

Optimal Redistributive Capital Taxation in a Neoclassical Growth Model*

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Forthcoming, *Journal of Public Economics*

First Draft: July 1996
This Draft: December 9, 1998

Abstract

This paper provides a counterexample to the simplest version of the redistribution models considered by Judd (1985) in which the government chooses an optimal distortionary tax on capitalists to finance a lump-sum payment to workers. I show that the steady-state optimal tax on capital income is generally non-zero when the capitalists' utility is logarithmic and the government faces a balanced-budget constraint. With log utility, agents' optimal decisions depend solely on the current rate of return, not any future rates of return or tax rates. This feature of the economy effectively deprives the government of a useful policy instrument because promises about future tax rates can no longer influence current allocations. When combined with a lack of other suitable policy instruments (such as government bonds), the result is an inability to decentralize the allocations that are consistent with a zero limiting capital tax. I show that the standard approach to solving the dynamic optimal tax problem yields the wrong answer in this (knife-edge) case because it fails to properly enforce the constraints associated with the competitive equilibrium. Specifically, the standard approach lets in an additional policy instrument through the back door.

Keywords: *Fiscal Policy, Optimal Taxation, Redistribution.*

JEL Classification: H21, H23, E13, E62.

* I am indebted to Kenneth Judd for many helpful insights and suggestions that have substantially improved this paper. I also thank Jess Benhabib, Nicholas Bull, Steven Cassou, Silke Fabian-Reeves, Timothy Fuerst, Gregory Huffman, Thomas Sargent, and an anonymous referee for their valuable comments. Of course, I take full responsibility for any errors. Part of this research was conducted while I was a national fellow at the Hoover Institution, whose hospitality is gratefully acknowledged.

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1 Introduction

A classic question in economics is the degree to which the government should undertake policies designed to achieve a more equitable distribution of income. A commonly used framework for such an analysis, motivated by the work of Kaldor (1956) and Pasinetti (1962), is one with two social classes: capitalists and workers. In the context of a standard neoclassical growth model, Judd (1985) considers the optimal distortionary tax on capitalists to finance a lump-sum redistribution to workers. He obtains the following striking result: A benevolent optimizing government will undertake no redistribution in the long-run, i.e., the steady-state optimal tax on capital income is zero. Judd's result is independent of the level of income inequality or the weight assigned to workers in the social welfare function.

A closely related paper is one by Chamley (1986). He shows that the steady-state optimal tax on capital income is zero in a representative-agent version of the neoclassical growth model.¹ Both Judd and Chamley claim that the zero limiting capital tax result does not depend on assumptions about the government's ability to borrow or lend.² This paper provides a counterexample to that claim. In particular, I use a version of Judd's model to show that the limiting optimal tax on capital income can be non-zero when capitalists' utility is logarithmic and the government faces a balanced-budget constraint.

Before laying out the details of the counterexample, it is worthwhile to review two principles from the theory of optimal taxation in a multi-good static economy that are often used to provide some intuition for the Judd-Chamley result. First, the static optimal taxation literature generally supports the principle of uniform commodity taxation.³ Applying this result in a dynamic economy simply requires that we make use of the standard Arrow-Debreu device of interpreting goods at different dates or different states of nature as different goods. In a dynamic economy, uniform commodity taxation can be accomplished by eliminating the saving distortion so that present and future consumption goods are taxed at the same rate. Second, the "production efficiency" principle of Diamond and Mirlees (1971) says that, absent

¹The zero limiting capital tax result is also discussed by Arrow and Kurz (1970, pp. 191-203) in the context of a representative agent model with inelastic labor supply and productive public expenditures.

²On p. 71, Judd (1985) states "[S]uppose that [the government] has only two instruments: capital income taxation and lump-sum transfers to, or taxation of, workers." Moreover, on p. 78 he states "The addition of a bond market changes no asymptotic result and is assumed away..." On p. 609, Chamley (1986) states: "[O]ne does not need to specify whether the budget is balanced in every period, and this constraint does not affect the main result..."

³See Sandmo (1976) for a survey of this literature. Besley and Jewitt (1995) provide a more recent analysis of the necessary and sufficient conditions for the optimality of uniform commodity taxation.

profits or monopoly power, taxes should be levied on final goods, not intermediate goods. In a dynamic economy, the capital stock (or its service flow) can be viewed as an intermediate good because it typically does not show up as a direct argument of the utility function.

In solving for the optimal tax policy, both Judd and Chamley follow the standard approach of imposing the private sector first-order conditions as constraints on the government's allocation problem. In general, the first-order conditions imply that agents' consumption and saving decisions depend on the anticipated trajectory of *future* tax rates. In such an environment, a levy on the initial capital stock is an efficient method of raising revenue because it does not distort private-sector decisions. The standard approach implies that a benevolent government (which can commit itself to following a pre-announced policy) will set the optimal capital tax to its maximum value (typically 100%) at the beginning of the time horizon, but promise a zero tax rate in the future. Revenue obtained from the initial levy is used to expand public expenditures (if they are endogenous), or if intertemporal borrowing and lending is allowed, to accumulate a stock of interest-bearing assets that helps finance future public expenditures.⁴

A key to understanding the intuition for the counterexample is an important point made by Stern (1992, p. 284). He notes that the extension of static optimal taxation theorems to dynamic environments generally requires very strong assumptions of perfect competition, rational expectations, and the existence of complete markets. It is well-known, for instance, that the absence of lump-sum transfers or government debt can prevent an overlapping generations economy from achieving the "golden rule" condition, thereby creating a motive for a saving distortion to address the resulting under- or over-accumulation of capital. Hence, one might anticipate that shutting down the market for government bonds in an infinite-horizon economy could affect the conditions under which the Judd-Chamley result holds. I show that this is indeed the case.⁵

The counterexample is derived using the simplest version of the various redistribution models considered by Judd (1985) in which workers do not save and capitalists do not work. Capitalists solve an intertemporal optimization problem while workers simply consume their

⁴The binding upper limit on the capital tax during the initial regime represents a particular type of restriction on the government's ability to borrow or lend. Specifically, it prevents the government from acquiring a stock of assets that is large enough to finance all future expenditures without ever having to resort to distortionary taxation. Such a restriction is necessary to make the problem interesting, but it is obviously weaker than the one I emphasize here which rules out a market for government bonds altogether.

⁵The notion that a period-by-period balanced-budget requirement may overturn the Judd-Chamley result is mentioned briefly by Jones, Manuelli, and Rossi (1997, p. 116) and Roubini and Milesi-Ferretti (1994, p. 5). Neither paper provides any formal analysis in support of the assertion, however.

wage income plus any transfers received from the government. Following Judd, I rule out a market for government bonds. I then specialize the capitalists' isoelastic utility function to the logarithmic case.

Given the above setup, I show that closed-form expressions for the capitalists' decision rules can be obtained without knowing anything about how tax rates will evolve in the future. This is due to the special property of log utility for which the income and substitution effects of future interest rate movements exactly cancel out.⁶ It turns out that at each point in time, capitalists consume a constant fraction of the productive capital stock. Their saving decision depends solely on the current rate of return, not any future rates of return or tax rates. I solve for the optimal tax policy by imposing agents' *decision rules* rather than their first-order conditions as constraints on the government's allocation problem. In contrast to Judd (1985), I find that the steady-state optimal tax on capital income is generally non-zero.

The economic intuition for the counterexample is straightforward. The fact that agents' decisions depend only on the current rate of return implies that an initial capital levy will be highly distortionary. This is especially true in a balanced-budget environment where revenue derived from the initial levy cannot be stored up to finance future transfers to workers. The government can therefore do better by spreading out its redistribution policy over the entire time horizon rather than front-loading the transfers to workers at the beginning of the horizon.

The technical problem with the standard approach that explains why it yields the wrong answer in the log-utility/balanced-budget case is more subtle. By imposing agents' first-order conditions as constraints on the government's allocation problem, the standard approach presumes the existence of "anticipation effects," whereby an announced trajectory of *future* tax rates can affect *current* allocations. In cases without anticipation effects (such as the counterexample), the government is effectively deprived of a useful policy instrument. The standard approach fails to properly restrict the government's choice of allocations to take this feature of the economy into account. Hence, it may not be possible to decentralize the chosen allocations with the existing set of policy instruments.

When applying the standard approach in Judd's model, the government's first-order conditions with respect to capital (k) and consumption (c) determine the optimal allocations for these variables. Given the allocations, the optimal tax rate is recovered from the capitalists'

⁶This property of log utility has been exploited in addressing other economic questions involving intertemporal optimization and planning. For examples, see Phelps and Pollak (1968), Levhari and Mirman (1980), and Barro (1998).

Euler equation. The validity of optimal tax rate computed in this way depends crucially on the assumption that there is a sufficient number of policy instruments to implement the independently chosen allocations for k and c as a competitive equilibrium. This assumption breaks down in the log-utility/balanced-budget case. Specifically, the unique competitive equilibrium has the property $c = \rho k$, where ρ is the capitalists' rate of time preference. This means that the government does not have the freedom to independently choose the allocations for k and c as is (incorrectly) assumed when deriving the government's first-order conditions.

By naively treating the allocations for capital and consumption as independent, the standard approach lets in an additional policy instrument through the back door. This serves to break the direct link between k and c . We can interpret the additional policy instrument as either government bonds (b) or a constant consumption tax ($\bar{\tau}_c$). With government bonds, the unique competitive equilibrium has the property $c = \rho(k + b)$. With a constant consumption tax, the unique competitive equilibrium has the property $c = \left(\frac{\rho}{1 + \bar{\tau}_c}\right)k$. Either setup allows the government to independently choose allocations for k and c . I show that by making either policy instrument explicit in the formulation of the government's problem, we can recover Judd's result even in the logarithmic case. In fact, since a constant consumption tax can implement the first-best allocations, the optimal capital tax is zero at all times, not just in the long-run.

The counterexample turns out to be a knife-edge result. Any small change in the capitalists' intertemporal elasticity of substitution away from one (the log case) will create anticipation effects such that an announced trajectory for the capital tax can implement a set of independently chosen allocations for k and c . This restores the validity of the crucial assumption that is made under the standard approach. Put another way, the government's ability to influence current allocations through promises about future tax rates vanishes only when utility is logarithmic. While it may seem odd that the trajectory of the optimal capital tax undergoes an abrupt change as the intertemporal elasticity of substitution crosses one, the level of social welfare is what matters for the government and this varies smoothly with the elasticity parameter.

A direct implication of the absence of anticipation effects is that the optimal open-loop tax policy is time consistent. In particular, when agents' decisions do not rely on any promises about future policy actions, the government perceives no gain (immediate or otherwise) from renegeing on a pre-announced plan. Hence, the solutions to the government's problem under

commitment and no-commitment are the same.

When the conditions needed for the counterexample hold, the steady-state optimal capital tax is increasing in the level of income inequality and the weight assigned to workers in the social welfare function. This result is reminiscent of an earlier body of research that explores redistributive fiscal policies in Solow-type growth models where agents' decision rules are characterized by fixed saving propensities.⁷ While fixed saving propensities can be reconciled with optimizing behavior for a restricted class of functional forms, capitalists in this model exhibit a variable saving rate that depends positively on the after-tax rate of return.

This paper also compliments research that identifies alternative theoretical justifications for non-zero limiting capital taxation in dynamic general equilibrium models. Some alternative justifications include: divergence between public and private rates of time preference (Arrow and Kurz, 1970), divergence from the golden rule in an overlapping generations economy (Auerbach, 1979, Atkinson and Sandmo, 1980, and King, 1980), private borrowing constraints (Hubbard and Judd, 1986, Aiyagari, 1995, and İmrohoroğlu, 1998), alternative formulations of the capital accumulation technology (Zhu, 1995), congestion externalities (Judd, 1999 and Corsetti and Roubini, 1996), pure profits (Jones, Manuelli, and Rossi, 1993, 1997), tax code restrictions (Stiglitz, 1987 and Jones, Manuelli, and Rossi, 1997), untaxed factors of production (Correia, 1996), imperfectly competitive product markets (Judd, 1997 and Guo and Lansing, 1998), lack of a commitment mechanism (Benhabib and Rustichini, 1997), cyclical dynamics associated with a non-attracting steady-state (Kemp, Long, and Shimomura, 1993), and political economy considerations (Huffman, 1996 and Krussell, Quadrini, and Ríos Rull, 1996).

The remainder of this paper is organized as follows. Section 2 describes a continuous time growth model that corresponds to the simplest version of the models considered by Judd (1985). Section 3 reproduces Judd's result using the standard approach and then provides a counterexample for the case of log utility. Section 4 examines the technical problem with the standard approach and shows how Judd's result can be recovered in the log case by introducing an additional policy instrument. Section 5 considers the robustness of the Judd-Chamley result in other economic environments. Section 6 concludes. An appendix discusses the time consistency issue.

⁷See, for example, Hamada (1967), Grieson (1975), Marglin (1976), Stanley (1978), Pestieau and Possen (1978), Stiglitz (1978), Boadway (1979), Homma (1981), and Hammer (1981), among others.

2 The Model

The model economy consists of a government, identical competitive firms, and two types of infinitely-lived, price-taking agents called workers and capitalists. For simplicity, the number of each type in the population is normalized to one. The model abstracts from uncertainty, technological progress, and population growth. Workers supply one unit of labor inelastically and incur a transaction cost for saving or borrowing small amounts. As a result, all wealth is concentrated in the hands of the capitalists, who do not work.⁸ In what follows, I describe each of these features of the economy in more detail.

2.1 Capitalists

The decision problem faced by capitalists is:

$$\max \int_0^{\infty} e^{-\rho t} u[c(t)] dt, \quad \rho > 0, \quad (1)$$

subject to

$$c(t) + i(t) = [1 - \tau_k(t)] r(t)k(t) + \tau_k(t)\delta k(t), \quad (2)$$

$$\dot{k}(t) = i(t) - \delta k(t), \quad \delta \in [0, 1], \quad k(0) \text{ given.} \quad (3)$$

where ρ is the constant rate of time preference, $c(t)$ is consumption, and $i(t)$ is investment. The utility function $u[c(t)]$ is assumed to be increasing and strictly concave in $c(t)$ with $\lim_{c \rightarrow 0} u'(c) = \infty$ and $\lim_{c \rightarrow \infty} u'(c) = 0$. The elasticity of marginal utility (the inverse of the intertemporal elasticity of substitution in consumption) is given by $\beta \equiv \frac{-cu''(c)}{u'(c)}$ and is assumed constant. Capitalists derive income by renting their capital stock $k(t)$ to competitive firms at the rate $r(t)$. Gross rental income is taxed at the rate $\tau_k(t)$. The term $\tau_k(t)\delta k(t)$ is a depreciation allowance, where δ is the constant depreciation rate. In making their decisions, capitalists take $r(t)$ and $\tau_k(t)$ as given. The capital stock evolves according to (3).

The current-value Hamiltonian for the capitalists' problem is

$$H = u(c) + \lambda(\hat{r}k - c). \quad (4)$$

where $\hat{r} \equiv (1 - \tau_k)(r - \delta)$ and, for ease of notation, the time dependence of all variables is now suppressed. The first-order necessary conditions are

$$\frac{\partial H}{\partial c} = u'(c) - \lambda = 0, \quad (5a)$$

⁸This is the setup used by Judd (1985, Section 3).

$$\frac{\partial H}{\partial k} = \lambda \hat{r} = \rho \lambda - \dot{\lambda}, \quad (5b)$$

$$\frac{\partial H}{\partial \lambda} = \hat{r}k - c = \dot{k}, \quad (5c)$$

where λ is the multiplier associated with (2). The transversality condition is $\lim_{t \rightarrow \infty} e^{-\rho t} u'(c) k = 0$. Since H is concave, the necessary conditions are also sufficient. Differentiating (5a) with respect to t and substituting in (5b) yields the following differential equation:

$$\dot{c} = \frac{c}{\beta} (\hat{r} - \rho). \quad (6)$$

The system of nonlinear differential equations described by (5c) and (6), together with the initial condition and the transversality condition, determine the capitalists' optimal decisions for consumption and saving over time. In general, the solution to this system depends on the anticipated trajectory of the after-tax rate of return \hat{r} . However, we shall see that when $\beta = 1$ (the log case), knowledge of this trajectory is not required.

2.2 Workers

Workers neither save nor borrow, but supply one unit of labor inelastically for all t . Their decision problem is trivial: at each point in time they consume all of their income. The workers' consumption x is given by

$$x = w + TR, \quad (7)$$

where w is the real wage and TR represents lump-sum transfers received from the government. The workers' lifetime utility is given by $\int_0^\infty e^{-\rho t} v(x) dt$, where the function $v(x)$ is assumed to be increasing and strictly concave in x , with $\lim_{x \rightarrow 0} v'(x) = \infty$, $\lim_{x \rightarrow \infty} v'(x) = 0$.⁹

2.3 Firms

Firms operate in competitive markets and produce goods using a technology that exhibits constant returns to scale in capital and labor inputs.¹⁰ Since the labor input is equal to one, k can also be interpreted as the capital-labor ratio. The production function $f(k)$ is assumed to be increasing and strictly concave in k with $\lim_{k \rightarrow 0} f'(k) = \infty$, and $\lim_{k \rightarrow \infty} f'(k) = 0$. Profits are given by $f(k) - rk - w$. Profit maximization implies

$$r = f'(k), \quad (8)$$

$$w = f(k) - f'(k)k. \quad (9)$$

⁹Judd (1985) assumes $v(\cdot) = u(\cdot)$. This affects no result.

¹⁰We may assume that firms are owned by capitalists but, since profits are zero in equilibrium, this affects no result.

2.4 Government

Following Judd (1985), I rule out a market for government bonds and assume that the government can commit itself to following a tax-transfer policy announced at $t = 0$. The government chooses trajectories for τ_k and TR to maximize a weighted sum of agents' lifetime utilities, subject to (5)-(9), and the following instantaneous budget constraint:

$$TR = \tau_k (r - \delta) k. \quad (10)$$

In order to isolate the pure redistribution motive for capital taxation, equation (10) includes only one type of public expenditures: lump-sum transfers to workers.¹¹ By substituting (10) into (7) and making use of the zero-profit condition $f(k) - rk - w = 0$, the workers' consumption can be written as

$$\begin{aligned} x &= (1 - \tau_k) w + \tau_k (f(k) - \delta k), \\ &= f(k) - \delta k - \hat{r}k. \end{aligned} \quad (7')$$

Equation (7') reveals the trade-off the government faces in deciding how much to tax capital income. On the one hand, an increase in τ_k reduces the workers' effective after-tax wage. On the other hand, an increase in τ_k provides revenue that can be transferred to workers. The optimal tax policy balances these two opposing forces to maximize social welfare.

3 The Steady-State Optimal Capital Tax

This section lays out the expressions that govern the steady-state optimal tax on capital income in the model. I begin by reproducing the zero limiting tax result of Judd (1985), assuming that $u(c)$ remains general within the class of isoelastic utility functions, i.e., $u(c) = \frac{c^{1-\beta}-1}{1-\beta}$.¹² Next, I use an alternative solution procedure to derive a counterexample for the case of $\beta = 1$, which corresponds to log utility.

¹¹In addition to transfers, Judd (1985, p. 71) allows for a constant government spending requirement G that acts as a drain on productive resources. He assumes that any revenue needed to finance G in excess of that collected by the (optimally chosen) capital tax can be obtained by imposing a non-distortionary tax on workers' labor income. I have omitted this feature of the model because it does not affect the steady-state optimal capital tax in Judd's analysis.

¹²Judd's result continues to hold when $\beta = \beta(c)$. See Kemp, Long, and Shimomura (1993, Section 2).

3.1 The Result of Judd (1985)

The dynamic optimal tax problem considered by Judd (1985) can be formulated as:

$$\max_{\hat{r}, k, c} \int_0^{\infty} e^{-\rho t} \{ \gamma v(f(k) - \delta k - \hat{r}k) + u(c) \} dt, \quad (11)$$

subject to

$$\dot{c} = \frac{c}{\beta} (\hat{r} - \rho), \quad (12)$$

$$\dot{k} = \hat{r}k - c, \quad k(0) \text{ given}, \quad (13)$$

$$\hat{r} \geq 0, \quad (14)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} u'(c)k = 0. \quad (15)$$

The parameter $\gamma \geq 0$ in (11) can be interpreted as a political weighting factor that controls how much the policymaker favors workers relative to capitalists.¹³ Judd's formulation follows the standard approach by imposing the capitalists' first-order conditions, equations (12) and (13), as constraints on the government's allocation problem. The optimal policy must also satisfy the constraint $\hat{r} \geq 0$ which implies $\tau_k \leq 1$. If this constraint were to be violated, capitalists would have no incentive to rent their capital to firms, but instead would simply let capital depreciate and write off the depreciation expense against their tax bill. Finally, equation (15) says that the optimal allocations must satisfy the transversality condition. The above constraints are intended to ensure that the government's chosen set of allocations can be implemented as a competitive equilibrium.

The current-value Hamiltonian for the government's problem is

$$H_g = \gamma v(f(k) - \delta k - \hat{r}k) + u(c) + q_1(\hat{r}k - c) + q_2 \frac{c}{\beta} (\hat{r} - \rho) + \eta \hat{r}, \quad (16)$$

where q_1 and q_2 are the multipliers associated with (12) and (13) and η is the Kuhn-Tucker multiplier on the inequality constraint (14). The first-order conditions for this problem are

$$\frac{\partial H_g}{\partial k} = \gamma v'(x) (f'(k) - \delta - \hat{r}) + q_1 \hat{r} = \rho q_1 - \dot{q}_1, \quad (17a)$$

$$\frac{\partial H_g}{\partial c} = u'(c) - q_1 + q_2 \frac{1}{\beta} (\hat{r} - \rho) = \rho q_2 - \dot{q}_2, \quad (17b)$$

$$\frac{\partial H_g}{\partial \hat{r}} = -\gamma v'(x)k + q_1 k + q_2 \frac{c}{\beta} + \eta = 0, \quad (17c)$$

$$\frac{\partial H_g}{\partial q_1} = \hat{r}k - c = \dot{k}, \quad (17d)$$

$$\frac{\partial H_g}{\partial q_2} = \frac{c}{\beta} (\hat{r} - \rho) = \dot{c}, \quad (17e)$$

¹³Judd (1985) also assigns the weight $(1 - \gamma)$ to capitalists in the social welfare function. I have omitted this feature of the model because only the relative weights matter.

together with the complementary slackness condition $\eta\hat{r} = 0$, and the transversality conditions $\lim_{t \rightarrow \infty} e^{-\rho t} q_1 k = 0$ and $\lim_{t \rightarrow \infty} e^{-\rho t} q_2 c = 0$. A potential problem with the standard approach (aside from the one brought out in the counterexample below) is that H_g may not be concave. This issue is generally ignored in the literature and the government's first-order conditions are simply assumed to be both necessary and sufficient for a global optimum.¹⁴

We are now in a position to demonstrate Judd's zero limiting capital tax result. In steady-state (when $\dot{k} = \dot{c} = \dot{q}_1 = \dot{q}_2 = 0$), equation (17e), which is simply the capitalists' Euler equation, implies $\hat{r} = \rho$. Using this result in the steady-state version of (17a) and assuming $\gamma v'(x) > 0$ (social welfare is increasing in the workers' consumption) yields $f'(k) - \delta - \hat{r} = 0$. Combining this expression with the definition $\hat{r} \equiv (1 - \tau_k)(f'(k) - \delta)$ yields $\tau_k(\infty) = 0$. This is the result given by Judd's Theorem 2: a benevolent optimizing government undertakes no redistribution in the long-run. Notice that Judd's result is independent of capital's share of total output $\frac{f'(k)k}{f(k)}$ (which determines the level of income inequality) and the weight γ assigned to workers in the social welfare function.

Although there is no redistribution in the long-run, Judd's Theorem 1 says that a government concerned about workers will employ unanticipated, temporary capital taxation for short-run redistributive purposes. This can be accomplished by setting $\tau_k = 1$ (or equivalently, $\hat{r} = 0$) at the beginning of the time horizon to take advantage of the fixed nature of the initial capital stock.

From a purely mathematical standpoint, the key to Judd's result is the form of government's first-order condition with respect to k , equation (17a). The crucial assumption that has been made here is that the government can choose an allocation for k , but that this choice does pin down an allocation for c . Rather, the allocation for c is assumed to be pinned down by equation (17b). Given these allocations, the optimal tax rate is recovered from the capitalists' Euler equation (17e). The validity of optimal tax rate computed in this way requires that there exists a sufficient number of policy instruments to implement the independently-chosen allocations for k and c as a competitive equilibrium. In what follows, I show that when $\beta = 1$ (the log case) these allocations cannot be decentralized unless the government has access to an additional policy instrument.

¹⁴The counterparts to equations (16) and (17a)-(17c) are Judd's equations (22) and (23a)-(23c) on pp. 71-72. Note that there are some typographical errors in the published versions of these equations in Judd (1985). Of course, these do not affect the main result.

3.2 A Counter Example: The Case of Log Utility

By taking the limit as $\beta \rightarrow 1$, the utility function $u(c) = \frac{c^{1-\beta}-1}{1-\beta}$ can be written as $u(c) = \log(c)$ and a unique closed-form solution to the capitalists' decision problem can be obtained.

Proposition 1. *When there is no market for government bonds and $u(c) = \log(c)$, the capitalists' unique decision rules are given by*

$$c = \rho k, \tag{18}$$

$$\dot{k} = k(\hat{r} - \rho), \tag{19}$$

where ρ is the rate of time preference and $\hat{r} \equiv (1 - \tau_k)(f'(k) - \delta)$.

Proof: The decision rules can be obtained by applying the method of undetermined coefficients. A guess for the consumption decision rule is $c = d_0 k$, where d_0 is a constant to be determined. Substituting the conjectured form into (6) yields $\dot{k} = \frac{k}{\beta}(\hat{r} - \rho)$. Using this expression for \dot{k} in (5c) and solving for consumption yields $c = \hat{r}k - \frac{k}{\beta}(\hat{r} - \rho)$. When $u(c) = \log(c)$, we have $\beta = 1$ and the original guess for c is confirmed with $d_0 = \rho$. The transversality condition is satisfied since $\lim_{t \rightarrow \infty} e^{-\rho t} \left(\frac{1}{\rho}\right) = 0$. \square

At this point, one might wonder whether the above decision rules would change if our initial guess had allowed capitalists to explicitly condition their decisions on the anticipated time path of the capital tax. The answer is no. Equations (18) and (19) characterize the unique competitive equilibrium for *any* trajectory of the future tax rate. The fact that we are able to solve for the decision rules without knowing anything about how the tax rate will evolve in the future is due to the special property of log utility. Specifically, the income and substitution effects of future interest rate movements exactly cancel so that capitalists only need to observe the current after-tax return in order to decide how much to consume and save.

Representative-agent versions of the neoclassical growth model may also exhibit the property that optimal decisions do not depend on the future return to capital. A number of such examples are provided by Kurz (1969) and Chang (1988), who consider the “inverse optimal problem.” This involves finding the appropriate set of preferences and technology to reconcile a given set of decision rules with optimizing behavior. For example, these authors show that the constant saving rate assumption of Solow growth models is consistent with optimizing behavior for a restricted class of functional forms. Unlike the Solow model, however, Proposition 1 implies that the saving rate in this model is not constant, but depends positively on the after-tax return \hat{r} . It is straightforward to show that the capitalists' saving rate is given by

$s = \frac{i}{c+i} = \frac{\hat{r}+\delta-\rho}{\hat{r}+\delta}$. This expression implies $\frac{\partial s}{\partial \hat{r}} = \frac{\rho}{(\hat{r}+\delta)^2} > 0$.¹⁵

Given Proposition 1, we can now proceed by imposing agents' *decision rules* rather than their first-order conditions as constraints on the government's allocation problem. Taking this approach, the current-value Hamiltonian for the government's problem becomes

$$H_g = \gamma v(f(k) - \delta k - \hat{r}k) + \log(\rho k) + q_1 k(\hat{r} - \rho) + \eta \hat{r}, \quad (20)$$

where (18) has been used to eliminate c and q_1 is the multiplier associated with (19). The first-order conditions are

$$\frac{\partial H_g}{\partial k} = \gamma v'(x)(f'(k) - \delta - \hat{r}) + \frac{1}{k} + q_1(\hat{r} - \rho) = \rho q_1 - \dot{q}_1, \quad (21a)$$

$$\frac{\partial H_g}{\partial \hat{r}} = -\gamma v'(x)k + q_1 k + \eta = 0, \quad (21b)$$

$$\frac{\partial H_g}{\partial q_1} = k(\hat{r} - \rho) = \dot{k}, \quad (21c)$$

together with $\eta \hat{r} = 0$ and $\lim_{t \rightarrow \infty} e^{-\rho t} q_1 k = 0$. As before, the government's first-order condition with respect to k is the key to determining the steady-state optimal capital tax. Notice, however, that (21a) differs in significant ways from the corresponding first-order condition (17a) that was obtained under the standard approach. This is because equation (21a), unlike (17a), respects the constraint $c = \rho k$ which is a necessary condition of the competitive equilibrium. The upshot is that equation (17a) is not valid when $\beta = 1$ because a crucial assumption needed to apply the standard approach (the independence of k and c) breaks down as $\beta \rightarrow 1$. This leads to the following proposition.

Proposition 2. *When there is no market for government bonds and $u(c) = \log(c)$, the sign of the steady-state optimal tax on capital income is governed by the following conditions:*

- (i) $\tau_k(\infty) > 0$ if $\rho > \frac{1}{\gamma v'(x)k}$,
- (ii) $\tau_k(\infty) = 0$ if $\rho = \frac{1}{\gamma v'(x)k}$,
- (iii) $\tau_k(\infty) < 0$ if $\rho < \frac{1}{\gamma v'(x)k}$.

Proof: In steady-state (when $\dot{k} = \dot{c} = \dot{q}_1 = 0$), equation (21c) implies $\hat{r} = \rho$. Using this result in the steady-state version of (21a) yields $\gamma v'(x)(f'(k) - \delta - \hat{r}) + \frac{1}{k} - \rho q_1 = 0$. Since $\hat{r} = \rho$, the constraint $\hat{r} \geq 0$ is not binding and (21b) implies $q_1 = \gamma v'(x)$. Substituting this expression

¹⁵The economy-wide saving rate is given by $\frac{i}{x+c+i} = \frac{k(\hat{r}-\rho)+\delta k}{f(k)}$ and is also not constant.

for q_1 into (21a) and solving for $f'(k) - \delta - \hat{r}$ yields

$$f'(k) - \delta - \hat{r} = \rho - \frac{1}{\gamma v'(x)k}. \quad (22)$$

From the definition of \hat{r} , the left side of (22) can be written as $\tau_k (f'(k) - \delta)$. Since $\hat{r} = \rho$, we know $f'(k) - \delta = \frac{\rho}{1 - \tau_k}$. For any $\tau_k \in (-\infty, 1)$, we have $f'(k) - \delta > 0$ and the sign of $\tau_k(\infty)$ will be governed by the right side of (22). \square

Proposition 2 says that the steady-state optimal tax on capital income is non-zero except when the parameters and long-run allocations exactly satisfy condition (ii). The economic intuition is straightforward. The fact that agents' decisions depend solely on the current after-tax return means that an initial capital levy will be highly distortionary. This is especially true when the government cannot engage in intertemporal borrowing or lending because revenue derived from the initial levy cannot be stored up to finance future transfers to workers. It turns out that the government can do better by spreading out its redistribution policy over the entire time horizon rather than front-loading the transfers to workers at the beginning of the horizon.¹⁶

Equation (22) implies that the optimal tax rate is more likely to be positive as agents become more impatient (ρ increases); the government places more weight on the welfare of the workers (γ increases); the consumption of the worker falls ($v'(x)$ increases); and capitalists acquire more wealth (k increases). A comprehensive sensitivity analysis must also take into account the dependence of $v'(x)$ and k on the parameters ρ and γ . In what follows, I consider a particular example with $v(x) = \log(x)$ and $f(k) = k^\theta$, where $\theta \in (0, 1)$.

Substituting the above functional forms into (22), together with $x = f(k) - \delta k - \hat{r}k$ and $\hat{r} = \rho$, yields the following expression for the steady-state capital stock:

$$k(\infty) = \left[\frac{1 + \gamma\theta}{\rho(1 + 2\gamma) + \delta(1 + \gamma)} \right]^{\frac{1}{1-\theta}}. \quad (23)$$

When $\delta = 0$, the above expression coincides exactly with the steady-state capital stock derived by Kemp, Long, and Shimomura (1993) in their analysis of the *time consistent*, feedback Stackelberg solution for this particular version of Judd's model.¹⁷ Since $k(\infty)$ uniquely pins down $\tau_k(\infty)$ via the steady-state condition $\hat{r} = \rho$, the steady-state capital tax implied

¹⁶While this result is demonstrated analytically by Proposition 2, I have also verified it numerically in a calibrated version of the model. Details are available upon request.

¹⁷When $\delta = 0$, equation (23) can be written as $k(\infty) = \left[\frac{\rho(1/\gamma + 2)}{1/\gamma + \theta} \right]^{\frac{1}{\theta-1}}$. By defining $\alpha \equiv 1/\gamma$, we obtain Kemp, Long, Shimomura's equation (A.20), p. 428.

by the open-loop Stackelberg solution considered here turns out to be identical to that of the feedback Stackelberg solution considered by Kemp, Long, and Shimomura (1993).¹⁸ The agreement between the two solution concepts is due to the absence of anticipation effects when $u(c) = \log(c)$.¹⁹

From the steady-state condition $\hat{r} = \rho$, we have $\tau_k(\infty) = 1 - \frac{\rho}{\theta k(\infty)^{\theta-1} - \delta}$. Combining this expression with (23) and rearranging yields

$$\tau_k(\infty) = \frac{\rho\gamma\theta - (1-\theta)(\rho + \delta)}{\rho\theta(1+2\gamma) - \delta(1-\theta)}. \quad (24)$$

Equation (24) says that the steady-state optimal capital tax depends on the parameters ρ , γ , θ , and δ in the following ways:

$$\begin{aligned} \frac{\partial \tau_k(\infty)}{\partial \rho} &= \frac{\delta(1-\theta)(1+\gamma\theta)}{[\rho\theta(1+2\gamma) - \delta(1-\theta)]^2} > 0, \\ \frac{\partial \tau_k(\infty)}{\partial \gamma} &= \frac{\rho\theta[\rho(2-\theta) + \delta(1-\theta)]}{[\rho\theta(1+2\gamma) - \delta(1-\theta)]^2} > 0, \\ \frac{\partial \tau_k(\infty)}{\partial \theta} &= \frac{\rho[\rho(1+2\gamma) + \delta(1+\gamma)]}{[\rho\theta(1+2\gamma) - \delta(1-\theta)]^2} > 0, \\ \frac{\partial \tau_k(\infty)}{\partial \delta} &= \frac{-\rho(1-\theta)(1+\gamma\theta)}{[\rho\theta(1+2\gamma) - \delta(1-\theta)]^2} < 0. \end{aligned}$$

An increase in the rate of time preference ρ reduces the perceived benefits of saving relative to consumption. This motivates the government to raise the tax rate. As γ goes up, the government increasingly favors workers and undertakes more redistribution. The parameter θ represents capital's share of total output. As θ increases, before-tax income inequality rises and the government chooses a higher tax rate. Finally, a higher depreciation rate δ means that capitalists need to save more in order to maintain the capital stock. A lower tax rate provides the incentive to do so.

Two interesting special cases occur when the government cares only about the workers ($\gamma = \infty$) or only about the capitalists ($\gamma = 0$). As $\gamma \rightarrow \infty$, we have $\tau_k(\infty) = \frac{1}{2}$. This result

¹⁸See Cruz (1975) and Kydland (1977) for formal descriptions of the open-loop and feedback solution concepts in dominant-player dynamic games.

¹⁹Pollak (1968) shows that log utility can eliminate a Strotz (1956)-type time inconsistency problem such that agents' "naive" and "sophisticated" optimal consumption plans coincide. In the terminology of Blackorby et al. (1973), the model with log utility exhibits the property that "the future is functionally separable from the present."

can be interpreted as the outcome of a political process in which the workers control the appointment of the policymaker.²⁰ The opposite case when $\gamma = 0$ implies $\tau_k(\infty) = \frac{-(1-\theta)(\rho+\delta)}{\rho\theta-\delta(1-\theta)}$. Finally, when $\gamma = \frac{(1-\theta)(\rho+\delta)}{\rho\theta}$ we have $\tau_k(\infty) = 0$.²¹

4 Reconciling the Different Results

The counterexample to Judd (1985) that occurs in the log-utility/balanced-budget case is very troubling because it shows that two seemingly equivalent ways of formulating the optimal tax problem can yield dramatically different results. In this section, I take a closer look at the standard approach used by Judd and explain where it goes wrong.

The technical problem with the standard approach outlined in section 3.1 is this: Imposing agents' first-order conditions as constraints on the government's allocation problem presumes the existence of anticipation effects which may not be a characteristic of the competitive equilibrium. In cases without anticipation effects (such as the counterexample), the government is effectively deprived of a useful policy instrument. The standard approach fails to properly restrict the government's choice of allocations to take this feature of the competitive equilibrium into account. Hence, it may not be possible to decentralize the chosen allocations with the existing set of policy instruments.

When applying the standard approach in Judd's model, the allocations for k and c are assumed to be independent for all β . However, as $\beta \rightarrow 1$, this assumption breaks down because the competitive equilibrium requires $c = \rho k$. By continuing to treat k and c as independent, the standard approach actually lets in an additional policy instrument through the back door. To see this, note that the steady-state versions of the standard approach's first-order conditions (17b) and (17c) can be written as

$$u'(c) = q_1 + \rho q_2, \quad (17b')$$

$$\gamma v'(x)k = q_1 k + q_2 \frac{c}{\beta}, \quad (17c')$$

where I have set $\eta = 0$ because the constraint $\hat{r} \geq 0$ is not binding in the steady-state. When $\beta = 1$ (the log case), we know from Proposition 1 that the unique competitive equilibrium has

²⁰Kaitala and Pohjola (1990) develop a model of a worker-controlled government in an economy with lump-sum taxes and linear utility.

²¹In Lansing (1999), I undertake a quantitative analysis of the optimal tax policy in a richer version of this model that allows for elastic labor supply, different numbers of workers and capitalists, technical progress, and aggregate uncertainty.

the property $c = \rho k$. Substituting this expression into (17c') with $\beta = 1$ and then combining with (17b') yields

$$\gamma v'(x) = u'(c). \quad (25)$$

Equation (25) tells us that in the long-run, the government makes transfers to workers in an amount necessary to equalize the social marginal utility of the two groups, just as a planner with access to lump-sum taxes would do. But this leads to the following question: If the budget is balanced according to (10) and the steady-state optimal capital tax is zero (as is claimed under the standard approach), then how can the government be making these transfers? The answer is that transfers are being accomplished using some other policy instrument—one that is not explicit but nevertheless admitted under the standard approach. I will show that the implicit policy instrument can be interpreted as either government bonds or a consumption tax. Either one will break the direct link between k and c and thereby restore the validity of the crucial independence assumption that is made under the standard approach.

Additional insight can be obtained by examining the so-called “implementability constraint” that is often used in dynamic optimal tax problems to collapse the necessary conditions of the competitive equilibrium into a single equation. In the appendix, I show that the capitalists’ first-order necessary conditions can be used to derive the following implementability constraint:

$$k(0) = c(0) \int_0^\infty e^{-\frac{\rho t}{\beta}} e^{\int_0^t \frac{(1-\beta)}{\beta} \hat{r}(s) ds} dt. \quad (26)$$

As $\beta \rightarrow 1$, the implementability constraint implies $k(0) = \frac{c(0)}{\rho}$, which is just the capitalists’ optimal decision rule at $t = 0$. Notice, however, that (26) *does not* enforce the constraint $k(t) = \frac{c(t)}{\rho}$ for $t > 0$, even though this is a necessary condition of the competitive equilibrium. This clearly illustrates the problem with the standard approach. As $\beta \rightarrow 1$, the implementability constraint imposes no restrictions whatsoever on the government’s choice of allocations for $t > 0$. Consequently, the chosen allocations cannot be decentralized with the existing set of policy instruments.

From (26) we see that any small change in β away from 1 will create anticipation effects, whereby future values of $\hat{r}(t)$ influence the capitalists’ current decisions. When $\beta \neq 1$, promises about future tax rates represent a policy instrument that can be exploited by the government to implement a set of independently chosen allocations for k and c . This policy instrument vanishes only as $\beta \rightarrow 1$. Although the counterexample set forth in Proposition 2 is a knife-

edge result, it highlights the importance of the intertemporal elasticity of substitution in problems involving dynamic optimal taxation. Indeed, Judd (1985, p. 69) acknowledges that the intertemporal elasticity is “an important factor since it influences the rate at which the capitalists respond to a tax increase and how they respond immediately to a tax change.”

As capitalists’ intertemporal elasticity of substitution in consumption crosses one, the trajectory of the optimal capital tax in this model undergoes an abrupt change. This result resembles one by Benhabib and Rustichini (1996) who use a generalized version of the Barro (1990) model to show that the trajectory of the optimal wealth tax changes abruptly as the elasticity of substitution between public and private capital in production crosses one (the Cobb-Douglas case). In both examples, the level of social welfare is what matters for the government and this continues to vary smoothly with the elasticity parameter. I will now demonstrate how Judd’s zero-tax result can be recovered even in log case by introducing an additional policy instrument.

4.1 Adding a Market for Government Bonds

When there exists a market for bonds, the government’s budget constraint becomes

$$TR + \hat{r}b = \tau_k (r - \delta) k + \dot{b}, \quad (27)$$

where b is the stock of bonds and \hat{r} is the corresponding after-tax return. Arbitrage requires the after-tax returns on capital and bonds to be equal at all times. By substituting (27) into (7) and making use of the zero-profit condition we obtain

$$x = f(k) - \delta k - \hat{r}(k + b) + \dot{b}. \quad (28)$$

The current value Hamiltonian for the capitalists’ problem becomes

$$H = u(c) + \lambda [\hat{r}(k + b) - c], \quad (29)$$

with $k(0)$ and $b(0)$ given.

Proposition 3. *When there exists a market for government bonds and $u(c) = \log(c)$, the capitalists’ unique decision rules are*

$$c = \rho a, \quad (30)$$

$$\dot{a} = a(\hat{r} - \rho), \quad (31)$$

where $a \equiv k + b$.

Proof: The proof follows the argument in Proposition 1 with the exception that the guess for the consumption decision rule is given by $c = d_0(k + b)$. \square

As in section 3.2, I will proceed by imposing agents' decision rules as constraints on the government's allocation problem. This ensures that the resulting allocations will respect the constraint $c = \rho a$, which is a necessary condition of the competitive equilibrium. Once again, since agents' decisions do not depend on any future policy actions, the optimal open-loop tax policy will be time consistent. The current-value Hamiltonian for the government's problem becomes

$$H_g = \gamma v(x) + \log(\rho a) + q_1 a (\hat{r} - \rho) + q_3 [\hat{r} a + x - f(a - b) + \delta(a - b)] + \eta \hat{r}, \quad (32)$$

where (30) has been used to eliminate c and q_1, q_3 are the multipliers associated with (31) and (28), respectively. The first-order conditions are

$$\frac{\partial H_g}{\partial a} = \frac{1}{a} + q_1 (\hat{r} - \rho) + q_3 (\hat{r} - f'(a - b) + \delta) = \rho q_1 - \dot{q}_1, \quad (33a)$$

$$\frac{\partial H_g}{\partial b} = q_3 (f'(a - b) - \delta) = \rho q_3 - \dot{q}_3, \quad (33b)$$

$$\frac{\partial H_g}{\partial \hat{r}} = q_1 a + q_3 a + \eta = 0, \quad (33c)$$

$$\frac{\partial H_g}{\partial x} = \gamma v'(x) + q_3 = 0, \quad (33d)$$

$$\frac{\partial H_g}{\partial q_1} = a (\hat{r} - \rho) = \dot{a}, \quad (33e)$$

$$\frac{\partial H_g}{\partial q_3} = \hat{r} a + x - f(a - b) + \delta(a - b) = \dot{b}, \quad (33f)$$

together with $\eta \hat{r} = 0$, $\lim_{t \rightarrow \infty} e^{-\rho t} q_1 a = 0$, and $\lim_{t \rightarrow \infty} e^{-\rho t} q_3 b = 0$.

Proposition 4. *When there exists a market for government bonds and $u(c) = \log(c)$, the optimal tax rate on capital income is zero for all $t \geq T$, where $t \in [0, T)$ represents the interval for which the constraint $\hat{r} \geq 0$ is binding.*

Proof: When the $\hat{r} \geq 0$ no longer binds, we have $\eta = 0$ and (33c) implies $q_1 + q_3 = 0$ for all $t \geq T$ since $a = 0$ can be ruled out by the presence of $\log(\rho a)$ in (32). Applying $q_1 + q_3 = 0$ in the combined equation (33a) + (33b) yields $q_1 = -q_3 = \frac{1}{\rho a}$. Substituting these values and $\dot{q}_1 = -\dot{q}_3 = -\frac{\dot{a}}{\rho a^2}$ back into (33a) and making use of (33e) yields $f'(k) - \delta - \hat{r} = 0$ which in turn implies $\tau_k(t \geq T) = 0$. \square

Proposition 4 says that the optimal capital tax drops sharply from 100 to 0% at $t = T$ and stays there for the remainder of the time horizon. This result, which is analogous to Theorem 2 in Chamley (1986), Theorem 8 in Judd (1999), and Section 6 in Xie (1998), is often referred

to as a “bang-bang” solution to the government’s optimal control problem. For $t \geq T$, we can combine (33d) and $q_3 = -\frac{1}{\rho a}$ from Proposition 4 to obtain

$$\gamma v'(x) = \frac{1}{\rho a} = \frac{1}{c} = u'(c), \quad (34)$$

which tells us that transfers are set to equalize the social marginal utility of the two groups. Since $\tau_k(t \geq T) = 0$ from Proposition 4, these transfers must be financed by the interest earned on a stock of government assets that has been accumulated during the interval of 100% capital taxation defined by $t \in [0, T)$. Since there are no anticipation effects, the optimal policy is time consistent: T is chosen once and never revised.²²

Proposition 4 shows that allowing the government to engage in intertemporal borrowing and lending recovers Judd’s result in the log case. A market for bonds is important because it allows the government to store up revenue from taxing capital income at the beginning of the time horizon. This, in turn, influences the optimal policy at the end of the horizon.²³ In the log case, the long-run level of government assets can be pinned down on the basis of steady-state considerations alone. This contrasts with the usual result in dynamic optimal tax problems where $b(\infty)$ depends on the initial level of debt and the entire trajectory of taxes and public expenditures from $t = 0$ until the steady state is reached.²⁴ To see that $b(\infty)$ is pinned down without having to consider the transition path, we can combine the steady-state versions of (33a)-(33f) to obtain

$$\gamma v'(x) - \frac{1}{\rho(k+b)} = 0, \quad (35a)$$

$$f'(k) - \delta - \rho = 0, \quad (35b)$$

$$\rho(k+b) + x - f(k) + \delta k = 0, \quad (35c)$$

which is a system of three equations in the three unknowns k , b , and x . When $v(x) = \log(x)$ and $f(k) = k^\theta$, we have

$$b(\infty) = \left[\frac{(1-\theta)(\rho+\delta) - \rho\gamma\theta}{\rho(1+\gamma)(\rho+\delta)} \right] \left(\frac{\theta}{\rho+\delta} \right)^{\frac{\theta}{1-\theta}}. \quad (36)$$

²²The length of the interval depends on the values of $b(0)$, $k(0)$, and model parameters.

²³A similar result obtains in models of optimal monetary policy. See, for example, Turnovsky and Brock (1980, pp. 198-200).

²⁴See Chamley (1985) for a discussion of the usual result. See Xie (1997, Section 6) for another example where $b(\infty)$ is pinned down on the basis of steady-state considerations alone.

It is straightforward to show that $\frac{\partial b(\infty)}{\partial \gamma} < 0$. As the policymaker increasingly favors workers, $b(\infty)$ is more likely to be negative, implying that the government is a net lender to the public. From equation (27) with $\tau_k(\infty) = 0$, we have $TR(\infty) = -(\theta k(\infty)^{\theta-1} - \delta) b(\infty)$ which shows that government lending ($b(\infty) < 0$) is necessary for a positive level of long-run transfers to workers.

4.2 Adding a Consumption Tax

When a constant consumption tax $\bar{\tau}_c$ is levied on capitalists, the government's budget constraint becomes

$$TR = \tau_k (r - \delta) k + \bar{\tau}_c c. \quad (37)$$

By substituting (37) into (7) we obtain

$$x = f(k) - \delta k - \hat{r}k + \bar{\tau}_c c. \quad (38)$$

The current value Hamiltonian for the capitalists' problem becomes

$$H = u(c) + \lambda [\hat{r}k - (1 + \bar{\tau}_c) c], \quad (39)$$

with $k(0)$ given.

Proposition 5. *When a constant consumption tax is levied on capitalists and $u(c) = \log(c)$, the capitalists' unique decision rules are*

$$c = \left(\frac{\rho}{1 + \bar{\tau}_c} \right) k, \quad (40)$$

$$\dot{k} = k(\hat{r} - \rho). \quad (41)$$

Proof: The proof follows the argument in Proposition 1. \square

The current-value Hamiltonian for the government's problem becomes

$$H_g = \gamma v \left(f(k) - \delta k - \hat{r}k + \frac{\bar{\tau}_c \rho k}{1 + \bar{\tau}_c} \right) + \log \left(\frac{\rho k}{1 + \bar{\tau}_c} \right) + q_1 k (\hat{r} - \rho) + \eta \hat{r}, \quad (42)$$

where (38) and (40) have been used to eliminate x and c and q_1 is the multiplier associated with (41). I will solve for the optimal policy by treating $\bar{\tau}_c$ as a control variable for the government and then verifying that the chosen allocations can indeed be implemented by a consumption tax that is constant over time. The first-order conditions are

$$\frac{\partial H_g}{\partial k} = \gamma v'(x) \left(f'(k) - \delta - \hat{r} + \frac{\bar{\tau}_c \rho}{1 + \bar{\tau}_c} \right) + \frac{1}{k} + q_1 (\hat{r} - \rho) = \rho q_1 - \dot{q}_1, \quad (43a)$$

$$\frac{\partial H_g}{\partial \hat{r}} = -\gamma v'(x)k + q_1 k + \eta = 0, \quad (43b)$$

$$\frac{\partial H_g}{\partial \bar{\tau}_c} = \gamma v'(x) \left[\frac{\rho k}{(1 + \bar{\tau}_c)^2} \right] - \frac{1}{1 + \bar{\tau}_c} = 0, \quad (43c)$$

$$\frac{\partial H_g}{\partial q_1} = k(\hat{r} - \rho) = \dot{k}, \quad (43d)$$

together with $\eta \hat{r} = 0$ and $\lim_{t \rightarrow \infty} e^{-\rho t} q_1 k = 0$.

Proposition 6. *When a constant consumption tax is levied on capitalists and $u(c) = \log(c)$, the optimal tax on capital income is zero for all $t \geq 0$ and the government can implement the first-best allocations.*

Proof: Rearrange (43c) to obtain

$$\gamma v'(x) = \frac{1 + \bar{\tau}_c}{\rho k} = \frac{1}{c} = u'(c), \quad (44)$$

which tells us that the optimal policy equalizes the social marginal utility of the two groups for all $t \geq 0$ not just for $t \geq T$ as in the case with bonds. A conjecture regarding the optimal policy is that $\hat{r} \geq 0$ (such that $\eta = 0$) for all $t \geq 0$. Applying this conjecture in (43b) and making use of (44) yields $\gamma v'(x) = q_1 = \frac{1}{c}$. Substituting this expression into (43a) together with $\frac{1}{k} = \frac{\gamma v'(x)\rho}{(1 + \bar{\tau}_c)}$ from (43c) yields

$$\dot{c} = c(f'(k) - \delta - \rho). \quad (45)$$

When the consumption tax is constant over time, the capitalists' Euler equation is given by $\dot{c} = c(\hat{r} - \rho)$. Comparing the Euler equation to (45) implies $\hat{r} = f'(k) - \delta$, or equivalently, $\tau_k(t \geq 0) = 0$. This verifies the conjecture $\hat{r} \geq 0$ and shows that the chosen allocations can indeed be implemented by a constant consumption tax. Since (44) and (45) coincide with the first-order conditions of the planner's problem, the chosen allocations are first best.²⁵ \square

Proposition 7 shows that introducing a consumption tax provides an alternative means of recovering Judd's result in the log case. The above result is actually stronger than Judd's since $\tau_k(t \geq 0) = 0$ and the equilibrium is first best. This is due to the fact that a constant consumption tax is nondistortionary when labor supply is completely inelastic. Thus, Proposition 7 can easily be extended to the more general case where $u(c) = \frac{c^{1-\beta}-1}{1-\beta}$.

²⁵The Hamiltonian for the social planner's problem is $H_p = \gamma v(x) + \ln c + q_1(f(k) - c - x - \delta k)$.

5 Other Economic Environments

5.1 Allowing Workers to Save

The counterexample set forth in Proposition 2 applies only to the simplest version of the various models considered by Judd (1985) in which workers do not have access to capital markets. Theorems 4 and 5 in Judd (1985) extend the zero limiting capital tax result to more general environments where workers can save. I will now consider whether a counterexample can arise in this case.

If workers can save, their budget constraint is

$$x + \dot{k}_w = \hat{r}k_w + w + TR, \quad k_w(0) \text{ given}, \quad (46)$$

where I now use k_w and k_c to represent the capital stocks owned by workers and capitalists, respectively. The current-value Hamiltonian for the workers' problem is

$$H_w = v(x) + \mu [\hat{r}k_w + w + TR - x], \quad (47)$$

where μ is the multiplier associated with (46). Workers take \hat{r} , w , and TR as given. Following Judd (1985, Section 4), I assume that workers and capitalists have the same constant rate of time preference ρ . This means that the steady-state distribution of capital across workers and capitalists will be indeterminate.²⁶

For capitalists', I assume log utility such that the optimal decision rules are again given by $c = \rho k_c$ and $\dot{k}_c = k_c(\hat{r} - \rho)$. The neoclassical production function is $f(K)$ where $K \equiv k_w + k_c$. Profit maximization implies $r = f'(K)$ and $w = f(K) - f'(K)K$. The government's budget constraint is

$$TR = \tau_k (r - \delta) K, \quad (48)$$

where a market for bonds is again ruled out and all tax revenue is transferred to workers in lump-sum. The workers' consumption and saving decisions satisfy the following necessary conditions

$$\dot{x} = \frac{x}{\sigma} (\hat{r} - \rho), \quad (49)$$

$$\dot{k}_w = f(K) - \delta K - \hat{r}k_c - x, \quad (50)$$

²⁶See Becker (1980) for a general proof of this result. Judd (1985, Section 5) allows for a non-constant rate of time preference that depends on agents' allocations. In this case, the model can pin down a unique steady-state distribution of capital, as shown by Epstein and Hynes (1983).

together with the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} v'(x) k_w = 0$, where $\sigma \equiv \frac{xv''(x)}{v'(x)}$ is the workers' elasticity of marginal utility, assumed to be constant.

When (49) and (50) are imposed as constraints on the government's allocation problem, together $c = \rho k_c$ and $\dot{k}_c = k_c(\hat{r} - \rho)$, the result is $\tau_k(\infty) = 0$, consistent with Judd's Theorem 4.²⁷ This result is not surprising because (49) and (50) presume the existence of anticipation effects, whereby promises about future tax rates can influence workers' current decisions. A government with the ability to commit itself to following a pre-announced policy (as is assumed here) can exploit these anticipation effects to implement a set of allocations that are consistent with $\tau_k(\infty) = 0$ even though bonds are ruled out. It is precisely the existence of anticipation effects that gives rise to a time inconsistency problem, however.

For a counterexample, the foregoing analysis suggests that we need to identify some combination of $v(x)$ and $f(K)$ that can eliminate workers' anticipation effects. This setup would effectively deprive the government of a useful policy instrument and again lead to the result $\tau_k(\infty) \neq 0$, provided, of course, that we continue to rule out other suitable policy instruments such as government bonds or a consumption tax. In an economy without anticipation effects, the worker's consumption decision rule would be independent of future tax rates and take the form $x = g(k_w, k_c)$. This feature of the competitive equilibrium would invalidate the crucial assumption regarding independence of x , k_w , and k_c that is made when deriving the government's first order conditions under the standard approach.

Thus far, I have been unable to identify an appropriate combination of $v(x)$ and $f(K)$ that can reconcile a decision rule of the form $x = g(k_w, k_c)$ with optimizing behavior on the part of the workers.²⁸ It appears possible to do so in principle, however, perhaps by modifying the structure of the model or by solving an inverse optimal problem of the type considered by Kurz (1969) and Chang (1988). I leave this as an open question for future research.

5.2 A Representative Agent: The Model of Chamley (1986)

At this point, it is natural to ask whether a counterexample can be found to the zero limiting capital tax result of Chamley (1986) who considers the problem of dynamic optimal taxation in a representative environment. I take up this question below.

In Chamley's model, the government chooses tax rates on labor and capital incomes to

²⁷The government's first-order condition with respect to k_w is: $q_3(f'(K) - \delta) = \rho q_3 - \dot{q}_3$, where q_3 is the multiplier associated with (50). In steady-state, with $\hat{r} = \rho$, we have $f'(K) - \delta - \hat{r} = 0$, which implies $\tau_k(\infty) = 0$.

²⁸I have tried decision rules of the form $x = d_0 k_w$ and $x = d_0 f(K)$, where $f(K) = (k_w + k_c)^\theta$.

finance an exogenous stream of public expenditures $\{g(t)\}_{t=0}^{\infty}$. The representative agent's budget constraint is

$$c + \dot{k} = \hat{r}k + \hat{w}l, \quad k(0) \text{ given}, \quad (51)$$

where c is consumption, l is labor effort, $\hat{r} \equiv (1 - \tau_k)(r - \delta)$ is the after-tax return on capital, and $\hat{w} \equiv (1 - \tau_l)w$ is the after-tax real wage. The current-value Hamiltonian for the representative agent's problem is

$$H_r = u(c, l) + \lambda [\hat{r}k + \hat{w}l - c], \quad (52)$$

where the utility function $u(c, l)$ is assumed to be increasing in c , decreasing in l , twice continuously differentiable, and concave. The production function is given by $f(k, l)$ and exhibits constant returns to scale in k and l . Profit maximization implies $r = f_k(k, l)$ and $w = f_l(k, l)$. The government's budget constraint can be written as

$$g = f(k, l) - \delta k - \hat{r}k - \hat{w}l, \quad (53)$$

where a market for bonds is again ruled out. The representative agent's consumption, labor supply, and saving decisions must satisfy the following necessary conditions:

$$\frac{\partial H_r}{\partial c} = u_c(c, l) - \lambda = 0 \quad (54a)$$

$$\frac{\partial H_r}{\partial l} = u_l(c, l) + \lambda \hat{w} = 0, \quad (54b)$$

$$\frac{\partial H_r}{\partial k} = \lambda \hat{r} = \rho \lambda - \dot{\lambda}, \quad (54c)$$

$$\frac{\partial H_r}{\partial \lambda} = \hat{r}k + \hat{w}l - c = \dot{k}, \quad (54d)$$

together with the transversality condition $\lim_{t \rightarrow \infty} e^{-\rho t} \lambda k = 0$.

The standard approach to solving the optimal tax problem imposes (53) and (54a)-(54d) as constraints on the government's allocation problem. Taking this approach, it is straightforward to show that the government's first-order condition with respect to k yields $f_k(k, l) - \delta - \hat{r} = 0$ in steady state. Given the definition of \hat{r} , we have $\tau_k(\infty) = 0$, thus confirming Theorem 1 in Chamley (1986).

Since Chamley's result is obtained using the standard approach, it presumes the existence of anticipation effects. In a balanced-budget environment, these anticipation effects are crucial because they allow the government to implement a set of independently chosen allocations for k , c , and l , despite having only two policy instruments: \hat{r} and \hat{w} . For a counterexample, we

require some combination of $u(c, l)$ and $f(k, l)$ that eliminates the anticipation effects. This setup would prevent the government from being able to decentralize the allocations implied by the standard approach.

As a candidate utility function, consider

$$u(c, l) = \log(c^\alpha - Bl^\alpha), \quad (55)$$

which is concave for $\alpha > 0$, but exhibits no income effect on labor supply.²⁹ By making the initial guess $\lambda = (d_0 k)^{-1}$ and then applying the method of undetermined coefficients in (54a)-(54d), we obtain $d_0 = \frac{\rho}{\alpha}$. The following system of nonlinear equations defines the unique competitive equilibrium allocations as functions of \hat{r} and \hat{w} :

$$B \left(\frac{l}{c} \right)^{\alpha-1} = \hat{w}, \quad (56a)$$

$$c = \rho k + \hat{w}l, \quad (56b)$$

$$\dot{k} = k(\hat{r} - \rho). \quad (56c)$$

Once again, due to the special property of log utility, private-sector decisions do not depend on any future policy actions. This feature is important for the purpose of finding a counterexample because it deprives the government of the ability to influence current allocations with promises about future tax rates. Xie (1997) uses a version of the above utility function with $\alpha = B = 1$ to demonstrate that the optimal open-loop tax policy in Chamley's model can be time consistent. His result follows directly from the absence of anticipation effects. Xie also shows that the optimal tax policy continues to imply $\tau_k(t \geq T) = 0$. His analysis allows a market for government bonds, however, and we know from section 4.1 that bonds can be crucial for achieving the zero tax result.

The question then arises: When combined with a continuously balanced government budget, can the above utility function overturn the Chamley (1986) result? Based on my analysis, the answer is no, but for an unexpected reason. It turns out that the model with (55) has a *continuum* of steady states. To see this, let us assume without loss of generality that $f(k, l) = k^\theta l^{1-\theta}$, where $\theta \in (0, 1)$.³⁰ Then, (56b) can be written as $\frac{c}{l} = \rho \left(\frac{k}{l} \right) + (1 - \tau_l)(1 - \theta) \left(\frac{k}{l} \right)^\theta$.

²⁹In a model with technological progress, this utility specification must be modified as follows: $U(c, l) = \ln [c^\alpha - B(e^{\mu t} l)^\alpha]$, where μ is the growth rate of technology. This setup, which can be motivated by a home production argument, ensures that l remains stationary along the model's balanced growth path.

³⁰It can be shown that the result holds for any $f(k, l)$ that is homogenous of degree one in k and l .

Substituting this expression into (56a) yields

$$B \left[\rho \left(\frac{k}{l} \right) + (1 - \tau_l)(1 - \theta) \left(\frac{k}{l} \right)^\theta \right]^{1-\alpha} = (1 - \tau_l)(1 - \theta) \left(\frac{k}{l} \right)^\theta. \quad (57)$$

The steady-state version of (56c) implies $\hat{r} = \rho$, which can be used to obtain

$$\left(\frac{k}{l} \right)^{1-\theta} = \frac{\theta(1 - \tau_k)}{\rho + \delta(1 - \tau_k)}. \quad (58)$$

Given any pair of values for the long-run tax rates, (57) and (58) represent a system of two equations in only one unknown $\frac{k}{l}$. Hence, when the utility function is given by (55), the model is incapable of pinning down a unique set of long-run allocations for k , c , and l . Consequently, the steady-state optimal tax rates are also indeterminate. It remains to be seen, therefore, whether an economically meaningful counterexample to Chamley's result can be constructed.

6 Conclusion

This paper shows that the combination of log utility and a continuously balanced government budget can overturn the zero limiting capital tax result in the simplest version of the capitalist-worker models considered by Judd (1985). With log utility, the government is effectively deprived of a useful policy instrument because promises about future tax rates do not influence current allocations. When this situation is combined with a lack of other suitable policy instruments (such as government bonds), the result is an inability to decentralize the allocations that are consistent with a zero limiting capital tax. The standard approach to solving the dynamic optimal tax problem yields the wrong answer in this (knife edge) case because it presumes the existence of anticipation effects which are not a feature of the competitive equilibrium. More generally, the standard approach is flawed because it does not guarantee that the government's chosen set of allocations can be decentralized with the existing set of policy instruments. In cases without anticipation effects (such as the counterexample), the standard approach can create an additional implicit policy instrument that the government does not truly have. Future research should be directed at developing a solution method that gives the right answer in all cases.

Despite its specialized nature, the counterexample is important for at least three reasons. First, it calls into question the robustness of optimal tax results that are derived using the standard approach. Second, despite some previous claims to the contrary, it shows that assumptions regarding the government's budget constraint are very important in problems

involving dynamic optimal taxation. In this regard, it can be argued that a balanced-budget requirement represents a closer approximation to actual constraints than one which allows the government to accumulate large budget surpluses financed by confiscatory capital taxation—a policy of doubtful political feasibility. Third, the counterexample provides an efficiency argument for the use of a capital tax that responds positively to the level of income inequality and the political influence of the poor.

A Appendix

A.1 Time Consistency of the Open-Loop Policy

In what follows, I derive some expressions that show how the trajectory of future tax rates affects capitalists' consumption and saving decisions, and thereby, their lifetime utility. I then demonstrate how the dependence on future tax rates vanishes completely with log utility. As a result, the optimal open-loop tax policy is time consistent.

When $u(c) = \frac{c^{1-\beta}-1}{1-\beta}$, the capitalists' necessary conditions are

$$\dot{c} = \frac{c}{\beta} (\hat{r} - \rho), \quad (\text{A.1})$$

$$\dot{k} = \hat{r}k - c, \quad k(0) \text{ given.} \quad (\text{A.2})$$

By integrating (A.2) and applying $\lim_{\bar{t} \rightarrow \infty} e^{-\int_0^{\bar{t}} \hat{r}(s) ds} k(\bar{t}) = 0$, we obtain the following present-value budget constraint:

$$k(0) = \int_0^\infty e^{-\int_0^t \hat{r}(s) ds} c(t) dt. \quad (\text{A.3})$$

Separating variables and integrating (A.1) yields

$$c(t) = c(0) e^{\int_0^t \frac{1}{\beta} [\hat{r}(s) - \rho] ds}. \quad (\text{A.4})$$

The above expression for $c(t)$ can be substituted into (A.3) to obtain

$$\begin{aligned} k(0) &= c(0) \int_0^\infty e^{\int_0^t \frac{1}{\beta} [(1-\beta)\hat{r}(s) - \rho] ds} dt, \\ &= c(0) \int_0^\infty e^{-\frac{\rho t}{\beta}} e^{\int_0^t \frac{(1-\beta)}{\beta} \hat{r}(s) ds} dt, \end{aligned} \quad (\text{A.5})$$

which is the so-called ‘‘implementability constraint’’ that corresponds to (26) in the text. This constraint collapses the capitalists' necessary conditions into a single equation.

The lifetime utility of a capitalist is given by $V_c = \int_0^\infty e^{-\rho t} \left(\frac{c(t)^{1-\beta}-1}{1-\beta} \right) dt$. Substituting the expression for $c(t)$ from (A.4) into the utility function yields

$$V_c = \frac{c(0)^{1-\beta}}{1-\beta} \int_0^\infty e^{-\frac{\rho t}{\beta}} e^{\int_0^t \frac{(1-\beta)}{\beta} \hat{r}(s) ds} dt - \frac{1}{\rho(1-\beta)}, \quad (\text{A.6})$$

where the constant term in (A.6) allows us to recover the log case by taking the limit as $\beta \rightarrow 1$.

We can now solve the implementability constraint for $c(0)$ and substitute the resulting expression into (A.6) to obtain

$$V_c = \frac{[\rho k(0)]^{1-\beta}}{\rho^{1-\beta}(1-\beta)} \left[\int_0^\infty e^{-\frac{\rho t}{\beta}} e^{\int_0^t \frac{(1-\beta)}{\beta} \hat{r}(s) ds} dt \right]^\beta - \frac{1}{\rho(1-\beta)}. \quad (\text{A.7})$$

In general, (A.7) implies that V_c depends on $k(0)$ and $\{\hat{r}(t)\}_{t=0}^\infty$. Notice, however, that when $\beta = 1$ we have $e^{\int_0^t \frac{(1-\beta)}{\beta} \hat{r}(s) ds} = 1$ such that the dependence on future tax rates vanishes completely. Kydland and Prescott (1977, p. 476) note that optimal open-loop policies will be time consistent (implying that a commitment technology is not needed) when agents' current decisions do not depend on future policy actions.³¹ By taking the limit as $\beta \rightarrow 1$, equation (A.7) can be written as

$$V_c = \frac{\log[\rho k(0)]}{\rho} = \frac{\log[c(0)]}{\rho}. \quad (\text{A.8})$$

Equations (A.7) and (A.8) show that the level of social welfare can vary smoothly as β crosses 1, even though the trajectory of the optimal capital tax does not.

³¹For other examples see Barro (1990), Benhabib and Velasco (1996), Benhabib, Rustichini, and Velasco (1996), Xie (1997), and Cassou and Lansing (1998a,b).

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