A Post-Pandemic New Normal for Interest Rates? Evidence from U.K. Index-Linked Debt

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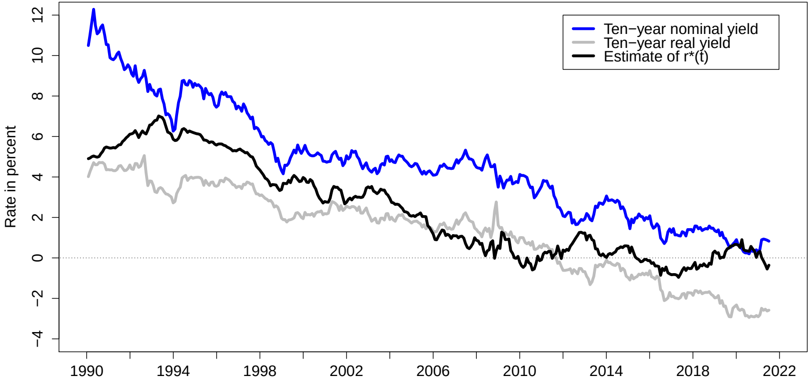
Motivation

- Before the global pandemic, interest rates around the world had fallen in the previous decades leading to a consensus about a lower new normal for interest rates.
- However, the post-pandemic sharp rise in global interest rates has raised questions about that consensus.
- We revisit this issue and offer a U.K. perspective using a dynamic term structure finance model estimated directly on prices of individual index-linked gilts since 1990.
- This allows us to provide a longer than 30-year perspective on changes in the natural real rate in the U.K., a major advanced economy.

Contribution

- We use a novel term structure model of real yields that accounts for bond-specific risk premiums taken from Christensen and Rudebusch (2019).
- Advantages of this approach:
  – No requirement of stable, correct macro specification;
  – No adjustment for bounds on nominal interest rates;
  – Available in real time.
- The resulting finance-based alternative measure of the steady-state level of the short-term real interest rate, r_t^*, offers new evidence that the equilibrium interest rate has gradually declined over the past three decades.
- Importantly, r_t^* in the U.K. has spiked sharply and more than expected since the end of our sample in mid-2021.

Long-Term Yields and Our Estimate of r_t^*

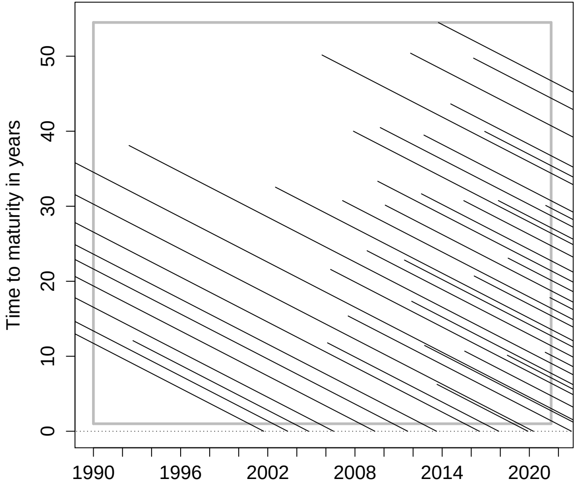


- Shown are U.K. ten-year nominal and real yields. Also shown is our estimate of r_t^*.
- Most of the net decline in long-term yields since 1990 is explained by a decline in r^*.

Outline

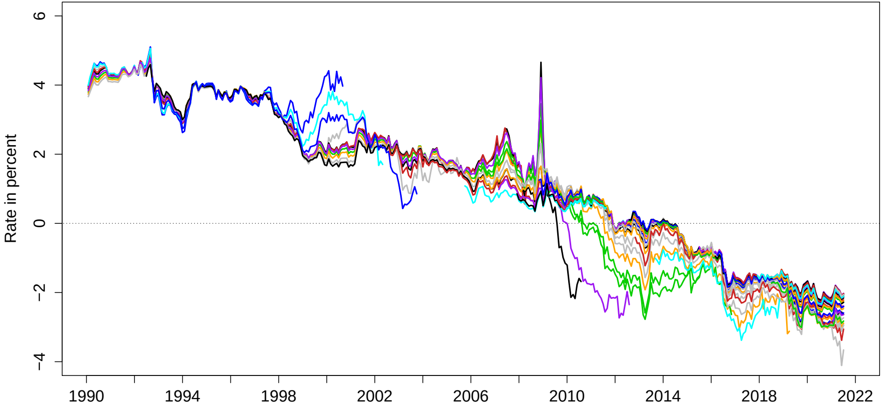
1. The inflation-indexed bond data
2. Identifying r_t^* with real bond prices
3. Convenience premium analysis
4. r_t^* analysis
5. Conclusion

Maturity Distribution of the U.K. Index-Linked Gilts



- Shown is the universe of index-linked gilts since 1990.
- Note the diverse set of bonds issued with maturities of up to 55 years.
- We use monthly bond prices from 1/90 to 6/21.

U.K. Index-Linked Gilt Yields



- Shown are the yield to maturity series for all 42 bonds.
- Some business cycle variation around general slow-moving downward trend.
- Empirical question: Why are U.K. real yields so low? Is there excess institutional demand for these bonds?

Outline

An Affine Model of Real Yields

- We follow the usual empirical finance approach that models bond prices with latent factors, denoted X_t, and the assumption of no residual arbitrage opportunities.
- We assume that X_t follows an affine Gaussian process
$$dX_t = K^{\mathbb{P}}(\theta^{\mathbb{P}} - X_t) + \Sigma dW_t^{\mathbb{P}}.$$
- Also, the instantaneous risk-free real rate, r_t, is affine:
$$r_t = \delta_0 + \delta_1 X_t.$$
- Finally, the risk premiums, Γ_t, are also affine
$$\Gamma_t = \gamma_0 + \gamma_1 X_t.$$
- Duffie and Kan (1996) show that these assumptions imply that zero-coupon real yields are also affine in X_t:
$$y_t(\tau) = -\frac{1}{\tau}A(\tau) - \frac{1}{\tau}B(\tau)' X_t,$$
where $A(\tau)$ and $B(\tau)$ are solutions to a system of ODEs.

Our Definition of r_t^*

Our definition of the equilibrium rate of interest r_t^* is

$$r_t^* = \frac{1}{5}\int_{t+5}^{t+10} E_t^{\mathbb{P}}[r_s]ds,$$

that is, the average expected real short rate over a five-year period starting five years ahead.

- This 5yr5yr forward average expected real short rate should be little affected by short-term transitory shocks and well positioned to capture persistent trends in the natural real rate.

"*The level of the real interest rate expected to prevail, say, five to 10 years in the future, after the economy has emerged from any cyclical fluctuations and is expanding at its trend rate.*" Laubach and Williams (2016, p. 57).

The Indexed-Linked Bond Discount Function

- We follow Andreasen, Christensen, and Riddell (RF, 2021) and discount the cash flow of a given index-linked bond i with a bond-specific function:

$$\bar{r}_t^i = r_t + \beta^i (1 - e^{-\lambda^{R,i}(t-t_0^i)}) X_t^R.$$

- Time since issuance, t-t_0^i, is a proxy for the notion that, as time passes, an increasing fraction of a given security is held in buy-and-hold investors' portfolios and not available for trading.

- Forward-looking investors factor this into their trading strategies, which determines X_t^R and the index-linked bond-specific risk premiums.

Note: This can be combined with any existing model of r_t.

Frictionless Dynamics

Proposition: If the risk-free real rate is defined by

$$r_t = L_t + S_t$$

and the $\mathbb{Q}$-dynamics of the factors $X_t = (L_t, S_t, C_t)$ are

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix} \left[\begin{pmatrix} \theta_1^{\mathbb{Q}} \\ \theta_2^{\mathbb{Q}} \\ \theta_3^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right] dt + \Sigma dW_t^{\mathbb{Q}},$$

where Σ is a constant matrix, then zero-coupon yields have the popular Nelson-Siegel factor structure:

$$y_t(\tau) = L_t + \Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\Big) S_t + \Big(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\Big) C_t - \frac{A(\tau)}{\tau}.$$

This defines the arbitrage-free Nelson-Siegel (AFNS) model class derived in Christensen, Diebold, and Rudebusch (JoE, 2011), which we refer to as the gilts-only (G-O) model.

The Augmented G-O-R Model

We refer to the G-O model augmented with the bond-specific risk factor X_t^R as the G-O-R model.

Its four state variables, $X_t = (L_t, S_t, C_t, X_t^R)$, have risk-neutral $\mathbb{Q}$-dynamics given by

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \kappa_R^{\mathbb{Q}} \end{pmatrix} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ \theta_R^{\mathbb{Q}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \right] dt + \Sigma \begin{pmatrix} dW_t^{L,\mathbb{Q}} \\ dW_t^{S,\mathbb{Q}} \\ dW_t^{C,\mathbb{Q}} \\ dW_t^{R,\mathbb{Q}} \end{pmatrix}.$$

<u>Note:</u> The bond-specific risk factor, X_t^R, is modeled as an independent Vasiček (1977) process under the $\mathbb{Q}$-measure.

In the G-O-R model, the cash flow from the bond indexed i is discounted with the following exponential-affine function

$$\begin{aligned} P^i(t_0^i, t, T) &= E_t^{\mathbb{Q}}\left[e^{-\int_t^T (r_s + \beta^i (1 - e^{-\lambda^{R,i}(s - t_0^i)}) X_s^R) ds} \right] \\ &= \exp\Big(B^i(t_0^i, t, T)' X_t + A^i(t_0^i, t, T) \Big). \end{aligned}$$

Now, consider the value of the inflation-indexed bond issued at time t_0 with maturity at $t+\tau$ that pays an annual coupon C semiannually.

Its clean price is

$$\begin{aligned}\overline{P}_t(t_0,\tau) \quad &= \quad C(t_1-t)E^{\mathbb{Q}}\left[e^{-\int_t^{t_1}\overline{r}(s,t_0)ds}\right]+\sum_{j=2}^{N}\frac{C}{2}E^{\mathbb{Q}}\left[e^{-\int_t^{t_j}\overline{r}(s,t_0)ds}\right] \\ &\quad +E^{\mathbb{Q}}\left[e^{-\int_t^{t+\tau}\overline{r}(s,t_0)ds}\right].\end{aligned}$$

Note only two minor omissions:

- We do not account for the lag in inflation indexation, but effect likely small, see Grishchenko and Huang (2013).
- We do not account for differences in the indexation lag, but effect likely small, see Christensen (2018).

Market Prices of Risk

To facilitate empirical implementation, we use the essentially affine risk premium specification introduced in Duffee (2002).

This implies that the risk premiums Γ_t are state-dependent

$$\Gamma_t = \gamma^0 + \gamma^1 X_t,$$

where $\gamma^0 \in \mathbf{R}^4$ and $\gamma^1 \in \mathbf{R}^{4\times 4}$ are unrestricted.

Thus, the unrestricted G-O-R model has $\mathbb{P}$-dynamics

$$\begin{pmatrix} dL_t \\ dS_t \\ dC_t \\ dX_t^R \end{pmatrix} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & \kappa_{14}^{\mathbb{P}} \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & \kappa_{24}^{\mathbb{P}} \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & \kappa_{34}^{\mathbb{P}} \\ \kappa_{41}^{\mathbb{P}} & \kappa_{42}^{\mathbb{P}} & \kappa_{43}^{\mathbb{P}} & \kappa_{44}^{\mathbb{P}} \end{pmatrix} \left(\begin{pmatrix} \theta_1^{\mathbb{P}} \\ \theta_2^{\mathbb{P}} \\ \theta_3^{\mathbb{P}} \\ \theta_4^{\mathbb{P}} \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \\ X_t^R \end{pmatrix} \right) dt + \Sigma dW_t^{\mathbb{P}}.$$

This is the transition equation in the Kalman filter estimation.

Measurement Equation and Model Estimation

- To make the fitted bond pricing errors comparable across maturities and time, we scale each bond price by its duration
$$\frac{\overline{P}_t(t_0,\tau)}{D_t(t_0,\tau)} = \frac{\widehat{P}_t(t_0,\tau)}{D_t(t_0,\tau)} + \varepsilon_t,$$
where $\widehat{P}_t(t_0,\tau)$ is the model-implied bond price and $D_t(t_0,\tau)$ is its duration, which is fixed and calculated before estimation.

- Due to the nonlinear measurement, the model is *robustly* estimated with the extended Kalman filter, see Andreasen, Christensen, and Rudebusch (JoE, 2019).

- For identification, the 38-year bond maturing 7/17/2024 (2.5%) has unit loading on the X_t^R risk factor, i.e., $\beta^i = 1$.

- State variables are assumed stationary throughout.

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Calculation of Convenience Premiums

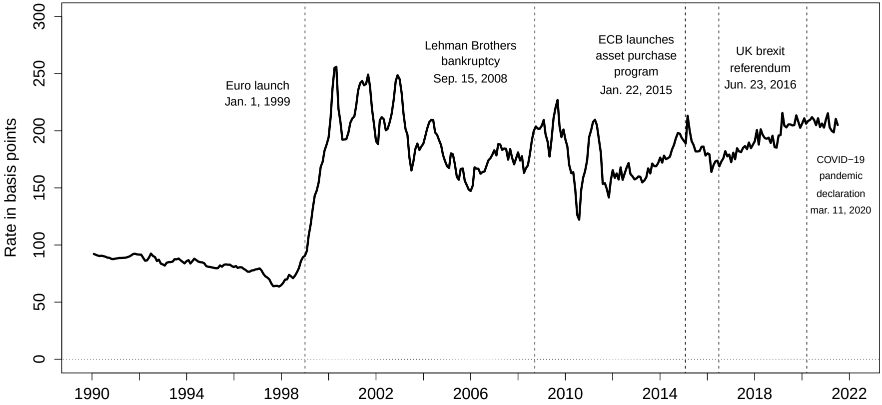
- First, we use the estimated parameters and the filtered states to calculate the fitted bond prices $\hat{P}_t$.
- Second, we calculate the matching frictionless price $\tilde{P}_t$ by switching off the bond-specific risk factor.
- These bond prices are then converted into yields to maturity by solving the fixed-point problem

$$\begin{aligned} P_t &= C(t_1 - t)\exp\{-(t_1 - t)y_t^c\} + \sum_{k=2}^{n} \frac{C}{2} \exp\{-(t_k - t)y_t^c\} \\ &\quad + \exp\{-(T - t)y_t^c\} \end{aligned}$$

  with the solutions denoted $\hat{y}_t^c$ and $\tilde{y}_t^c$, respectively.
- The convenience premium for the ith bond is then defined as

$$\Psi_t^i \equiv \tilde{y}_t^{FL,i} - \hat{y}_t^{FIT,i} \quad \text{(frictionless - fitted yield).}$$

Average Index-Linked Gilt Convenience Premium



- Shown is the average estimated liquidity premium for the parsimonious AFNS-R model with diagonal $K^{\mathbb{P}}$ and Σ.
- It averaged less than 100 bps in the 1990-1998 period.
- It then spiked and has averaged about 200 bps since.
- Is the euro introduction in 1999 a factor behind this?

Full Sample Regression Analysis

To begin, we run regressions of the form:

$$\bar{\Psi}_t = \alpha + \delta_{euro} d_t^{euro} + \sum_{l=0}^{L} \delta_l X_{t-l} + \varepsilon_t,$$

where

- d_t^{euro} ia a dummy variable taking the value of one from Jan. 1999 onwards;
- X_t is a vector of control variables;
- L is the number of lags included;
- ε_t is a random residual.

Full Sample Regression Results

	1	2	3
α	81.91	38.82	-23.00
	(0.00)	(0.09)	(0.08)
δ_{euro}	108.70	97.33	80.75
	(0.00)	(0.00)	(0.00)
Controls	No	All	All
L	-	12	24
Adj. R^2	0.86	0.94	0.92
DW	0.20	0.56	2.19

- Clear evidence of a significant *positive* effect on the index-linked gilt convenience premium following the launch of the euro in Jan. 1999 — about 80-100 bps.
- Adding controls does not alter that conclusion. Neither does an expanding window analysis (see paper).
- Remaining question: Do regulatory or institutional features create excess demand for these bonds?

Outline

Term Premium Calculation

We define real term premiums in the standard way:

$$TP_t(\tau) = y_t(\tau) - \frac{1}{\tau}\int_t^{t+\tau} E_t^{\mathbb{P}}[r_s]ds,$$

where

- $y_t(\tau)$ refers to the adjusted frictionless zero-coupon yield;
- $E_t^{\mathbb{P}}[r_s]$ is the expected future frictionless short rate.
- Note: $\mathbb{P}$-dynamics are important.
- To accommodate the documented break in X_t^R, we model it as an independent factor under the $\mathbb{P}$-measure.

Model Selection in the G-O-R Model

For yield decompositions, the $\mathbb{P}$-dynamics of the state variables are crucial.

To find the appropriate $\mathbb{P}$-dynamics, we use a general-to-specific strategy starting from an unrestricted $K^{\mathbb{P}}$:

$$K^{\mathbb{P}} = \begin{pmatrix} \kappa_{11}^{\mathbb{P}} & \kappa_{12}^{\mathbb{P}} & \kappa_{13}^{\mathbb{P}} & 0 \\ \kappa_{21}^{\mathbb{P}} & \kappa_{22}^{\mathbb{P}} & \kappa_{23}^{\mathbb{P}} & 0 \\ \kappa_{31}^{\mathbb{P}} & \kappa_{32}^{\mathbb{P}} & \kappa_{33}^{\mathbb{P}} & 0 \\ 0 & 0 & 0 & \kappa_{44}^{\mathbb{P}} \end{pmatrix}.$$

In each step, the parameter with the lowest t-statistic is eliminated.

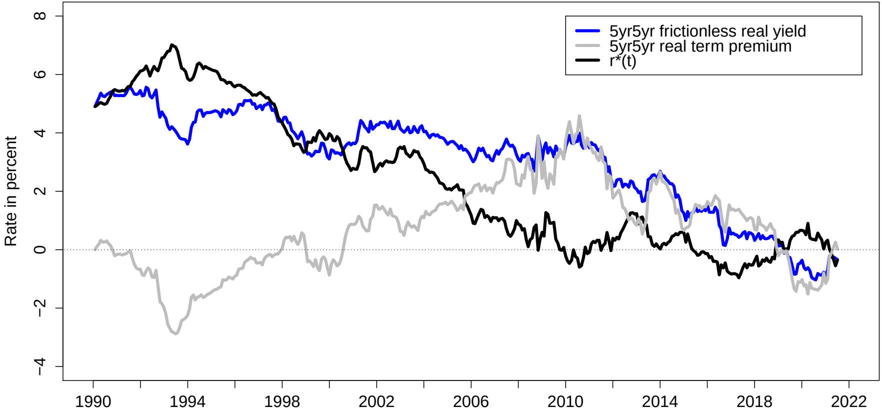
As in Christensen et al. (2014), the Bayesian Information Criterion and marginal likelihood ratio tests are used to find the optimal stopping point.

Preferred G-O-R Model Specification

$K^{\mathbb{P}}$	L_t	S_t	C_t	X_t^R	$\theta^{\mathbb{P}}$	Σ
L_t	0.0049	0	0	0	0.0631	0.0039
	(0.0157)				(0.0635)	(0.0001)
S_t	0	0.4915	0.2863	0	-0.0177	0.0197
		(0.0698)	(0.0641)		(0.0151)	(0.0009)
C_t	0	0	0.2126	0	-0.0247	0.0217
			(0.0661)		(0.0240)	(0.0009)
X_t^R	0	0	0	0.0689	0.0911	0.0634
				(0.0620)	(0.0756)	(0.0054)

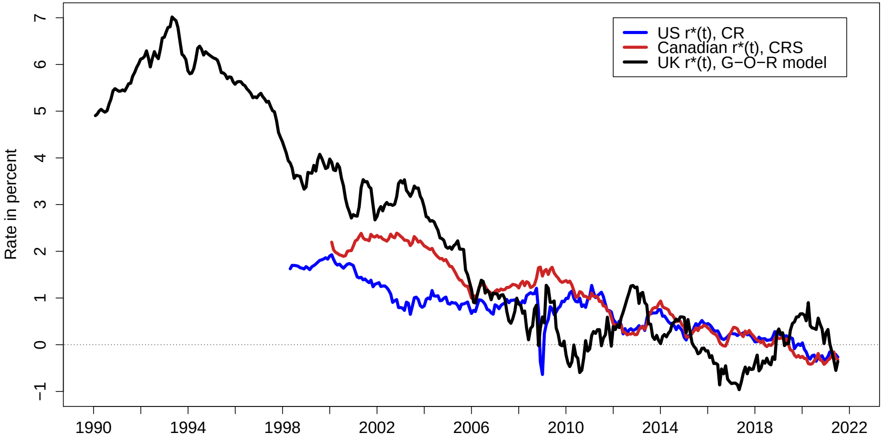
- Only dynamic interaction is between the frictionless slope and curvature factors.
- The frictionless level factor is close to a unit-root process.

5yr5yr Real Yield Decomposition



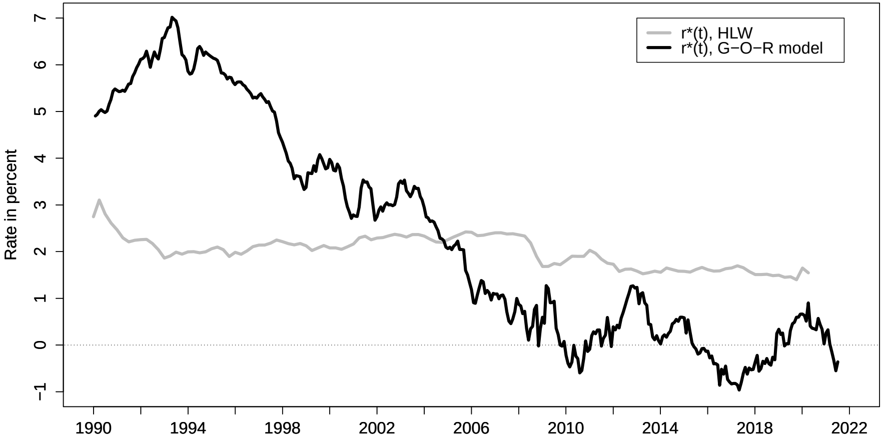
- Shown is the decomposition of the 5yr5yr real yield from the preferred G-O-R model.
- The real term premium has pronounced countercyclical variation but no long-term trend.
- In contrast, the r_t^* estimate has a persistent downward trend.

Comparison with Foreign Market-Based r_t^* Estimates



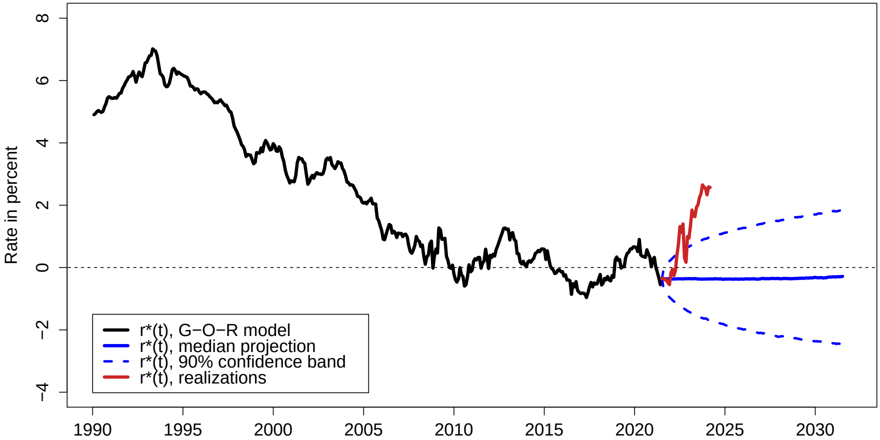
- Shown are market-based r_t^* estimates from the U.S. and Canada taken from Christensen and Rudebusch (2019) and Christensen et al. (2021), resp.
- Our U.K. r_t^* estimate exhibits a similar secular decline.

Comparison with a Macro-Based Estimate of r_t^*



- Shown is a macro-based estimate of r_t^* taken from Holston et al. (HLW) (2017).
- Our finance-based estimate of r_t^* exhibits a pronounced secular decline unlike the macro-based estimate.

Ten-Year Projections of r_t^*



- Following Christensen et al. (JME, 2015), we use simulations to generate probability-based projections of r_t^* implied by the G-O-R model as of June 30, 2021.
- The median path indicated no change.
- Realizations have been much above expected levels.

Outline

Conclusion

- To complement earlier empirical work based on macroeconomic models and data, we estimate the equilibrium real rate using only prices of index-linked gilts.
- By adjusting for both convenience and real term premiums, we uncover investors' expectations for the underlying frictionless real short rate for the five-year period starting five years ahead.
- The resulting finance-based measure of r_t^* offers new evidence that the equilibrium interest rate in the U.K. has steadily declined the past three decades.
- Realizations since June 2021 have been well above expected levels.
- Hence, higher rate levels may be here to stay in the U.K.