DYNAMIC CONDITIONAL BETA

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ARE BETAS CONSTANT?

- LEAST SQUARES MODELS ARE USED IN COUNTLESS EMPIRICAL STUDIES IN FINANCE AND ECONOMICS
- RARELY IS THE HYPOTHESIS THAT BETAS ARE CONSTANT GIVEN CAREFUL SCRUTINY
- WHAT TOOLS DO WE HAVE?

HAMILTON SEMINAL CONTRIBUTION

* Regime switching models
  + These allow the betas to switch from one value to another
  + The trigger can be simply a constant probability
  + Or a set of observables
  + There can be multiple states
  + These can be estimated with a Kalman Filter
* What is wrong with these powerful models?
  + Nothing. But only one beta has ever(?) been time varying. Number of discrete states is limited. Specification search is potentially complex.
  + Test of no switching is hard to get sized correctly

MODELLING TIME VARYING BETA

- ROLLING REGRESSION
- INTERACTING VARIABLES WITH TRENDS, SPLINES OR OTHER OBSERVABLES
- TIME VARYING PARAMETER MODELS BASED ON KALMAN FILTER
- STRUCTURAL BREAK AND REGIME SWITCHING MODELS
- EACH OF THESE SPECIFIES CLASSES OF PARAMETER EVOLUTION THAT MAY NOT BE CONSISTENT WITH ECONOMIC THINKING OR DATA.

THE BASIC IDEA OF DCB

- IF $\left(y_t, x_t\right), \; t = 1, \ldots, T$ is a collection of k+1 random variables that are distributed as

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} \Big| \mathbb{F}_{t-1} \sim N\left(\mu_t, H_t\right) = N\left(\begin{pmatrix} \mu_{y,t} \\ \mu_{x,t} \end{pmatrix}, \begin{pmatrix} H_{yy,t} & H_{yx,t} \\ H_{xy,t} & H_{xx,t} \end{pmatrix}\right)$$

- Then

$$y_t \Big| x_t, \mathbb{F}_{t-1} \sim N\left(\mu_{y,t} + H_{yx,t} H_{xx,t}^{-1}\left(x_t - \mu_{x,t}\right), H_{yy,t} - H_{yx,t} H_{xx,t}^{-1} H_{xy,t}\right)$$

- Hence: $\beta_t = H_{xx,t}^{-1} H_{xy,t}$

HOW TO ESTIMATE H

* Econometricians have developed a wide range of approaches to estimating large covariance matrices. These include
  + Multivariate GARCH models such as VEC and BEKK
  + Constant Conditional Correlation models
  + Dynamic Conditional Correlation models
  + Dynamic Equicorrelation models
  + Multivariate Stochastic Volatility Models
  + Many many more
* Exponential Smoothing with prespecified smoothing parameter.

MLE

- $$L\left(\theta, \ddot{y}, \ddot{x}\right) = L_{y|x}\left(\theta, \ddot{y}, \ddot{x}\right) + L_x\left(\theta, \ddot{x}\right), \quad \ddot{y} \equiv \left\{y_1, ..., y_T\right\}$$

- When all betas are DCBs. Then estimate Multivariate GARCH and compute coefficients.

$$y_t - x_t'\beta\left(\hat{\theta}\right)_t \equiv w_t, \quad L_{y|x}\left(\theta\right) = -.5\sum_{t=1}^{T}\left(\log(h_{w,t}) + \frac{w_t^2}{h_{w,t}}\right)$$

- When no betas are DCBs, then estimate regression with heteroskedastic errors.

$$y_t = x_t'\beta + u_t, \quad V_{t-1}\left(u_t\right) = h_{u,t}, \quad L_{y|x}\left(\theta\right) = -.5\sum_{t=1}^{T}\left(\log(h_{u,t}) + \frac{u_t^2}{h_{u,t}}\right)$$

- When some betas are DCBs, then subtract time varying coefficients and estimate constant ones.

IS BETA CONSTANT?

- For none of these methods will beta appear constant.

- In the one regressor case this requires the ratio of $h_{yx,t} / h_{xx,t}$ to be constant.

- This is a non-nested hypothesis

  + Either $h_{yx,t} = \beta h_{xx,t}$

  + Or it is given by a bivariate GARCH model

NON-NESTED HYPOTHESES

* Model Selection based on information criteria
    + Two possible outcomes
* Artificial Nesting
    + Four possible outcomes
* Testing equal closeness- Quang Vuong
    + Three possible outcomes

CLASSIFICATION:

* Models may be nested

* Models may be non-nested

* Models may be *partially nested* or *overlapping.* In this case there are some parameter values that are nested and others that are not.

OVERLAPPING

- Constant beta and Dynamic Beta models are Overlapping.

- If there is no heteroskedasticity, then the models are the same and are thus partially nested.

- To deal with this point of overlap, it is sufficient to verify that there is heteroskedasticity. Effectively this is a sequential approach as recommended by Vuong.

COMPARISON OF PENALIZED LIKELIHOOD

- Select the model with the highest value of penalized log likelihood. Choice of penalty is a finite sample consideration- all are consistent.

- Estimate model with constant betas and heteroskedastic residuals

- Estimate model with DCB and heteroskedastic residuals.

- Compare information criteria.

ARTIFICIAL NESTING

* Consider the model:

$$y_t = \beta' x_t + (\gamma \circ \beta_t)' x_t + v_t$$

* If gamma is zero, the parameters are constant
* If beta is zero, the parameters are time varying.
* If both are non-zero, the nested model may be entertained.

MULTIFACTOR ASSET PRICING

MULTIFACTOR ASSET PRICING EXAMPLE

* Excess Returns of one asset regressed on risk factors should have insignificant alpha.

* Do FF factors price individual stocks and other asset classes?

* Are the betas constant?

* Examine *daily industry* returns *1963 -2011* and *FF three factors* from Ken French website.

MULTI-FACTOR PRICING KERNEL

- Standard Asset Pricing Theory

$$m_t = a_t - b_t ' f_t$$

$$1 = E_{t-1}\left(m_t\left(r_t^f + 1\right)\right), \quad 1 + r_t^f = 1/E_{t-1}\left(m_t\right)$$

$$E_{t-1}\left(r_t - r_t^f\right) = \left(1 + r_t^f\right)Cov_{t-1}\left(r_t, f_t\right)b_t$$

$$= \beta_{r,f,t}\left(1 + r_t^f\right)Var_{t-1}\left(f_t\right)b_t$$

$$= \beta_{r,f,t}E_{t-1}\left(f_t\right)$$

- Implies DCB should price assets – expected return is linear in dynamic conditional beta

$$\beta_{r,f,t} = Cov_{t-1}\left(r_t, f_t\right)\left[V_{t-1}\left(f_t\right)\right]^{-1}$$

MULTI-FACTOR PRICING MODEL

✖ Estimate

$$r_t^j - r_t^f = \alpha_j + \beta_{j,m}\left(r_t^m - r_t^f\right) + \beta_{j,hml}r_t^{hml} + \beta_{j,smb}r_t^{smb} + \sqrt{h_t^j}\,\varepsilon_t^j$$

+ By OLS with constant coefs and robust s.e.

+ By GJR-GARCH with constant coefficients

+ By DCB with DCC for the factors. Bivariate DCC parameters for the correlations between factor and dependent variable are restricted to equal factor parameters.

+ NESTED DCB includes each factor with constant coefficient and time varying coefficient
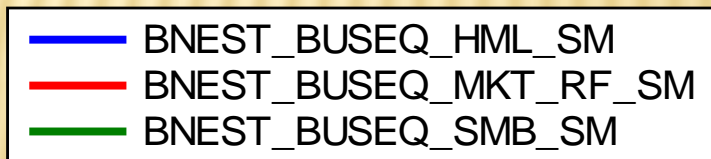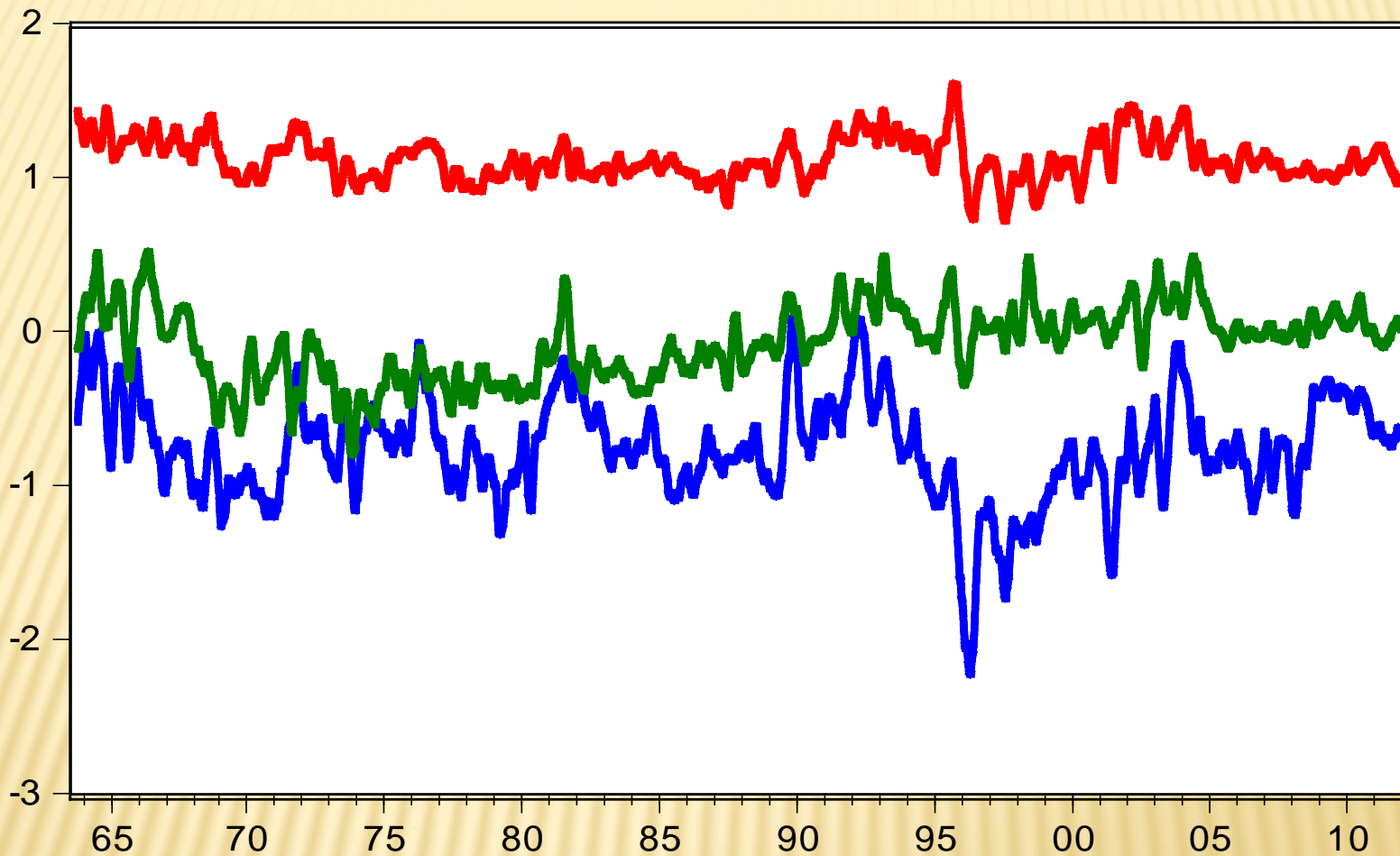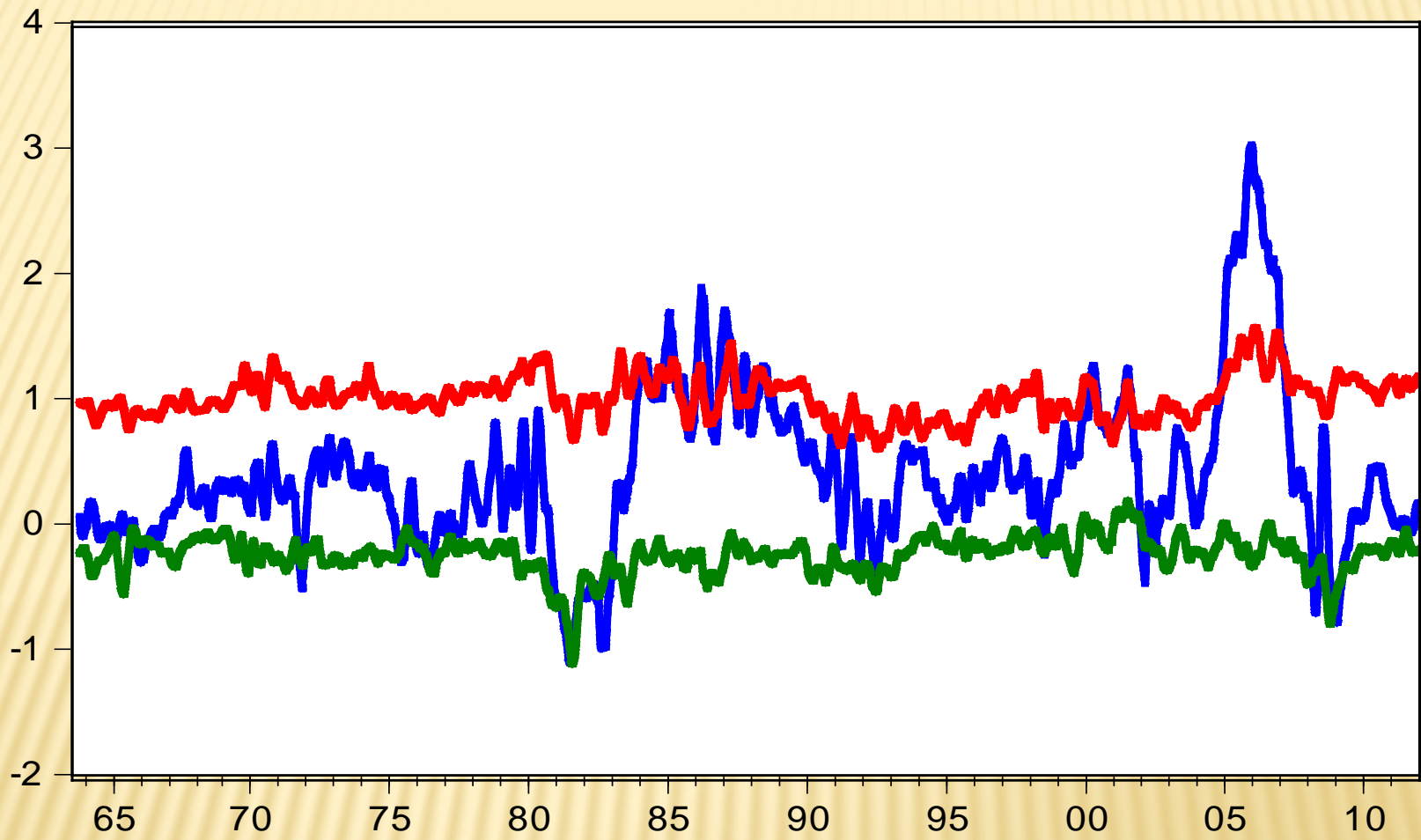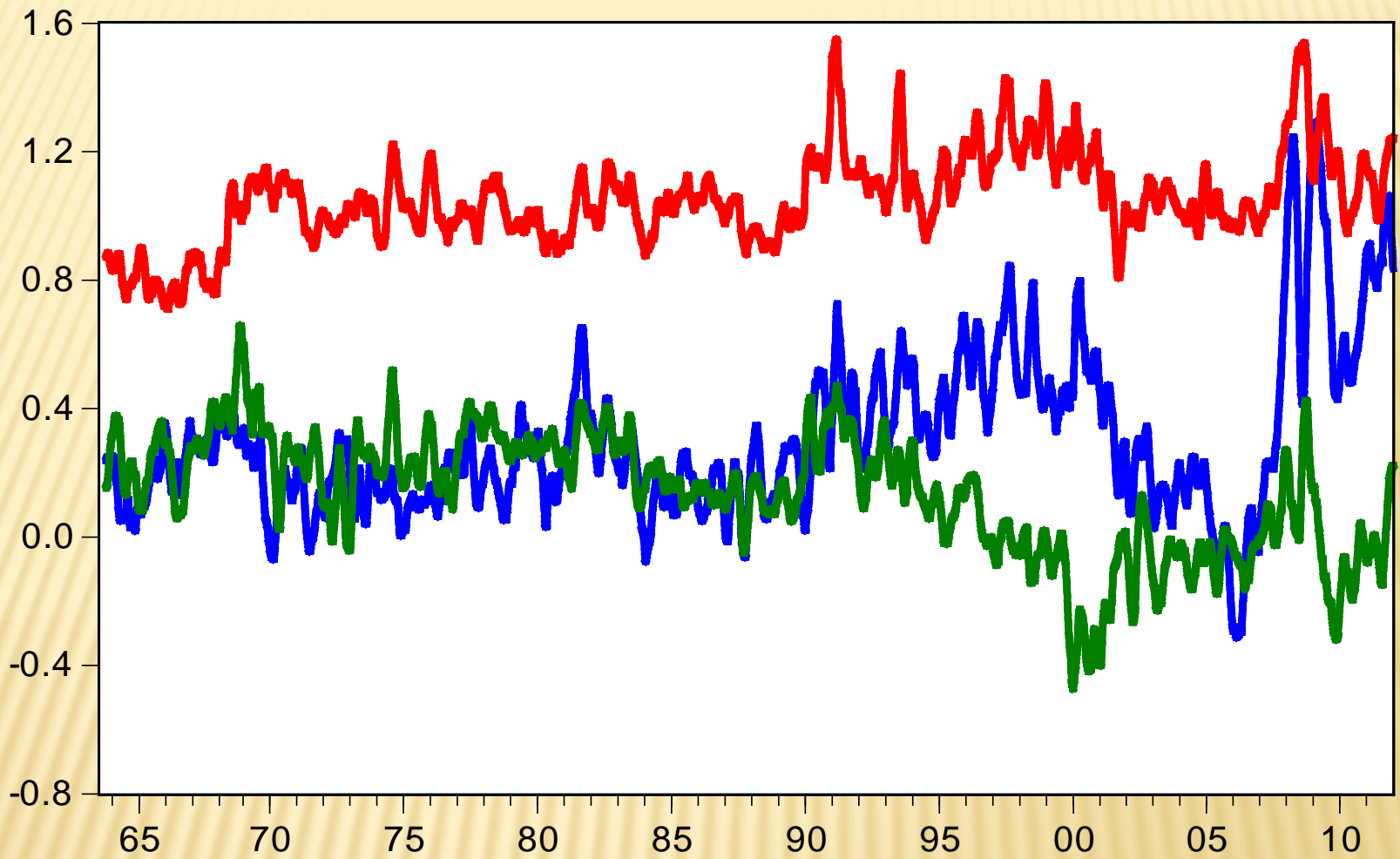
BUT DO THESE DYNAMIC CONDITIONAL BETAS PRICE ASSETS?

TESTING THAT ALPHA =0

NAMEN	TSTAT_DCB	TSTAT_NEST	TSTAT_GARCH	TSTAT_OLS
Buseq	0.54	0.98	1.63	2.46
Chems	-0.67	-0.12	0.76	0.87
Durbl	-2.60	-2.81	-2.70	-2.52
Enrgy	0.06	0.54	2.16	0.85
Hlth	2.81	3.83	5.04	3.42
Manuf	-3.27	-3.06	-1.82	-0.35
Money	-2.69	-2.76	-2.01	-3.10
Nodur	1.19	1.71	4.15	3.74
Other	-1.47	-1.91	-2.60	-2.75
Shops	0.65	1.19	2.08	1.76
Telcm	-0.88	-0.38	0.69	-0.14
Utils	-3.17	-3.23	-2.55	-0.39

GLOBAL SYSTEMIC RISK

A SIMPLE STRESS TEST

* How much will total equity value of a financial firm fall when the global equity market falls a certain amount?

* Answer is the beta of the stock.

* This will be related to systemic risk and the *SRISK* measure of capital shortfall.

TIME ZONES
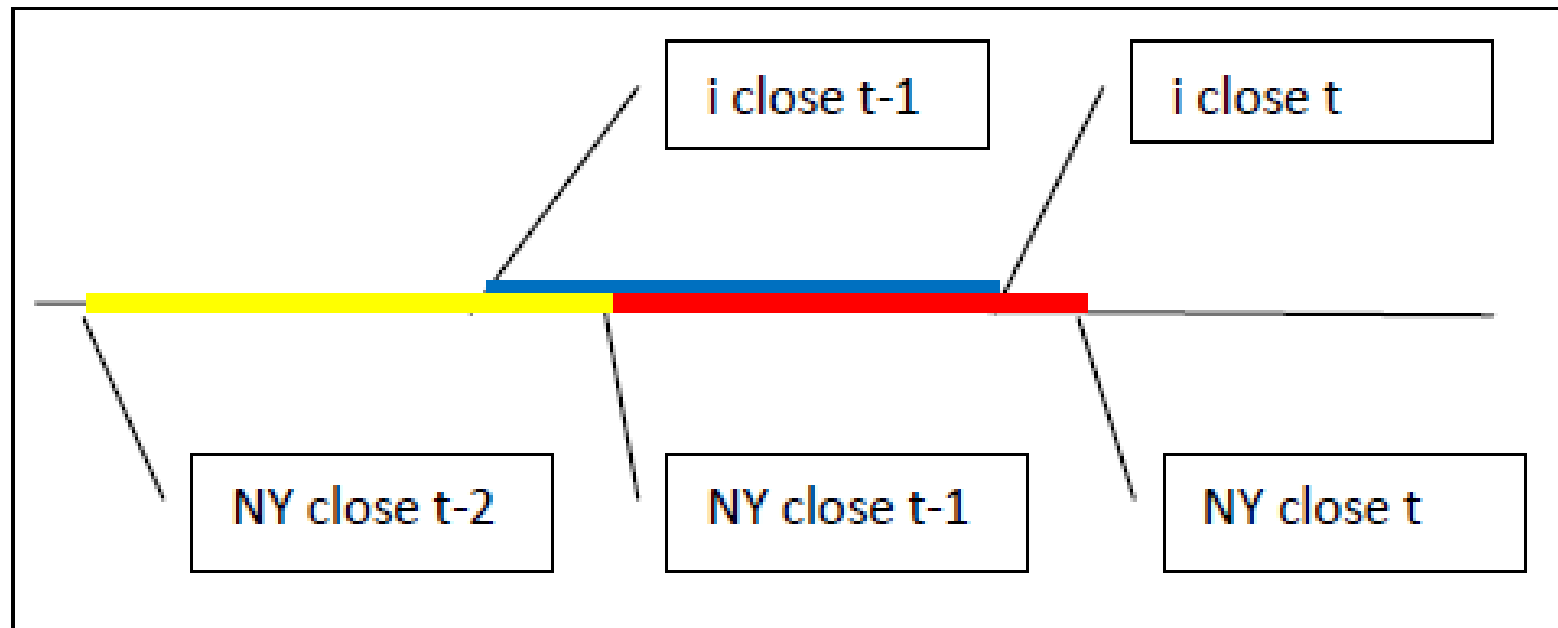
Figure 1

MODEL

× Condition on t-2

$$\begin{pmatrix} R_{i,t} \\ R_{m,t} \\ R_{m,t-1} \end{pmatrix} \Big| \mathbb{F}_{t-2} \sim N\left(0, H_t\right)$$

× The equation $\quad R_{i,t} = \beta_{i,t} R_{m,t} + \gamma_{i,t} R_{m,t-1} + u_{i,t}$

× But u can be an MA(1) and GARCH. In fact, it must have MA(1) if R_i is to be a Martingale difference.

NESTED MODEL

- Combining the constant beta and dynamic conditional beta into one regression:

$$R_{i,t} = \left(\phi_1 \beta_{i,t} + \phi_2 \right) R_{m,t} + \left(\phi_3 \gamma_{i,t} + \phi_4 \right) R_{m,t-1} + u_t$$

- Where u will be an MA(1) GARCH

V-LAB

- For 1200 global financial institutions we update weekly estimates of SRISK. These now use Nested Dynamic Conditional Beta with MA(1) and GARCH.

- http://vlab.stern.nyu.edu

BETA FOR DEUTSCHE BANK
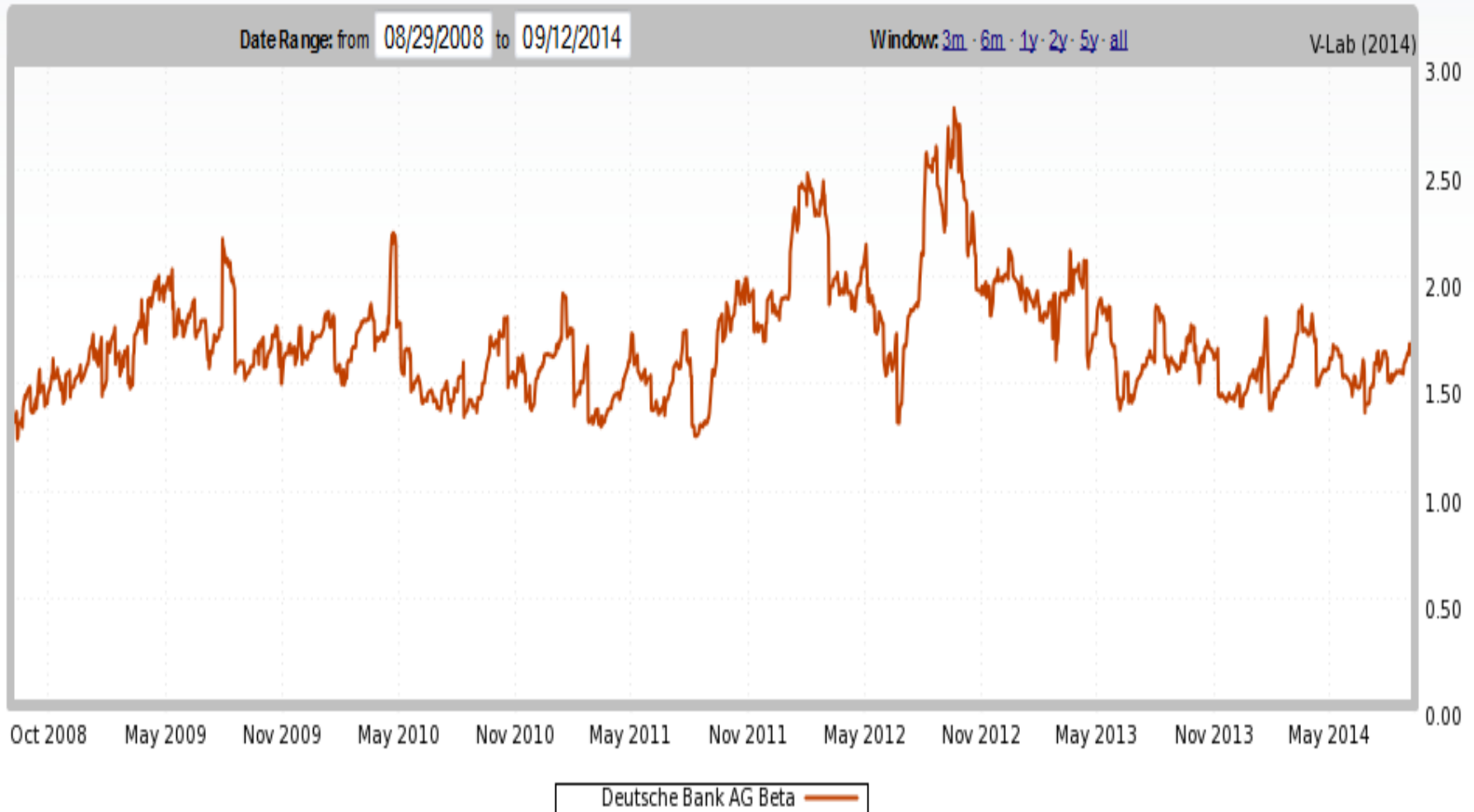
BETA FOR CREDIT AGRICOLE

Credit Agricole SA Beta
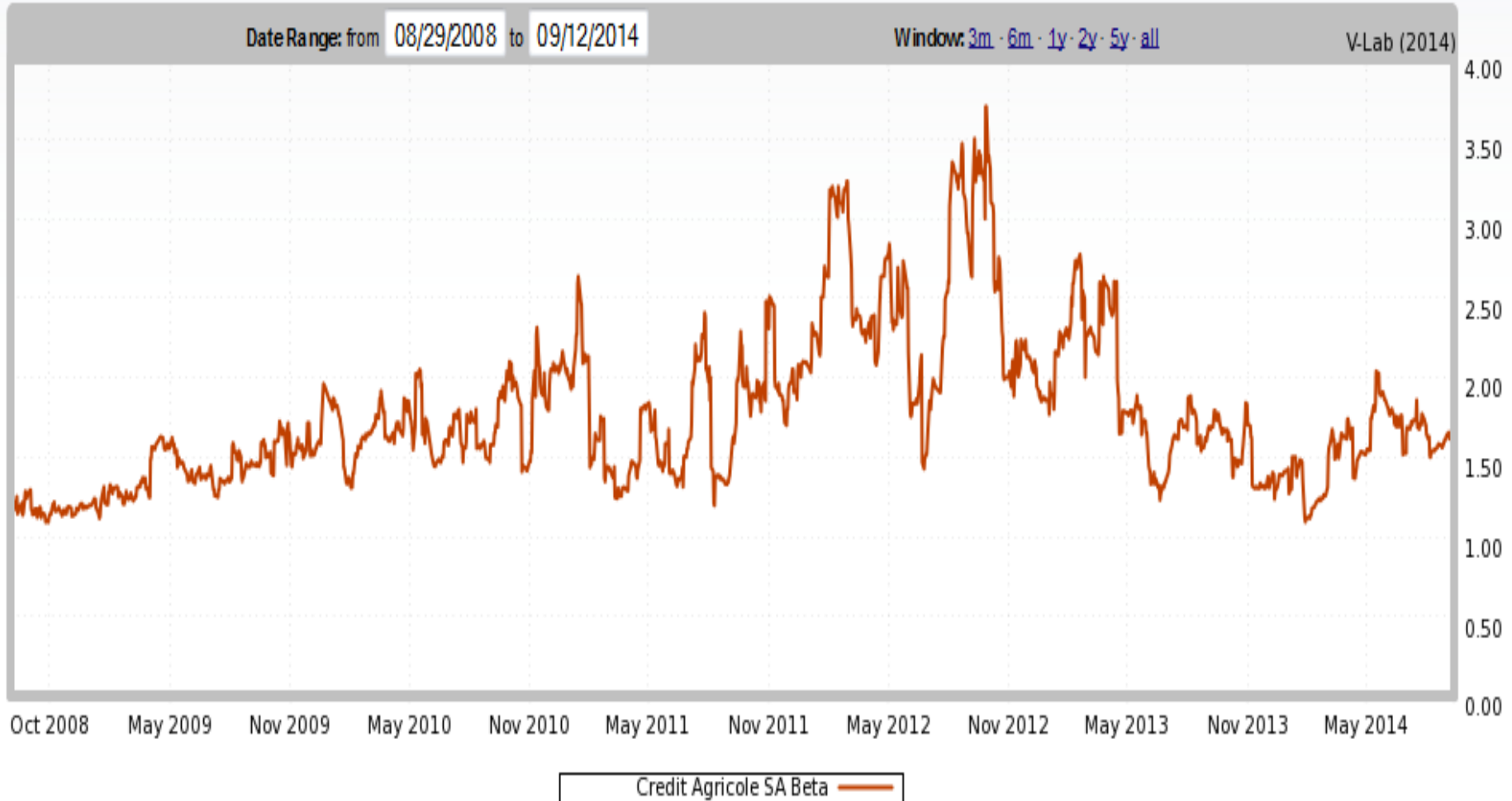
BETA FOR BARCLAYS
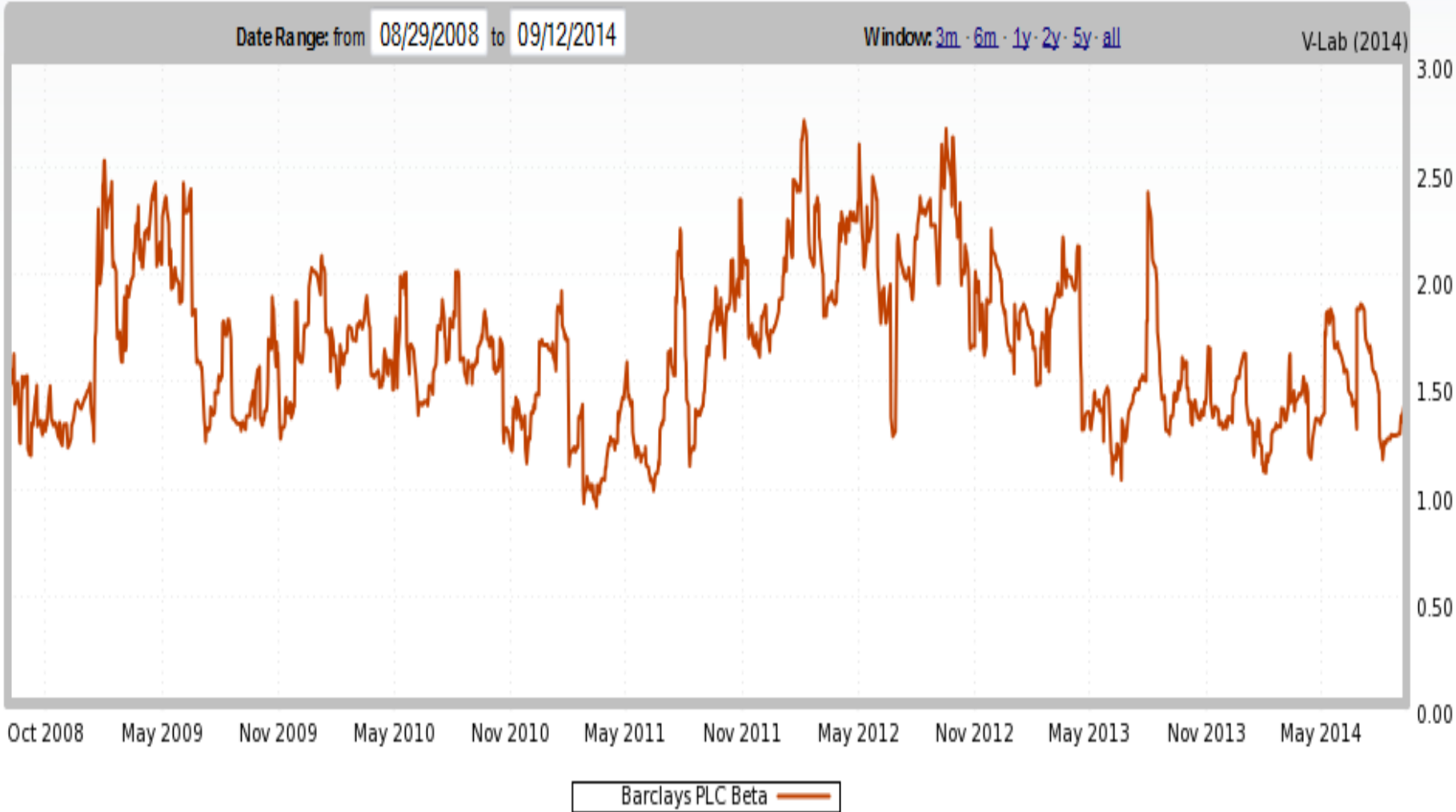
BETA FOR UNICREDIT
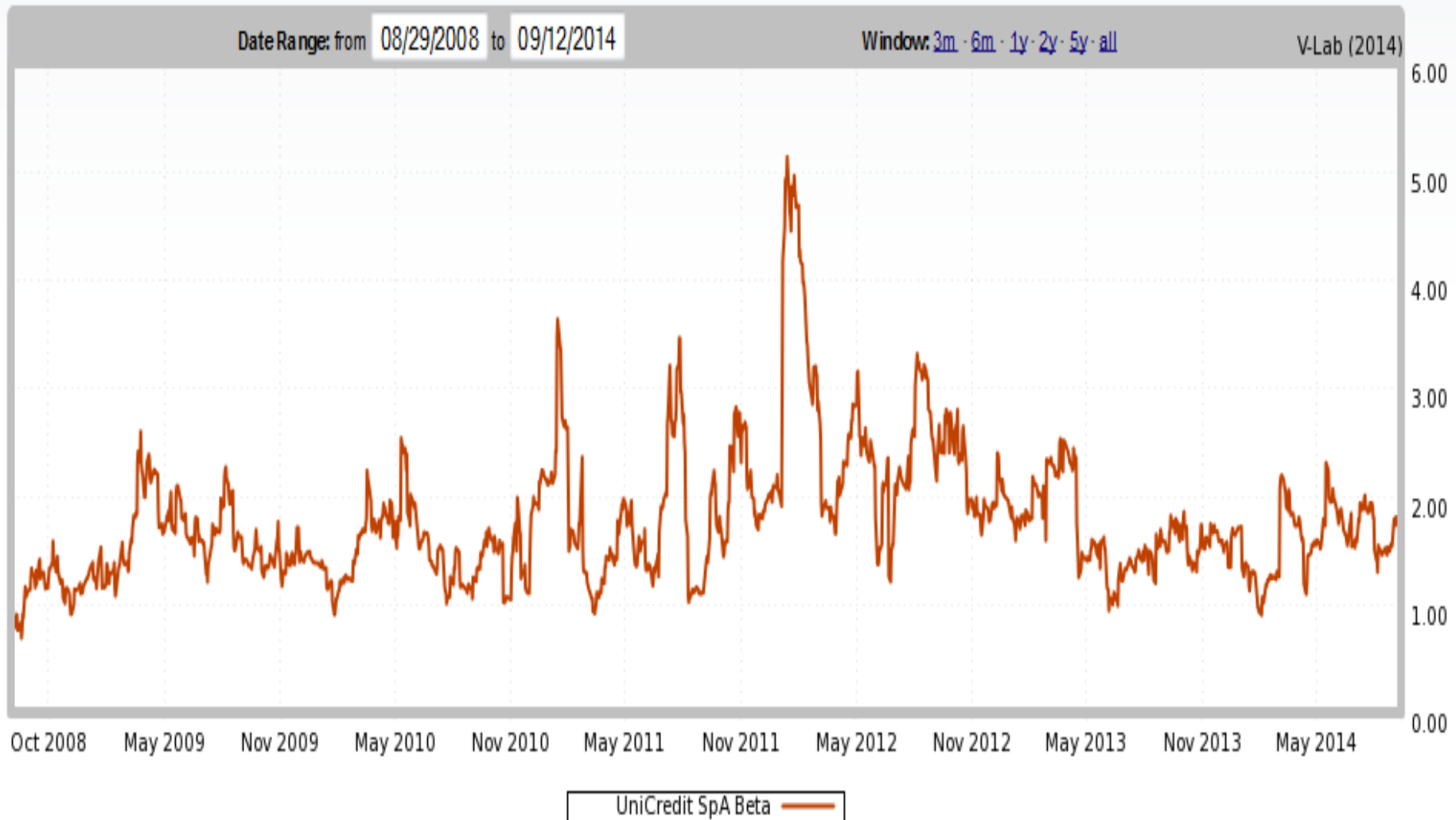
BETA FOR SANTANDER
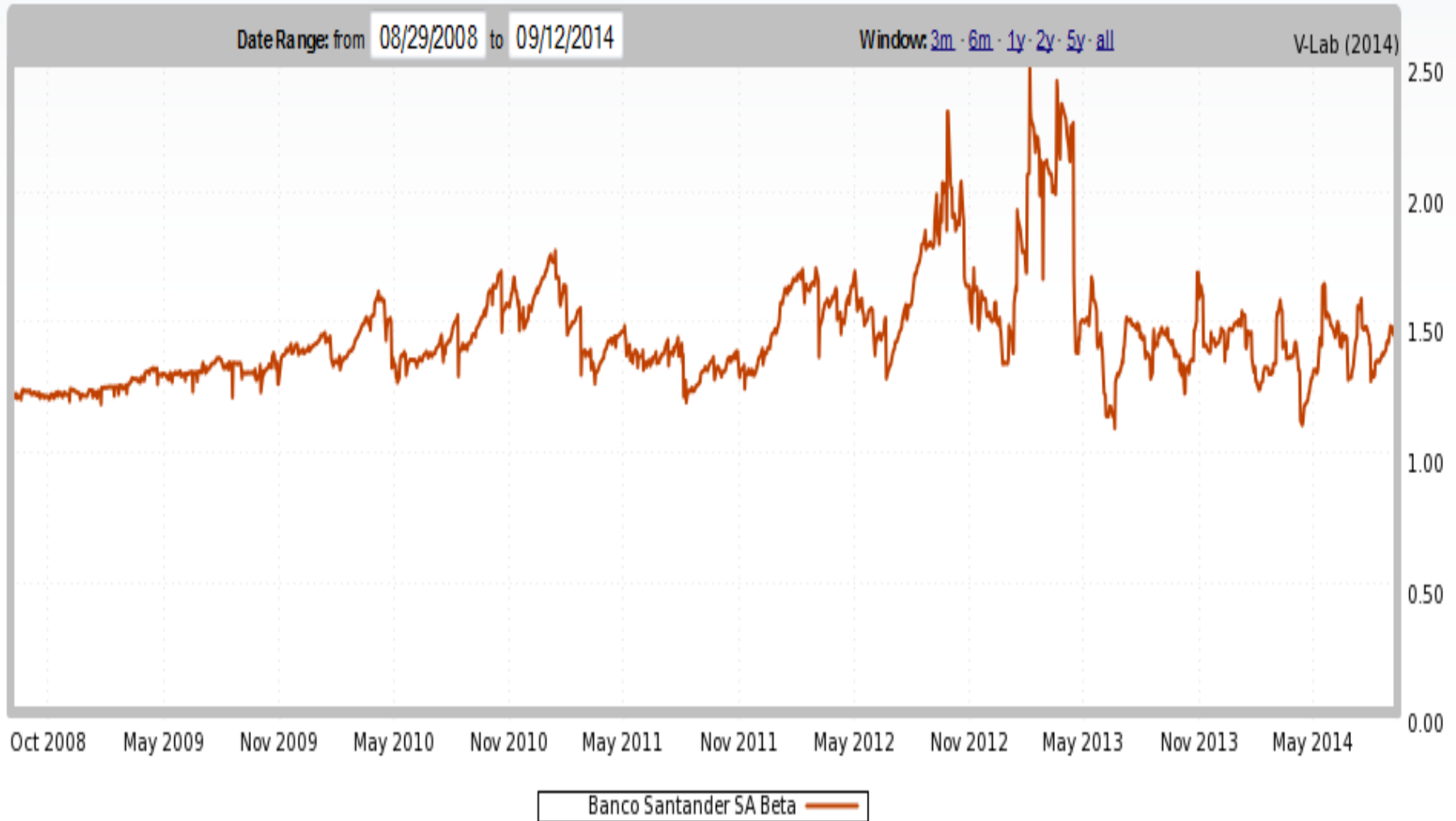
PIRAEUS BANK – MAXIMUM BETA
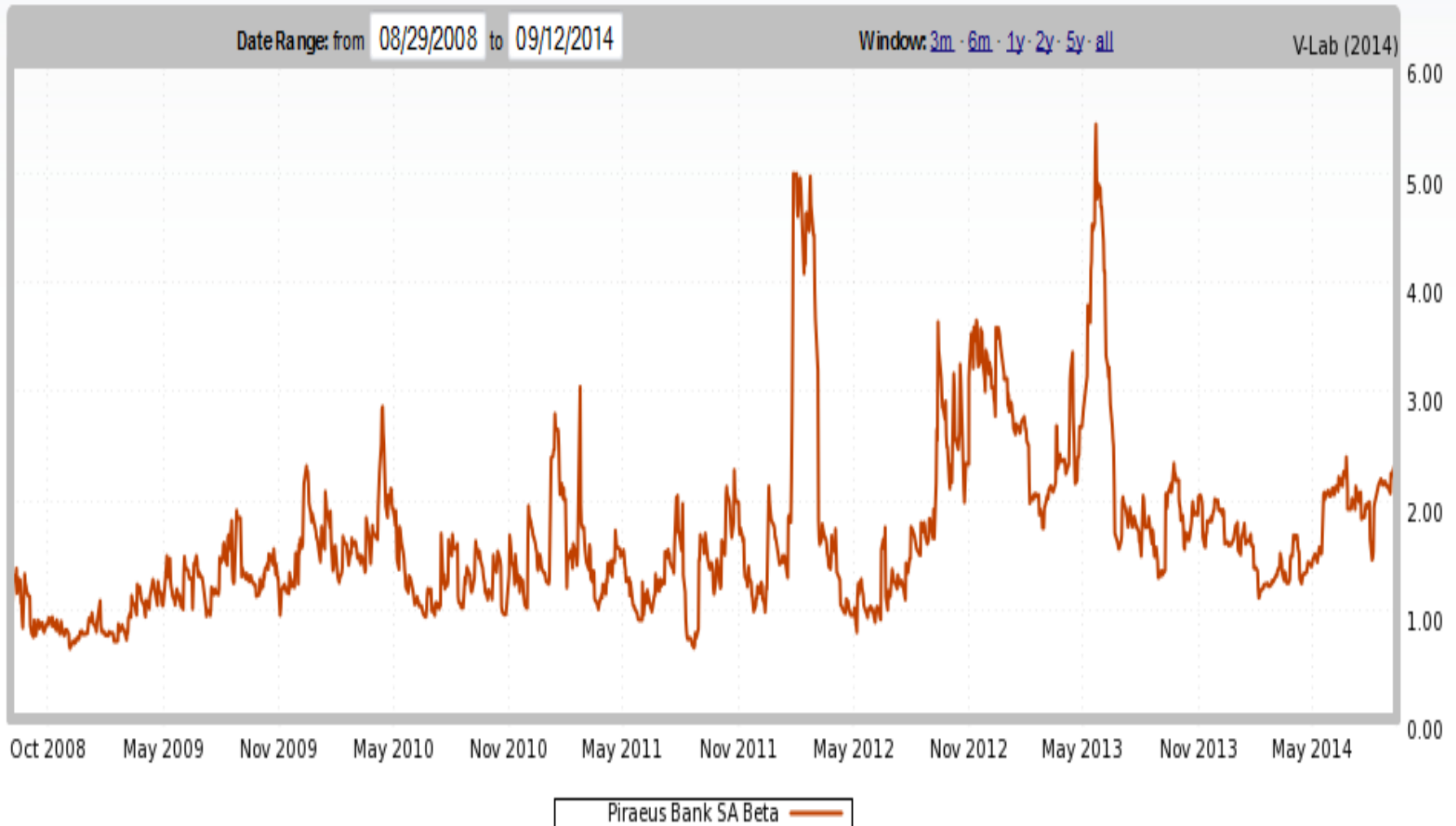
SRISK

- *SRISK=k*LIABILITIES-(1-k)*EQUITY*(1-LRMES)*
  - *k is a capital cushion that financial institutions need to function normally. We use 5.5% for Europe, 8% for GAAP.*
  - *LRMES is the expected equity loss if there is another financial crisis. It is the estimated expected shortfall for the firm when the global equity market declines 40% over the next six months.*

WHY IS THIS A MEASURE OF SYSTEMIC RISK?

- If we have a financial crisis, then all firms with positive SRISK will try simultaneously to raise capital and the only source is likely to be taxpayers. The bigger SRISK, the more serious the threat to financial stability.

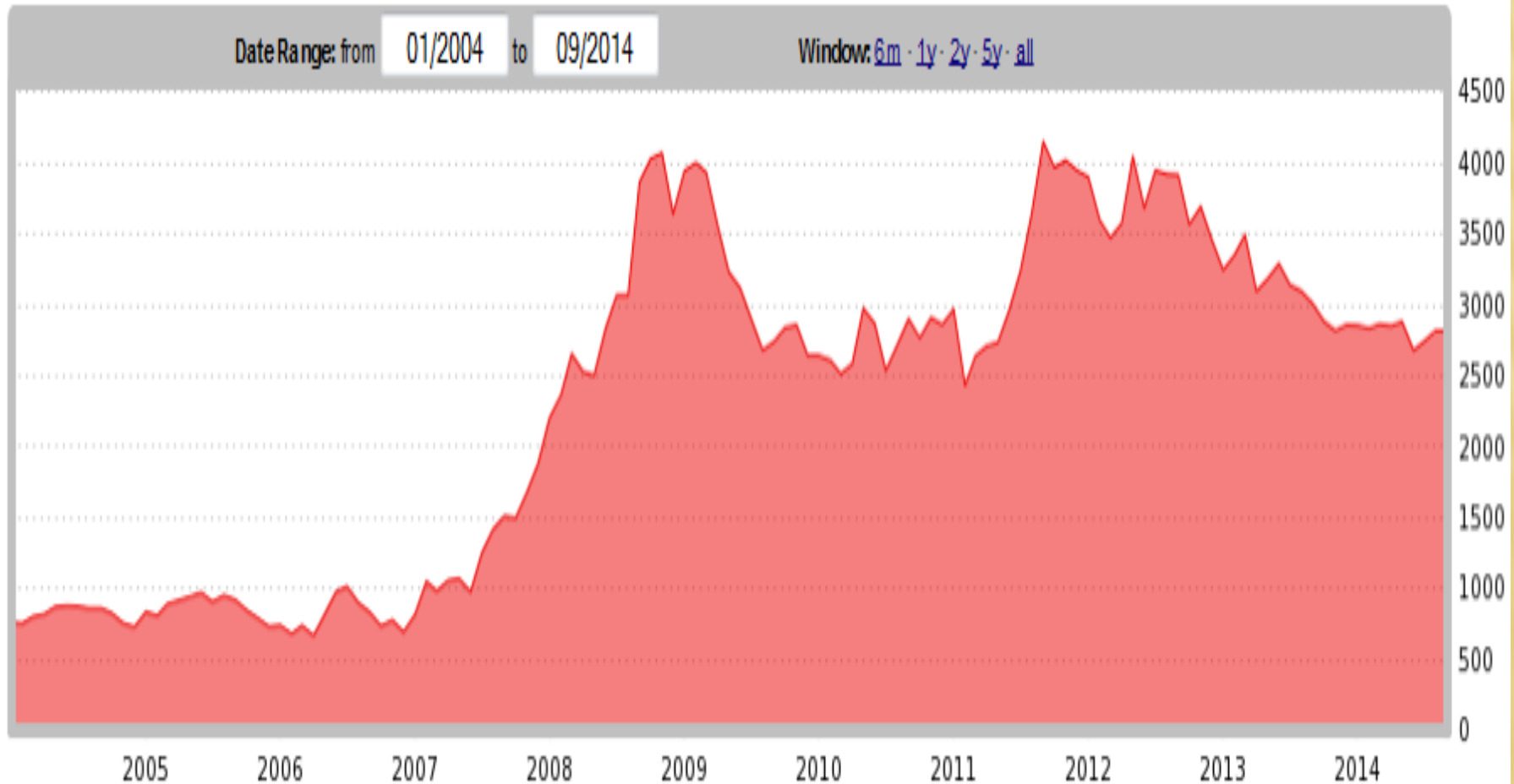- SRISK is estimated conditional on an endogenous variable – a stress test does not indicate causality.

US SRISK – LAST 10 YEARS

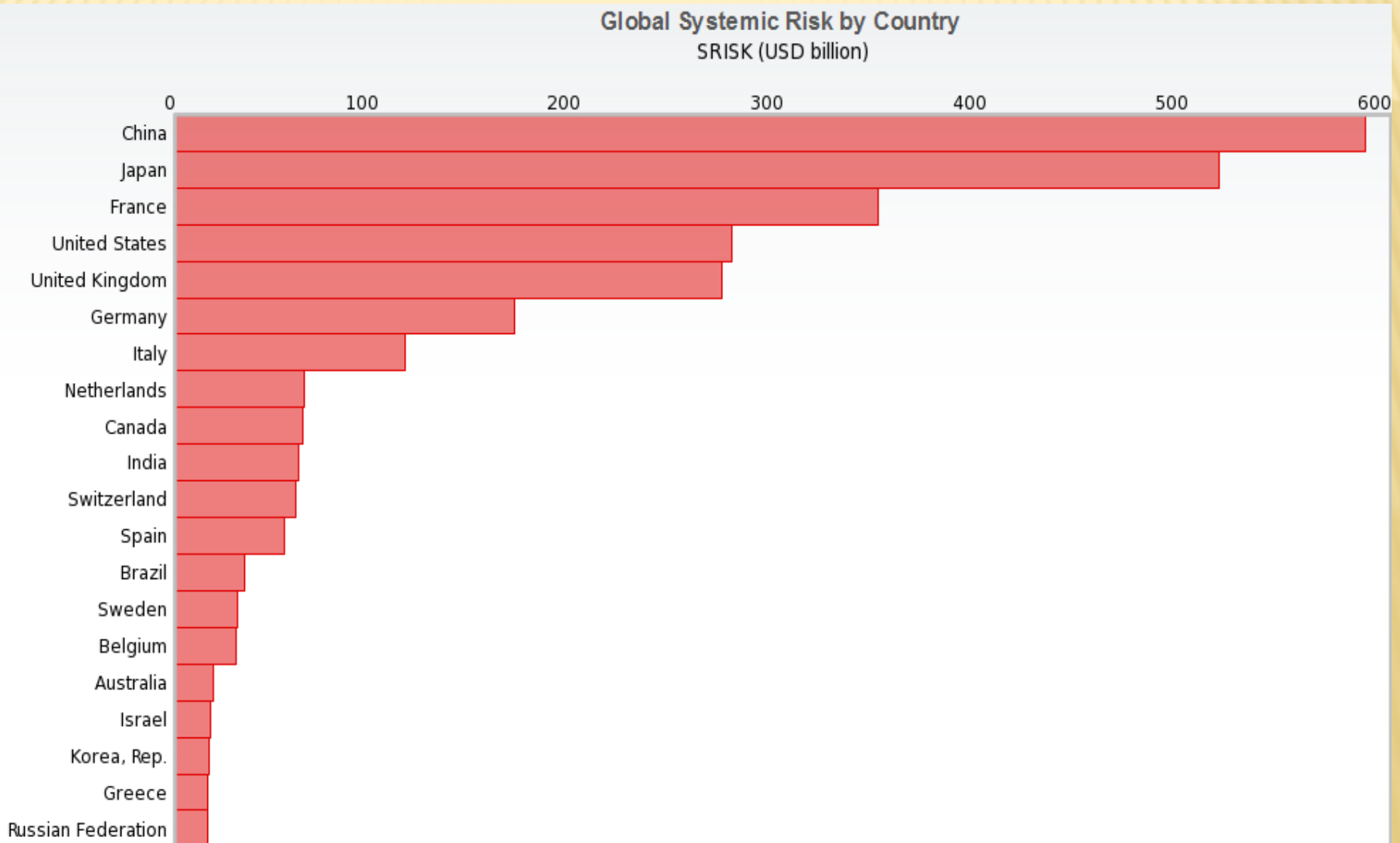Risk Analysis Overview - United States Financials Total SRISK (US$ billion)
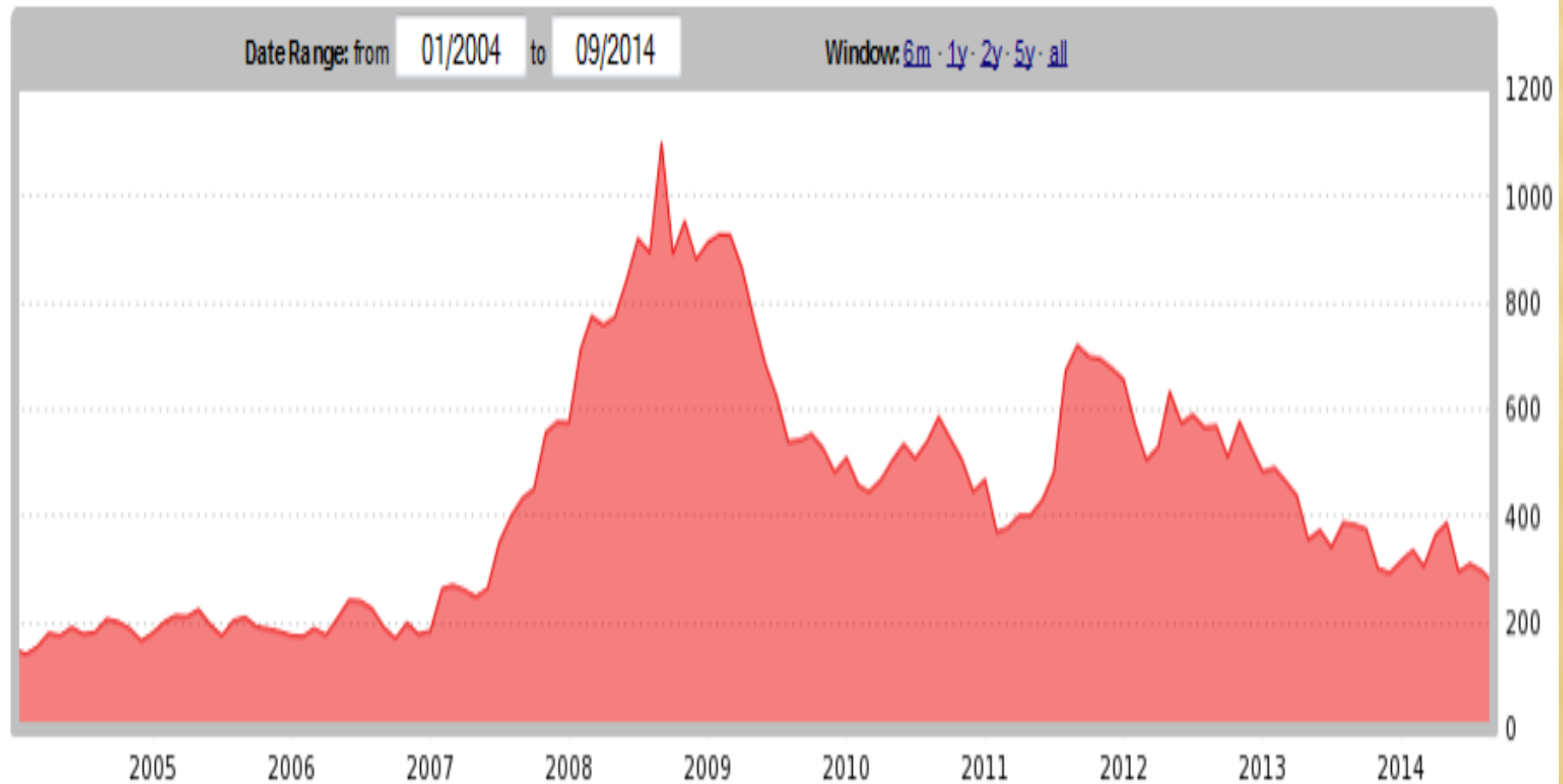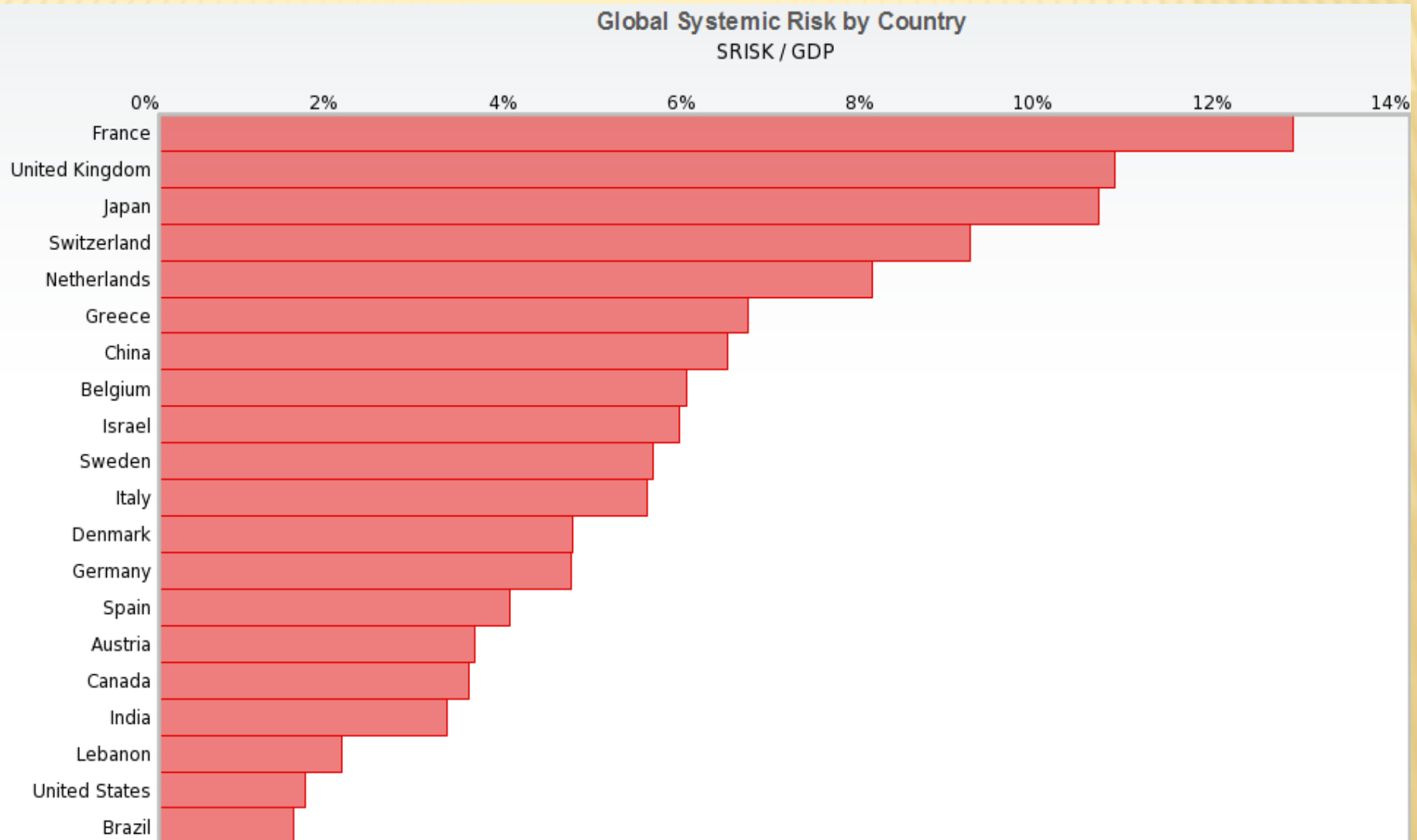
WORLD SRISK – LAST 10 YEARS

Risk Analysis Overview - World Financials Total SRISK (US$ billion)
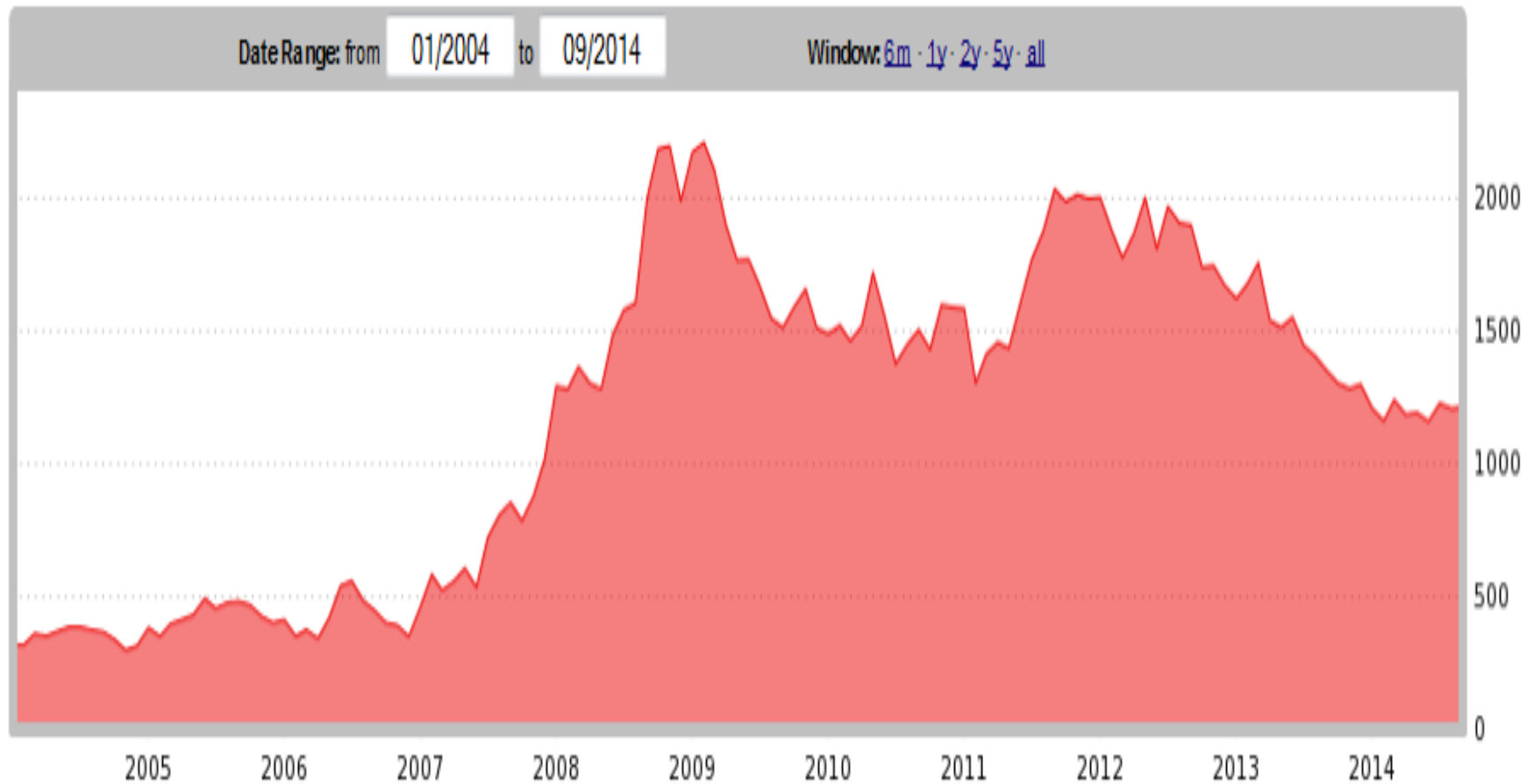
WHERE IS THE RISK TODAY?

Global Systemic Risk by Country
SRISK (USD billion)

RELATIVE TO GDP

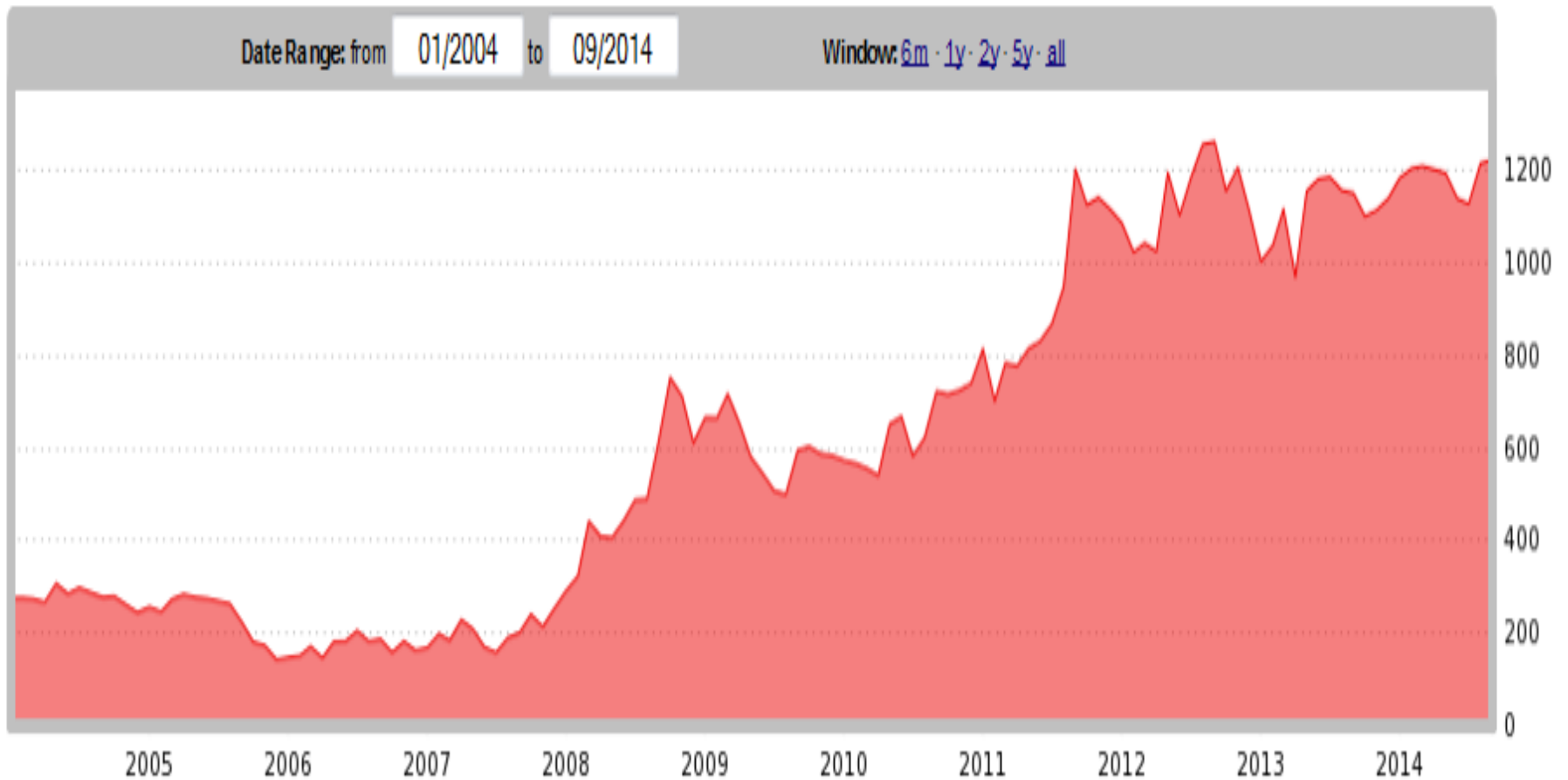Global Systemic Risk by Country
SRISK / GDP

EUROPE SINCE 2004

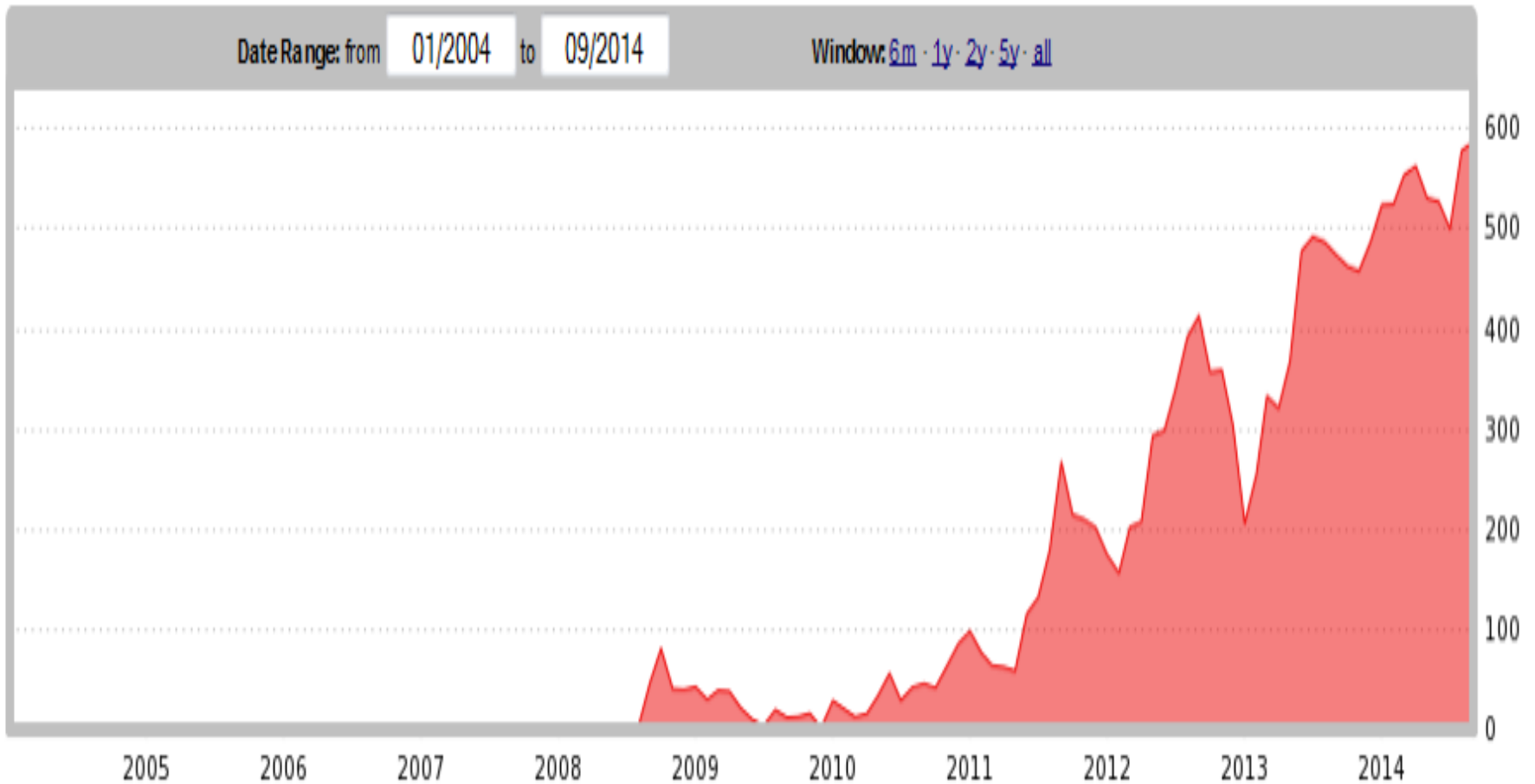Risk Analysis Overview - Europe Financials Total SRISK (US$ billion)

ASIA SINCE 2004

Risk Analysis Overview - Asia Financials Total SRISK (US$ billion)

CHINA SINCE 2004

Risk Analysis Overview - China Financials Total SRISK (US$ billion)

CONCLUSIONS

- Simple general approach to estimating a wide range of models in finance.

- Need experience and Monte Carlo evidence

- How sensitive are the results to the models used for covariance estimation?

- Can we interpret the changes in beta and rely on them for out of sample decision making?

A. CALDER 1973, *LA PORTE DE L'ESPACE*