# Pacific Basin Working Paper Series


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Working Paper No. PB01-09

**Center for Pacific Basin Monetary and Economic Studies**
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**September 2001**

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# Testing for Contagion using Correlations: Some Words of Caution*

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**Abstract**

Tests for contagion in financial returns using correlation analysis are seriously affected by the size of the 'non-crisis' and 'crisis' periods. Typically the 'crisis' period contains relatively few observations which seriously affects the power of the test.

JEL Classification: G14

Keywords: contagion, correlations, heteroskedasticity

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## 1. Introduction

A popular and straightforward test for the presence of contagion in financial asset returns has been to examine changes in the correlations of those returns between ‘crisis’ and ‘non-crisis’ periods. A statistically significant change in the correlation is interpreted as evidence of contagion. This approach has been applied in the equity market literature, (for example Baig and Goldfajn (1998) and references cited in Forbes and Rigobon (1999)) and to some extent to yields and currency market data (Ellis and Lewis (2000)). Here we outline this basic test of contagion, including the heteroskedasticity correction developed by Forbes and Rigobon, and examine its properties. In particular we address the limitations of the power of this test. Using an application to equity market data during the US stock market crash of 1987 we show that the Forbes and Rigobon results consistently over-reject the hypothesis of contagion due in large part to the comparison of a large sample of non-crisis period data to a small sample of crisis period data. We suggest that as the relative size of the crisis sample is likely to be small in general, the correlation test is an unattractive approach to the problem.

## 2. Correlations as a measure of contagion

The preferred test of contagion is to examine the relationship between returns in different markets using a simple linear model as follows.

$$y_{i,t} = \beta y_{j,t} + \varepsilon_{i,t} \tag{1}$$

If there is a change in the relationship between the returns, $y_{i,t}$ and $y_{j,t}$ at some point, as given by a significant change in $\beta$, this is evidence of contagion. This simple test is complicated by the fact that in the move between a non-crisis and crisis period the volatility of the error term, $\varepsilon_{i,t}$, usually changes also, that is the assumption of homoskedasticity is violated. In order to avoid this problem the test can be respecified in terms of testing for a statistical change in the correlation coefficient, $\rho_i$, between the two periods. However, when examining a relationship such as (1) over a data sample involving a crisis, one may well infer that there are also changes in the variance structure of the data from the crisis period to the non-crisis period, and in general that the variance of the series will increase with the advent of a crisis. To allow for this Forbes and Rigobon introduce a simple correction:

$$\rho_i^* = \frac{\rho_i}{\sqrt{1+\delta[1-(\rho_i)^2]}} \tag{2}$$

where $d = \frac{s_j^h}{s_j^l} - 1$ is the relative increase in the variance of the equity returns from the low volatility non-crisis period, $s_j^l$, to the high volatility crisis period, $s_j^h$.

Forbes and Rigobon compare two periods under the null hypothesis that the same model holds throughout (that is contagion is evidenced by a rejection of the null). Hence the $s_j^l$ used in their test is drawn from the entire sample period, *including the high volatility period*, and the variance, $s_j^h$ is drawn from a subperiod of the data. The general approach here is to first control for common effects between markets using a VAR. Practically, the process is implemented by running a VAR analysis between the two stock index returns for the total period as in (3) below.

$$\begin{aligned} y_{i,t} &= B(L)y_{j,t-1} + \varepsilon_{i,t} \\ y_{j,t} &= B(L)y_{i,t-1} + \varepsilon_{j,t} \end{aligned} \tag{3}$$

The correlations between the error terms $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ are then examined for contagion by calculating $\rho_i^{*,h}$ for the high volatility period and $\rho_i^{*,l}$ for the low volatility period. In practice Forbes and Rigobon draw the errors for the crisis period and the subsequent value $\sigma_i^h$ from the sub-period results of (3). The test for contagion is then:

$$H_0 : \rho_i^{*,h} \leq \rho_i^{*,l}$$
$$H_1 : \rho_i^{*,h} > \rho_i^{*,l}$$

In fact the heteroskedasticity correction they propose is only necessary if one draws all the information from a system estimated across both crisis and non-crisis periods under the null of no contagion. A stronger test is to estimate crisis and non-crisis VARs separately and test the null of whether the unadjusted correlation coefficients are significantly different across the two samples. This has not generally been done due to limited degrees of freedom in a VAR for the crisis period.

## 3. Sample selection and the Fisher transformation

To conduct the test of $H_0$ versus $H_1$, the Fisher transformation on the calculated correlation coefficients is required to achieve a distribution closer to the normal. The resulting mean and standard deviations are expressed as:

$$\overline{\rho}_i^* = \frac{1}{2}\ln\left(\frac{1+\rho_i^*}{1-\rho_i^*}\right) \tag{4}$$

and

$$s_i = \sqrt{\frac{1}{n_i - 3}} \tag{5}$$

where $n_i$ is the number of observations in the sample. The two-sample test on independent means is performed as:

$$t = \frac{\overline{r}_i^{*,h} - \overline{r}_i^{*,l}}{\sqrt{s_h^2 + s_l^2}} \tag{6}$$

where $s_h$ and $s_l$ are the standard deviations of the correlation coefficients estimated during the subperiods of relative turmoil and stability respectively.

However, this transformation is an asymptotic result, and best suited for sample sizes of $n_i$>50 (see for example Kendall and Stuart (1979)). The literature applies this test to rather smaller subsamples, in particular tests are undertaken between non-turmoil periods of up to 500 observations and turmoil periods of as few as 10 observations. A simple Monte Carlo experiment demonstrates the undesirable power properties of this application.

The test of correlation coefficients is performed on errors from VARs of the form given in (3) where the errors are normally distributed. Hence in our experiment we simulate 2 populations of bivariate normal observations for a given correlation matrices P1 and P2.

The correlations matrices are chosen arbitrarily so that the correlation coefficients $\rho_1$ and $\rho_2$ are statically different for 2 populations at 1% level of significance. These two population correlation coefficients then represent a 'non-crisis' and 'crisis' period for our purposes. If we randomly sample from each of the populations we can see that with reasonable size samples our ability to produce reliable estimates of the true correlation coefficients and most importantly their standard errors is good. However, as the sample size declines the estimates of the standard errors are seriously affected. Table 1 sets out these results, Experiments 5, 6 and 7 use some of the actual sample sizes tested in Forbes and Rigobon.

Further, if we examine the t-test of the significance between the two correlation coefficients, we see that if one sample is large, and the other fairly small then the test has very little power, that is our ability to reject the null hypothesis can be seriously affected by the sample size. A chart of the dependence of the standard error of the correlation coefficient is given in Figure 1 – with rapidly increasing standard errors associated with decreasing sample size the chances of rejecting the null hypothesis become vanishingly small.

**Table 1: Monte Carlo experiments on testing correlations**

| | Sample size1 | Sample size 2 | $?_1$ | $s_1$ | $?_2$ | $s_2$ | t-test on $?_2$-$?_1$ |
|---|---|---|---|---|---|---|---|
| True values | 2,000 | 2,000 | 0.31 | 0.022 | 0.50 | 0.022 | 7.37 |
| Simulated values for Experiment 1 | 1,000 | 1,000 | 0.31 | 0.032 | 0.50 | 0.032 | 5.22 |
| Experiment 2 | 500 | 500 | 0.31 | 0.045 | 0.50 | 0.044 | 3.70 |
| Experiment 3 | 500 | 300 | 0.31 | 0.045 | 0.50 | 0.058 | 3.21 |
| Experiment 4 | 500 | 100 | 0.31 | 0.045 | 0.50 | 0.101 | 2.12 |
| Experiment 5 | 467 | 22 | 0.31 | 0.046 | 0.49 | 0.229 | 1.03 |
| Experiment 6 | 509 | 10 | 0.31 | 0.044 | 0.49 | 0.377 | 0.70 |
| Experiment 7 | 466 | 35 | 0.31 | 0.046 | 0.50 | 0.176 | 1.30 |

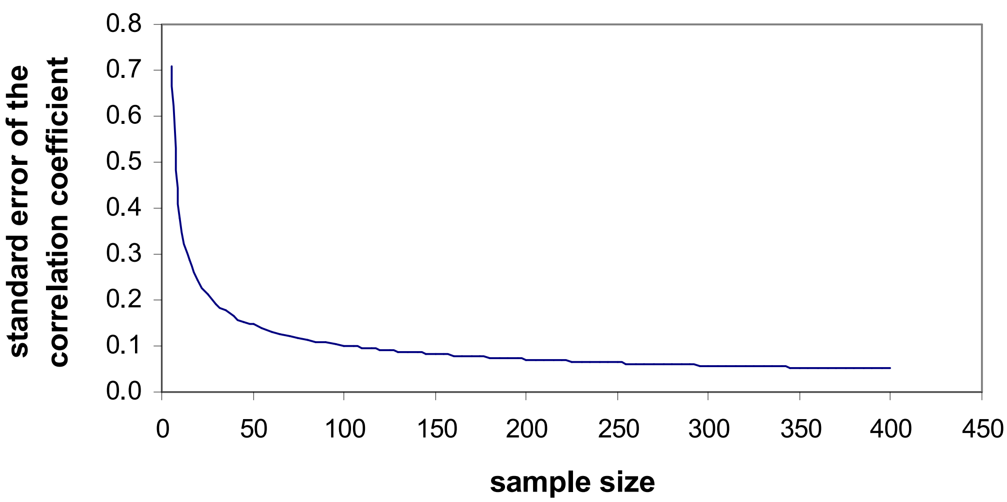


**Figure 1: The effect of sample size on the calculated standard error**

## 4. An example – contagion from the US stock market crash:

In this section we demonstrate the consequences of this problem for an example drawn from Forbes and Rigobon. By increasing the sample size for the crisis period we find more evidence of contagion than did Forbes and Rigobon. This is a strong result given that the volatility increase is now smaller than in their sample as the crisis period contains some observations previously classified as 'non-crisis'.

Table 2 reports the results of correlation tests for contagion between the US stock market index and 9 other developed stock market indices. All data are daily returns taken from the Thomson Financial Datastream stock market indices. Forbes and Rigobon propose a non-crisis period from 1-January-1986 to the 16-October-1987, a total of 466 observations. We also consider this as the stable period. The crisis or turmoil period in Forbes and Rigobon consists of 35 observations from 17-October-1987 to 4-December-1987. To increase this sample size to one with more appropriate power properties we extend the turmoil period to 30-April-1988, a total of 142 observations. We estimate VAR model for the full period, and calculate the stable period correlation coefficient, and heteroskedasticity adjusted crisis period correlation coefficients for a short and long crisis periods. The correlation tests for contagion are conducted for the short and long crisis periods using heteroskedasticity-adjusted correlation coefficients.

The results in Table 2 show that there is evidence of contagion in 6 of the 9 countries examined using the longer crisis period with adjusted correlation coefficients, whereas the short period with adjustments presents no evidence at all demonstrating the seriousness of the power problem discussed above.

**Table 2: Correlation test for contagion in the US stock market crash: $n_1$ is non crisis period, $n_2$ is short crisis period, $n_3$ long crisis period**

| Country | Non-crisis $n_1$=466 ($\sigma$=0.046) | Correlation coefficient (std err) Crisis $n_2$=35 ($\sigma$=0.177) | | Crisis $n_3$=142 ($\sigma$=0.085) | | Test of contagion t-test using $\rho_{stab}$ & $\rho_1^{adj}$ | Contag? | t-test using $\rho_{stab}$ & $\rho_2^{adj}$ | Contag? |
|---|---|---|---|---|---|---|---|---|---|
| | $\rho_{stab}$ | $\rho_1$ | $\rho_1^{adj}$ | $\rho_2$ | $\rho_2^{adj}$ | | | | |
| Canada | 0.610 | 0.800 | 0.253 | 0.778 | 0.388 | 0.48 | No | -3.13 | No |
| France | 0.122 | 0.658 | 0.227 | 0.500 | 0.319 | -2.42 | No | 2.14 | Yes |
| Germany | 0.076 | 0.564 | 0.217 | 0.345 | 0.265 | 0.64 | No | 2.01 | Yes |
| Netherlands | 0.266 | 0.676 | 0.237 | 0.597 | 0.348 | 0.77 | No | 0.92 | No |
| Switzerland | 0.072 | 0.750 | 0.239 | 0.524 | 0.334 | -0.31 | No | 2.84 | Yes |
| U.K. | 0.238 | 0.636 | 0.242 | 0.558 | 0.348 | 0.89 | No | 1.22 | No |
| Australia* | 0.118 | 0.859 | 0.218 | 0.796 | 0.360 | 0.05 | No | 2.66 | Yes |
| Hong Kong* | 0.052 | 0.209 | 0.168 | 0.296 | 0.253 | 0.65 | No | 2.14 | Yes |
| Japan* | 0.144 | 0.855 | 0.214 | 0.697 | 0.345 | 0.39 | No | 2.21 | Yes |

* Lagged returns were used for these countries to adjust for time period differences.

## 5. Conclusion

The correlation test for contagion as proposed in the existing literature, and popularised in Forbes and Rigobon (1999) suffers from substantial power problems when applied to the typical situation of a large ‘non-crisis’ sample and a small ‘crisis’ sample. A simple example shows that when the crisis sample period is extended (to include what were previously defined as non-crisis observations) the correlation test finds statistically signficant evidence of contagion, where it did not with the shorter crisis period. For this reason we caution against using this method of identifying contagious episodes, as the biases in constructing the test statistics and choosing the sample period are not well defined.

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