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Arbitrage and Efficient Markets Interpretations of Purchasing Power Parity: Theory and Evidence

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The theory of Purchasing Power Parity was the first well-developed theory of exchange rate determination. Although the efficient market approach is an important theoretical advance over the conventional arbitrage interpretation of purchasing power parity, many of the empirical implications of the two approaches are similar. As a result, at this time, the empirical evidence supports both views.

The adoption of more flexible exchange rates in the early 1970s spurred both theoretical and empirical research on purchasing power parity (PPP). The theoretical work refined existing ideas about the theory and led to a new version of PPP based on efficient commodity markets. The empirical research created an impressive body of evidence. This article reviews the theory behind two major approaches to purchasing power parity, the arbitrage and efficient markets approaches, and discusses the evidence relevant to each.

The arbitrage approach is discussed first. In spite of a widespread belief that arbitrage has failed, particularly during the current float, the evidence provides substantial support for an arbitrage interpretation of purchasing power parity. The efficient commodity market approach to purchasing power parity initially proposed by Richard Roll (1979) is the newest version of PPP, and it is discussed more thoroughly. Although the efficient market approach is an important theoretical advance over the conventional arbitrage interpretation of purchasing power parity, many of the empirical implications of the two approaches are similar. As a result, at this time, the empirical evidence supports both views.

I. Arbitrage

Theory

The arbitrage version of purchasing power parity was the first well-developed theory of the determination of exchange rates. Although the roots of the theory go back at least to the period when gold from the New World began to influence prices in Europe, Gustav Cassel (1916) is generally credited with the first formal statement of the theory. The name, purchasing power parity, comes from Cassel's basic idea that exchange rates should, in time, adjust so that a given amount of currency buys the same bundle of goods in all countries. In other words, exchange rates tend to settle at the point where the purchasing power of a currency is the same, or at parity, in all countries.[1]

As an example, start with a single commodity. It might be a quart of milk, a Sony Walkman®, a gallon of gasoline or a bushel of number 2 red wheat. Ignoring information and transaction costs, with effective arbitrage, the cost of buying

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the good in the United States at time t, p(H,t), should equal the cost of the good in Great Britain at time t, p(F,t), converted to dollars using the dollar price of the pound at time t, S(t). That is, p(H,t) should equal S(t)p(F,t). This is commonly referred to as the law of one price. The law of one price implies that the domestic price of foreign exchange S(t) equals the domestic price of the product p(H,t) divided by the foreign price p(F,t). If the product were wheat and the countries the United States and Great Britain, then the dollar price of pound sterling should equal the dollar price of wheat divided by the pound price of wheat.

The arbitrage interpretation of purchasing power parity rests on a weaker version of the law of one price that does not require zero information and transaction costs. For some goods, p(H,t) may be less than S(t)p(F,t). For actual or potential exports by the United States, the price differential would reflect the information and transaction costs associated with shipping goods to Great Britain. For actual or potential imports, the excess of p(H,t) over S(t)p(F,t) reflects the cost of moving the goods from the U.K. to the U.S. If the information and transaction costs are roughly the same in both directions, then the price in the U.S. of a broadly based *bundle* of goods, P(H,t), should tend to equal the price of that bundle in the U.K., P(F,t), converted into dollars at the going exchange rate, S(t). If there are goods for which the information and transaction costs exclude any possibility of international trade, then this version of PPP implicitly assumes that there is no systematic difference in their relative prices between any two countries.

$$S(t) = \frac{P(H,t)}{P(F,t)} \tag{1}$$

Equation 1 describes absolute purchasing power parity. That is, it describes the relation between the level of exchange rates and relative price levels. This version of the theory is not widely used for at least three reasons. First, in spite of relatively little research, there is a general consensus that it is not very accurate. Second, while price indices are easy to find for almost all countries, information about the price of identical bundles of goods in different countries is difficult to locate. Third, for many purposes, it is the change in exchange rates that is important, not the level.

For these reasons, almost all empirical work on PPP has concentrated on the relative version of the theory, which explains changes in the exchange rate. Let S(0) be the exchange rate in some base period, and P(H,0) and P(F,0) be the domestic and foreign price of the broadly based bundle of goods in the base period. The relative version of PPP says that the change in the exchange rate from the base period 0 to some later period t equals the relative change in the price of the bundle of goods in the two countries.

$$\frac{S(t)}{S(0)} = \frac{P(H,t)/P(F,t)}{P(H,0)/P(F,0)} \tag{2}$$

The right hand side of this equation can be rearranged into a more familiar form — a ratio of price indices. With a little manipulation, the right hand side of equation 2 becomes [P(H,t)/P(H,0)]/[P(F,t)/P(F,0)]. The numerator of this ratio is simply a price index for the United States, P^H, and the denominator a price index for the foreign country, P^F. Both indices have the same base period and use identical weights. Equation 3 uses these price indices to describe the relative version of purchasing power parity.[2]

$$\frac{S(t)}{S(o)} = \frac{P^H}{P^F} \tag{3}$$

Most empirical research on PPP involves regressing the log of the ratio of exchange rates on the log of a ratio of price indices:

$$\ln(s) = \alpha + \beta \ln\left(\frac{P}{P^*}\right) + z \tag{4}$$

where z is an error term; ln(x) is the natural log of x; S equals S(t)/S(0); P is a domestic price index; P* a foreign price index; and the price indexes usually are consumer or wholesale indexes, or GNP deflators not based on identical bundles of goods.[3] The usual interpretation of equation 4 is that it supports PPP when estimates of α are not different from zero, estimates of β are not significantly different from one, and the R^2 is high.[4]

Evidence

Most of the evidence concerning the arbitrage version of purchasing power parity has come either from estimating equations like 4 or analyzing the behavior of real exchange rates (which are actual exchange rates divided by the rates implied by PPP). This section concentrates on regression results. The behavior of real exchange rates is covered in the section dealing with the evidence for efficient commodity markets. For an extensive review of the results of regression analysis, see Officer (1976). Dornbusch (1985) provides a briefer review that covers most of the relevant research through 1984.

The general consensus on this empirical research is that, while regression results may provide some support for PPP during the 1920s, they provide almost no support for the theory during the 1970s.[5] However, this conclusion is too negative for two reasons. First, recent evidence not available to Officer or Dornbusch supports PPP. Second, in many cases, the rejection of PPP is based on a misinterpretation of the regression results.

As an example of some of the evidence not available to Officer or Dornbusch, Mark Rush and Steven Husted (1985) report long-run support for PPP between the U.S. and several countries. For other combinations of countries, their results are mixed. In addition, Craig Hakkio (1984) combines time series and cross section analysis, and obtains results that provide strong support for PPP. Although Tahmoures Parsai's (1982) research indicates that other factors influence exchange rates, his estimates of the relationship between price levels and exchange rates also support PPP and are not sensitive to the inclusion of other variables. As Paul Krugman (1978) points out "... one must be cautious in determining the extent of and the reasons for failure of PPP to hold, for the world has laid statistical traps for the unwary."

The following sections use the arbitrage approach to PPP to examine why PPP might appear to fail and to show how these apparent failures can be statistical traps. They also review the evidence concerning the relative importance of the various sources for failure. The last section provides some examples of how regressions can be misinterpreted.

Different Weights

From an arbitrage point of view, the weights in price indices must be the same. Using consumer or wholesale indices or GNP deflators violates this requirement. The following example illustrates the problem. Suppose the United States produces only wheat and Great Britain produces only cloth. Some real shock causes the price of wheat to rise ten percent in both countries and the price of cloth to fall ten percent. If the law of one price holds, then PPP holds and the exchange rate should not change.

But consider what happens if one tests PPP using equation 4 and GNP deflators. The GNP deflator in the U.S. rises ten percent because it contains only wheat. The GNP deflator for the U.K. falls ten percent because it contains only cloth. The exchange rate is constant, but the ratio of the price indices rises. Because the indices do not have identical weights, estimates of equation 4 can reject purchasing power parity even though the theory holds exactly.

From an arbitrage perspective, different weights introduce a form of measurement error into relative price levels. As an example, suppose the variance in the ratio of price indices, σ^2, comes from two independent sources: pure monetary shocks for which PPP holds exactly, σ_M^2, and movements in the ratio of price indices that come from changes in relative prices with unequal weights, σ_W^2.

$$\sigma^2 = \sigma_M^2 + \sigma_W^2$$

Under these conditions, ordinary least squares yields the following estimate for B:

$$\text{plim } \hat{B} = \frac{1.0}{1.0 + \frac{\sigma_W^2}{\sigma_M^2}} \qquad (5)$$

As inflationary shocks dominate measurement error, estimates of B and the R^2 approach unity. But as monetary shocks decline relative to the measurement error, R^2 declines and estimates of B approach zero even though PPP in the form of equation 3 holds exactly regardless of the relative importance of monetary shocks and measurement error.

For a number of years, the Federal Statistical Office of Germany has used identical bundles of goods to calculate absolute purchasing power parities for several countries.[6] John Mussachia (1984) compares the results of testing PPP with this data and conventional price indices. The results suggest that, except perhaps for very stable relative price levels, different weights are not a major source for the observed errors in purchasing power parity.

Simultaneity

Even if purchasing power parity held exactly and there were no problems with price indices, tests of equation 4 still could yield a low R^2 and estimates of β close to zero. Under the arbitrage version of PPP, neither price levels nor exchange rates are exogenous variables. As a result, there is the possibility of bias due to simultaneous equations. Krugman (1978) provides a simple example of simultaneous equations bias in PPP. In his model, the central bank attempts to stabilize the exchange rate by expanding the domestic money supply as the domestic price of foreign exchange falls. This stabilization policy biases the estimate of B toward zero because it causes the error term in equation 4 to be correlated with the ratio of price levels, violating one of the assumptions of ordinary least squares (OLS) regression.

Two stage least squares (2SLS) is the standard way to deal with this problem. The first stage of 2SLS develops a proxy variable. If this variable is a good proxy for the original explanatory variable, e.g., p^H/p^F, and it is also independent of the error term in the original regression, then substituting the proxy for the original explanatory variable in the second stage regression eliminates the correlation with the error term and eliminates the bias.

Although OLS estimates of equation 4 are subject to bias due to simultaneous equations, this bias does not appear to be a major reason that regressions often fail to support PPP. Measurement error due to unequal weights and some of the other sources for errors in PPP described below also introduce bias and cause the error term in equation 4 to be correlated with the ratio of price levels. These other sources for bias appear to be more important for two reasons. If the conventional arbitrage version of PPP were correct and simultaneous equations bias *per se* were the problem with the regressions, then the real exchange rate would not behave as though it were very close to a random walk. In addition, when the test equation for PPP is reformulated so as to reduce the bias from these other sources, two stage and ordinary least squares yield essentially the same results.[7]

Information and Transaction Costs

Tradables. In discussions of PPP, it is customary to divide goods into two categories: tradables, for which information and transaction costs as well as other impediments are zero, and nontradables, for which these impediments effectively prevent trade. The assumption of no impediments for tradables is analytically convenient, but not very accurate. Transaction costs and tariffs introduce errors into the law of one price even for widely traded goods such as wheat and oil. Although these impediments can introduce errors into PPP, the errors are bounded. Once the pound price of wheat converted into dollars at the going exchange rate exceeds the dollar price of wheat by the cost of shipping wheat plus any tariff, arbitrage presumably prevents the next shock from widening that gap. (See Aizenman, 1984a and 1984b, for a detailed discussion of how transaction costs introduce errors into PPP and how these errors can bias the estimate of B toward zero.) As a result, if the errors in PPP were primarily the result of the effects of information and transaction costs for tradables, then real exchange rates should not behave like random walks.

Work by Richard Roll (1979), Michael Darby (1980), John Pippenger (1982), and Michael Adler and Bruce Lehman (1983) indicates that real exchange rates behave randomly, which implies that the predictive error in PPP is unbounded. Although some new evidence presented below indicates that the errors are bounded, the boundaries appear to be very wide and/or very weak. The behavior of real exchange rates, therefore, suggests that the errors in purchasing power parity are not primarily due to the effects of trade impediments on tradables.

Dynamics. Purchasing power parity is usually viewed as primarily a theory of the long-run determination of exchange rates. Actual and parity rates can diverge in the short-run, but in the long-run they tend to converge.[8] Almost every asset model of the exchange rate implies this kind of behavior. Indeed, many asset models assume PPP fails completely in the short-run but holds exacty in the long-run.

A dynamic interpretation of PPP implies that equation 4 is misspecified. In a dynamic framework, the current exchange rate depends on both current and lagged relative price levels and, perhaps, lagged exchange rates. See Hodgson and Phelps (1975) for an attempt to estimate a dynamic version of equation 4.

If market forces tend to bring actual and parity rates into equality in the long-run, then changes in the deviation from PPP must be correlated. Suppose the actual rate is above the rate implied by PPP. If the error is random, then that gap is as likely to increase as decrease. Any move above parity is as likely to be followed by a further move away from as a move toward parity, and the changes in the error are uncorrelated. But if there are market forces at work bringing actual and parity rates together, then the gap is more likely to decrease than increase. Beyond some point, any move above parity eventually is followed by a movement back toward parity, and there is negative serial correlation in the changes in the error. Since, as mentioned earlier, the predictive errors for PPP behave almost like random walks, a dynamic version of equation 4 does not appear to be appropriate.

The evidence concerning the behavior of real exchange rates raises serious questions about the view that purchasing power parity is essentially a long-run theory. Although there is evidence that real rates do not behave exactly like random walks, the deviation from a random walk is so slight that it does not indicate any strong tendency for actual and parity rates to converge in the long-run. Opponents of PPP will be tempted to interpret this pattern as evidence that the theory does not hold much better in the long-run than in the short-run. However, the efficient commodity market model of purchasing power parity discussed below suggests a different interpretation. From that perspective, the observed behavior of real exchange rates suggests that commodity markets influence exchange rates in both the long-run and short-run.

Nontradables. As mentioned earlier in discussing PPP, it is convenient to divide goods into two groups: tradables with no impediments and nontradables where transaction costs or trade restrictions effectively prohibit trade. For tradables, the law of one price holds and so does equation 3 as long as the bundle contains only tradables. When price indices contain nontradables, real shocks can cause PPP to fail.

Take concrete as an example of a nontradable. Suppose some shock raises the price of concrete in the U.S. and lowers the price of concrete in the U.K., but all other prices in both countries are unchanged. With no change in the prices of traded goods, the exchange rate is unchanged. But a price index including concrete rises in the U.S. and falls in the U.K. Purchasing power parity fails because the change in relative prices between tradables and nontradables is different in the two countries.

The distinction between the structure of the errors for tradables and nontradables is important. If the errors in PPP are due primarily to shocks that affect tradables, then the errors are bounded. If the errors are due primarily to changes in relative prices for nontradables, no such restriction applies. A given shock might raise the relative price of concrete in the U.S., but the next shock might either accentuate or offset the effect of the first shock.[9]

From an arbitrage perspective, changes in capital flows, tastes or technology can introduce large persistent errors into PPP by causing relative prices between tradables and nontradables to change differently in different countries. This interpretation of the effects of such shocks helps explain why it is so difficult to find any empirical regularity between a given type of shock and the error in PPP. Under some circumstances a larger capital flow might cause the relative price of concrete to rise in a country; under others, the relative price might fall.

Changes in relative prices for nontradables not only introduce errors into PPP, they also bias the estimate of B toward zero. Suppose the variance in the ratio of price indices is σ^2 and part of this variance comes from purely monetary shocks, σ_M^2, for which PPP holds perfectly. In addition, there is another element, σ_R^2, that comes from real shocks. If these different sources for the variance in the ratio of price indices are uncorrelated, then σ^2 equals σ_M^2

plus σ_R^2, and a variation of equation 5′ describes the estimate of B.[10]

$$\text{plim } \hat{B} = \frac{1.0}{1.0 + \frac{\sigma_R^2}{\sigma_M^2}}. \qquad (5')$$

As inflationary shocks dominate real shocks, the R^2 and estimate of B approach unity. As monetary shocks disappear, the R^2 and estimate of B approach zero even though PPP holds perfectly for monetary shocks and real shocks have not increased. In other words, under these conditions, regression results do not depend on just the effectiveness of arbitrage and PPP, they also depend on the degree of monetary coordination in the two countries. On the one hand, the real shocks can be relatively large, but if the differences in the rates of inflation are also very large, then the R^2 and B are close to unity. On the other hand, even if the errors in PPP due to real shocks are very small, a sufficient degree of monetary coordination can make the ratio σ_R^2/σ_M^2 such that the R^2 and B are not statistically different from zero. As a result, PPP can appear to fail when the errors are relatively small, and to succeed even though the errors are relatively large.

Since the behavior of real exchange rates is very close to a random walk, from an arbitrage perspective, the errors in purchasing power parity appear to be dominated by changes in relative prices for nontradables. Some shock raises the relative price of haircuts or concrete in the United States, but not in Great Britain. If the price of traded goods remains constant, the U.S. price level rises relative to the price level in the U.K., but the exchange rate does not change. If the next shock is as likely to reinforce as reverse the first, then the errors in PPP behave like a random walk.

This interpretation of the error structure must be taken as tentative for several reasons. First, direct tests of the effectiveness of arbitrage for traded goods suggest that the law of one price does not hold as a reasonable approximation even for traded goods. See, for example, Peter Isard (1977) and J. David Richardson (1978). These results, however, are suspect because they are based on subcategories such as leather products in price indices in different countries that do not refer to identical, or even very similar, products. In addition, when the indices are wholesale, they tend to reflect posted prices rather than the actual prices at which trade takes place. When market prices for individual products such as Malaysian rubber are used, the results provide more support for arbitrage. See, for example, Liliane Crouhy-Veyrac, Michel Crouhy and Jacques Melitz (1980) and Aris Protopapadakis and Hans Stoll (1984).

Another problem with this interpretation of the errors is that almost everything is tradable. Concrete is traded internationally and tourists get haircuts. If almost everything is tradable, but the boundaries generated by impediments are very wide and not very rigid for many commodities, then the boundaries for real exchange rates could be quite wide and not very rigid. In that case, real exchange rates would behave like a random walk with wide and flexible boundaries, which is consistent with evidence discussed later. Errors of this type would not eliminate the kind of bias described in equation 5′; they would just make the problem more complex.

The efficient commodity market model discussed below provides still another possible interpretation of the observed errors in PPP. In that context, efficient international speculation in commodities in the absence of trade generates a random walk in real exchange rates.

Examples

Paul De Grauwe, Marc Janssens and Hilde Leliaert (1982), De Grauwe and Marc Rosiers (1984) and Davutyan and Pippenger (1985) show that the predictive errors for PPP tend to be relatively large when there are large differences in the rates of inflation. If, as seems likely, these errors are the result of changes in the relative prices for nontradables, then regression estimates of equation 4 will give the best results when, in terms of the predictive error, PPP works the worst. The reason is that, even though monetary instability tends to increase σ_R^2, it also makes σ_R^2/σ_M^2 very small.[11]

The dependence of regression results on the degree of monetary coordination not only leads to a misinterpretation of the evidence, it also invites specification search. Advocates of PPP can find episodes where regressions appear to support the theory, and those who oppose it can find situations in which the same regressions appear to reject PPP.

Estimates of equation 4 for the United States and Canada during the 1970s and early 1980s in the first half of Table 1 provide an example of the importance of relative monetary stability, and illustrate how specification search can influence regression results. An examination of the regression errors for France from the 1920s and 1970s illustrates why it is incorrect to conclude that PPP worked during the 1920s but failed during the 1970s.

The first half of Table 1 shows estimates of equation 4 using monthly data from January 1972 to December 1977, and January 1978 to February 1984. During the first period, price levels in the two countries moved together very closely. Wholesale prices in Canada rose only five percent more than in the United States. For that period, both β and the R^2 are effectively zero. During the later period, the Canadian price level rose 15 percent more than the price level in the United States. For that period, the R^2 is respectable and the estimate of β is not statistically different from unity. Using the usual criteria of R^2 and β, anyone wishing to reject PPP could use the earlier period and anyone wishing to support PPP could use the later period.

Although estimates for the earlier period appear to reject PPP and estimates for the later period support the theory, this interpretation of the evidence is misleading. Although the R^2 and β are closer to unity for the later period, this is primarily because there is more variability in both exchange rates and relative price levels during the later period. In other words, σ^2_M is larger in the second period. The fact that the standard errors are identical in the two periods means that the amount of variation in the exchange rate that cannot be explained by PPP is identical in the two cases, which indicates that σ^2_R is the same in both periods. PPP worked just as well in the earlier period as in the later. The difference between the two periods is primarily that σ^2_R/σ^2_M is smaller in the second period because σ^2_M is larger.

The bottom half of Table 1 shows Frenkel's results for France in the 1920s and 1970s using two stage least squares. Based on the estimates of R^2 and β, the results for the 1920s appear to support purchasing power parity while those for the 1970s reject the theory. The widespread belief that PPP worked in the 1920s but failed in the 1970s is based on similar results for a number of countries.[12]

However, if one interprets the standard errors of the regression as an index of the effects of real shocks, the evidence does not support the conclusion that purchasing power parity worked in the 1920s and failed in the 1970s. Indeed, those errors suggest just the opposite. The standard error for France in the 1920s is 0.054, but it falls to 0.029 in the 1970s.[13] The large R^2 and β during the 1920s is simply a reflection of the fact that a very large proportion of the variability in the exchange rate can

TABLE 1

Monthly Estimates of Equation 4 Using Wholesale Indices

Country	Period	α	β	R^2/ Standard Error	Durbin Watson	p
Canada	Jan. 1972 Dec. 1977	−0.02 (0.01)	0.25 (0.16)	0.03 0.010	1.07	0.82
	Jan. 1978 Feb. 1984	−0.15 (0.00)	0.82 (0.12)	0.37 0.010	1.62	0.73
France	Feb. 1921 May 1925	1.183 (0.157)	1.091 (0.109)	n.a. 0.054	1.70	0.58
	June 1973 July 1979	−1.52 (0.03)	−0.18 (0.37)	n.a. 0.029	2.26	0.86

Sources: Canada, Davutyan and Pippenger (1985) Table 5. France, Frenkel (1981) Tables 1 and 2.

Note: Canadian estimates use SAS autoreg corrected for one period serial correlation. French estimates use two stage least squares. Standard errors in parentheses. No base period.

be explained by monetary shocks. In other words, the R^2 and estimates of B are close to one for the 1920s because σ^2_M is large, not because σ^2_R is small. The fact that the absolute size of the standard error is smaller during the 1970s means that amount of the variability in the exchange rate that cannot be explained by PPP is smaller during the 1970s. Since there was much more monetary coordination during the 1970s, this result indicates that the R^2 and estimate of B for that period are low because σ^2_M is low, not because the errors due to real shocks, σ^2_R, are large. If purchasing power parity was a success in the 1920s, it did not collapse in the 1970s.

The widespread belief that PPP collapsed in the last decade is based on a serious misinterpretation of the evidence that ignores the econometric traps involved in estimating purchasing power parity.

II. Efficient Commodity Markets

A number of studies referred to earlier indicate that real exchange rates behave like a random walk. To explain these random walks, Roll (1979) developed a theory based on speculation in efficient international commodity markets. Roll's theory expands the traditional view of purchasing power parity in two ways. It uses speculation rather than arbitrage and stresses intertemporal transactions. Since most international trade involves time and some element of speculation, this approach is a significant advance in terms of realism over the traditional arbitrage approach to purchasing power parity.[14]

Under the arbitrage approach, a trader buys a good this month at home and sells it this month in another country. Since the presence of risk is never mentioned in such an analysis, there is an implicit assumption that all prices are known with certainty. In Roll's model, there is no physical transfer of commodities. Instead, speculators in one country speculate on changes in exchange rates and changes in commodity prices in the other country.

Intertemporal Speculation without Trade

As an example of Roll's approach, consider a speculator who buys a commodity in a foreign country in month $t-1$ for sale in that country the next month t. If $p(F,t-1)$ is the cost of the good in the foreign countrty in $t-1$ and $S(t-1)$ is the domestic price of foreign exchange that month, then the domestic price of the foreign good in $t-1$ is $S(t-1)p(F,t-1)$. The return from the sale of the commodity is $S(t)p(F,t)$, where $S(t)$ is the exchange rate in t and $p(F,t)$ is the price the speculator receives for the good in the foreign country. Since the natural log of the return over the cost is approximately the percentage difference between the two, the gross rate of return from this transaction is

$$\ln\left[\frac{S(t)p(F,t)}{S(t-1)p(F,t-1)}\right] \tag{6}$$

Whether or not the speculator engages in such a transaction depends on the net return, which is the difference between the return from foreign speculation and a similar domestic transaction. Let $p(H,t-1)$ be the domestic price of the good in $t-1$ and $p(H,t)$ the price in t. Under these conditions, equation 7 describes the net return r_s from intertemporal international speculation.

$$r_s = \ln\left[\frac{S(t)p(F,t)}{S(t-1)p(F,t-1)}\right] - \ln\left[\frac{p(H,t)}{p(H,t-1)}\right]$$

$$= \ln\left[\frac{S(t)p(F,t)}{p(H,t)}\right] - \ln\left[\frac{S(t-1)p(F,t-1)}{p(H,t-1)}\right] \tag{7}$$

If international commodity speculation is efficient, then, based on the information available in period $t-1$, the expected net return should be zero.[15]

$$E\left[\frac{r_s}{Z(t-1)}\right] = 0 \tag{8}$$

where E is the expectations operator and $Z(t-1)$ is the information available in $t-1$.

Equations 7 and 8 imply equation 9, where u_t is an uncorrelated random variable with zero mean.

$$\ln\left[\frac{S(t)}{p(H,t)/p(F,t)}\right] = \ln\left[\frac{S(t-1)}{p(H,t-1)/p(F,t-1)}\right] + u_t \quad (9)$$

In the terminology of efficient markets, equation 9 means all the information relevant for determining the real exchange rate next period is already fully reflected in the current real exchange rate.

Consider the following implication of equation 9. Suppose the price of wheat in Canada this month times the current price of the Canadian dollar does not equal the current price of wheat in the United States. According to equation 9, that difference is as likely to increase as to decrease in the next month. Given efficient international speculation without trade, market forces do not work toward restoring the law of one price. Since this is true for every commodity, it holds for arbitrary bundles of commodities.[16] As a result, there are no market forces at work restoring long-run equality between actual exchange rates and the rates implied by purchasing power parity. Real exchange rates perform a random walk because, no matter what the gap between the actual and parity rate is in one period, the gap is as likely to grow as to shrink in the next period.[17]

Intertemporal Speculation with Trade

Without trade, speculators can only guess whether an expected change in a price at home $p(H,t)/p(H,t-1)$ will equal the domestic value of the change in the price of the same good in a foreign country — $S(t)p(F,t)/S(t-1)p(F,t-1)$. In this type of speculation, the level of the exchange rate is irrelevant. Halving or doubling $S(t)$ and $S(t-1)$ does not alter $S(t)p(F,t)/S(t-1)p(F,t-1)$. When speculation involves trade, the returns depend on the level of exchange rates. If the price of pound sterling rises with no change in the product price in the U.S. or U.K., it becomes relatively more profitable to buy in the U.S. "this" period for sale in the U.K. in the "next" period.

Consider an exporter who buys a good at home this period, ships it, and sells it abroad next period. Using the earlier notation, the gross return from this transaction is $\ln - \{[S(t)p(F,t)]/p(H,t-1)\}$. The net return, which is the incentive for such activity, depends on the return from similar domestic transactions. If the speculator buys the good at home one month and sells it at home next month, the return is $\ln[p(H,t)/p(H,t-1)]$. The net return from speculation with trade, r_T, is the difference between these two gross returns.

$$r_t = \ln\left[\frac{S(t)p(F,t)}{p(H,t-1)}\right] - \ln\left[\frac{p(H,t)}{p(H,t-1)}\right] = \ln\left[\frac{S(t)p(F,t)}{p(H,t)}\right] \quad (10)$$

The net return is the percentage error in the law of one price. If $S(t)p(F,t)/p(H,t)$ is unity, the law of one price holds and the return from additional intertemporal international trade is zero.

The arbitrage version of the law of one price is based on international trade at known prices within a given time period where, ignoring transaction costs, arbitrage eliminates any net return. An efficient market version involves intertemporal trade with expected prices where the *expected* net return is zero given all currently available information.

$$E\left[\frac{r_t}{Z(t-1)}\right] = 0 \quad (11)$$

Equations 10 and 11 imply the conventional law of one price with an error term that reflects the uncertainty about future prices.

$$\ln[p(H,t)] = \ln[S(t)p(F,t)] + e_t \quad (12)$$

where e_t is an uncorrelated random variable with zero mean.

Although the argument has been developed in terms of a single commodity, exactly the same reasoning applies to any arbitrary bundle of commodities. In an efficient market without transaction costs, the expected return from buying any bundle at home this period and selling it abroad next period cannot exceed the expected return from buying at home and selling at home.[18] Efficient commodity markets with trade imply the absolute version of purchasing power parity with an error term.

$$\ln[S(t)] = \ln\left[\frac{P(H,t)}{P(F,t)}\right] + \gamma_t \qquad (13)$$

where P(H,t) and P(F,t) are the home and foreign price of an identical bundle of goods, and γ_t is an uncorrelated random variable with zero mean.

Since the discussion has ignored the transaction costs associated with trade, the source of the error term γ in equation 13 is the same as the source for the error u in equation 9. They both come from imperfect information. In equation 9, imperfect information generates a random walk in real exchange rates because expected returns depend on expected changes in prices and exchange rates. With trade, expected returns depend on the level of prices and exchange rates, and so deviations of the actual rate from the rate implied by parity are uncorrelated. If they were correlated, expected net returns from trade would not be zero and trade in international commodity markets would not be efficient.

Recognizing the information and transaction costs associated with trade provides a link between the arbitrage version of PPP and the efficient market interpretation with trade. In the conventional arbitrage version of PPP, these costs introduce errors that are larger in the short-run than in the long-run. An efficient market interpretation of PPP with trade essentially adds an error term like γ to the arbitrage version.[19] Equation 14 describes the relative version of an efficient market interpretation of PPP with trade and transactions costs.

$$\ln\left[\frac{S(t+1)}{S(t)}\right] = \ln\left(\frac{P^H}{P^F}\right) + v_t + g_t \qquad (14)$$

where both v and g are error terms with negative serial correlation.[20] The term g has negative serial correlation because it represents the temporary deviations of the actual rate from parity generated by imperfect information.[21] The term v has negative serial correlation because it is due to transaction costs that allow only limited deviations between actual exchange rates and those implied by PPP. This negative serial correlation is reinforced when these costs are effectively zero in the long-run. In the case where transaction costs are zero in the long-run, deviations of the actual rate from parity are not only bounded, they also tend to disappear in the long-run.

To see the relation between an efficient market interpretation of PPP with trade and Roll's interpretation without trade, consider the following example. Suppose Roll's speculation in wheat between the U.S. and Canada generates a random walk for the real wheat exchange rate between the two countries. If $/$C is the U.S. price of the Canadian dollar, W is wheat in the U.S. and WC is wheat in Canada, then speculation without trade causes ($/$C)/[($/W)/($C/WC)] to perform a random walk. As a result, in the absence of any other influences, the real wheat exchange rate will drift off toward plus or minus infinity in time.

But long before that happens, trade takes place. Suppose this morning the price of wheat in Winnipeg converted to U.S. dollars is less than the price expected next week in Chicago. If the price difference exceeds the transportation costs, there is an incentive to buy wheat in Winnipeg, load it on a train and ship it to Chicago for sale next week. From that point on, real exchange rates no longer behave like a random walk. Any further downward movement in the real wheat exchange rate is resisted by wheat moving from Canada to the United States. The shipments of wheat put upward pressure on Canadian wheat prices, downward pressure on wheat prices in United States, and increase the demand for Canadian dollars. Since the same argument holds for every commodity, efficient international commodity markets with trade imply that changes in real exchange rates should show evidence of negative serial correlation and should not be random walks.

Evidence

Roll (1979), Darby (1980) and Mussachia (1984) analyze monthly real exchange rates for many countries during the 1970s while Pippenger (1982) and Adler and Lehman (1983) use annual data over long periods.[22] The tests include regressions, autocorrelations and spectral analysis, and in each case real exchange rates appear to behave as though they were random walks. Although a random walk is consistent with efficient international commodity markets without trade, trade should impose boundaries on real exchange rates. The evidence presented next suggests that such boundaries exist.

These tests combine autocorrelation and spectral analysis with a technique used by Roll (1979). Roll tests his model by calculating the means of regression coefficients for many pairs of countries. The advantage of this approach is that it can reveal patterns that are so weak that they are not observable for any given pair. A regression coefficient might be statistically insignificant for 20 different pairs of countries, but if it is positive for all of them then it is almost certainly positive. Unfortunately, Roll's regressions were not designed to test for the presence of the kind of barriers that exist with trade. Since autocorrelation and spectral analysis are natural ways to test for such barriers, Tables 2 and 3 apply Roll's technique to autocorrelation and spectral estimates respectively.

Autocorrelation

As pointed out earlier, one implication of both the arbitrage view of PPP and efficient markets with trade is that real exchange rates are bounded by "reflecting barriers" and changes in real exchange rates have negative serial correlation. Although Roll (1979), Darby (1980) and Mussachia (1984) all find no evidence of negative serial correlation for monthly data in the 1970s, combining the results from several countries suggests that reflecting barriers do exist.

The technique is simple: obtain the autocorrelation estimates for 13 lags for 24 real exchange rates using wholesale indices and end-of-month exchange rates from the International Financial Statistics tape for 1976.7 to 1983.12.[23] Compute the average autocorrelation estimate at each lag using the 24 pairs of countries and, in addition, take the mean of these averages. The reason for computing the mean of the averages at the various lags is that reflecting barriers are probably not identical for the various countries; their differences would lead to different lag structures. If the series are true random walks, there should be no evidence of either negative or positive correlation. If there are reflecting barriers, then there should be some evidence of negative serial correlation.

Table 2 shows the average autocorrelation estimates. For these countries the real exchange rate is not a random walk. Five of the lags are significant at the one percent level,[24] but there is no clear pattern of negative serial correlation because two of these estimates are positive. The mean of the 13 autocorrelation estimates, however, is negative and significant at the ten percent level. The average autocorrelations are not consistent with a random walk, but

TABLE 2

Average Autocorrelation Estimates

Lag	Estimate	t-Statistic
1	−0.065	−3.29***
2	−0.017	−0.92
3	0.028	1.24
4	−0.021	−1.29
5	0.017	0.82
6	−0.063	3.80***
7	0.058	2.83***
8	0.019	1.56*
9	−0.018	−1.52*
10	0.052	2.59***
11	0.028	1.12
12	−0.012	−0.59
13	−0.100	4.36***
Mean	−0.007	−1.41*

*Significant at ten percent level, single-tailed.
**Significant at five percent level, single-tailed.
***Significant at one percent level, single-tailed.

they provide only weak support for the existence of reflecting barriers. Spectral analysis yields stronger results.

Spectral Analysis

One natural interpretation of the concept of the short-run is that it refers to short cycles. A similar relationship between cycle length and the length of the run holds for the intermediate and long-run. For income and employment, the short-run might refer to cycles of up to two years and the long-run to cycles longer than the business cycle. In the context of highly organized markets such as the foreign exchange market, the short-run is more likely to refer to a period of a few days or a few months at most. Cycles as long as a couple of years almost certainly would correspond to the long-run, and the concept of the intermediate-run would apply to cycles from a few months up to perhaps a year. Given this association between the length of the run and the length of cycles, spectral analysis allows us to see how much of the variance in a variable, such as the change in the real exchange rates, comes from the short-run, intermediate-run, and long-run.[25] If changes in real exchange rates are uncorrelated, as implied by a random walk, then the short-run, intermediate-run and long-run all contribute equally to the variance. In Figure 1, which shows average estimates for spectral density, that implication of a random walk is shown by the solid horizontal line at $1/\pi$ or 0.318.[26]

If there are barriers that restrict long-run movements in real exchange rates, they would reduce the long-run component of the variance for changes in real exchange rates. As an example, suppose the traditional dynamic view of PPP is correct. In the short-run, a variety of shocks drive actual rates away from PPP, but in the long-run, market forces bring actual and parity rates back into equality. In that case, there are short-run changes in the real exchange rate, but no long-run changes because in the long-run the real exchange rate is constant at 1.0. In other words, none of the variance in changes in real exchange rates comes from the long-run. A dynamic interpretation of PPP implies that the spectral density estimates in the figure are above $1/\pi$ for short cycles and below $1/\pi$ at long cycles.

Pippenger (1982) shows that spectral density estimates for annual changes in real exchange rates are essentially constant regardless of the length of

TABLE 3

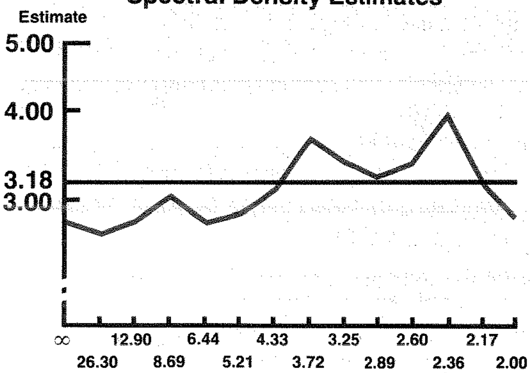
Average Spectral Density Estimates for 24 Countries

Cycle Length in Months	Estimate	Estimate $-1/\pi$	t-Statistic
2.00	0.286	−0.032	−1.24
2.17	0.319	0.001	0.04
2.36	0.394	0.076	4.21***
2.60	0.337	0.019	1.03
2.89	0.326	0.008	0.81
3.25	0.339	0.021	1.16
3.72	0.364	0.046	2.01**
4.33	0.310	−0.008	−0.40
5.21	0.287	−0.031	−1.61*
6.49	0.280	−0.038	−2.17
8.69	0.303	−0.015	−0.76
12.99	0.279	−0.039	−2.77***
26.31	0.269	−0.049	−2.75***
∞	0.280	−0.038	−1.57*

*Significant at ten percent level, single-tailed.
**Significant at five percent level, single-tailed.
***Significant at one percent level, single-tailed.

Chart 1

Spectral Density Estimates



the cycle. Since Mussachia (1984) obtains similar results for monthly data during the 1970s, an approach like the one used for autocorrelations is applied to the spectral estimates. That is, the estimate for the two-month cycle is the mean of the spectral density estimates for the twenty-four real exchange rates in that cycle.

The broken line in the figure shows the average special density estimates for the countries used earlier. These estimates and their deviation from $1/\pi$ are given in Table 3. If there were no reflecting barriers and real exchange rates perform a random walk, then the spectral density estimates should not be significantly different from $1/\pi$. If there were reflecting barriers, the estimates should be above $1/\pi$ at the shorter cycles and below $1/\pi$ for long cycles.

The pattern for the spectral estimates in the figure allows one to reject the idea that the real exchange rate performs a random walk. Instead, it supports a dynamic interpretation of PPP. There is a clear tendency for the estimates to lie below $1/\pi$ for the longest cycles. Table 3 shows that, although the estimate at the shortest cycle is below $1/\pi$ (although not significantly so at even the ten percent level), the next six estimates all are above $1/\pi$. At the seven longest cycles, all estimates are below $1/\pi$ with two significant at the ten percent level, one at the five percent level, and two estimates significantly below $1/\pi$ at the one percent level. For these countries as a group and for this time period, real exchange rates do not behave as a random walk. The spectral density estimates strongly support the existence of elastic reflecting barriers that restrain long-run movements in real exchange rates. These barriers may be quite wide and very elastic, but they do appear to exist. The pattern shown in the figure and Table 3 does not refute Roll's basic idea of efficient international commodity markets, it simply indicates that, beyond some point, trade limits the movement in real exchange rates.

III. Accept or Reject?

In most people's mind, the decision to accept or reject a theory involves two closely related, but different, issues. The first is whether the theory is the best available and the second is whether it is accurate. There is a good deal of support for the arbitrage and efficient market interpretations of purchasing power parity. After allowing for the economic and econometric effects of information and transaction costs, the evidence supports the basic implication of purchasing power parity — that substantial and prolonged changes in relative price levels are associated with roughly proportional changes in exchange rates.

Even more important, no theory can explain either the level or change in exchange rates over time and across space as well as purchasing power parity. The only serious contender is the asset approach to exchange rates and, at this time, that approach has failed.[27] There is no choice. In the strict sense, we must accept purchasing power parity because it yields the best predictions.

Most of the objections to PPP are related to the accuracy of the theory. Even if it is the best available, many people are unwilling to accept a theory unless it achieves some minimal level of accuracy. Performing only slightly better than demon chance is not good enough. The problem with this aspect of acceptance is that it is almost entirely subjective. Is the glass half full or half empty? Is an error of ten percent large or small?

Table 4 illustrates the problem. It shows the "real" German mark price of the United States dollar, French franc, British pound and Canadian dollar from 1975 to 1985 using identical bundles.[28]

At one extreme, from 1975 to 1985, the actual mark price of the French franc rose only four percent more than implied by PPP. At the other, the mark price of United States' dollars rose 56 percent more than implied by PPP.[29] For these countries on average, the actual rate rose 28 percent more than predicted by PPP. Relative PPP as an explanation of exchange rates certainly is not impressive for this time period and these countries.

The errors for absolute PPP in Table 4 range from a minus 22 percent for Great Britain in January 1977 to 59 percent for the U.S. in January 1985. That is, in January 1985, the actual mark price of the dollar was 59 percent higher than predicted by purchasing power parity based on the bundle of goods used by the German Federal Statistical Office. Although individual errors are quite large, the average error for each of the four countries over the 10 years is much smaller. They range from −2 percent for France to 10 percent for Canada. The average error for all the countries combined over the 10 years is only 2 percent. Deviations from absolute PPP can be very large, but, on average, the theory is amazingly accurate.[30]

Whether or not the occasionally large errors justify rejecting purchasing power parity, or the small average error warrants acceptance, is up to each individual to decide. The way one uses PPP will play an important role in that decision. For policymakers, the potential for large errors means potentially serious mistakes when policy is based primarily on PPP. For scientific purposes, the occasionally large errors are challenges for future research rather than potential disasters.

TABLE 4

Real German Exchange Rates Using Identical Bundles

Period	DM/US	DM/FF	DM/UK	DM/CAN	AVERAGE
			Relative		
Jan 1975 to Jan 1985	0.56	0.04	0.19	0.35	0.28
			Absolute		
Jan 1975	0.92	0.93	0.70	1.00	0.89
Jan 1976	1.02	1.05	0.88	1.15	1.03
Jan 1977	0.97	0.92	0.78	1.08	0.94
Jan 1978	0.88	0.89	0.82	0.91	0.88
Jan 1979	0.81	0.94	0.79	0.79	0.83
Jan 1980	0.81	0.99	0.94	0.79	0.88
Jan 1981	1.02	1.06	1.28	0.99	1.09
Jan 1982	1.11	1.03	1.15	1.13	1.11
Jan 1983	1.19	0.98	1.00	1.21	1.09
Jan 1984	1.40	0.96	1.08	1.41	1.21
Jan 1985	1.59	1.01	1.01	1.52	1.28
Average	1.06	0.98	0.95	1.10	1.02

Country Pairs: DM/US, DM/FF, DM/UK, DM/CAN, AVERAGE

Data: Absolute PPP, German Federal Statyistical Office. Actual exchange rates, end of month from IFS tape.

IV. Summary

The evidence supporting the arbitrage version of purchasing power parity is stronger than generally realized. Rejection of the theory often rests on a misinterpretation of the evidence. Regressions can yield low coefficients and R^2s even though the predictive errors are relatively small. In addition, in the absence of rapid inflation, the average predictive error for absolute PPP appears to be quite small.

Those who insist on a high degree of accuracy might reject the theory because individual predictive errors are sometimes very large. In terms of relative predictive power, however, one must choose between the arbitrage and efficient commodity markets versions of PPP. Over time and space, no other theory about exchange rates is as consistent with the evidence. The only other serious contender, the asset approach, has failed so far.

Accepting either an arbitrage or efficient market version of purchasing power parity implies nothing about the direction of causation. In addition, acceptance is not an assertion that other influences are not important. The exceptionally strong dollar in the 1980s suggests that other factors are indeed important. One of the advantages of the arbitrage approach is that it provides a way of thinking about how real shocks, such as changes in capital flows or technology, drive actual rates away from the rates implied by PPP.

Whether an arbitrage or efficient markets approach to purchasing power parity is the right choice is less clear. Standard interpretations of the arbitrage version imply that the errors in PPP should be primarily short-run in nature. The evidence, however, indicates that the predictive errors are almost as large in the long-run as in the short-run. The fact that the predictive error behaves in the fashion of a random walk — with wide elastic reflecting barriers — tends to favor the efficient market interpretation. But Roll developed the efficient commodity market model in order to explain random walks in exchange rates, so random behavior does not constitute a true test of the theory. Until some new implications of the efficient commodity market model are derived and tested, the evidence appears to support both the arbitrage and efficient market approaches.

The choice between the two models is important. The arbitrage version is consistent with the attempt to build asset models to explain the behavior of exchange rates. Since the conventional arbitrage version of PPP is essentially a theory about the long-run behavior of exchange rates, and the asset approach concentrates on the short-run, there is no inherent conflict between the two. The efficient commodity markets model, however, implies that commodity markets play a key role in the short-run determination of exchange rates. This approach is inconsistent with most existing asset models of the exchange rate because they exclude any role for efficient commodity markets in the short-run determination of exchange rates.

FOOTNOTES

1. For a more thorough review of the theory underlying PPP, see Lawrence Officer (1976) and Rudiger Dornbusch (1985).

2. Although the relative version of PPP in general requires weaker assumptions than the absolute version, it does involve at least one important assumption that the absolute form does not require. Relative PPP implicitly assumes that the base period describes an equilibrium or normal situation.

3. Although equation 4 is the basic test equation, several studies include lags, e.g., John Hodgson and Patricia Phelps (1975), or other explanatory variables, e.g., Richard Dino (1977).

4. In many cases, the left hand side of the equation is simply $\ln[S(t)]$ and α is an estimate of the log of the base period exchange rate. In that case, a nonzero estimate for α does not reject PPP.

5. See in particular Jacob Frenkel (1981).

6. For a description of this data, see W. Kohlhammer (1970).

7. See Nurhan Davutyan and John Pippenger (1984).

8. This dynamic view of PPP implicitly assumes that transaction costs decline with the length of the run. The discussion of the nature of costs by Armen Alchian (1959) suggests a number of reasons for this decline.

9. Since the difference between tradables and nontradables is one of degree, not kind, this argument overstates the case. The basic point, however, is valid. The structure of the error terms should be substantially different depending on whether it is related to tradables or nontradables.

10. Since real and monetary shocks can be, and apparently are, correlated, the problem is more complex than in this simple example.

11. The effects of transaction costs on tradables, which is what Aizenman (1984a and b), De Grauwe, Janssens and Leliaert (1982) and De Grauwe and Rosiers (1984) stress, and different weights, reinforce the bias from changes in relative prices for nontradables.

12. Although similar results hold for a number of countries, they do not hold for all. Price levels in Canada and the United States moved together very closely in both the 1920s and early 1970s, and estimates of R^2 and β reject PPP in both periods. In addition, estimates for inflationary countries in the 1970s such as Israel, Argentina and Brazil yield results that are similar to the results for France in the 1920s. See Davutyan and Pippenger (1985).

13. This result is not particular to France. The average standard error for the regressions that Frenkel reports for the 1920s is 0.102, but it falls to 0.029 for the 1970s.

14. See Alan Shapiro (1983) for a discussion of efficient commodity markets and purchasing power parity.

15. If there is no risk premium and futures prices equal expected prices, then a similar argument holds for a form of international arbitrage without trade.

16. Instead of the arbitrage approach to PPP used here, Roll (1979, p. 142) uses a welfare approach. "When relative prices are not assumed to be constant, the continuously compounded rate of inflation must be measured by another log price change, that of the price index relevant to the speculator's purchasing power."

17. Technically, the error is a martingale. But because it is more widely recognized, the term random walk is used throughout instead of the more accurate martingale.

18. With transaction costs, the expected net return would have to at least cover those costs before goods would be shipped.

19. When international trade involves buying either at home or abroad in t−1 for sale at home in t, there is no international uncertainty and γ disappears.

20. Since the errors are correlated with both sides of equation 18, from an econometric perspective it would be more accurate to write this equation as $[\ln\{[S(t+1)/S(t)]/(p^H/p^F)\} = v_t + g_t$.

21. More formally, g has first order negative serial correlation because it is the first difference of an uncorrelated random variable γ.

22. Roll's data cover more than the 1970s. They run from 1957 to 1976.

23. The countries are Argentina, Australia, Brazil, Canada, Germany, Italy, Israel, Japan, U.K., and U.S. To avoid any undue weight on inflationary episodes, for Argentina, Brazil, and Israel, only real rates with the U.S. are used. The time period, number of lags and countries were selected before the tests were conducted.

24. The t−tests are based on the observed standard deviation, not the theoretical standard deviation which would assume independence.

25. See Jenkins and Watts (1968) for a detailed discussion of spectral analysis.

26. Spectral density is the normalized spectrum. It has the same relation to the spectrum that autocorrelation has to autocovariance. When frequency is measured in radians, the observed frequencies run from 0 to π. Since the estimates of the spectral density must sum to unity, the estimates must equal $1/\pi$ to be constant across frequency.

27. See, for example, Graham Hacche and John Townend (1983) and Waseem Khan and Thomas Willett (1984).

28. The series start in 1975 because there is a break in the German data in 1974.

29. For the U.S. dollar, relative PPP even gets the direction wrong. It predicts a 21 percent fall in the mark value of the dollar when the value of the dollar actually rises 35 percent.

30. With rapid inflation, the average predictive errors are much larger. See Davutyan and Pippenger (1985), Table 2.

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