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Has There Been a Change in the Natural Rate of Unemployment?

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Many economists argue that inflation begins to pick up when the unemployment rate falls below a critical level known as the natural rate. Those who worry that there is a risk of faster inflation argue that unemployment has fallen below this level. Others suggest that the natural rate has declined and hence that the inflation risk is less. The results of this paper do not support the argument that the natural rate has declined. The slow response of inflation to changes in unemployment can explain why the low rate of unemployment since 1987 has not led to faster inflation. However, this slow response of inflation to employment suggests that if inflation is allowed to rise, it would take a long period of slow growth to bring it down again.

In the last two years, the rate of unemployment in the U.S. has declined significantly. During 1989, unemployment averaged 5¼ percent of the civilian labor force, more than one percentage point below its level in the first half of 1987 and lower than at any time since 1974.

As the unemployment rate fell, concern increased that the associated high level of economic activity would lead to a pick-up in inflation. Many economists argue that the unemployment rate is an indicator of the strength of aggregate demand, and that inflation tends to increase when unemployment is low. These economists hypothesize that there is a critical level of the unemployment rate below which wages and prices tend to accelerate and above which they tend to decelerate. This critical level is called the "natural rate of unemployment." Those who worry that there is a risk that inflation will pick up argue that unemployment has fallen below this natural rate.

Until recently, most estimates of the natural rate of unemployment placed it around six percent.[1] With a natural rate in this range, inflation pressures should have begun developing in the second half of 1987, when the unemployment rate fell to 5.9 percent from 6.4 percent in the first half. Concerned that the risk of faster inflation was increasing, the Federal Reserve began to tighten policy in mid-1987. In 1988, after a brief pause following the stock market crash, the Federal Reserve continued the process of policy tightening that it had begun in 1987.

Despite the low level of unemployment, however, the increase in inflation since 1987 has been relatively modest. This raises the possibility that the natural rate of unemployment may be less than six percent,[2] or even that there is no critical level of unemployment below which inflation necessarily worsens. This article seeks to throw light on this issue by estimating the natural rate and examining whether it has changed in recent years.

The first section reviews the theory of the link between inflation and unemployment. Particular attention is paid to the role of "supply-side" or "relative-price" shocks in the inflation process. Section II estimates four alternative empirical equations linking inflation to the unemployment rate and to relative-price shocks. The equations are estimated over a series of different sample periods to test whether the natural rate of unemployment has changed through time. Section III summarizes the empirical results and discusses their implications for policy. The results of this paper do not support the argument that the natural rate has declined. The slow response of inflation to changes in unemployment can explain why the low rate of unemployment since 1987 has not led to faster inflation. However, this slow response of inflation to unemployment suggests that if inflation *is* allowed to rise, it would take a long period of slow growth to bring it down again.

I. Inflation and Unemployment: Theory

The notion that low levels of unemployment are associated with high inflation is generally known as the "Phillips curve."[3] Although there is widespread agreement that unemployment and inflation are related, at least in the short run, the direction of causation is disputed. Most economists argue that if unemployment falls below its natural level, this causes inflation to increase. However, some theorists maintain that causation runs from inflation to unemployment.[4] The empirical models in this paper are based on the first hypothesis.

The theory that low unemployment causes faster inflation views the unemployment rate as an indicator of the demand for labor relative to the supply. When the demand for labor is strong, wages tend to be bid up. Because the prices of goods and services are set as a mark-up over their costs of production, most of which comprise wage costs, wage inflation leads to overall price inflation.[5]

In the labor market, there always is a number of persons who are searching for employment even when there are jobs vacant. This is because individual jobs and workers are unique, and matching unemployed persons to vacant jobs requires a process of search on both sides.[6] In a changing economy, persons continually are entering and leaving the unemployment pool and firms are creating and filling job vacancies. This observation means that even when the supply of and demand for labor are in overall equilibrium at going wages, there will be a certain remaining level of unemployment.

The "natural rate" of unemployment reflects this "equilibrium" quantity of joblessness. When unemployment is below (or above) its natural rate, wages tend to rise more (or less) rapidly. If the mark-up of prices over production costs remains constant, (or, *a fortiori,* if it increases when aggregate demand is strong[7]), the rate of overall price inflation also will be inversely related to the unemployment rate.

This inverse relation between inflation and unemployment may be represented algebraically as:

$$\pi_t = f(U_t - UNAT) \tag{1}$$

where $f'() < 0$. In this equation, π_t represents the inflation rate, and U_t and $UNAT$ represent the actual and the natural rates of unemployment, respectively.

Inflation Expectations

In negotiating wages, workers and employers pay attention not only to the state of labor supply and demand in their own markets, but also to the rate of overall inflation they expect in the future. If prices are expected to rise, workers demand wage increases to maintain their real incomes. Employers are willing to meet these demands, because they expect to be able to pass their higher costs on to their customers, and fear that, if they do not, they will lose their best workers. Hence, an expectation that prices and wages will rise tends to be self-fulfilling, even when unemployment is at its natural rate.

This argument leads to the "expectations-augmented" Phillips curve, which may be written as:

$$\pi_t = E_{t-1}\pi_t + f(U_t - UNAT) \tag{2}$$

where $E_{t-1}\pi_t$ represents the rate of inflation in period t that was expected in period t-1. When unemployment is below (above) the natural rate, actual inflation will exceed (fall short of) the rate that was expected.

Under simple assumptions about how price expectations are formed, this model implies that inflation will increase continually if unemployment remains below its natural level. This will be the case, for example, under the common assumption that people raise their expectations of future inflation when current inflation is higher than they had expected. This assumption about expectations may be written as

$$E_{t-1}\pi_t = E_{t-2}\pi_{t-1} + g(\pi_{t-1} - E_{t-2}\pi_{t-1}) \tag{3}$$

where $g'() > 0$. This assumption implies that the rate of inflation expected for the current period is a function only of past inflation. If this is a linear function, it may be written:

$$E_{t-1}\pi_t = \sum_{s=1}^{S} w_s \cdot \pi_{t-s} \tag{4}$$

The rate of inflation expected in the current period is a weighted average of the rates of inflation experienced in the past.[8] This is known as the "adaptive expectations hypothesis." Substituting equation (4) into equation (2) and assuming that the function $f()$ in equation (2) is linear yields:

$$\pi_t = \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i} - UNAT) \qquad (5)$$

It seems plausible that if inflation has been constant in the past and unemployment is at its natural level, then it will remain constant in the future. This assumption implies that the weights (w_s) in equation (5) sum to one. In this case, a level of unemployment below the natural rate not only adds to current inflation, but also causes inflation to continue rising in the future.

Even if unemployment remains at its natural level, this does not guarantee a *low* inflation rate. Any rate of inflation, if it is anticipated, is compatible with unemployment at its natural level. Thus, although a decline in the unemployment rate below its natural rate will lead to faster-than-expected inflation, there is no *long-run* relationship between the level of unemployment and the rate of inflation.

The assumption that agents form inflation expectations adaptively implies that the only information they use in forming expectations is the past behavior of inflation. However, the theory of rational expectations suggests that, since agents know inflation is related to the unemployment rate, they should base their inflation expectations on their forecasts of unemployment rather than on past inflation. Despite this reservation, models that assume adaptive expectations appear to fit the data reasonably well.[9]

An alternative explanation why past inflation appears to affect current inflation is that wage contracts are made for several years and are not all negotiated simultaneously.[10] When a firm and its employees are negotiating a new contract, they recognize that contracts made in the past in related industries will be in effect for at least part of their contract period. If those other contracts contained wage increases, new contracts likely will call for similar increases even if economic conditions have changed. As a result, inflation will tend to persist once it has begun.

Recently, some economists have suggested that changes in the actual level of unemployment may affect the equilibrium rate at which inflation remains constant. According to this "hysteresis" theory,[11] although an increase in unemployment may lower inflation initially, it later leads to a corresponding increase in the natural rate, as employers and workers become accustomed to higher rates of unemployment. As a result, its effect on inflation is only temporary. This hysteresis hypothesis may be tested by examining whether the coefficients on the unemployment rate sum to zero in equation (5).

Supply Shocks

The average price level also may be influenced by shocks that affect the prices of particular commodities. For example, a rise in the prices of imported raw materials adds directly to costs of production and hence to the average price level. As prices adjust to the higher level, there will be an increase in the measured inflation rate. This suggests that equation (5) should be extended to include the effects of such "relative-price" or "supply" shocks.

On several occasions in the last two decades, changes in the overall inflation rate have been attributed to changes either in the real exchange value of the dollar or in the real price of oil. The real exchange rate measures the price of U.S.-produced goods relative to foreign-produced goods. A decline in the real exchange rate could add to U.S. inflation, by raising the cost of imports and reducing the pressures on domestic producers to keep their prices low.[12] The real price of oil represents the price of oil relative to the general level of prices in the U.S. A rise in the price of oil adds directly to costs of production in the U.S. Thus, both the real exchange rate and the real price of oil are prime candidates for inclusion in equation (5).

This discussion suggests an estimating equation of the form:

$$\pi_t = \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i} - UNAT) + \sum_{g=0}^{G} b_{Og} \cdot SHKOIL_{t-g} + \sum_{h=0}^{H} b_{Xh} \cdot SHKEX_{t-h} \qquad (6)$$

where $SHKOIL_{t-g}$ represents the *change* in the real price of oil and $SHKEX_{t-h}$ the *change* in the real exchange rate.[13] The distributed lags on these variables capture the idea that the effects of supply shocks do not occur instantaneously. Several previous researchers[14] have estimated equations similar to equation (6) and have found that relative-price shocks have a significant impact on the measured inflation rate.

Supply Shocks and Expectations

Equation (6) implicitly assumes that the impact of past inflation on current inflation is the same regardless whether that past inflation was the result of excess demand or of relative-price shocks. In particular, equation (6) implies that a single relative-price shock, unless offset by a change in the unemployment rate, will lead to a *permanent* change in the rate of inflation. This is because such a shock not only has a direct impact on prices but also has an indirect effect via expectations.[15]

This implication does not seem plausible. Economic agents generally would recognize that the rise in prices following an increase in the price of oil, for example, is a "one-time" effect, and so would not change their longer-run inflation expectations. Hence, although such a shock would have a permanent effect on the *level* of prices, the associated speed-up in *inflation* should be only temporary. This suggests that equation (6) may not be a fully satisfactory model of inflation in periods (such as the 1970s and 1980s) in which there were significant relative-price shocks.

To incorporate the restriction that relative price shocks do not affect the inflation rate in the long run, two alternative modifications of equation (6) will be estimated. The first approach maintains the assumption that expected inflation is a weighted average of past inflation, but imposes the restriction that the coefficients on the shock variables in equation (6) sum to zero:

$$\pi_t = \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i} - UNAT) + \sum_{n=0}^{N} c_{On} \cdot SHKOIL_{t-n} + \sum_{m=0}^{M} c_{Xm} \cdot SHKEX_{t-m} \quad (7)$$

where[16] $\Sigma c_{On} = \Sigma c_{Xm} = 0$. This restriction implies that although relative-price shocks affect the average level of prices and so change the inflation rate temporarily, they have no long-run effect on inflation.

The second approach modifies equation (4) so that changes in inflation resulting from relative-price shocks do not affect inflation expectations and so do not pass through into future inflation:

$$E_{t-1}\pi_t = \sum_{s=1}^{S} w_s \left[\pi_{t-s} - \left(\sum_{j=0}^{J} d_{Oj} \cdot SHKOIL_{t-s-j} + \sum_{k=0}^{K} d_{Xk} \cdot SHKEX_{t-s-k}\right)\right] \quad (4')$$

where d_{Oj} and d_{Xk} represent the direct effects of relative price shocks on inflation. This equation says that expected inflation is a weighted average of past inflation, excluding that part of past inflation that was due to relative-price shocks. This model of expectations yields an estimating equation of the form:

$$\pi_t = \sum_{s=1}^{S} w_s \left[\pi_{t-s} - \left(\sum_{j=0}^{J} d_{Oj} \cdot SHKOIL_{t-s-j} + \sum_{k=0}^{K} d_{Xk} \cdot SHKEX_{t-s-k}\right)\right] + \sum_{i=0}^{I} a_i \cdot (U_{t-i} - UNAT) + \sum_{j=0}^{J} d_{Oj} \cdot SHKOIL_{t-j} + \sum_{k=0}^{K} d_{Xk} \cdot SHKEX_{t-k} \quad (8)$$

where no restrictions are placed on the d_{Oj} and d_{Xk} coefficients.

Equation (8) also may be written as follows:

$$\pi_t = \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i} - UNAT) + \sum_{j=0}^{J} d_{Oj} \cdot (SHKOIL_{t-j} - \sum_{s=0}^{S} w_s \cdot SHKOIL_{t-s-j}) + \sum_{k=0}^{K} d_{Xk} \cdot (SHKEX_{t-k} - \sum_{s=0}^{S} w_s \cdot SHKEX_{t-s-k}) \quad (9)$$

In this form, the sums of the coefficients on the shock variables are $\Sigma d_{Oj} \cdot (1 - \Sigma w_s)$ and $\Sigma d_{Xk} \cdot (1 - \Sigma w_s)$, respectively. Since $\Sigma w_s = 1$, these coefficient sums are equal to zero. Thus, equations (7) and (9) are similar since both specify that current inflation is influenced by a distributed lag of past relative-price shocks with coefficients summing to zero. Equation (7) may be viewed as a generalization of equation (9) that puts fewer restrictions on the shapes of the distributed lags on the shock variables. Thus, the c_{On} and c_{Xm} coefficients in equation (7) are combinations[17] of the underlying parameters (w_s and d_{Oj}, and w_s and d_{Xk}, respectively), representing both the direct and indirect effects of relative-price shocks.

One objection that may be raised to this approach is that it is inconsistent to assume, as in equation (4′), that agents can distinguish between price increases resulting from supply shocks and those due to other factors, but cannot see that future inflation also will be affected by the unemployment rate. This objection may be less serious in practice, however, because relative-price shocks often have been sufficiently large that the public probably was able to recognize them as one-time events. This is particularly true of the oil-price shocks in 1974 and 1979.

Demographic Shifts

The natural rate of unemployment may vary as conditions in the labor market change. Faster technological change, for example, may lead to more job-changing and hence a higher natural unemployment rate. Demographic changes have similar effects. Because young persons have fewer skills and less work experience than adults, and also move in and out of the work force more often, they have a higher unemployment rate. If the proportion of young workers in the labor force declines, the measured unemployment rate will fall, but this does not imply an increase in inflationary pressure, but rather a decline in the natural rate. This argument implies that the unemployment measure used in estimating the Phillips relation should be adjusted for demographic changes.[18]

If l_{it} represents the proportion of the labor force that is in the ith population group at date t, and u_{it} is the unemployment rate for that group, the total unemployment rate, U_t, may be decomposed as follows:

$$U_t = \sum_{i=1}^{I}\bar{l}_i \cdot \bar{u}_i + \sum_{i=1}^{I}\bar{u}_i \cdot (l_{it} - \bar{l}_i) + \sum_{i=1}^{I}\bar{l}_i \cdot (u_{it} - \bar{u}_i) + \sum_{i=1}^{I}(u_{it} - \bar{u}_i)(l_{it} - \bar{l}_i) \qquad (10)$$

where $\bar{l}_i$ is the average proportion of the labor force in the ith group over a period of years, and $\bar{u}_i$ is the average group unemployment rate.

The first term on the right side of equation (10) is the average unemployment rate over the period. The remaining terms decompose the difference between the actual rate at date t and the average rate into three components. The second term, $\Sigma\, \bar{u}_i \cdot (l_{it} - \bar{l}_i)$, measures how the overall unemployment rate would have changed if the group unemployment rates had remained constant and only the structure of the population had changed. This component measures the size of purely demographic effects on the unemployment rate. Conversely, the third component, $\Sigma\, \bar{l}_i \cdot (u_{it} - \bar{u}_i)$, shows how overall unemployment would have changed as a result of changes in the group unemployment rates, if the demographic structure of the labor force had remained unchanged. The final term represents ambiguous "cross" effects, some of which also may be demographic.[19] Since their net contribution to overall unemployment is small, these cross effects are grouped with the demographic effects.

The "demographically-adjusted" unemployment rate is constructed by subtracting the demographic and "cross" effects from the measured unemployment rate. Thus:

$$U^*_t = U_t - \sum_{i=1}^{I}\bar{u}_i \cdot (l_{it} - \bar{l}_i) - \sum_{i=1}^{I}(u_{it} - \bar{u}_i) \cdot (l_{it} - \bar{l}_i) = \sum_{i=1}^{I}\bar{l}_i \cdot u_{it} \qquad (11)$$

Chart 1A

Unemployment Rate: Actual vs Adjusted

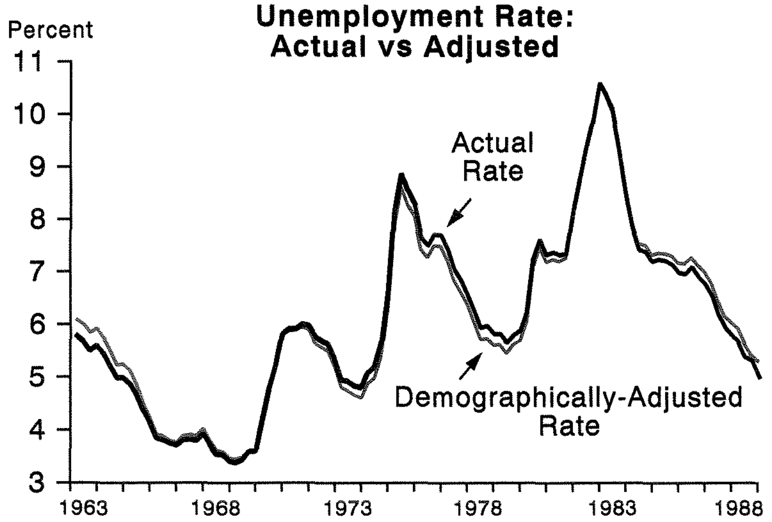
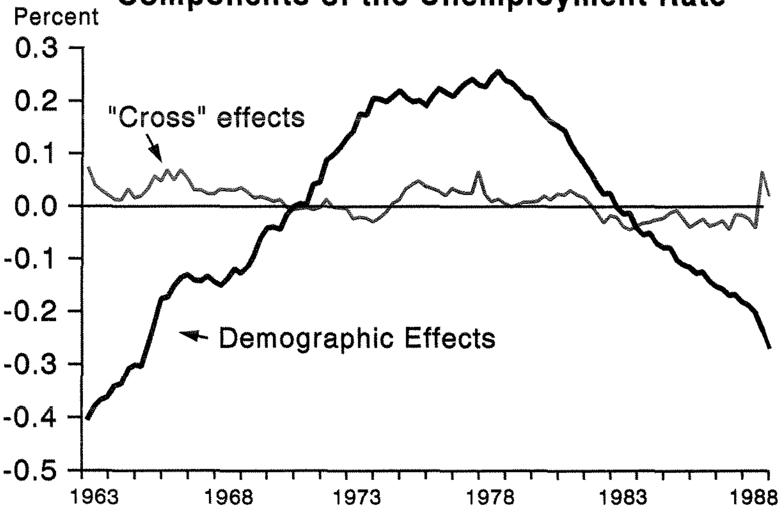


Chart 1B

Components of the Unemployment Rate



In constructing the data series of U_t^*, the population was divided into eight age-sex groups: teens, young adults, prime-age adults and seniors.[20]

Chart 1 shows the components of the unemployment rate. The upper panel compares the actual unemployment rate with the adjusted rate defined in equation (11). Most of the variation in the unemployment rate since 1960 has been due to factors that affected all population groups rather than to demographic changes and cross effects. However, the lower panel shows that purely demographic factors did have a significant effect, raising the overall jobless rate by almost 0.7 percentage point between 1960 and 1978 and reducing it by 0.5 percentage point since then.

II. Empirical Analysis

The coefficients of the equations developed in the previous section were estimated using the demographically-adjusted series (U_t^*) constructed in equation (11). Thus, the equations estimated were:

$$\begin{aligned}\pi_t = {} & \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i}^* - UNAT^*) \\ & + \sum_{g=0}^{G} b_{Og} \cdot SHKOIL_{t-g} \\ & + \sum_{h=0}^{H} b_{Xh} \cdot SHKEX_{t-h} \end{aligned} \qquad (6a)$$

$$\begin{aligned}\pi_t = {} & \sum_{s=1}^{S} w_s \cdot \pi_{t-s} + \sum_{i=0}^{I} a_i \cdot (U_{t-i}^* - UNAT^*) \\ & + \sum_{n=0}^{N} c_{On} \cdot SHKOIL_{t-n} \\ & + \sum_{m=0}^{M} c_{Xm} \cdot SHKEX_{t-m} \end{aligned} \qquad (7a)$$

$$\begin{aligned}\pi = {} & \sum_{s=1}^{S} w_s [\pi_{t-s} - (\sum_{j=0}^{J} d_{Oj} \cdot SHKOIL_{t-s-j} \\ & + \sum_{k=0}^{K} d_{Xk} \cdot SHKEX_{t-s-k})] \\ & + \sum_{i=0}^{I} a_i \cdot (U_{t-i}^* - UNAT^*) \\ & + \sum_{j=0}^{J} d_{Oj} \cdot SHKOIL_{t-j} + \sum_{k=0}^{K} d_{Xk} \cdot SHKEX_{t-k} \end{aligned} \qquad (8a)$$

The coefficients on the relative price variables (SHKOIL and SHKEX) are unrestricted in equation (6a), are constrained to sum to zero in equation (7a), and are subject to nonlinear restrictions in equation (8a). To assess the role of the shock variables, an equation that excludes these variables also was estimated. The equations were estimated over a sample period from the first quarter of 1963 to the fourth quarter of 1988, a total of 104 quarterly observations. Since the equations are nonlinear in *UNAT**, and equation (8a) includes restrictions on the relative-price shock coefficients, nonlinear regression estimation was employed.[21]

The measure of inflation used is the annualized quarterly growth rate of the fixed-weight GNP price index. Unemployment is measured by the civilian unemployment rate.[22] The real price of oil is the ratio of the producers' price of crude petroleum to the producer price index for finished goods, and the real exchange rate is the Federal Reserve's multilateral trade-weighted value of the dollar deflated by the ratio of a trade-weighted average of foreign consumer price indexes to the U.S. fixed-weight GNP price index. The equations also include dummy variables (NIXON and NIXOFF) to capture the effects of the imposition and removal of price controls by the Nixon Administration in the early 1970s.[23]

The estimates of *UNAT** represent the demographically-adjusted natural rate. These are converted to estimates of the actual natural rate by adding the difference between U_t and U_t^*. For a given estimate of $UNAT^*$, Chart 1 implies that the natural rate has fallen since 1978, as the proportion of the population in groups that have higher-than-average unemployment rates has declined.

Full Sample Estimates

Earlier estimates of inflation equations have assumed that the distributed lag on past inflation is long. Gordon and King,[24] for example, used a twenty-four quarter lag distribution constrained to lie on a fourth-order polynomial with a zero end-point constraint. In a later paper, Gordon[25] used four-quarter averages of the inflation rate extending back six years (24 quarters).

To determine the appropriate lag lengths, equation (6a) was estimated using a series of alternative lag distributions on past inflation from four to twenty-four quarters. In these regressions, no restrictions were placed on any coefficients.[26] Table 1 shows the results of these regressions. The estimates reported in this table represent the *cumulative sums* of the coefficients on lagged inflation out to each indicated lag length.[27] Regardless of the lag length chosen, the estimated sum of all the coefficients (the last figure reported in each column) is not significantly different from one. However, in each case, the sum of the coefficients reaches unity by the fifth or sixth lag, suggesting that current inflation is not affected by inflation more than five or six quarters in the past. Although some of the coefficients for more distant lags are statistically significant when lags longer than twelve quarters are introduced,[28] several of the later coefficients in these equations take negative values;[29] which seems implausible. Column I of Table 1 shows the result of estimating the equation assuming a 24-quarter lag, but imposing the restriction that this lag follows a polynomial.[30] This restriction smoothes the estimated lag distribution and has the effect of making it appear longer. However, the hypothesis that the data satisfy this restriction may be rejected.

The results in Table 1 suggest that the finding of long distributed lags in earlier research[31] may be due to the use of overly restrictive polynomial specifications. Hence, the equations in Tables 2 and 3 were estimated using a five-quarter lag on past inflation, with no polynomial restriction.

Table 2 shows the results of estimating the alternative models over the 1963 to 1988 sample period. Model I refers to an equation that excludes the relative-price shock variables. Models II and III refer to equations (6a) and (7a), which are identical except that Model III constrains the coefficients on *SHKOIL* and *SHKEX* to sum to zero. Model IV refers to equation (8a), which imposes additional restrictions on the relative-price coefficients. F_1 is the F-statistic that tests the restriction that the coefficients on lagged inflation sum to one. In all four equations, this restriction may be accepted.[32, 33] F_2 is an F-statistic that tests for the presence of first- through fourth-order autocorrelation in the residuals.[34] In every case, the hypothesis that the residuals are autocorrelated may be rejected.

The coefficients on *SHKOIL* and *SHKEX* in Table 2

Table 1

Alternative Distributed Lags on Past Inflation

Dependent Variable: Inflation in Fixed-Weight GNP Index

Lagged Values of the Dependent Variable	Unrestricted Regressions								Restricted Regression
	A	B	C	D	E	F	G	H	I
1	0.55 (0.10)	0.52 (0.11)	0.51 (0.11)	0.55 (0.11)	0.56 (0.11)	0.44 (0.12)	0.45 (0.12)	0.44 (0.13)	0.46 (0.10)
2	0.67 (0.11)	0.64 (0.12)	0.60 (0.12)	0.62 (0.12)	0.63 (0.13)	0.40 (0.13)	0.36 (0.14)	0.36 (0.15)	0.50 (0.09)
3	0.66 (0.11)	0.63 (0.12)	0.59 (0.13)	0.64 (0.13)	0.63 (0.14)	0.46 (0.15)	0.42 (0.16)	0.42 (0.17)	0.55 (0.08)
4	0.99 (0.08)	0.92 (0.13)	0.87 (0.14)	0.88 (0.15)	0.91 (0.15)	0.68 (0.15)	0.55 (0.18)	0.54 (0.19)	0.60 (0.08)
5		1.01 (0.08)	0.90 (0.13)	0.91 (0.14)	0.93 (0.15)	0.76 (0.14)	0.68 (0.18)	0.67 (0.18)	0.64 (0.08)
6			1.02 (0.08)	1.08 (0.13)	1.10 (0.11)	1.05 (0.14)	0.91 (0.17)	0.90 (0.18)	0.69 (0.07)
8				1.03 (0.08)	1.06 (0.15)	1.19 (0.13)	1.14 (0.14)	1.14 (0.15)	0.77 (0.07)
12					1.02 (0.08)	0.65 (0.11)	0.71 (0.11)	0.70 (0.12)	0.91 (0.08)
16						1.14 (0.08)	1.04 (0.12)	1.03 (0.13)	1.02 (0.09)
20							1.17 (0.09)	1.19 (0.13)	1.11 (0.09)
24								1.15 (0.15)	1.17 (0.10)

Notes: 1. Sample period is 1963.1–1988.4.
2. Figures in parentheses are standard errors.
3. In the equation in column (I), the summed coefficients (that is, the figures reported in the table) are constrained to follow a second-order polynomial.

represent the estimated sums of the distributed lag coefficients on these variables.[35] The figures in parentheses below these coefficients are the standard errors of these sums and those in brackets are F-statistics that test the joint hypothesis that the individual lag coefficients are all zero.

The alternative estimates of the demographically-adjusted natural rate ($UNAT^*$) range from 5.99 percent in Model II to 6.20 percent in Model IV. In the fourth quarter of 1988, the demographically-adjusted unemployment rate (U^*) exceeded the measured rate by 0.26 percentage point, so these estimates of $UNAT^*$ imply actual values of the natural rate between 5.73 and 5.94 percent.[36] These estimates are similar to estimates developed in earlier research by Gordon.[37] The finding that the results are not sensitive to the model used to capture the effects of relative price shocks adds to one's confidence in the estimate. On the other hand, in all cases, the one-standard-error confidence interval on the estimated natural rate is nearly one percentage point wide, which may be too wide for the unemployment rate to be a useful signal to policy-makers.

The response of inflation to divergences between the actual and natural unemployment rates depends on the coefficients on lagged inflation and on the current and lagged unemployment rate. Dynamic simulations of the equations were used to estimate the response of inflation to temporary and permanent divergences between the natural unemployment rate and the actual rate. These simulations showed that, in Model IV, for example, a one-percentage-point decline in the unemployment rate that lasts for a single quarter will raise the inflation rate by 0.11 percentage point. This increase in inflation will be permanent unless it is later offset by an equal single-quarter rise in unemployment. If the unemployment rate remains one percentage point below the natural rate permanently, the inflation rate will increase by 0.11 percentage point every quarter. The other estimated models yielded similar results.

The coefficients on current and lagged unemployment are of opposite signs, implying that the initial impact on inflation of a higher unemployment rate is less than its long-run impact. The hysteresis hypothesis referred to earlier suggests that the inflation rate changes only in response to a *change* in the unemployment rate and that inflation may remain constant at any *level* of unemployment. Although the coefficients on current and lagged unemployment have different signs, they are not equal. The estimated sum of these coefficients ranges from −0.23 to −0.27 and is significantly different from zero. This means that the *level* of unemployment does have a significant impact on the inflation rate. Contrary to the hysteresis hypothesis, an unemployment rate that remains constant below the natural level does cause inflation to increase continually.

Table 2

Alternative Phillips Curve Models

Dependent Variable: Inflation in Fixed-Weight GNP Index

Sample Period 1963.1–1988.4

Independent Variables	Model I (No Shocks)	Model II (Equation 6a)	Model III (Equation 7a)	Model IV (Equation 8a)
UNAT*	6.18*	5.99*	6.18*	6.20*
	(0.46)	(0.53)	(0.44)	(0.43)
U*	−1.55*	−1.39*	−1.36*	−1.29*
	(0.34)	(0.41)	(0.41)	(0.39)
U*(−1)	1.29*	1.16*	1.09*	1.03*
	(0.34)	(0.41)	(0.41)	(0.40)
Σ SHKOIL	—	0.0045	0**	0.0091*
		(0.0067)		(0.0038)
		[2.06]	[2.88]	[6.10]*
Σ SHKEX	—	−0.034	0**	−0.044
		(0.025)		(0.038)
		[0.83]	[0.72]	[1.41]
NIXON	−0.75	−1.00	−0.86	−0.80
	(0.68)	(0.69)	(0.68)	(0.67)
NIXOFF	2.08*	1.91*	2.16*	2.18*
	(0.55)	(0.67)	(0.64)	(0.60)
F_1	0.0	0.12	0.64	0.18
F_2	1.99	1.53	1.45	1.48
SEE	1.17	1.16	1.16	1.14

* Significant at 5% level
** Constrained

In Model II, the hypotheses that the sums of the coefficients on *SHKOIL* and *SHKEX* are both zero may be accepted, implying that relative-price shocks have only temporary effects on inflation. Models III and IV impose this restriction. In Model III, the hypothesis that all the coefficients on *SHKOIL* are individually zero can be rejected at the six percent level of significance ($F = 2.88$). Moreover, the coefficient on the current value of *SHKOIL* is positive and significant (t-statistic = 2.18), indicating that changes in the real price of oil have a significant temporary effect on the inflation rate. This conclusion is confirmed by Model IV, which also shows a positive and significant temporary impact of oil prices on inflation.

Although the effects of changes in the exchange rate are less significant, they go in the direction predicted by theory. In Model III, the sum of the coefficients on the current and twelve lagged values of *SHKEX* is constrained to zero, but the estimated coefficient-sum out to the eighth lag is negative and significant at the five percent level, implying that a real appreciation of the dollar reduces inflation temporarily. However, the hypothesis that all the coefficients on *SHKEX* are individually zero cannot be rejected (F = 0.72). In the more restrictive Model IV, the sum of the coefficients on *SHKEX* (which in this model represent only the *direct* effect of exchange-rate changes) is negative and significant at the 13-percent level (one-tailed test) and the hypothesis that these coefficients are all individually zero may be rejected at the 20-percent level of significance (F = 1.41). Thus, there is only weak evidence that changes in the real exchange rate have effects on inflation.

Rolling Regressions

To examine whether the natural rate has changed over time, the equations were estimated over fifteen overlapping twelve-year sample periods, beginning with 1963.1 to 1974.4 and ending with 1977.1 to 1988.4. The twelve-year length of these sample periods was chosen to provide a reasonable number of degrees of freedom and ensure that even the earliest sample periods included a number of observations after the shift to flexible exchange rates in 1971 and the first oil-price shock in 1974. The lag lengths determined from the full 1963–88 sample period were used for these "rolling regressions." Table 3 shows the estimates of Model IV.[38]

This table suggests that the estimated natural rate tends to be *higher* for samples that include the 1980s. The estimates of *UNAT** range around six percent for samples ending before 1980,[39] but are closer to seven percent for samples ending between 1980 and 1988. The estimate of *UNAT** over the final period implies a natural rate in the fourth quarter of 1988 of 6.8 percent, more than 1½ percentage points above the actual rate in the quarter and ¾ percentage point above the estimate from the full sample period.

The estimates of the impact of relative-price shocks on inflation are mixed. In all sample periods, the impact of these shocks has the sign predicted by theory. On a one-tailed test, the hypothesis that oil-price shocks initially have a positive impact on inflation (that is, the coefficient on the contemporaneous value of *SHKOIL* is positive) may be accepted at the 10-percent level in nine sample periods. The corresponding hypothesis that the sum of the coefficients on *SHKEX* is negative also may be accepted in nine sample periods.[40] Although relative-price shocks probably have affected overall inflation in the directions expected from theory, these effects have not been consistent.

Table 3

Rolling Regressions: Model IV

SAMPLE	UNAT*	SHKOIL	SHKEX
1963.1-74.4	5.08*	0.016	−0.55*
	(0.41)	(0.012)	(0.23)
		[1.89]	[1.58]
1964.1-75.4	8.02	0.012	−0.25
	(6.54)	(0.012)	(0.22)
		[0.89]	[1.39]
1965.1-76.4	6.42*	0.023*	−0.46*
	(1.65)	(0.011)	(0.17)
		[4.41]*	[1.97]
1966.1-77.4	6.27*	0.015	−0.45*
	(0.86)	(0.009)	(0.12)
		[3.17]	[2.54]*
1967.1-78.4	5.72	0.015	−0.33*
	(0.64)	(0.008)	(0.08)
		[3.39]	[2.47]*
1968.1-79.4	6.14*	0.013	−0.32*
	(0.63)	(0.008)	(0.08)
		[2.67]	[2.27]*
1969.1-80.4	7.57*	0.011	−0.21
	(2.30)	(0.009)	(0.11)
		[1.55]	[0.94]
1970.1-81.4	7.05*	0.009	−0.09
	(0.87)	(0.007)	(0.12)
		[1.36]	[0.97]
1971.1-82.4	7.04*	0.007	−0.12
	(0.44)	(0.007)	(0.09)
		[0.90]	[0.91]
1972.1-83.4	7.26*	0.007	−0.15*
	(0.36)	(0.006)	(0.07)
		[1.17]	[1.11]
1973.1-84.4	7.19*	0.008	−0.14*
	(0.34)	(0.006)	(0.07)
		[2.07]	[1.14]
1974.1-85.4	7.06*	0.011*	−0.13*
	(0.35)	(0.005)	(0.06)
		[3.99]*	[1.50]
1975.1-86.4	6.85*	0.008	−0.04
	(0.35)	(0.007)	(0.05)
		[1.22]	[0.62]
1976.1-87.4	6.94*	0.009*	−0.04
	(0.38)	(0.004)	(0.05)
		[4.97]*	[0.61]
1977.1-88.4	7.06*	0.008*	−0.03
	(0.27)	(0.004)	(0.03)
		[3.98]*	[0.50]

Note: * = Significant at 5% level

Simulations

To provide a further test of the alternative models, out-of-sample simulations of each of the four models were conducted. For purposes of this simulation exercise, the models were re-estimated[41] over the period from 1963.1 to 1984.4, and simulated to 1988.4.

Table 4 compares the error statistics for forecasts of the quarterly *change* in inflation obtained from static and dynamic simulations of the four models.[42] The static simulations are out-of-sample fitted values of the estimated equations. The dynamic simulations were begun in the first quarter of 1963. In the early years of the simulations, the simulated quarterly changes in inflation depend on actual inflation before 1963 as well as on current and past values of the unemployment rate and the shock variables. However, the effect of inflation before 1963 gradually dies out and by 1980 is effectively zero.

The errors from the dynamic simulations are in most cases smaller than those from the static simulations. In the dynamic simulations, the simulated change in the inflation rate depends only on its underlying determinants (current and lagged unemployment and relative price shocks) and is not affected by lagged actual inflation. The results in Table 4 suggest that this may be a superior forecasting procedure.

The errors from both the static and dynamic simulations of Model III and Model IV are lower than those from Model II. This result supports the argument that relative-price changes should have no permanent effect on inflation. In addition, the errors from Model IV are lower than those from Model III, suggesting that the Model IV specification, in which changes in inflation associated with relative-price shocks are explicitly "purged" from the lagged dependent variable, is a better specification.

Table 5

Sources of Inflation 1985–1988

	I	II	III	IV
1984 Inflation	3.69	3.69	3.69	3.69
+ Unemployment Effect	−0.55	−0.34	−0.37	−0.38
+ Oil Price Effect	0	−0.15	+0.02	−0.03
+ Exchange Rate Effect	0	−0.57	−0.20	−0.00
+ Error	+0.14	+0.65	+0.15	+0.00
= 1985 Inflation	3.28	3.28	3.28	3.28
+ Unemployment Effect	−0.46	−0.27	−0.29	−0.30
+ Oil Price Effect	0	−0.74	+0.05	−0.31
+ Exchange Rate Effect	0	+2.15	+1.33	+1.09
+ Error	−0.18	−1.77	−1.73	−1.11
= 1986 Inflation	2.65	2.65	2.65	2.65
+ Unemployment Effect	+0.35	+0.31	+0.32	+0.28
+ Oil Price Effect	0	+0.80	−0.16	+0.83
+ Exchange Rate Effect	0	+2.38	+2.10	+1.70
+ Error	+0.95	−2.18	−0.95	−1.50
= 1987 Inflation	3.96	3.96	3.96	3.96
+ Unemployment Effect	+0.64	+0.42	+0.55	+0.49
+ Oil Price Effect	0	−0.21	+0.13	−0.70
+ Exchange Rate Effect	0	−0.40	−0.64	−0.66
+ Error	−0.13	+0.71	+0.47	+1.38
= 1988 Inflation	4.47	4.47	4.47	4.47

Minor discrepancies in table are due to rounding.

Table 4

Out-of-Sample Simulation Errors

(1985.1–1988.4)

	Static Simulations				Dynamic Simulations			
	I	II	III	IV	I	II	III	IV
Mean Error	0.11	−0.42	−0.29	−0.17	0.03	−0.17	−0.12	−0.03
Root Mean Squared Error	0.64	1.31	0.99	0.92	0.68	1.14	0.96	0.70
Mean Absolute Error	0.51	0.99	0.76	0.70	0.56	0.92	0.70	0.55

Note: Errors shown are for simulations of the one-quarter change in the annualized inflation rate.

However, Model I, which omits the relative-price variables entirely, appears to predict inflation as well as Model IV. This finding, together with the rolling regression results in Table 3, which showed that the effects of the shock variables were sporadic, suggests that even the direct impact of relative-price shocks on inflation is small, except in periods when these shocks are unusually large (as in the 1974 and 1979 oil shocks). Thus, Table 4 seems to suggest that the role of relative-price shocks in the inflation process probably has been over-emphasized in earlier research and in media discussions of the sources of inflation.

The dynamic simulations make it possible to decompose the change in the inflation rate into its underlying sources. These decompositions are shown in Table 5. For each year since 1985, the change in annual inflation is separated into the portions due to past and present divergences of unemployment from its natural level, to oil-price and exchange-rate shocks, and to the cumulative simulation error.

Inflation declined by about one percentage point between 1984 and 1986. All four models attribute a significant portion of this decline to the high level of unemployment relative to its natural rate. Conversely, the models agree in attributing much of the 1.8-percentage-point increase in inflation in 1986–88 to the low rate of unemployment in those years.

As in Table 4, the simulation errors are larger for Model II than for the other three models, again suggesting that the former is an inappropriate specification. The decompositions suggest that the errors in Models II, III, and IV are largely due to the exchange rate variable. In every case, the annual error is opposite in sign and of a similar magnitude to the contribution of the exchange rate variable, implying that the error would be reduced by omitting that variable.[43]

III. Summary and Conclusions

This paper has examined the link between inflation and the rate of unemployment. The feature of the inflation equations estimated in this paper that distinguishes them from other equations in the literature is that relative-price shocks are constrained to have only temporary effects on the inflation rate. In addition, inflation expectations are proxied by a distributed lag on past inflation that is much shorter than in earlier studies.

As pointed out earlier, the failure of inflation to pick up significantly since 1987, despite the decline in unemployment, has led some economists to lower their estimates of the natural rate. However, apart from the effects of the change in the age-sex structure of the labor force, the results of this paper do not support the hypothesis that the natural rate has declined. If anything, the results of the rolling regressions in Table 3 suggest that the (demographically-adjusted) natural rate of unemployment has been higher in the 1980s. However, the estimates of the natural rate in these regressions are subject to an uncomfortably wide margin of uncertainty.

This paper has not investigated the causes of any such change in the natural rate. One possible cause is the apparent increase in the rate of technological change during the last decade, as a result of advances in computer technology and the response of the economy to the oil-price shocks of the 1970s. More rapid change either in methods of production or in the types of goods and services being produced would be expected to add to job-changing and hence to the level of normal unemployment.

A second important result of this paper concerns the role of relative-price shocks. Commentary on inflation in the media frequently focuses on the role of these shocks in influencing the inflation rate, and earlier research generally has found their effect on inflation to be statistically significant. The empirical estimates in this paper suggest that these shocks may raise or lower the *level* of prices and so cause temporary changes in measured inflation. However, these shocks do not influence the rate of inflation over longer periods. In all the equations estimated, we can accept the hypothesis that shocks have no long-run effect on inflation.

Moreover, the estimates suggest that even the short-run effects of relative-price changes have been sporadic. In many sample periods, we cannot reject the hypothesis that the shocks had no impact on inflation, even in the short run. Also, in out-of-sample simulations, inclusion of these shocks did not improve forecasts of the inflation rate. These empirical results suggest that the role of these shocks in causing inflation has been over-emphasized in earlier research.

The results of this paper suggest that since the Fall of 1988, the gap between the actual unemployment rate and the natural rate has ranged between ¾ and 1½ percentage points.[44] In view of this gap, why has inflation remained relatively subdued? The primary reason[45] appears to be that the increase in inflation in response to low unemployment occurs quite slowly. Simulations of the estimated equations indicate that a permanent one-percentage point gap between the natural and actual unemployment rates would cause inflation to increase by 0.11 percentage point per quarter or less than one-half percentage point per year.

Unemployment has been below its natural rate only since mid-1987. Between the fourth quarters of 1986 and 1988, inflation increased by about 1½ percentage points. The dynamic simulations reported in Table 5 indicate that the low rate of unemployment contributed between one and 1¼ percentage points of this increase. Thus, the relatively modest pick-up in inflation may be explained without invoking a decline in the natural rate.

At the same time, the estimated equations suggest that if the unemployment rate were to remain permanently at its present 5¼ percentage level, inflation would continue to increase by about ½ percentage point each year. The slow response of inflation to a rate of unemployment above or below the natural rate means that the costs of low levels of unemployment, in terms of rising inflation, are initially small, and policy-makers may be tempted to ignore them. However, once inflation has been allowed to increase to "unacceptable" levels, this slow response means that bringing it down again will require either a lengthy period in which unemployment is held above the natural rate or a shorter period of excessively high unemployment. The high costs associated with either course suggest that it is more prudent to move against rising inflation before it reaches unacceptable levels.

NOTES

1. The estimate of "high-employment GNP" by the Commerce Department, for example, is computed as the level of output that the economy would produce at six percent unemployment, on the presumption that a lower jobless rate would be associated with increasing inflation.

2. Commenting on the report of an unemployment rate of 5.3 percent in June 1989, Michael Boskin, the Chairman of the President's Council of Economic Advisers, said, "I'm pleased that unemployment remains low, and I don't see the current level of unemployment as inflationary." *The Wall Street Journal*, Monday, July 10, 1989, page 2.

3. A.W. Phillips, "The Relationship between the Unemployment Rate and the Rate of Change in Money Wage Rates in the United Kingdom, 1861–1957," *Economica,* November 1957, pp 283–299.

4. These theorists argue that higher-than-expected inflation leads to a decline in the unemployment rate. These alternative views of the theoretical underpinnings of the Phillips curve are illustrated in two popular macroeconomics textbooks. Dornbusch and Fischer develop a model of unemployment and inflation in which causation runs from the unemployment rate, which serves as a proxy for the strength of demand in the economy, to the rate of inflation. See Rudiger Dornbusch and Stanley Fischer, *Macroeconomics,* Fourth Edition, McGraw-Hill, 1987, Chapters 13–14. The causation runs in the opposite direction in the "new classical" model presented by Hall and Taylor. In this model, unexpected changes in the inflation rate lead to changes in the unemployment rate. See Robert E. Hall and John B. Taylor, *Macroeconomics: Theory, Performance and Policy*, Norton, 1986, Chapter 13.

5. This argument assumes implicitly that the excess demand for labor by firms is the result of an increase in demand for their products. Clearly, if higher nominal wage rates reflect increases in labor productivity, they will not spill over into higher prices.

6. See Edmund S. Phelps, "Introduction: The New Microeconomics in Employment and Inflation Theory," in Phelps (Editor), *Microeconomic Foundations of Employment and Inflation Theory,* W.W. Norton, New York, 1970.

7. See the discussion of price determination in Flint Brayton and Eileen Mauskopf, "The Federal Reserve Board MPS Quarterly Econometric Model of the US Economy," *Economic Modelling,* July 1985, pp 202–203.

8. A special case arises when the function $g()$ in equation (3) is linear. In this case, the weights w_s in equation (4) decline geometrically.

9. For a discussion of alternative models of inflation expectations, see Adrian Throop, "An Evaluation of Alternative Models of Expected Inflation," *Economic Review*, Federal Reserve Bank of San Francisco, Summer 1988.

10. John B. Taylor, "Staggered Wage Setting in a Macro Model," *American Economic Review,* Vol 69 (May 1979), pp 108–113.

11. For an example of this approach, see Olivier J. Blanchard and Lawrence H. Summers, "Hysteresis and the European Unemployment Problem," in Stanley Fischer (Editor), *NBER Macroeconomics Annual 1986,* National Bureau of Economic Research, 1986, and Robert J. Gordon, "Hysteresis in History: Was There Ever a Phillips Curve?," *American Economic Review,* May 1989.

12. A decline in the real exchange rate also tends to increase GNP growth by increasing exports and reducing imports. In the Phillips curve, these "aggregate-demand" effects of exchange rate changes will be captured through changes in the unemployment rate.

13. This specification implies that the direct effect of relative-price shocks is on the *level* of prices. An increase in the level of the price of imported oil, for example, adds to the average *level* of prices in the U.S. Hence, the average *inflation rate,* which is the dependent variable in

equation (6), is influenced by the *growth rate* of oil prices.

14. Robert Gordon has contributed extensively to this literature. See, for example, Robert J. Gordon, "Understanding Inflation in the 1980s," *Brookings Papers on Economic Activity,* 1:1985, pp 263–302.

15. An arithmetic example may clarify the argument. Consider a simplified version of equation (6):

$$\pi_t = \pi_{t-1} + a \cdot (U_t - UNAT) + b_O \cdot SHKOIL_t + b_X \cdot SHKEX_t \qquad (6')$$

Suppose unemployment is at the natural rate ($U_t = UNAT$) and initially there are no shocks ($SHKOIL_t = SHKEX_t = 0$). This implies a constant inflation rate ($\pi_t = \pi_{t-1}$). A one-time one percent increase in the price of oil means that $SHKOIL_t$ rises from 0 to 1 for a single quarter and then declines back to 0. The direct effect of this shock is to raise the inflation rate by b_O percentage point. However, because of the presence of the lagged inflation term (with a coefficient of one) in equation (6′), the inflation rate also will be b_O higher in the next quarter and in all future quarters. Thus, a temporary shock leads to permanently higher inflation. Similarly, suppose the exchange rate begins to fall steadily at one percent per quarter, so that $SHKEX_t$ decreases permanently from 0 to −1. The inflation rate will rise by b_X percent in the first quarter, by an *additional* b_X percentage point in the second quarter, and so on. Thus, a permanent shock leads to continually rising inflation.

16. In equation (7) different symbols are used for the coefficients on the shock variables (c_{On} and c_{Xm} in place of b_{Og} and b_{Xh}) to signify that these coefficients are constrained to sum to zero, whereas those in equation (6) are not.

17. In equation (9), the distributed lags on the two relative price variables run from 0 to S+J and S+K, respectively. In equation (7) these lags run from 0 to N and M, respectively. If equation (7) is interpreted as a generalization of (9), this implies that N = S+J and M = S+K. In the empirical estimations, this implication was used as a guide in choosing the lengths of the estimated distributed lags.

18. See Robert J. Gordon, "Inflation, Flexible Exchange Rates, and the Natural Rate of Unemployment," in Martin N. Baily, ed., *Workers, Jobs and Inflation,* Washington D.C., The Brookings Institution, 1982.

19. In cases where changes in the size of individual population groups are associated with unemployment-rate changes in the same direction, this term probably captures demographic factors, as increases in the supply of particular groups of workers lead to more unemployment. For both young adults and older workers, the correlation between unemployment and labor force share is positive over the 1963 to 1988 period, suggesting a demographic effect of this kind. However, for prime age women and for male teenagers, this correlation is strongly negative. In these cases, the causation may be reversed, with strong labor demand leading both to lower jobless rates and to greater labor force participation.

20. A similar approach is used by the Congressional Budget Office. See Appendix B, "Estimates of Potential Output," in Congressional Budget Office, *The Economic and Budget Outlook: An Update,* August 1987. However, CBO estimates the natural rate from the raw data rather than from the adjusted data.

21. The restrictions that the coefficients on lagged inflation sum to unity and that the coefficients on each of the shock variables in equation (7a) sum to zero were imposed using a technique suggested by Scadding. See John L. Scadding, "Simple Technique for Imposing Restrictions on Sums of PDL Coefficients," Appendix 1 in Rose McElhattan, "The Response of Real Output and Inflation to Monetary Policy," *Economic Review,* Federal Reserve Bank of San Francisco, Summer 1981.

22. The demographic adjustments to the unemployment rate data were made using the same 1963–88 sample period as the regression estimates.

23. The definitions of these variables were adopted from Robert J. Gordon and Stephen R. King, "The Output Cost of Disinflation in Traditional and Vector Autoregressive Models," *Brookings Papers on Economic Activity,* I:1982. NIXON is defined as 0.8 for the five quarters from 1971.3 to 1972.3 and NIXOFF is defined as 0.4 in 1974.2 and 1975.1 and as 1.6 in 1974.3 and 1974.4.

24. Gordon and King, cited in note 23.

25. Gordon, "Understanding Inflation in the 1980s." See note 14 for full citation.

26. These preliminary regressions also include a constant term, the current and four lagged values of *SHKOIL,* the current and eight lagged values of *SHKEX,* and the current and one lagged value of the unemployment rate variable.

27. For estimation, the equations were transformed as follows:

$$\begin{aligned}\pi_t &= a_1\pi_{t-1} + a_2\pi_{t-2} + \ldots + a_T\pi_{t-T} \ldots \\ &= b_1(\pi_{t-1} - \pi_{t-2}) + b_2(\pi_{t-2} - \pi_{t-3}) \\ &\quad + \ldots b_{T-1}(\pi_{t-T+1} - \pi_{t-T}) + b_T\pi_{t-T} + \ldots\end{aligned}$$

In this form, $b_s = \sum_{t=1}^{S} a_t$. That is, the estimated coefficients (b_s) are the cumulative sums of the underlying parameters (a_t).

28. Specifically, when more than 12 lagged values of inflation are included in the equation, one can reject the hypothesis that coefficients beyond the fifth lag are all zero. For example, for the regression that includes 24 lags of the inflation rate, (column H) the F-statistic (with 19 and 63 degrees of freedom) for the hypothesis that the coefficients for lags beyond the fifth are all zero is 2.04, indicating that this hypothesis may be rejected with 97 percent confidence. On the other hand, in the regression that includes only 12 lags (column E), the F-statistic for the hypothesis that the coefficients on the sixth through twelfth

lags are all zero is only 0.83, implying that this hypothesis may not be rejected.

29. In columns G, H and I of Table 1, the sum of coefficients out to lag 12 is less than that out to lag 8.

30. Specifically, the cumulative sums of the coefficients (that is, the figures reported in the table) are constrained to follow a second-order polynomial. This restrictive specification was adopted because third-, fourth- and fifth-order polynomials yielded estimated lag distributions in which the coefficient sums did not converge as the lag length was extended.

31. See Gordon and King, cited above, and Adrian Throop, "A Macroeconomic Model of the U.S. Economy," Working Paper 88–06, *Working Papers in Applied Economic Theory,* Federal Reserve Bank of San Francisco, 1988.

32. F_1 is less than one in all cases, compared to its critical value at the 5 percent level of 2.5.

33. However, it should be mentioned that the estimate of *UNAT** is sensitive to the imposition of this restriction. In Model III, for example, the relaxation of the restriction reduces the estimate of *UNAT** by two standard errors from 6.18 to 5.33 percent.

34. A.C. Harvey, *The Econometric Analysis of Time Series,* New York: John Wiley and Sons, 1981, pp 276–77.

35. In Model III, the equations include the current and two lagged values of *SHKOIL* and the current and twelve lagged values of *SHKEX;* that is, N = 2 and M = 12. In Model IV, the direct effects of the shocks are represented by the current value of *SHKOIL* and the current and eight lagged values of *SHKEX;* that is J = 0 and K = 8. As implied in note 17, J < N and K < M.

36. The differences between these alternative estimates are less than one standard error.

37. See, for example, the estimates by Gordon cited in Appendix B, "Estimates of Potential Output," in Congressional Budget Office, *The Economic and Budget Outlook: An Update,* August 1987.

38. The estimates of Models I, II, and III are similar to those from Model IV and are available from the author.

39. The estimates of *UNAT** are unexpectedly low and high in the 1963–74 and 1964–75 periods respectively. These estimates may be biased by the effects of the 1973–74 oil shock.

40. However, the hypothesis that all the coefficients on *SHKEX* are individually zero may be rejected with 90 percent confidence in only four sample periods.

41. The estimated coefficients over this sample period are not significantly different from those reported in Table 2.

42. In the static simulations, the errors in predicting the change in inflation are the same as those in predicting its level, because the equations include the lagged level of inflation. In the dynamic simulations, the simulated level of inflation depends on actual inflation in the quarters before the simulation begins. This means that the error in predicting the level of inflation depends on the starting date of the simulation. Hence, it is more appropriate to compare the errors in predicting the change in inflation, which do not depend on the starting date of the simulations.

43. Recall that the real exchange rate is only marginally significant in the equations reported in Table 2.

44. After adjusting for demographic change, Models I, III, and IV all imply a natural rate of six percent in the fourth quarter of 1988, when estimated over the full sample period. When estimated over the 1977–1988 period, these models imply a rate of 6¾ percent.

45. The results in Tables 4 and 5 do not show a preponderance of negative errors that would suggest that the response of inflation to the unemployment rate in recent years has been atypical.