with a half-life of 45 months, the one-sector model requires firms to change prices much less

frequently - on average once every 20-21 months. The volatility of the real exchange rate is also much higher in the heterogeneous economy (by a factor that ranges from 2.5 to more than 5, depending on the specification of the model). These results obtain in the *absence* of strategic complementarities in price-setting decisions. Thus, heterogeneity in price stickiness can lead nominal disturbances to have very persistent effects on real exchange rates, without the need for the propagation mechanism that such complementarities provide.

Our quantitative findings reveal that the counterfactual one-sector world economy is a poor representation of the multi-sector model. As a result of cross-sectional aggregation of sectoral exchange rates with heterogeneous dynamics, the aggregate real exchange rate in the multi-sector economy displays much richer dynamics than the real exchange rate in the counterfactual one-sector model. As our analytical results show, the volatility and persistence of real exchange rates are convex functions of the frequency of price adjustments, which leads the counterfactual one-sector model to understate both quantities relative to the underlying heterogeneous economy.

We start by presenting our multi-sector general equilibrium model in Section 2. It has two countries trading intermediate goods produced by monopolistically competitive firms, which are divided into sectors that differ in the frequency of price changes. Firms can price-discriminate across the two countries, and set prices in the currency of the market in which the good is sold. Consumers supply labor to these intermediate firms and consume the non-traded final good, which is produced by competitive firms that bundle the intermediate goods from the two countries.

Using common assumptions about preferences and nominal shocks, Section 3 presents analytical results that show that the volatility and persistence of the aggregate real exchange rate in the multi-sector economy are larger than in the counterfactual one-sector model. It provides a decomposition of such difference in persistence into two terms: an *aggregation effect* - defined as the difference between the persistence of the aggregate real exchange rate of the heterogeneous economy and the (weighted) average persistence of sectoral exchange rates; and a *counterfactuality effect* - defined as the difference between the former weighted average and the persistence of the real exchange rate in the counterfactual one-sector world economy. This decomposition clarifies the role of heterogeneity and aggregation in accounting for the difference in real exchange rate persistence across the two world economies.

Section 4 presents results from simulations of the model using the cross-sectional distribution of the frequency of price changes from Nakamura and Steinsson (2008). It shows that in response to monetary shocks our multi-sector model generates much higher volatility and persistence than the counterfactual one-sector model. The counterfactuality effect accounts for well over 90% of this difference in persistence, whereas the aggregation effect explains the residual difference. We also

present several robustness exercises, and find that our results can survive important departures from the baseline specification.

In Section 5 we use our structural model to revisit the empirical literature on heterogeneity, aggregation and the real exchange rate. Using the same data as in Imbs et al. (2005a), and our structural model as a guide, we estimate and decompose the total effect of heterogeneity on persistence into the aggregation and counterfactuality effects defined in accordance with our theory. While the former effect only depends on estimates of persistence of real exchange rates for which we have data, the latter requires an estimate of the persistence of the real exchange rate in the counterfactual one-sector economy. We show how this can be obtained by applying a Mean Group estimator for panel datasets with heterogeneous dynamics (Pesaran and Smith 1995).

We estimate a half-life of 46 months for aggregate real exchange rates, replicating exactly the results of Imbs et al. (2005a). The average of estimated half-lives of the underlying sector-country real exchange rates that comprise the panel dataset is very close to the aggregate, at 43 months. In turn, the half-life for the counterfactual one-sector real exchange rate process obtained with the Mean Group estimator is 26 months (also replicating exactly the findings of Imbs et al. 2005a).<sup>1</sup> Around 90% of the difference between the estimated half-life for aggregate real exchange rates and the half-life of the counterfactual real exchange rate obtained with the Mean Group estimator is attributed to the counterfactuality effect. The aggregation effect accounts for the remaining fraction.

Our results reconcile the apparently conflicting findings of the empirical literature on heterogeneity, aggregation and the real exchange rate that gave rise to the so-called “PPP Strikes Back debate” (Imbs et al. 2005a,b and Chen and Engel 2005). The bottom line is that different papers have measured different objects. In particular, Chen and Engel (2005) measure the “aggregation bias” as the difference between the persistence of the aggregate real exchange rate and the average persistence across its underlying components, and find it to be small. This is what we define in our model as the aggregation effect. We also find it to be small, both in our simulated model and in the data used in Imbs et al. (2005a).<sup>2</sup> In contrast, Imbs et al. (2005a) measure what they term a “dynamic aggregation bias” or “dynamic heterogeneity bias” as the difference between estimates of persistence of aggregate real exchange rates (or estimates based on panel-data methods that impose homogeneous dynamics across units of the panel), and estimates of persistence based on methods that account for heterogeneous dynamics. They find this “bias” to be large. Our structural model

---

<sup>1</sup>We simulate a version of the model with equally-weighted sectors and apply the same estimation methods to model-generated data (to make the results comparable to those of the empirical exercise). The results are very similar to those obtained with the actual data.

<sup>2</sup>Crucini and Shintani (2008) measure aggregation bias in a similar way, and also find it to be small in their data.

allows an interpretation of the results obtained with their preferred statistical method, which turns out to measure what we define in our model as the total effect of heterogeneity; that is, the *sum* of the aggregation and counterfactual effects. We find the latter to be large, both in our simulated model and in the data.

Finally, to obtain a more comprehensive picture of how the model fares when confronted with other dimensions of the PPP puzzle, in Section 6 we investigate the properties of prices, nominal and real exchange rates. In particular, we focus on the volatility of these variables, and on the correlation between real and nominal exchange rates. While falling short of matching those features of the data as well as it matches the persistence of the aggregate real exchange rate, we find that our multi-sector model performs better than its one-sector counterpart in essentially all of those dimensions. We also analyze some cross-sectional implications of our model for the dynamic properties of sectoral real exchange rates, and relate our results to the findings of Kehoe and Midrigan (2007).

We conclude that our multi-sector sticky-price model can produce an aggregate real exchange rate that is quite volatile, and as persistent as in the data. However, our findings still leave open a series of important questions. These include the role of monetary policy and its transmission mechanism, the interplay of the various shocks that can hit the economy, and the stability of our findings across different policy regimes. We conclude in Section 7 with a discussion of some of these issues.

Our paper is naturally related to the growing literature that focuses on the aggregate implications of heterogeneity in price setting.<sup>3</sup> It contributes to the body of work that uses dynamic sticky-price models to study the persistence of real exchange rates, such as Bergin and Feenstra (2001), Kollman (2001), Chari et al. (2002), Benigno (2004), Steinsson (2008), Johri and Lahiri (2008), and Martinez-Garcia and Søndergaard (2008). There is also a connection between the results from our multi-sector model, and the findings of the literature on cross-sectional aggregation of time-series processes (e.g. Granger and Morris 1976; Granger 1980; Zaffaroni 2004). Our focus on economic implications as opposed to purely statistical aspects of aggregation links our work with Abadir and Talmain (2002). A specific version of our model is close to Kehoe and Midrigan (2007), who analyze cross-sectional implications for sectoral real exchange rates. Finally, our paper shares with Ghironi and Melitz (2005) and Atkeson and Burstein (2008) the themes of heterogeneity and real exchange rate dynamics. However, while we focus on the PPP puzzle in a sticky-price model, they emphasize productivity shocks in flexible price models.

---

<sup>3</sup>Carvalho and Schwartzman (2008) provide detailed references.

## 2 The model

The world economy consists of two symmetric countries, *Home* and *Foreign*. In each country, identical infinitely lived consumers supply labor to intermediate firms that they own, invest in a complete set of state-contingent assets, and consume a non-traded final good. The latter is produced by competitive firms that bundle varieties of intermediate goods produced in the two countries. The monopolistically competitive intermediate firms that produce these varieties are divided into sectors that differ in their frequency of price changes. Labor is the variable input in the production of intermediate goods, which are the only goods that are traded. Intermediate producers can price-discriminate across countries, and set prices in local currency.

The Home representative consumer maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t C_t + E_t [\Theta_{t,t+1} B_{t+1}] \leq W_t N_t + B_t + T_t,$$

where  $E_t$  is the familiar time- $t$  expectations operator,  $C_t$  is consumption of the final good,  $N_t$  is labor,  $P_t$  is the price of the final good,  $W_t$  is the nominal wage,  $T_t$  stands for profits received from Home intermediate firms, and  $\beta$  is the time-discount factor.  $B_{t+1}$  stands for the state contingent value of the portfolio of financial securities held by the consumer at the beginning of  $t+1$ . Complete markets allow agents to choose the value of  $B_{t+1}$  for each possible state of the world at all times, and a no-arbitrage condition requires the existence of a nominal stochastic discount factor  $\Theta_{t,t+1}$  that prices in period  $t$  any financial asset portfolio with value  $B_{t+1}$  at the beginning of period  $t+1$ . To avoid cluttering the notation we omit explicit reference to the different states of nature. Finally,  $\sigma^{-1}$  denotes the intertemporal elasticity of substitution and  $\gamma^{-1}$  is the Frisch elasticity of labor supply.

To rule out ‘‘Ponzi Schemes,’’ agents’ financial wealth must be, at all times and states, large enough to avoid default:

$$B_t \geq - \sum_{s=0}^{\infty} E_t [\Theta_{t,t+s} (W_{t+s} N_{t+s} + T_{t+s})] \geq -\infty,$$

where  $\Theta_{t,t} = 1$ , and  $\Theta_{t,t+s} \equiv \prod_{r=t+1}^{t+s} \Theta_{r-1,r}$  for  $s > 0$ .

The first order conditions for the consumer's problem are:

$$\frac{C_t^{-\sigma}}{C_{t+s}^{-\sigma}} = \frac{\beta^s}{\Theta_{t,t+s}} \frac{P_t}{P_{t+s}}, \quad (1)$$

$$C_t^\sigma N_t^\gamma = \frac{W_t}{P_t},$$

where (1) holds for each future state of nature. The solution must also satisfy a transversality condition:

$$\lim_{s \rightarrow \infty} E_t [\Theta_{t,t+s} B_{t+s}] = 0.$$

The Foreign consumer solves an analogous problem. She maximizes:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{*1-\sigma} - 1}{1-\sigma} - \frac{N_t^{*1+\gamma}}{1+\gamma} \right),$$

subject to the flow budget constraint:

$$P_t^* C_t^* + E_t \left[ \Theta_{t,t+1}^* \frac{B_{t+1}^*}{\mathcal{E}_t} \right] \leq W_t^* N_t^* + \frac{B_t^*}{\mathcal{E}_t} + T_t^*, \quad (2)$$

where a “\*” superscript denotes the Foreign counterpart of the corresponding Home variable, and  $\mathcal{E}_t$  is the nominal exchange rate, defined here as the price of the Foreign currency in terms of the Home currency.  $\mathcal{E}_t$  is thus quoted in units of Home currency per unit of the Foreign currency. Without loss of generality and for simplicity, we assume that the complete set of state-contingent assets are denominated in the Home currency. As a result, in the budget constraint (2)  $B_t^*$  appears divided by the nominal exchange rate, to convert the value of the portfolio into Foreign currency.

The optimality conditions are:

$$\frac{C_t^{*-\sigma}}{C_{t+s}^{*-\sigma}} = \frac{\beta^s}{\Theta_{t,t+s}^*} \frac{\mathcal{E}_t P_t^*}{\mathcal{E}_{t+s} P_{t+s}^*}, \quad (3)$$

$$C_t^{*\sigma} N_t^{*\gamma} = \frac{W_t^*}{P_t^*},$$

where, again, (3) holds for each future state of nature, and a transversality condition:

$$\lim_{s \rightarrow \infty} E_t [\Theta_{t,t+s}^* B_{t+s}^*] = 0.$$

The stochastic discount factor has to be the same for both countries, since assets are freely traded and there are no arbitrage opportunities. Letting  $Q_t \equiv \mathcal{E}_t \frac{P_t^*}{P_t}$  denote the real exchange rate,

from equations (1) and (3) this implies:

$$Q_{t+s} = Q_t \frac{C_t^{-\sigma} C_{t+s}^{*-\sigma}}{C_{t+s}^{-\sigma} C_t^{*-\sigma}}. \quad (4)$$

Iterating equation (4) backwards and assuming  $Q_0 \frac{C_0^{-\sigma}}{C_0^{*-\sigma}} = 1$ , yields:

$$Q_t = \frac{C_t^{*-\sigma}}{C_t^{-\sigma}}.$$

The Home final good is produced by a representative competitive firm that bundles varieties of intermediate goods from both countries. Each variety is produced by a monopolistically competitive firm. Intermediate firms are divided into sectors indexed by  $k \in \{1, \dots, K\}$ , each featuring a continuum of firms. To highlight the role of heterogeneity in price stickiness, across sectors these intermediate firms only differ in their pricing practices, as we detail below. Overall, firms are indexed by the country where they produce, by their sector, and are further indexed by  $j \in [0, 1]$ . The distribution of firms across sectors is given by sectoral weights  $f_k > 0$ , with  $\sum_{k=1}^K f_k = 1$ .

The technology employed to produce the final good is given by:

$$Y_t = \left( \sum_{k=1}^K f_k^\eta Y_{k,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (5)$$

$$Y_{k,t} = \left( \omega^{\frac{1}{\rho}} Y_{H,k,t}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} Y_{F,k,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (6)$$

$$Y_{H,k,t} = \left( f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{H,k,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (7)$$

$$Y_{F,k,t} = \left( f_k^{\frac{\theta-1}{\theta}} \int_0^1 Y_{F,k,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad (8)$$

where  $Y_t$  denotes the Home final good,  $Y_{k,t}$  is the aggregation of the Home and Foreign intermediate goods produced by sector  $k$  to be sold in Home,  $Y_{H,k,t}$  and  $Y_{F,k,t}$  are the aggregation of intermediate varieties produced by firms in sector  $k$  in Home and Foreign, respectively, to be sold in Home, and  $Y_{H,k,j,t}$  and  $Y_{F,k,j,t}$  are the varieties produced by firm  $j$  in sector  $k$  in Home and Foreign to be sold in Home. Finally,  $\eta \geq 0$  is the elasticity of substitution across sectors,  $\rho \geq 0$  is the elasticity of substitution between Home and Foreign goods,  $\theta > 1$  is the elasticity of substitution within sectors, and  $\omega \in [0, 1]$  is the steady-state share of domestic inputs.

The maximization problem of a representative Home final good producing firm is:

$$\begin{aligned} \max \quad & P_t Y_t - \left( \sum_{k=1}^K f_k \int_0^1 (P_{H,k,j,t} Y_{H,k,j,t} + P_{F,k,j,t} Y_{F,k,j,t}) dj \right) \\ \text{s.t.} \quad & (5)-(8). \end{aligned}$$

The first order conditions, for  $j \in [0, 1]$  and  $k = 1, \dots, K$ , are given by:

$$Y_{H,k,j,t} = \omega \left( \frac{P_{H,k,j,t}}{P_{H,k,t}} \right)^{-\theta} \left( \frac{P_{H,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t, \quad (9)$$

$$Y_{F,k,j,t} = (1 - \omega) \left( \frac{P_{F,k,j,t}}{P_{F,k,t}} \right)^{-\theta} \left( \frac{P_{F,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t. \quad (10)$$

The underlying price indices are:

$$P_t = \left( \sum_{k=1}^K f_k P_{k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \quad (11)$$

$$P_{k,t} = \left( \omega P_{H,k,t}^{1-\rho} + (1 - \omega) P_{F,k,t}^{1-\rho} \right)^{\frac{1}{1-\rho}}, \quad (12)$$

$$P_{H,k,t} = \left( \int_0^1 P_{H,k,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (13)$$

$$P_{F,k,t} = \left( \int_0^1 P_{F,k,j,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (14)$$

where  $P_t$  is the price of the Home final good,  $P_{k,t}$  is the price index of sector  $k$  intermediate goods sold in Home,  $P_{H,k,t}$  is the price index for sector  $k$  Home-produced intermediate goods sold in Home, and  $P_{H,k,j,t}$  is the price charged in the Home market by Home firm  $j$  from sector  $k$ .  $P_{F,k,t}$  is the price index for sector  $k$  Foreign-produced intermediate goods sold in Home, and  $P_{F,k,j,t}$  is the price charged in the Home market by Foreign firm  $j$  from sector  $k$ . Both  $P_{H,k,j,t}$  and  $P_{F,k,j,t}$  are set in the Home currency.

With an analogous maximization problem, the Foreign final firm chooses its demands for intermediate inputs from Foreign ( $Y_{F,k,j,t}^*$ ) and Home ( $Y_{H,k,j,t}^*$ ) producers:

$$Y_{F,k,j,t}^* = \omega \left( \frac{P_{F,k,j,t}^*}{P_{F,k,t}^*} \right)^{-\theta} \left( \frac{P_{F,k,t}^*}{P_{k,t}^*} \right)^{-\rho} \left( \frac{P_{k,t}^*}{P_t^*} \right)^{-\eta} Y_t^*, \quad (15)$$

$$Y_{H,k,j,t}^* = (1 - \omega) \left( \frac{P_{H,k,j,t}^*}{P_{H,k,t}^*} \right)^{-\theta} \left( \frac{P_{H,k,t}^*}{P_{k,t}^*} \right)^{-\rho} \left( \frac{P_{k,t}^*}{P_t^*} \right)^{-\eta} Y_t^*. \quad (16)$$

The Foreign price indices are analogous to the Home ones (equations (11)-(14)):

$$P_t^* = \left( \sum_{k=1}^K f_k P_{k,t}^{*1-\eta} \right)^{\frac{1}{1-\eta}}, \quad (17)$$

$$P_{k,t}^* = \left( \omega P_{F,k,t}^{*1-\rho} + (1 - \omega) P_{H,k,t}^{*1-\rho} \right)^{\frac{1}{1-\rho}}, \quad (18)$$

$$P_{H,k,t}^* = \left( \int_0^1 P_{H,k,j,t}^{*1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (19)$$

$$P_{F,k,t}^* = \left( \int_0^1 P_{F,k,j,t}^{*1-\theta} dj \right)^{\frac{1}{1-\theta}}, \quad (20)$$

where  $P_t^*$  is the price of the Foreign final good,  $P_{k,t}^*$  is the price index of sector  $k$  intermediate goods sold in Foreign,  $P_{F,k,t}^*$  is the price index for sector  $k$  Foreign-produced intermediate goods sold in Foreign, and  $P_{F,k,j,t}^*$  is the price charged in the Foreign market by Foreign firm  $j$  from sector  $k$ .  $P_{H,k,t}^*$  is the price index for sector  $k$  Home-produced intermediate goods sold in Foreign, and  $P_{H,k,j,t}^*$  is the price charged in the Foreign market by Home firm  $j$  from sector  $k$ . Both  $P_{F,k,j,t}^*$  and  $P_{H,k,j,t}^*$  are set in the Foreign currency.

For ease of reference, we refer to  $P_{H,k,t}$ ,  $P_{F,k,t}$ ,  $P_{H,k,t}^*$ ,  $P_{F,k,t}^*$  as *country-sector* price indices, and to  $P_{k,t}$ ,  $P_{k,t}^*$  as *sectoral price indices*. We can then define the *sectoral real exchange rate* for sector  $k$  as the ratio of sectoral price indices in a common currency:

$$Q_{k,t} \equiv \mathcal{E}_t \frac{P_{k,t}^*}{P_{k,t}}.$$

For analytical tractability, we assume that intermediate firms set prices as in Calvo (1983). The frequency of price changes varies across sectors, and it is the only source of (ex-ante) heterogeneity. Thus, sectors in the model are naturally identified with their frequency of price changes. In each period, each firm  $j$  in sector  $k$  changes its price independently with probability  $\alpha_k$ .<sup>4</sup> To keep track of the sectors, we order them in terms of increasing price stickiness, so that  $\alpha_1 > \dots > \alpha_K$ .

Each time Home firm  $j$  from sector  $k$  adjusts, it chooses prices  $X_{H,k,j,t}$ ,  $X_{H,k,j,t}^*$  to be charged in the Home and Foreign markets, respectively, with each price being set in the corresponding local currency. Thus, its maximization problem is:

$$\begin{aligned} \max E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1 - \alpha_k)^s & \left( X_{H,k,j,t} Y_{H,k,j,t+s} + \mathcal{E}_{t+s} X_{H,k,j,t}^* Y_{H,k,j,t+s}^* - W_{t+s} N_{k,j,t+s} \right) \\ \text{s.t. } & (9), (16), \\ & Y_{H,k,j,t} + Y_{H,k,j,t}^* = N_{k,j,t}^\chi, \end{aligned} \quad (21)$$

where  $N_{k,j,t}$  is the amount of labor it employs, and  $\chi$  determines returns to labor.

---

<sup>4</sup>Woodford (2008) shows that this model is a good approximation to a carefully microfounded model in which firms set prices subject to information frictions. Furthermore, in closed economies heterogeneity in price setting has similar implications in a large class of models that includes various sticky-price and sticky-information specifications. For a detailed analysis of such models, and additional references, see Carvalho and Schwartzman (2008). Nakamura and Steinsson (2007) find that heterogeneity in price stickiness also has similar implications in state-dependent pricing models.

The first order conditions are:

$$X_{H,k,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1 - \alpha_k)^s \Lambda_{H,k,t+s} \left( \chi N_{k,j,t+s}^{\chi-1} \right)^{-1} W_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1 - \alpha_k)^s \Lambda_{H,k,t+s}},$$

$$X_{H,k,j,t}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1 - \alpha_k)^s \Lambda_{H,k,t+s}^* \left( \chi N_{k,j,t+s}^{\chi-1} \right)^{-1} W_{t+s}}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s} (1 - \alpha_k)^s \mathcal{E}_{t+s} \Lambda_{H,k,t+s}^*},$$

where:

$$\Lambda_{H,k,t} = \omega \left( \frac{1}{P_{H,k,t}} \right)^{-\theta} \left( \frac{P_{H,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t,$$

$$\Lambda_{H,k,t}^* = (1 - \omega) \left( \frac{1}{P_{H,k,t}^*} \right)^{-\theta} \left( \frac{P_{H,k,t}^*}{P_{k,t}^*} \right)^{-\rho} \left( \frac{P_{k,t}^*}{P_t^*} \right)^{-\eta} Y_t^*.$$

An analogous maximization problem for the Foreign firms yields:

$$X_{F,k,j,t}^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s}^* (1 - \alpha_k)^s \Lambda_{F,k,t+s}^* \left( \chi N_{k,j,t+s}^{*\chi-1} \right)^{-1} W_{t+s}^*}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s}^* (1 - \alpha_k)^s \Lambda_{F,k,t+s}^*},$$

$$X_{F,k,j,t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s}^* (1 - \alpha_k)^s \Lambda_{F,k,t+s} \left( \chi N_{k,j,t+s}^{*\chi-1} \right)^{-1} W_{t+s}^*}{E_t \sum_{s=0}^{\infty} \Theta_{t,t+s}^* (1 - \alpha_k)^s \mathcal{E}_{t+s}^{-1} \Lambda_{F,k,t+s}},$$

where:

$$\Lambda_{F,k,t}^* = \omega \left( \frac{1}{P_{F,k,t}^*} \right)^{-\theta} \left( \frac{P_{F,k,t}^*}{P_{k,t}^*} \right)^{-\rho} \left( \frac{P_{k,t}^*}{P_t^*} \right)^{-\eta} Y_t^*,$$

$$\Lambda_{F,k,t} = (1 - \omega) \left( \frac{1}{P_{F,k,t}} \right)^{-\theta} \left( \frac{P_{F,k,t}}{P_{k,t}} \right)^{-\rho} \left( \frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t.$$

We focus on a symmetric equilibrium in which, conditional on time  $t$  information, the joint distribution of future variables that matter for price setting is the same for all firms in sector  $k$  in a given country that change prices in period  $t$ . Therefore, they make the same pricing decisions, and choose prices that we denote by  $X_{H,k,t}$ ,  $X_{H,k,t}^*$ , and  $X_{F,k,t}$ ,  $X_{F,k,t}^*$ . The country-sector price indices can thus be written as:

$$P_{H,k,t} = \left( \alpha_k X_{H,k,t}^{1-\theta} + (1 - \alpha_k) P_{H,k,t-1}^{1-\theta} \right)^{\frac{1}{1-\theta}},$$

$$P_{H,k,t}^* = \left( \alpha_k X_{H,k,t}^{*1-\theta} + (1 - \alpha_k) P_{H,k,t-1}^{*1-\theta} \right)^{\frac{1}{1-\theta}},$$

and likewise for  $P_{F,k,t}$  and  $P_{F,k,t}^*$ .

Finally, the model is closed by a monetary policy specification that ensures existence and unique-

ness of the rational expectations equilibrium. We consider different specifications in subsequent sections. Equilibrium is characterized by the optimality conditions of the consumers' utility maximization problem and of every firm's profit maximization problem, and by market clearing in assets, goods and labor markets.

We solve the model by log-linearizing around a zero inflation steady state. Due to symmetry, the steady state around which we work is such that prices for all intermediate firms, levels of employment and allocations of consumption, imports and exports are the same for both countries. Additionally, the common preferences assumption implies that, in steady state, the real exchange rate  $Q$  equals 1. The derivations of the steady state and the log-linear approximation are in a supplementary appendix available upon request. Throughout the rest of the paper, lowercase variables denote log-deviations from the steady-state.

## 2.1 The counterfactual one-sector world economy

We also build a counterfactual world economy with only one-sector of intermediate firms in each country. The model is exactly the same as before, except that the frequency of price changes,  $\bar{\alpha}$ , is set equal to the average frequency of adjustments in the multi-sector economies:  $\bar{\alpha} = \sum_{k=1}^K f_k \alpha_k$ . In terms of notation, we differentiate the variables in these one-sector economies from the corresponding variables in the heterogeneous economies by adding a "1 sec" superscript. We refer to this economy as the *counterfactual one-sector world economy*.

## 3 Analytical results

In this section we make a set of simplifying assumptions to deliver analytical results. This allows us to characterize the dynamic properties of aggregate and sectoral real exchange rates, and to compute different measures of persistence and volatility explicitly.

We leave the specification of monetary policy implicit by postulating that the growth rate of nominal aggregate demand in each country follows a first-order autoregressive (*AR*) process. This specification, common in the Monetary Economics literature, fits the data well. It can be justified through a cash-in-advance constraint when money growth itself follows an *AR*(1), or as the result of a monetary policy rule. Denoting nominal aggregate demand in Home and Foreign, respectively, by  $Z_t \equiv P_t Y_t$ ,  $Z_t^* \equiv P_t^* Y_t^*$ , our assumption is:

$$\begin{aligned}\Delta z_t &= \rho_z \Delta z_{t-1} + \sigma_{\varepsilon_z} \varepsilon_{z,t}, \\ \Delta z_t^* &= \rho_z \Delta z_{t-1}^* + \sigma_{\varepsilon_z} \varepsilon_{z,t}^*,\end{aligned}$$

where  $\rho_z$  denotes the autocorrelation in nominal aggregate demand growth, and  $\varepsilon_{z,t}$  and  $\varepsilon_{z,t}^*$  are purely monetary, uncorrelated, zero-mean, unit-variance *i.i.d.* shocks. For expositional simplicity, we assume that  $\rho_z \in (1 - \alpha_1, 1 - \alpha_K)$ .<sup>5</sup>

In addition, we impose restrictions on some parameters, as follows. We assume logarithmic consumption utility ( $\sigma = 1$ ), linear disutility of labor ( $\gamma = 0$ ), and linear production function ( $\chi = 1$ ). These assumptions give rise to no strategic complementarity nor substitutability in price setting in the context of closed-economy models, i.e. to a Ball and Romer (1990) index of real rigidities equal to unity. We refer to this case as the one of *strategic neutrality* in price setting. We drop these simplifying assumptions in Section 4.

Under these assumptions, in the Appendix we obtain explicit expressions for the processes followed by the aggregate and sectoral real exchange rates, and prove the following:

**Proposition 1** *Under the assumptions above, sectoral real exchange rates follow AR(2) processes:*

$$(1 - \rho_z L)(1 - \lambda_k L) q_{k,t} = \varphi_k u_t, \quad k = 1, \dots, K,$$

where  $\lambda_k \equiv 1 - \alpha_k$  is the per-period probability of no price adjustment for a firm in sector  $k$ ,  $u_t \equiv \sigma_{\varepsilon_z} (\varepsilon_{z,t} - \varepsilon_{z,t}^*)$  is white noise,  $\varphi_k \equiv \lambda_k - (1 - \lambda_k) \frac{\rho_z \beta \lambda_k}{1 - \rho_z \beta \lambda_k}$ , and  $L$  is the lag operator.

The dynamic properties of sectoral real exchange rates depend on the frequency of price adjustments in the sector, as well as on the persistence of shocks hitting the two economies.<sup>6</sup> Aggregating the sectoral exchange rates, we obtain the following result, from the work of Granger and Morris (1976):

**Corollary 1** *The aggregate real exchange rate follows an ARMA( $K + 1, K - 1$ ) process:*

$$(1 - \rho_z L) \prod_{k=1}^K (1 - \lambda_k L) q_t = \left( \sum_{k=1}^K \prod_{j \neq k} (1 - \lambda_j L) f_k \varphi_k \right) u_t.$$

The aggregate real exchange rate naturally depends on the whole distribution of the frequencies of price adjustments across sectors, as well as on the shocks hitting the two countries. Because it follows a possibly high order ARMA, the dynamics of the aggregate real exchange rate can be quite different from those of the underlying sectoral real exchange rates.

<sup>5</sup>This restriction is consistent with empirical estimates of  $\rho_z$  and microeconomic evidence on the frequency of price changes. Generalizing our results to the case in which  $\rho_z \in [0, 1)$  is straightforward.

<sup>6</sup>When  $\rho_z = 0$ , this simplified version of our model produces sectoral real exchange rate dynamics that coincide with those in Kehoe and Midrigan (2007). When we simulate our model with values of  $\rho_z$  estimated with data on nominal income or monetary aggregates, it generates a relationship between the frequency of price changes and the autocorrelation of sectoral real exchange rates that closely matches their empirical findings. However, even in this case we also find that the model falls short of reproducing the large comovements between the nominal and real exchange rates in sectors with relatively small degrees of price stickiness. We return to this issue in Section 6.

Finally, the real exchange rate in the counterfactual one-sector world economy can be obtained as a degenerate case in which all firms belong to a single sector, with frequency of price adjustments equal to the average frequency of the heterogeneous economy:

**Corollary 2** *The real exchange rate of the counterfactual one-sector world economy follows an AR(2) process:*

$$(1 - \rho_z L) (1 - \bar{\lambda} L) q_t^{1\text{sec}} = \varphi u_t,$$

where  $\bar{\lambda} \equiv \sum_{k=1}^K f_k \lambda_k$  and  $\varphi \equiv \bar{\lambda} - (1 - \bar{\lambda}) \frac{\rho_z \beta \bar{\lambda}}{1 - \rho_z \beta \bar{\lambda}}$ .

### 3.1 Persistence

We are interested in analyzing the persistence of deviations of the real exchange rate from PPP. In this subsection we focus on measures of persistence used in the literature for which we can obtain analytical results. In particular, we focus on the cumulative impulse response, the largest autoregressive root, and the sum of autoregressive coefficients. The cumulative impulse response ( $\mathcal{CIR}$ ) is defined as follows. Let  $IRF_t(q)$ ,  $t = 0, 1, \dots$  denote the impulse response function (to a unit impulse) of the  $q_t$  process. Then,  $\mathcal{CIR}(q) \equiv \sum_{t=0}^{\infty} IRF_t(q)$ . The largest autoregressive root ( $\mathcal{LAR}$ ) for a process  $q_t$  with representation  $\tilde{A}(L)q_t = \tilde{B}(L)u_t$ ,  $\mathcal{LAR}(q)$ , is simply the largest root of the  $\tilde{A}(L)$  polynomial. Finally, the sum of autoregressive coefficients ( $\mathcal{SAC}$ ) of such a process is  $\mathcal{SAC}(q) \equiv 1 - \tilde{A}(1)$ . In Section 4 we use simulations of the model to assess the quantitative importance of our analytical findings in terms of these and other measures of persistence, such as the half-life.<sup>7</sup>

Let  $\mathcal{P}$  denote one such measure of persistence. We prove the following:

**Proposition 2** *For the measures of persistence  $\mathcal{P} = \mathcal{CIR}, \mathcal{LAR}, \mathcal{SAC}$ :*

$$\mathcal{P}(q) > \mathcal{P}(q^{1\text{sec}}).$$

**Proposition 2** shows that a simple model with sectoral heterogeneity stemming solely from differences in the frequency of price changes can generate an aggregate real exchange rate that is more persistent than the real exchange rate in a one-sector version of the world economy with the same average frequency of price changes.

As will become clear, the main determinant of this result is the fact that the counterfactual one-sector model is a poor representation the multi-sector model. **Corollary 1** shows that as a

---

<sup>7</sup>The literature focuses mainly on the half-life of estimated real exchange rate processes, and on the first autocorrelation under the assumption of AR(1) specifications for the purpose of providing analytical results. However, the latter becomes less meaningful as one moves away from AR(1) specifications as we do in our model. Moreover, beyond the AR(1) case it is quite difficult to obtain analytical results for the half-life.

result of cross-sectional aggregation of sectoral exchange rates with heterogeneous dynamics, the aggregate real exchange rate in the multi-sector economy follows a much richer stochastic process than the real exchange rate in the counterfactual one-sector model. Moreover, the persistence of real exchange rates under these commonly used measures is a convex function of the frequency of price adjustments. Thus, the counterfactual one-sector model understates the persistence of the real exchange rate relative to the underlying heterogeneous economy.

Our next result will prove helpful in understanding the source of that difference in persistence. For any measure of persistence  $\mathcal{P}$ , we define the *total heterogeneity effect under  $\mathcal{P}$*  to be the difference between the persistence of the aggregate real exchange rate in the heterogeneous economy,  $q_t$ , and the persistence of the real exchange rate in the counterfactual one-sector world economy,  $q_t^{1\text{sec}}$ :

$$\text{total heterogeneity effect under } \mathcal{P} \equiv \mathcal{P}(q) - \mathcal{P}(q^{1\text{sec}}).$$

We can rewrite the total heterogeneity effect by adding and subtracting the weighted average of the persistence of the sectoral exchange rates,  $\sum_{k=1}^K f_k \mathcal{P}(q_k)$ , to obtain the following decomposition:

$$\begin{aligned} \text{total heterogeneity} \\ \text{effect under } \mathcal{P} \end{aligned} = \left( \mathcal{P}(q) - \sum_{k=1}^K f_k \mathcal{P}(q_k) \right) + \left( \sum_{k=1}^K f_k \mathcal{P}(q_k) - \mathcal{P}(q^{1\text{sec}}) \right). \quad (22)$$

In (22), the first term in parentheses is what we refer to as the *aggregation effect under  $\mathcal{P}$* : the difference between the “persistence of the average” and the “average of the persistences”:

$$\text{aggregation effect under } \mathcal{P} \equiv \mathcal{P}(q) - \sum_{k=1}^K f_k \mathcal{P}(q_k). \quad (23)$$

Since the aggregate real exchange rate is equal to the weighted average of the sectoral exchange rates, the measure in (23) is indeed purely a result of aggregation.

The second term in the decomposition (22) is the difference between the weighted average of the persistence of sectoral real exchange rates in the heterogeneous economy, and the persistence of the real exchange rate in the counterfactual one-sector world economy:

$$\text{counterfactuality effect under } \mathcal{P} \equiv \sum_{k=1}^K f_k \mathcal{P}(q_k) - \mathcal{P}(q^{1\text{sec}}). \quad (24)$$

Our next result gives substance to the decomposition in (22), by showing that both the aggregation and the counterfactuality effects are positive:

**Proposition 3** For the measures of persistence  $\mathcal{P} = CIR, \mathcal{L}AR, SAC$ :

$$\begin{aligned} \text{aggregation effect under } \mathcal{P} &> 0, \\ \text{counterfactual effect under } \mathcal{P} &\geq 0. \end{aligned}$$

In particular:<sup>8</sup>

$$\begin{aligned} CIR &: \begin{aligned} \text{aggregation effect} &> 0, \\ \text{counterfactual effect} &> 0; \end{aligned} \\ \mathcal{L}AR &: \begin{aligned} \text{aggregation effect} &> 0, \\ \text{counterfactual effect} &> 0; \end{aligned} \\ SAC &: \begin{aligned} \text{aggregation effect} &> 0, \\ \text{counterfactual effect} &= 0. \end{aligned} \end{aligned}$$

For almost every case, both the aggregation and the counterfactual effects are strictly positive. The only exception is when persistence is measured through the sum of autoregressive coefficients, in which case the counterfactual effect is zero. This is a general result for measures of persistence that are linear in the autoregressive coefficients - the sum of autoregressive coefficients is an example - and is formalized below:

**Lemma 1** For measures of persistence that are linear in the autoregressive coefficients:

$$\sum_{k=1}^K f_k \mathcal{P}(q_k) = \mathcal{P}(q^{1\text{sec}}),$$

so that the counterfactual effect is zero.

**Lemma 1** follows directly from the fact that in the  $AR(2)$  process followed by the real exchange rate in the counterfactual one-sector world economy, the autoregressive coefficient at each lag equals the weighted average of the corresponding autoregressive coefficients of the sectoral exchange rates. Thus, for the special case of linear measures of persistence, the counterfactual effect is zero, and the aggregation effect equals the total heterogeneity effect. We use this result in Section 5, when we revisit the empirical literature on aggregation and the real exchange rate.

### 3.2 Volatility

The other dimension of the PPP puzzle is the high volatility of the real exchange rate. Comparing the real exchange rate in the counterfactual one-sector world economy, and the sectoral real exchange rates in the heterogeneous economy, the following result holds:

---

<sup>8</sup>Below we omit the “under  $\mathcal{P}$ ” qualifier, since it is clear from the context.

**Proposition 4** Let  $\mathcal{V}(q_k)$  denote the unconditional variance of the  $q_{k,t}$  process. Then:

$$\sum_{k=1}^K f_k \mathcal{V}(q_k) > \mathcal{V}(q^{1\text{sec}}).$$

**Proposition 4** shows that the average volatility of sectoral real exchange rates in the multi-sector world economy exceeds the volatility of the real exchange rate in the counterfactual one-sector model. However, this is not a comparison between the latter and the aggregate real exchange rate in the multi-sector model. In the simulations of the model analyzed in Section 4, we find that the volatility of the aggregate real exchange rate also exceeds that of the real exchange rate in the counterfactual one-sector world economy.

### 3.3 A limiting result

This subsection shows that a “suitably heterogeneous” multi-sector world economy can generate an aggregate real exchange rate that is arbitrarily more volatile and persistent than the real exchange rate in the counterfactual one-sector world economy.<sup>9</sup> We consider the effects of progressively adding more sectors, and assume that the frequency of price changes for each new sector is drawn from  $(0, 1 - \delta)$  for arbitrarily small  $\delta > 0$ , according to some distribution with density  $g(\alpha|b)$ , where  $\alpha$  is the frequency of price changes and  $b$  is a parameter. For  $\alpha \approx 0$  such density is assumed to be approximately proportional to  $\alpha^{-b}$ , with  $b \in (\frac{1}{2}, 1)$ .<sup>10</sup> The shape of this distribution away from zero need not be specified, and moreover it yields a strictly positive average frequency of price changes:  $\bar{\alpha} = \int_0^{1-\delta} g(\alpha|b) \alpha d\alpha > 0$ .

We prove the following:

**Proposition 5** Under the assumptions above:

$$\begin{aligned} \mathcal{V}\left(\frac{1}{K} \sum_{k=1}^K q_{k,t}\right) &\xrightarrow{K \rightarrow \infty} \infty, \\ CIR\left(\frac{1}{K} \sum_{k=1}^K q_{k,t}\right) &\xrightarrow{K \rightarrow \infty} \infty, \\ \mathcal{V}(q^{1\text{sec}}), CIR(q^{1\text{sec}}) &< \infty. \end{aligned}$$

The results in **Proposition 5** follow from the fact that, under suitable assumptions, the aggregate real exchange rate converges to a non-stationary process. It inherits some features of unit-root

<sup>9</sup>We build on the work of Granger (1980), Granger and Joyeux (1980), Zaffaroni (2004) and others.

<sup>10</sup>Thus, we approximate a large number of potential new sectors by a continuum, and replace the general  $f_k$  distribution by this semi-parametric specification for  $g(\alpha|b)$ , based on Zaffaroni (2004). An example of a parametric distribution that satisfies this restriction is a Beta distribution with suitably chosen support and parameters.

processes, such as infinite variance and persistence, due to the relatively high density of very persistent sectoral real exchange rates embedded in the distributional assumption for the frequencies of price changes. However, the process does not have a unit root, since none of the sectoral exchange rates actually has one. Moreover, the limiting process remains mean reverting in the sense that its impulse response function converges to zero as  $t \rightarrow \infty$ .<sup>11</sup> In contrast, the limiting process for the real exchange rate in the counterfactual one-sector world economy remains stationary, since  $\bar{\alpha} > 0$  and as such, it has both finite variance and persistence.

In qualitative terms, **Propositions 2-5** provide an affirmative answer to the question of whether a model with heterogeneity in price stickiness *can* solve the PPP puzzle. However, to answer the more relevant question of whether a version of the model parametrized to match the microeconomic evidence on the frequency of price changes *does* in fact account for the puzzle we must go beyond qualitative results. We turn to that question next.

## 4 Quantitative analysis

In this section we analyze the quantitative implications of our model through simulations. We describe our parametrization, starting with how we use the recent microeconomic evidence on price setting to specify the cross-sectional distribution of price stickiness. We then present the quantitative results for our baseline specification, and consider alternative configurations as robustness checks. In particular, we consider the case in which monetary policy follows an interest rate rule subject to persistent shocks, and allow for productivity shocks. We also consider the case of strategic neutrality in price setting.

### 4.1 Parametrization

#### 4.1.1 Cross-sectional distribution of price stickiness

A series of recent papers have documented several features of price-setting behavior in modern industrial economies using disaggregated price data that underlies consumer price indices (e.g. Bils and Klenow 2004, and Nakamura and Steinsson 2008 for the U.S. economy; Dhyne et al. 2006, and references cited therein for the Euro area; Gagnon 2007 for Mexico). In turn, Gopinath and Rigobon (2008) document price-setting practices using disaggregated price data on U.S. imports and exports.

In our model, whenever a firm changes its prices it sets one price for the domestic market and another price for exports, and for simplicity we impose the same frequency of price adjustments

---

<sup>11</sup>Such properties characterize the so-called *fractionally integrated processes*. See, for example, Granger and Joyeux (1980).

in both cases.<sup>12</sup> In addition, we also assume the same cross-sectional distribution of the frequency of price changes in both countries. As a result, we must choose a single suitable distribution to parametrize the model.

We analyze our model having in mind a two-country world economy with the U.S. and the rest of the world. Since the domestic market is relatively more important for firms decisions (due to a small import share), we favor a distribution for the frequency of price changes across sectors that reflects mainly domestic rather than export pricing decisions. Due to our assumption of symmetric countries, we also favor distributions that are representative of price-setting behavior in different developed economies. Finally, and perhaps most importantly, we want to relate our results to the empirical PPP literature, which most often focuses on real exchange rates based on consumer price indices (CPIs). As a result, we choose to use the statistics on the frequency of price changes reported by Nakamura and Steinsson (2008).

We work with the statistics on the frequency of regular price changes - those that are not due to sales or product substitutions - for the 272 categories of goods and services analyzed by Nakamura and Steinsson (2008). To make the model computationally manageable, in our benchmark specification we aggregate those 272 categories into 36 sectors, according to the frequency of price changes. In particular, we consider frequencies in the range that corresponds to prices changing, on average, every month, to prices changing on average once every 36 months. Recall that we order the sectors in terms of increasing price stickiness, and thus we set  $\alpha_k = \frac{1}{k}$ , for  $k = 1, \dots, 36$ . We aggregate the CPI expenditure weights accordingly. Specifically, we add the weights of categories that have an average duration of price spells between zero and one month (inclusive) and assign the sum to  $f_1$ ; the sum for categories with an expected duration of price spells between one (exclusive) and two months (inclusive) is assigned to the second sector, and so on. We proceed in this fashion until the 35<sup>th</sup> sector. Finally, we aggregate all the remaining categories, which have mean durations of price rigidity of 3 years and beyond, into the last sector, which receives a weight of 4.2%. Given this distribution, the average frequency of price changes is  $\bar{\alpha} = \sum_{k=1}^K f_k \alpha_k = 0.226$ , which implies that prices change on average once every 4.4 months.

#### 4.1.2 Remaining parameters

In our baseline specification we fix the remaining structural parameters as follows. We set the intertemporal elasticity of substitution  $\sigma^{-1}$  to 1/3, unit labor supply elasticity ( $\gamma = 1$ ), and the

---

<sup>12</sup>Benigno (2004) studies a one-sector model in which he allows the frequency of price changes for those two pricing decisions to differ and also incorporates asymmetry in the frequency of price changes across countries. He shows that when this leads to different frequencies of price changes within a same country (due to differences in frequencies for varieties produced by local versus foreign firms), the real exchange rate becomes more persistent.

usual extent of decreasing returns to labor ( $\chi = 2/3$ ). The consumer discount factor  $\beta$  implies a time-discount rate of 2% per year.

For the final good technology parameters, we set the elasticity of substitution between varieties of the same sector to  $\theta = 10$ . We set the elasticity of substitution between Home and Foreign goods to  $\rho = 1.5$ , and the share of domestic goods at  $\omega = 0.9$ .<sup>13</sup> The elasticity of substitution between varieties of different sectors should be smaller than within sectors, and so we assume a unit elasticity of substitution across sectors,  $\eta = 1$  (i.e. the aggregator that converts sectoral into final output is Cobb-Douglas).

Finally, to specify the process for nominal aggregate demand, the literature usually relies on estimates based on nominal GDP, or on monetary aggregates such as M1 or M2. With quarterly data, estimates of  $\rho_z$  typically fall in the range of 0.4 to 0.7.<sup>14</sup> This maps into a range of 0.74–0.89 at a monthly frequency, and so we set  $\rho_z = 0.8$ . The standard deviation of the shocks is set at  $\sigma_{\varepsilon_z} = 0.58\%$  (1% at a quarterly frequency), also in line with the same estimation results.

## 4.2 Quantitative results

Table 1 presents the quantitative results of our simulated model. The first column shows the statistics computed for the aggregate real exchange rate, and the middle column presents the cross-sectional weighted average of the same statistics at the sectoral level. Finally, the last column contains the statistics for the real exchange rate of the counterfactual one-sector world economy. We present results for the measures of persistence for which we provided analytical derivations in Section 3, and also for the half-life ( $\mathcal{HL}$ ) - reported in months - and the first-order autocorrelation ( $\rho_1$ ). We also present results for a measure of volatility (the standard deviation) of the real exchange rate.<sup>15</sup>

Table 1 shows that the model with heterogeneity can generate a highly volatile and persistent real exchange rate. In particular, at 45 months the half-life of deviations from PPP falls well within the “consensus view” of 3 to 5 years reported by Rogoff (1996). In contrast, the counterfactual one-

---

<sup>13</sup>Atkeson and Burstein (2008) set the import share to 16.5%, while Chari et al. (2002) and Steinsson (2008) use 1.6%. Most references in the literature choose values in this range.

<sup>14</sup>See, for instance, Mankiw and Reis (2002).

<sup>15</sup>Since under our baseline specification the real exchange rates no longer follow the exact processes derived in Section 3, we compute  $\mathcal{SAC}$ ,  $\mathcal{LAR}$ ,  $\rho_1$ , and  $\mathcal{V}$  through simulation. Specifically, we simulate 150 replications of our economy and construct time series for aggregate, sectoral and one-sector economy real exchange rates with 1500 observations each. After dropping the first 100 observations to eliminate possible effects from the initial steady-state conditions, we compute the statistics for each replication and then average across the 150 replications. While  $\rho_1$ , and  $\mathcal{V}$  are computed directly, for  $\mathcal{SAC}$  and  $\mathcal{LAR}$  we rely on fitting  $AR(p)$  processes. In particular, we fit an  $AR(30)$  process to the aggregate real exchange rate, and  $AR(10)$  processes for the sectoral exchange rates and for the real exchange rate in the counterfactual one-sector world economy. The reported results are quite robust to increasing the number of lags. Finally,  $\mathcal{CTR}$  and  $\mathcal{HL}$  are computed directly from the impulse response functions implied by the solution of the model.

Table 1: Results from the Baseline Specification

Persistence measures:	$\mathcal{P}(q)$	$\sum f_k \mathcal{P}(q_k)$	$\mathcal{P}(q^{1\text{sec}})$
<i>CIR</i>	79.9	75.4	20.2
<i>SAC</i>	0.98	0.97	0.95
<i>LAR</i>	0.94	0.92	0.86
<i>HL</i>	45	44.3	14
$\rho_1$	0.98	0.98	0.97
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\sum f_k \mathcal{V}(q_k)^{1/2}$	$\mathcal{V}(q^{1\text{sec}})^{1/2}$
	0.10	0.11	0.04

sector economy produces much less volatility and persistence, with a half-life only slightly exceeding one year. In short, the total heterogeneity effect is quite large.<sup>16</sup>

The difference between the first two columns yields the aggregation effect, whereas the difference between the last two columns equals the counterfactuality effect. We focus on the cumulative impulse response and the half-life, since these are the measures of persistence that best capture the features of the impulse response functions. The bottom line is clear: the aggregation effect is small, while the counterfactuality effect is large. For example, in terms of the half-life the counterfactuality effect accounts for  $\frac{44.3-14}{45-14} \approx 97.7\%$  of the total heterogeneity effect; for the cumulative impulse response, the corresponding figure is 92.3%.

### 4.3 Robustness checks

#### 4.3.1 Strategic neutrality in price setting

As a first robustness check of our parametrization, we run the same analysis imposing the restrictions on parameter values that underscore our analytical results from Section 3.<sup>17</sup> That is, we look at the quantitative implications of our model in the case of strategic neutrality in price setting.

Despite the change in the parametrization, the essence of our results is not affected, as shown in Table 2. Note that in this case the results are exact, since we know the processes followed by each of the variables from **Proposition 1**, and **Corollaries 1** and **2**.<sup>18</sup> The aggregate real exchange rate

<sup>16</sup>We also computed the up-lives and quarter-lives of the real exchange rates, following Steinsson (2008). Our multi-sector model produces an up-life of 27 months and a quarter-life of 65 months. Steinsson (2008) estimates the up-life of the trade-weighted U.S real exchange rate to be 28.4 months, and the quarter-life to be 76 months. In contrast, the real exchange rate in the counterfactual one-sector world economy has an up-life of 9 months, and a quarter-life of 18 months.

<sup>17</sup>Recall that these are  $\sigma = 1$ ,  $\gamma = 0$ , and  $\chi = 1$ . Under these assumptions, the additional structural parameters have no effect on the dynamics of real exchange rates.

<sup>18</sup>The only exceptions are the first autocorrelation for the aggregate real exchange rate and the volatilities, which for simplicity are calculated through simulations, as outlined in footnote 15.

in the heterogeneous economy is still much more volatile and persistent than in the counterfactual one-sector world economy. In particular, the half-lives are similar to the ones that result from our baseline specification, and a larger fraction of the difference is explained by the counterfactuality as opposed to the aggregation effect.<sup>19</sup>

Persistence measures:	$\mathcal{P}(q)$	$\sum f_k \mathcal{P}(q_k)$	$\mathcal{P}(q^{1\text{sec}})$
<i>CIR</i>	87.3	65.2	22.2
<i>SAC</i>	$\approx 1$	0.85	0.85
<i>LAR</i>	.97	0.89	0.8
<i>HL</i>	47	34	15
$\rho_1$	0.99	0.96	.97
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\sum f_k \mathcal{V}(q_k)^{1/2}$	$\mathcal{V}(q^{1\text{sec}})^{1/2}$
	0.06	0.06	0.02

### 4.3.2 Different shocks

We consider a specification with an explicit description of monetary policy, and later also add productivity shocks. We assume that in each country monetary policy is conducted according to an interest rate rule subject to persistent shocks:

$$I_t = \beta \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left( \frac{GDP_t}{GDP_t^n} \right)^{\phi_Y} e^{v_t},$$

where  $I_t$  is the short term nominal interest rate in Home,  $GDP_t$  is gross domestic product,  $GDP_t^n$  denotes gross domestic product when all prices are flexible,  $\phi_\pi$  and  $\phi_Y$  are the parameters associated with Taylor interest rate rules, and  $v_t$  is a persistent shock with process  $v_t = \rho_v v_{t-1} + \sigma_{\varepsilon_v} \varepsilon_{v,t}$ , where  $\varepsilon_{v,t}$  is a zero mean, unit variance *i.i.d.* shock, and  $\rho_v \in [0, 1)$ . The policy rule in Foreign is analogous, and we assume that the shocks are uncorrelated across countries. We set  $\phi_\pi = 1.5$ ,  $\phi_y = .5/12$ , and  $\rho_v = 0.965$ .<sup>20</sup> The remaining parameter values are unchanged from the baseline specification.

<sup>19</sup>Strategic complementarities in price setting are known to amplify the real effects of monetary shocks in closed-economy models (e.g. Woodford 2003, chapter 3). They are also common in open-economy sticky-price models that try to produce persistent real exchange rates (e.g. Bergin and Feenstra 2001; Steinsson 2008). Carvalho (2006) shows that complementarities in price-setting amplify the role of heterogeneity in price stickiness in generating monetary non-neutrality. The results of this subsection emphasize that such propagation mechanism is not required in order to obtain substantial persistence.

<sup>20</sup>Recall that the parameters are calibrated to the monthly frequency, and so this value for  $\rho_v$  corresponds to an autoregressive coefficient of 0.9 at a quarterly frequency. We specify the size of the shocks to be consistent with the estimates of Justiniano et al. (2008), and thus set the standard deviation to 0.2% at a quarterly frequency.

The results are in Table 3.<sup>21</sup> The model with heterogeneity still produces a significantly more volatile and persistent real exchange rate than in the counterfactual one-sector world economy. Moreover, the counterfactuality effect is substantially more important than the aggregation effect. In fact, for this version of the model the aggregation effect is *negative* under both the half-life and the cumulative impulse response.<sup>22</sup>

Persistence measures:	$\mathcal{P}(q)$	$\sum f_k \mathcal{P}(q_k)$	$\mathcal{P}(q^{1\text{sec}})$
<i>CIR</i>	49.2	53.3	29.1
<i>SAC</i>	0.98	0.97	0.96
<i>LAR</i>	0.96	0.98	0.96
<i>HL</i>	38	39.9	21
$\rho_1$	0.98	0.96	0.96
Volatility measure:	$\mathcal{V}(q)^{1/2}$	$\sum f_k \mathcal{V}(q_k)^{1/2}$	$\mathcal{V}(q^{1\text{sec}})^{1/2}$
	0.07	0.08	0.01

We also consider a version of the model with interest rate and productivity shocks.<sup>23</sup> We introduce the latter by changing the production function in (21) to:

$$Y_{H,k,j,t} + Y_{H,k,j,t}^* = A_t N_{k,j,t}^X,$$

where  $A_t$  is a productivity shock. It evolves according to:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_{\varepsilon_A} \varepsilon_{A,t},$$

where  $\rho_A \in [0, 1)$  and  $\varepsilon_{A,t}$  is a zero mean, unit variance *i.i.d.* shock. An analogous process applies to  $A_t^*$ , and once more we assume that the shocks are independent across countries.

We keep the same specification for the monetary policy rule, and set  $\rho_A = 0.965$ . To determine the relative size of the shocks we rely on the estimates obtained by Justiniano et al. (2008), and set  $\sigma_{\varepsilon_v} = 0.12\%$ , and  $\sigma_{\varepsilon_A} = 0.52\%$ . The remaining parameter values are unchanged from the baseline parametrization. The heterogeneous world economy still produces a significantly more volatile and persistent real exchange rate than the counterfactual one-sector world economy. As an example,

<sup>21</sup>We compute these statistics based on simulations, following the methodology outlined in footnote 15.

<sup>22</sup>Recall that our analytical results of Section 3 showing that both effects are positive apply under the assumptions highlighted in that section.

<sup>23</sup>Steinsson (2008) studies real exchange rate dynamics in one-sector sticky-price models with a rich set of shocks that includes both interest rate and productivity shocks. He finds substantial persistence in response to productivity and other disturbances that he terms “Phillips curve” shocks, but not in response to monetary shocks.

the half-life of the aggregate real exchange rate in the multi-sector world economy is around 34 months, the average of sectoral half-lives is 32.7 months, and the half-life of the real exchange rate in the counterfactual one-sector world economy is 21.8 months.

We also consider several (unreported) alternative specifications and parametrizations. We find that the results with shocks to the interest rate rule and productivity shocks are somewhat more sensitive to the details of the specification than under nominal aggregate demand shocks. On the one hand, they are still robust to the absence of strategic interactions in price-setting decisions (i.e., they hold under strategic neutrality in price setting). On the other, they can be relatively sensitive to the exogenous persistence of monetary and productivity shocks. The results do naturally vary conditional on each type of shock. The source of persistence in the interest rate rule - persistent shocks versus interest rate smoothing - also matters somewhat.<sup>24</sup>

Uncovering the reasons for such differences in results is an interesting endeavor for future research. In particular, it would be valuable to investigate the “demand block” of the model, especially since the forward looking “IS curve” has only weak empirical support (e.g. Fuhrer and Rudebusch 2004). In another direction, while it is a strength that our model can produce significantly volatile and persistent real exchange rates in response to purely monetary disturbances, it would be interesting to introduce additional shocks and analyze in more detail the differences between unconditional results and those conditional on particular shocks.<sup>25</sup>

### 4.3.3 Business cycle statistics

As an additional robustness check, this subsection presents results of the model for several business cycle statistics commonly analyzed in the literature. Table 4 presents the business cycle statistics obtained from our multi-sector and one-sector models under the baseline parametrization, and compares them with the data reported by Chari et al. (2002).<sup>26</sup> It shows that our multi-sector model produces reasonable results along these 21 business cycle dimensions, and actually improves upon its one-sector counterpart in most cases: 14 out of 19 (the two models yield the same results for two of the statistics). The improvements are well divided between statistics related to the volatility and the persistence of macroeconomic variables.

Among the failures of both models, it is worth noting the very high cross-correlation between

---

<sup>24</sup>Chari et al.(2002) find that their one-sector sticky-price model with a policy rule that features interest rate smoothing fails to generate reasonable business cycle behavior, in particular in terms of the persistence of deviations of the real exchange rate from PPP.

<sup>25</sup>Most of the empirical literature on the dynamics of real exchange rates refers to unconditional results, although there are exceptions, such as Eichenbaum and Evans (1995).

<sup>26</sup>All statistics are based on logged and HP-filtered quarterly data for the period 1972:1 to 1994:4. For further details on the data, refer to the Chari et al. (2002). To match their data set, we simulate 100 replications of our model with a time series of 264 months (or 88 quarters). We compute quarterly quantities by averaging the artificial data over non-overlapping 3-month periods. Finally, we HP-filter the quarterly simulated data.

the real exchange rate and relative consumption, which is clearly at odds with the data. This discrepancy is expected, given our assumption of complete asset markets, and the results of Backus and Smith (1993). The cross-correlations between Home and Foreign consumption and GDP are significantly lower than in the data. Chari et al. (2002) address this result by assuming that monetary shocks are correlated across countries. In unreported results, we also find that this change improves the performance of both models in terms of those statistics, without jeopardizing the other results. Finally, in terms of exchange rates and prices, the multi-sector model improves significantly upon the one-sector model. However, it still produces a too volatile price ratio. We discuss this finding in more detail in Section 6.

Table 4: Business Cycle Statistics

Statistic	Data (CKM)	Multi-sector	One-sector
<i>Standard deviations relative to GDP</i>			
Price Ratio	0.71	3.81	6.08
Nominal Exchange Rate	4.67	7.08	7.51
Real Exchange Rate	4.36	3.96	3.09
Consumption	0.83	0.97	0.85
Employment	0.67	1.40	1.31
Net Exports	0.11	0.17	0.29
<i>Autocorrelation</i>			
Price Ratio	0.87	0.90	0.90
Nominal Exchange Rate	0.86	0.86	0.85
Real Exchange Rate	0.83	0.84	0.74
GDP	0.88	0.85	0.78
Consumption	0.89	0.84	0.76
Employment	0.90	0.84	0.77
Net Exports	0.82	0.92	0.84
<i>Cross-correlations</i>			
Home and Foreign GDP	0.60	0.04	-0.07
Home and Foreign Consumption	0.38	0.11	0.29
Home and Foreign Employment	0.39	0.19	0.21
Net Exports and GDP	-0.41	0.32	0.65
RER and GDP	0.08	0.65	0.70
RER and Net Exports	0.14	0.33	0.82
RER and Relative Consumption	-0.35	0.99	0.99
Nominal and Real Exchange Rates	0.99	0.90	0.62

#### 4.3.4 How many sectors do we need?

As a final robustness check, this subsection poses the question of how many sectors we need to obtain a good approximation of our multi-sector economy. Our baseline parametrization considers

36 sectors that match the empirical distribution of price stickiness in Nakamura and Steisson (2008). This provides discipline to our analysis, in that it leaves no room for using such distribution to target particular results. However, handling so many sectors can be computationally costly. It is thus desirable to investigate whether we can produce similar results with fewer sectors.

To that end, we consider versions of the model with only two or three sectors. In the two-sector model, we choose the sectoral weights and frequencies of price changes to match the following moments of the distribution of price stickiness from our baseline parametrization: average frequency of price changes ( $\sum_{k=1}^K f_k \alpha_k$ ), cross-sectional average of the expected durations of price spells ( $\bar{d} \equiv \sum_{k=1}^K f_k \alpha_k^{-1}$ ), and cross-sectional standard deviation of the expected durations of price spells ( $\sqrt{\sum_{k=1}^K f_k (\alpha_k^{-1} - \bar{d})^2}$ ). In the three-sector model, in addition to those moments we also match the skewness of the cross-sectional distribution of expected durations of price spells ( $\sum_{k=1}^K f_k (\alpha_k^{-1} - \bar{d})^3$ ).<sup>27</sup>

We present our findings in Figure 1. It shows the impulse response functions of the aggregate real exchange rate to a nominal income shock in Home in our baseline multi-sector economy, and in the two- and three-sector economies described above.<sup>28</sup> The three-sector economy provides a very good approximation to our multi-sector economy, whereas the response of the real exchange rate in the two-sector economy is not as similar. This can be confirmed by inspection of the up-life, half-life, and quarter life of deviations from PPP in the three economies (not reported).

One should, of course, be careful when drawing general lessons from this exercise. Nevertheless, we conclude that in the context of our baseline multi-sector model a carefully parametrized three-sector economy can provide a good approximation of a much more heterogeneous economy, and remain consistent with key features of the cross-sectional distribution of price stickiness.

## 5 Revisiting the empirical literature on heterogeneity and aggregation

The empirical relevance of heterogeneity and aggregation in accounting for the persistence of the aggregate real exchange rate has been the subject of intense debate. In particular, while some studies find that they play at most a small role (e.g. Chen and Engel 2005, Crucini and Shintani 2008), Imbs et al. (2005a) conclude that heterogeneity can explain why the aggregate real exchange rate is so persistent.

In this section we use our structural model to interpret the apparently conflicting findings in

---

<sup>27</sup>The choice of moments to be matched is based on the analytical results obtained in Carvalho and Schwartzman (2008).

<sup>28</sup>The value of all other parameters is the same as in the baseline parametrization.

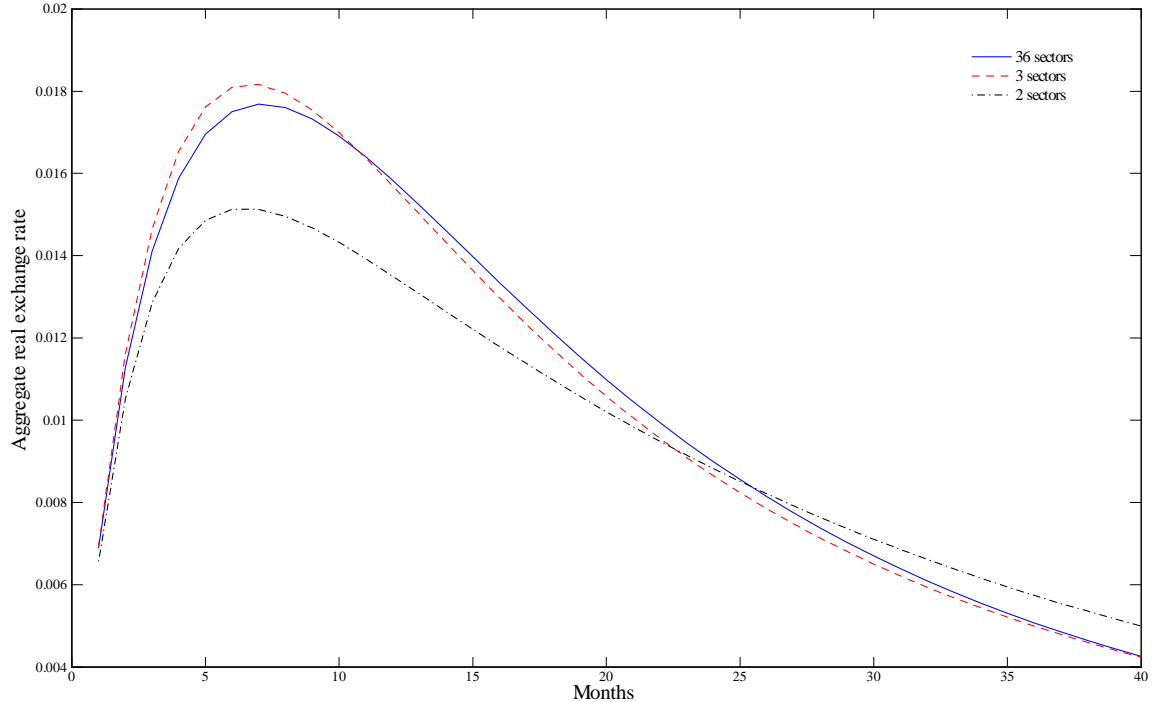


Figure 1: Impulse response functions of aggregate real exchange rate in multi-sector economies

this empirical literature. The bottom line is that different papers have measured different objects. The strand of the literature that finds a small role for heterogeneity and aggregation measures their effects through the difference between the persistence of the aggregate real exchange rate and the average persistence across its underlying components. This corresponds to what we define in our model as the aggregation effect. In turn, Imbs et al. (2005a) measure the effects of heterogeneity and aggregation through econometric methods that, as we show below, uncover what we define as the total heterogeneity effect.

We start by noticing that the empirical literature usually relies on a stylized framework when it comes to illustrating the effects of heterogeneity and aggregation analytically. In particular, it is common to assume that sectoral real exchange rates follow  $AR(1)$  processes, and use the first autocorrelation as a measure of persistence. In this context, the effect of heterogeneity is measured as the difference between the first autocorrelation of the aggregate real exchange rate, and the average of the first autocorrelations of the underlying sectoral exchange rates. From **Lemma 1**, such analytical exercise does not allow a distinction between the aggregation effect and the total heterogeneity effect, since the first autocorrelation of an  $AR(1)$  process equals the autoregressive coefficient, and thus the counterfactual effect is zero.

Our structural model uncovers an important distinction between the aggregation effect and

the total heterogeneity effect whenever the measure of persistence is a non-linear function of the autoregressive coefficients of the relevant process. Given that the empirical literature on PPP usually focuses on such non-linear measures of persistence (e.g., the half-life), it is important to keep in mind that, conceptually, the comparison between the persistence of the aggregate real exchange rate and the average persistence across its underlying components uncovers only what we term the aggregation effect.

In contrast, the empirical analysis of Imbs et al. (2005a) focuses on the comparison between estimates of persistence of aggregate real exchange rates (or estimates based on panel-data methods that impose homogeneous dynamics across units of the panel), and estimates of persistence based on a Mean Group (MG) estimator for panel data sets with heterogeneous dynamics (Pesaran and Smith 1995). We show below that, under our structural model, such a comparison uncovers the sum of the aggregation and counterfactuality effects, i.e. the total heterogeneity effect.

To be more precise in our description of the empirical implementation of Imbs et al. (2005a), assume that sectoral real exchange rates follow autoregressive processes of order  $p$  ( $AR(p)$ ), with sector specific coefficients:

$$q_{k,t} = \phi_{k,1}q_{k,t-1} + \phi_{k,2}q_{k,t-2} + \dots + \phi_{k,p}q_{k,t-p} + \varepsilon_{k,t},$$

where  $\varepsilon_{k,t}$  is an *i.i.d.* shock. The  $AR(p)$  real exchange rate process constructed on the basis of the MG estimator, denoted  $q_t^{MG}$ , is given by:

$$q_t^{MG} = \phi_1^{MG}q_{t-1} + \phi_2^{MG}q_{t-2} + \dots + \phi_p^{MG}q_{t-p} + \varepsilon_t^{MG},$$

where  $\varepsilon_t^{MG}$  is an *i.i.d.* shock, and  $\phi_i^{MG} = \frac{1}{K} \sum_{k \in K} \hat{\phi}_{k,i}$ , with  $\hat{\phi}_{k,i}$  denoting the OLS estimate of the  $i^{th}$  autoregressive coefficient for the  $k^{th}$  cross-sectional unit of the panel of sectoral real exchange rates.<sup>29</sup> In words,  $q_t^{MG}$  is an  $AR(p)$  process with autoregressive coefficients given by the cross-sectional averages of the (estimated) autoregressive coefficients of the sectoral real exchange rates, where the averages are taken for each of the  $p$  lags. The comparison made by Imbs et al. (2005a) is between the estimated persistence of the aggregate real exchange rate,<sup>30</sup> and the persistence of the MG-based real exchange rate.

An interpretation of the MG-based real exchange rate follows under our structural model and its counterfactual one-sector world economy (in the case of equal sectoral weights). In that case,

---

<sup>29</sup>For simplicity and consistency with the empirical implementation of Imbs et al. (2005) we assume equal sectoral weights.

<sup>30</sup>Or, alternatively, the persistence estimated from a panel of sectoral exchange rates with methods that impose homogeneous dynamics across all units of the panel.

under the simplifying assumptions of Subsection 3, sectoral exchange rates follow  $AR(2)$  processes:

$$q_{k,t} = (\rho_z + \lambda_k) q_{k,t-1} - \rho_z \lambda_k q_{k,t-2} + \varphi_k u_t.$$

Thus, applying the MG estimator in the population yields  $\rho_z + \frac{1}{K} \sum_{k=1}^K \lambda_k$  as the cross-sectional average of the *first* autoregressive coefficients, and  $-\rho_z \frac{1}{K} \sum_{k=1}^K \lambda_k$  as the cross-sectional average of the *second* autoregressive coefficients. It turns out that these are exactly the autoregressive coefficients of the  $AR(2)$  process followed by the aggregate real exchange rate in the counterfactual one-sector world economy.

We conclude that the comparison between the persistence of the aggregate real exchange rate in the heterogeneous world economy and the persistence implied by the MG estimator uncovers the total heterogeneity effect rather than the aggregation effect. In the next subsection we apply this insight to obtain an empirical decomposition of the total heterogeneity effect into the aggregation and counterfactuality effects.

## 5.1 Estimation results

We revisit the empirical literature on heterogeneity and aggregation, having as a guide the results of the previous subsection. We start with the Eurostat data used in the estimation of Imbs et al. (2005a).<sup>31</sup> Table 5 presents our replication of some of the results of that paper in the first and last columns. The first column shows the results obtained with application of a standard fixed effects estimator to a panel of aggregate real exchange rates - consisting of up to 19 sectors for 11 countries, while the last column presents our results for the Mean Group (MG) estimator of Pesaran and Smith (1995).<sup>32</sup> The middle column, in turn, presents the estimates for the cross-sectional average across units of the panel. To construct these estimates we run separate OLS regressions for an  $AR(19)$  for each panel unit (sector-country). The choice of lags matches that of the MG estimator in Imbs et al. (2005a). For each of these series, the relevant persistence statistics are calculated on the basis of the estimated autoregressive coefficients, and then averaged to yield the result presented in the table.

The results show that the total heterogeneity effect is indeed large. Once we account for heterogeneity, the  $\mathcal{HL}$  drops from 46 months to 26 months. The aggregation effect is, however, only

---

<sup>31</sup>The authors make the data available on their websites. It consists of sectoral and aggregate price indices and exchange rates for 11 european countries versus the U.S., and up to 19 goods categories per country. The frequency is monthly, from January 1981 through December 1995. The countries are Belgium, Germany, Denmark, Spain, Italy, France, Greece, Netherlands, Portugal, Finland, and the U.K. The goods categories are bread, meat, dairy, fruits, tobacco, alcohol, clothing, footwear, rents, fuel, furniture, domestic appliances, vehicles, public transportation, communications, sound, leisure, books, hotels.

<sup>32</sup>These results match those in Imbs et al. (2005a) exactly - refer to their Table II, first line, and Table III line 4. We also found very similar results for some of the other estimators that they report.

Table 5: Decomposition of the Total Heterogeneity Effect - Eurostat Data

Data	Panel, aggregate	Panel, sectoral	Panel, sectoral
Estimation method	Fixed Effects	OLS	MG
Equal-weight model:	$\mathcal{P}(q)$	$\frac{1}{K} \sum \mathcal{P}(q_k)$	$\mathcal{P}(q^{1\text{sec}})$
Persistence measures:			
$CTR$	64.39	59.48	33.19
$SAC$	0.98	0.97	0.97
$LAR$	0.97	0.94	0.95
$HL$	46	43.16	26

a small part of this difference. Indeed, the counterfactuality effect is responsible for  $\frac{43.16-26}{46-26} \approx 86\%$  of the total heterogeneity effect.<sup>33</sup> For the cumulative impulse response the corresponding figure is  $\frac{59.48-33.19}{64.39-33.19} \approx 84\%$ . These results are similar to the ones reported in Section 4, which are based on the solution of our parametrized model.<sup>34</sup>

To complement our analysis we apply the same estimation methods used for the actual data to artificial data generated by the model with equal sectoral weights. We focus on the case of strategic neutrality in price setting, since it is the one for which the MG estimator recovers the dynamics of the counterfactual one-sector world economy. We generate the data as outlined in footnote 15, and estimate  $AR$  processes to obtain the measures of persistence. We limit the length of the time-series to 180 observations to match the size of the Eurostat sample. Under strategic neutrality we know the exact order of the process followed by each of the variables (from **Proposition 1** and **Corollaries 1** and **2**). For the sectoral exchange rates and for the real exchange rate in the counterfactual one-sector world economy, we fit  $AR(2)$  processes. For the aggregate real exchange rate of the heterogeneous economy, we fit an  $AR(30)$  process to approximate the high order ARMA model.<sup>35</sup> We also apply the MG estimator to the panel of sectoral exchange rates. For each simulation, we compute the measures of persistence, and then average across the replications, after discarding the ones that generate non-stationary processes.<sup>36</sup> The results are presented in Table 6.

<sup>33</sup>To replicate the results in Imbs et al. (2005a), in Table 5 we use the actual aggregate series available in Eurostat. To be consistent with the model, we also analyze aggregate real exchange rates for each country constructed by equally weighting the percentage change of the real exchange rates for the goods that comprise the underlying sectoral panel. Applying a fixed effects estimator to the resulting panel of country real exchange rates, we estimate a half-life of 39 months. Alternatively, when we estimate separate  $AR$  specifications for each country, compute each half-life and then take a simple average, we obtain an average half-life of 43 months.

<sup>34</sup>We find similar results when we consider the “preferred” specification of Imbs et al. (2005a), based on Mean Group estimators with correction for common correlated effects (MG-CCE). Specifically, for the half-life we find that the counterfactuality effect explains 92% of the total heterogeneity effect, and for the cumulative impulse response the corresponding figure is 89%.

<sup>35</sup>The results are not very sensitive to increasing the number of lags.

<sup>36</sup>Although the model implies stationary real exchange rates, explosive roots may arise when we estimate the  $AR$  models with the small samples that we generate to match the length of the time-series in the actual data (180

Table 6: Decomposition of the Total Heterogeneity Effect - Simulated Data

Economy	Heterog.	Heterog.	Heterog.	One-sector
Data	Single, aggreg.	Panel, sectoral	Panel, sectoral	Single, aggreg.
Estim. method	OLS	OLS	MG	OLS
Equal-weight model:	$\mathcal{P}(q)$	$\frac{1}{K} \sum \mathcal{P}(q_k)$	$\mathcal{P}(q^{1\text{sec}})$	$\mathcal{P}(q^{1\text{sec}})$
Persist. meas:				
<i>CTR</i>	73.4	60.8	35.5	36.9
<i>SAC</i>	0.98	0.97	0.97	0.97
<i>LAR</i>	0.96	0.89	0.87	0.87
<i>HL</i>	43.1	31.6	20.9	21.8

The similarity with the results based on actual data is impressive. The total heterogeneity effect is, again, large: accounting for heterogeneity brings the  $\mathcal{HL}$  from 43.1 months to 21.8 months in the actual one-sector world economy. The MG estimator indeed generates results that are very close to the counterfactual one-sector world economy.

We also apply those estimation methods to data generated by the model under the baseline specification, which departs from the case of strategic neutrality in price setting. Due to sectoral interdependences, real exchange rates do not follow processes that are as simple as the ones derived in Section 3. In fact, in the absence of strategic neutrality in price setting the joint dynamics of all variables are, in general, described by a vector-autoregression. Nevertheless, we find that the MG estimator still gets close to uncovering the dynamics of the real exchange rate in the counterfactual one-sector world economy in these cases.

## 5.2 Bottom line

Imbs et al. (2005a) conclude that their empirical results show a large role for what they term a “dynamic aggregation bias” or “dynamic heterogeneity bias” in accounting for the PPP puzzle. In contrast, Chen and Engel (2005) and Crucini and Shintani (2008) find that the “aggregation bias” defined as the difference between the persistence of the aggregate real exchange rate and the average of the persistences of the underlying sectoral real exchange rates is small.<sup>37</sup>

As Chen and Engel (2005) and Crucini and Shintani (2008), we find the difference between the persistence of the aggregate real exchange rate and the average of the persistences of the underlying sectoral real exchange rates - what defines the aggregation effect - to be small, both in

observations).

<sup>37</sup>The discussion in Engel and Chen (2005) around measurement error in the data used in early working paper versions of Imbs et al. (2005a) was essentially superseded by their use of revised data. The debate involved other methodological issues that we do not address. A summary of the questions involved is provided by Imbs et al. (2005b).

the quantitative results of our model and in the data. At the same time, as Imbs et al. (2005a), we find that the difference between the persistence of the aggregate real exchange rate and the persistence of the counterfactual real exchange rate - what we refer to as the total heterogeneity effect - is large, both in the quantitative results of our model and in the data.

Our structural model provides an interpretation of the different measures of the effects of heterogeneity and aggregation used in the empirical literature. It shows that they serve to estimate conceptually different objects. Moreover, we find that both empirical measures accord well with the quantitative predictions of our model. Thus, we conclude that the different findings of the existing empirical literature are not in conflict.

## 6 Pieces of the Puzzle

In this section we confront the results of our multi-sector model with other dimensions of the PPP puzzle. In particular, we disentangle the properties of prices, nominal and real exchange rates. In the data, real and nominal exchange rates are highly correlated, and almost equally volatile. In turn, the ratio of national price levels (“price ratio”) is sluggish, and much smoother.

Using the Eurostat data from the previous section, for each country, we compute the standard deviation of price ratios, nominal and real exchange rates, and the correlations between nominal and real exchange rates, and between price ratios and nominal exchange rates.<sup>38</sup> We perform the same calculations with simulated data from the baseline specification, for both the multi-sector and one-sector models.<sup>39</sup> The results are presented in Table 7. The first two columns show statistics for, respectively, the average across all countries, and the average excluding Greece and Portugal.<sup>40</sup> The last two columns show the average across simulations of the statistics for the multi-sector economy, and its counterfactual one-sector version.

Our multi-sector model brings the level of volatility of the real exchange rate closer to the data, and generates a large increase in the cross-correlation between real and nominal exchange rates. In terms of the price ratio, the improvements are less substantial. Nevertheless, the multi-sector model still produces a slightly less volatile price ratio, which is also less correlated with the nominal exchange rate. While falling short of matching these additional features of the data as well as it matches the persistence of the aggregate real exchange rate, our multi-sector model improves on

---

<sup>38</sup>All variables are in logarithms.

<sup>39</sup>In order to match the size of the dataset, we generate samples with 180 observations each, and report averages across 150 replications.

<sup>40</sup>During the sample period, these two countries had much higher inflation rates than the U.S. economy. As a result, their statistics deviate substantially from those of the other 9 countries. For example, for Greece and Portugal the standard deviations of the nominal exchange rate are 0.43 and 0.28, the standard deviations of the price ratio are 0.53 and 0.38, and the cross correlations between real and nominal exchange rates are -0.44 and -0.12, respectively.

Table 7: Pieces of the Puzzle

	11 countries	9 countries	Multi-sector	One-sector
<i>Standard deviation:</i>				
Nom. exchange rate	0.22	0.19	0.22	0.19
Price ratio	0.14	0.07	0.17	0.18
Real exchange rate	0.18	0.18	0.09	0.04
<i>Cross-correlations:</i>				
Nominal - RER	0.70	0.91	0.66	0.33
Nominal - P.Ratio	-0.46	-0.37	-0.92	-0.97

its one-sector counterpart in most dimensions of the PPP puzzle.

The relatively modest improvement in terms of the behavior of the price ratio suggests that, relative to the data, prices in both models still move “too much” to offset movements in the nominal exchange rate induced by the monetary shocks. Kehoe and Midrigan (2007) discuss this issue in a model that is similar to ours when nominal aggregate demand follows a random walk ( $\rho_z = 0$ ). They analyze a cross-section of sectoral real exchange rates, and find that in the data the degree of comovement between nominal, aggregate and sectoral real exchange rates is uniformly high, irrespective of the degree of sectoral price stickiness. In contrast, their model predicts that the degree of comovement should be strongly correlated with the extent of price stickiness.

As we previously mentioned (footnote 6), with  $\rho_z > 0$  our model can account for the relationship between the first-order autocorrelation of sectoral real exchange rates and the extent of sectoral price stickiness that Kehoe and Midrigan (2007) document. However, it still falls short of producing as much comovement between nominal, aggregate and sectoral real exchange rates as they document for sectors in which prices change more frequently.

This deficiency of the benchmark economy can be reasonably well addressed with a version of our multi-sector model that includes a source of real rigidity in the sense of Ball and Romer (1990). We developed a version of the model with segmented labor markets.<sup>41</sup> If labor cannot move freely, the adjustment process of firms in different sectors tends to be more similar, since firms that adjust more frequently choose to react relatively less to shocks when compared to the model with an economy-wide labor market. Labor market segmentation can induce strategic complementarities in price setting (Woodford 2003, ch. 3), and amplify the degree of comovement between sectoral real exchange rates.

Figure 2 presents a simulation for some of the 36 sectoral exchange rates and the aggregate real exchange rate in our benchmark economy. The lack of comovement for the flexibly priced sectors

<sup>41</sup>The model with labor segmentation is developed in a separate appendix available upon request.

that we refer to is clear from the lines corresponding to the real exchange rates of sectors 1 and 2 - the most flexible sectors. However, it also shows that the extent of comovement with the aggregate real exchange rate is already somewhat high for sectors 5-36 (we only display these two sectors to avoid cluttering the graph). Figure 3 shows the analogous results for the model with segmented labor, under the same shocks. We use the same parametrization as for the benchmark economy. It is clear that real rigidities increase the extent of comovement between sectoral real exchange rates. More importantly, the change is relatively more pronounced for the more flexibly priced sectors.

The results of the model along this cross-sectional dimension can be improved further by introducing mechanisms that render the adjustment of prices to nominal shocks more sluggish. A variety of such mechanisms has been studied in the context of closed-economy models. For instance, one could consider allowing some price adjustments to be based on indexation to past inflation. Crucini et al. (2008) explore an interesting alternative, by developing a model that combines sticky prices with sticky information, and show that it fares better in explaining the cross-section of the dynamics of sectoral real exchange rates.

## 7 Conclusion

We show that a multi-sector model with heterogeneity in price stickiness parametrized to match the microeconomic evidence on price setting in the U.S. economy can produce very volatile and persistent real exchange rates. In turn, a counterfactual one-sector version of the world economy that features the same average frequency of price changes fails to do so. We conclude that the empirical properties of deviations from PPP only warrant the “puzzle” adjective if seen under the lens of such a one-sector model.

Our findings still leave open a series of important questions. In our heterogeneous model, as in the data, aggregate and sectoral real exchange rates are highly persistent, even for sectors in which prices change somewhat frequently. Despite the relative uniformity in persistence, the failure of the one-sector model in matching the data shows that the heterogeneity in the frequency of price adjustments is crucial for our results. This highlights the importance of investigating further the reasons for persistence being high across sectors. Our results with the baseline and alternative specifications point to the importance of the properties of shocks, the nature of the systematic component of monetary policy, the strength of pricing complementarities, and the details of the “demand side” of our structural model. There is clearly more work to be done in this direction.

The large persistence of the aggregate real exchange rate in the multi-sector economy depends on at least *some* sectors displaying a low frequency of price adjustment.<sup>42</sup> Our parametrized

---

<sup>42</sup>If the frequency of price changes is uniformly high, the model behaves similarly to a one-sector model with a

model features a distribution of the frequency of price changes derived from the recent evidence on price setting, which documents the existence of sectors in which prices are indeed quite sticky. In this paper, we highlight the fact that our results hold even in the case of strategic neutrality in price setting - i.e. when firms' pricing decisions are unrelated. Thus, we do not explore pricing complementarities, which are well known to strengthen the real effects of monetary shocks in one-sector closed-economy models (e.g. Woodford 2003, chapter 3). Moreover, as Carvalho (2006) shows, such complementarities amplify the magnitude and persistence of the real effects of monetary shocks even more in the presence of heterogeneity in price stickiness. The reason is that the sectors in which prices are relatively more sticky end up having a disproportionate aggregate effect. Such interdependence in pricing decisions also has important quantitative effects in terms of real exchange rate dynamics in the presence of heterogeneity. In particular, pricing complementarities generate stronger comovement between sectoral real exchange rates, with a relatively more pronounced effect in sectors in which price changes are more frequent.

For analytical tractability, in this paper we model price stickiness as in Calvo (1983), and assume that the sectoral frequencies of price adjustment are constant. In closed economies, heterogeneity in price setting has similar effects in a much larger class of models that includes various sticky-price and sticky-information specifications. While these results suggest that the nature of nominal frictions is not a crucial determinant of the effects of heterogeneity, it seems worthwhile to assess whether the results of this paper do in fact hold in models with different nominal frictions. In particular, one such class of models involves endogenous, optimal pricing strategies, chosen in the face of explicit information and/or adjustment costs.<sup>43</sup> The importance of our assumption of local currency pricing, and more generally, the stability of our findings across different policy regimes can also be assessed with models that feature fully endogenous pricing decisions, along the lines of Gopinath et al. (2007).

Finally, another important line of investigation refers to the source of heterogeneity in sectoral exchange rate dynamics. While we emphasize heterogeneity in price stickiness, an additional, potentially important source of heterogeneity is variation in the dynamic properties of sectoral shocks. It has been emphasized in recent work on the dynamics of international relative prices (e.g. Ghironi and Melitz 2005, and Atkeson and Burstein 2008), but to our knowledge a quantitative analysis in the context of the PPP puzzle has yet to be undertaken.

---

high frequency of price adjustment, and fails to generate volatile and persistent real exchange rates.

<sup>43</sup>More specifically, "menu cost" models (e.g. Barro 1972), models with information costs as in Reis (2006), and models with both adjustment and information frictions, as in Bonomo and Carvalho (2004) and Woodford (2008).

# A Appendix

## A.1 Proofs of Propositions

**Proposition 1** *Under the assumptions of Section (3), sectoral real exchange rates follow AR(2) processes:*

$$(1 - \rho_z L)(1 - \lambda_k L) q_{k,t} = \varphi_k u_t,$$

where  $\lambda_k \equiv 1 - \alpha_k$  is the per-period probability of no price adjustment for a firm in sector  $k$ ,  $u_t \equiv \sigma_{\varepsilon_z} (\varepsilon_{z,t} - \varepsilon_{z,t}^*)$  is a white noise process,  $\varphi_k \equiv \lambda_k - (1 - \lambda_k) \frac{\rho_z \beta \lambda_k}{1 - \rho_z \beta \lambda_k}$ , and  $L$  is the lag operator.

**Proof.** From the optimal price equations:

$$\begin{aligned} x_{H,k,t} &= (1 - \beta(1 - \alpha_k)) E_t \sum_{s=0}^{\infty} \beta^s (1 - \alpha_k)^s [c_{t+s} + p_{t+s}] \\ &= (1 - \beta(1 - \alpha_k)) E_t \sum_{s=0}^{\infty} \beta^s (1 - \alpha_k)^s z_{t+s} \\ &= z_t + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t - z_{t-1}), \end{aligned}$$

and analogously:

$$\begin{aligned} x_{F,k,t} &= z_t + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t - z_{t-1}), \\ x_{H,k,t}^* &= z_t^* + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t^* - z_{t-1}^*), \\ x_{F,k,t}^* &= z_t^* + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t^* - z_{t-1}^*). \end{aligned}$$

This implies that the country-sector price indices follow:

$$\begin{aligned} p_{H,k,t} &= (1 - \alpha_k) p_{H,k,t-1} + \alpha_k \left( z_t + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t - z_{t-1}) \right), \\ p_{F,k,t} &= (1 - \alpha_k) p_{F,k,t-1} + \alpha_k \left( z_t + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t - z_{t-1}) \right), \\ p_{H,k,t}^* &= (1 - \alpha_k) p_{H,k,t-1}^* + \alpha_k \left( z_t^* + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t^* - z_{t-1}^*) \right), \\ p_{F,k,t}^* &= (1 - \alpha_k) p_{F,k,t-1}^* + \alpha_k \left( z_t^* + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t^* - z_{t-1}^*) \right), \end{aligned}$$

and that sectoral price indices evolve according to:

$$\begin{aligned} p_{k,t} &= (1 - \alpha_k) p_{k,t-1} + \alpha_k \left[ z_t + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t - z_{t-1}) \right], \\ p_{k,t}^* &= (1 - \alpha_k) p_{k,t-1}^* + \alpha_k \left[ z_t^* + \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta(1 - \alpha_k) \rho_z} (z_t^* - z_{t-1}^*) \right]. \end{aligned}$$

Therefore, sectoral real exchange rates follow:

$$\begin{aligned} q_{k,t} &= e_t + p_{k,t}^* - p_{k,t} \\ &= e_t + \alpha_k \left( \frac{z_t^* - z_t}{1 - \beta(1 - \alpha_k)\rho_z} (\Delta z_t^* - \Delta z_t) \right) + (1 - \alpha_k) q_{k,t-1} - (1 - \alpha_k) e_{t-1}. \end{aligned} \quad (25)$$

In turn, the nominal exchange rate can be written as:

$$e_t = q_t + p_t - p_t^* = c_t - c_t^* + p_t - p_t^* = z_t - z_t^*. \quad (26)$$

Substituting (26) into (25) and simplifying yields:

$$q_{k,t} = (1 - \alpha_k) q_{k,t-1} + \left( 1 - \alpha_k - \alpha_k \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta (1 - \alpha_k) \rho_z} \right) \Delta e_t.$$

Finally, note that the nominal exchange rate evolves according to:

$$\begin{aligned} e_t &= z_t - z_t^* \\ &= (1 + \rho_z) (z_{t-1} - z_{t-1}^*) - \rho_z (z_{t-2} - z_{t-2}^*) + \sigma_{\varepsilon_z} (\varepsilon_{z,t} - \varepsilon_{z,t}^*) \\ &= (1 + \rho_z) e_{t-1} - \rho_z e_{t-2} + \sigma_{\varepsilon_z} (\varepsilon_{z,t} - \varepsilon_{z,t}^*), \end{aligned}$$

so that:

$$\Delta e_t = \rho_z \Delta e_{t-1} + u_t,$$

where  $u_t \equiv \sigma_{\varepsilon_z} (\varepsilon_{z,t} - \varepsilon_{z,t}^*)$  is a white noise process. As a result, we can write:

$$(1 - \rho_z L) (1 - \lambda_k L) q_{k,t} = \varphi_k u_t,$$

where  $\lambda_k \equiv 1 - \alpha_k$ , and  $\varphi_k \equiv \left( 1 - \alpha_k - \alpha_k \frac{\rho_z \beta (1 - \alpha_k)}{1 - \beta (1 - \alpha_k) \rho_z} \right)$ . ■

**Corollary 1** *The aggregate real exchange rate follows an ARMA( $K + 1, K - 1$ ) process:*

$$(1 - \rho_z L) \prod_{k=1}^K (1 - \lambda_k L) q_t = \left[ \sum_{k=1}^K \prod_{j \neq k}^K (1 - \lambda_j L) f_k \varphi_k \right] u_t.$$

**Proof.** This is a standard result in aggregation of time-series processes (Granger and Morris 1976). The aggregate real exchange rate is given by:

$$q_t = \sum_{k=1}^K f_k q_{k,t}.$$

From the result of **Proposition 1**, multiply each sectoral real exchange rate equation by its re-

spective sectoral weight to obtain:

$$f_k (1 - \rho_z L) (1 - \lambda_k L) q_{k,t} = f_k \varphi_k u_t.$$

Multiplying each such equation by all  $(K - 1)$   $L$ -polynomials of the form  $(1 - \lambda_m L)$ ,  $m \neq k$  and adding them up yields:

$$(1 - \rho_z L) \prod_{k=1}^K (1 - \lambda_k L) q_t = \left[ \sum_{k=1}^K \prod_{m \neq k}^K (1 - \lambda_m L) f_k \varphi_k \right] u_t,$$

so that  $q_t$  follows an  $ARMA(K + 1, K - 1)$ . ■

**Corollary 2** *The aggregate real exchange rate of the counterfactual one-sector world economy follows an AR(2) process:*

$$(1 - \rho_z L) (1 - \bar{\lambda} L) q_t^{1\text{sec}} = \varphi u_t,$$

where  $\bar{\lambda} \equiv \sum_{k=1}^K f_k \lambda_k$  and  $\varphi \equiv \bar{\lambda} - (1 - \bar{\lambda}) \frac{\rho_z \beta \bar{\lambda}}{1 - \rho_z \beta \bar{\lambda}}$ .

**Proof.** From **Corollary 1**, the real exchange rate in a one-sector world economy with frequency of price changes equal to  $\bar{\alpha}$  - probability of no-adjustment equal to  $\bar{\lambda} = 1 - \bar{\alpha}$  - follows:

$$(1 - \rho_z L) (1 - \bar{\lambda} L) q_t = (1 - \bar{\lambda} L) \left( \bar{\lambda} - (1 - \bar{\lambda}) \frac{\rho_z \beta \bar{\lambda}}{1 - \rho_z \beta \bar{\lambda}} \right) u_t.$$

■

**Proposition 2** *For the measures of persistence  $\mathcal{P} = CIR, \mathcal{LAR}, SAC$ :*

$$\mathcal{P}(q) > \mathcal{P}(q^{1\text{sec}}).$$

**Proof.** We prove separate results for each measure of persistence.

*CIR:*

Recall that we denote the impulse response function of the  $q_t$  process to a unit impulse by  $IRF_t(q)$ . In turn, let  $SIRF_t(q)$  denote the “scaled impulse response function,” i.e. the impulse response function to one standard deviation shock. Since  $q_t = \sum_{k=1}^K f_k q_{k,t}$ ,  $SIRF_t(q) = \sum_{k=1}^K f_k SIRF_t(q_k)$ . So, the impulse response function of the  $q_t$  process to a unit impulse, which is simply the scaled impulse response function normalized by the initial impact of the shock, can be written as:

$$IRF_t(q) = \frac{\sum_{k=1}^K f_k SIRF_t(q_k)}{\sum_{k=1}^K f_k SIRF_0(q_k)}. \quad (27)$$

From (27), the cumulative impulse response for  $q_t$  is:

$$CIR(q) = \sum_{t=0}^{\infty} IRF_t(q) = \frac{\sum_{k=1}^K f_k \sum_{t=0}^{\infty} SIRF_t(q_k)}{\sum_{k=1}^K f_k SIRF_0(q_k)}. \quad (28)$$

From the processes in **Proposition 1** we can compute  $\sum_{t=0}^{\infty} SIRF_t(q_k)$ , and  $SIRF_0(q_k)$ :

$$\sum_{t=0}^{\infty} SIRF_t(q_k) = \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k} \frac{1}{(1 - \lambda_k)(1 - \rho_z)}, \quad (29)$$

$$SIRF_0(q_k) = \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k}. \quad (30)$$

Substituting (29) and (30) into (28) yields:

$$CIR(q) = \frac{\sum_{k=1}^K f_k \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k} \frac{1}{(1 - \lambda_k)(1 - \rho_z)}}{\sum_{k=1}^K f_k \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k}}.$$

Note that  $\frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k}$  is increasing in  $\lambda_k$ , so that  $\tilde{f}_k \equiv \frac{f_k \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k}}{\sum_{k=1}^K f_k \frac{\lambda_k (1 - \rho_z \beta)}{1 - \rho_z \beta \lambda_k}}$  are sectoral weights obtained through a transformation of  $f_k$ , which attaches higher weight to higher  $\lambda_k$ 's. The fact that  $\frac{1}{(1 - \lambda_k)(1 - \rho_z)}$  is also increasing, and moreover convex, in  $\lambda_k$  thus implies the following inequalities:

$$\underbrace{\sum_{k=1}^K \tilde{f}_k \frac{1}{(1 - \lambda_k)(1 - \rho_z)}}_{CIR(q)} > \underbrace{\sum_{k=1}^K f_k \frac{1}{(1 - \lambda_k)(1 - \rho_z)}}_{\sum_{k=1}^K f_k CIR(q_k)} > \underbrace{\frac{1}{\sum_{k=1}^K f_k (1 - \lambda_k)(1 - \rho_z)}}_{CIR(q^{1 \text{ sec}})}. \quad (31)$$

This proves that  $CIR(q) > CIR(q^{1 \text{ sec}})$ .

**LAR:**

We order the sectors in terms of price stickiness, starting from the most flexible:  $\alpha_k > \alpha_{k+1}$  ( $\lambda_k < \lambda_{k+1}$ ). Moreover, recall that we assume  $\rho_z \in (1 - \alpha_1, 1 - \alpha_K)$ . Thus, based on **Proposition 1** and **Corollaries 1** and **2**, we obtain directly the following results:

$$\begin{aligned} \mathcal{LAR}(q) &= \lambda_K, \\ \mathcal{LAR}(q_k) &= \max\{\lambda_k, \rho_z\}, \\ \mathcal{LAR}(q^{1 \text{ sec}}) &= \max\{\bar{\lambda}, \rho_z\} = \max\left\{\sum_{k=1}^K f_k \lambda_k, \rho_z\right\}. \end{aligned}$$

Therefore:

$$\mathcal{LAR}(q) > \sum_{k=1}^K f_k \mathcal{LAR}(q_k) > \mathcal{LAR}(q^{1 \text{ sec}}). \quad (32)$$

**SAC:**

From **Corollary 1**:

$$\begin{aligned}
\mathcal{SAC}(q) &= 1 - (1 - \rho_z) \prod_{k=1}^K (1 - \lambda_k) \\
&> 1 - (1 - \rho_z) \prod_{k=1}^K (1 - \lambda_k)^{f_k} \\
&> 1 - (1 - \rho_z) \sum_{k=1}^K f_k (1 - \lambda_k) \\
&= \sum_{k=1}^K f_k (1 - (1 - \rho_z) (1 - \lambda_k)) = \sum_{k=1}^K f_k \mathcal{SAC}(q_k) \\
&= 1 - (1 - \rho_z) \left( 1 - \sum_{k=1}^K f_k \lambda_k \right) \\
&= 1 - (1 - \rho_z) (1 - \bar{\lambda}) = \mathcal{SAC}(q^{1\text{sec}}).
\end{aligned} \tag{33}$$

■

**Proposition 3** For the measures of persistence  $\mathcal{P} = \mathcal{CIR}, \mathcal{LAR}, \mathcal{SAC}$ :

$$\begin{aligned}
\text{aggregation effect under } \mathcal{P} &> 0, \\
\text{counterfactuality effect under } \mathcal{P} &\geq 0.
\end{aligned}$$

In particular:<sup>44</sup>

$$\begin{aligned}
\mathcal{CIR} &: \begin{aligned} &\text{aggregation effect} > 0, \\ &\text{counterfactuality effect} > 0; \end{aligned} \\
\mathcal{LAR} &: \begin{aligned} &\text{aggregation effect} > 0, \\ &\text{counterfactuality effect} > 0; \end{aligned} \\
\mathcal{SAC} &: \begin{aligned} &\text{aggregation effect} > 0, \\ &\text{counterfactuality effect} = 0. \end{aligned}
\end{aligned}$$

**Proof.** The proof is a by-product of the proof of **Proposition 2**, equations (31), (32), and (33). ■

**Lemma 1** For measures of persistence that are linear in the autoregressive coefficients:

$$\sum_{k=1}^K f_k \mathcal{P}(q_k) = \mathcal{P}(q^{1\text{sec}}),$$

so that the counterfactuality effect is zero.

---

<sup>44</sup>Below we omit the “under  $\mathcal{P}$ ” qualifier, since it is clear from the context.

**Proof.** Since all variables involved follow  $AR(2)$  processes, any linear measure of persistence is characterized by two real numbers  $c_1$  and  $c_2$ , such that the persistence for any given process equals the inner product between these two numbers and the two autoregressive coefficients. From **Proposition 1**, the persistence of  $q_{k,t}$  under a linear measure is thus:

$$\mathcal{P}(q_k) = c_1(\rho_z + \lambda_k) - c_2(\rho_z \lambda_k).$$

The weighted average of sectoral persistences is then:

$$\begin{aligned} \sum_{k=1}^K f_k \mathcal{P}(q_k) &= \sum_{k=1}^K f_k (c_1(\rho_z + \lambda_k) - c_2(\rho_z \lambda_k)) \\ &= c_1 \left( \rho_z + \sum_{k=1}^K f_k \lambda_k \right) - c_2 \left( \rho_z \sum_{k=1}^K f_k \lambda_k \right) \\ &= c_1(\rho_z + \bar{\lambda}) - c_2(\rho_z \bar{\lambda}) = \mathcal{P}(q^{1\text{sec}}). \end{aligned}$$

■

**Proposition 4** Let  $\mathcal{V}(q_k)$  denote the unconditional variance of the  $q_{k,t}$  process. Then:

$$\sum_{k=1}^K f_k \mathcal{V}(q_k) > \mathcal{V}(q^{1\text{sec}}).$$

**Proof.** We prove that  $\mathcal{V}(q_k)$  is convex in  $\lambda_k$ . From **Proposition 1**, the variance of  $q_{k,t}$  is:

$$\text{Var}(q_{k,t}) = \frac{(1 + \rho_z \lambda_k) \varphi_k^2 \sigma_{\varepsilon_z}^2}{(1 - \rho_z \lambda_k) \left[ (1 + \rho_z \lambda_k)^2 - (\rho_z + \lambda_k)^2 \right]},$$

where  $\varphi_k \equiv \lambda_k - (1 - \lambda_k) \frac{\rho_z \beta \lambda_k}{1 - \rho_z \beta \lambda_k}$ . We differentiate twice with respect to  $\lambda_k$ , and show that the result is positive. Due to the extremely long resulting expressions, for these steps we rely on Wolfram *Mathematica*, by Wolfram Research. The code file is available upon request. ■

**Proposition 5** Under the assumptions of Subsection 3.3:

$$\begin{aligned} \mathcal{V} \left( \frac{1}{K} \sum_{k=1}^K q_{k,t} \right) &\xrightarrow{K \rightarrow \infty} \infty, \\ CIR \left( \frac{1}{K} \sum_{k=1}^K q_{k,t} \right) &\xrightarrow{K \rightarrow \infty} \infty, \\ \mathcal{V}(q^{1\text{sec}}), CIR(q^{1\text{sec}}) &< \infty. \end{aligned}$$

**Proof.** For convenience, we reproduce here the required assumptions. The frequency of price changes  $\alpha_k$  for each new sector is drawn from  $(0, 1 - \delta)$  for arbitrarily small  $\delta > 0$ , according to some distribution with density  $g(\alpha|b)$ , where  $\alpha$  is the frequency of price changes and  $b$  is a parameter.

For  $\alpha \approx 0$  such density is assumed to be approximately proportional to  $\alpha^{-b}$ , with  $b \in (\frac{1}{2}, 1)$ . Moreover, it yields a strictly positive average frequency of price changes:  $\bar{\alpha} = \int_0^{1-\delta} g(\alpha|b) \alpha d\alpha > 0$ . For simplicity we focus on the case of  $\rho_z = 0$ , and then extend the argument to the general case.

For each  $q_{k,t}$  process  $(1 - \lambda_k L) q_{k,t} = \varphi_k u_t$  with  $\varphi_k \equiv \lambda_k$  and  $\alpha_k = 1 - \lambda_k$  drawn from  $g(\alpha|b)$ , define an auxiliary  $\tilde{q}_{k,t}$  process satisfying:

$$(1 - \lambda_k L) \tilde{q}_{k,t} = \tilde{\varphi} u_t,$$

where  $\tilde{\varphi} < \delta$ . Since  $\tilde{\varphi}$  is independent of  $\lambda_k$ , these  $\tilde{q}_{k,t}$  processes satisfy the assumptions in Zaffaroni (2004), and application of his Theorem 4 yields:

$$\mathcal{V} \left( \frac{1}{K} \sum_{k=1}^K \tilde{q}_{k,t} \right) \xrightarrow{K \rightarrow \infty} \infty.$$

Finally, since the  $\alpha_k$ 's have support  $(0, 1 - \delta)$  for small  $\delta > 0$ ,  $\mathcal{V} \left( \frac{1}{K} \sum_{k=1}^K q_{k,t} \right) > \mathcal{V} \left( \frac{1}{K} \sum_{k=1}^K \tilde{q}_{k,t} \right)$ , which proves that  $\mathcal{V} \left( \frac{1}{K} \sum_{k=1}^K q_{k,t} \right) \xrightarrow{K \rightarrow \infty} \infty$ . Analogously, application of Zaffaroni's (2004) result to the spectral density of the limiting process at frequency zero shows that it is unbounded. In turn, the fact that the spectral density at frequency zero for  $AR(p)$  processes is an increasing monotonic transformation of the cumulative impulse response (e.g. Andrews and Chen 1994) implies  $CIR \left( \frac{1}{K} \sum_{k=1}^K q_{k,t} \right) \xrightarrow{K \rightarrow \infty} \infty$ .

The results for the real exchange rate in the limiting counterfactual one-sector world economy follow directly from the fact that  $\bar{\alpha} = \int_0^{1-\delta} g(\alpha|b) \alpha d\alpha > 0$ , so that it follows a stationary  $AR$  process.

Finally, Zaffaroni's (2004) extension of his results to  $ARMA(p, q)$  processes implies that **Proposition 5** also holds for  $\rho_z > 0$ . ■

## References

- [1] Abadir, K. and G. Talmain (2002), “Aggregation, Persistence and Volatility in a Macro Model,” *Review of Economic Studies* 69: 749-779.
- [2] Andrews, D. and H. Chen (1994), “Approximately Median-Unbiased Estimation of Autoregressive Models,” *Journal of Business & Economic Statistics* 12: 187-204.
- [3] Atkeson, A. and A. Burstein (2008), “Pricing-to-Market, Trade Costs, and International Relative Prices,” *American Economic Review*, forthcoming.
- [4] Backus, D. and G. Smith (1993), “Consumption and Real Exchange Rates in Dynamic Economies with Non-Traded Goods,” *Journal of International Economics* 35: 297-316.
- [5] Ball, L. and D. Romer (1990), “Real Rigidities and the Non-Neutrality of Money,” *Review of Economic Studies* 57: 183-203.
- [6] Barro, R. (1972), “A Theory of Monopolistic Price Adjustment,” *Review of Economic Studies* 39: 17-26.
- [7] Benigno, G. (2004), “Real Exchange Rate Persistence and Monetary Policy Rules,” *Journal of Monetary Economics* 51: 473-502.
- [8] Bergin, P. and R. Feenstra (2001), “Pricing-to-Market, Staggered Contracts, and Real Exchange Rate Persistence,” *Journal of International Economics* 54: 333–359.
- [9] Bills, M. and P. Klenow (2004), “Some Evidence on the Importance of Sticky Prices,” *Journal of Political Economy* 112: 947-985.
- [10] Bonomo, M. and C. Carvalho (2004), “Endogenous Time-Dependent Rules and Inflation Inertia,” *Journal of Money, Credit and Banking* 36: 1015-1041.
- [11] Calvo, G. (1983), “Staggered Prices in a Utility Maximizing Framework,” *Journal of Monetary Economics* 12: 383-98.
- [12] Carvalho, C. (2006), “Heterogeneity in Price Stickiness and the Real Effects of Monetary Shocks,” *Frontiers of Macroeconomics: Vol. 2: Iss. 1, Article 1*.
- [13] Carvalho, C. and F. Schwartzman (2008), “Heterogeneous Price Setting Behavior and Aggregate Dynamics: Some General Results,” mimeo available at <http://www.princeton.edu/~cvianac/research.html>.

- [14] Chari, V., P. Kehoe, and E. McGrattan (2002), “Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?,” *Review of Economic Studies* 69: 533-63.
- [15] Chen, S. and C. Engel (2005), “Does ‘Aggregation Bias’ Explain the PPP Puzzle?,” *Pacific Economic Review* 10: 49-72.
- [16] Crucini, M. and M. Shintani (2008), “Persistence in Law-of-One-Price Deviations: Evidence from Micro-data,” *Journal of Monetary Economics* 55: 629-644.
- [17] Crucini, M., M. Shintani, and T. Tsuruga (2008), “Accounting for Persistence and Volatility of Good-level Real Exchange Rates: The Role of Sticky Information,” Bank of Japan IMES Discussion Paper No. 2008-E-5.
- [18] Dhyne, E., L. Álvarez, H. Le Bihan, G. Veronese, D. Dias, J. Hoffman, N. Jonker, P. Lünne-  
mann, F. Rumler and J. Vilmunen (2006), “Price Changes in the Euro Area and the United  
States: Some Facts from Individual Consumer Price Data,” *Journal of Economic Perspectives*  
20: 171-192.
- [19] Eichenbaum, M. and C. Evans (1995), “Some Empirical Evidence on the Effects of Monetary  
Shocks on Exchange Rates,” *Quarterly Journal of Economics* 110: 975-1010.
- [20] Fuhrer, J. and G. Rudebusch (2004), “Estimating the Euler Equation for Output,” *Journal of  
Monetary Economics* 51: 1133-1153.
- [21] Gagnon, E. (2007), “Price Setting During Low and High Inflation: Evidence from Mexico,”  
International Finance Discussion Papers 896, Board of Governors of the Federal Reserve Sys-  
tem.
- [22] Ghironi, F. and M. Melitz (2005), “International Trade and Macroeconomic Dynamics with  
Heterogeneous Firms,” *Quarterly Journal of Economics* 120: 865-915.
- [23] Granger, C. (1980), “Long Memory Relationships and the Aggregation of Dynamic Models,”  
*Journal of Econometrics* 14: 227-238.
- [24] Granger, C and R. Joyeux (1980), “An Introduction to Long Memory Time Series Models and  
Fractional Differencing,” *Journal of Time Series Analysis* 1: 15-29.
- [25] Granger, C. and M. Morris (1976), “Time Series Modelling and Interpretation,” *Journal of  
the Royal Statistical Society, Series A, Vol. 139: 246-257.*

- [26] Gopinath, G., O. Itskhoki, and R. Rigobon (2007), “Currency Choice and Exchange Rate Pass-through,” NBER Working Paper 13432.
- [27] Gopinath, G. and R. Rigobon (2008), “Sticky Borders,” *Quarterly Journal of Economics*, forthcoming.
- [28] Imbs, J., H. Mumtaz, M. Ravn and H. Rey (2005a), “PPP Strikes Back: Aggregation and the Real Exchange Rate,” *Quarterly Journal of Economics* 120: 1-43.
- [29] \_\_\_\_\_ (2005b), “Aggregation Bias’ DOES Explain the PPP Puzzle,” NBER Working Paper 11607.
- [30] Johri, A. and A. Lahiri (2008), “Persistent Real Exchange Rates,” mimeo available at <http://www.econ.ubc.ca/alahiri/Research.htm>.
- [31] Justiniano, A., G. Primiceri, and A. Tambalotti (2008), “Investment Shocks and Business Cycles,” mimeo available at <http://faculty.wcas.northwestern.edu/~gep575/research.html>.
- [32] Kehoe, P. and V. Midrigan (2007), “Sticky Prices and Sectoral Real Exchange Rates,” Federal Reserve Bank of Minneapolis Working Paper 656.
- [33] Kollman, R. (2001), “The Exchange Rate in a Dynamic-Optimizing Business Cycle Model with Nominal Rigidities: A Quantitative Investigation,” *Journal of International Economics* 55: 243-262.
- [34] Mankiw, G. and R. Reis (2002), “Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve,” *Quarterly Journal of Economics* 117: 1295-1328.
- [35] Martinez-Garcia, E. and J. Søndergaard (2008), “The Real Exchange Rate in Sticky Price Models: Does Investment Matter?,” Federal Reserve Bank of Dallas, GMPI Working Paper No. 17.
- [36] Nakamura, E. and J. Steinsson (2007), “Monetary Non-Neutrality in a Multi-Sector Menu Cost Model,” mimeo available at <http://www.columbia.edu/~js3204/cu-papers.html>.
- [37] \_\_\_\_\_ (2008), “Five Facts About Prices: A Reevaluation of Menu Cost Models,” forthcoming, *Quarterly Journal of Economics*.
- [38] Pesaran, M. and R. Smith (1995), “Estimating Long-Run Relationships from Dynamic Heterogeneous Panels,” *Journal of Econometrics* 68: 79-113.
- [39] Reis, R. (2006), “Inattentive Producers,” *Review of Economic Studies* 73: 793-821.

- [40] Rogoff, K. (1996), “The Purchasing Power Parity Puzzle,” *Journal of Economic Literature* 34: 647–668.
- [41] Steinsson, J. (2008), “The Dynamic Behavior of the Real Exchange Rate in Sticky-Price Models,” *American Economic Review* 98: 519-533.
- [42] Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.
- [43] \_\_\_\_\_ (2008), “Information-Constrained State-Dependent Pricing,” mimeo available at <http://www.columbia.edu/~mw2230/>.
- [44] Zaffaroni, P. (2004), “Contemporaneous Aggregation of Linear Dynamic Models in Large Economies,” *Journal of Econometrics* 120: 75-102.

### Sectoral Real Exchange Rates

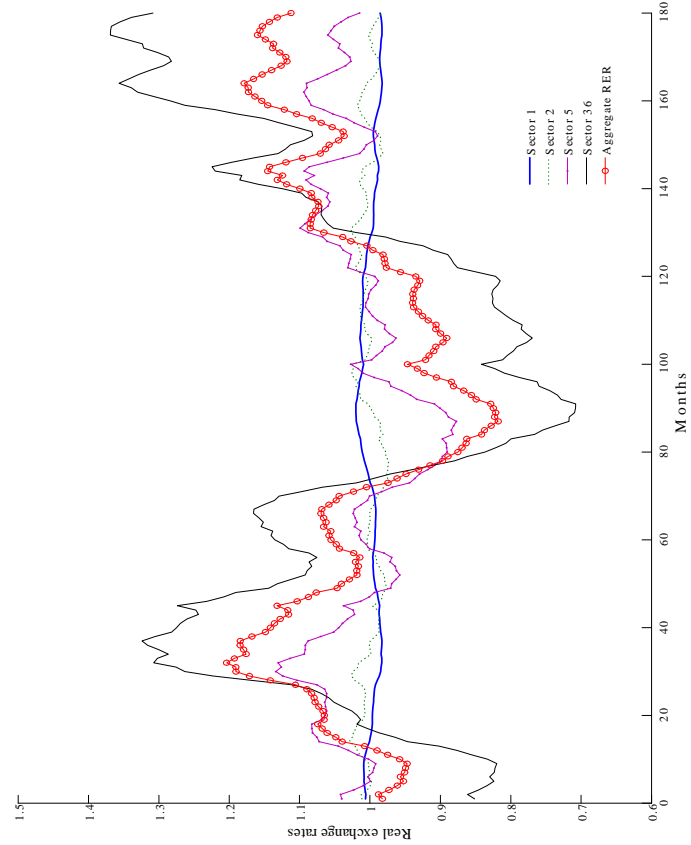
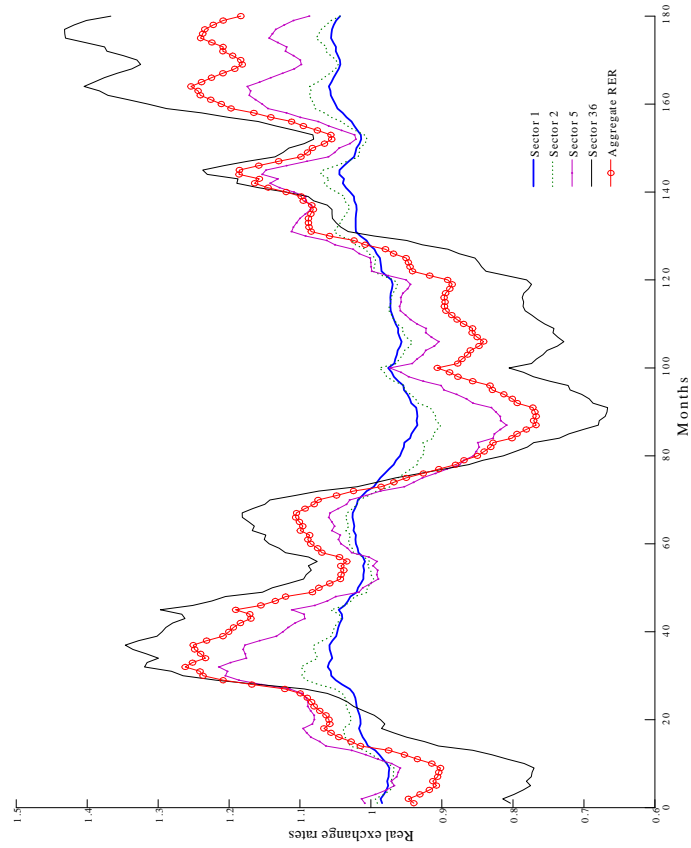


Figure 2: Baseline specification

Figure 3: Segmented labor markets