APPENDIX TO "RESERVE REQUIREMENTS AND OPTIMAL CHINESE STABILIZATION POLICY"

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ABSTRACT. This appendix shows some additional details of the model and equilibrium conditions in Chang, Liu, Spiegel, and Zhang (2017).

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I. Optimizing conditions

I.1. **Household decisions.** The household optimizing conditions are summarized by the following equations:

$$\Lambda_t = \frac{1}{C_t}, \quad (1)$$

$$w_t = \frac{\Psi H_t^\eta}{\Lambda_t}, \quad (2)$$

$$1 = \mathrm{E}_t \beta R_t \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}, \quad (3)$$

$$1 = \frac{\Psi_m}{\Lambda_t M_t / P_t} + \mathrm{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}, \quad (4)$$

$$1 = q_t^k \left[1 - \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2 - \Omega_k \left(\frac{I_t}{I_{t-1}} - g_I\right)\frac{I_t}{I_{t-1}}\right] + \beta \mathrm{E}_t q_{t+1}^k \frac{\Lambda_{t+1}}{\Lambda_t} \Omega_k \left(\frac{I_{t+1}}{I_t} - g_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \quad (5)$$

$$q_t^k = \beta \mathrm{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}[q_{t+1}^k(1-\delta) + r_{t+1}^k], . \quad (6)$$

where Λ_t denotes the Lagrangian multiplier for the budget constraint, $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the inflation rate from period $t-1$ to period t, and $q_t^k \equiv \frac{\Lambda_t^k}{\Lambda_t}$ is Tobin's q, with Λ_t^k being the Lagrangian multiplier for the capital accumulation equation.

I.2. **Optimal debt contracts.** Under the loan contract characterized by $\bar{\omega}_{jt}$ and B_{jt}, the expected nominal income for a firm in sector j is given by

$$\begin{aligned}
&\int_{\overline{\omega}_{jt}}^{\infty} \tilde{A}_{jt}\omega_{jt}(N_{j,t-1} + B_{jt})dF(\omega) - (1 - F(\overline{\omega}_{jt}))Z_{jt}B_{jt} \\
&= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})[\int_{\overline{\omega}_{jt}}^{\infty} \omega dF(\omega) - (1 - F(\overline{\omega}_{jt}))\overline{\omega}_{jt}] \\
&\equiv \tilde{A}_{jt}(N_{j,t-1} + B_{jt}) f(\overline{\omega}_{jt}),
\end{aligned}$$

where $f(\overline{\omega}_{jt})$ is the share of production revenue going to the firm under the loan contract.

The expected nominal income for the lender is given by,

$$\begin{aligned}
&(1 - F(\overline{\omega}_{jt}))Z_{jt}B_{jt} + \int_0^{\overline{\omega}_{jt}} \{(1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt}) \\
&\qquad + l_j[Z_{jt}B_{jt} - (1 - m_j)\tilde{A}_{jt}\omega(N_{j,t-1} + B_{jt})]\}dF(\omega) \\
&= \tilde{A}_{jt}(N_{j,t-1} + B_{jt})\{[1 - (1 - l_j)F(\overline{\omega}_{jt})]\bar{\omega}_{jt} + (1 - m_j)(1 - l_j)\int_0^{\overline{\omega}_t} \omega dF(\omega)\} \\
&\equiv \tilde{A}_{jt}(N_{j,t-1} + B_{jt}) g_{jt}(\overline{\omega}_{jt}),
\end{aligned} \quad (7)$$

where $g_j(\overline{\omega}_{jt})$ is the share of production revenue going to the lender. Note that

$$f(\overline{\omega}_{jt}) + g_{jt}(\overline{\omega}_{jt}) = 1 - m_j \int_0^{\overline{\omega}_{jt}} \omega dF(\omega) + l_j \int_0^{\overline{\omega}_{jt}} [\overline{\omega}_{jt} - (1-m_j)\omega] dF(\omega). \tag{8}$$

The optimal contract is a pair $(\bar{\omega}_{jt}, B_{jt})$ chosen at the beginning of period t to maximize the borrower's expected period t income,

$$\max \tilde{A}_{jt}(N_{j,t-1} + B_{jt}) f(\overline{\omega}_{jt}) \tag{9}$$

subject to the lender's participation constraint

$$\tilde{A}_{jt}(N_{j,t-1} + B_{jt}) g_j(\overline{\omega}_{jt}) \geq R_{jt} B_{jt}. \tag{10}$$

The optimizing conditions for the contract characterize the relation between the leverage ratio and the productivity cut-off

$$\frac{N_{j,t-1}}{B_{jt} + N_{j,t-1}} = -\frac{g_j'(\overline{\omega}_{jt})}{f'(\overline{\omega}_{jt})} \frac{\tilde{A}_{jt} f(\overline{\omega}_{jt})}{R_{jt}}. \tag{11}$$

II. Summary of equilibrium conditions

On a balanced growth path, output, consumption, investment, real bank loans and real wage rates all grow at a constant rate g. To obtain balanced growth, we make the stationary transformations

$$\begin{aligned}
&y_t^f = \frac{Y_t^f}{g^t}, \gamma_{mt} = \frac{\Gamma_t}{g^t}, c_t = \frac{C_t}{g^t}, i_t = \frac{I_t}{g^t}, k_t = \frac{K_t}{g^t}, gdp_t = \frac{GDP_t}{g^t}, \lambda_t = \Lambda_t g^t, \\
&y_{st} = \frac{Y_{st}}{g^t}, y_{pt} = \frac{Y_{pt}}{g^t}, k_{st} = \frac{K_{st}}{g^t}, k_{pt} = \frac{K_{pt}}{g^t}, \tilde{w}_t = \frac{w_t}{g^t}, \tilde{w}_{s,e,t} = \frac{w_{s,e,t}}{g^t}, \tilde{w}_{p,e,t} = \frac{w_{p,e,t}}{g^t}, \\
&n_{st} = \frac{N_{st}}{P_t g^t}, n_{pt} = \frac{N_{pt}}{P_t g^t}, b_{st} = \frac{B_{st}}{P_t g^t}, b_{pt} = \frac{B_{pt}}{P_t g^t}, m_t = \frac{M_t}{P_t g^t}, m_t^s = \frac{M_t^s}{P_t g^t}.
\end{aligned}$$

On the balanced growth path, the transformed variables, the interest rate and the inflation rate are all constants.

The balanced growth equilibrium is summarized by the following equations:

1) Households.

$$k_t = \tfrac{1-\delta}{g} k_{t-1} + i_t[1 - \tfrac{\Omega_k}{2}(\tfrac{i_t g}{i_{t-1}} - g)^2], \tag{A1}$$

$$\lambda_t = \tfrac{1}{c_t}, \tag{A2}$$

$$\tilde{w}_t = \Psi \tfrac{H_t^\eta}{\lambda_t}, \tag{A3}$$

$$1 = \mathrm{E}_t \beta \tfrac{\lambda_{t+1}}{\lambda_t} \tfrac{R_t}{\pi_{t+1} g}, \tag{A4}$$

$$1 = \tfrac{\Psi_m}{\lambda_t m_t} + \mathrm{E}_t \beta \tfrac{\lambda_{t+1}}{\lambda_t} \tfrac{1}{\pi_{t+1} g}, \tag{A5}$$

$$1 = q_t^k [1 - \tfrac{\Omega_k}{2}(\tfrac{i_t g}{i_{t-1}} - g)^2 - \Omega_k(\tfrac{g i_t}{i_{t-1}} - g)\tfrac{i_t g}{i_{t-1}}] + \beta \mathrm{E}_t q_{t+1}^k \tfrac{\lambda_{t+1}}{\lambda_t g} \Omega_k(\tfrac{i_{t+1} g}{i_t} - g)(\tfrac{i_{t+1} g}{i_t})^2, \tag{A6}$$

$$q_t^k = \beta \mathrm{E}_t \tfrac{\lambda_{t+1}}{\lambda_t g}[(1-\delta) q_{t+1}^k + r_{t+1}^k]. \tag{A7}$$

2) Firms and banks.

$$y_{st} = A_t^m \bar{A}_s k_{st}^{1-\alpha} (H_{st}^\theta)^\alpha, \tag{A8}$$

$$\tilde{w}_t H_{st} = \alpha\theta(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}), \tag{A9}$$

$$\tilde{w}_{s,e,t} = (\tfrac{n_{s,t-1}}{\pi_t g} + b_{st})\alpha(1-\theta), \tag{A10}$$

$$k_{st} r_t^k = (1-\alpha)(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}), \tag{A11}$$

$$\tilde{A}_{st} = \tfrac{p_{st} y_{st}}{\frac{n_{s,t-1}}{\pi_t g} + b_{st}}, \tag{A12}$$

$$\tilde{A}_{st}(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}) g_s(\overline{\omega}_{st}) = b_{st} R_{st}, \tag{A13}$$

$$\tfrac{\frac{n_{s,t-1}}{\pi_t g}}{\frac{n_{s,t-1}}{\pi_t g} + b_{st}} = -\tfrac{g_s'(\overline{\omega}_{st}) f(\overline{\omega}_{jt})}{f'(\overline{\omega}_{st})} \tfrac{\tilde{A}_{st}}{R_{st}}, \tag{A14}$$

$$n_{st} = \tilde{w}_{s,e,t} + \xi_s \tilde{A}_{st}(\tfrac{n_{s,t-1}}{\pi_t g} + b_{st}) f(\overline{\omega}_{st}), \tag{A15}$$

$$y_{pt} = A_t^m \bar{A}_p k_{pt}^{1-\alpha} (H_{pt}^\theta)^\alpha, \tag{A16}$$

$$\tilde{w}_t H_{pt} = \alpha\theta(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A17}$$

$$\tilde{w}_{p,e,t} = \alpha(1-\theta)(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A18}$$

$$r_t^k k_{pt} = (1-\alpha)(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}), \tag{A19}$$

$$\tilde{A}_{pt} = \tfrac{p_{pt} y_{pt}}{\frac{n_{p,t-1}}{\pi_t g} + b_{pt}}, \tag{A20}$$

$$\tilde{A}_{pt}(\tfrac{n_{p,t-1}}{\pi_t g} + b_{pt}) g_p(\overline{\omega}_{pt}) = b_{pt} R_{pt}. \tag{A21}$$

$$\frac{\frac{n_{p,t-1}}{\pi_t g}}{\frac{n_{p,t-1}}{\pi_t g}+b_{pt}} = -\frac{g_p'(\overline{\omega}_{pt})f(\overline{\omega}_{pt})}{f'(\overline{\omega}_{pt})}\frac{\tilde{A}_{pt}}{R_{pt}}, \tag{A22}$$

$$n_{pt} = \tilde{w}_{p,e,t} + \xi_p \tilde{A}_{pt}(\frac{n_{p,t-1}}{\pi_t g} + b_{pt})f(\overline{\omega}_{pt}), \tag{A23}$$

$$(R_{st}-1)(1-\tau_t) = R_t - 1, \tag{A24}$$

$$R_{pt} = R_t. \tag{A25}$$

3) Pricing, market clearing and monetary policy.

$$p_{wt} = \frac{\epsilon-1}{\epsilon} + \frac{\Omega_p}{\epsilon}\frac{1}{y_t}[(\frac{\pi_t}{\pi}-1)\frac{\pi_t}{\pi}c_t - \beta \mathrm{E}_t\frac{\lambda_{t+1}}{\lambda_t}(\frac{\pi_{t+1}}{\pi}-1)\frac{\pi_{t+1}}{\pi}c_{t+1}], \tag{A26}$$

$$\ln\left(\frac{\mu_t}{\mu}\right) = \psi_{mp}\ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \psi_{my}\ln\left(\frac{\gamma_t}{\bar{\gamma}}\right), \tag{A27}$$

$$\ln\left(\frac{\tau_t}{\tau}\right) = \psi_{\tau p}\ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \psi_{\tau y}\ln\left(\frac{\gamma_t}{\bar{\gamma}}\right), \tag{A28}$$

$$\mu_t = \frac{m_t^s}{m_{t-1}^s}, \tag{A29}$$

$$\gamma_t = \frac{GDP_t}{GDP_{t-1}}, \tag{A30}$$

$$m_t^s = m_t + d_t - b_{pt}, \tag{A31}$$

$$g_t = gdp_t g^c, \tag{A32}$$

$$\begin{aligned} y_t^f = {} & i_t + c_t + g_t + c_t\frac{\Omega_p}{2}(\frac{\pi_t}{\pi}-1)^2 + \tilde{A}_{st}(\frac{n_{s,t-1}}{\pi_t g} + b_{st})m_s\int_0^{\overline{\omega}_{st}}\omega dF(\omega) \\ & + \tilde{A}_{pt}(\frac{n_{p,t-1}}{\pi_t g} + b_{pt})m_p\int_0^{\overline{\omega}_{pt}}\omega dF(\omega), \end{aligned} \tag{A33}$$

$$y_t^f = \gamma_t, \tag{A34}$$

$$\gamma_{mt} = \left(\phi y_{st}^{\frac{\sigma_m-1}{\sigma_m}} + (1-\phi)y_{pt}^{\frac{\sigma_m-1}{\sigma_m}}\right)^{\frac{\sigma_m}{\sigma_m-1}}, \tag{A35}$$

$$y_{st} = \phi^{\sigma_m}\left(\frac{p_{st}}{p_{wt}}\right)^{-\sigma_m} y_t, \tag{A36}$$

$$y_{pt} = (1-\phi)^{\sigma_m}\left(\frac{p_{pt}}{p_{wt}}\right)^{-\sigma_m} y_t, \tag{A37}$$

$$\frac{k_{t-1}}{g} = k_{st} + k_{pt}, \tag{A38}$$

$$H_t = H_{st} + H_{pt}, \tag{A39}$$

$$gdp_t = g_t + i_t + c_t. \tag{A40}$$

where

$$f(\overline{\omega}_{st}) = \tfrac{1}{k-1}\omega_m^k \overline{\omega}_{st}^{1-k}, \tag{A41}$$

$$f'(\overline{\omega}_{st}) = -\omega_m^k \overline{\omega}_{st}^{-k}, \tag{A42}$$

$$g_s(\overline{\omega}_{st}) = \omega_m \tfrac{k}{k-1}(1-l_s)(1-m_s) + l_s\overline{\omega}_{st} + (1-l_s)[1 - \tfrac{(1-m_s)k}{k-1}]\omega_m^k \overline{\omega}_{st}^{1-k}, \tag{A43}$$

$$g_s'(\overline{\omega}_{st}) = l_s + (1-l_s)(1-m_s k)\omega_m^k \overline{\omega}_{st}^{-k}, \tag{A44}$$

$$f(\overline{\omega}_{pt}) = \tfrac{1}{k-1}\omega_m^k \overline{\omega}_{pt}^{1-k}, \tag{A45}$$

$$f'(\overline{\omega}_{pt}) = -\omega_m^k \overline{\omega}_{pt}^{-k}, \tag{A46}$$

$$g_p(\overline{\omega}_{pt}) = \omega_m \tfrac{k}{k-1}(1-l_p)(1-m_p) + l_p\overline{\omega}_{pt} + (1-l_p)[1 - \tfrac{(1-m_p)k}{k-1}]\omega_m^k \overline{\omega}_{pt}^{1-k}, \tag{A47}$$

$$g_p'(\overline{\omega}_{pt}) = l_p + (1-l_p)(1-m_p k)\omega_m^k \overline{\omega}_{pt}^{-k}, \tag{A48}$$

$$\textstyle\int_0^{\overline{\omega}_{st}} \omega dF(\omega) = \tfrac{k}{k-1}(\omega_m - \omega_m^k \overline{\omega}_{st}^{1-k}), \tag{A49}$$

$$\textstyle\int_0^{\overline{\omega}_{pt}} \omega dF(\omega) = \tfrac{k}{k-1}(\omega_m - \omega_m^k \overline{\omega}_{pt}^{1-k}). \tag{A50}$$

The system of 40 equations from (A1) to (A40) determine the equilibrium solution for the 40 endogenous variables summarized in the vector,

$$\begin{aligned}
&[y_t^f, \gamma_{mt}, c_t, i_t, g_t, gdp_t, k_t, \lambda_t, \tilde{q}_t^k, H_t, H_{st}, H_{pt},\\
&y_{st}, y_{pt}, k_{st}, k_{pt}, n_{st}, n_{pt}, b_{st}, b_{pt}, \tilde{A}_{st}, \tilde{A}_{pt}, \overline{\omega}_{st}, \overline{\omega}_{pt},\\
&m_t, m_t^s, \mu_t, \gamma_t,\\
&\tilde{w}_t, \tilde{w}_{s,e,t}, \tilde{w}_{p,e,t}, r_t^k, R_t, R_{st}, R_{pt}, \pi_t, p_{wt}, p_{st}, p_{pt}, \tau_t]
\end{aligned}$$

III. Macro effects of SOE-specific TFP shocks

In the text, we discuss the macro implications of an aggregate TFP shock and a POE-specific TFP shock. Here, we summarize our findings about the macro effects an SOE-specific TFP shock.

Table 1 shows the volatilities of several macroeconomic variables and welfare under the benchmark policy regime as well as under the three alternative optimal policy rules.

The table shows that, in response to a positive SOE TFP shock, the individually optimal τ rule and μ rule both call for a tightening of monetary policy. Since the shock raises GDP and lowers inflation under the benchmark policy, the central bank raises τ in response to the increase in GDP growth and lowers τ to accommodate the decline in inflation. Similarly, when the central bank can optimally choose the reaction coefficients in the μ rule, it reduces money growth in response to the acceleration in GDP growth and raises money growth to mitigate declines in inflation. Under the jointly optimal rule, the optimal reaction coefficients in both policy rules indicate more aggressive monetary policy responses. Acting alone, each of the two alternative optimal rules (i.e., the τ rule and the μ rule) can better stabilize real GDP and inflation fluctuations and also improve welfare relative to the benchmark policy regime. The jointly optimal rule leads to further welfare improvement relative to the individually optimal rules. In this sense, adjusting reserve requirements is a complementary policy instrument to the conventional policy instrument (money supply growth) in the economy buffeted by SOE-specific TFP shocks. This finding is qualitatively similar to that under POE-specific TFP shocks, as we discuss in the text. In both cases, the sector-specific shocks call for resource reallocation across sectors, and the use of reserve requirements in combination with money supply adjustments helps enhance welfare and macro stability.

Figure 1 shows the impulse responses to a positive SOE TFP shock. In the benchmark economy with a constatnt τ and calibrated parameters in the μ rule (the black solid lines), the shock raises real GDP and lowers inflation. It also raises SOE output and reduces POE output relative to their steady-state levels.

Under the optimal τ rule (the red dashed lines), the central bank responds to the increase in real GDP by tightening reserve requirements (as shown in Table 1). On the other hand, the central bank also responds to the decline in inflation by easing reserve requirements, which stimulates aggregate demand. Overall, the leads to a small decline in τ. These policy reactions help stabilize fluctuations in both real GDP and inflation. The decline in τ also reallocates resources from the POE sector to the SOE sector, which experiences a positive TFP shock. Although the lower τ reduces the funding costs for the SOE sector and thus reduces the incidence of SOE failures, it also stimulates aggregate demand. Through the financial accelerator that operates in the POE sector (but not for SOEs), the expansion in aggregate demand raises POE output relative to SOE output despite the improvement in SOE productivity and the reallocation effects.

Under the optimal μ rule (the blue dashed lines), the central bank reduces money supply to dampen the increase in real GDP and raises money supply to stimulate aggregate demand and therefore mitigate the declines in inflation. The policy is very effective for stabilizing both real GDP and inflation, as shown in Table 1. Similar to the case with the optimal τ rule, POE output is more responsive to the aggregate demand expansion than SOE output because the financial accelerator is operating in the POE sector but not in the SOE sector.

Finally, under the jointly optimal rule (the magenta dashed lines), the central bank reacts to the SOE TFP shock by aggressively cutting both mony growth and reserve requirements. These policy reactions further improves welfare relative to the individually optimal policies.

References

Chang, C., Z. Liu, M. M. Spiegel, and J. Zhang (2017): "Reserve Requirements and Optimal Chinese Stabilization Policy," Federal Reserve Bank of San Francisco Working Paper 2016-10.

Table 1. Volatilities and welfare under alternative policy rules: SOE-specific TFP shock

Variables	Benchmark	Optimal τ rule	Optimal money rule	Jointly optimal rule
Policy rule coefficients				
ψ_{mp}	-0.65	-0.65	-54.63	-128.10
ψ_{my}	0.30	0.30	-3.11	-62.29
$\psi_{\tau p}$	0.00	49.58	0.00	1000.00
$\psi_{\tau y}$	0.00	7.56	0.00	69.25
Volatility and welfare				
GDP	1.460%	1.425%	1.413%	1.403%
π	0.560%	0.086%	0.032%	0.045%
C	1.139%	1.103%	1.086%	1.093%
H	0.289%	0.283%	0.262%	0.302%
R	0.139%	0.157%	0.136%	0.153%
Y_s	7.304%	7.329%	7.257%	7.640%
Y_p	1.423%	1.352%	1.337%	1.577%
Welfare gains	—	0.0260%	0.0265%	0.0272%

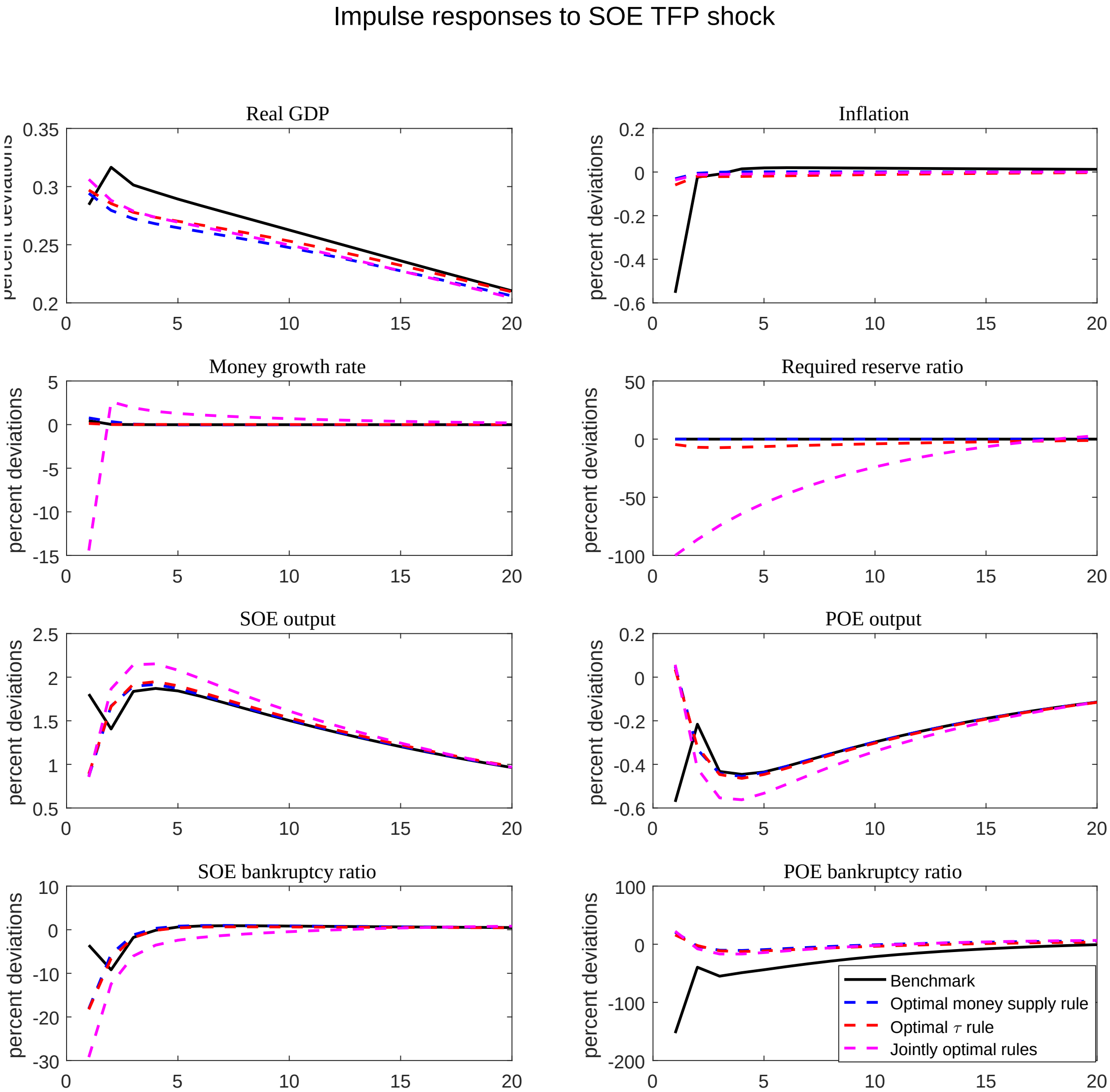

FIGURE 1. Impulse responses to a positive SOE TFP shock under alternative policy rules. Benchmark rule: black solid lines; optimal money growth rule: blue dashed lines; optimal reserve requirement rule: red dashed lines; jointly optimal rule: magenta dashed lines. The horizontal axes show the quarters after the impact period of the shock. The units on the vertical axes are percent deviations from the steady state levels.