

The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks[†]

By GLENN D. RUDEBUSCH AND ERIC T. SWANSON*

The term premium in standard macroeconomic DSGE models is far too small and stable relative to the data—an example of the “bond premium puzzle.” However, in endowment economy models, researchers have generated reasonable term premiums by assuming investors have recursive Epstein-Zin preferences and face long-run economic risks. We show that introducing Epstein-Zin preferences into a canonical DSGE model can also produce a large and variable term premium without compromising the model’s ability to fit key macroeconomic variables. Long-run nominal risks further improve the model’s empirical fit, but do not substantially reduce the need for high risk aversion. (JEL E13, E31, E43, E44)

The term premium on long-term nominal bonds compensates investors for inflation and consumption risks over the lifetime of the bond. A large finance literature finds that these risk premiums are substantial and vary significantly over time (e.g., Campbell and Shiller 1991; Cochrane and Piazzesi 2005); however, the economic forces that can justify such large and variable term premiums are less clear. Piazzesi and Schneider (2007) provide some economic insight into the source of a large positive mean term premium in a consumption-based asset pricing model of an endowment economy. Their analysis relies on two crucial features: first, the structural assumption that investors have Epstein-Zin recursive utility preferences;¹ and second, an estimated reduced-form process for consumption and inflation with the feature that positive inflation surprises lead to lower future consumption growth. With these two elements, they show that investors require a premium for holding nominal bonds because a positive inflation surprise

*Rudebusch: Economic Research, Federal Reserve Bank of San Francisco, 101 Market St., San Francisco, CA 94105 (e-mail: glenn.rudebusch@sfrb.org); Swanson: Economic Research, Federal Reserve Bank of San Francisco, 101 Market St., San Francisco, CA 94105 (e-mail: eric.swanson@sfrb.org). We are grateful for helpful comments and suggestions from John Campbell, Ivan Jaccard, Hagen Kim, Sharon Kozicki, Jean-Marc Natal, Monika Piazzesi, Martin Schneider, David Vestin, Mike Woodford, Moto Yogo, Stan Zin, two anonymous referees, and participants at the AEA Meetings, NBER EFG Meeting, NBER Asset Pricing Meeting, NBER Monetary Economics Meeting, NBER Summer Institute, NBER DSGE Workshop, the SED Meetings, SCE Meetings, Bank of Canada conference on Fixed Income Markets, National Bank of Belgium conference on Monetary and Financial Economics, and seminar participants at the Bank of Italy, European Central Bank, Federal Reserve Bank of Chicago, Federal Reserve Bank of Kansas City, Swedish Riksbank, University of California at San Diego, and University of Indiana. The views expressed in this paper are those of the authors and do not necessarily reflect the views of other individuals within the Federal Reserve System.

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¹ Krepes and Porteus (1978) provide the original theoretical framework for these preferences. Epstein and Zin (1989) and Weil (1989) provide extended results and applications for a special case of this framework.

is followed by declines in a bond's value and in consumption. In such a situation, bondholders' wealth decreases just as their marginal utility rises, so they require a premium to compensate for this risk. Bansal and Shaliastovich (2010) use a similar framework—characterized by both Epstein-Zin preferences and reduced-form consumption and inflation empirics—to match the level and variation of risk premiums in bond and foreign exchange markets.

An advantage of these endowment economy studies is that bond prices depend only on the joint stochastic process for consumption and inflation, regardless of the production sector that generates that stochastic process. Nevertheless, there are several reasons to reconsider these questions in a more structural framework that includes production, such as the standard dynamic stochastic general equilibrium (DSGE) framework used in macroeconomics. First, endowment economy studies rely on reduced-form empirical specifications for consumption and inflation that may change over time or in response to policy interventions. For example, if the central bank announces a change to its monetary policy rule, a reduced-form model will be unable to assess the implications of that change for bond prices or for consumption and inflation without several years of data with which to estimate the new reduced-form relationship, whereas a structural model can assess the effects of such changes immediately. Second, structural DSGE models can potentially answer many interesting questions, such as *why* consumption, inflation, and long-term bond yields co-move in a certain way and how that co-movement may change in response to policy interventions or structural shifts. Third, endowment economy studies may assume a functional form for preferences and a consumption or inflation process that are difficult to reconcile within standard macroeconomic models.² Finally, asset prices may provide important information about macroeconomic models. As Cochrane (2007) emphasizes, a total failure of macroeconomic models to explain even the most basic asset pricing facts may be a sign of fundamental flaws in the model. Since DSGE models are widely used by researchers in academia and at central banks and other policy institutions, bringing these models into agreement with basic bond pricing facts, as we start to do in this paper, should be a source of reassurance to many macroeconomists.

The equity premium has generally received more attention in the literature than the term premium, but there are several advantages to focusing on the term premium, as we do in this paper. From a practical perspective, central banks use the yield curve to measure expectations about monetary policy and inflation, so understanding movements in bond risk premiums is important for monetary policy. The value of long-term bonds outstanding in the United States (and in other countries) is also far larger than the value of equities, so the term premium applies to a larger class of securities. From a theoretical perspective, bonds are also extremely simple to model, consisting of a known nominal payment on a default-free government obligation; equities, in contrast, require taking a stand on how to model dividends, leverage,

²For example, Campbell and Cochrane (1999) assume a very large consumption habit, which Lettau and Uhlig (2000); Jermann (1998); and Boldrin, Christiano, and Fisher (2001) show is inconsistent with the empirical volatility of consumption in standard macroeconomic models, because households with those preferences endogenously choose a very smooth path for consumption. See also Rudebusch and Swanson (2008a), who raise a similar point regarding Wachter (2006) in the case of consumption and inflation.

capital adjustment costs, capital mobility, intangible capital, growth options, and the like. Moreover, the term premium provides a metric, above and beyond the equity premium, with which to assess model performance: for example, Boldrin, Christiano, and Fisher (2001) show that the presence of capital immobility in a two-sector DSGE model can account for the equity premium because it increases the variance of the price of capital and its covariance with consumption. However, this mechanism cannot explain a long-term bond premium, which involves the valuation of a fixed nominal payment on a default-free government bond. Finally, the nominal long-term bond premium is closely related to the behavior of inflation and nominal rigidities, which are crucial and still unresolved aspects of the current generation of DSGE models.

The basic form of our DSGE model closely follows the standard specification of these models in the literature (e.g., Woodford 2003; Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007) and, notably, contains an important role for nominal rigidities in order to endogenously describe the behavior of inflation, short-term nominal interest rates, and long-term nominal bonds. We evaluate the model based on its ability to match both basic macroeconomic moments (e.g., the standard deviations of consumption and inflation) and basic bond pricing moments (e.g., the means and volatilities of the yield curve slope and bond excess holding period returns). In order to match the bond pricing facts, we augment the standard DSGE model in two ways. First, we assume that households in the model have Epstein-Zin preferences, so risk aversion can be modeled independently from the intertemporal elasticity of substitution.³ Second, we assume that agents in the model face long-run economic risks, as in Bansal and Yaron (2004).⁴ However, because we are pricing a nominal asset, we consider not just long-run real risk but also long-run *nominal* risk, in the sense that the central bank's long-run inflation objective may vary over time, as in Gürkaynak, Sack, and Swanson (2005).⁵

We show that our model can replicate the basic bond pricing moments without compromising its ability to fit the macroeconomic facts. Intuitively, our model is identical to first order to standard macroeconomic DSGE representations because the first-order approximation to Epstein-Zin preferences is the same as the first-order approximation to standard expected utility preferences. Furthermore, the macroeconomic moments of the model are not very sensitive to the additional second-order and higher-order terms introduced by Epstein-Zin preferences, while risk premiums are unaffected by first-order terms and completely determined by those second- and higher-order terms. Therefore, by varying the Epstein-Zin risk-aversion parameter while holding the other parameters of the model constant, we are able to fit the asset pricing facts without compromising the model's ability to

³van Binsbergen et al. (2010) also price bonds in a DSGE model with Epstein-Zin preferences, but their model treats inflation as an exogenous stochastic process, and thus has some of the same limitations as the reduced-form studies of Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2010).

⁴Bansal and Yaron (2004) show that uncertainty about the economy's long-run growth prospects can play an important role in generating sizable equity risk premiums.

⁵Gürkaynak, Sack, and Swanson (2005) find that the excess sensitivity of long-term bond yields to macroeconomic announcements appears to be due to financial markets expecting some degree of pass-through from near-term inflation to the long-term inflation outlook.

fit the macroeconomic data.⁶ Long-run nominal risk improves the model's ability to fit the data, but does not substantially reduce the need for a high level of household risk aversion.

Our results have implications for both the macroeconomics and finance literatures. For macroeconomics, our analysis suggests a straightforward modification of standard DSGE models that helps bring those models into agreement with basic bond pricing facts. For finance, our analysis helps to illuminate the earlier reduced-form results with a structural economic interpretation. We also suggest a resolution to a long-standing puzzle in the bond pricing literature (Backus, Gregory, and Zin 1989, and den Haan 1995); namely, why does the yield curve slope upward? If interest rates are low during a recession, then bond prices should be high when consumption is low; as a result, long-term bonds should carry an insurance-like, negative risk premium and the yield curve should, counterfactually, slope downward. In our DSGE model, the yield curve slopes upward because technology shocks cause *inflation* to rise persistently when consumption falls, so long-term nominal bonds *lose* rather than gain value in recessions, implying a positive risk premium. More generally, any shock that causes inflation to move persistently and inversely to output, including a markup shock or an oil price shock, will tend to imply such a positive term premium.

The remainder of the paper proceeds as follows. Section I generalizes a standard DSGE model to the case of Epstein-Zin preferences. Section II presents results for this model, and shows how it is able to match the term premium without impairing the fit to macroeconomic data. Section III extends the model to include long-run real and nominal risks, which improves the model's overall fit to the data. Section IV concludes. Two technical appendices provide additional details of the preference specification and how DSGE models in general can be extended to the case of Epstein-Zin preferences and solved.

I. A DSGE Model with Epstein-Zin Preferences

In this section, we generalize the simple, stylized DSGE model of Woodford (2003) to the case of Epstein-Zin preferences. We show how to price long-term nominal bonds in this model and define the term premium and other measures of long-term bond risk.

⁶This is in contrast to DSGE models with habit preferences—see Rudebusch and Swanson (2008a). Intuitively, households with habits primarily dislike very sudden changes in consumption, and these changes are the easiest for households to offset by varying labor supply or savings in a DSGE model. In contrast, a household with Epstein-Zin preferences is concerned about changes in consumption over medium and long horizons (through the value function V) as well as short horizons. While the household would like to offset fluctuations in consumption by varying hours or savings, its ability to offset longer-horizon changes in consumption are much more limited than its ability to smooth out very sudden changes. Including long-run risks in the model makes longer-run changes in consumption even more important and makes it even harder for households to offset shocks in the model.

A. Epstein-Zin Preferences

It is standard in macroeconomic models to assume that a representative household chooses state-contingent plans for consumption, c , and labor, l , so as to maximize expected utility:

$$(1) \quad \max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

subject to an asset accumulation equation, where $\beta \in (0, 1)$ is the household's discount factor and the period utility kernel $u(c_t, l_t)$ is twice-differentiable, concave, increasing in c , and decreasing in l . The maximand in equation (1) can be expressed in first-order recursive form as:

$$(2) \quad V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1},$$

where the household's state-contingent plans at time t are chosen so as to maximize V_t .

In this paper, we follow the finance literature and generalize (2) to an Epstein-Zin specification:

$$(3) \quad V_t \equiv u(c_t, l_t) + \beta (E_t V_{t+1}^{1-\alpha})^{1/(1-\alpha)},$$

where the parameter α can take on any real value.⁷ If $u \geq 0$ everywhere, then the proof of Theorem 3.1 in Epstein and Zin (1989) shows that there exists a solution V to (3) with $V \geq 0$. If $u \leq 0$ everywhere, then it is natural to let $V \leq 0$ and reformulate the recursion as:

$$(4) \quad V_t \equiv u(c_t, l_t) - \beta [E_t (-V_{t+1})^{1-\alpha}]^{1/(1-\alpha)}.$$

The proof in Epstein and Zin (1989) also demonstrates the existence of a solution V to (4) with $V \leq 0$ in this case.⁸ When $\alpha = 0$, both (3) and (4) reduce to the standard case of expected utility (2). When $u \geq 0$ everywhere, higher values of α correspond to greater degrees of risk aversion. When $u \leq 0$ everywhere, the opposite is true: higher values of α correspond to lesser degrees of risk aversion.

Traditionally, Epstein-Zin preferences over consumption streams have been written as:

$$(5) \quad \tilde{V}_t \equiv [c_t^\rho + \beta (E_t \tilde{V}_{t+1}^{\tilde{\alpha}})^{\rho/\tilde{\alpha}}]^{1/\rho},$$

⁷The case $\alpha = 1$ corresponds to $V_t = u(c_t, l_t) + \beta \exp(E_t \log V_{t+1})$ for the case $u \geq 0$, and $V_t = u(c_t, l_t) - \beta \exp[E_t \log(-V_{t+1})]$ for $u \leq 0$.

⁸For simplicity, we exclude here the case where u may take on both positive and negative values. We note, however, that for local approximations around a deterministic steady state with V sufficiently far from zero in steady state, this case does not present any particular difficulties.

but by setting $V_t = \tilde{V}_t^\rho$ and $\alpha = 1 - \tilde{\alpha}/\rho$, this can be seen to correspond to (3). The advantage of the form (3) is that it more easily allows us to consider utility kernels $u(c_t, l_t)$ with general functional forms that include labor.

The key advantage of using Epstein-Zin utility (3) is that it breaks the linkage between the intertemporal elasticity of substitution and the coefficient of relative risk aversion that has long been noted in the literature regarding expected utility (2)—see, e.g., Mehra and Prescott (1985) and Hall (1988). In (3), the intertemporal elasticity of substitution over deterministic consumption paths is exactly the same as in (2), but now the household's risk aversion to uncertain lotteries over V_{t+1} can be amplified by the additional parameter α , a feature which is crucial for allowing us to fit both the asset pricing and macroeconomic facts below.⁹

We turn now to the utility kernel u . In earlier versions of this paper (Rudebusch and Swanson 2008b), we used the additively separable utility kernel featured in Woodford's (2003) textbook. However, that specification is not consistent with balanced growth in general, so we extend our analysis here to the following functional form:

$$(6) \quad u(c_t, l_t) \equiv \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-l_t)^{1-\chi}}{1-\chi},$$

where $\varphi, \chi, \chi_0 > 0$,¹⁰ the household's per-period time endowment is unity, and Z_t is an aggregate productivity trend (that may be either deterministic or stochastic). This specification follows Campbell and Ludvigson (2001), who argue that there is little empirical evidence of nonseparability between consumption and leisure, while the intertemporal elasticity of substitution in consumption seems to be substantially less than unity.¹¹ We show in Appendix A how these preferences over market goods and market hours follow from a model in which households receive utility from both market goods and nonmarket goods, or "home production". The additive separability between consumption and leisure in (6) is not crucial for our analysis, but it simplifies the expressions for the intertemporal elasticity of substitution, $1/\varphi$, the Frisch labor supply elasticity, $(1-l)/\chi l$ (where l denotes the steady-state value of l_t), the coefficient of relative risk aversion (discussed in Section IIB, below), and the stochastic discount factor, which will be helpful when discussing intuition for our results below. Finally, additive separability in (6) makes the model less homogeneous with respect to shocks, and it is the nonhomogeneities of the model that allow it to generate endogenous time-variation in the term premium, as we will discuss.

⁹Indeed, the linearization or log-linearization of (3) is exactly the same as that of (2), which turns out to be very useful for matching the model to macroeconomic variables, since models with (2) are already known to be able to fit macroeconomic quantities reasonably well. We will return to this point in Section II, below.

¹⁰If $\varphi, \chi < 1$, then (6) is everywhere positive and V is defined by (3). If $\varphi, \chi > 1$, then (6) is everywhere negative and V is defined by (4). For simplicity, we do not consider the case where φ and χ lie on opposite sides of unity, but this does not pose a particular problem for local approximations around a steady state when V is sufficiently far from zero. Restrictions can also be imposed on c_t or l_t to ensure that $u \geq 0$ or $u \leq 0$ holds.

¹¹See the discussion in Campbell and Ludvigson (2001) and the empirical evidence in Kiley (2010) and Campbell and Mankiw (1990). The usual King-Plosser-Rebelo (1988) preferences require that the IES equal unity or that consumption and leisure be nonseparable.

B. The Household's Optimization Problem

Households in the model are representative and choose state-contingent consumption and labor plans to maximize the Epstein-Zin recursive functional (3) subject to an intertemporal flow budget constraint, specified below. We will solve the household's optimization problem as a Lagrange problem with the states of nature explicitly specified. To that end, let $\mathbf{s}^0 \in S_0$ denote the initial state of the economy at time 0, let $\mathbf{s}_t \in S$ denote the realizations of the shocks that hit the economy in period t , and let $\mathbf{s}^t \equiv \{\mathbf{s}^{t-1}, \mathbf{s}_t\} \in S_0 \times S^t$ denote the initial state and history of all shocks up through time t . We refer to \mathbf{s}^t as the "state of the world at time t ." We define \mathbf{s}_{t-1}^t to be the projection of the history \mathbf{s}^t onto its first t components; that is, \mathbf{s}_{t-1}^t is the history \mathbf{s}^t as it would have been viewed at time $t - 1$, before time- t shocks have been realized. If \mathbf{s}^t and \mathbf{s}^{t-1} are arbitrary elements of $S_0 \times S^t$ and $S_0 \times S^{t-1}$, respectively, we will say that $\mathbf{s}^t \supseteq \mathbf{s}^{t-1}$ if $\mathbf{s}_{t-1}^t = \mathbf{s}^{t-1}$.

Households have access to an asset whose price is given by $p_{s^t} > 0$ in each period t and state of the world \mathbf{s}^t . In each period t , households choose the quantity of consumption c_{s^t} , labor l_{s^t} , and asset holdings a_{s^t} that will carry through to the next period, subject to a constraint that the household's asset holdings a_{s^t} are always greater than some lower bound $\underline{a} \ll 0$, which does not bind in equilibrium but rules out Ponzi schemes. Households are price takers in consumption, asset, and labor markets, and face a price per unit of consumption of P_{s^t} , and nominal wage rate w_{s^t} . Households own an aliquot share of firms and receive a per period lump sum transfer from firms in the amount d_{s^t} . The household's flow budget constraint is thus:

$$(7) \quad p_{s^t} a_{s^t} + P_{s^t} c_{s^t} = w_{s^t} l_{s^t} + d_{s^t} + p_{s^t} a_{s_{t-1}^t}.$$

The household's optimization problem is to choose a sequence of vector-valued functions, $[c_t(\mathbf{s}^t), l_t(\mathbf{s}^t), a_t(\mathbf{s}^t)]: S_0 \times S^t \rightarrow [\underline{c}, \infty) \times [0, \bar{l}] \times [\underline{a}, \infty)$, so as to maximize (3) subject to the sequence of budget constraints (7). For clarity in what follows, we assume that \mathbf{s}^0 and \mathbf{s}_t can take on only a finite number of possible values (i.e., S_0 and S have finite support), and we let $\pi_{s^t|\mathbf{s}^t}$, $\tau \geq t \geq 0$ denote the probability of realizing state \mathbf{s}^τ at time τ conditional on being in state \mathbf{s}^t at time t .

The household's optimization problem can be formulated as a Lagrangean, where the household chooses state-contingent plans for consumption, labor, and asset holdings, $(c_{s^t}, l_{s^t}, a_{s^t})$, that maximize V_0 subject to the infinite sequence of state-contingent constraints (3) and (7); that is, maximize

$$(8) \quad \mathcal{L} \equiv V_{s^0} - \sum_{t=0}^{\infty} \sum_{\mathbf{s}^t} \mu_{s^t} \left\{ V_{s^t} - u(c_{s^t}, l_{s^t}) - \beta \left(\sum_{\mathbf{s}^{t+1}} \pi_{\mathbf{s}^{t+1}|\mathbf{s}^t} V_{\mathbf{s}^{t+1}}^{1-\alpha} \right)^{1/(1-\alpha)} \right\} \\ - \sum_{t=0}^{\infty} \sum_{\mathbf{s}^t} \lambda_{s^t} \{ p_{s^t} a_{s^t} + P_{s^t} c_{s^t} - w_{s^t} l_{s^t} - d_{s^t} - p_{s^t} a_{s_{t-1}^t} \}.$$

The household's first-order conditions for (8) are then:

$$\frac{\partial \mathcal{L}}{\partial c_{s^t}} : \mu_{s^t} u_1(c_{s^t}, l_{s^t}) = P_{s^t} \lambda_{s^t},$$

$$\frac{\partial \mathcal{L}}{\partial l_{s^t}} : -\mu_{s^t} u_2(c_{s^t}, l_{s^t}) = w_{s^t} \lambda_{s^t},$$

$$\frac{\partial \mathcal{L}}{\partial a_{s^t}} : \lambda_{s^t} p_{s^t} = \sum_{s^{t+1} \supseteq s^t} \lambda_{s^{t+1}} p_{s^{t+1}},$$

$$\frac{\partial \mathcal{L}}{\partial V_{s^t}} : \mu_{s^t} = \beta \pi_{s^t | s_{t-1}^t} \mu_{s_{t-1}^t} \left(\sum_{\tilde{s}^t \supseteq s_{t-1}^t} \pi_{\tilde{s}^t | s_{t-1}^t} V_{\tilde{s}^t}^{1-\alpha} \right)^{\alpha/(1-\alpha)} V_{s^t}^{-\alpha}; \quad \mu_{s^0} = 1,$$

where u_1 and u_2 denote the partial derivatives of u with respect to its first and second arguments, respectively. Letting $(1 + r_{s^{t+1}}) \equiv p_{s^{t+1}}/p_{s^t}$, the gross rate of return on the asset, making substitutions, and defining the discounted Lagrange multipliers $\tilde{\lambda}_{s^t} \equiv \beta^{-t} \pi_{s^t | s^0} \lambda_{s^t}$ and $\tilde{\mu}_{s^t} \equiv \beta^{-t} \pi_{s^t | s^0} \mu_{s^t}$, these become:

$$(9) \quad \frac{\partial \mathcal{L}}{\partial c_{s^t}} : \tilde{\mu}_{s^t} u_1(c_{s^t}, l_{s^t}) = P_{s^t} \tilde{\lambda}_{s^t}$$

$$(10) \quad \frac{\partial \mathcal{L}}{\partial l_{s^t}} : -\tilde{\mu}_{s^t} u_2(c_{s^t}, l_{s^t}) = w_{s^t} \tilde{\lambda}_{s^t}$$

$$(11) \quad \frac{\partial \mathcal{L}}{\partial a_{s^t}} : \tilde{\lambda}_{s^t} = \beta E_{s^t} \tilde{\lambda}_{s^{t+1}} (1 + r_{s^{t+1}})$$

$$(12) \quad \frac{\partial \mathcal{L}}{\partial V_{s^t}} : \tilde{\mu}_{s^t} = \tilde{\mu}_{s_{t-1}^t} (E_{s_{t-1}^t} V_{s^t}^{1-\alpha})^{\alpha/(1-\alpha)} V_{s^t}^{-\alpha}; \quad \tilde{\mu}_{s^0} = 1,$$

where E_{s^t} denotes the expected value conditional on being in state s^t . These first-order conditions are very similar to the expected utility case except for the introduction of the additional Lagrange multipliers $\tilde{\mu}_{s^t}$, which translate utils at time t into utils at time 0, allowing for the “twisting” of the value function by $1 - \alpha$ that takes place at each time $1, 2, \dots, t$. Note that in the expected utility case, $\tilde{\mu}_{s^t} = 1$ for every s^t , and equations (9)–(12) reduce to the standard optimality conditions. Similarly, linearizing (9)–(12) separates out $\tilde{\mu}_{s^t}$ and causes α to drop out, so to first order, (9)–(12) are identical to the expected utility case.

Substituting out for $\tilde{\lambda}_{s^t}$ and $\tilde{\mu}_{s^t}$ in (9)–(12), we get the household's intratemporal and intertemporal (Euler) optimality conditions:

$$\frac{-u_2(c_{s^t}, l_{s^t})}{u_1(c_{s^t}, l_{s^t})} = \frac{w_{s^t}}{P_{s^t}}$$

$$u_1(c_{s^t}, l_{s^t}) = \beta E_{s^t} (E_{s^t} V_{s^{t+1}}^{1-\alpha})^{\alpha/(1-\alpha)} V_{s^{t+1}}^{-\alpha} u_1(c_{s^{t+1}}, l_{s^{t+1}}) (1 + r_{s^{t+1}}) P_{s^t} / P_{s^{t+1}}.$$

Finally, let $p_{s^t}^{s^\tau}$, $t \leq \tau$, denote the price at time t in state s^t of a state-contingent bond that pays \$1 at time τ in state s^τ , and 0 otherwise. If we insert this state-contingent security into the household's optimization problem. We see that, for $t < \tau$,

$$(13) \quad p_{s^t}^{s^\tau} = \beta E_{s^t} (E_{s^t} V_{s^{t+1}}^{1-\alpha})^{\alpha/(1-\alpha)} V_{s^{t+1}}^{-\alpha} \frac{u_1(c_{s^{t+1}}, l_{s^{t+1}})}{u_1(c_{s^t}, l_{s^t})} \frac{P_{s^t}}{P_{s^{t+1}}} p_{s^{t+1}}^{s^\tau}.$$

That is, the household's (nominal) stochastic discount factor at time t in state s^t for stochastic payoffs at time $t + 1$ is given by

$$(14) \quad m_{s^t, s^{t+1}} \equiv \left(\frac{V_{s^{t+1}}}{(E_{s^t} V_{s^{t+1}}^{1-\alpha})^{1/(1-\alpha)}} \right)^{-\alpha} \frac{\beta u_1(c_{s^{t+1}}, l_{s^{t+1}})}{u_1(c_{s^t}, l_{s^t})} \frac{P_{s^t}}{P_{s^{t+1}}}.$$

Despite the twisting of the value function by $1 - \alpha$, the price $p_{s^t}^{s^\tau}$ nevertheless satisfies the standard relationship,

$$\begin{aligned} p_{s^t}^{s^\tau} &= E_{s^t} m_{s^t, s^{t+1}} m_{s^{t+1}, s^{t+2}} p_{s^{t+2}}^{s^\tau} \\ &= E_{s^t} m_{s^t, s^{t+1}} m_{s^{t+1}, s^{t+2}} \cdots m_{s^{t-1}, s^t}, \end{aligned}$$

and the asset pricing equation (13) is linear in the future state-contingent payoffs, so we can price any compound security by summing over the prices of its individual constituent state-contingent payoffs.

C. The Firm's Optimization Problem

To model nominal rigidities, we assume that the economy contains a continuum of monopolistically competitive intermediate goods firms indexed by $f \in [0, 1]$ that set prices according to Calvo contracts and hire labor from households in a competitive labor market. Firms have identical Cobb-Douglas production functions:

$$(15) \quad y_t(f) = A_t k_t(f)^{1-\eta} (Z_t l_t(f))^\eta,$$

where we suppress the explicit state-dependence of the variables in this equation and in the remainder of the paper to simplify notation. In (15), Z_t is the aggregate productivity trend, A_t a stationary aggregate productivity shock that affects all firms, and $k_t(f)$ and $l_t(f)$ denote the firm's capital and labor inputs. The technology shock A_t follows an exogenous AR(1) process:

$$(16) \quad \log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A,$$

with $|\rho_A| < 1$, and where ε_t^A denotes an independently and identically distributed white noise process with mean zero and variance σ_A^2 .

We follow Woodford (2003) and Altig et al. (2011) and assume that capital is firm-specific, which those authors show is important for generating inflation

persistence in the model consistent with the data.¹² By increasing the persistence of inflation to more realistic levels, firm-specific capital stocks also help the model match the nominal term premium in the data. We also follow Woodford (2003) and abstract from endogenous capital accumulation for simplicity. Endogenous capital would introduce two more state variables into the model (lagged capital and, for investment adjustment costs, lagged investment) and would make modeling firm-specific capital difficult if not impossible in our nonlinear framework.¹³ Moreover, Woodford (2003) finds that the basic business cycle features of the model with fixed firm-specific capital are very similar to a model with endogenous capital and investment adjustment costs, so we do not view this simplifying assumption as substantially altering the business cycle correlations of our model.¹⁴ Finally, for consistency with balanced growth, we assume that each firm's capital stock $k_t(f)$ grows with the productivity trend Z_t , so that $k_t(f) = \bar{k} Z_t$.

Firms set prices according to Calvo contracts that expire with probability $1 - \xi$ each period. When the Calvo contract expires, the firm can reset its price freely, and we denote the price that firm f sets in period t by $p_t(f)$. There is no indexation, so the price $p_t(f)$ is fixed over the life of the contract. In each period $\tau \geq t$ that the contract remains in effect, the firm must supply whatever output is demanded at the contract price $p_t(f)$, hiring labor $l_\tau(f)$ from households at the market wage w_τ .

Firms are collectively owned by households and distribute profits and losses back to households each period. When a firm's price contract expires, the firm chooses the new contract price $p_t(f)$ to maximize the value to shareholders of the firm's cash flows over the lifetime of the contract (equivalently, the firm chooses a state-contingent plan for prices that maximizes the value of the firm to shareholders):

$$(17) \quad E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} [p_t(f) y_{t+j}(f) - w_{t+j} l_{t+j}(f)],$$

where $m_{t,t+j}$ is the representative household's stochastic discount factor from period t to $t + j$.

The output of each intermediate firm f is purchased by a perfectly competitive final goods sector that aggregates the continuum of intermediate goods into a single final good using a CES production technology:

$$(18) \quad Y_t = \left[\int_0^1 y_t(f)^{1/(1+\theta)} df \right]^{1+\theta}.$$

¹² See particularly Woodford (2003, 158–173). Unlike Woodford, but consistent with Altig et al. (2011), we assume that labor input is homogeneous across firms.

¹³ In a nonlinear model, endogenous firm-specific capital does not have a known recursive formulation. Altig et al. (2011) show that the linearized version of the model is first-order recursive and thus can be modeled and solved using standard methods.

¹⁴ In fact, our model is equivalent to one with investment adjustment costs (e.g., Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007; Altig et al. 2011), but with the adjustment costs being large enough that investment is always equal to steady-state investment.

Each intermediate firm f thus faces a downward-sloping demand curve for its product

$$(19) \quad y_t(f) = \left(\frac{p_t(f)}{P_t} \right)^{-(1+\theta)/\theta} Y_t,$$

where P_t is the CES aggregate price per unit of the final good

$$(20) \quad P_t \equiv \left[\int_0^1 p_t(f)^{-1/\theta} df \right]^{-\theta}.$$

Differentiating (17) with respect to $p_t(f)$ yields the standard firm optimality condition

$$(21) \quad p_t(f) = \frac{(1 + \theta) E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} mc_{t+j}(f) y_{t+j}(f)}{E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j} y_{t+j}(f)},$$

where $mc_t(f)$ denotes the marginal cost for firm f at time t :

$$(22) \quad mc_t(f) \equiv \frac{w_t l_t(f)}{\eta y_t(f)}.$$

D. Aggregate Resource Constraints and Government

To aggregate firm-level variables to aggregate quantities, it is useful to define cross-sectional price dispersion, Δ_t :

$$(23) \quad \Delta_t^{1/\eta} \equiv (1 - \xi) \sum_{j=0}^{\infty} \xi^j p_{t-j}(f)^{-(1+\theta)/\theta\eta},$$

where the parameter η in the exponent is due to the firm-specificity of capital. We define L_t , the aggregate quantity of labor demanded by firms, by

$$(24) \quad L_t \equiv \int_0^1 l_t(f) df.$$

Then L_t satisfies

$$(25) \quad Y_t = \Delta_t^{-1} A_t K_t^{1-\eta} (Z_t L_t)^\eta,$$

where $K_t = Z_t \bar{k}$ is the aggregate capital stock. We normalize the representative households to have unit mass, so equilibrium in the labor market requires that $L_t = l_t$, labor demand equals the aggregate labor supplied by households.

To allow for potentially important effects of fiscal policy on the yield curve, we assume a fiscal authority that levies lump sum taxes G_t on households and destroys the resources it collects.¹⁵ Government consumption $G_t = g_t Z_t$ grows with the economy, with g_t following an exogenous AR(1) process:

$$(26) \quad \log(g_t/g) = \rho_G \log(g_{t-1}/g) + \varepsilon_t^G,$$

where $|\rho_G| < 1$, g is the steady-state level of G_t/Z_t , and ε_t^G denotes an independently and identically distributed government consumption shock with mean zero and variance σ_G^2 .

We also assume that some output each period must be devoted to maintaining and increasing the capital stock, and we denote this output by $I_t = \bar{k}(1 + \gamma - \delta)Z_t$, where δ is the depreciation rate of capital and γ is the growth rate of Z_t . The economy's aggregate resource constraint implies that

$$(27) \quad Y_t = C_t + I_t + G_t,$$

where aggregate consumption C_t equals c_t , the representative household's consumption. Including I_t and G_t in (27) helps the model to match time variation in the term premium below, because it makes the model less homogeneous in response to shocks.

Finally, there is a monetary authority that sets the one-period continuously compounded nominal interest rate i_t according to a Taylor-type policy rule,

$$(28) \quad i_t = \rho_i i_{t-1} + (1 - \rho_i)[r^* + \log \bar{\pi}_t + g_y(Y_t - Y_t^*)/Y_t^* \\ + g_\pi(\log \bar{\pi}_t - \log \pi^*)] + \varepsilon_t^i,$$

where r^* denotes the steady-state real interest rate; Y_t^* is the trend level of output yZ_t (where y denotes the steady-state level of Y_t/Z_t); π^* is the steady-state rate of inflation; ε_t^i is an independently and identically distributed stochastic monetary policy shock with mean zero and variance σ_i^2 ; and ρ_i , g_y , and g_π are parameters.¹⁶ The variable $\bar{\pi}_t$ denotes a geometric moving average of inflation:

$$(29) \quad \log \bar{\pi}_t = \theta_\pi \log \bar{\pi}_{t-1} + (1 - \theta_\pi) \log \pi_t,$$

¹⁵Ex ante, fiscal policy and monetary policy could be very important drivers of the term premium. Ex post, we will show in Section IID that technology shocks are by far the most important driver of the term premium. Nevertheless, this is a result of the paper and not something assumed ex ante. In addition, the presence of government purchases makes the model less homogeneous, which, as discussed in Section IIE, helps to make the term premium more variable over time.

¹⁶In equation (28) (and equation (28) only), we express i_t , π_t , and $1/\beta$ in annualized terms, so that the coefficients g_π and g_y correspond directly to the estimates in the empirical literature. We also follow the literature by assuming an "inertial" policy rule with independently and identically distributed policy shocks, although there are a variety of reasons to be dissatisfied with the assumption of AR(1) processes for all stochastic disturbances except the one associated with short-term interest rates. Indeed, Rudebusch (2002, 2006) and Carrillo, Fève, and Matheron (2007) provide strong evidence that an alternative policy specification with serially correlated shocks and little gradual adjustment is more consistent with the dynamic behavior of nominal interest rates.

where current-period inflation $\pi_t \equiv P_t/P_{t-1}$, and we set $\theta_\pi = 0.7$ so that the geometric average in (29) has an effective duration of about four quarters, which is typical in estimates of the Taylor rule.¹⁷

E. Productivity Trend and Balanced Growth

The variables Y_t , C_t , I_t , G_t , K_t , and w_t can be normalized by the productivity trend Z_t to yield a stationary system with a nonstochastic steady state. The productivity trend Z_t may be either deterministic or stochastic. The macroeconomic data do not provide clear evidence in favor of either a trend-stationary or difference-stationary specification for Z_t (see, e.g., Christiano and Eichenbaum 1990), and it is also true that the yield curve does not provide clear evidence in favor of either of these specifications (if it did, the question of whether US GDP is trend- or difference-stationary would have been resolved already).¹⁸ Since the data do not clearly distinguish between the trend- and difference-stationary specifications for Z_t , we take Z_t to be a deterministic trend in our baseline results and estimation below (that is, $Z_t = \gamma Z_{t-1}$, $\gamma > 1$) and let the stochastic process for A_t capture transitory dynamics in productivity. However, when we consider long-run risks, below, we will switch to a persistent difference-stationary specification for Z_t , described in detail there.

F. Long-Term Bonds and the Term Premium

The price of any asset in the model equals the sum of the stochastically discounted state-contingent payoffs of the asset. For example, the price of a default-free n -period zero-coupon bond that pays \$1 at maturity satisfies

$$(30) \quad p_t^{(n)} = E_t[m_{t+1} p_{t+1}^{(n-1)}],$$

where $m_{t+1} \equiv m_{t,t+1}$; $p_t^{(n)}$ denotes the price of the bond at time t ; and $p_t^{(0)} \equiv 1$, i.e., the time- t price of \$1 delivered at time t is \$1. The continuously compounded yield to maturity on the n -period zero-coupon bond is defined to be

$$(31) \quad i_t^{(n)} \equiv -\frac{1}{n} \log p_t^{(n)}.$$

In the US data, the benchmark long-term bond is the 10-year Treasury note, so we will focus on the model-implied yield to maturity of a 40-quarter zero-coupon

¹⁷Including the usual four-quarter moving average of inflation in the policy rule adds three lags (π_{t-1} , π_{t-2} , and π_{t-3}) as state variables, while our geometric average adds only one lag ($\bar{\pi}_{t-1}$). All results are very similar for either specification.

¹⁸Labadie (1994) finds that the slope of the yield curve depends on whether consumption is modeled as a trend-stationary or difference-stationary process. However, the differences she highlights are for a stationary AR(1) model versus an AR(2) model with a unit root (i.e., an ARIMA(1,1,0) model). The term structure differences between an AR(1) model with a near unit root and an AR(1) model with an exact unit root are negligible in Labadie's own formulas (as are the differences between an AR(2) model with a near unit root and an AR(2) model with an exact unit root). More generally, the term structure implications of the consumption process are continuous in the roots of the lag polynomial governing that process, including at and around unity. Thus, the term structure does not readily distinguish between the cases where Z_t is a process with a unit root or merely a near unit root.

bond in our analysis below.¹⁹ Note that even though the nominal bond in our model is default-free, it is still risky in the sense that its price can co-vary with the household's marginal utility of consumption. For example, when inflation is expected to be higher in the future, then the price of the bond generally falls, because households discount its future nominal coupons more heavily. If times of high inflation are correlated with times of low output (as is the case for technology shocks in the model), then households regard the nominal bond as being very risky, because it loses value at exactly those times when the household values consumption the most. Alternatively, if inflation is not very correlated with output and consumption, then the bond is correspondingly less risky. In the former case, we would expect the bond to carry a substantial risk premium (its price would be lower than the risk-neutral price), while in the latter case we would expect the risk premium to be smaller.

In the literature, the risk premium or term premium on a long-term bond is typically expressed as the difference between the yield on the bond and the unobserved risk-neutral yield for that same bond. To define the term premium in our model, then, we first define the *risk-neutral* bond price, $\hat{p}_t^{(n)}$:

$$(32) \quad \hat{p}_t^{(n)} \equiv e^{-i_t} E_t \hat{p}_{t+1}^{(n-1)},$$

where again $\hat{p}_t^{(0)} \equiv 1$. Equation (32) is the expected present discounted value of the bond's terminal payment, where the discounting is performed using the risk-free rate rather than the stochastic discount factor. The implied term premium on the bond is then given by

$$(33) \quad \psi_t^{(n)} \equiv \frac{1}{n} (\log \hat{p}_t^{(n)} - \log p_t^{(n)}),$$

which is the difference between the observed yield to maturity on the bond and the risk-neutral yield to maturity. When reporting our results below, we multiply (33) by 400 in order to report the term premium in units of annualized percentage points rather than logs.

The term premium (33) can also be expressed more directly in terms of the stochastic discount factor, which can be useful for gaining intuition about how the term premium is related to the economic shocks driving the DSGE model. First,

$$\begin{aligned} (34) \quad p_t^{(n)} - \hat{p}_t^{(n)} &= E_t m_{t+1} p_{t+1}^{(n-1)} - E_t m_{t+1} E_t \hat{p}_{t+1}^{(n-1)}, \\ &= \text{cov}_t(m_{t+1}, p_{t+1}^{(n-1)}) + E_t m_{t+1} E_t (p_{t+1}^{(n-1)} - \hat{p}_{t+1}^{(n-1)}), \\ &= \text{cov}_t(m_{t+1}, p_{t+1}^{(n-1)}) + e^{-i_t} E_t (p_{t+1}^{(n-1)} - \hat{p}_{t+1}^{(n-1)}), \\ &= E_t \sum_{j=0}^{n-1} e^{-i_{t,j+1}} \text{cov}_{t+j}(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}), \end{aligned}$$

¹⁹In Rudebusch and Swanson (2008b), we derive and emphasize results for a *generalized consol*, which pays a geometrically declining coupon in every period *ad infinitum*. The advantage of a generalized consol is that it has a first-order recursive form that fits very naturally with standard macroeconomic models. In the present paper, we report results only for a 40-quarter zero-coupon bond, and refer the interested reader to Rudebusch and Swanson (2008b) for the generalized consol results, which are very similar.

where $i_{t,t+j} \equiv \sum_{m=0}^{j-1} i_{t+m}$, and the last line of (34) follows from forward recursion. Equation (34) makes it clear that, even though the bond price depends only on the one-period-ahead covariance between the stochastic discount factor and next period's bond price, the bond *risk premium* depends on this covariance over the entire lifetime of the bond. The term premium is given by

$$\begin{aligned}
 (35) \quad \psi_t^{(n)} &= \frac{1}{n} (\log \hat{p}_t^{(n)} - \log p_t^{(n)}), \\
 &\approx \frac{1}{n\bar{p}^{(n)}} (\hat{p}_t^{(n)} - p_t^{(n)}), \\
 &= \frac{-1}{n\bar{p}^{(n)}} E_t \sum_{j=0}^{n-1} e^{-i_{t,t+j+1}} \text{cov}_{t+j}(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}),
 \end{aligned}$$

where $\bar{p}^{(n)}$ denotes the nonstochastic steady-state bond price.²⁰ Intuitively, the term premium is larger the more negative the covariance between the stochastic discount factor and the price of the bond over the lifetime of the bond.

G. Alternative Measures of Long-Term Bond Risk

Although the term premium is conceptually the cleanest measure of the riskiness of long-term bonds in our model, it is not directly observed in the data and must be inferred using term structure models or other methods. Accordingly, the literature has also focused on two directly observable empirical measures that are closely related to the term premium: the slope of the yield curve and the excess return to holding the long-term bond for one period.

The slope of the yield curve is simply the difference between the yield to maturity on the long-term bond and the one-period risk-free rate, i_t . The slope is an imperfect measure of the riskiness of the long-term bond because it can vary in response to shocks even if all investors in the model are risk-neutral. However, on average, the slope of the yield curve equals the term premium, and the volatility of the slope provides us with a noisy measure of the volatility of the term premium.

The excess return to holding the long-term bond for one period is given by

$$(36) \quad x_t^{(n)} \equiv \frac{p_t^{(n-1)}}{p_{t-1}^{(n)}} - e^{i_{t-1}}.$$

The first term on the right-hand side of (36) is the gross return to holding the long-term bond and the second term is the gross one-period risk-free rate. As with

²⁰The first-order approximation on the second line of (35) is useful for gaining intuition. However, when we solve for the term premium in the model numerically, below, our solution will include the second- and third-order as well as the first-order terms.

the yield curve slope, the excess return in (36) is an imperfect measure of the term premium because it can vary in response to shocks even if investors are risk-neutral. However, the mean and standard deviation of the excess holding period return are directly observable and are popular measures of the deviation of long-term bond markets from risk neutrality.

H. Model Solution Method

A technical issue in solving the model above arises from its relatively large number of state variables: A_{t-1} , G_{t-1} , i_{t-1} , Δ_{t-1} , $\bar{\pi}_{t-1}$; and the three shocks, ε_t^A , ε_t^G , and ε_t^i , make a total of eight.²¹ Because of this high dimensionality, discretization and projection methods are computationally infeasible, so we solve the model using the standard macroeconomic technique of approximation around the nonstochastic steady state—so-called perturbation methods. However, a first-order approximation of the model and bond price (i.e., a linearization or log-linearization) eliminates the term premium entirely, because equations (30) and (32) are identical to first order. A second-order approximation to the solution of the model and bond price produces a term premium that is nonzero but constant (a weighted sum of the variances σ_A^2 , σ_G^2 , and σ_i^2). Since our interest in this paper is not just in the level of the term premium but also in its volatility and variation over time, we compute a third-order approximate solution to the model and bond price around the nonstochastic steady state using the algorithm of Swanson, Anderson, and Levin (2006). For the baseline model above with eight state variables, a third-order accurate solution can be computed in about two minutes on a standard laptop computer, and for the more complicated specifications we consider below with long-run risks, a third-order solution can be computed in less than 20 minutes. Additional details of this solution method are provided in Swanson, Anderson, and Levin (2006) and Rudebusch, Sack, and Swanson (2007).

To compute long-term zero-coupon bond prices and their measures of risk, it is most efficient to solve the model in two stages. First, we solve for the macroeconomic variables, including the stochastic discount factor, to third order. Then, in a second stage, we use the stochastic discount factor from the first step to solve for the bond prices in equations (30) and (32) to third order for each maturity ranging from 1 to 40 quarters.

Once we have computed an approximate solution to the model and bond prices, we compare the model and the data using a standard set of macroeconomic and financial moments, such as the standard deviations of consumption, labor, and other variables, and the means and standard deviations of the term premium and other measures of long-term bond risk described above. One method of computing these moments is by simulation, but this method is slow and, for a nonlinear model, the simulations can sometimes diverge to infinity. We thus compute these moments in closed form, using perturbation methods. In particular, we compute the

²¹ The number of state variables can be reduced a bit by noting that G_t and A_t are sufficient to incorporate all of the information from G_{t-1} , A_{t-1} , ε_t^G , and ε_t^A , but the basic point remains valid, namely, that the number of state variables in the model is large from a computational point of view.

unconditional standard deviations and unconditional means of the variables of the model to second order.²² For the term premium, the unconditional standard deviation is zero to second order, so we compute the unconditional standard deviation of the term premium to third order. This method yields results that are extremely close to those that arise from simulation, while at the same time being quicker and more numerically robust.

II. Comparing the Epstein-Zin DSGE Model to the Data

We now investigate whether the standard DSGE model, extended to the case of Epstein-Zin preferences as developed in the previous section, is consistent with the most basic features of the macroeconomic and financial market data. We first investigate the behavior of the model under a baseline set of parameter values and then search over those parameters to find the best possible fit of the model to the data.

A. Model Parameterization

The baseline parameter values that we use for our simple New Keynesian model are reported in Table 1 and are fairly standard in the literature (e.g., Smets and Wouters 2007, and Levin et al. 2006). We set the growth rate of productivity, γ , to 1 percent per year. The household's discount factor, β , is set to imply a value for $\tilde{\beta} \equiv \beta\gamma^{-\varphi}$ of 0.99, consistent with an annual real interest rate of about 4 percent. The depreciation rate δ is set to 0.02, implying a steady-state investment-output ratio of 22.5 percent. We set χ_0 to imply steady-state household hours worked, l , equal to 1/3 of the time endowment. The steady-state capital-output ratio is set to 2.5, and government purchases consume 17 percent of output in steady state.

We set the curvature of household utility with respect to consumption, φ , to 2, implying an intertemporal elasticity of substitution (IES) in consumption of 0.5, which is consistent with estimates in the micro literature (e.g., Vissing-Jørgensen 2002), but we will also estimate this parameter below. The curvature of household utility with respect to labor, χ , is set to imply a Frisch elasticity of labor supply of 2/3, also in line with estimates from the microeconomics literature (e.g., Pistaferri 2003). We set the Epstein-Zin parameter α to imply a coefficient of relative risk aversion of 75 (and we discuss this parameter in detail in the next section).

We set firms' output elasticity with respect to labor, η , to 2/3, firms' steady-state markup, θ , to 0.2 (implying a price-elasticity of demand of 6), and the Calvo frequency of price adjustment, ξ , to 0.75 (implying an average price contract duration of four quarters), all of which are standard in the literature.

The monetary policy rule coefficients, ρ_i , g_π , and g_y , are taken from Rudebusch (2002) and are typical of those in the literature. We normalize the central bank's target inflation rate π^* to 0 (equivalently, one could assume that nominal price contracts

²²To compute the standard deviations of the variables to second order, we compute a fourth-order accurate solution to the unconditional covariance matrix of the variables and then take the square root along the diagonal. Note that a third-order accurate solution for X and Y is sufficient to compute the product $E[XY]$ to fourth order, when X and Y have zero mean (as in a covariance).

TABLE 1—BASELINE PARAMETER VALUES FOR THE SIMPLE DSGE MODEL

γ	1.0025	φ	2	ρ_i	0.73	ρ_A	0.95
$\tilde{\beta}$	0.99	χ	3	g_π	0.53	ρ_G	0.95
δ	0.02	CRRA	75	g_y	0.93	σ_A^2	0.005 ²
l	1/3	η	2/3	π^e	0	σ_G^2	0.004 ²
$\bar{K}/(4\bar{Y})$	2.5	θ	0.2			σ_i^2	0.003 ²
\bar{G}/\bar{Y}	0.17	ξ	0.75				
Memo:							
χ_0	1.224						
α	−148.3						
IES	0.5						
Frisch	2/3						

in the economy are indexed to the steady-state inflation rate). The shock persistences ρ_A and ρ_G are set to 0.95 and the shock variances σ_A^2 , σ_G^2 , and σ_i^2 are set to 0.005², 0.004², and 0.003², respectively, consistent with the estimates in Smets and Wouters (2007), Levin et al. (2006), and Rudebusch (2002).

B. The Coefficient of Relative Risk Aversion

The coefficient of relative risk aversion measures the household’s aversion to gambles over wealth. Because we are interested in pricing a risky asset in the model (a long-term nominal bond), the degree of household risk aversion is a crucial parameter. In previous studies of Epstein-Zin preferences in an endowment economy, computing household risk aversion is typically straightforward because those studies exclude labor and are homogeneous; that is, the quantity of consumption demanded in each period depends linearly on the household’s beginning-of-period wealth. Risk aversion in that setting is typically measured as $-WV''/V'$, where W denotes household wealth, V the household’s value function, and V' the derivative with respect to wealth. However, when household labor supply is endogenous, closed-form solutions for V do not exist in general, so computing risk aversion is more difficult than in the homogeneous consumption-only case. In this paper, we use closed-form expressions for risk aversion derived by Swanson (2009, forthcoming) for the more general case where the model is nonhomogeneous and households can vary their labor supply.²³

²³For the additively separable utility kernel considered above, Swanson (2009) shows that the household’s consumption-based coefficient of relative risk aversion is given by

$$\text{CRRA} = \frac{\gamma}{1 + \frac{\gamma}{\chi} \frac{1-l}{l}} + \alpha \frac{1-\gamma}{1 + \frac{1-\gamma}{1-\chi} \frac{1-l}{l}}.$$

In Rudebusch and Swanson (2008b), we measured risk aversion using what we called the *quasi*-coefficient of relative risk aversion, or quasi-CRRA, which was the coefficient of relative risk aversion holding household labor fixed. (For the utility kernel above, the quasi-CRRA is $\gamma + \alpha(1 - \gamma)$.) In general, the consumption-based coefficient of relative risk aversion is less than the quasi-CRRA because the household can vary its hours worked to help insure itself from the outcome of gambles over asset values—see Swanson (2009, forthcoming).

In the baseline parameterization of our model given in Table 1, we set the coefficient of relative risk aversion to 75. This may seem like a high baseline value, but we will also estimate a high value for risk aversion below, and it will be helpful for gaining intuition for our results to have a baseline coefficient of risk aversion that is similarly high.²⁴ We also investigate to what extent the need for high risk aversion can be reduced by appealing to long-run risks in the model, such as a long-run risk to inflation.

There are a number of possible explanations for why representative-agent models such as ours seem to require high levels of risk aversion to match asset prices. Barillas, Hansen, and Sargent (2009) show that high risk aversion in an Epstein-Zin specification is isomorphic to a model in which households have low risk aversion but a moderate degree of uncertainty about the economic environment. In other words, one of the reasons our simple DSGE model requires high risk aversion to fit the data is that households in our model have perfect knowledge about the equations of the model, the model's parameter values, and so on. Thus, the quantity of risk in our model is much smaller than in the US economy and, as a result, the household's aversion to risk in our model must be correspondingly higher to fit the data. Campanale, Castro, and Clementi (2010) also emphasize that the quantity of consumption risk in a standard DSGE model is very small, and thus the required risk aversion to match asset prices is correspondingly large. In addition, Malloy, Moskowitz, and Vissing-Jørgensen (2009) show that the consumption of stockholders is more volatile than the consumption of nonstockholders. As a result, the required level of risk aversion in a representative-agent model like ours is higher than the level of risk aversion that would be required in a model that recognized that asset holders have more variable consumption than households that do not hold assets. In other words, our simple, representative-agent DSGE model, again, may understate the true quantity of risk that bondholders in the US economy face. Since our model understates the quantity of risk faced by US households, it requires a higher degree of risk aversion to match the risk premiums in the data.

C. Model Results

We report various model-implied moments in Table 2, along with the corresponding empirical moments for quarterly US data from 1961 to 2007. The first set of rows reports a set of very basic macroeconomic moments that the model should be able to match, while the second set of rows reports a set of very basic financial moments.²⁵ Additional information about the model moments and parameter values are reported in the last two sets of rows of the table.

The empirical moments in the first column of Table 2 are relatively standard and were computed as follows: consumption, C , is real personal consumption

²⁴ Other studies using Epstein-Zin preferences also typically estimate a high coefficient of relative risk aversion: Piazzesi and Schneider (2007) estimate a value of 57, and van Binsbergen et al. (2010) a value of about 80.

²⁵ We omit output from the macro moments because our simple DSGE model has fixed capital and investment, so output and consumption behave very similarly. The standard deviation of the long-term bond yield, $i^{(40)}$, has some elements of both a "macro" and a "finance" moment, but we will classify it as a macro moment for the purposes of the table and this discussion.

TABLE 2—EMPIRICAL AND MODEL-BASED UNCONDITIONAL MOMENTS

Unconditional moment	US data, 1961–2007	Model with expected utility preferences	Model with Epstein-Zin preferences	Model with EZ preferences (best fit)
SD [C]	0.83	1.32	1.38	1.10
SD [L]	1.71	1.57	1.56	2.42
SD [w^r]	0.82	1.17	1.23	1.13
SD [π]	2.52	1.75	1.81	2.67
SD [i]	2.71	1.63	1.68	3.09
SD [r]	2.30	1.19	1.23	1.80
SD [$i^{(40)}$]	2.41	0.84	0.91	2.14
Mean [$\psi^{(40)}$]	1.06	0.007	0.375	1.12
SD [$\psi^{(40)}$]	0.54	0.001	0.047	0.47
Mean [$i^{(40)} - i$]	1.43	−0.030	0.337	1.02
SD [$i^{(40)} - i$]	1.33	0.91	0.91	1.31
Mean [$x^{(40)}$]	1.76	0.012	0.639	2.02
SD [$x^{(40)}$]	23.43	8.99	9.81	20.64
Distance to:				
Macro moments		5.63	5.18	1.16
Finance moments		8.85	5.19	0.32
All moments		14.48	10.37	1.49
Memo:				
IES		0.5	0.5	0.11
CRRA		6/7	75	110
Frisch		2/3	2/3	0.28
ξ		0.75	0.75	0.78
ρ_A		0.95	0.95	0.96
σ_A		0.005	0.005	0.005

Notes: All variables are quarterly values expressed in percent. Inflation, interest rates, the term premium (ψ), and excess holding period returns (x) are expressed at an annual rate.

expenditures of nondurables and services from the US national income and product accounts; labor, L , is total hours of production workers from the Bureau of Labor Statistics (BLS); and the real wage, w^r , is total wages and salaries of production workers from the BLS divided by total production worker hours and deflated by the GDP price index. Standard deviations were computed for logarithmic deviations of each series from a Hodrick-Prescott trend and reported in percentage points. Standard deviations for inflation, interest rates, and the term premium were computed for the raw series rather than for deviations from trend. Inflation, π , is the annualized rate of change in the quarterly GDP price index from the Bureau of Economic Analysis. The short-term nominal interest rate, i , is the end-of-month federal funds rate from the Federal Reserve Board, reported in annualized percentage points. The short-term real interest rate, r , is the short-term nominal interest rate less the realized quarterly inflation rate at an annual rate. The ten-year zero-coupon bond yield, $i^{(40)}$, is the end-of-month ten-year zero-coupon bond yield taken from Gürkaynak, Sack,

and Wright (2007). The term premium on the ten-year zero-coupon bond, $\psi^{(40)}$, is the term premium computed by Kim and Wright (2005), in annualized percentage points.²⁶ The yield curve slope and one-period excess holding return are calculated from the data above and are reported in annualized percentage points.

The second column of Table 2 reports results for the baseline version of our model with expected utility preferences (that is, all parameters are the same as in Table 1, except that $\alpha = 0$).²⁷ The model does a reasonable job of matching the basic macroeconomic moments in the first seven rows of the table—indeed, this is one of the main reasons these models have become so widely used in macroeconomics. However, the term premium implied by the expected utility version of the model is both too small in magnitude—less than one basis point—and far too stable, with an unconditional standard deviation less than one-tenth of one basis point. This basic finding of a term premium that is too small and far too stable relative to the data is extremely robust with respect to wide variation of the model's parameters (see Rudebusch and Swanson 2008a, for additional discussion and sensitivity analysis).

The third column of Table 2 reports results from the baseline parameterization of the model with Epstein-Zin preferences and a relative risk aversion coefficient of 75 ($\alpha = -148$). Note that the model fits all of the macroeconomic variables just as well as the expected utility version of the model; that is, even for relatively high levels of risk aversion, the dynamics of the macroeconomic variables implied by the model are largely unchanged, a finding that also has been noted by Tallarini (2000) and Backus, Routledge, and Zin (2007). This is a straightforward implication of two features of the model. First, the linearization or log-linearization of Epstein-Zin preferences (3) is exactly the same as that of standard expected utility preferences (2), so to first order, these two utility specifications are the same. Second, the shocks that we consider in the model, and are standard in macroeconomics, have standard deviations of only about 1 percent or less, so a linear approximation to the model is typically very accurate.²⁸

For asset prices, however, the implications of the Epstein-Zin and expected utility versions of the model are very different, since risk premia in the model are entirely determined by second-order and higher-order terms. With Epstein-Zin preferences, the mean term premium is 37.5 basis points—about 50 times higher than under expected utility—and the standard deviation of the term premium is 4.7 bp, compared to 0.1 bp for expected utility. The fit of the model to the yield curve slope and excess holding period return show similarly marked improvements.

²⁶ Kim and Wright (2005) use an arbitrage-free, three-latent-factor affine model of the term structure to compute the term premium. Alternative measures of the term premium using a wide variety of methods produce qualitatively similar results in terms of the overall magnitude and variability—see Rudebusch, Sack, and Swanson (2007) for a detailed discussion and comparison of several methods.

²⁷ The coefficient of relative risk aversion is not equal to φ in the expected utility case when households can vary their labor supply—see Swanson (forthcoming) for details. For the functional form of utility used in this model, the coefficient of relative risk aversion is $6/7$ rather than 2.

²⁸ As the magnitude of α increases, second-order terms in the model become relatively more important for the macroeconomic variables. Yet even for the case $\alpha = -148$, second-order terms do not have a very large effect on the macroeconomic moments in the second column of Table 2. Intuitively, this is because V is both “twisted” and “untwisted” by the factor $(1 - \alpha)$, so that much of the curvature that α introduces into the model is effectively neutralized. As a result, the parameter α only affects the macro variables in the model through its effect on uncertainty, precautionary motives, and the like, and this has only a small effect on the unconditional moments we report in Table 2.

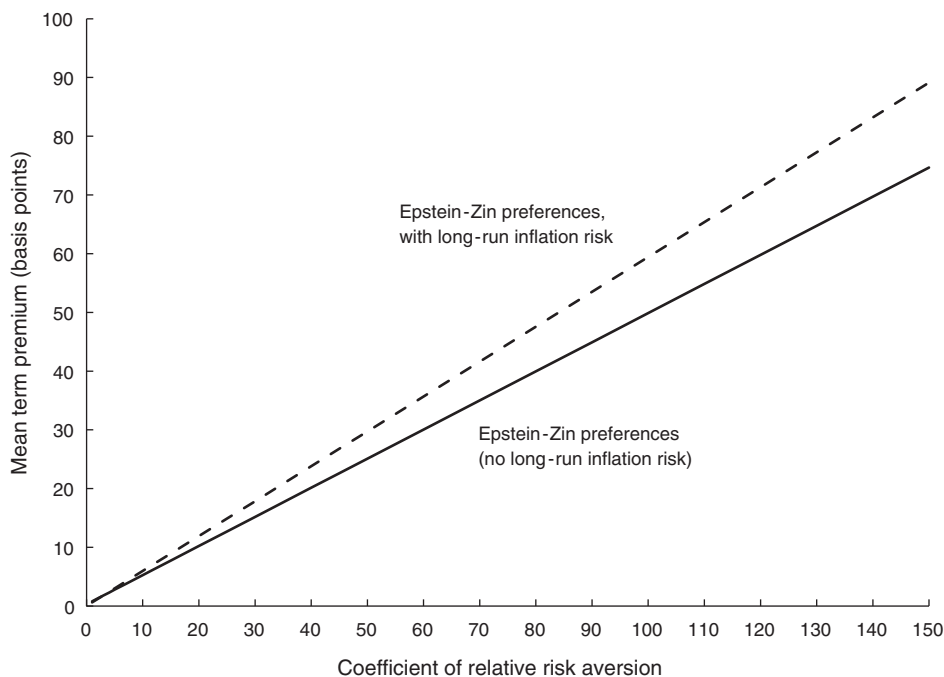


FIGURE 1. MEAN TERM PREMIUM WITH VARYING AMOUNTS OF RISK AVERSION

Note: The solid and dashed lines show the mean 10-year term premium as a function of risk aversion in our DSGE model without and with long-run inflation risk, respectively.

Figure 1 graphically illustrates the effects of increasing the coefficient of relative risk aversion in the model by plotting the mean term premium, $\psi^{(40)}$, as a function of the coefficient of relative risk aversion, holding all the other parameters of the model fixed at their baseline values. As the coefficient of relative risk aversion increases, the mean term premium rises steadily, so that essentially any level of the term premium can be attained by making households sufficiently risk-averse. The special case of expected utility preferences corresponds to the bottom-left corner of the solid line, with risk aversion equal to $6/7$ and a mean term premium of less than 1 bp.

The last column of Table 2 reports results from the “best fit” parameterization of the model with Epstein-Zin preferences, where we search over a wide range of values to find the parameterization that provides the closest joint fit of the model to both the macroeconomic and financial moments in the data. The computational time required to solve the model for any given set of parameter values can be up to several minutes, so it is generally infeasible to estimate all the parameters of the model using maximum likelihood or Bayesian methods. Instead, we search over a range of values for the six parameters listed at the bottom of Table 2 that are among the most interesting and important for the term premium—namely, the IES, coefficient of relative risk aversion, Frisch elasticity, ξ , ρ_A , and σ_A —to find the best fit to the macroeconomic and financial moments in Table 2 using a generalized method

of moments procedure.²⁹ The distance between the model and the empirical macro moments, finance moments, and both sets of moments together are reported in the third set of rows in the table, above the estimated parameter values.

The fit of the model to the data in the final column of Table 2 is strikingly good—indeed, the model is only a few basis points away from any of the finance moments in the data, with a mean term premium of 112 basis points and a term premium standard deviation of 47 bp. To achieve this fit, the estimation procedure picks a high value for risk aversion, 110, and low values for the IES, 0.11, and Frisch elasticity, 0.28. All else equal, a high value for the CRRA improves the model's ability to fit the finance moments in the data, as we saw in the third column. The low value for the IES helps to make consumption less volatile and real interest rates more volatile, both of which improve the fit to the macro moments in the data. The higher interest rate volatility also increases bond price volatility and improves the model's fit to the finance moments. The low IES also has a side effect of increasing the volatility of labor, as households seek to smooth consumption more aggressively. The low value for the Frisch elasticity helps to reduce this variability of hours. The estimated values for ξ , ρ_A , and σ_A are all close to their baseline values and are standard in the literature.

D. The Importance of Technology Shocks for the Term Premium

We can gain insight into what features of the model are the most important for the term premium by examining the model's impulse responses to shocks. The first column of Figure 2 reports the responses of consumption, inflation, the long-term bond price, and the term premium in the model to a positive one standard deviation shock to technology, using the “best fit” parameterization from Table 2. The second and third columns report analogous impulse responses for one standard deviation shocks to government spending and monetary policy, respectively.³⁰ These impulse responses demonstrate that the correlations between consumption, inflation, and the long-term bond price depend on the type of structural shock.

Over the period 1952–2005, Piazzesi and Schneider (2007) find that a surprise increase in US inflation, which lowers the value of nominal bonds, was typically followed by lower consumption going forward. This relationship implies that long-term nominal bonds lose value precisely when households desire consumption the most, resulting in a positive term premium (cf. equation (35)). In the first column of Figure 2, we can see that a technology shock in our structural model has this same

²⁹We define the “best fit” to be the set of parameters that matches the equally weighted sum of squared deviations from the 13 moments in the first column of Table 2 as closely as possible (with one exception; we divide the standard deviation of the excess holding period return $x^{(40)}$ by 10 to give it roughly as much weight as the other moments in the column). The best fit is computed using a hill-climbing procedure that proceeds from an initial guess in the direction of steepest descent. Multiple initial guesses were tried over a wide range of parameter values, including values for the IES greater than unity. We constrain the estimation to a maximum value of 110 for risk aversion because larger values yield only minuscule additional improvement in fitting the finance moments. Note that minimizing the equal-weighted distance to the 13 moments in Table 2 provides us with a consistent estimator of our parameters, though it is not efficient.

³⁰These impulse responses are computed using the methods described in Rudebusch, Sack, and Swanson (2007). For consumption, inflation, and the long-term bond price, the first-order terms are dominant and provide a very good approximation, so only those (linear) terms are plotted. For the term premium, the impulse response is zero to first and second order, so the third-order terms are dominant and are plotted in the figure.

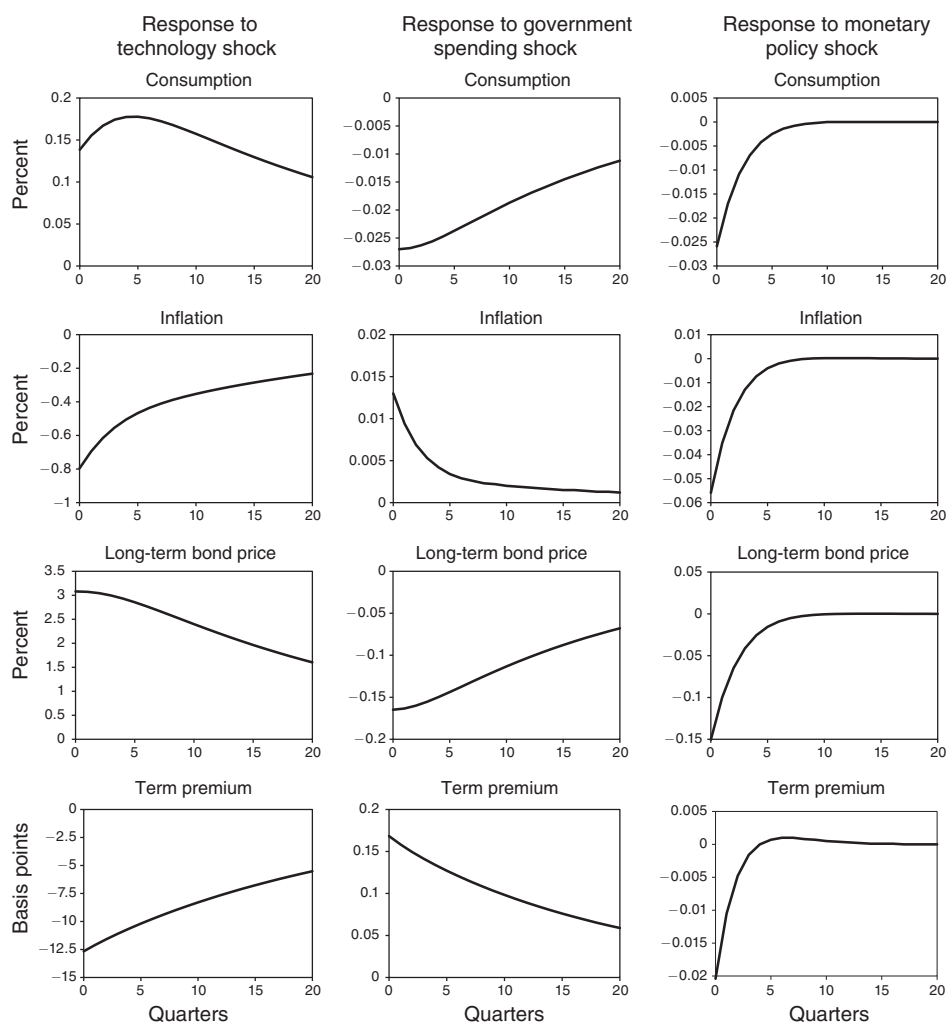


FIGURE 2. IMPULSE RESPONSES TO STRUCTURAL SHOCKS

Note: Impulse responses of consumption, inflation, long-term bond prices, and term premiums to positive one standard deviation shocks to technology, government spending and monetary policy.

feature: an increase in technology causes consumption to rise as households feel wealthier, and also causes marginal cost and inflation to fall (and the fall in inflation causes nominal bond prices to rise). In contrast to technology shocks, monetary policy shocks in our model imply a correlation between consumption and inflation that is exactly the opposite: the rise in short-term interest rates causes bond prices to fall and also consumption and output to fall through intertemporal substitution; the fall in output in turn lowers marginal cost and inflation. Thus, the relationship between consumption and inflation depends critically on the nature of the underlying shocks driving the economy. The reduced-form correlations that Piazzesi and Schneider estimate suggest that technology and/or government spending shocks predominated over their sample as a whole.

While technology shocks are the largest source of macroeconomic fluctuations in our DSGE model, their importance for the model's term premium is even greater. As can be seen in Figure 2, all three shocks imply a negative covariance between the stochastic discount factor and the long-term bond price, and hence a positive term premium, but that covariance is both much larger and much longer-lasting for the technology shock. (This is primarily driven by the large and long-lasting effect that the technology shock has on inflation in the model, which in turn has a large effect on the long-term bond price.) As a result, the technology shock is far more important for the term premium than are the other two shocks, since its impact on the sum of the covariances in equation (35) is so much larger. Thus, in our standard DSGE model, technology shocks and the negative correlation between inflation and consumption that they generate are crucial for matching the term premium.

This observation provides an answer to a bond-pricing puzzle that dates back to Backus, Gregory, and Zin (1989) and den Haan (1995)—namely, why does the yield curve slope upward? According to the traditional line of thinking, if interest rates are low in a recession (when consumption is low), then long-term bond prices should be high in a recession and hence long-term bonds should carry an insurance-like, negative risk premium; that is, the yield curve should slope downward rather than upward. In our New Keynesian DSGE model, a negative technology shock causes inflation to rise persistently at the same time that consumption falls, so long-term nominal bonds *lose* value rather than gain value in recessions.³¹ Thus, our model resolves the puzzle by appealing to the behavior of inflation following a technology shock in standard New Keynesian DSGE models.³² More generally, any supply-type shock that causes inflation to move persistently and inversely to output, such as a technology shock, a markup shock, or an oil price shock, will have similar implications for the term premium.

Moreover, while Piazzesi and Schneider (2007) show that consumption and inflation were negatively correlated over the 1952–2004 period, Benigno (2007) notes that this correlation has varied substantially over time. In particular, the correlation was large and negative in the 1970s and early 1980s, but much smaller in the 1990s and 2000s. This suggests that the relative importance of technology shocks (or supply shocks more generally) may have been larger in the 1970s and early 1980s than over the rest of the sample. If this is true, then our model predicts that we should see this also in the yield curve, with a much larger term premium in the 1970s and early 1980s than in the rest of the sample, and, in fact, the data seem to be consistent with this view (e.g., Kim and Wright 2005, Rudebusch, Sack, and Swanson 2007).

A final point to note is that, not only does our simple model predict a term premium that is positive on average, it also implies that the term premium is countercyclical—that is, technology shocks in Figure 2 cause the term premium to rise at the same time that they cause consumption and output to fall—consistent with a widely-held view in the macro-finance literature

³¹ This same intuition holds for positive government spending shocks as well as negative technology shocks, but the effects of government spending shocks on the term premium in the model are quantitatively smaller.

³² Note that this analysis is for long-term *nominal* bonds rather than real bonds. If real interest rates are lower in recessions, then the traditional line of reasoning still implies that the real yield curve should slope downward. In fact, the evidence in Evans (1998) and Appendix B of Piazzesi and Schneider (2007) suggests that this is the case.

that risk premiums should be and are higher in recessions (e.g., Campbell and Cochrane 1999, Cochrane 2007, Piazzesi and Swanson 2008). Thus, not only is the term premium in our model large and variable, it is consistent with this key business-cycle correlation.

E. Time-Varying Term Premia and Heteroskedasticity

Epstein-Zin preferences also greatly improve the model's ability to generate a term premium that varies over time. For the case of expected utility, the term premium in the model varies by less than 0.1 basis points, and this result is robust to varying the model's parameters over very wide ranges. In contrast, our baseline Epstein-Zin specification produces a term premium with an unconditional standard deviation of about 5bp, and the "best fit" parameterization does even better, generating a term premium with a roughly 50bp standard deviation.

In order for the model to generate appreciable time-variation in the term premium, either the stochastic discount factor or the asset return, or both, must display conditional heteroskedasticity.³³ In our model, the exogenous driving shocks (technology, government purchases, monetary policy) are all homoskedastic, but our DSGE model endogenously generates conditional heteroskedasticity in the stochastic discount factor and other variables. Intuitively, a second-order or higher-order solution to the stochastic discount factor (and other variables) in the model includes terms of the form $x_{t-1}\varepsilon_t$, the product of a state variable and a shock, and the conditional variance of these terms varies with the state of the economy x .

With expected utility preferences, these second-order terms and the conditional heteroskedasticity generated by the model are small. But with Epstein-Zin preferences, the size of these higher-order terms is much greater and leads to substantial endogenously-generated heteroskedasticity in the stochastic discount factor, and hence time-variation in the term premium. To demonstrate this, we start by simulating the third-order solution to our baseline model forward for 1,200 quarters, and drop the first 200 quarters to minimize the effect of initial conditions. We plot the 40-quarter realized standard deviation of the simulated stochastic discount factor as the solid black line in Figure 3. For comparison, we also simulate the first-order (i.e., log-linear) solution to our model forward for the same 1,200 periods, using the same set of shock realizations, and plot the realized standard deviation of the stochastic discount factor from that simulation as the solid gray line in Figure 3. As can be seen in the figure, the third-order stochastic discount factor is both more volatile on average and more heteroskedastic than the linearized stochastic discount factor. The third-order series also tests significantly for GARCH at the 1 percent level, while the linearized series shows no evidence of GARCH.³⁴

³³To see this, note that one measure of the risk premium on an asset is $e^{-i} p_{t+1} - E_t(m_{t+1} p_{t+1}) = -\text{cov}_t(m_{t+1}, p_{t+1})$, the time- t risk-neutral price of the asset less the actual price. If the stochastic discount factor and asset price are both conditionally homoskedastic, then so is the conditional covariance and hence the risk premium.

³⁴By contrast, neither simulated consumption nor simulated inflation show evidence of GARCH. Even though the model generates a small degree of conditional heteroskedasticity through second-order and higher order terms of the form $x_{t-1}\varepsilon_t$, this heteroskedasticity is apparently too small and subtle in these two series to be picked up by a simple univariate GARCH procedure. Like the simulations in the next paragraph, this suggests that the

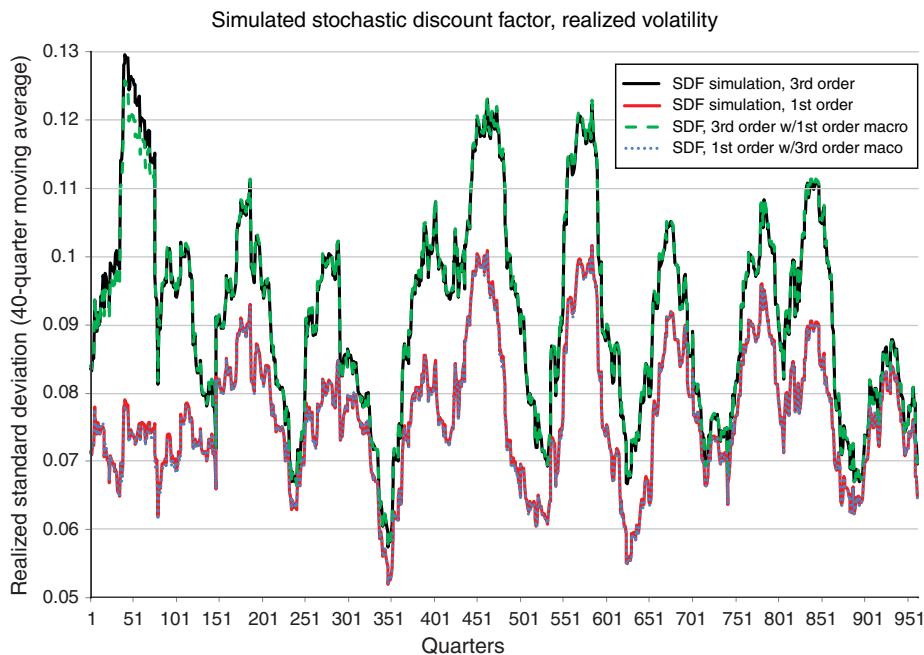


FIGURE 3. REALIZED VOLATILITY OF STOCHASTIC DISCOUNT FACTOR SIMULATIONS

Notes: Model solution simulated forward for 1,200 periods, with first 200 periods dropped. Realized standard deviation computed as 40-quarter trailing moving average. Solid black line denotes realized standard deviation of stochastic discount factor from simulation of third-order model solution. Solid gray line denotes same for first-order model solution, with identical set of shock realizations. Dashed gray line denotes realized standard deviation from simulation using third-order solution for stochastic discount factor but first-order solution for every model variable other than the SDF. Dotted light gray line denotes realized standard deviation from simulation using first-order solution for stochastic discount factor but third-order solution for every model variable other than the SDF. See text for details.

To further isolate the source of the heteroskedasticity in the stochastic discount factor, we simulate the model using the linearized solution for every variable (consumption, inflation, output, labor, etc.) *except* the stochastic discount factor, for which we use the third-order solution. We plot the realized volatility from this series as the dashed gray line in Figure 3. Note that this series looks very similar to the full third-order simulation (the black line), despite the fact that every variable in the model other than the stochastic discount factor is linear and homoskedastic. Finally, we perform the reverse experiment, simulating the model using the third-order solution for every variable except the stochastic discount factor, which we linearize. The realized volatility of the SDF in this case is given by the dotted light gray line in Figure 3, and looks very similar to the completely linearized model, with no evidence of heteroskedasticity. Thus, conditional heteroskedasticity in the stochastic discount factor in our model comes from the higher-order terms in the stochastic discount factor itself, rather than from a counterfactually large degree of

heteroskedasticity in the SDF comes from the higher order terms of the SDF itself rather than from a counterfactually large degree of conditional heteroskedasticity in consumption, inflation, or any other variable.

model-implied heteroskedasticity in consumption, inflation, or any other macroeconomic variable.

The nonhomogeneities of the model are crucial for producing this heteroskedasticity. In a homogeneous, consumption-only model, in which household utility is homothetic, production is Cobb-Douglas, technology is a random walk, and consumption equals output, there is no heteroskedasticity in the model or in the stochastic discount factor. This is because, in a homogeneous model, the household's stochastic discount factor is the same no matter whether the initial level of technology is high or low.

In our nonhomogeneous model, the stochastic discount factor depends very much on whether the initial level of technology is high or low. In particular, government spending and investment in the model are like a lump sum—they must be paid by households whether the level of technology (and hence output) is high or low. If initial technology is low, future technology shocks become substantially more risky to households, because a further decline in output implies a proportionally larger effect on consumption after the quasi lump-sum government and investment costs are paid.³⁵ Thus, the conditional variance of the stochastic discount factor is substantially higher in low-technology states of the world than in high-technology states, even though the technology shocks themselves are homoskedastic.

While this effect is present in nonhomogeneous expected utility models as well, it is greatly amplified by the Epstein-Zin exponent α in the stochastic discount factor (14). In that equation, consumption growth, inflation, and the value function V all turn out to have a similar, relatively small amount of conditional heteroskedasticity in the model. Thus, what drives the heteroskedasticity of the SDF is the large Epstein-Zin coefficient $\alpha \approx -150$, which amplifies the heteroskedasticity of V by a factor of about 150. The economic story for this amplification is risk aversion. When technology is low, consumption volatility is slightly higher, for the reasons discussed above. Households in the model are very risk averse (have large α), so they view bad outcomes as being very costly, especially when technology and consumption are already low. Thus, in a low-technology state of the world, a small increase in consumption volatility leads to a substantial increase in SDF volatility. This increases the conditional covariance of the SDF with the bond price in (35), and drives the term premium higher.

III. Long-Run Risk

The preceding section demonstrates that a standard DSGE model with Epstein-Zin preferences can match both the basic macroeconomic and financial moments in the data. This success stands in sharp contrast to habit-based specifications, which

³⁵The assumption in our model that G and I are roughly fixed are similar to standard assumptions in medium-scale macro models such as Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Altig et al. (2011). In Smets-Wouters (2007), G evolves according to an exogenous process just like (26), and in Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Altig et al. (2011), I is subject to adjustment costs that make investment close to fixed in the short run (indeed, our model is equivalent to one in which the investment adjustment costs are large, and hence investment is essentially fixed in response to shocks). Thus, the channel that we describe here is operative in these models in general.

Jermann (1998), Lettau and Uhlig (2000), and Rudebusch and Swanson (2008a) found failed in the DSGE setting despite their successes in endowment economy studies such as Campbell and Cochrane (1999) and Wachter (2006). However, the fit in the last two columns of Table 2 comes at the cost of a high value for risk aversion. In this section, we examine to what extent a long-run risk in the model (such as long-run productivity risk or long-run inflation risk) can help the model fit the data with less reliance on a high coefficient of relative risk aversion.

A. Long-Run Productivity Risk

Since Bansal and Yaron (2004), the finance literature has often stressed the importance of long-run risk in consumption growth for generating high-risk premia with lower levels of risk aversion. Bansal and Shaliastovich (2010) find that a small but highly persistent long-run risk to consumption growth can account for a variety of risk premium puzzles in an endowment economy framework. In our DSGE setting, consumption is not an exogenous process, but we can model long-run real risk in the economy with a small but highly persistent component of productivity growth, as in Croce (2008). We thus assume that the productivity trend Z_t is no longer deterministic, but instead follows a persistent difference-stationary process:

$$(37) \quad \log Z_t/Z_{t-1} = (1 - \rho_z)\gamma + \rho_z \log Z_{t-1}/Z_{t-2} + \varepsilon_t^z,$$

with a mean growth rate of γ , persistence $\rho_z \in [0, 1)$, and independently and identically distributed disturbance ε_t^z . Productivity in the model is then the sum of two components: A_t and Z_t , the former stationary and specified as before, and the latter difference-stationary and highly persistent, but small.

Surprisingly, we find that introducing this long-run real risk (37) into the model does not help match the term premium and, in fact, drives the term premium in the wrong direction. Intuitively, shocks to ε_t^z have a very large wealth effect because productivity growth has such a large effect on the present value of household wage income. Since leisure is a normal good in our model, households want to work fewer hours when there is a positive shock to ε_t^z , and so the real wage must rise substantially in order to keep employment in line with consumption and output after the shock. This drives up marginal cost and inflation in response to a positive ε_t^z shock, exactly the opposite effect that a positive shock to ε_t^A would have (in the case of the latter shock, there is only a temporary increase in real wages, so there is a strong substitution effect that offsets the wealth effect and allows marginal cost and inflation both to fall). As discussed at length in Section IID, the correlation between inflation and consumption is crucial for the term premium in the model, and a positive correlation between inflation and consumption, as implied by shocks to ε_t^z , results in a negative term premium.

This observation is similar to a point raised by Labadie (1994) but stands in sharp contrast to the endowment economy findings of Bansal and Shaliastovich (2010). The difference between Bansal and Shaliastovich's specification and ours is that they assume the correlation between consumption and inflation is constant

(and negative) no matter what the endowment process. In our structural model, one cannot change the technology process so substantially without also changing the correlation between consumption and inflation. When we include the persistent process (37) for productivity, we reduce the negative correlation between consumption and inflation in the model, and the term premium falls, driving us further away from the data. This remains true for any ρ_z above zero and any variance of ε_t^z .

Thus, long-run productivity risk does not help our standard DSGE model to match the nominal bond data—in fact, it worsens the fit—even at high levels of risk aversion.

B. Long-Run Inflation Risk

Since Bansal and Yaron (2004), the finance literature has focused on the importance of long-run consumption risk for asset pricing, but there has been little attention devoted to long-run nominal risks, such as time-variation in the central bank's inflation target, even though such risk would be very relevant for pricing nominal bonds. Thus, despite the fact that long-run productivity risk did not help our model match the data, there are good reasons to think that long-run nominal risk might be more successful. Empirically, financial market perceptions of the long-run inflation rate in the United States appear to have varied considerably in recent decades: Kozicki and Tinsley (2001) show that survey data on long-run inflation expectations have varied substantially over the past 50 years, Rudebusch and Wu (2007, 2008) estimate a similar degree of variation in a macro-finance no-arbitrage model, and Gürkaynak, Sack, and Swanson (2005) find that the “excess sensitivity” of long-term bond yields to macroeconomic announcements is consistent with financial markets perceiving the long-run inflation rate in the economy to be less than perfectly anchored.

Long-run inflation risk also has a number of advantages over long-run consumption risk from the point of view of modeling the term premium. First, estimates of the low-frequency component of consumption are extremely imprecise, so it is very difficult to test empirically the direct predictions of a Bansal-Yaron long-run consumption risk model with observable macroeconomic variables. In contrast, survey data on long-run inflation expectations are readily available and show considerable variation. Second, the idea that long-term nominal bonds are risky because of uncertainty about future monetary policy and long-run inflation is intuitively appealing. Third, estimates of the term premium in the finance literature are low in the 1960s, high in the late 1970s and early 1980s, and then low again in the 1990s and 2000s (e.g., Kim and Wright 2005), which suggests that inflation and inflation variability are highly correlated with the term premium, at least over these longer, decadal samples. Modeling the linkage between long-run inflation risk and the term premium thus seems to be a promising avenue for understanding and modeling long-term bond yields.

Following the empirical evidence in Gürkaynak, Sack, and Swanson (2005), we assume that π_t^* loads to some extent on the recent history of inflation:

$$(38) \quad \pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \vartheta_{\pi^*} (\bar{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}.$$

There are two main advantages to using specification (38) rather than a simple random walk or AR(1) specification with $\vartheta_{\pi^*} = 0$. First, (38) allows long-term inflation expectations to respond to current news about inflation and economic activity in a manner that is consistent with the bond market responses documented by Gürkaynak, Sack, and Swanson (2005). Thus, $\vartheta_{\pi^*} > 0$ seems to be consistent with the data. Second, if $\vartheta_{\pi^*} = 0$, then even though π_t^* varies over time, it does not do so systematically with output or consumption. As a result, long-term bonds are not particularly risky, in the sense that their returns are not very correlated with the household's stochastic discount factor. In fact, long-term bonds even have some elements of insurance in this case, because a negative shock to $\varepsilon_t^{\pi^*}$ leads the monetary authority to raise interest rates and depress output at the same time that it causes long-term bond yields to fall and bond prices to rise, which results in a negative term premium on the bond. By contrast, if $\vartheta_{\pi^*} > 0$, then a negative technology shock today raises inflation and long-term inflation expectations and depresses bond prices at exactly the same time that it depresses output, which makes holding long-term bonds riskier and helps the model to match the positive mean term premium we see in the data.

We add equation (38) to our DSGE model from the preceding section, setting the baseline value of $\vartheta_{\pi^*} = 0.01$, consistent with the high-frequency bond market evidence in Gürkaynak, Sack, and Swanson (2005). We set the baseline values for ρ_{π^*} and σ_{π^*} equal to 0.99 and 5 basis points, respectively, consistent with the Bayesian DSGE model estimates in Levin et al. (2006).

As can be seen in Figure 1, the effects of long-run inflation risk on the term premium are substantial. As the coefficient of relative risk aversion varies along the horizontal axis, holding the other parameters of the model fixed at their baseline values, the term premium is always higher for the version of the model that includes long-run inflation risk. That is, by making long-term bonds in the model riskier, the model can generate any given level of the term premium with a lower value for risk aversion than was possible without long-run inflation risk.

Table 3 reports the macroeconomic and financial moments that result from introducing long-run inflation risk into our DSGE model. The first column repeats the empirical moments from the US data, and the second column reports results for a version of the model with long-run inflation risk and expected utility preferences (i.e., with the parameters of the model set to their baseline values, except for $\alpha = 0$). The introduction of time-variation in π^* makes inflation and the nominal short- and long-term interest rates more volatile in the model, improving the fit to the macroeconomic data. However, the fit of the model to the finance moments is essentially no better than without long-run risk—the term premium remains less than one basis point, and its standard deviation is still a small fraction of one basis point (and this result is robust to varying the parameters of the model over very wide ranges). Intuitively, long-run inflation risk increases the quantity of nominal bond risk in the model, but households simply aren't risk-averse enough for that greater quantity of risk to have a noticeable effect on bond prices.

With Epstein-Zin preferences, long-run inflation risk has more substantial effects, as can be seen in Figure 1 and also the last column of Table 3, which reports results for a “best fit” version of the model with Epstein-Zin preferences and long-run

TABLE 3—EMPIRICAL AND MODEL-BASED UNCONDITIONAL MOMENTS WITH LONG-RUN RISK

Unconditional moment	US data, 1961–2007	Model with EU preferences and LR π^* risk	Model with EU preferences and LR π^* risk (best fit)
SD $[C]$	0.83	1.52	0.82
SD $[L]$	1.71	1.79	2.31
SD $[w^r]$	0.82	1.29	1.01
SD $[\pi]$	2.52	2.54	2.79
SD $[i]$	2.71	2.44	3.20
SD $[r]$	2.30	1.41	1.95
SD $[i^{(40)}]$	2.41	1.59	2.15
Mean $[\psi^{(40)}]$	1.06	0.005	1.00
SD $[\psi^{(40)}]$	0.54	0.002	0.29
Mean $[i^{(40)} - i]$	1.43	−0.048	0.88
SD $[i^{(40)} - i]$	1.33	1.03	1.42
Mean $[x^{(40)}]$	1.76	0.008	1.74
SD $[x^{(40)}]$	23.43	12.88	20.99
Distance to:			
Macro moments		2.24	0.89
Finance moments		7.85	0.44
All moments		10.09	1.33
Memo:			
IES		0.5	0.09
CRRA		6/7	110
Frisch		2/3	0.28
ξ		0.75	0.76
ρ_A		0.95	0.95
σ_A		0.005	0.005
ρ_{π^*}		0.99	0.995
ϑ_{π^*}		0.01	0.003
σ_{π^*}		5 bp	7 bp

Notes: All variables are quarterly values expressed in percent. Inflation, interest rates, the term premium (ψ), and excess holding period returns (x) are expressed at an annual rate.

inflation risk. Here, the best fit is computed by searching over values for ρ_{π^*} , ϑ_{π^*} , and σ_{π^*} as well as the IES, risk aversion, Frisch elasticity, ξ , ρ_A , and σ_A to minimize the distance to the empirical moments in the first column. Not surprisingly, the model with Epstein-Zin preferences can achieve a remarkable fit to the data, even slightly better than in Table 2. However, the estimated risk aversion coefficient, remains as high as in Table 2—the case of no long-run risk. Although the model could fit the data just as well as in Table 2, with slightly lower risk aversion, it instead chooses the same high value of risk aversion, with a better fit to the macroeconomic data. Thus, including long-run inflation risk in the model improves the model's ability to fit the data, but does not substantially reduce the level of risk aversion required to do so.

C. Price-Level Targeting

Long-run inflation risk makes nominal long-term bonds more risky, but one can also ask the opposite question: what would the yield curve look like in the absence of any long-run inflation risk, such as when the central bank is committed to a price level target? In the United States and Great Britain during the gold standard, it has often been noted that nominal yield curves were flat or even downward-sloping on average (e.g., Wood 1983, Barsky and De Long 1991). Our simple DSGE model is consistent with this prediction, as we now show.

Let \bar{P} denote the price level to which the central bank is committed. To model price-level targeting, we set g_y in the Taylor-type policy rule (28) to zero, and we let the geometric weight θ_π in equation (29) approach unity. When $\theta_\pi \ll 1$, the central bank makes essentially no effort to offset inflation deviations from target that occurred more than a few quarters in the past, but as $\theta_\pi \rightarrow 1$, the central bank responds to a much longer moving average of inflation, which implies that it will work to offset past deviations of inflation from target. In the limit as $\theta_\pi \rightarrow 1$, the central bank sets interest rates in proportion to the deviation of the price level P_t from the initial level of prices in the distant past, \bar{P} . Under this monetary policy, price-level risk in the very long run is minimized, although at any finite horizon there is some probability that the price level will lie slightly above or below the target price level \bar{P} , due to shocks.

In Table 4, we report the financial moments implied by the model with $g_y = 0$ and $\theta_\pi = 0.7, 0.95, 0.99$, and 0.999 . All of the other parameters of the model are fixed at their “best fit” estimated values from Table 2. The first column of Table 4 ($g_y = 0, \theta_\pi = 0.7$) corresponds to the best fit parameterization of the model but with a monetary policy rule that does not respond to the output gap. The financial moments in this column are slightly smaller than the corresponding numbers in the last column of Table 2 due to the lower volatility of the short-term interest rate and inflation that occurs when the central bank does not respond to the output gap.

The second, third, and fourth columns of Table 4 consider successively stricter price-level targeting policies by the central bank. Looking across these columns, the term premium and other risk premium measures for the 40-quarter nominal bond fall substantially as the monetary authority pursues a stricter price-level targeting policy. For $\theta_\pi = 0.999$ in the last column, the term premium is essentially zero and constant over time. Although not reported in the table, increasing the policy response coefficient on inflation g_π can reduce these numbers even further, to produce a slightly negative term premium and yield curve slope. Thus, eliminating long-term inflation risk from the model essentially eliminates the riskiness of long-term nominal bonds, consistent with the evidence on long-term bond yields during the gold standard.

IV. Conclusions

In sharp contrast to our earlier work with habits (Rudebusch and Swanson 2008a), here we have found that introducing Epstein-Zin preferences into a DSGE model is a very successful strategy for matching both the basic macroeconomic and financial moments in the data. Intuitively, households with habits primarily dislike

TABLE 4—FINANCIAL MOMENTS UNDER PRICE-LEVEL TARGETING

Unconditional moment	Model with EZ preferences and inflation target ($g_y = 0, \theta_\pi = 0.7$)	Model with EZ preferences and weak price level target ($g_y = 0, \theta_\pi = 0.95$)	Model with EZ preferences and stricter price level target ($g_y = 0, \theta_\pi = 0.99$)	Model with EZ preferences and stricter price level target ($g_y = 0, \theta_\pi = 0.999$)
Mean[$\psi^{(40)}$]	0.99	0.42	0.12	0.01
SD[$\psi^{(40)}$]	0.26	0.11	0.03	0.00
Mean[$i^{(40)} - i$]	0.91	0.40	0.12	0.01
SD[$i^{(40)} - i$]	0.65	0.49	0.16	0.00
Mean[$x^{(40)}$]	1.77	0.83	0.25	0.03
SD[$x^{(40)}$]	12.37	6.14	1.88	0.31

very sudden changes in consumption, and these changes are the easiest to offset by varying labor supply or savings in a DSGE model. In contrast, a household with Epstein-Zin preferences is concerned about changes in consumption over medium and long horizons (through the value function V) as well as short horizons. While the household would like to offset fluctuations in consumption by varying hours or savings, its ability to offset longer-horizon changes in consumption are much more limited than its ability to smooth out very sudden changes.

Our otherwise standard macroeconomic model with Epstein-Zin preferences produces a large and variable term premium, generalizing earlier results in the finance literature for an endowment economy. Our model also offers a structural explanation for why the yield curve slopes upward (technology or supply-type shocks that cause inflation and output to move in opposite directions), and endogenously generates conditional heteroskedasticity in the stochastic discount factor—and hence a time-varying term premium—even though the exogenous shocks in the model are homoskedastic. Introducing long-run inflation risk into the model allows the model to fit the data better, but does not substantially reduce the need for a high level of household risk aversion.

Of course, many unresolved issues remain for exploration. Although we have restricted attention in this paper to a simple, stylized DSGE model along the lines of Woodford (2003), there is no reason why the methods of this paper cannot be applied to larger, more empirically relevant DSGE models such as Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Altig et al. (2011); indeed, preliminary research that we have conducted indicates that all of the basic conclusions in the present paper carry over to these larger scale and more fully specified models. A related next step would be to go beyond matching sample moments and perform full econometric estimation (and inference) of a DSGE model with Epstein-Zin preferences, as in van Binsbergen et al. (2010), but extended to include intrinsic nominal rigidities and endogenous inflation. Examining to what extent a DSGE model can jointly explain the risk premiums on equity, real bonds, nominal bonds, and uncovered interest parity violations would also be very interesting. Finally, the relationship between the variability or uncertainty surrounding the central bank's inflation objective and the size and variability of the term premium warrants further study, in our view. In short, there appear to be many fruitful avenues for future research in this area.

APPENDIX A: HOUSEHOLD UTILITY AND BALANCED GROWTH

Following Campbell and Ludvigson (2001), suppose that households have preferences over the consumption of market and nonmarket goods each period, $c_{m,t}$ and $c_{nm,t}$, given by

$$(39) \quad \frac{c_{m,t}^{1-\varphi_m}}{1-\varphi_m} + \theta_{nm} \frac{c_{nm,t}^{1-\varphi_{nm}}}{1-\varphi_{nm}},$$

where $\varphi_m, \varphi_{nm}, \theta_{nm} > 0$ are parameters. The economy faces production technologies for per capita market output and nonmarket consumption given by

$$(40) \quad y_{m,t} = k_{m,t}^{1-\alpha_m} (A_{m,t} l_t)^{\alpha_m},$$

$$(41) \quad c_{nm,t} = k_{nm,t}^{1-\alpha_{nm}} (A_{nm,t} (1-l_t))^{\alpha_{nm}},$$

where the household's per-period time endowment is unity, l_t denotes hours devoted to market work, $k_{m,t}$ and $k_{nm,t}$ denote the per capita capital stocks in the market and nonmarket sectors, $A_{m,t}$ and $A_{nm,t}$ denote the level of productivity in the market and nonmarket sectors, and $\alpha_m, \alpha_{nm} \in (0, 1)$ are parameters. As discussed by Campbell and Ludvigson (2001), the utility function (39) does not include an explicit term for leisure, the idea being that leisure is not valued for its own sake, but rather for what the household does with those leisure hours.

Market and nonmarket capital evolve according to

$$(42) \quad k_{m,t} = (1-\delta)k_{m,t-1} + i_{m,t},$$

$$(43) \quad k_{nm,t} = (1-\delta)k_{nm,t-1} + i_{nm,t},$$

where $i_{m,t}$ and $i_{nm,t}$ denote per-capita investment in each type of capital; and $\delta > 0$ denotes the capital depreciation rate, equal across sectors. Note that, in the main text, investment is assumed to be a fixed share of output each period. The resource constraint for market goods is given by

$$(44) \quad y_{m,t} = c_{m,t} + i_{m,t} + i_{nm,t} + g_{m,t},$$

where $g_{m,t}$ denotes government consumption of market goods. Note that investment in both the market and nonmarket capital stocks is made with market goods.

A balanced growth path in this economy is one in which all of the above variables grow at constant rates, and labor l_t and the marginal product of capital $(1-\alpha_m)k_{m,t}^{-\alpha_m}(A_{m,t}l_t)^{\alpha_m}$ are constant. Let γ denote the growth rate of $A_{m,t}$. Then $k_{m,t}$ and hence $y_{m,t}$ and $i_{m,t}$ (from (40) and (42)), and $c_{m,t}$, $i_{nm,t}$, and $g_{m,t}$ (from (44)) must also grow at rate γ . It follows that $k_{nm,t}$, $A_{nm,t}$, and $c_{nm,t}$ (from (43) and (41)) must also grow at rate γ . From the household's marginal hours decision, we have

$$(45) \quad \alpha_m k_{m,t}^{1-\alpha_m} A_{m,t}^{\alpha_m} l_t^{\alpha_m-1} c_{m,t}^{-\varphi_m} = \theta_{nm} (1-\alpha_{nm}) k_{nm,t}^{1-\alpha_{nm}} A_{nm,t}^{\alpha_{nm}} (1-l_t)^{\alpha_{nm}-1} c_{nm,t}^{-\varphi_{nm}},$$

from which it follows that $\varphi_m = \varphi_{nm}$ in order for balanced growth to be satisfied. Although this is a strong restriction on the utility kernel (39), it is no more so than the usual King-Plosser-Rebelo restrictions that either consumption and hours be nonseparable or that $\varphi_m = 1$ (and both of these restrictions are rejected by Kiley (2010) and Campbell and Mankiw (1990), while the restriction $\varphi_m = 1$ is rejected by Campbell and Ludvigson (2001), Campbell and Mankiw (1989), and Woodford (2003), among others). The specification (39) allows for both additive separability and an intertemporal elasticity of substitution different from unity.

Defining $\varphi \equiv \varphi_m = \varphi_{nm}$ and substituting (41) into (39), the household's period utility is given by

$$(46) \quad \frac{c_{m,t}^{1-\varphi}}{1-\varphi} + \theta_{nm} k_{nm,t}^{(1-\alpha_{nm})(1-\varphi)} A_{nm,t}^{\alpha_{nm}(1-\varphi)} \frac{(1-l_t)^{\alpha_m(1-\varphi)}}{1-\varphi}.$$

Finally, defining $\chi \equiv 1 - \alpha_m(1 - \varphi)$, $\chi_0 \equiv \theta_{nm} \bar{k}(1 - \chi)/(1 - \varphi)$, and $Z_t \equiv A_{nm,t}$, we have

$$(47) \quad u(c_{m,t}, l_t) \equiv \frac{c_{m,t}^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-l_t)^{1-\chi}}{1-\chi},$$

as in (6).

APPENDIX B: EQUATIONS OF THE MODEL

The following equations show exactly how we incorporate Epstein-Zin preferences into our otherwise standard DSGE model in first-order recursive form, and how to compute bond prices and the term premium for the generalized consol bond used in Rudebusch and Swanson (2008b). Trending variables are normalized by an appropriate power of Z_t to make the model stationary around a nonstochastic steady state, and the variable DZ_{t+1} denotes Z_{t+1}/Z_t . (To consider a simpler version of the model without balanced growth, simply delete all the occurrences of DZ .) The Mathematica-style syntax of these equations is consistent with the Perturbation AIM algorithm of Swanson, Anderson, and Levin (2006), which we use to solve this system to third order around the nonstochastic steady state.

(* Value function and Euler equation *)

```
V[t] == C[t]^(1-phi)/(1-phi) + chi0*(LMax-L[t])^(1-chi)/(1-chi) + beta*Vkp[t],
C[t]^phi == beta*(Exp[Int[t]/pi[t+1]])*C[t+1]^phi*DZ[t+1]^phi
*(V[t+1]*DZ[t+1]^(1-phi)/Vkp[t])^alpha,
```

(* The following two equations define the E-Z-W-K-P certainty equivalent term

$Vkp[t] = (E_t V[t+1]^{1-\alpha})^{1/(1-\alpha)}$. It takes two equations to do this because Perturbation AIM sets the expected value of all equations equal to zero, $E_t F(\text{variables}) = 0$.

Thus, the first equation below defines $Valphaexp[t] = E_t V[t+1]^{1-\alpha}$. The second equation then takes the $(1-\alpha)$ th root of this expectation.

Finally, the scaling and unscaling of $Valphaexp[t]$ by the constant $VAIMSS$ and $DZBar$ improves the numerical behavior of the model; without it, the steady-state value of $Valphaexp$ can be minuscule (e.g., 10^{-50}), which requires Mathematica to use astronomical levels of precision to solve. *)

```
Valphaexp[t] == (V[t+1]*DZ[t+1]^(1-phi)/VAIMSS/DZBar^(1-phi))^(1-alpha),
Vkp[t] == VAIMSS*DZBar^(1-phi)*Valphaexp[t]^(1/(1-alpha)),
```



```
(* Price-setting equations *)
zn[t] == (1+theta) * MC[t] * Y[t] + xi * beta * (C[t+1]/C[t])^phi * DZ[t+1]^phi
      * (V[t+1] * DZ[t+1]^(1-phi) / Vkp[t])^alpha * pi[t+1]^((1+theta)/theta/eta) * zn[t+1],
zd[t] == Y[t] + xi * beta * (C[t+1]/C[t])^phi * DZ[t+1]^phi
      * (V[t+1] * DZ[t+1]^(1-phi) / Vkp[t])^alpha * pi[t+1]^(1/theta) * zd[t+1],
p0[t]^(1+(1+theta)/theta * (1-eta)/eta) == zn[t] / zd[t],
pi[t]^(-1/theta) == (1-xi) * (p0[t] * pi[t])^(-1/theta) + xi,

(* Marginal cost and real wage *)
MC[t] == wreal[t] / eta * Y[t]^((1-eta)/eta) / A[t]^(1/eta) / KBar^((1-eta)/eta),
chi0 * (LMax-L[t])^chi / C[t]^phi == wreal[t],

(* Output equations *)
Y[t] == A[t] * KBar^(1-eta) * L[t]^eta / Disp[t],
Disp[t]^(1/eta) == (1-xi) * p0[t]^(-(1+theta)/theta/eta)
      + xi * pi[t]^((1+theta)/theta/eta) * Disp[t-1]^(1/eta),
C[t] == Y[t] - G[t] - IBar, (* aggregate resource constraint, no adj costs *)

(* Monetary Policy Rule *)
Log[piavg[t]] == rhoinflav * Log[piavg[t-1]] + (1-rhoinflav) * Log[pi[t]],
4*Int[t] == (1-taylrho) * (4*Log[1/beta * DZBar^phi] + 4*Log[piavg[t]]
      + taylpi * (4*Log[piavg[t]] - pistar[t]) + tayly * (Y[t] - YBar) / YBar)
      + taylrho * 4*Int[t-1] + eps[Int][t], (* multiply Int, infl by 4 to put at annual rate *)

(* Exogenous Shocks *)
Log[A[t]/ABar] == rhoa * Log[A[t-1]/ABar] + eps[A][t],
Log[DZ[t]/DZBar] == rhoz * Log[DZ[t-1]/DZBar] + eps[Z][t],
Log[G[t]/GBar] == rhog * Log[G[t-1]/GBar] + eps[G][t],
pistar[t] == (1-rhopistar) * piBar + rhopistar * pistar[t-1] + gssload * (4*Log[piavg[t]] - pistar[t])
      + eps[pistar][t],

(* Term premium and other auxiliary finance equations *)
Intr[t] == Log[Exp[Int[t-1]]/pi[t]], (* ex post real short rate *)
pricebond[t] == 1 + consoldelta * beta * (C[t+1]/C[t])^phi * DZ[t+1]^phi
      * (V[t+1] * DZ[t+1]^(1-phi) / Vkp[t])^alpha / pi[t+1] * pricebond[t+1],
pricebondrn[t] == 1 + consoldelta * pricebondrn[t+1] / Exp[Int[t]],
ytm[t] == Log[consoldelta * pricebond[t] / (pricebond[t-1])] * 400, (* yield in annualized pct *)
ytmrn[t] == Log[consoldelta * pricebondrn[t] / (pricebondrn[t-1])] * 400,
termprem[t] == 100 * (ytm[t] - ytmrn[t]), (* term prem in annualized basis points *)
ehpr[t] == ((consoldelta * pricebond[t] + Exp[Int[t-1]]) / pricebond[t-1] - Exp[Int[t-1]]) * 400,
slope[t] == ytm[t] - Int[t] * 400
```

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