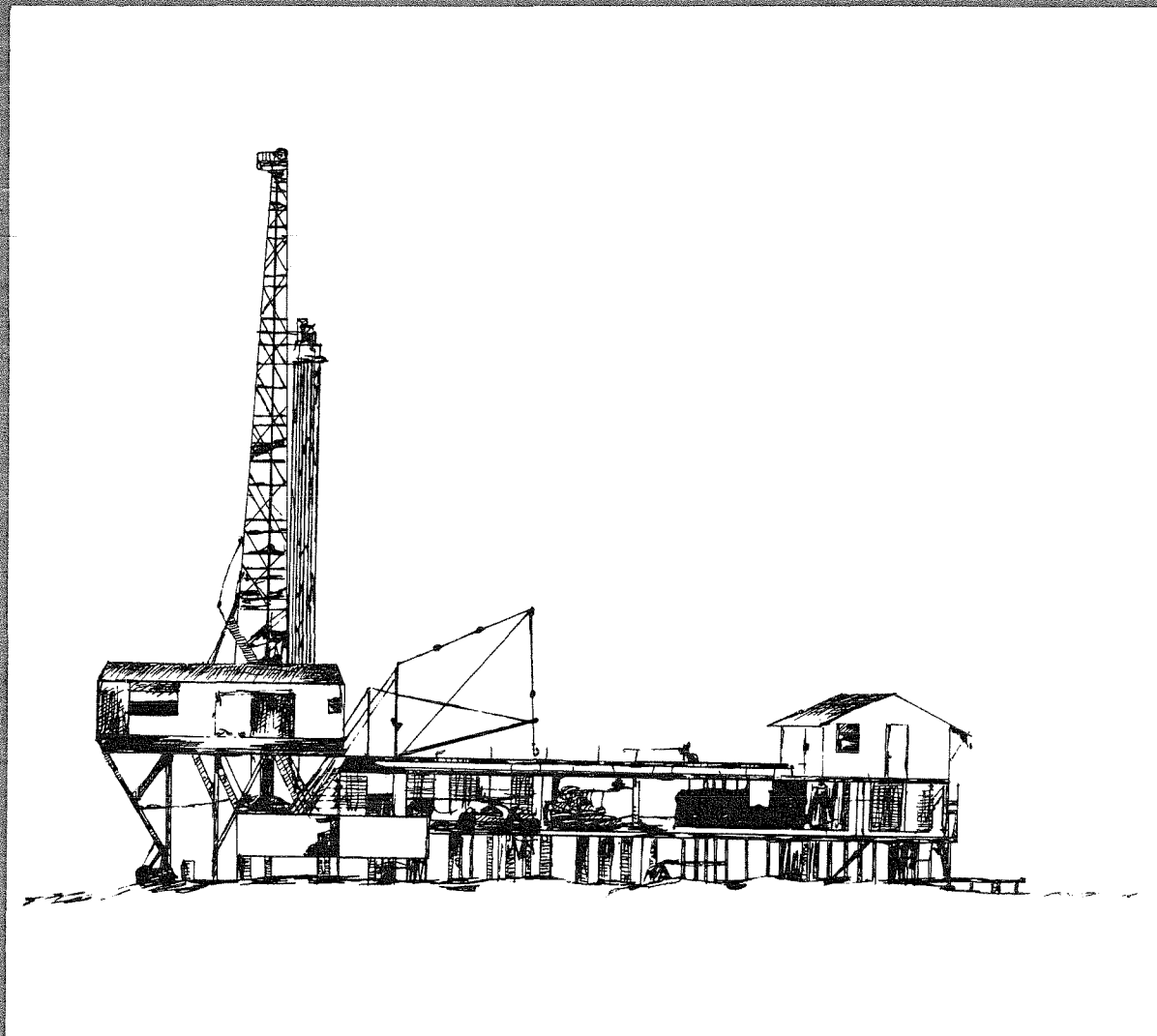


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WINDFALL PROFITS,
INTEREST RATES
AND
INDEX NUMBERS

Interest Rate Forecasts and Market Efficiency

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Increasingly volatile financial markets have put a premium on accurate forecasts of interest rates. To the extent, however, that market participants base their decisions on available interest-rate forecasts, the value of these forecasts to an individual investor is diminished because security prices would already tend to reflect that information. At the extreme, the *efficient-market* hypothesis asserts that the market efficiently utilizes all available information in the pricing of securities, so that market participants generally can not profit from more accurate forecasts than those already incorporated in security prices.

This article examines the degree to which the Treasury-bill market efficiently utilizes all available information so as to incorporate the best possible interest-rate forecast into current market prices. In other words, it evaluates the applicability of the *efficient-market* hypothesis to the Treasury-bill market. Two types of independent forecasts are examined: an autoregressive forecasting equation based on the past history of the bill rate, and the forecast of a selected panel of market professionals. Statistical tests are used to determine whether all useful information contained in these two forecasts is efficiently incorporated into Treasury-bill market prices. The market's forecast is derived from the "forward rate" implied by the term structure of yields.

If the market is not efficient, a group of investors could improve their returns by altering the maturity of their investments in light of

superior interest rate forecasts. Near a peak in the business cycle, for example, if such forecasts correctly foresee a larger decline in interest rates than anticipated by the market, then investors should buy securities with a maturity longer than their expected investment periods. The yields obtained by investors would be greater than those obtainable if they had chosen maturities equal to their investment period, due to larger capital gains than had been anticipated by the market. Alternatively, if the available forecasts correctly foresee interest rates falling more slowly than the market does, the investors utilizing those forecasts should buy maturities shorter than their investment periods. This is because investors would obtain a higher return from "rolling over" a series of short-term securities than from buying maturities equal to their investment periods.

Whether or not investors can profit by "speculating" on interest rates, through holding other than their normally preferred maturities, depends critically on whether available interest-rate forecasts are more accurate than the market's. If the market is efficient, the information in these forecasts would already be incorporated into the prices of securities, and therefore nothing would be gained. In that case, investors would be better off by simply "hedging" their positions with maturities equal to their planned investment periods, thereby avoiding possible risk.

This study shows that professional analysts' prediction of the Treasury-bill rate two quarters ahead are significantly more accurate than market predictions. This indicates that the market does not efficiently utilize all available information in making bill-rate forecasts. By

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making use of the information contained in the analysts' forecast, an investor in Treasury bills could have improved his return. We show that the analysts' ability to "beat the market" stems from a better utilization of information about past movements in the bill rate, and also from a more efficient utilization of other sorts of information.

Section I explains the concept of market efficiency and examines the forecasting accuracy of the market, compared with that of both a panel of professional analysts and a simple autoregressive forecasting equation based on the past history of the bill rate. In Section II, statistical tests for market inefficiency are de-

scribed and performed, on the basis of these three forecasts. Both the analysts' forecast and the forecast from the autoregressive forecasting equation are found to contain useful information which is not fully incorporated into the market's forecast — indicating market inefficiency. Section III shows that the analysts' forecast contains information similar to that in the autoregressive forecasting equation, plus other useful information which is also not fully incorporated into the market's forecast. Investors could have traded on both types of information to improve their returns in the period examined. Section IV provides a summary and some further conclusions.

I. The Concept of Market Efficiency

Efficient financial markets exist when the prices, or yields, of securities fully reflect all available information relevant for their valuation. In the case of riskless fixed-income securities, such as Treasury bills, the relevant information consists of expectations about the future course of interest rates. Investors then bid the prices of securities to the point where expected holding-period yields for securities of different maturities are roughly the same, given these expectations. For example, given the current six-month bill rate, the price and yield of a nine-month bill depends upon the expected three-month rate for six months ahead. New information can develop, but when it does it is rapidly reflected in revised expectations and in the prices of securities. Consequently, in an efficient Treasury-bill market, investors would not have significant opportunities for making profits on the basis of information about future interest rates¹.

The hypothesis of an efficient market is an extreme one, and therefore could not be expected to be literally true. Past tests of the efficient-market hypothesis have thus attempted to pinpoint the level of information at which the hypothesis breaks down. In these tests, all available information can be separated into three distinct types, or subsets, as shown in Table 1. The first subset consists of

the past history of rates of return, or prices. A test of whether the market efficiently utilizes information in the past history of rates of return, or prices, is called a test of *weak-form* efficiency. The second information subset consists of any other information publicly available at little or no cost — such as government statistics on other relevant variables. Tests of whether the market efficiently utilizes this kind of information in securities pricing are called *semistrong-form* tests. The third information subset consists of information that is privileged or available only at significant cost. Tests of whether the market efficiently utilizes this kind of information, so that profits cannot be made from trading on it, are called *strong-form* tests.

Table 1
Types of Market Efficiency

Subset of Information	Test of Whether Particular Information Subset Is Efficiently Utilized by Market
1. Past History of Prices, or Rates of Return	Weak-Form Test
2. All Other Publicly Available Information	Semistrong-Form Test
3. Privileged or Costly Information	Strong-Form Test

The three subsets of information exhaust the universe of all available information; thus, a market is fully efficient only if it passes all three kinds of tests. According to previous studies, the Treasury-bill market tends to be efficient in the weak-form sense of utilizing information in the past behavior of the bill rate. However, the evidence has generally been confined to very near-term market forecasts of only up to a few months ahead.² Also, little evidence is available on the bill market's efficiency in the strong or semi-strong form sense of incorporating other available information useful for forecasting.³

In this study, we consider two specific types of information that the Treasury-bill market could utilize in formulating a two-quarter ahead forecast of the 3-month bill rate. The first is simply an autoregressive forecast based on the past history of the bill rate. The yield on Treasury bills may vary somewhat predictably over time, due to the business cycle and/or predictable patterns in monetary policy. To pass the weak-form test, the market's forecast of the future bill rate needs to take into account any systematic behavior evidenced by its past history.⁴

For the period 1951-IV through 1969-III, we estimated a simple autoregressive forecast that could have been used by the market. In this period, quarterly movements in the 3-month bill rate followed a significant autoregressive pattern, i.e., past movements of the rate were significantly related to future movements. An equation explaining quarterly changes in the bill rate contained significant lags at 1, 2, 3, and 6 quarters, as well as a significant constant term indicating a positive time trend. We then obtained autoregressive forecasts for the period 1970-I through 1979-III by reestimating this equation on a growing sample of available observations and computing two quarter ahead forecasts from the latest coefficient estimates at each point in time.⁵ Whether or not the market efficiently utilized the information contained in this autoregressive forecast is a test of weak-form efficiency.

The second type of information is the average interest-rate forecast made by a panel

of professional analysts, and compiled by the *Goldsmith-Nagan Bond and Money Market Letter* since September 1969.⁶ The forecast period is again 1970-I through 1979-III. This period begins with the first Goldsmith-Nagan sampling of professional forecasts and continues through the quarter just prior to the Federal Reserve's October 1979 shift in operating procedures, which emphasized controlling bank reserves for achieving its monetary objectives. We excluded later data on the ground that both the market and professional forecasters had to go through a learning experience which reduced the forecasting accuracy of each by perhaps differing amounts.

Professional analysts use, either directly or indirectly, information in the past history of the bill rate — but they undoubtedly use other sources of information as well. So the analysts' forecasts contain information relevant to all three kinds of efficiency. However, it is not possible here to distinguish between a semi-strong and strong test of efficiency. Some interest-rate forecasts in the Goldsmith-Nagan sample are not widely circulated, being privileged or costly to obtain, while other information may be publicly available. Since we cannot discriminate between these two types of information in the analysts' forecasts, we simply use the term "strong-form efficiency" to refer to efficient market use of all types of information besides the past history of the rate.

In order to test the bill market's efficiency, we need a measure of the market's forecast of the 3-month bill rate for two quarters ahead. This can be obtained from the term structure of Treasury bill rates — specifically, from the market's two quarter ahead "forward rate." This is the interest rate on a 3-month Treasury bill two quarters ahead that would be required to equalize expected returns on 6- and 9-month bills over a 9-month holding period.⁷ The forward rate also contains a "liquidity premium," which compensates investors in the longer-term security for their sacrifice of liquidity. So an adjustment for that premium must be made to provide an estimate of the market's forecast of the bill rate. An Appendix develops the concept of the forward rate more

precisely, and details the technique used to estimate the liquidity premium.

For a preliminary view of market efficiency, we can compare the market's two-quarter ahead forecast of the 3-month Treasury bill rate (F_{t+2}^m) with the professional analysts' forecast (F_{t+2}^{gn}) and the forecast from the autoregressive equation (F_{t+2}^{ar}). To measure forecast accuracy, we use the mean squared error (MSE), the arithmetic average of the squares of the forecast errors — or better still, the root mean squared error (RMSE), since it measures the error in the same units as the forecasted variable (Table 2). As a baseline for comparison, we use the RMSE for a forecast of no change.

After extraction of the estimated liquidity premium, the RMSE of the market's forecast, 1.24 percentage points, is only slightly lower than that for the forecast of no change — but substantially higher than the RMSEs of the analysts' and autoregressive forecasts. Moreover, the approach used to estimate the market's liquidity premium is more likely to have understated rather than overstated the true difference between forecast errors, as shown in the Appendix. These differences thus suggest the possibility of market inefficiency. If the estimated difference between the RMSEs of the autoregressive forecast and the market's forecast were statistically significant, then the condition of weak-form efficiency would not be met. An autoregressive forecast based on available past information on the

Table 2
Accuracy of Forecasts
Root Mean Squared Error (RMSE)
1970I - 1979III
(percentage points)

Forecast of No Change (i_t)	1.25
Market's Forecast (F_{t+2}^m) (Adjusted for Liquidity Premium as Estimated from Appendix Table 1, Eq. 3)	1.24
Analysts' Forecast (F_{t+2}^{gn})	1.10
Autoregressive Forecast (F_{t+2}^{ar})	.94

Treasury-bill yields would have provided a means for investors to "beat the market". By altering the maturity of their holdings in light of a superior forecast they could have improved their returns.

The RMSE of the analysts' forecast is lower than the market's, but higher than that of the autoregressive forecast. Thus, the analysts may not have made full use of the available autoregressive information. Still, the analysts' forecast could contain other information besides that embodied in the past history of the rate. If an investor could have profited from trading on such other information, the market would also be inefficient in a strong-form sense.⁸ The next two sections provide tests of forecast accuracy that distinguish between strong-form and weak-form inefficiency.

II. Evidence of Market Inefficiency

We first test for weak-form inefficiency by examining whether the autoregressive forecast could have been systematically used to reduce the error in the market's forecast. This could have been done if the market's forecast error is at least partially explained by the difference between the autoregressive forecast and the market's forecast. Symbolically, this relationship is:

$$i_{t+2} - F_{t+2}^m = B_1(F_{t+2}^{ar} - F_{t+2}^m) + e_{t+2}, \quad (1)$$

where $i_{t+2} - F_{t+2}^m$ = the market's forecast error for two quarters ahead;

F_{t+2}^{ar} = the autoregressive forecast for quarter $t+2$ made at time t ;

F_{t+2}^m = the market's forecast for quarter $t+2$ made at time t ;

e_{t+2} = a random error term.

Note that this equation can be rearranged to give the optimal forecast of the interest rate as a weighted average of the two forecasts. That is, it implies:

$$i_{t+2} = B_1 F_{t+2}^{ar} + (1 - B_1) F_{t+2}^m + e_{t+2} \quad (2)$$

In the case of weak-form inefficiency, the autoregressive forecast would receive a significant weight. On the other hand, if there were no weak-form inefficiency, the weight of the autoregressive forecast would be insignificantly different from zero, and the weight of the market's forecast would be close to one. Thus, the test of weak-form inefficiency is that the estimated value of B_1 be significantly different from zero, so that the autoregressive forecast could have been used to improve upon the accuracy of the market's forecast.

F_{t+2}^{ar} and F_{t+2}^m are apt to be highly correlated in equation (2), tending to increase the standard errors of the estimated coefficients. So it is preferable to test for weak-form inefficiency by estimating equation (1); in addition, this equation constrains the weights to add up to unity. We estimated this and other equations using ordinary least squares, with a correction for a moving-average pattern of serial correlation in the error term.⁹ The estimate of equation (1) is:

$$i_{t+2} - F_{t+2}^m = .644 (F_{t+2}^{ar} - F_{t+2}^m) \quad (3)$$

(.163)

$$\bar{R}^2 = .467 \quad \text{S.E.} = .900$$

The value of B_1 , at .664, is more than four times its estimated standard error, given in the parentheses, indicating that it is indeed significantly different from zero. Thus, the autoregressive forecast could have been used to reduce the market's forecast error, confirming the existence of weak-form inefficiency. In fact, in an optimal forecast combining the two, the autoregressive forecast would be given a larger weight (.664) than the market's forecast (.336). Also note that the unexplained forecast error has been reduced to .90 percentage points (equals the equation's standard error, S.E.) from 1.24 percentage points (equals RMSE in Table 2).

A similar test can be performed with the forecast of the Goldsmith-Nagan panel of analysts, F_{t+2}^{gn} , as the alternative to the market's forecast. When the difference between the analysts' and the market's forecast is used to explain the market's forecast error, we obtain the following estimated equation:

$$i_{t+2} - F_{t+2}^m = 1.15 (F_{t+2}^{gn} - F_{t+2}^m) \quad (4)$$

(.178)

$$\bar{R}^2 = .474 \quad \text{S.E.} = .893$$

The estimated value of the B_1 coefficient, at 1.15, is over six times its estimated standard error. It is clearly significantly different from zero and also not significantly different from one. Thus, only the analysts' forecast should be used in an optimal forecast combining both, because the market's forecast provides little or no additional information.

This test also supports a finding of market inefficiency. Whether it is of the weak or strong form, or both, depends upon the type of information embodied in the analysts' forecast. If the analysts' forecast were based only on the historical behavior of the bill rate, only weak-form inefficiency would be confirmed. But if it contains other information as well, both weak and strong-form inefficiency would be indicated.

Two additional tests can be made for the presence of strong-form inefficiency. The first explains the forecast error of the autoregressive equation by the difference between the analysts' forecast and the autoregressive forecast. The significant coefficient estimated in this equation, given below, indicates that the analysts' forecast can be used to explain an important part of the error in the autoregressive forecast. Therefore, that forecast contains other useful information besides the past history of rates.

$$i_{t+2} - F_{t+2}^{ar} = .685 (F_{t+2}^{gn} - F_{t+2}^{ar}) \quad (5)$$

(.187)

$$\bar{R}^2 = .155 \quad \text{S.E.} = .862$$

A second test for confirming the existence of strong-form inefficiency involves generaliz-

ing the form of the original test. That is, the error in the market's forecast might be explained by *both* the difference between the autoregressive and the market's forecast and the difference between the analysts' and the market's forecast. In symbols,

$$i_{t+2} - F_{t+2}^m = B_1 (F_{t+2}^{ar} - F_{t+2}^m) + B_2 (F_{t+2}^{gn} - F_{t+2}^m) + e_{t+2} \quad (6)$$

or in the alternative form,

$$i_{t+2} = B_1 F_{t+2}^{ar} + B_2 F_{t+2}^{gn} + (1 - B_1 - B_2) F_{t+2}^m + e_{t+2} \quad (7)$$

If both B_1 and B_2 were significantly different from zero, then both weak and strong-form inefficiency would be confirmed. We again choose the first form of the equation for estimation, in order to reduce the extent of multicollinearity among the independent variables. The estimate of the equation in this form is:

$$i_{t+2} - F_{t+2}^m = .657 (F_{t+2}^{gn} - F_{t+2}^m) + .321 (F_{t+2}^{ar} - F_{t+2}^m) \quad (8)$$

(.337)
(.201)

$$\bar{R}^2 = .498 \quad \text{S.E.} = .873$$

Both of the estimated coefficients are larger

than their standard errors and significantly different from zero at the 5-percent level, indicating both weak- and strong-form inefficiency.

To summarize the results so far, we have found that two different types of information could have been used to improve upon the market's forecasts of the 3-month Treasury bill rate, as embodied in the forward rate. The significance of an autoregressive equation in explaining the market's forecast error indicates weak-form inefficiency, and the added significance of the analysts' forecast confirms strong-form inefficiency. By incorporating the information contained in an autoregressive model and the analysts' forecast, the RMSE for a two-quarter ahead forecast of the 3-month Treasury bill rate could have been reduced from the market's 1.24 percentage points to .87 percentage points (equal to S.E. of equation (8)). In fact, when all three forecasts are combined into an optimal forecast, as in equation (7), the weight attached to the market's forecast is not significantly different from zero, at .022 (equals $1 - .657 - .321$). Thus, the autoregressive forecast and the analysts' forecast contain all the useful information embodied in the market's forecast, plus other useful information as well.

III. Information in the Analysts' Forecast

The previous section tested whether information from a simple autoregressive model that *could have been* used to predict the Treasury-bill rate was fully incorporated into the market's forecast, as embodied in the forward rate. However, some judgment enters into the construction of even such a simple autoregressive forecast. Specifically, if the time pattern of rate movements is unstable, the forecasting power of the estimated autoregressive equation could depend importantly upon the period of estimation chosen. Therefore, we should also test whether the autoregressive information that was *actually*

used by the professional analysts was efficiently incorporated into the market's forecast. In fact, we shall see that the autoregressive information contained in the analysts' forecast is substantially the same as that in our autoregressive equation.

To approach this question, we decomposed both the professional analysts' forecast and the market's forecast into the portion related to current and past bill rates (extrapolative component) and the remainder that is not so related (autonomous component).¹⁰ The difference between the two forecasts can then be decomposed into the difference between

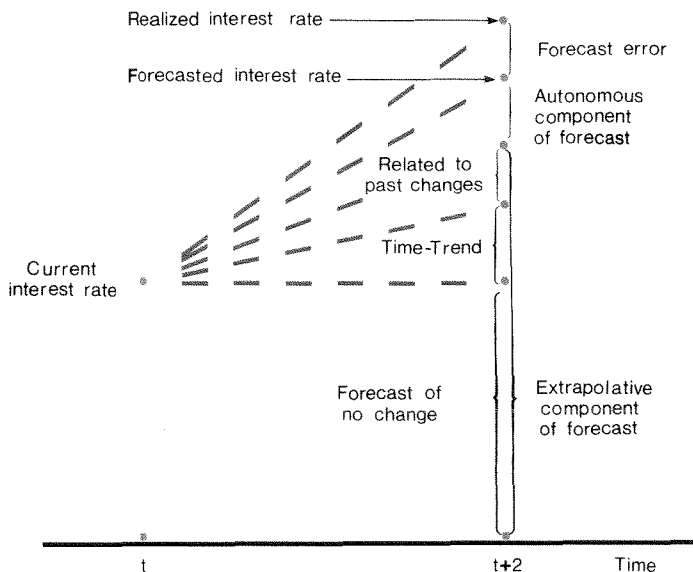
the extrapolative components of the two forecasts plus the difference between the autonomous components. If the difference between the *extrapolative* components were statistically significant in explaining the market's forecast error, this would indicate that autoregressive information actually incorporated into the analysts' forecasts was useful in predicting the interest rate, but nevertheless was not fully incorporated into the market's forecast — an instance of weak-form inefficiency. Similarly, if the difference between the two *autonomous* components were significant in explaining the market's forecast error, this would mean that the market did not incorporate other useful information available to the analysts — an instance of strong-form inefficiency.

The extrapolative components of the two forecasts were estimated by regressing the difference between each forecast and the current rate on lagged quarterly differences in the

bill rate. Such components can be divided conceptually into three parts (Figure 1). The first is simply the current interest rate, or a prediction of no change corresponding to a random-walk hypothesis. If there were no significant coefficients in the above regression, the estimated extrapolative component would contain only the current interest rate. The second component is a time trend, or drift factor, indicated by a significant constant term in the above regression. The third component is the part of the forecast related to past changes in the interest rate, indicated by any significant coefficients on past changes in the rate.

The best equation explaining the difference between the market's two-quarter ahead forecast of the bill rate and the current rate, chosen on the basis of a minimum standard error, contained no constant term and no lagged changes in the bill rate. Thus the extrapolative component of the market's forecast, E_{t+2}^m , is estimated to be simply a

Figure 1
Elements of a Forecast



Note: For simplicity, this illustration assumes that elements of the forecast other than the current interest rate all contribute to reducing the forecast error. Of course, this need not be true in general. For example, the time trend, the part of the extrapolative component related to past changes, and the autonomous component could all be negative, instead of positive as assumed here. In that case, and given the same realized interest rate, the forecasted interest rate would be below, rather than above, a forecast of no change; and the forecast error would be increased, rather than reduced, by these three elements.

forecast of no change, equal to the current interest rate (i_t). Symbolically,

$$E_{t+2}^m = i_t \quad (9)$$

In contrast, the extrapolative component of the analysts' two quarter-ahead forecast, E_{t+2}^{gn} , estimated in the same fashion, was found to be more complex. It contains a constant term, indicating a positive time trend in the interest rate of 54 basis points per year, and significant lags at 1, 3, 4, and 5 quarters on past changes in the bill rate. Specifically, the extrapolative component of the analysts' forecast is estimated to be:

$$\begin{aligned} E_{t+2}^{gn} = & i_t + .270 - .196(i_t - i_{t-1}) \quad (10) \\ & + .186(i_{t-3} - i_{t-4}) \\ & + .306(i_{t-4} - i_{t-5}) + .200(i_{t-5} - i_{t-6}) \end{aligned}$$

The autonomous components, denoted by A_{t+2}^m and A_{t+2}^{gn} , are simply the difference between the respective forecast and its extrapolative element. It follows that the difference between the analysts' forecast and the market's forecast equals the sum of the differences between the autonomous and extrapolative components; or

$$\begin{aligned} F_{t+2}^{gn} - F_{t+2}^m = & (A_{t+2}^{gn} + E_{t+2}^{gn}) \quad (11) \\ & - (A_{t+2}^m + E_{t+2}^m) \\ = & (A_{t+2}^{gn} - A_{t+2}^m) \\ & + (E_{t+2}^{gn} - E_{t+2}^m) \end{aligned}$$

To test for both weak and strong-form inefficiency in terms of the autoregressive information actually used by analysts in the Goldsmith-Nagan survey, we substitute equation (11) into equation (4) and reestimate the latter. The resulting estimated equation is:

$$i_{t+2} - F_{t+2}^m = 1.10 (A_{t+2}^{gn} - A_{t+2}^m) \quad (12)$$

(.200)

$$1.35 (E_{t+2}^{gn} - E_{t+2}^m)$$

(.377)

$$\bar{R}^2 = .464 \quad \text{S.E.} = .902$$

The significant coefficient on the difference between the two extrapolative components

shows that the analysts made better use of information in the bill rate's past history than did the market. In addition, the significant coefficient on the difference between the two autonomous elements indicates that the analysts made superior use of other information besides past rates. The first finding confirms weak-form inefficiency, and the second substantiates a strong form of inefficiency.

A final point of interest is whether the extrapolative component of the analysts' forecast utilizes available information on past bill-rate movements as efficiently as possible. To examine this question, we simply added to the right-hand side of equation (12) the difference between the forecast from the autoregressive equation and the market's forecast, and then reestimated the equation. This gives

$$i_{t+2} - F_{t+2}^m = .650 (A_{t+2}^{gn} - A_{t+2}^m) \quad (13)$$

(.348)

$$+ .766 (E_{t+2}^{gn} - E_{t+2}^m) + .323$$

(.584)

$$+ .323 (F_{t+2}^{ar} - F_{t+2}^m)$$

(.215)

$$\bar{R}^2 = .484 \quad \text{S.E.} = .885$$

The estimated standard error of the equation is reduced somewhat by the addition of the difference between these two forecasts. However, neither the coefficient on this variable nor the coefficient on the difference between the extrapolative elements is significantly different from zero at the 5-percent level. Thus, neither the extrapolative component of the analysts' forecast nor the autoregressive equation's forecast significantly reduces the market's forecasting error once the other is already being utilized. Both contain a positive time trend and extrapolate from past changes in the bill rate, in contrast to the prediction of no change in the extrapolative element of the market's forecast. On the basis of this evidence, we conclude that the analysts have utilized the available information on past movements in the bill rate about as efficiently as possible in their forecast.

These analysts also efficiently incorporated other publicly available information into the autonomous component of their forecast. As Friedman (1980) has shown, the forecasting errors of the Goldsmith-Nagan panel for short-term interest rates have not been significantly

related to costlessly available information on past values of macroeconomic series affecting the interest-rate outlook. These series include the unemployment rate, industrial production, price inflation, the money stock, and the Federal government's deficit.

IV. Summary and Conclusions

We have investigated whether the Treasury-bill market has efficiently utilized all available information, so as to incorporate the best possible two-quarter ahead forecast of the 3-month interest rate into its pricing of Treasury bills. A forecast by a panel of professional analysts was used as a measure of all available information, and the subset of information relating to the bill rate's past history was estimated by a simple autoregressive forecasting equation. We found that the analysts' forecast contained a similar extrapolative component, based on past movements in the bill rate. Either this extrapolative component of the analysts' forecast or a forecast from the autoregressive equation could have been used to reduce the error in the market's forecast significantly; but both contained substantially the same information. In addition, the analysts' forecast contained other useful information for explaining a portion of the market's forecast error. Altogether, the tests performed indicate the existence of both weak-form and strong-form inefficiency.

Earlier studies of the efficiency of the Treasury-bill market, focusing on shorter term interest-rate forecasts, have usually indicated weak-form efficiency. In contrast, our results for a two-quarter ahead forecast of the bill rate show the existence of weak-form inefficiency. This reflects the positive time trend found in both the extrapolative component of the

analysts' forecast and the autoregressive forecasting equation, but not in the market's forecast. The market's forecast of the 3-month bill rate failed to incorporate the upward drift in the bill rate attributable to rising inflation in the forecast period, even though this drift could have been extrapolated from past data.

In testing for a stronger form of efficiency in the bill market, earlier studies have generally used current and past values of relevant macroeconomic variables to measure other available information besides the past history of the bill rate. This study used the non-extrapolative component of the professional analysts' forecast for this purpose. We found that this component also contributed significantly to reducing the market's forecast error, indicating strong-form inefficiency as well.

The difference between the overall forecast error of the analysts and the market in the 1970's was a modest 14 basis points, as measured by the root mean square error. Nevertheless, our results indicate that the above types of market inefficiency would have allowed an investor to trade on the information contained in the analysts' forecast. An investor could have improved his overall returns with a strategy of shortening maturities when the analysts' forecast was above the market's and lengthening them when the opposite was true.

Appendix

Estimation of the Market's Forecast

The market's two-quarter ahead forecast of the 3-month Treasury bill rate is embodied in the differential between yields on 6- and 9-month bills. To see this, first consider that market arbitrage makes the yield on a 9-month Treasury bill equal to the expected return on a 3-month Treasury bill that is rolled over twice, plus premiums for the sacrifice of liquidity. In algebraic terms, the yield on the 9-month Treasury bill is:

$$(1 + {}_3i_t)^3 = (1 + {}_1i_t)(1 + {}_1i_{t+1}^e + {}_1p_{t+1}) \quad (1)$$

$$(1 + {}_1i_{t+2}^e + {}_1p_{t+2})$$

where i = yield at a quarterly rate,
 p = liquidity premium,
 left subscript = maturity of security in quarters,
 right subscript = time at which investment in security begins in quarters,
 superscript "e" = interest rate expected by the market as of time t .

Similarly, for a 6-month Treasury bill:

$$(1 + {}_2i_t)^2 = (1 + {}_1i_t)(1 + {}_1i_{t+1}^e + {}_1p_{t+1}) \quad (2)$$

Dividing (1) by (2) and rearranging terms, we obtain equation (3) for the market's two-quarter ahead "forward rate." This is the expected return (including a liquidity premium) on a 3-month Treasury bill two quarters ahead that is required to bring about equality between the expected returns on 6- and 9-month bills over a 9-month holding period.

$$\frac{(1 + {}_3i_t)^3}{(1 + {}_2i_t)^2} - 1 = {}_1i_{t+2}^e + {}_1p_{t+2} = {}_1f_{t+2} \quad (3)$$

To evaluate the accuracy and information content of the market's two-quarter ahead forecast, it is necessary to separate the liquidity premium from the forward rate. The approach used here assumes that market expectations

are generally unbiased, so that the realized interest rate is approximately equal to the anticipated interest rate plus a random error:

$${}_1i_{t+2} = {}_1i_{t+2}^e + v. \quad (4)$$

The market's forward rate (f) is composed of an anticipated interest rate and a liquidity premium (p), or:

$${}_1f_{t+2} = {}_1i_{t+2}^e + {}_1p_{t+2} \quad (5)$$

Subtracting (4) from (5), we obtain the liquidity premium as equal to the difference between the market's forward rate and the realized interest rate plus the random (expectational) error:

$${}_1p_{t+2} = {}_1f_{t+2} - {}_1i_{t+2} + v \quad (6)$$

If the liquidity premium were constant, it could be estimated simply from the average difference between the forward rate and subsequently realized interest rate. But previous research indicates that liquidity premiums are in fact somewhat variable. Three not mutually exclusive hypotheses have received support in the literature.

The first hypothesis views short-term securities as better substitutes for money balances than longer-term securities. If this is so, the liquidity premium would tend to be affected by the level of interest rates. When interest rates are high, the foregone interest income in holding money is larger; and so the public desires to exchange part of its money balances for securities. However, if the public prefers short to long securities in this exchange, prices of short-term securities would be bid up relative to those on longer-term ones. This would increase long-term interest rates relative to returns on short-term securities, or in other words increase the liquidity premium. Thus, if short-term securities are better substitutes for money than longer-term ones, liquidity premiums would vary positively with the level of interest rates.¹¹

Two other hypotheses relate to the fact that liquidity premiums on longer-term securities compensate investors for exposure to the risk of capital losses. The greater the probable variation in interest rates, the larger would be such risk. If the variation in interest rates is expected to approximate its recent variance, liquidity premiums ought to vary positively with that variance. Moreover, at times when interest rates are abnormally low (high), there is a large likelihood of unanticipated increases (decreases) producing unanticipated capital losses (gains) for holders of longer-term securities. Therefore, liquidity premiums might also be expected to vary inversely with the height of interest rates relative to recently experienced levels.¹²

These three hypotheses were tested with available data for the period 1963-IV through 1979-III (Table A.1). Following equation (6), we regressed the difference between the two-quarter ahead forward rate and the realized Treasury-bill rate on the current level of the Treasury-bill rate (i), the difference between its current level and a weighted average of its

value over the previous 2 years ($i - \bar{i}$), and the standard deviation of the bill rate over the same period (SD).¹³ When no explanatory variables other than a constant term are included, the constant term is significantly positive — indicating a positive average liquidity premium of 54 basis points.

The level of the bill rate (reflecting the effect of better substitutability between short-term securities and money) and the deviation of the bill rate from its recent trend (proxying for the probability of unanticipated capital gains on long-term securities) show significant effects when entered together. So also does the moving standard deviation of the bill rate when entered alone. But when all three variables are entered together, only the standard deviation retains at least marginal significance. Also the equation containing only the moving standard deviation of the bill rate explains more of the variation in the liquidity premium than does the equation containing the other two variables. Mainly on statistical grounds, we chose this equation ((3) in Table A.1) for separating out the liquidity premium from the

Table A.1.
Estimation of Liquidity Premium in the Two-Quarter Ahead
Three-Month Forward Rate (percentage points)
1963IV - 1979III

	Constant	i	$i - \bar{i}$	SD	\bar{R}^2	D.W.	S.E.
(1)	.540 (.134)**				.000	1.28*	1.17
(2)	-1.30 (.565)**	3.54 (1.03)**	-.443 (.153)**		.148	1.67**	1.08
(3)	-1.56 (.565)**			.162 (.0424)**	.175	1.53**	1.07
(4)	-1.49 (.578)**	-.0909 (.120)		.195 (.0611)**	.169	1.62**	1.07
(5)	-1.49 (.582)**		-.0746 (.126)	.158 (.0431)**	.166	1.62**	1.07
(6)	-1.58 (.560)**	-.602 (.645)	.545 (.675)	.409 (.273)	.165	1.51**	1.07

Note: Standard errors are in the parentheses. ** indicates a regression coefficient that is significantly different from zero at the 1- percent level on the basis of a single-tailed test; and * indicates significance at the 5-percent level. The same symbols on the Durbin-Watson statistic (D.W.) indicate the absence of significant serial correlation in the residuals at the 5- and 1-percent levels, respectively.

forward rate to arrive at an estimate of the market's anticipated 3-month bill rate. Because this equation has a minimum standard error, the resulting estimate of the market's forecast has a minimum forecast error. However, equations incorporating the other variables did about as well; and our results are not particularly sensitive to this choice.

The procedure used to estimate the liquidity premium assumes that the market's true expectational error, v , is uncorrelated with the variables used to model the liquidity premium. To the extent that this condition is not met, the estimated liquidity premium captures a portion of the market's true expectational error. This portion of the true expectational error would then be removed from the estimate of the market's forecast when the estimated liquidity premium is subtracted from the forward rate. As a simple example of the problem, if the true expectational error is positively biased, the procedure would overestimate the liquidity premium and correspondingly underestimate the size of the market's forecast errors.

We investigated the seriousness of this problem by regressing the difference between the forecast of the Goldsmith-Nagan panel and the realized interest rate on the variables used in Table A.1 to explain the liquidity premium. The estimated coefficients from these regressions carried the same signs as those in Table A.1 and were frequently statistically significant, indicating that these variables are capable of proxying for a portion of the analysts' forecast errors. However, the size of these coefficients was generally considerably smaller than those in Table A.1., as would be true if these variables were also related to the market's liquidity premium.

To minimize inclusion of expectational errors, we estimated the liquidity-premium models on all available data going back to 1963, rather than just on the forecast period of 1971-I thru 1979-III. This reduced contamination of the estimated liquidity premium with

expectational error, because the additional data allowed more of an *ex ante* estimation. A purely *ex post* fit of the forward rate's forecasting error against the liquidity-premium variables would have generated a larger bias.

Even so, our estimate of the liquidity premium appears to have picked up some of the true expectational error in the market's forecast. This was tested for by removing from both the analysts' forecast and the market's forward rate the portion of the respective forecast error associated with the moving standard deviation of the bill rate (and a constant term) during 1971-I thru 1979-III. These adjusted forecasts were obtained by regressing each forecast error on the moving standard deviation of the bill rate, and subtracting the values predicted by these regression equations from each respective forecast. The procedure removes a similar amount of true expectational error from both forecasts, but also an estimated liquidity premium from the market's forward rate. The RMSE's of these adjusted forecasts are 1.01 and 1.18 percentage points for the analysts' and the forward rate, respectively. The difference of 17 basis points exceeds the 14-basis-point difference between the RMSE's of the analysts' forecast and our estimate of the market's forecast, suggesting that our estimate of the latter tends to underestimate the size of its forecasting error.

The procedure used for estimating the liquidity premium has thus tended to understate the difference in true expectational errors between the market's forecast and the two alternative forecasts. A possible bias in the opposite direction could result from the omission of an important variable for explaining the liquidity premium. However, we have already considered all such possibilities. On balance, it is more likely that our estimate of the market's liquidity premium has understated, rather than overstated, the true difference between the forecast errors of the market and those of alternative forecasts.

FOOTNOTES

1. Tests of market efficiency were first applied to the stock market. Surveys of this literature are contained in Cootner (1964), Fama (1970), and Lorie and Hamilton (1973, Ch. 4).

2. The weak form of the efficient-market hypothesis has been tested in several ways in the bill market. One approach is to determine whether the forward rate applicable to any particular point in time follows a random walk. If the market's adjustment to new information is virtually instantaneous, as in an efficient market, successive changes in the forward rate applicable to any particular period should be random. This approach is followed in Shiller (1973) and Roll (1970).

A second approach to testing the weak form is to determine whether the market reacts appropriately to any autocorrelation in the bill rate. While changes in the forward rate applicable to any particular time period should be random, it does not follow that for weak-form efficiency the spot rate should also follow a random walk. Indeed, in an efficient market the forward rate should extrapolate any systematic autocorrelation that tends to occur in the spot rate. Evidence of weak-form efficiency in market forecasts of up to a few months ahead is contained in Hamburger and Platt (1975), Fama (1975b), and Fildes and Fitzgerald (1980).

Fama (1975a, 1977) takes a rather different approach by focusing on the relationship between the nominal Treasury-bill rate and the subsequently observed inflation rate. For weak-form efficiency, the nominal interest rate would summarize all the information about future inflation rates contained in the time series of past inflation rates, so long as the expected real return is constant. Fama argues that the Treasury-bill market is efficient in this sense, and also that expected real returns on Treasury bills are approximately constant. Carlson (1977), Joines (1977), and Nelson and Schwert (1977) present contrary evidence on the constancy of the expected real rate, but their evidence with respect to weak-form efficiency is not conclusive.

3. Hamburger and Platt (1975) conduct a test of the semistrong form of efficiency by investigating whether variables other than current and past levels of interest rates are better than forward rates in predicting future bill rates. They considered such potential predictors as the current and past values of personal income and three alternative monetary aggregates, but found that the root-mean-squared error for such forecasts was no smaller than from a forecast using the forward rate, consistent with the semistrong form of efficiency.

4. For the market to be considered efficient, the Treasury-bill rate does not necessarily have to follow a random walk, in which the change in the return from one period to the next is completely random. A random walk would be consistent with weak-form efficiency in stock and bond markets because short-period returns on these securities are dominated by capital gains or losses resulting from changes in

market prices. Any systematic pattern in expected short-period returns would quickly be eliminated as investors bid prices up or down in attempts to profit from them. In contrast, the returns on 3-month Treasury bills held to maturity cannot be affected by this kind of speculation, because the price at the end of the 3 months is fixed contractually. Therefore, even a fully anticipated time pattern in 3-month Treasury bill yields is not likely to be arbitrated away in an efficient market. However, an efficient market *would* take such a pattern into account in its bill-rate forecasts.

5. The data sample was extended back to 1951-IV using Salomon Brothers, *An Analytical Record of Yields and Yield Spreads*. The best autoregressive equation for explaining quarterly changes in the 3-month Treasury bill rate during 1951-IV thru 1969-III was found to be:

$$\begin{aligned} \Delta i_t = & .118 + .239 \Delta i_{t-1} - .241 \Delta i_{t-2} \\ & (.055) (1.19) \quad (.120) \\ & - .126 \Delta i_{t-3} - .338 \Delta i_{t-6} \\ & (.124) \quad (.122) \end{aligned}$$

$$\bar{R}^2 = .193 \quad \text{S.E.} = .449 \quad \text{D.W.} = 1.93$$

Standard errors are indicated in the parenthesis. A two-step ahead forecast from this autoregressive model of quarterly changes was found to be considerably more accurate than a one-step ahead forecast from past changes over six-month periods. Hence, the two-step ahead forecast (with coefficients reestimated at each point in time) was used for F_{t+2}^{ar} .

6. The author wishes to thank Mr. Peter Nagan of *The Goldsmith-Nagan Bond and Money Market Letter* for permitting use of this data.

7. A large body of literature exists on the term structure of interest rates. Useful surveys are contained in Dodds and Ford (1974), Malkiel (1966), and Van Horne (1966).

An alternative measure of the market's forecast can be obtained from yields on Treasury-bill futures contracts. We did not examine the accuracy of this type of forecast because a futures market in 3-month Treasury bills has existed only since January 1976, reducing the number of observations by more than half. For a comparison of yields on futures contracts and implied forward rates, see Lang and Rasche (1978) and Poole (1978).

8. Prell (1973) compared the accuracy of forecasts by the Goldsmith-Nagan panel of analysts with those of no change and an autoregressive model for 1970 through 1973. In that period, the RMSE of the Goldsmith-Nagan panel for a two-quarter ahead prediction of the 3-month bill rate was smaller than for both of these forecasts. However, for long-term bond yields, the panel's forecasts were no more accurate than a forecast of no change. Prell (1973) did not compare the accuracy of the analysts' forecast with those of the market implied by the forward rate, as would be necessary for a test of bill-market effi-