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Work in progress

Comments welcome

MONETARY POLICY IN AN ESTIMATED STOCHASTIC DYNAMIC GENERAL EQUILIBRIUM MODEL OF THE EURO AREA*

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Abstract

This paper, first, develops and estimates a stochastic dynamic general equilibrium (SDGE) model with sticky prices and wages for the euro area. The model incorporates various other features such as habit formation, costs of adjustment in capital accumulation and variable capacity utilisation and is estimated using seven key macro-economic variables: GDP, consumption, investment, prices, real wages, employment and the nominal interest rate. The introduction of eight orthogonal structural shocks (including productivity, labour supply, preference, cost-push and monetary policy shocks) allows for an empirical investigation of the effects of such shocks and of their contribution to business cycle fluctuations in the euro area. For example, it is found that productivity shocks account for only 10 percent of the long run variance in output. Using the estimated model, the paper then analyses the output (real interest rate) gap, defined as the difference between the actual and the flexible-price level of output (real interest rate). Finally, the estimated model is also used to analyse optimal monetary policy.

Key words: SDGE models; monetary policy; euro area

JEL-classification: E4-E5

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1. Introduction

In this paper we present and estimate a stochastic dynamic general equilibrium (SDGE) model for the euro area. Following Christiano, Eichenbaum and Evans (CEE, 2001) it features a relatively large number of frictions that appear to be necessary to capture the empirical persistence in the main euro area macro-economic data. Many of these frictions have become quite standard in the SDGE literature. Following Kollman (1997) and Erceg, Henderson and Levin (2000), the model exhibits both sticky nominal prices and wages that adjust following a Calvo mechanism. However, the introduction of partial indexation of the prices and wages that can not be re-optimised results in a more general dynamic inflation and wage specification that will also depend on past inflation. Following Greenwood, Hercowitz, and Huffmann (1988) and King and Rebelo (2000) the model incorporates a variable capital utilisation rate. This tends to smooth the adjustment of the rental rate of capital in response to changes in output. As in CEE (2001), the cost of adjusting the utilisation rate is expressed in terms of consumption goods. We also follow CEE (2001) by modelling the cost of adjusting the capital stock as a function of the change in investment, rather than the level of investment as is commonly done. Finally, external habit formation in consumption is used to introduce the necessary empirical persistence in the consumption process (See Fuhrer (2000) and McCallum and Nelson (1999)).

While the model used in this paper has many elements in common with that used in CEE (2001), the analysis differs in two main respects. First, we introduce a full set of structural shocks to the various structural equations.[1] Next to two "supply" shocks, a productivity and a labour supply shock, we add two "demand" shocks, a preference shock and a cost-of-capital (or investment) shock, a "cost-push" shock (modelled as a shock to the mark-up equation) and two monetary policy shocks. We estimate the parameters of the model and the stochastic processes governing the structural shocks using seven key macro-economic time series in the euro area: real GDP, consumption, investment, the GDP deflator, the real wage, employment and the nominal short-term interest rate. The estimation methodology can be viewed as a constrained maximum likelihood procedure, whereby the usual likelihood criterion is augmented by a condition that the model-based cross-covariances can not be too different from their empirical counterparts estimated using an unrestricted VAR.

Overall, our estimation procedure yields a plausible set of estimates for the structural parameters of the sticky price and wage SDGE model. In contrast to the results of CEE (2001) for the US, we find that there is a considerable degree of price stickiness in the euro area. This feature appears to be important to account for the empirical persistence of euro area inflation in spite of the presence of sticky wages and variable capacity utilisation which tend to introduce stickiness in marginal costs. At this point it is not clear whether this difference is a result of structural differences between the US and the euro area, small differences in the underlying structural model or differences in the estimation methodology.

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[1] CEE (2001) only consider the effects of a monetary policy shock.

The introduction and estimation of a set of orthogonal structural shocks also allows us to examine the relative contribution of the various shocks to the empirical dynamics of the macro economic time series in the euro area. One of our findings is that in spite of the fact that we estimate the productivity parameter to be the most persistent structural shock, it accounts for only about 10 percent of the long run variance in output. In contrast to some of the results in the identified VAR literature, the "demand" shocks dominate the "supply" shocks in explaining output at all frequencies. A dominant factor in explaining the variance of inflation are the so-called "cost-push" shocks. However, the medium-term component of inflation is mostly driven by monetary policy shocks.

Our estimates of the effects of a temporary monetary policy shock are very much in line with the existing evidence on the euro area (e.g. Peersman and Smets (2000)). However, the important difference with the estimated effects of a persistent policy shock underlines the crucial importance of the combination of forward-looking pricing behaviour and the persistence of the shocks for assessing the effects of monetary policy.

Our second contribution in this paper is to analyse optimal monetary policy within this model. As a benchmark, we, first, analyse the response of the observable variables to the various shocks in a flexible price and wage economy. We find that due to the sluggish response of the two demand components in the estimated model, the natural real interest rate typically rises very significantly in response to positive "demand" shocks, while it falls in response to the "supply" shocks. In contrast to Neiss and Nelson (2000), we find that the natural rate is much more volatile than the observed one. The natural output level on the other hand does not respond very much to "demand" shocks.

Following Rotemberg and Woodford (1998) and Woodford (2001), we then use a quadratic approximation to the utility of the representative consumer to analyse optimal monetary policy in the estimated sticky price and wage model. The resulting policy trade-offs are quite complicated. A comparison of the outcome under the estimated policy reaction function with the outcome under optimal policy under commitment and the flexible price and wage outcome, shows that the interest rate response to the various "supply" and "demand" shocks is much stronger under commitment to the optimal loss function than under the historical reaction function. As a result, the counterfactual outcome for output and inflation under the optimal policy would have been much closer to the outcome in a flexible price and wage economy.

The rest of the paper is structured as follows. Section 2 presents the derivation of the linearised model. In Section 3, we, first, discuss how we estimate the model and, then, present the main results. Section 4 contains an analysis of optimal monetary policy in the estimated model. Finally, Section 5 reviews some of the main conclusions that we can draw from the analysis and contains suggestions for further work.

2. An SDGE model for the euro area

In this section we derive and present the linearised SDGE model that we estimate in Section 3. The model is an application of the real business cycle (RBC) methodology to an economy with sticky prices and wages[2]. Households maximise a utility function with three arguments (goods, money and leisure) over an infinite life horizon. Consumption appears in the utility function relative to the time-varying external habit variable.[3] Labour is differentiated over households, so that there is some monopoly power over wages which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo. Households allocate wealth among cash on the one hand and riskless bonds on the other hand. Households also rent capital services to firms and decide how much capital to accumulate given certain capital adjustment costs. As the rental price of capital goes up, the capital stock can be used more intensively according to some cost schedule.[4] Firms produce differentiated goods, decide on labour and capital inputs, and set prices, again according to the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that can not be freely set, are partially indexed on past inflation rates. Prices are therefore set in function of current and expected marginal costs, but also of the past inflation rate. The marginal costs depend on wages and the rental rate of capital.

Detailed derivations are presented in the appendix (to be completed). In this Section we sketch out the main building blocks.

2.1 The household sector

There is a continuum of households indicated by index τ. Households differ in that they supply a differentiated type of labour. So, each household has a monopoly power over the supply of its labour. Each household τ maximises an intertemporal utility function given by:

$$(1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau$$

where β is the discount factor and the instantaneous utility function is separable in consumption, labour (leisure) and real balances:

$$(2) \qquad U_t^\tau = \varepsilon_t^B \left(\frac{1}{1-\sigma_c} \left(C_t^\tau - H_t \right)^{1-\sigma_c} - \frac{\varepsilon_t^L}{1+\sigma_l} \left(\ell_t^\tau \right)^{1+\sigma_l} + \frac{\varepsilon_t^Q}{1-\sigma_m} \left(\frac{Q_t^\tau}{P_t} \right)^{1-\sigma_m} \right)$$

Utility depends positively on the consumption of goods, C_t^τ, relative to an external habit variable, H_t, positively on real cash balances, Q_t^τ / P_t and negatively on labour supply ℓ_t^τ. σ_c is the coefficient of relative risk aversion of households or the inverse of the intertemporal elasticity of substitution; σ_l represents the inverse of the elasticity of work effort with respect to the real wage, and σ_m represents the inverse of the elasticity of money holdings with respect to the interest rate.

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2 This model is a version of the model considered in Kollmann (1997) and features monopolistic competition in both the goods and labour markets. A similar model was discussed in Dombrecht and Wouters (2000). A closed economy version is analysed in Erceg, Henderson and Levin (2000). In addition, several features of CEE (2001) are introduced.

3 Habit depends on lagged aggregate consumption which is unaffected by any one agent's decisions. Abel calls this the "catching up with the Joneses" effect. See Abel (1990 .

4 See King and Rebelo (1998).

Equation (2) above also contains three preference shocks: ε_t^B represents a general shock to preferences that affects the intertemporal substitution of households; ε_t^L represents a shock to the labour supply in the economy and ε_t^Q is a money demand shock.

The external habit stock is assumed to be proportional to aggregate past consumption:

(3) $$H_t = hC_{t-1}$$

Households maximise their objective function subject to an intertemporal budget constraint which is given by:

(4) $$\frac{Q_t^\tau}{P_t} + b_t \frac{B_t^\tau}{P_t} = \frac{Q_{t-1}^\tau}{P_t} + \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau$$

Households hold their financial wealth in the form of cash balances Q_t and bonds B_t. Bonds are one-period securities with price b_t. Current income and financial wealth can be used for consumption and investment in physical capital.

Household's total income is given by:

(5) $$Y_t^\tau = (w_t^\tau l_t^\tau + A_t^\tau) + (\varepsilon_t^k r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau) K_{t-1}^\tau) + Div_t^\tau$$

Total income consists of three components: labour income plus the net cash inflow from participating in state-contingent securities ($w_t^\tau l_t^\tau + A_t^\tau$); the return on the real capital stock minus the cost associated with variations in the degree of capital utilisation ($\varepsilon_t^k r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau) K_{t-1}^\tau$) and the dividends derived from the imperfect competitive intermediate firms (Div_t^τ).

Following CEE (2001), we assume that there exist state-contingent securities that insure the households against variations in household specific labour income. As a result, the first component in the household's income will be equal to aggregate labour income and the marginal utility of wealth will be identical across different types of households.[5]

The income from renting out capital services depends not only on the level of capital that was installed last period, but also on its utilisation rate (z_t). As in CEE (2001), it is assumed that the cost of capital utilisation is zero when capital utilisation is one ($\psi(1) = 0$). The introduction of a stochastic shock to the rental rate of the effective capital stock (ε_t^k) is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium. In a fully-fledged model, the production of capital goods and the associated investment process could be modelled in a separate sector. In such a case, imperfect information between the capital producing borrowers and the financial intermediaries could give rise to a stochastic external finance premium. For example, in Bernanke, Gertler and Gilchrist (1998), the deviation from the perfect capital market assumptions generates deviations between the return on financial assets and equity that are related to the net worth position of the firms in

[5] See CEE (2001) for a more complete analysis.

their model. Here, we implicitly assume that the deviation between the two returns can be captured by a stochastic shock, whereas the steady-state distortion due to such informational frictions is zero.

Next we discuss each of the household decisions in turn.

2.1.1 Consumption and savings behaviour

The maximisation of the objective function (1) subject to the budget constraint (4) with respect to consumption and holdings of bonds yields the following first-order conditions for consumption:

(6) $$E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t}\frac{(1+i_t)P_t}{P_{t+1}}\right]=1$$

where i_t is the nominal rate of return on bonds ($R_t = 1+i_t = 1/b_t$) and λ_t is the marginal utility of consumption, which is given by:[6]

(7) $$\lambda_t = \varepsilon_t^b \left(C_t - H_t\right)^{-\sigma_c}$$

Equations (6) and (7) extend the usual first-order condition for consumption growth by taking into account the existence of external habit formation.

The demand for cash that follows from the household's optimisation problem is given by:

(8) $$\varepsilon_t^q \left(\frac{Q_t}{P_t}\right)^{-\sigma_m} = \left(C_t - H_t\right)^{-\sigma_c} - \frac{1}{1+i_t}$$

Real cash holdings depend positively on consumption (relative to the habit) with an elasticity equal to σ_c / σ_m and negatively on the nominal interest rate. In what follows we will take the nominal interest rate as the central bank's policy instrument. Due to the assumption that consumption and cash holdings are additively separable in the utility function, cash holding will not enter in any of the other structural equations. Equation (8) then becomes completely recursive to the rest of the system of equations.[7]

2.1.2 Labour supply decisions and the wage setting equation

Households act as price-setters in the labour market. Following Kollmann (1997) and Erceg, Henderson and Levin (2000), we assume that wages can only be optimally adjusted after some random "wage-change signal" is received. The probability that a particular household can change its nominal wage in period t is constant and equal to $1-\xi_w$. A household τ which receives such a signal in period t, will thus set a new nominal wage, $\tilde{w}_t^\tau$, taking into account the probability that it will not be re-optimized in the

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[6] Here we have already used the fact that the marginal utility of consumption is identical across households.

[7] In the rest of the paper, we will ignore equation (8). In future work we intend to examine the implications of more general money demand functions for the behaviour of money in this kind of SDGE models. See Casares (2001) and Ireland (2001) for models in which money balances enter the aggregate demand equation. Nelson (2000) and CEE (2001) consider costs in adjusting cash balances. In that case, expectations of future marginal utility of consumption and the nominal interest rate enter the money demand equation, which allows for a potentially more interesting informational role for money balances.

near future. In addition, we allow for a partial indexation of the wages that can not be adjusted to past inflation. More formally, the wages of households that can not re-optimise adjust according to:

(9) $$W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}^\tau$$

where γ_w is the degree of indexation. When $\gamma_w = 0$, there is no indexation and the wages that can not be re-optimised remain constant. When $\gamma_w = 1$, there is perfect indexation to past inflation.

Households set their nominal wages to maximise their intertemporal objective function subject to the intertemporal budget constraint and the demand for labour which is determined by:

(10) $$l_t^\tau = \left(\frac{W_t^\tau}{W_t}\right)^{-\frac{1+\lambda_w}{\lambda_w}} L_t$$

where aggregate labour demand, L_t, and the aggregate nominal wage, W_t, are given by the following Dixit-Stiglitz type aggregator functions:

(11) $$L_t = \left[\int_0^1 \left(l_t^\tau\right)^{\frac{1}{1+\lambda_w}} d\tau\right]^{1+\lambda_w},$$

(12) $$W_t = \left[\int_0^1 \left(W_t^\tau\right)^{-1/\lambda_w} d\tau\right]^{-\lambda_w}.$$

This maximisation problem results in the following mark-up equation for the re-optimised wage:

(13) $$\frac{\tilde{w}_t}{P_t} = (1+\lambda_w)\frac{E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i l_{t+i}^\tau U_{t+i}^\ell}{E_t \sum_{i=0}^{\infty} \beta^i \xi_w^i \left(\frac{P_t / P_{t-1}}{P_{t+i}/P_{t+i-1}}\right)^{\gamma_w} l_{t+i}^\tau U_{t+i}^C}$$

where U_{t+i}^l is the marginal disutility of labour and U_{t+i}^C is the marginal utility of consumption. Equation (13) shows that the nominal wage at time t of a household τ that is allowed to change its wage is set so that the present value of the marginal return to working is a mark-up over the present value of marginal cost (the subjective cost of working).[8] When wages are perfectly flexible ($\xi_w = 0$), the real wage will be a constant mark-up over the ratio of the marginal disutility of labour and the marginal utility of an additional unit of consumption.

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[8] Standard RBC models typically assume an infinite supply elasticity of labour in order to obtain realistic business cycle properties for the behaviour of real wages and employment. An infinite supply elasticity limits the increase in marginal costs and prices following an expansion of output in a model with sticky prices, which helps to generate real persistence of monetary shocks. The introduction of nominal-wage rigidity in this model makes the simulation outcomes less dependent on this assumption, as wages and the marginal cost become less sensitive to output shocks, at least over the short term.

Given equation (12), the law of motion of the aggregate wage index is given by:

(14) $$\left(W_t\right)^{-1/\lambda_w} = \xi\left(W_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w}\right)^{-1/\lambda_w} + (1-\xi)\left(\tilde{w}_t\right)^{-1/\lambda_w}$$

2.1.3 Investment and capital accumulation

Finally, households own the capital stock, a homogenous factor of production, which they rent out to the firm-producers of intermediate goods at a given rental rate of $\varepsilon_t^k r_t^k$. They can increase the supply of rental services from capital either by investing in additional capital (I_t), which takes one period to be installed or by changing the utilisation rate of already installed capital (z_t). Both actions are costly in terms of foregone consumption (see the intertemporal budget constraint (4) and (5)).[9]

Households choose the capital stock, investment and the utilisation rate in order to maximise their intertemporal objective function subject to the intertemporal budget constraint and the capital accumulation equation which is given by:

(15) $$K_t = K_{t-1}[1-\tau] + \left[1 - S\left(I_t / I_{t-1}\right)\right] I_t\,,$$

where I_t is gross investment, τ is the depreciation rate and the adjustment cost function $S(.)$ is a positive function of changes in investment.[10] $S(.)$ equals zero in steady state with a constant investment level. In addition, we assume that the first derivative also equals zero around equilibrium, so that the adjustment costs will only depend on the second-order derivative as in CEE (2001).

The first-order conditions result in the following equations for the real value of capital, investment and the rate of capital utilisation:

(16) $$Q_t = E_t\left[\beta \frac{\lambda_{t+1}}{\lambda_t}\left(Q_{t+1}(1-\tau) + \varepsilon_{t+1}^k z_{t+1} r_{t+1}^k\right)\right],$$

(17) $$Q_t S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \beta E_t\, Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = 1$$

(18) $$r_t^k = \Psi'\left(z_t\right)$$

Equation (16) states that the value of installed capital depends on the expected future value taking into account the depreciation rate and the expected future return as captured by the rental rate times the expected rate of capital utilisation.

The first order condition for the utilisation rate (18) equates the cost of higher capital utilisation with the rental price of capital services. As the rental rate increases it becomes more profitable to use the capital

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9 This specification of the costs is preferable above a specification with costs in terms of a higher depreciation rate (see King and Rebelo (2000), or Greenwood, Hercowitz, and Huffman (1988), Dejong, Ingram and Whiteman (2000)) because the costs are expressed in terms of consumption goods and not in terms of capital goods. This formulation limits further the increase in marginal cost of an output expansion (See CEE (2001)).

10 See CEE (2001).

stock more intensively up to the point were the extra gains match the extra output costs. One implication of variable capital utilisation is that it reduces the impact of changes in output on the rental rate of capital and therefore smooths the response of marginal cost to fluctuations in output.[11]

2.2 Technologies and firms

The country produces a single final good and a continuum of intermediate goods indexed by *j* where *j* is distributed over the unit interval ($j \in [0,1]$). The final-good sector is perfectly competitive. The final good is used for consumption and investment by the households. There is monopolistic competition in the markets for intermediate goods: each intermediate good is produced by a single firm.

2.2.1 Final-good sector

The final good is produced using the intermediate goods in the following technology:

$$(19) \qquad Y_t = \left[\int_0^1 \left(y_t^j \right)^{1/(1+\lambda_{p,t})} dj \right]^{1+\lambda_{p,t}}$$

where y_t^j denotes the quantity of domestic intermediate good of type *j* that is used in final goods production, at date t. $\lambda_{p,t}$ is a stochastic parameter which determines the time-varying mark-up in the goods market. Shocks to this parameter will be interpreted as "cost-push" shocks.

The cost minimisation conditions in the final goods sector can be written as:

$$(20) \qquad y_t^j = \left(\frac{p_t^j}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t$$

and where p_t^j is the price of the intermediate good j and P_t is the price of the final good. Perfect competition in the final goods market implies that the latter can be written as:

$$(21) \qquad P_t = \left[\int_0^1 \left(p_t^j \right)^{-1/\lambda_{p,t}} dj \right]^{-\lambda_{p,t}}$$

2.2.2 Intermediate goods producers

Each intermediate good j is produced by a firm j using the following technology:

$$(22) \qquad y_t^j = \varepsilon_t^a \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} ,$$

[11] Another assumption which will tend to have the same effect is that capital is perfectly mobile between firms. This is a rather strong hypothesis. Recently, Woodford (2000) has illustrated how this assumption can be relaxed in a model with sticky prices and adjustment costs in investment. The hypothesis has important consequences for the estimation of the degree of price stickiness. With capital specific to the firm, firms will be more reluctant to change the price of their good as the resulting demand response will have a much stronger impact on the marginal cost of production. The assumption of capital mobility across firms therefore bias the estimated degree of price stickiness upwards.

where ε_t^a is the productivity shock, $\tilde{K}_{j,t}$ is the effective utilisation of the capital stock given by $\tilde{K}_{j,t} = z_t K_{j,t-1}$ and $L_{j,t}$ is an index of different types of labour used by the firm given by (11).

Cost minimisation implies:

(23) $$\frac{W_t L_{j,t}}{r_t^k \tilde{K}_{j,t}} = \frac{1-\alpha}{\alpha}$$

Equation (23) implies that the capital-labour ratio will be identical across intermediate goods producers and equal to the aggregate capital-labour ratio. As the production function exhibits constant returns to scale, the firm's average and marginal cost are equal and given by:

(24) $$MC_t = AC_t = \frac{1}{\varepsilon_t^a} W_t^{1-\alpha} r_t^{k\,\alpha} (\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)})$$

This implies that the marginal cost, too, is independent of the intermediate good produced.

Nominal profits of firm j are then given by:

(25) $$\pi_t^j = (p_t^j - MC_t)\left(\frac{p_t^j}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} (Y_t)$$

Each firm j has market power in the market for its own good and maximises expected profits using a discount rate $(\beta\rho_t)$ which is consistent with the pricing kernel for nominal returns used by the shareholders-households: $\rho_{t+k} = \frac{\lambda_{t+k}}{\lambda_t}\frac{1}{P_{t+k}}$.

As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price-change signal". The probability that a given price can be re-optimised in any particular period is constant $(1-\xi_p)$. Following CEE (2001), prices that do not receive a price signal are indexed to last period's inflation rate.[12] Profit optimisation by producers that are "allowed" to re-optimise their prices at time t results in the following first-order condition:

(26) $$E_t \sum_{i=0}^{\infty} \beta^i \xi_p{}^i \lambda_{t+i} y_{t+i}^j \left(\frac{\tilde{p}_t^j}{P_t}\left(\frac{(P_{t-1+i}/P_{t-1})}{P_{t+i}/P_t}\right)^{\gamma_p} - (1+\lambda_{p,t+i}) mc_{t+i}\right) = 0$$

Equation (27) shows that the price set by firm j, at time t, is a function of expected future marginal costs. The price will be a mark-up over these weighted marginal costs. If prices are perfectly flexible

[12] Erceg, Henderson and Levin (2000) use indexation to the average steady state inflation rate. Allowing for indexation of the non-optimised prices on lagged inflation, results in a linearised equation for inflation that is an average of expected future inflation and lagged inflation. This result differs from the standard Calvo model that results in a pure forward looking inflation process. The more general inflation process derived here results however from optimising behaviour and this makes the model more robust for policy and welfare analysis. Another consequence of this indexation is that the price dispersion between individual prices of the monopolistic competitors will be much smaller compared to a constant price setting behaviour. This will also have important consequences for the welfare evaluation of inflation costs.

($\xi_p = 0$), the mark-up is a constant and equal to $1+\lambda_{p,t+i}$. With sticky prices the mark-up becomes variable over time when the economy is hit by exogenous shocks. A positive demand shock lowers the mark-up and stimulates employment, investment and real output.

The definition of the price index in equation (21) implies that its law of motion is given by:

(27) $$(P_t)^{-1/\lambda_{p,t}} = \xi_p \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} \right)^{-1/\lambda_{p,t}} + (1-\xi_p)\left(\tilde{p}_t^j\right)^{-1/\lambda_{p,t}} .$$

2.3 Market equilibrium

The final goods market is in equilibrium if production equals demand by households for consumption and investment and the government:

(30) $$Y_t = C_t + G_t + I_t + \psi(z_t)K_{t-1}$$

The capital rental market is in equilibrium when the demand for capital by the intermediate goods producers equals the supply by the households. The labour market is in equilibrium if firms' demand for labour equals labour supply at the wage level set by households.

The interest rate is determined by a reaction function that describes monetary policy decisions. This rule will be discussed in the following sections of the paper. In order to maintain money market equilibrium, the money supply adjusts endogenously to meet the money demand at those interest rates.

In the capital market, equilibrium means that the government debt is held by domestic investors at the market interest rate i_t.

2.4 The linearised model

For the empirical analysis of Section 3 we linearise the model equations described above around the non-stochastic steady state. Below we summarise the resulting linear rational expectations equations. The ^ above a variable denotes its log deviation from steady state. Variables dated at time t+1 refer to the rational expectation of those variables.

The *consumption equation* with external habit formation is given by:

(31) $$\hat{C}_t = \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}\hat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1-h}{(1+h)\sigma_c}(\hat{\varepsilon}_t^b - \hat{\varepsilon}_{t+1}^b)$$

When $h = 0$, this equation reduces to the traditional forward-looking consumption equation. With external habit formation, consumption depends on a weighted average of past and expected future consumption. Note that in this case the interest elasticity of consumption depends not only on the intertemporal elasticity of substitution, but also on the habit persistence parameter. A high degree of habit persistence will tend to reduce the impact of the real rate on consumption for a given elasiticity of substitution.[13]

The *investment equation* is given by:

(32) $$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}\hat{I}_{t+1} + \frac{1}{\varphi(1+\beta)}\hat{Q}_t$$

where $\varphi = 1/\overline{S}''$. As discussed in CEE (2001), modelling the capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks including monetary policy shocks.

The corresponding *Q equation* is given by:

(33) $$\hat{Q}_t = -(\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}\hat{Q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}\left[(1+\psi)\,\hat{r}^k_{t+1}\right] + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}\varepsilon^k_t$$

where $\beta = 1/(1-\tau+\bar{r}_k)$ and $\psi = \frac{\psi'(1)}{\psi''(1)}$ is the inverse of the elasticity of the capital utilisation cost function. The current value of the capital stock depends negatively on the ex-ante real interest rate, and positively on the expected future value and the expected rental rate. Variable capital utilisation implies that the value of capital and thus investment is more sensitive to changes in the expected rental rate.

The *capital accumulation equation* is standard:

(34) $$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\hat{I}_{t-1}$$

With partial indexation, the *inflation equation* becomes a more general specification of the standard new-Keynesian Phillips curve:

(35) $$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p}\hat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\hat{r}^k_t + (1-\alpha)\hat{w}_t - \hat{\varepsilon}^a_t + \hat{\lambda}_{p,t}\right]$$

Inflation depends on past and expected future inflation and the current marginal cost, which itself is a function of the rental rate on capital, the real wage and the productivity parameter. When $\gamma_p = 0$, this equation reverts to the standard purely forward-looking Phillips curve. In other words, the degree of indexation determines how backward looking the inflation process is. The elasticity of inflation with respect to changes in the marginal cost depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$), this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

Similarly, partial indexation of nominal wages results in the following real wage equation:

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[13] Without loss of generality, in the empirical estimation the shock variables will be normalised to have a unitary impact on the right-hand side variables.

(36)
$$\hat{w}_t = \frac{\beta}{1+\beta}\hat{w}_{t+1} + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{\beta}{1+\beta}\hat{\pi}_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\hat{\pi}_t + \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w}\left[\hat{w}_t - \sigma_L\hat{L}_t - \frac{\sigma_c}{1-h}(\hat{C}_t - h\hat{C}_{t-1}) - \hat{\varepsilon}_t^L\right]$$

The real wage is a function of expected and past real wages and the expected, current and past inflation rate where the relative weight depends on the degree of indexation of the non-optimised wages. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labour market. The size of this effect will be greater, the smaller the degree of wage rigidity, the lower the demand elasticity for labour and the lower the inverse elasticity of labour supply (the flatter the labour supply curve).

The equalisation of marginal cost implies that, for a given installed capital stock, *labour demand* depends negatively on the real wage (with a unit elasticity) and positively on the rental rate of capital:

(37) $$\hat{L}_t = -\hat{w}_t + (1+\psi)\hat{r}_t^k + \hat{K}_{t-1}$$

The *goods market equilibrium condition* can be written as:

(38) $$\hat{Y}_t = (1-\tau k_y - g_y)\hat{C}_t + \tau k_y \hat{I}_t + g_y \hat{G}_t = \hat{a}_t + \alpha\hat{K}_{t-1} + \alpha\psi\hat{r}_t^k + (1-\alpha)\hat{L}_t$$

Finally, the model is closed by adding a monetary policy reaction function:

(39) $$\hat{R}_t = \rho\hat{R}_{t-1} + (1-\rho)\{\overline{\pi}_t + r_\pi(\hat{\pi}_{t-1} - \overline{\pi}_t)\} + r_{\pi e}(\hat{\pi}_{t+1} - \hat{\pi}_t) + r_y(\hat{Y}_t - \hat{Y}_{t-1}) + \varepsilon_t^R$$

which we use in the estimation of the model. We assume that there are two monetary policy shocks: one is a persistent shock to the inflation objective; the other is a temporary interest rate shock.

Equations (31) to (39) determine the nine endogenous variables: $\hat{\pi}_t$, $\hat{w}_t$, $\hat{K}_{t-1}$, $\hat{Q}_t$, $\hat{I}_t$, $\hat{C}_t$, $\hat{R}_t$, $\hat{r}_t^k$, $\hat{L}_t$ of our model. The stochastics of the system of linear rational expectations equations is driven by eight exogenous shock variables: $\hat{\varepsilon}_t^L$, ε_t^a, ε_t^b, $\hat{G}_t$, $\overline{\pi}_t$, ε_t^R, ε_t^k, $\hat{\lambda}_{p,t}$. In the empirical section these shock variables will generally be assumed to follow an independent first-order autoregressive stochastic process.

3. Estimation results

In this section we, first, discuss how we estimate the structural parameters and the processes governing the eight structural shocks. Next, we present the main estimation results.

3.1 Estimation methodology

In order to estimate the parameters of the SDGE model presented in section 2, we use data over the period 1973-1999 on seven key macro-economic variables in the euro area: real GDP, real consumption, real investment, the GDP deflator, real wages, employment and the nominal interest rate.[14] As we do not have good measures of the area-wide capital stock, the value of capital or the rental rate on capital, we will assume these variables are not observed. As discussed above, the seven observable variables need to be explained in terms of eight structural shocks.

There are various ways of estimating or calibrating the parameters of a linearised SDGE model. Geweke (1999) distinguishes between the strong and the weak econometric interpretation of SDGE models.[15] Following Sargent (1989), a number of authors have estimated the structural parameters of SDGE models using maximum likelihood methods.[16] This is an example of the first approach. These maximum likelihood methods usually consist of four steps. In the first step, the linear rational expectations model is solved for the reduced form state equation in its predetermined variables. This can be done using standard methods for solving linear rational expectations models. In the second step, the model is written in its state space form. This involves augmenting the state equation in the predetermined variables with an observation equation which links the predetermined state variables to observable variables. In this step, the researcher needs to take a stand on the form of the measurement error that enters the observation equations.[17] The third step consists of using the Kalman filter to form the likelihood function. In the final step, the parameters are estimated by maximising the likelihood function.

The attractions of full maximum likelihood estimation are clear. When successful, it provides a full characterisation of the data generating process and allows for proper specification testing and forecasting. However, given the high number of parameters to be estimated and the highly non-linear nature of the optimisation problem, full maximum likelihood estimation is in practice often fraught with difficulties. Also in our case, we found it very difficult to achieve convergence.

An alternative way of calibrating the parameters of a SDGE model is to choose the parameters in such a way that selected theoretical moments given by the model match as closely as possible those observed in the data. This is an example of what Geweke (1999) has called the weak econometric interpretation of SDGE models.[18] One way of achieving this, is by minimising some distance function between the theoretical and empirical moments of interest. For example, recently, a number of researchers have estimated the parameters in monetary SDGE models by minimising the difference between an empirical and the theoretical impulse response to a monetary policy shock (Rotemberg and Woodford (1998) and

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[14] The dataset used is the one constructed in Fagan et al (2001). All variables are treated as deviations around the sample mean. Real variables are detrended by a linear trend.

[15] Geweke (1999) discusses and criticises both classical and Bayesian approaches to SDGE model calibration. In this paper we only use classical approaches.

[16] For a recent discussion, see Ireland (1999).

[17] Recently, Ireland (1999) has suggested a way of combining the power of SDGE theory with the flexibility of vector autoregressive time-series models by proposing to model the residuals in the observation equations (which capture the movements in the data that the theory can not explain) as a general VAR proces. This proposed method admits that while SDGE models may be powerful enough to account for and explain many key features of the data, they remain too stylised to possibly capture all of the dynamics that can be found in the data.

[18] It is in line with Kydland and Prescott's (1996) emphasis on the fact that the model economy is intended to "mimic the world along a carefully specified set of dimensions".

CEE (2001)). As we have introduced a full set of structural shocks, we can be a bit more ambitious and try to match the full cross covariance function in the data, for example, estimated on the basis of an unrestricted VAR. The advantage of this approach is that moment estimators are often more robust than the full-information maximum likelihood estimators discussed above. In addition, these estimation methods allow the researcher to focus on the characteristics in the data for which the SDGE model, which is necessarily an abstraction of reality, is most relevant. However, as pointed out by Geweke (1999), this weak econometric interpretation of SDGE models is not necessarily less stringent than the full maximum likelihood criterion: in spite of the focus on a restricted set of moments, the model is assumed to account for all aspects of the observed data series and these aspects are used in calculating the moments of interest.

In this paper, as a first step, we take an intermediate position between the weak and the strong econometric interpretation of SDGE models. In order to estimate the parameters of the model and the stochastic processes governing the structural shocks, we minimise a weighted average of the likelihood function based on a Kalman filter and a distance function between the theoretical and empirical cross-covariances of the observable variables. This can be seen as a constrained maximum likelihood estimation, where the constraint is given by the ability of the model to capture the cross-covariances in the data. The weight on the moment criterion ensures that the cross-covariances delivered by our estimated model are not too far away from those estimated in the data. This is important as a crucial question in this research is to what extent SDGE models can replicate the broad cross correlations and persistence in the data.[19] A second advantage of putting weight on the moment criterion is that it appears to make the estimation of the parameters much more robust. This is particularly important because of the relatively large number of parameters that we try to estimate. The focus on the likelihood criterion, on the other hand, ensures that the estimated model is not totally at odds with other aspects of the data as captured by the likelihood function. In particular, this is important as we are interested in retrieving best estimates of the structural shocks and their contribution to the historical time series.

More formally, we minimise the following criterion, which is a weighted average of the Gaussian log likelihood function and a weighted sum of the absolute deviations between the model-based cross-covariances and the empirical ones based on an unrestricted VAR:

(40) $$L^* = \omega\{-T\ln 2\pi - 0.5T\ln|V| - 0.5\sum_{t=0}^{T-1} u_t' V^{-1} u_t\} + (1-\omega)\{\sum_{i,j,l} \frac{\left|\text{cov}_{i,j,l}^m - \text{cov}_{i,j,l}^d\right|}{std\ \text{cov}_{i,j,l}^d}\}$$

where u_t and V are the prediction errors of the observable variables derived from a Kalman filter and their covariance matrix respectively, and $\text{cov}_{i,j,t}^m$ and $\text{cov}_{i,j,t}^d$ are the model and data based cross covariances of data series i and j at lag l, where l goes from –12 to +12. Below we will assume equal weight on both criteria. The absolute deviation between the model-based and data-based covariances is weighted by their empirical standard deviation. In other words, the moment criterion tries to make sure

[19] For a recent example and application to the euro area, see Casares (2001).

that the theoretical cross-covariances lie within a one-standard deviation of the empirical cross covariances.[20]

In order to calculate the empirical log likelihood, we use the Kalman filter as modified by Sargent (1989) to take into account serial correlation in the observation error. We allow for first-order serial correlation in both the structural shocks and the observation errors. We assume, however, that all structural and observation shocks are independent of each other.[21] Let X_t be the vector of predetermined state variables in our linearised model. This vector will not only include the predetermined state variables such as the capital stock and the interest rate, but also various lags of some of the endogenous variables such as inflation, investment, etc. and one lag of the structural shocks. Using a standard solution algorithm to solve the system of rational expectations equations, the state equation can then be written in its companion form as:

(41) $$X_{t+1} = \tilde{A} X_t + \tilde{\varepsilon}_{t+1}$$

where $\tilde{\varepsilon}_t$ is the vector of disturbances to the structural shocks and is assumed to have a diagonal covariance matrix.

The observation equation is given by:

(42) $$Y_t = \tilde{B} X_t + \tilde{\nu}_t ,$$

where $\tilde{\nu}_t$ is a vector of observations errors of dimension seven and is assumed to follow a first-order autoregressive process with a diagonal covariance matrix. Given the sample of observable variables, these equations can then be used to derive the prediction errors of the observables and their covariance matrix using the modified Kalman filter in Sargent (1989).

3.2 Parameter estimates

As it turned out to be difficult to estimate all parameters simultaneously, we partition the model parameters into two groups. The first group is calibrated ex ante and assumed to be fixed when we estimate the second group of parameters. The discount factor, β, is calibrated to be 0.99, which implies an annual steady state real interest rate of 4%. The depreciation rate, τ, is set equal to 0.025, which implies an annual depreciation on capital equal to 10 percent. We set $\alpha = 0.35$, which roughly implies a steady state share of labour income in total output of 65%. The share of steady-state consumption in total output is assumed to be 0.6, while the share of steady-state investment is assumed to be 0.22. This corresponds more or less with the average share of output and investment in total euro area output over the estimation period. Note that the latter implies a steady-state capital output ratio of about 2.2.

[20] For the use of similar moment condition, see DeJong, Ingram and Whiteman (1996).

[21] The assumption of uncorrelated shocks is consistent with a large part of the literature. It is different from Ireland (2000) who assumes a general VAR process for the measurement error. However, given the dimension of our system, Ireland's proposal turned out to be impractical in our case.

In addition, we fixed three parameters that in the estimation were converging to extreme values. The elasticity of the utilisation cost function was converging to zero. We put $1/\psi = 0.2$. The other parameters did not appear to be very sensitive to changes in this parameter. A similar phenomenon was noticed by CEE (2001) who set this parameter equal to 0.01, which implies that the rental rate is almost fixed. Our parameter is closer to the one estimated by King and Rebelo (2000). The parameter governing the indexation of nominal wages to past inflation, γ_w, had the tendency to become bigger than one, which appears to be implausible. We therefore set it equal to one. Finally, the mark-up parameter in the labour market, λ_w , turned out to be very imprecisely estimated and did not significantly affect the other estimation results. We, therefore, set it equal to 1.7.

The remaining parameters were estimated by minimising (40). We now discuss the results reported in Table 1. First, we estimate a considerable degree of wage and price stickiness. The average duration of wage contracts is estimated to be one year, whereas the average duration of the price contracts is much longer at two and a half years. The greater stickiness in prices relative to wages is somewhat counterintuitive, but turns out to be a very robust outcome of the estimated model. Our estimate of price stickiness is considerably larger than that found in Gali, Gertler and Lopez-Salido (2000). It also contrasts with the finding in CEE (2001) that the degree of price stickiness is much less than that in wages (for the US economy). At this point it is not clear whether these differences are due to structural differences in the US versus the euro area, or differences in the structure of the model and the estimation methodology. One important difference is that in the CEE model, interest rate changes have an immediate cost-push effect on marginal cost. As a result, prices can actually rise following a monetary policy tightening. This may go a long way in explaining the very sluggish empirical response of prices following a monetary policy shock. We also find that there is imperfect indexation of prices to past inflation, whereas CEE assumes perfect indexation. Our estimate of the degree of indexation implies that the weight on past inflation is about 0.40.

Second, our estimate of the intertemporal elasticity of substitution (1/σ) is higher than one. It is smaller than the estimates found in Rotemberg and Woodford (1997). It is different from most of the RBC literature which assumes an elasticity of substitution equal or below one. However, one needs to be careful when making such comparisons, as we have assumed external habit formation which turns out to be significant. The external habit stock is estimated to be about two thirds of past consumption, which is similar to the estimates reported in CEE (2001). Disregarding the consumption shocks, our consumption equation (31) can be written as:

$$\hat{C}_t = h\hat{C}_{t-1} - \frac{1-h}{\sigma_c}\sum_{i=0}^{\infty}(\hat{R}_{t+i} - \hat{\pi}_{t+1+i})$$

Our estimates of σ_c and h thus imply that an expected one percent increase in the short-term interest rate for four quarters has an impact on consumption of about 0.50.

Third, our estimate of the adjustment cost parameter is very similar to the one estimated in CEE (2001). It implies that investment increases by 0.22 percent following a one percent increase in the current price of installed capital.

Fourth, our estimate of σ_l is around 10 and relatively large. This implies a relatively small elasticity of the labour supply. In contrast to much of the RBC literature, we do not need a large labour supply elasticity to reduce the impact of output changes on wages and the marginal cost, because there are various other mechanisms such as the nominal wage stickiness and the variable capacity utilisation which will tend to reduce this sensitivity of the marginal cost.

In addition, our estimation delivers a plausible estimate of the long and short-run reaction function of the monetary authorities. Obviously, as there was no single monetary policy in the euro area over the estimation period, these results need to be taken with a grain of salt. The estimates imply that in the long run the response of interest rates to inflation was greater than one, thereby satisfying the so-called Taylor principle. In addition, we also find a significant positive short-term reaction to an increase in expected inflation and the real growth rate. Finally, in agreement with the large literature on estimated interest rate rules, we also find evidence of substantial degree of interest rate smoothing.

The estimates of the stochastic processes governing the structural shocks show that, with the exception of the mark-up shock which has a small negative autoregressive component, all shocks exhibit significant positive serial correlation. As it is commonly assumed in the literature, the productivity shock exhibits the highest persistence, while the cost of capital, labour supply and preference shocks have autoregressive parameters in the range of 0.60 to 0.70.[22] The estimated variance of the government spending shock converged to zero and as a result was set to zero in the estimation procedure. As a result, the discrepancy between observed output and the sum of consumption and investment is accounted for by observation error.

Finally, regarding the observation errors, next to the significant observation error in the output series which captures the unexplained part of output due to net foreign trade and government spending, we only find significant observation error in the investment and the real wage equation. In the case of investment, this observation error is quite persistent. Overall, this observation error explains only a small part of the overall variance.

3.3 Assessing the performance of the model: the cross-covariance function

The discussion in the previous section shows that the model is able to deliver reasonable estimates of the underlying parameters. In this Section we analyse how well the estimated model fits the empirical cross covariance function. Graph 1 summarises this fit by plotting the model-based cross-covariances together with a one standard error confidence band around the empirical ones. Overall, there are very few cases of

[22] Note that we have imposed an autoregressive parameter of 0.975 on the inflation target shock, while the temporary monetary policy shock is assumed to be white noise.

the 49 graphs in which the model-based cross-covariance lies outside the one-standard band of the empirical cross-covariance.

More specifically, focusing first on the upper left nine graphs, it is clear that the model is able to capture the considerable degree of persistence in output and inflation. The model also captures the hump-shaped positive correlation between current output and future inflation. As noted by Fuhrer and Moore (1995) and Gali and Gertler (2000), the purely forward-looking Phillips curve predicts that inflation should lead output, whereas in the data the opposite seems to be the case. The combination of some backward-looking-ness in the inflation equation and a sluggish response of the marginal cost to changes in output explains why our model is capable of capturing this feature of the data. The correlation of output and inflation with the short-term nominal interest rate is less well-captured. In particular, the estimated variance of the interest rate is too small, and the model seems to have problems fitting the negative correlation between current interest rates and future output and inflation.

The model also captures the persistence in the other variables very well. The model-based covariances of output and inflation with employment and real wages are very close to the empirical ones. In particular, the model has no problem explaining the postive correlation between current output and future wages.

One feature that the model has problems capturing is the strong positive correlation between consumption and investment. This is in part due to the relatively strong crowding out effects that are present in the model.

3.4 The structural shocks

3.4.1 Impulse response analysis

Graphs 2 to 9 plot the impulse responses to the various structural shocks. Note that these impulse responses are obtained with the estimated monetary policy reaction function. In Section 4 we will discuss the responses under optimal monetary policy.

Graph 2 shows that following a positive productivity shock output, consumption and investment rise, while employment falls. Also the utilisation rate of capital falls. As pointed out by Gali (1999), this is consistent with estimated impulse responses of identified productivity shocks in the US and is in contrast to the predictions of the standard RBC model without nominal rigidities. The implications for inflation under the estimated policy reaction function are clear. Due to the rise in productivity, the marginal cost falls on impact. As monetary policy does not immediately respond to offset this fall in marginal cost, inflation starts falling gradually. The estimated reaction of monetary policy on a productivity shock is in line with similar results for the US as presented in Ireland (1999) and Gali, Lopez-Salido and Valles (2000) (at least for the pre-Volcker period). Finally, note that the real wage only gradually rises following the positive productivity shock.

Graph 3 shows the effects of a negative labour supply shock. The effects of this supply shock on output, inflation and the interest rate are very similar to those of a negative productivity shock. The main qualitative differences are that, first, employment also falls in line with output and, second, that the real wage rises significantly. It is this significant rise in the real wage that leads to a rise in the marginal cost and an increase in inflation. At this point, it is worth noting that the qualitative impact of a wage push shock would be very similar.

It is interesting to note that a preference shock, while increasing consumption and output significantly, has almost no impact on investment (Graph 4). Most of the increase in capacity necessary to satisfy increased demand is delivered by an increase in the utilisation of installed capital and an increase in employment. As in empirical impulse responses typically strong accelerator effects are found, this points to a problem in the underlying model. Due to strong crowding out effects, the accelerator effects in these models are not strong enough. Increased consumption demand puts pressure on the prices of production factors: both the rental rate on capital and the real wage rise, putting upward pressure on the marginal cost and inflation. An investment boom driven by a reduction in the cost of capital (Graph 5) has very similar effects, but does lead to stronger crowding out effects on consumption.

Graph 6 plots the effects of a cost-push shock. The increase in inflation leads to a fall in real wages. Output and employment fall. Somewhat surprising is the large fall in investment.

Finally, Graphs 7 and 8 plot the effects of the two monetary policy shocks. An important observation is that the private sector is able to distinguish between the temporary and the persistent policy shock. The temporary shock leads to a rise in the nominal and real short-term interest rate. This leads to a hump-shaped fall in output, consumption and investment. In line with the stylised facts following a monetary policy shock, real wages fall. The maximum effect on investment is about twice as large as that on consumption. Overall, these effects are consistent with the evidence on the euro area (Peersman and Smets, 2001), although the price effects in the model are somewhat larger than estimated in some identified VARs.

The effects of a persistent change in the inflation objective are strikingly different in two respects. First, there is no liquidity effect, as nominal interest rates start falling immediately as a result of the reduced inflation expectations. This is in line with the arguments made in Gali (2000) that the presence (or lack thereoff) of a liquidity effect following a monetary policy shock will depend on the persistence of the shock. Second, because the change in policy is implemented gradually and expectations have time to adjust, the output effects of the change in inflation is much smaller. In order to see the effects of imperfect information, Graph 9 also plots the effects of a persistent fall in the inflation objective, when the private agents can not distinguish between the two monetary policy shocks.[23] Imperfect information implies that the effects of a reduction in the inflation objective on inflation are slower and smaller and the resulting sacrifice ratio higher.

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[23] We use the techniques of Svensson and Woodford (2000). For an application, see Ehrman and Smets (2001). It is important to note that the model is estimated under the assumption that the agents in the economy know the structure of the economy. In future research, we intend to investigate the implications of estimating the model consistent with the view that the private agents can not distinguish between the two shocks. This assumption may have an impact on some of the parameters in the model.

3.4.2 Variance decomposition

The contribution of each of the structural shocks to the variance of the endogenous variables at various horizons is shown in Table 2. Let us first focus on the determinants of output. Our model is able to explain 96 percent of the total long-run variance; observations errors accounting for the remaining 4 percent. The most important shocks are the preference and the labour supply shocks which each account for about one quarter of the long run variance. The productivity, investment, mark-up and policy shocks each account for between 10 and 15 percent of the long run variance. That both "supply shocks", the productivity and labour shock, account for only 32 percent in the long run seems to run counter to the results from identified VAR studies that those shocks account for most of the long-run variance (E.g. Shapiro and Watson, 1989 and Blanchard and Quah, 1989). However, it should be noted that in those studies it is assumed that only supply shocks affect output in the long run. Here all shocks are stationary. In the context of a standard RBC model, Ireland (2001) tests various stochastic processes for productivity shocks and finds that while the productivity process is very persistent, it is stationary. At the four and ten quarter horizon, the dominance of "demand shocks" is even greater. At the one year horizon, the preference, investment and interest rate shock account for more than three quarters of the variance in output.[24] This result confirms the conjecture made in Gali (2000) that the negative correlation between output and employment in response to a productivity shock raises serious doubts about the quantitative significance of productivity shocks as a source of aggregate fluctuations in industrialised countries.

Turning to the determinants of inflation, we find that even in the long run, about 60 percent of the variance is accounted for by cost-push shocks. The inflation objective shock, the second most important determinant, explains only 11 percent. This dominance is even greater at shorter horizons. To some extent, this finding is not very surprising. Inflation is empirically a quite volatile process. At the same time, however, we estimate inflation to be a very sluggish process, which only responds very gradually to current and expected changes in the marginal cost. It is therefore not very surprising that one needs quantitatively important "cost push" shocks to account for the behaviour of volatile prices. Of course, these shocks could capture a whole range of shocks that are not accounted for in the stylised model such as changes in oil prices, terms-of-trade shocks, changes in taxes, etc. It is also important to note that this decomposition does not say anything about the fundamentally monetary nature of inflation in the long run. As the steady state of our model is deterministic (even changes in the inflation objective are ultimately temporary), it is to be expected that the long-run variance will be determined by temporary shocks.

Under the estimated reaction function, the nominal interest rate is mainly determined by the "demand" shocks and the interest rate shock. The impact of the "supply" shocks on the interest rate is negligible.

It is worth noting that while productivity shocks explain an important part of the variance in employment in the short run, it is mainly the labour supply shock and the preference shock that play an important role

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[24] We put "demand" in brackets because in our model the preference shock will also have an effect on the supply side.

in the long run. Also worth noting is that the "cost push" shock is the second most important shock (after the labour supply shock) in explaining the variance in real wages.

Finally, overall temporary interest rate shocks seem to account for a non-negligible part in the variation of economic activity and inflation.

3.4.3 Historical decomposition

Graphs 10 and 11 summarise the historical contribution of the various structural shocks to output and inflation developments.[25] This decomposition is based on our best estimates of the various shocks.[26] While obviously such a decomposition must be treated with caution, it helps in understanding how the estimated model interprets specific movements in the observed data and therefore can shed some light on its plausibility.

Focusing on the decomposition of inflation first, it is clear that in line with the results from the variance decomposition the short-run variability in inflation is mostly accounted for by "cost-push" shocks. In part these shocks seem to come from oil price changes. For example, their contribution is high in 1974, 1979 and 1986. However, the secular part in inflation is mostly driven by monetary policy shocks. Adding the contributions of the temporary interest rate shock and the more persistent shock to the inflation objective shock in Graph 10, it is clear that monetary policy has contributed quite significantly to the surge in inflation in the second half of the 1970s. Subsequently, it has generated two periods of disinflation: the first one at the end of the 1970s and the beginning of the 1980s; the second one starting in 1989 and ending in 1994. This contribution of monetary policy is also reflected in real wages (not shown), which rise very significantly in the second half of the 1970s and fall subsequently. Finally, the various "supply" and "demand" shocks lead to a clear cyclical pattern in inflation with peaks in 1982 and 1992 and troughs in 1979, 1987 and 1999.

The relative role of the various shocks during the 1970s is also clear from the decomposition of output. The most important determinants of the recession of 1974 are the positive oil price shocks and to a lesser extent other supply and demand shocks. It is, however, striking how those shocks continue to contribute negatively to output during most of the 1970s. The economic recovery in 1975-76 is mostly due to loose monetary policy, which then contributed to the persistently high inflation of the 1970s. Most of the variation in output since the mid 1980s seems to be due to the various supply and demand shocks, although the monetary policy tightening during the ERM crisis of 1992 has clearly contributed to the recession of following that period.

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[25] The graphs with our best estimates of the structural shocks themselves are given in the appendix.

[26] Note that because of the observation error in output, it is not possible to do a perfect decomposition in terms of the structural shocks and the observation errors only.

4. Optimal monetary policy

In this section, we analyse optimal monetary policy in the estimated model of Section 3. First, we discuss the response of the economy to the estimated structural shocks when prices and wages are flexible (Section 4.1). Next, we analyse optimal monetary policy (Sections 4.2 and 4.3).

4.1 The flexible price and wage economy

As a benchmark for the analysis of optimal monetary policy, it is useful to describe how the economy would respond if both nominal prices and wages were perfectly flexible. In a simple version of this model with only nominal price rigidities and no "cost-push" shocks, Woodford (2001) has pointed out that optimal monetary policy will be able to replicate the flexible price equilibrium, thereby restoring the first best. As discussed in the next section, this will not be the case in our set-up. It is nevertheless interesting to analyse how output and the interest rate would respond to the various shocks in such an environment, as it allows us to understand what factors drive changes in the natural output or interest rate level.

Graphs 12 to 16 show the impulse responses to the five structural shocks. There is no point in discussing monetary policy in this set-up, as monetary policy will be neutral. We simply assume that monetary policy stabilises the price level.

With flexible prices and wages, output jumps up immediately in response to a productivity shock. In our model the natural interest rate falls dramatically. As pointed out by Neiss and Nelson (2001), the response of the interest rate will partly depend on the form of capital adjustment costs. As in our model, both consumption and investment respond quite sluggishly due to habit persistence and the adjustment costs in the change in investment respectively, a large decrease in the real rate is necessary to ensure that demand catches up with the increased supply. A similar reasoning explains why the real interest rate needs to rise in response to a negative labour supply shock.

Somewhat surprisingly, the natural output level responds negatively to a positive preference shock. This is in part because higher consumption reduces marginal utility and therefore leads to a fall in labour supply (or a rise in the real wage). As a result, the real interest rate needs to rise quite significantly in order to crowd out investment. Given the limited response of the natural output level in response to a positive investment shock, such crowding out is also important in that case.

In sum, due to the sluggishness of both demand components in our model, the natural real interest rate needs to rise quite significantly in response to positive "demand" shocks, while it needs to fall in response to positive "supply" shocks.

Finally, Graph 16 shows the effect of a "cost-push" shock in the flexible price and wage economy. This shock has a very large, but temporary negative effect on output and the real wage. Correspondingly, the natural real rate increases very strongly for one period. Note that both the typical "cost push" shock and the "labour supply shock" have very large effects on the natural output and interest rate level. This is in part a result of the relatively small estimated sensitivity of prices to marginal cost and of wages to the deviations of the wage from its flexible price level. This small sensitivity implies that in order to have a significant empirical impact on respectively price and wages, the estimated variance of those shocks need to be very high. It is interesting to note that in spite of the fact that we allow for measurement error in both wages and inflation, the estimation procedure attributes most of the short-run variation in those variables to the structural shocks.

One reason why it is interesting to analyse the response of the flexible price and wage economy to the various shocks is that several authors have argued that the output gap (defined as the deviation of actual output from its flexible price equilibrium) or the interest rate gap (defined as the deviation of the real interest rate from its flexible price equilibrium) could be useful indicators for monetary policy.[27] In order to do so, it is, however, important to take a firm stance on the nature of the structural shocks and, in particular, the "wage" and "price" shocks. If we interpret the "price" shock as coming from inefficient variations in the mark-up in the goods market (as we do in Section 2), then it makes sense to exclude those shocks from the calculation of the natural output and interest rate level. In a flexible price equilibrium, monetary policy will not be able to offset such "cost-push" shocks. However, in the sticky price and wage economy, monetary policy could be used to offset the effects of those shocks on output. In that case, those shocks will generate a trade-off between output gap stabilisation and inflation stabilisation. A similar reasoning holds for shocks to the wage equation. If we interpret this shock as a labour supply shock coming from changes in preferences (as we do in Section 2), then it makes sense to include the effects of those shocks in the calculation of the natural output level. As a result, those shocks will not give rise to a policy trade-off between wage and output gap stabilisation.

Of course, our identification of "price" shocks as inefficiencies and "wage" shocks as changing preferences is somewhat arbitrary. In reality, it is likely that a part of the "price" shocks are in fact technology shocks or terms-of-trade shocks that do have an impact on the natural output level, while a part of the "wage" shocks are due to inefficient changes in the market power of unions. We leave it for future research to try to distinguish between these shocks.

Graphs 17 and 18 plot our historical estimate of the natural output level and real interest rate under the assumption that the "price" shocks are inefficient "cost-push" shocks, while the "wage" shocks are labour supply shocks coming from changed preferences. In addition, we also plot those estimates under the assumption that the "wage" shocks are inefficient "wage-push" shocks.

Graph 17 shows that in accordance with the discussion above it is mostly the "supply" shocks that drive the natural level of output. If we exclude the "wage" shocks, the natural output level is quite smooth. Most movements in output can be explained by changes in the natural rate. Large gaps only appear during troughs and peaks. Adding the labour supply shocks leads to a much more volatile pattern of the natural level of output. Looking through the short-term deviations, the 1970s are mostly characterised by a positive output gap, while since the late 1980s the output gap is mostly negative. Graph 18 which depicts the interest rate gap and its components is less clear due to the large variability of the natural interest rate.

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[27] See, for example, Woodford (2000), Neiss and Nelson (2001) and Gali (2000).

Overall, this analysis underlines the practical difficulties with using either of the concepts as indicators for monetary policy. While they are theoretically appealing, both gaps very much depend on the identification of the shocks and the underlying model. Small changes in the interpretation of shocks can have large effects on the estimates. In particular, the interest rate gap seems to be very sensitive to this problem.

4.2 Welfare maximisation

In order to evaluate optimal monetary policy in the estimated sticky price and wage model, we follow Rotemberg and Woodford (1999), Woodford (1999), Erceg, Henderson and Levin (2000), Amato and Laubach (2000) and Steinsson (2000) by assuming that the monetary authorities maximise a quadratic approximation to the representative agent's utility function.

As shown in the appendix (to be completed), the quadratic welfare function can be written as follows:[28]

$$
\begin{aligned}
W_t - W_t^* \cong\ & -0.5U'C\{-\frac{1-\sigma_c}{1-h}\left[\hat{c}_t - h\hat{c}_{t-1} + \frac{\sigma_c(1-h)}{(\sigma_c-1)(1-\rho_b)}\tilde{\varepsilon}_t^b\right]^2 \\
& + \frac{(1-\alpha)(1+\sigma_l)}{c_y}\left[\hat{l}_t + \frac{1}{(1+\sigma_l)(1-\rho_b)}\tilde{\varepsilon}_t^b + \frac{1}{1+\sigma_l}\hat{\varepsilon}_t^l\right]^2 \\
& + \frac{\lambda_p}{c_y(1+\lambda_p)}\operatorname{var} y(j) \\
& + \frac{(1-\alpha)}{c_y}\left(\frac{\lambda_w}{1+\lambda_w} + \sigma_l\right)\operatorname{var} l(i)\}
\end{aligned}
\tag{43}
$$

with

$$
\begin{aligned}
\operatorname{var} y(j) =\ & \left(\frac{1+\lambda_p}{\lambda_p}\right)^2 E \operatorname{var}\log(p(j)) = \\
& \left(\frac{1+\lambda_p}{\lambda_p}\right)^2\left[\frac{(1-\xi_p\gamma_p)\xi_p}{(1-\xi_p)^2}\pi_p^2 + \frac{\xi_p^2\gamma_p^2 + (1-\xi_p)\xi_p\gamma_p}{(1-\xi_p)^2}\Delta\pi_p^2\right]
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
\operatorname{var} l(i) =\ & \left(\frac{1+\lambda_w}{\lambda_w}\right)^2 E \operatorname{var}\log(w(i)) = \\
& \left(\frac{1+\lambda_w}{\lambda_w}\right)^2\left[\begin{array}{l}\frac{\xi_w}{(1-\xi_w)^2}\pi_w^2 + \frac{\xi_w^2\gamma_w^2 + (1-\xi_w)\xi_w\gamma_w}{(1-\xi_w)^2}\pi_p^2 + \\ \frac{\xi_w^2\gamma_w^2 + (1-\xi_w)\xi_w\gamma_w}{(1-\xi_w)^2}\Delta\pi_p^2 - \frac{2\xi_w\gamma_w}{(1-\xi_w)^2}\pi_p\pi_w\end{array}\right]
\end{aligned}
\tag{45}
$$

---

[28] At this point, we have not been able to write the first two terms in terms of an appropriately defined output gap as was done in Erceg, Henderson and Levin (2000). The introduction of capital accumulation and investment implies that consumption is not equal to output and complicates such a calculation. Although we ignore first-order terms in (43), we also have not yet been able to show that these terms indeed cancel out as is the case in Woodford (1999). Below we show that the relative weight on the first two terms is small compared to that on the two dispersion terms, so that the results on optimal monetary policy would not be very much affected if we only concentrated on the latter terms.

where π_p and π_w denote respectively goods price and wage inflation. In deriving this welfare function we have assumed that there are subsidies so that the average mark-up in both goods and labour markets is zero.

The first two terms in the loss function (43) derive from the preferences of agents to reduce the variability in labour. The weight on these terms is, however, relatively small. The more important terms in the loss function are the last two terms (given by equations (44) and (45)) which result from the attempt of the monetary authorities to minimise the distortions due to an inefficient allocation of the production of differentiated goods or types of labour when prices and wage can not adjust instantaneously. As in Woodford (1999), the monetary authority would like to stabilise goods price inflation, in order to minimise the distortions in the goods market. However, due to introduction of indexation of non-optimised prices to past inflation, a second term in the change in inflation also enters expression (44). Note that when $\gamma_p = 0$, this term drops out. This results echos results obtained in Amato and Laubach (2000) and Steinsson (2000). Equation (45) derives a similar expression for the distortions in the labour market. As in Erceg, Henderson and Levin (2000), the main term is the stabilisation of wage inflation. However, because of the indexation of wages to past inflation, also inflation and the change in inflation and a covariance term enter the expression.

Overall, the expression for the loss function shows that the monetary authority may face many potential policy trade-offs. First, as in Clarida, Gali and Gertler (1999), "cost-push" shocks may create a trade-off between the stabilisation of employment and the stabilisation of inflation. However, as noted below the empirical relevance of this trade off appears relatively limited. Second, as in Erceg, Henderson and Levin (2000), the presence of both nominal price and wage rigidities creates a potential trade-off between the stabilisation of goods price inflation and wage inflation. Finally, the introduction of partial indexation may introduce an additional trade-off between the stabilisation of inflation and the change in inflation.

Table 3 gives the weights on the various terms, using the estimated values for the various parameters. It turns out that the weight on the terms related to labour market inefficiencies is larger than the weight on the terms related to the goods market inefficiencies (about 3 to 7). As discussed in Erceg, Henderson and Levin (2000) and Benigno (2000), one determinant of the relative weight is the relative degree of stickiness. Ceteris paribus, the weight on the more sticky sector should be larger. In our case this would imply a larger weight on the goods market. However, there is an additional difference due to the elasticity of the labour supply, which affects the losses from inefficiencies in the labour market, but not those from the goods market. As we have estimated the elasticity of the labour supply to be relatively small (or σ_l to be relatively large), sector-specific variations in labour demand are very costly. This explains why the weight on the labour market terms is more than twice as large than those on the goods market terms. However, because inflation and the change in inflation enter both terms, their weights nevertheless dominate those of wage inflation.

Finally, Table 3 also shows that the weight on the various inflation terms dominate those on employment and consumption. However, note that if one wants to write the loss function in terms of annualised inflation, the corresponding weights need to be divided by a factor of 16.

4.3 The response under optimal policy

Graphs A.3 to A.7 in the appendix give the impulse responses to the various structural shocks when the monetary authorities minimise loss function (42) augmented with a small weight on interest rate variability under respectively commitment and discretion. For the discussion here, it is useful to look at Graph 19 to 21 which compare the response of output, inflation and the real interest rate under three different monetary policy regimes (the historical reaction function, optimal policy under commitment and optimal policy under discretion) with the flexible price outcome.

A number of observations are worth making. First, comparing the responses to the "supply" and "demand" shocks under commitment with those under the estimated policy reaction function and the flexible price outcome, it is clear that the interest rate response under commitment is much stronger than the historical one. Overall, the outcome under commitment is closer to the flexible price outcome. This would be even more the case, if the weight on interest rate stabilisation was further reduced.

Second, the value of commitment in responding to the "supply" and "demand" shock primarily lies in the fact that the central bank is able to achieve an outcome that is closer to the flexible price outcome with an interest rate path that is initially less pronouned, but more persistent. The stabilisation bias under discretion is clear from the graphs. For example, in response to a productivity shock, both output and inflation are closer to the flexible price outcome under commitment versus discretion, while the nominal and real interest rate move more. It is interesting to note that under the estimated historical rule, inflation is much better behaved following a productivity shock than under discretion. This is probably due to the implicit assumption in the former case that the central bank can commit to the estimated rule.

Third, the response of the economy to the cost-push shock is relatively similar under commitment than under the historically estimated rule.

Finally, it is also interesting to examine the counterfactual outcome in terms of output, inflation and interest rates that would have been achieved if the monetary authorities had followed the optimal policy under commitment. The results are summarised in Graphs 22 and 23. Three observations are worth making. First, under the optimal policy the level of output would have been quite different than the one actually observed. It is striking that the counterfactual output path seems to follow quite closely a smoothed version of the natural output level depicted in Graph 17. Second, with the exception of the first oil crisis in 1974, counterfactual inflation appears to be quite well anchored around the average inflation rate over the period. The short-run volatility in inflation reflects primarily the impact effect of the "cost-push" shocks. Finally, surprisingly the nominal interest rate is not all that different from the historically observed one. This similar path for the nominal interest rate implies, however, a quite different path for the ex-ante real interest rate, given the very different behaviour in inflation and inflation expectations.

5. Conclusions

Recently a new generation of small-scale monetary business cycle models generally referred to as New-Keynesian or New Neoclassical Synthesis models have been developed (Goodfriend and King (1997), Rotemberg and Woodford (1997) and Clarida, Gali and Gertler (1999)). Gali (2000) highlights some of the new findings, ideas or features of these models relative to the traditional Keynesian literature. The monetary SDGE model used in this paper shares the essential features of this class of models (in particular the sticky, but forward-looking price setting). Following Christiano, Eichenbaum and Evans (2001), the model used in this paper also features a relatively large number of additional frictions that are necessary to capture the empirical persistence and covariances in the main macro-economic data of the euro area. These frictions include sticky, but forward-looking nominal wage setting, variable capital utilisation, adjustment costs in capital accumulation and habit formation in consumption. Finally, the model also includes a full set of structural shocks -- two "supply" shocks (a productivity and labour supply shock), two "demand" shocks (a preference and cost-of-capital shock), a shock to the price mark-up and two monetary policy shocks --, to account for the stochastics in the empirical data. These extensions of the canonical two-equation model allow us to (i) estimate the model using the main euro area macro data on output, inflation, real wages, investment, consumption, the short-term interest rate and employment; (ii) to examine the sources of business cycle dynamics in the euro area; and (iii) to analyse some of the new features of this class of models, highlighted by Gali (2000), in an empirically plausible set-up. Regarding the latter, it is worth recalling what we have learned from performing this exercise.

The forward-looking behaviour of inflation. The parameter estimates in this paper suggest that there is a considerable degree of price and wage stickiness in the euro area. As a result, prices respond only slowly to changes in expected marginal costs, while wages adjust only slowly to deviations from their efficient levels. Both price and wage inflation also depend to some extent on past inflation which introduces a backward-looking component. Nevertheless, the forward-looking component clearly dominates, in particular in the price setting equation. Despite the forward-looking nature of inflation, our model has no problems with capturing the empirical fact that changes in output lead changes in inflation. This is mainly a result of the fact that marginal costs respond only sluggishly to changes in output (Gali and Gertler (2000)).

The concept of the output gap (and interest rate gap). In the canonical model of Woodford (1999), the concept of the output gap – defined as the deviation of actual output from its flexible price and wage equilibrium value – plays a central role, both as a force driving underlying developments in inflation (through its effect on marginal cost) and as a policy target. A similar role can also be assigned to the real interest rate gap (Neiss and Nelson (2000), Woodford (2000)). In our estimated model which features a larger number of shocks arising from both technologies and preferences and inefficiencies, it is less clear what the appropriate output gap is from a monetary policy perspective. Clearly, all "non-monetary" shocks will potentially affect output and the real rate in a flexible price and wage economy. We show that due to the sluggishness in the two demand components, the natural real interest rate rises significantly in response to positive "demand" shocks, while it falls in response to positive "supply" shocks. In contrast, the natural output level only responds significantly to the "supply" shocks due to a low estimated coefficient of risk aversion. The highest volatility in output and interest rates in a flexible price economy would, however, be due to the "cost-push" shocks, which we assume to arise from inefficient variations in the mark-up. In a sticky price and wage economy monetary policy could try to offset the effects on output of shocks that arise from inefficiencies. It could therefore be argued that the appropriate target level of output for monetary policy should only take into account that part of the natural level of output that is driven by shocks arising from preferences and technologies. As in the limit all the shocks that we estimate could be interpreted as arising from either preferences and technologies or inefficiencies, this raises an important identification problem for empirical estimates of the appropriate natural level of output or the natural real interest rate. In this paper we illustrate this problem by calculating two different estimates of these concepts based on different assumptions regarding the nature of the labour supply shock. We show that these two estimates differ dramatically.

The transmission of monetary policy shocks and the liquidity effect. Our estimates of the effects of a *temporary* monetary policy shock are very much in line with the existing evidence for the euro area (e.g. Peersman and Smets (2000)). It leads to a rise in the nominal and real interest rate, a hump-shaped fall in output, consumption and investment with the latter responding significantly stronger and a gradual fall in marginal costs and prices. However, the effects of a *persistent* monetary policy shock are strikingly different in two respects. First, in line with the arguments made in Gali (2000) there is no liquidity effect as the fall in the nominal component outweighs the rise in the real component of the short-term interest rate. Second, because the change in policy is credible and implemented gradually, expectations have time to adjust and the output effects are much smaller. These findings underline the importance of forward-looking pricing behaviour and the persistence of the shocks for assessing the effects of monetary policy changes.

The transmission of non-monetary shocks. Gali (1999) emphasised that in models with sticky prices, unless monetary policy is sufficiently accommodating, employment is likely to drop in the short run in response to a favourable productivity shock. Our estimates of the effect of a positive productivity shock confirm this significant negative effect on employment under the estimated policy reaction function. It is worth noting that due to the low estimated labour supply elasticity, productivity shocks have a negligible effect on employment even in the flexible price and wage economy. Gali (2000) also conjectured that the empirical procyclicality of employment raised serious doubts about the quantitative significance of productivity shocks as a source of aggregate fluctuations. Our results indeed suggest that, in contrast to many identified VAR studies, the productivity shocks only account for 10 percent of the long-run output variance. Instead, preference shocks are the most important source of variation in output at all horizons. Although this shock would have no significant impact on output in the flexible price and wage economy, it does have strong effects on output in the sticky price and wage economy as long as monetary policy does not respond strong enough.

A utility based welfare analysis of monetary policies. One of the big advantages of the monetary SDGE models is that alternative monetary policies can be evaluated using the representative agents' utility as a welfare measure. In this paper we follow Rotemberg and Woodford (1997), Erceg, Henderson and Levin (2000) and present some preliminary results deriving a quadratic approximation to the utility of the representative consumer. The resulting trade-offs are quite complicated. First, because of the presence of quantitatively important "cost-push" shocks, there is a trade-off between output and inflation stabilisation as in the canonical New-Keynesian model of Clarida, Gali and Gertler (1999). Second, as in Erceg, Henderson and Levin (2000), there is a trade-off between wage, inflation and output stabilisation due to the presence of both price and wage rigidities. Finally, because of imperfect indexation, also changes in inflation will play a role in the objective function of the central bank (See Amato and Laubach (2000) and Steinsson (2000)). As in other papers we find that the dominant weight in the calibrated loss function based on the estimated parameters is on price and wage inflation. Using this objective function we find that the optimal policy response to the "supply" and "demand" shocks is much stronger than the estimated policy reaction function, but not so strong as to replicate the flexible price outcome. In contrast, the response to the "cost-push" shock under the optimal policy is quite similar to the one under the estimated policy reaction function. One implication of the problems with identifying the exact nature of the structural shocks and their effect on the appropriate output gap for monetary policy purposes is that it may be better to focus solely on the stabilisation of price and wage inflation, in particular as the weights on these components in the loss function dominate the weights on output and employment stabilisation.

Gains from commitment. The presence of a trade-off between price and wage inflation does create a value of commitment. In general, we find that as in the canonical model of Clarida, Gali and Gertler (1999), the optimal policy under commitment does involve a degree of mean reversion in the price level which is absent when the monetary authorities optimise under discretion.

Overall, the preliminary results presented in this paper show that an estimated version of the monetary SDGE model with sticky prices and wages can be used for monetary policy analysis in an empirically plausible set-up. At the same time, the analysis in this paper is still preliminary in many respects and needs to be further improved in a number of dimensions.

First, in order to estimate the model's parameters we have used a constrained maximum likelihood procedure, whereby the usual likelihood criterion based on a Kalman filter is augmented by a condition that the model-based cross-covariances can not be too different from their empirical counterparts estimated using an unrestricted VAR. One of the main reasons for adding the constraint on the cross-covariances is that it improves the robustness of the estimation procedure. Further work is necessary to examine the statistical properties of this estimator. In particular, we need to derive test statistics to examine the significance of the various parameters, the specification of the model and possibly its stability. In addition, when estimating the model, we have implicitly assumed that the agents in the economy have perfect information regarding the shocks hitting the economy. A more realistic assumption would be to estimate the model under the assumption that those agents (like the econometrician) only observe the observable variables. An interesting question is then to what extent imperfect information regarding the nature of the monetary policy shocks could account for the empirical persistence in the inflation process (as, for example, in Erceg and Levin (2000)).

Second, the robustness of the estimation results to various perturbations in the structure of the model needs to be examined. As in CEE (2001), it would be interesting to see which of the various frictions are crucial for capturing the persistence and covariances in the data. Even more important in our set-up is a further examination and identification of the various structural shocks. As discussed above, the identification of some of the shocks as arising from changing technologies and preferences and others from inefficiencies is largely arbitrary. In reality it is very likely that those shocks are a mixture of both. For example, the cost-of-capital shocks could partly arise from a stochastic external finance premium and inefficiencies in capital markets and partly from stochastic variations in the depreciation rate of newly installed capital, a technological factor. Although it will be very difficult to empirically distinguish between the various types of shocks without using additional information, a correct identification may be very important for defining the appropriate output gap and optimal monetary policy.

Third, in this paper we have only started to think about optimal monetary policy within the estimated model. A deeper analysis of the welfare function and the various trade-offs faced by the monetary authorities would be very welcome. The basic intuition that monetary authorities want to stabilise price and wage inflation to avoid relative price and wage distortions is, however, very strong.

Appendix

[to be completed]

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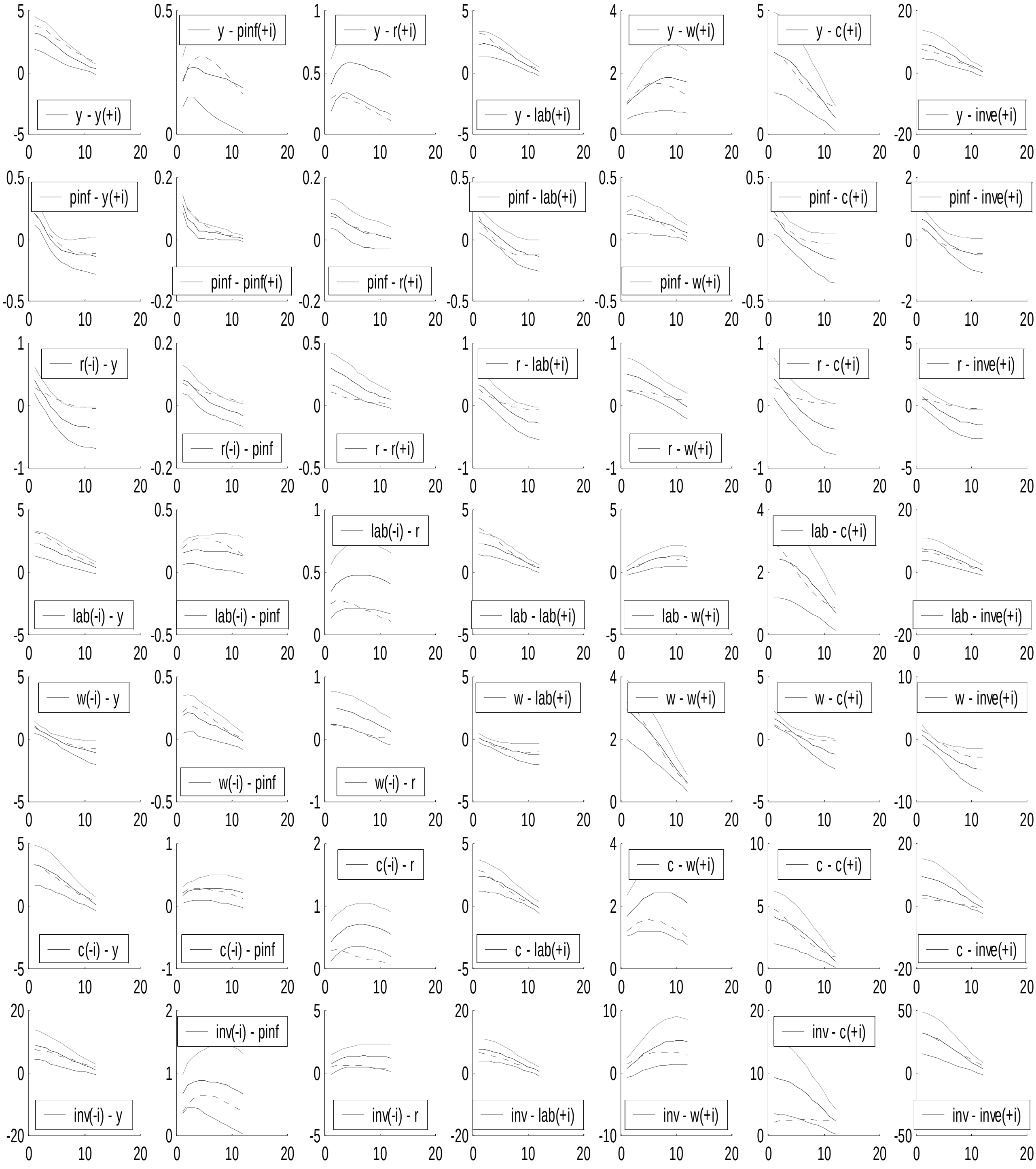
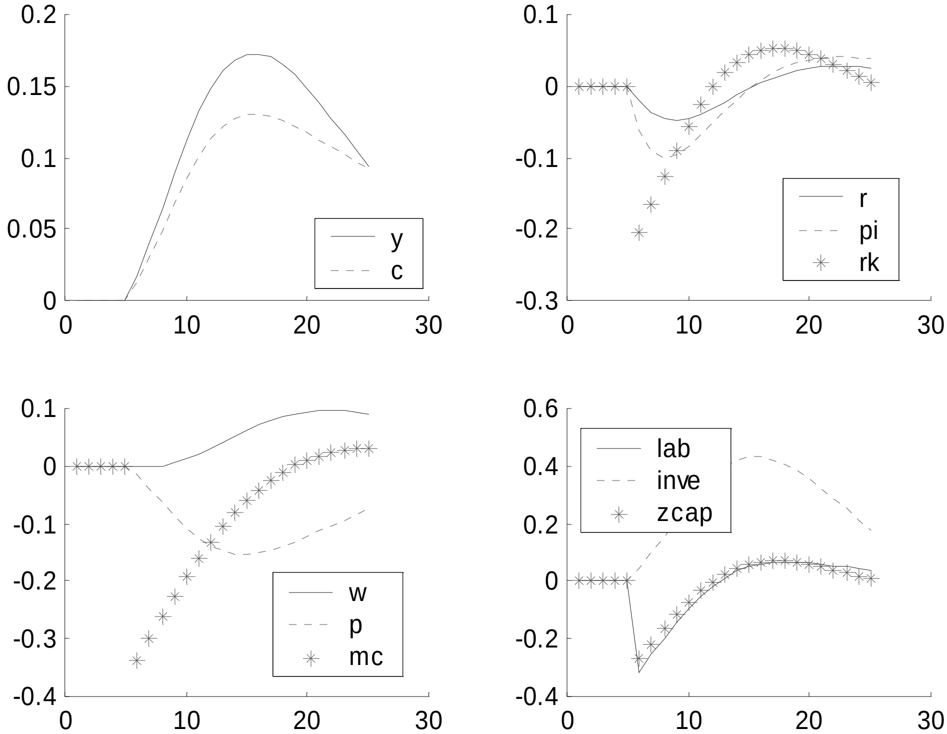
GRAPH 1 : Cross covariance functions

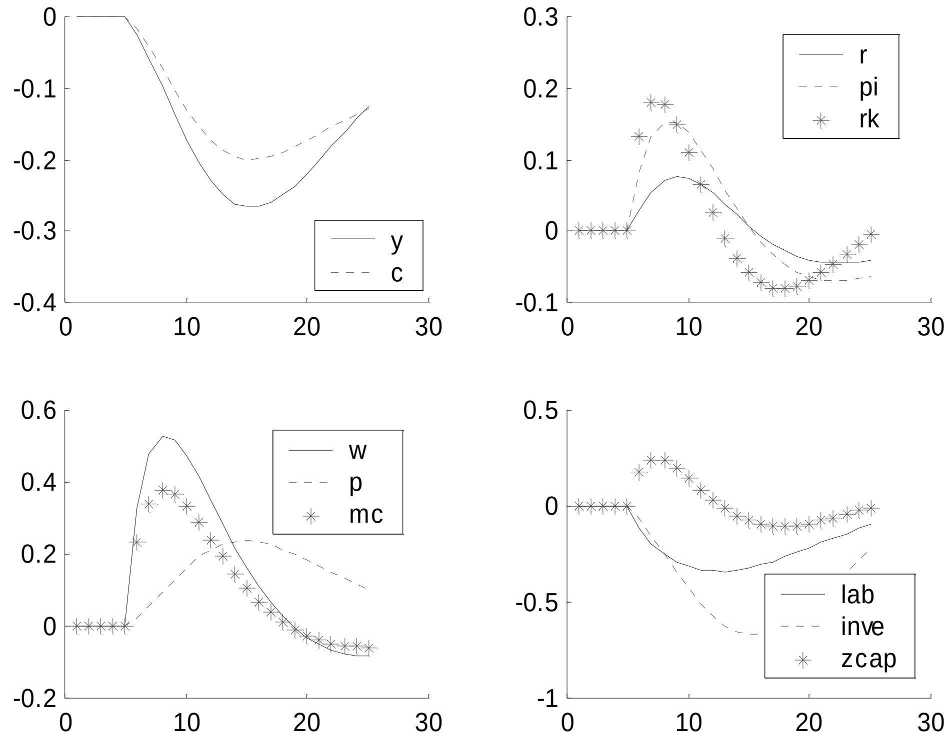
(straight lines = VAR +/- one s.e.; dotted line = Estimated theoretical model)



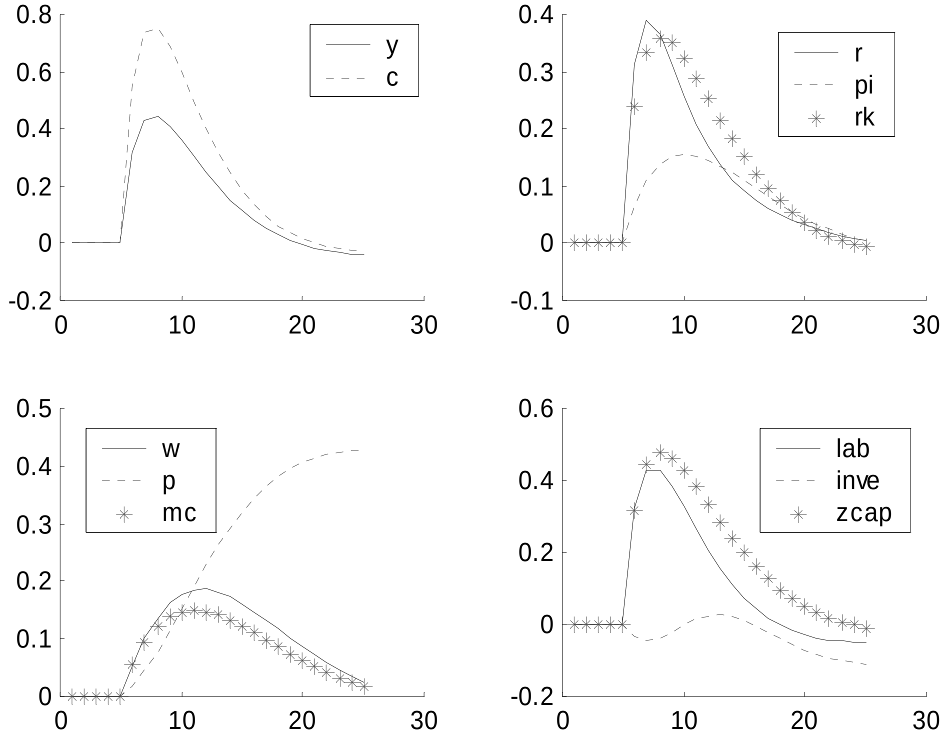
GRAPH 2 : Productivity shock



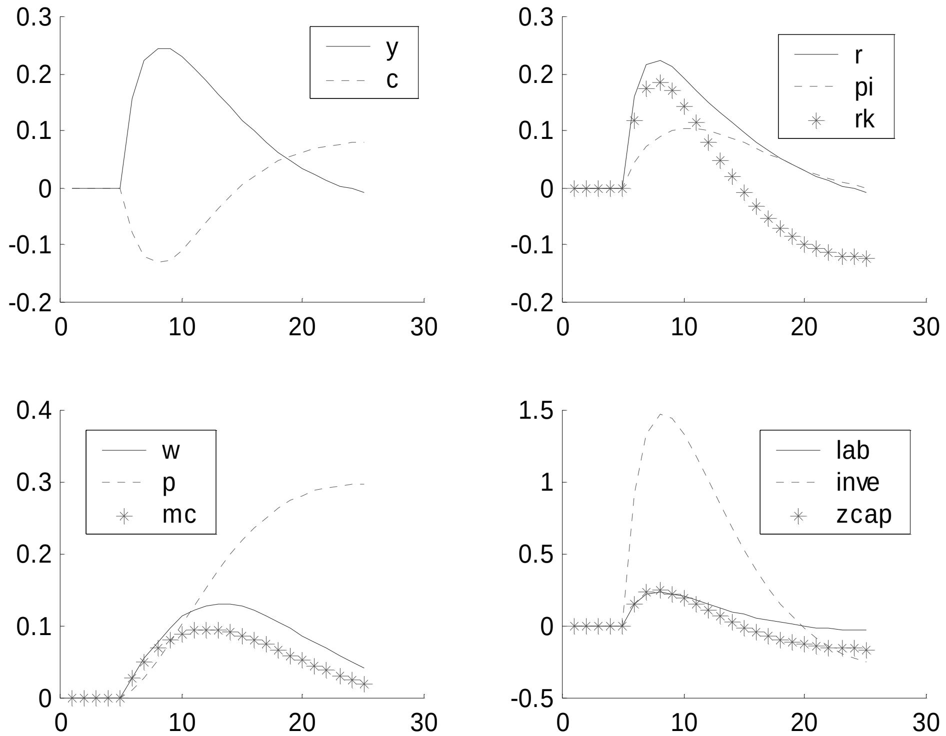
GRAPH 3 : Labour supply shock



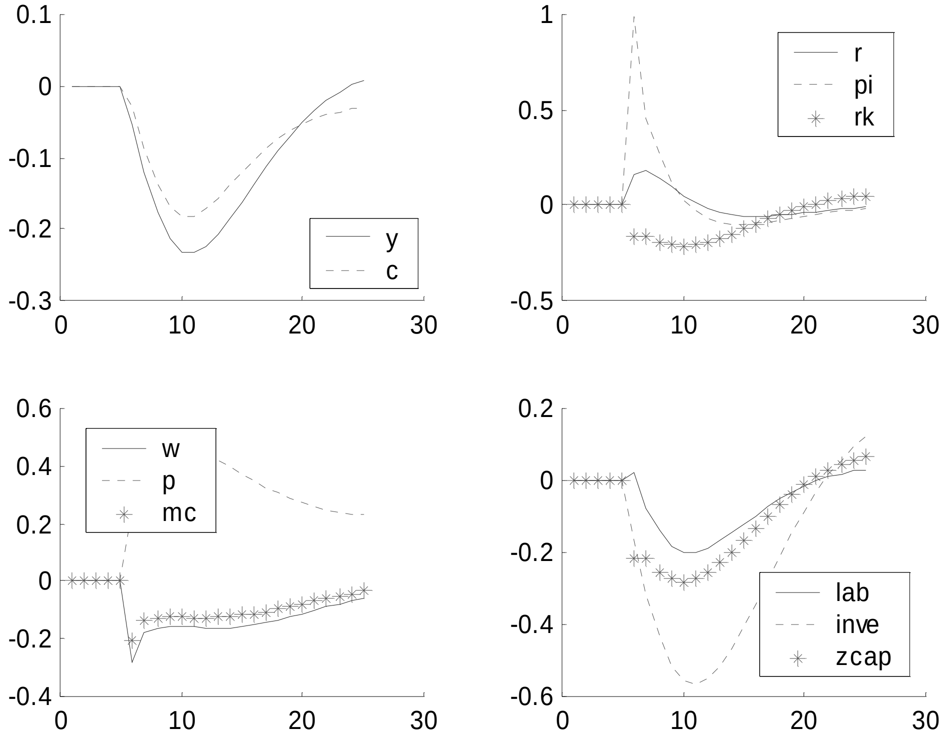
GRAPH 4 : Preference shock



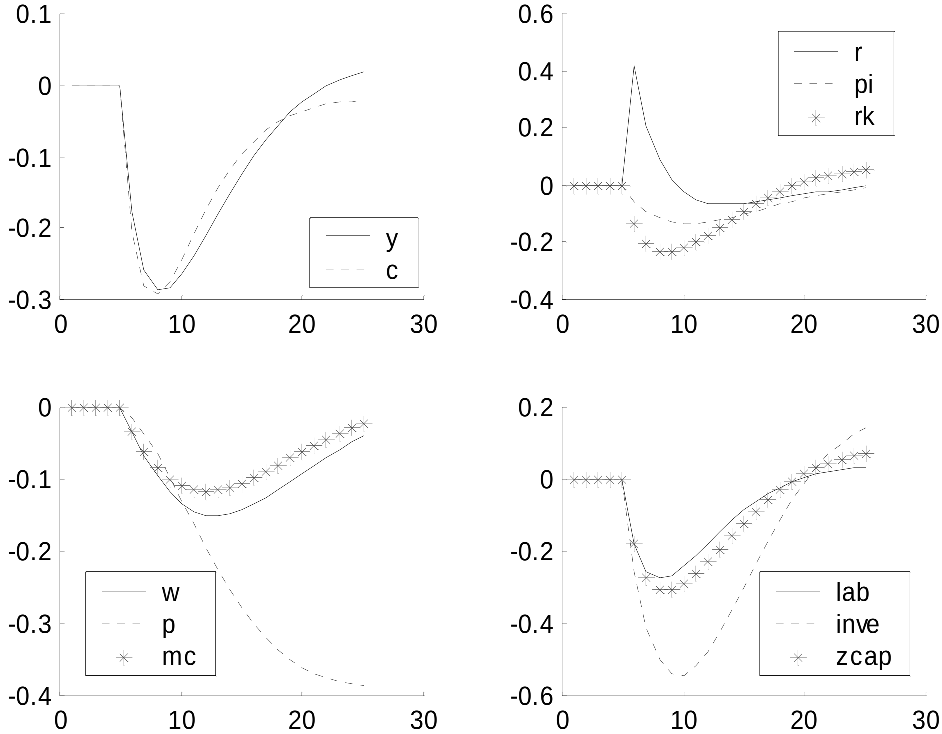
GRAPH 5 : Investment shock



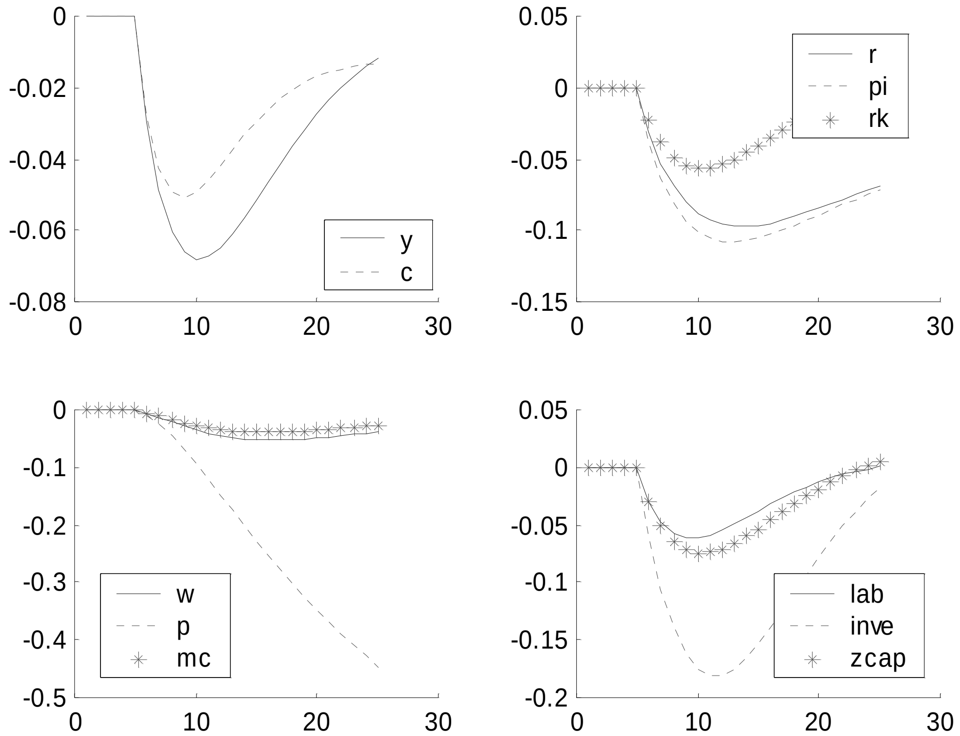
GRAPH 6 : Price mark-up shock



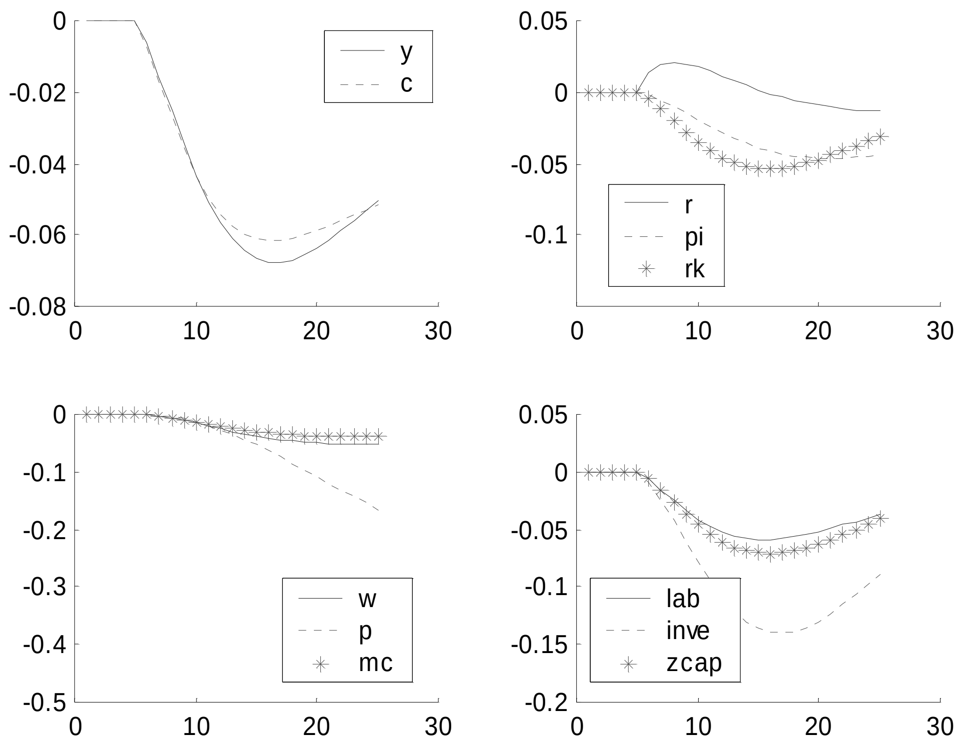
GRAPH 7 : Interest rate shock



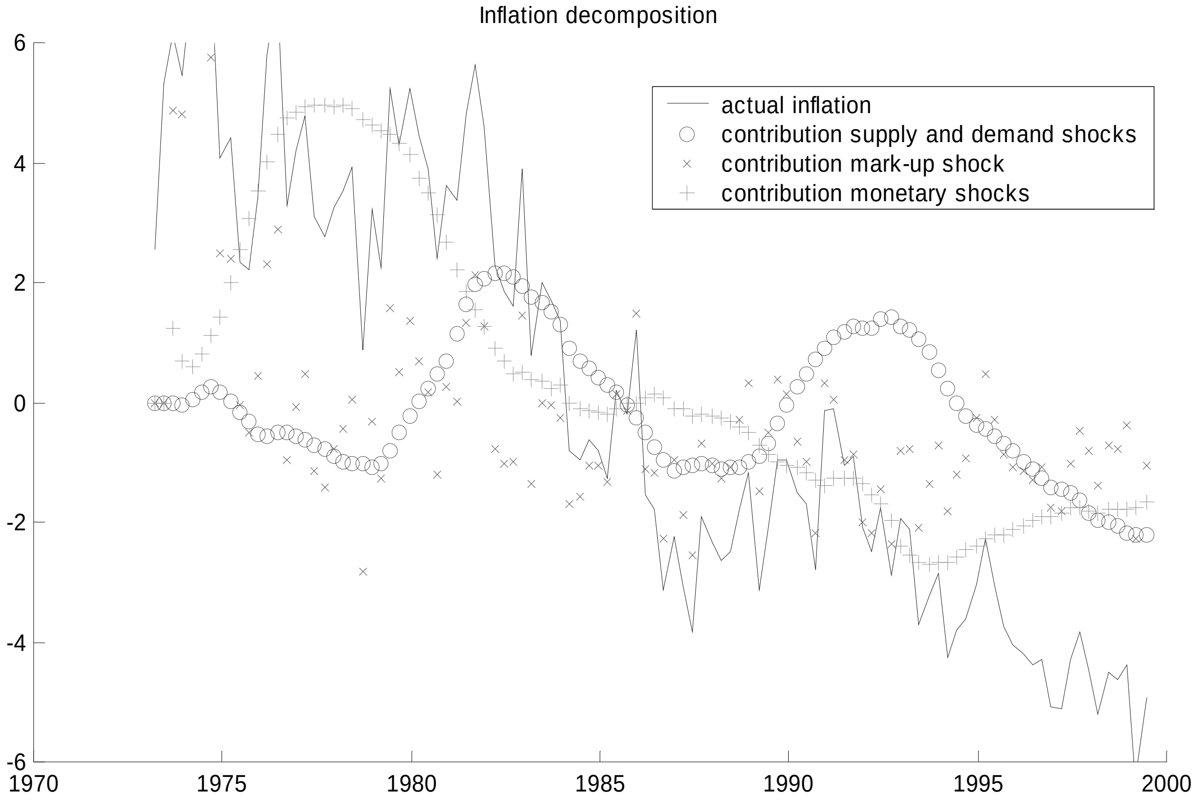
GRAPH 8 : Inflation objective shock



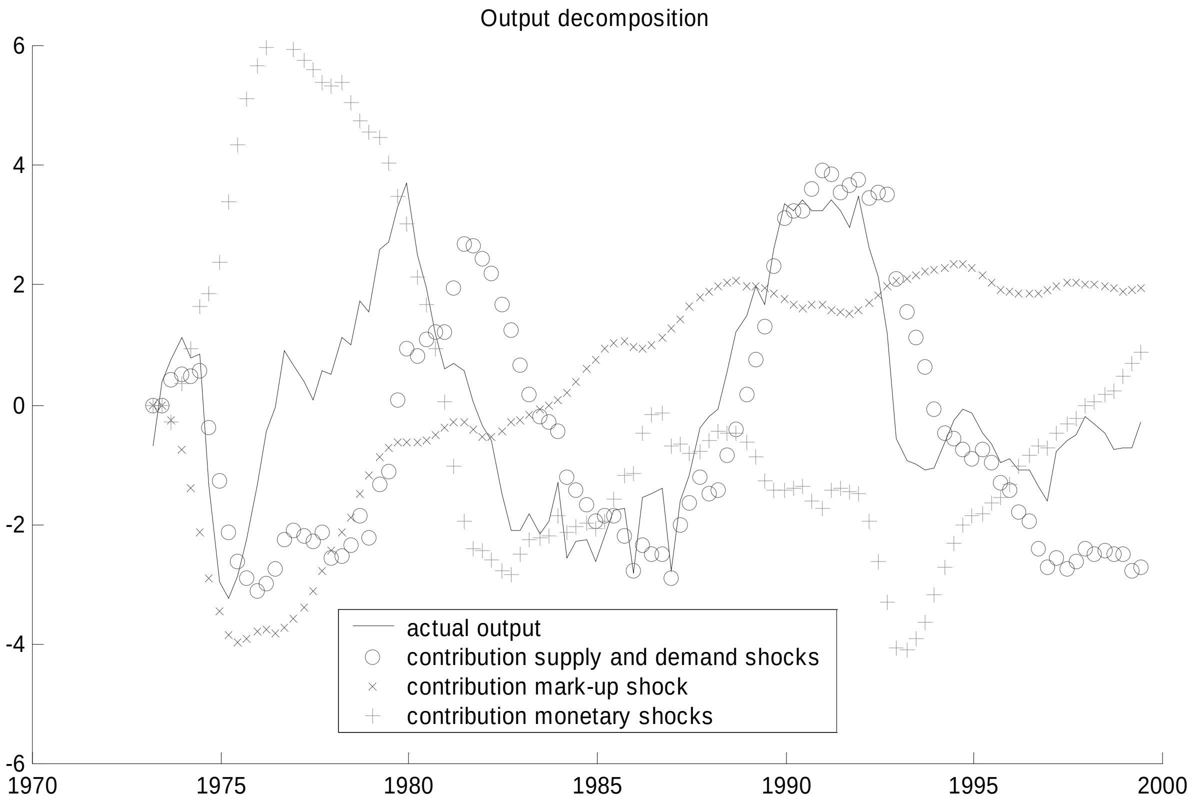
GRAPH 9 : Inflation objective shock under uncertainty



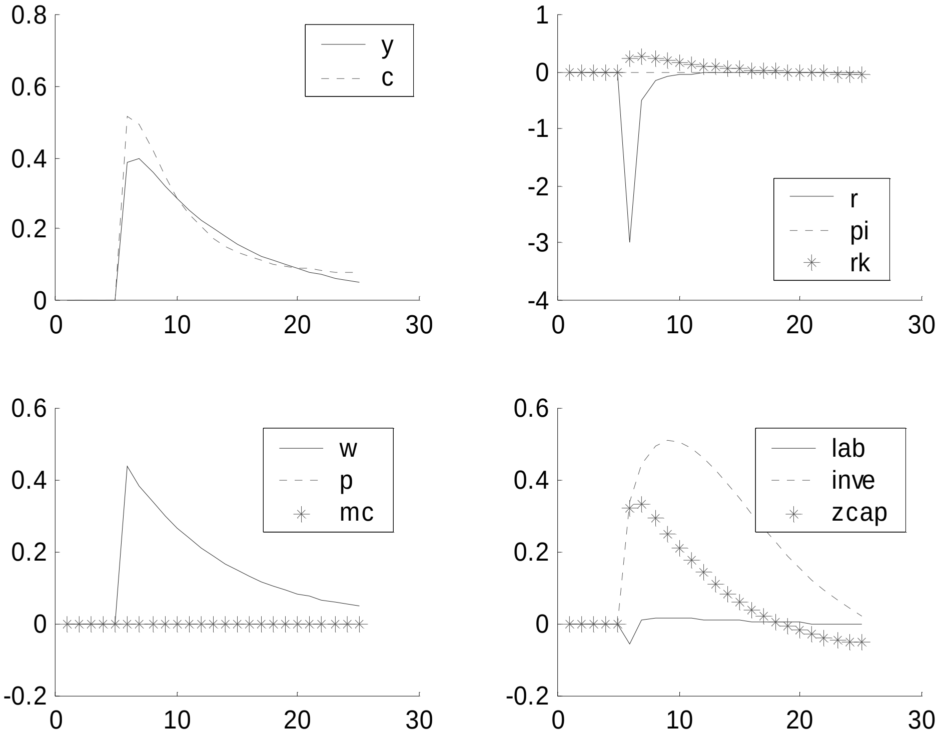
GRAPH 10 : Historical decomposition of inflation



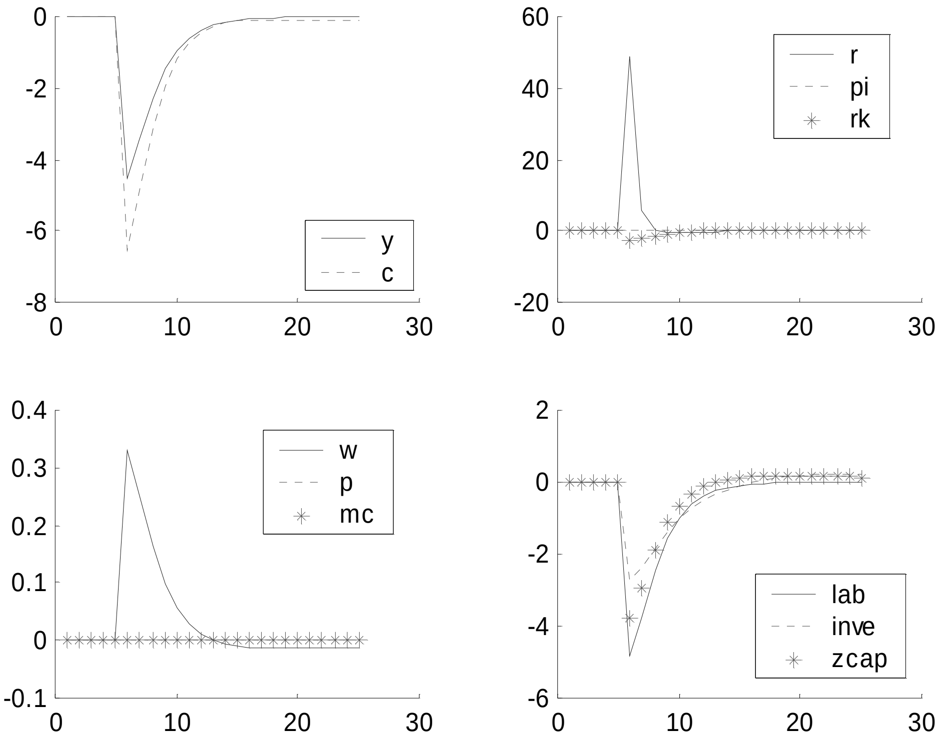
GRAPH 11 : Historical decomposition of output



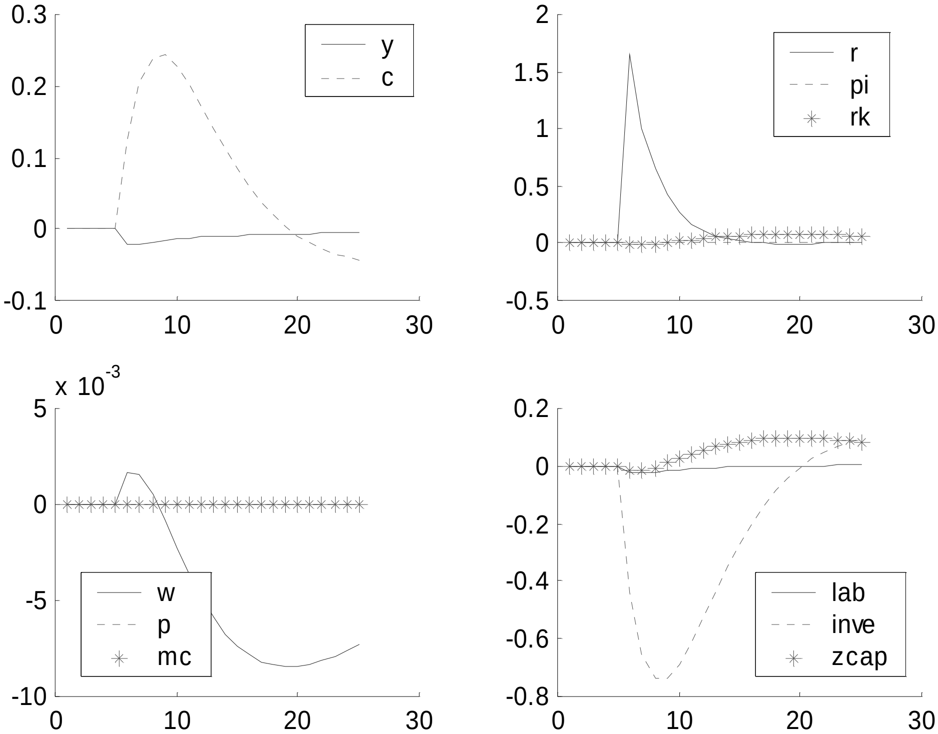
GRAPH 12 : Productivity shock in the flexible price-wage model



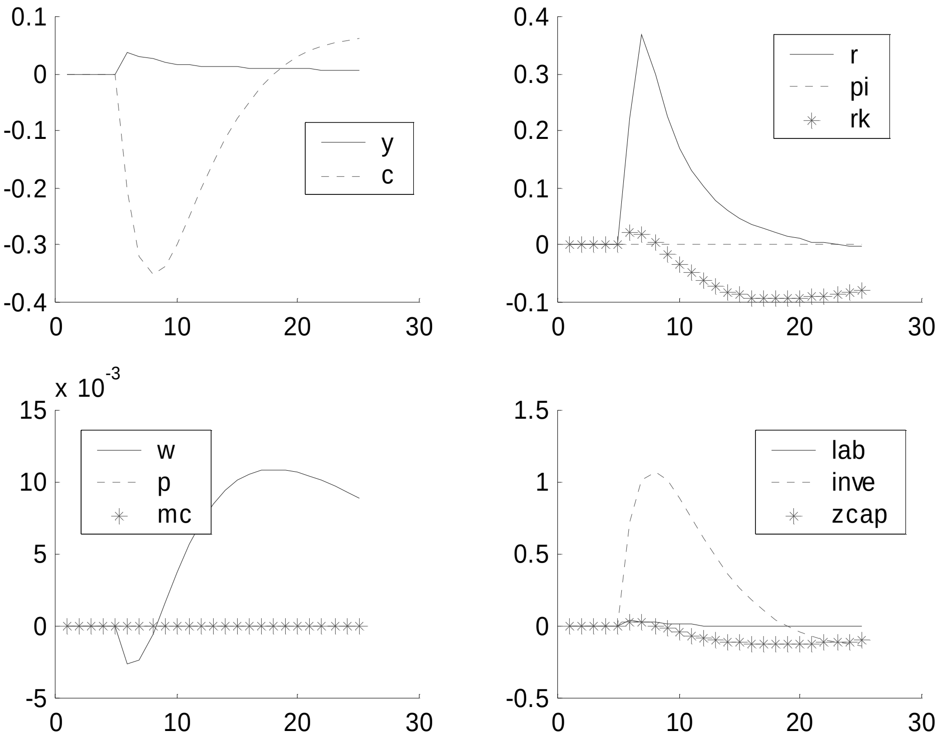
GRAPH 13 : Labour supply shock in the flexible price-wage model



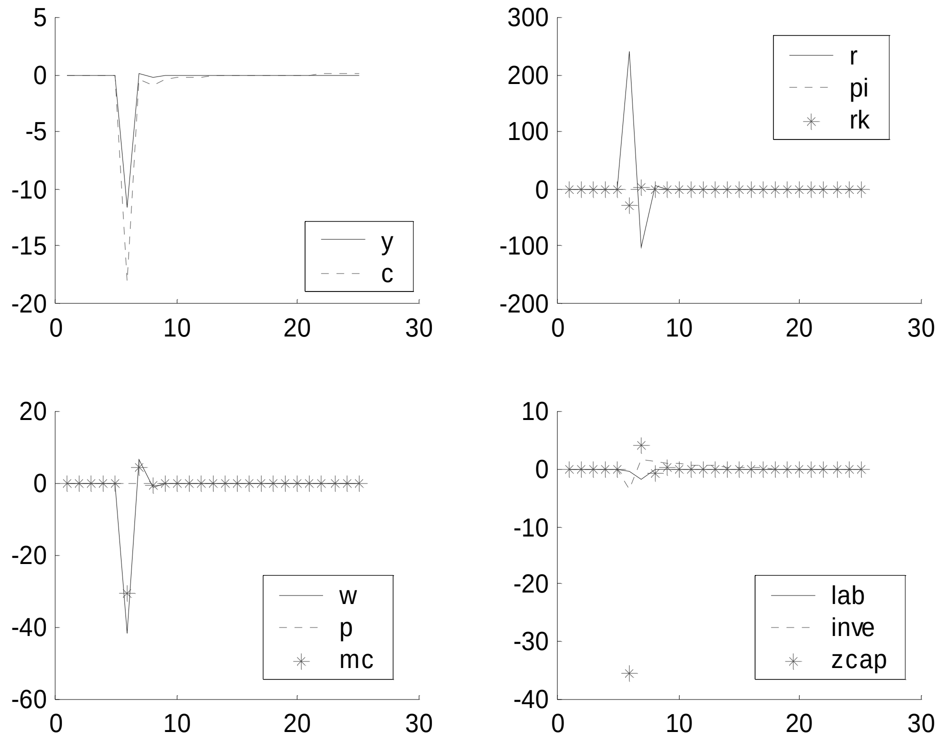
GRAPH 14 : Preference shock in the flexible price-wage model



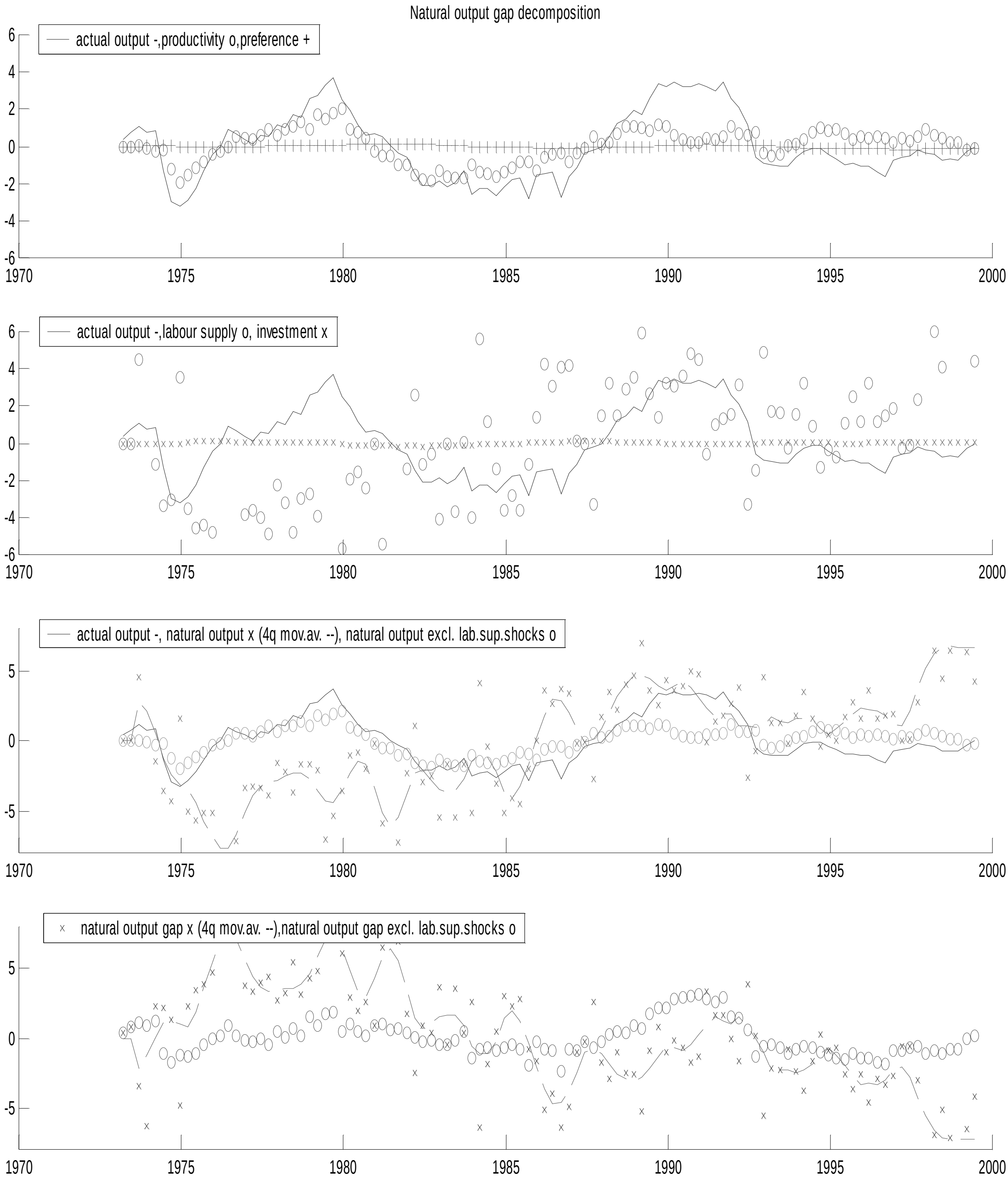
GRAPH 15 : Investment shock in the flexible price-wage model



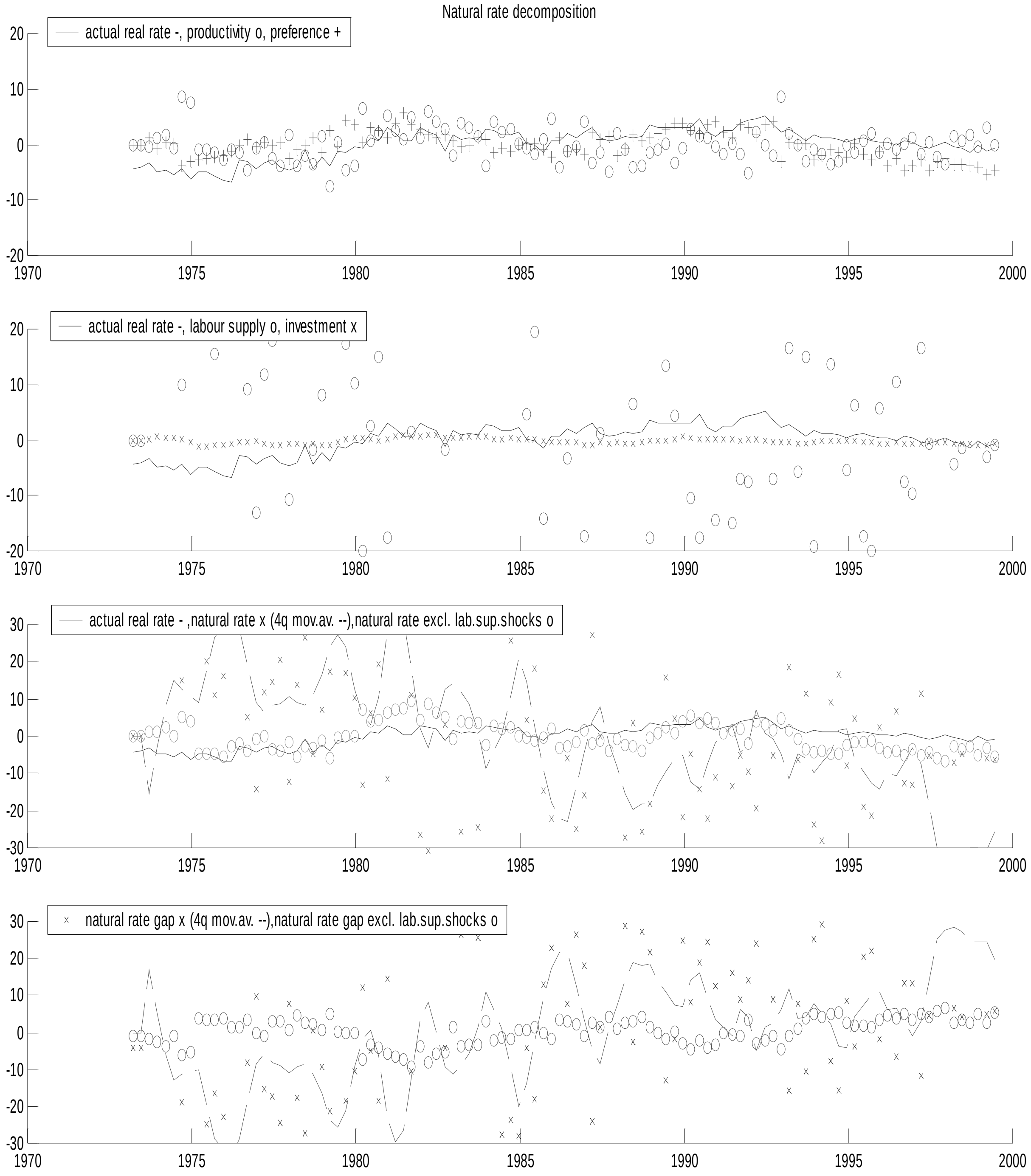
GRAPH 16 : Price mark-up shock in the flexible price-wage model



GRAPH 17: Natural output decomposition and output-gap in the flexible economy

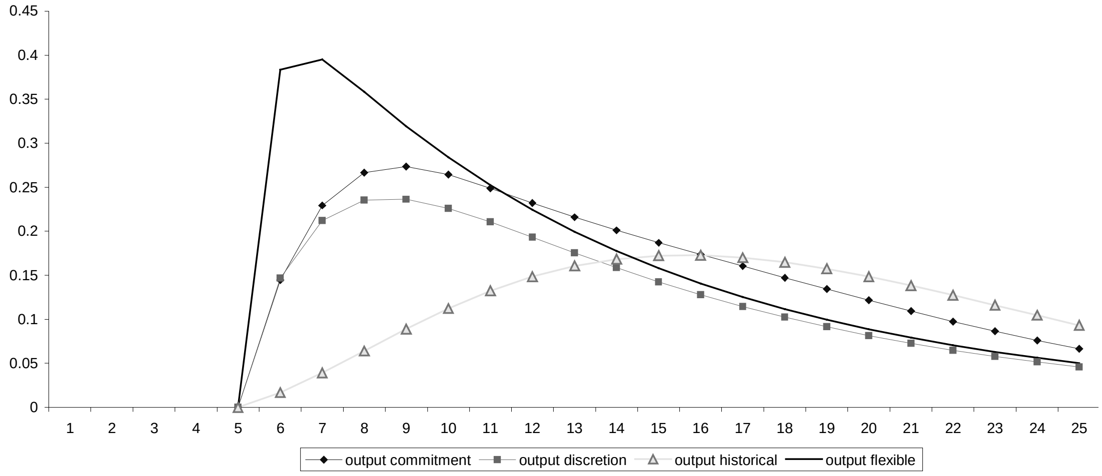


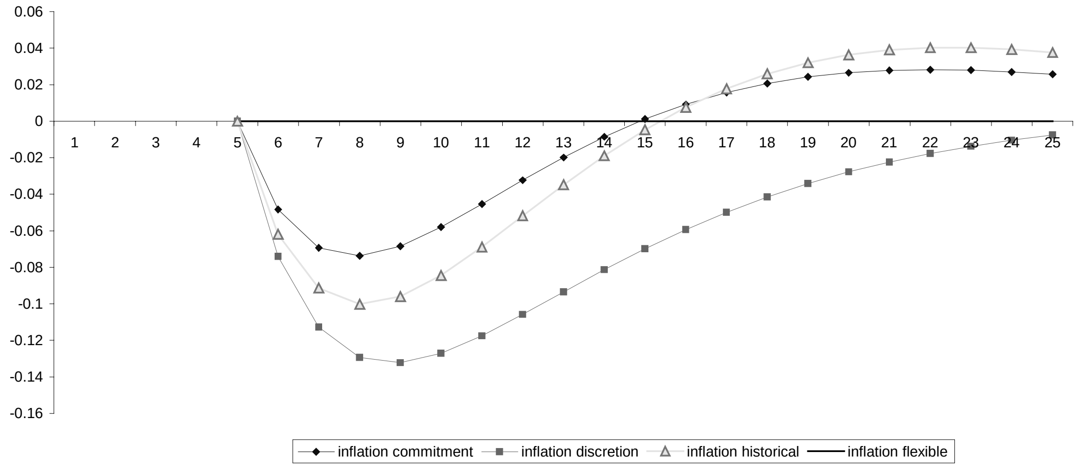
GRAPH 18 : Natural real rate decomposition and rate-gap in the flexible economy

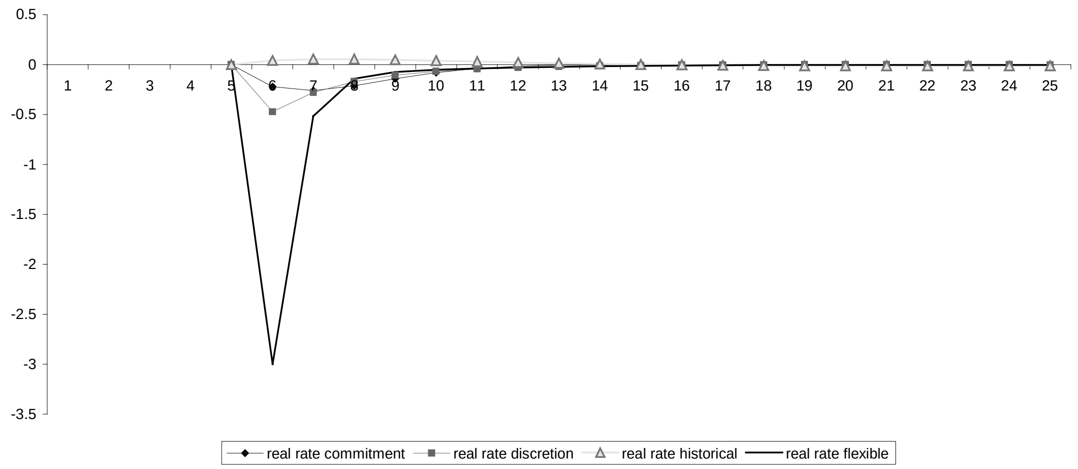


GRAPH 19 : Output, inflation and real rate for a productivity shock

Sticky price-wage model with alternative monetary policies versus flexible model

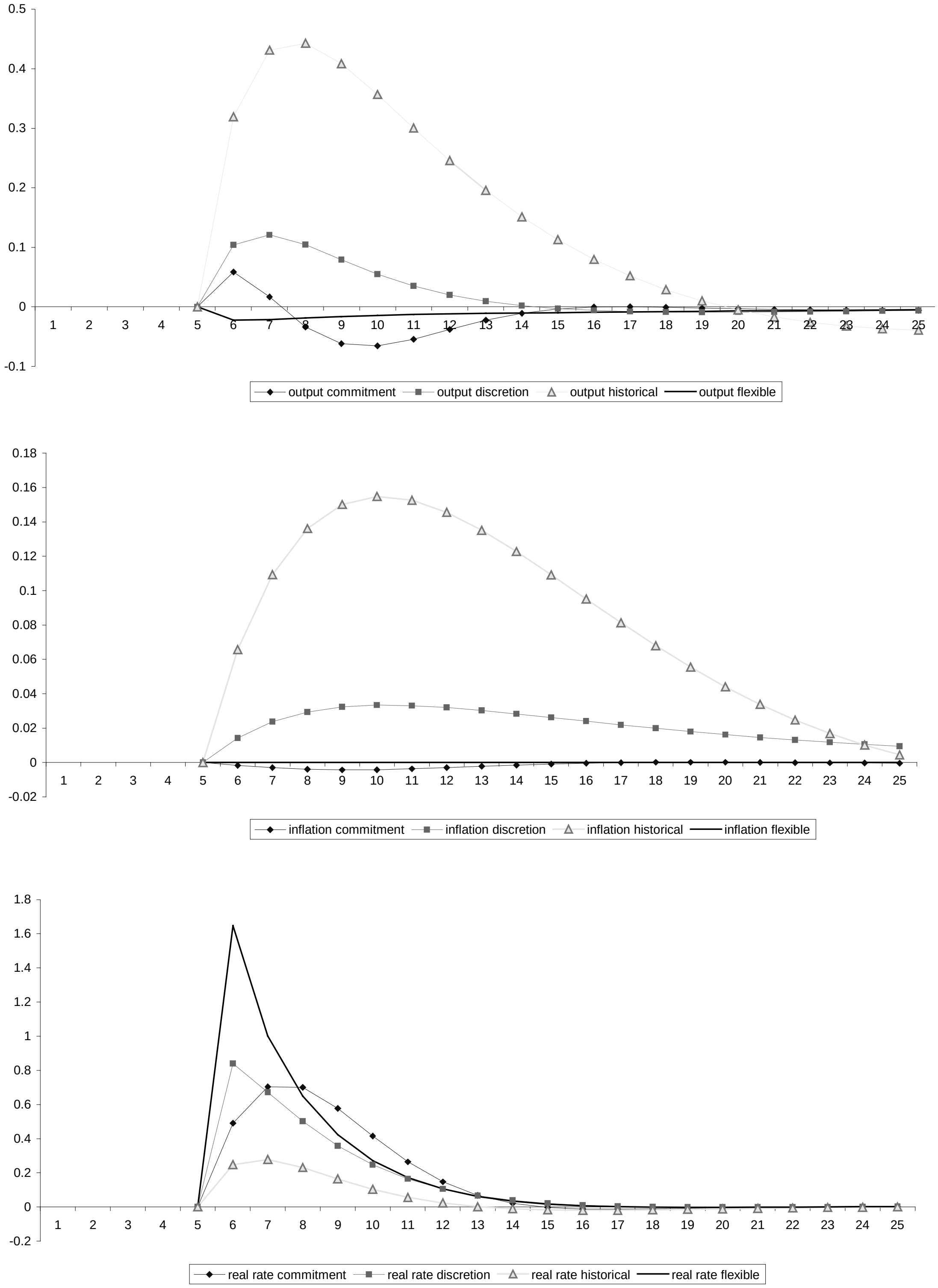






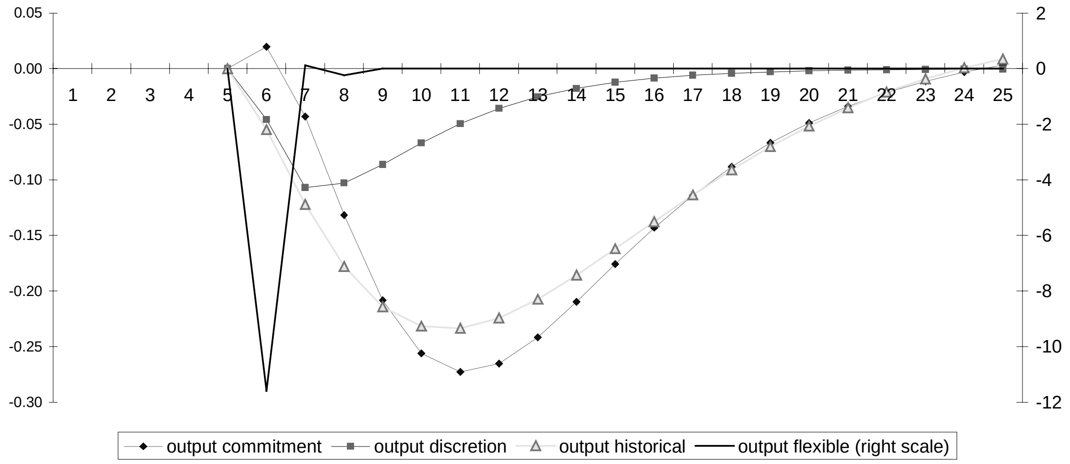
GRAPH 20 : Output, inflation and real rate for a preference shock

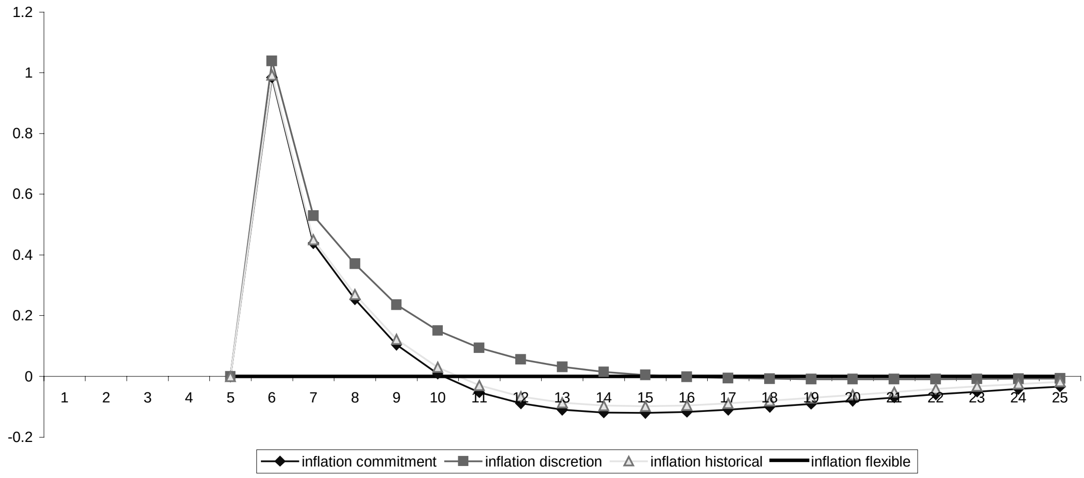
Sticky price-wage model with alternative monetary policies versus flexible model

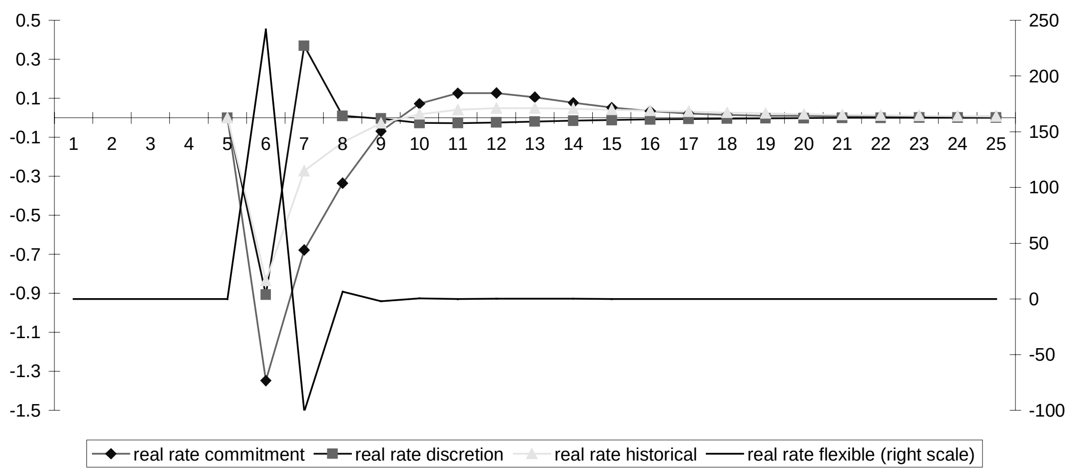


GRAPH 21 : Output, inflation and real rate for a price mark-up shock

Sticky price-wage model with alternative monetary policies versus flexible model







GRAPH 22 : Actual versus optimal policy outcomes

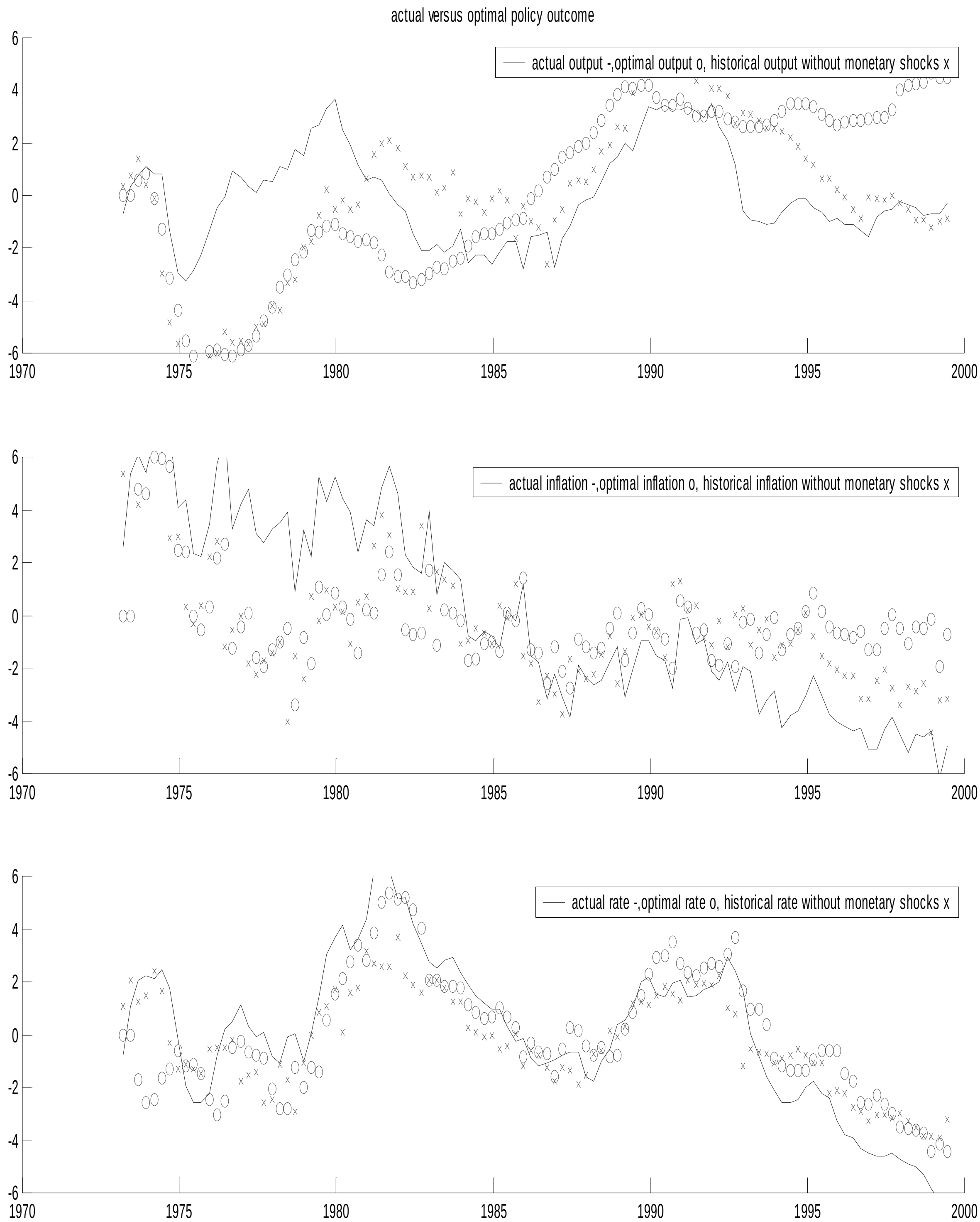


Table 1

Parameter estimates

Imposed parameters		Estimated parameters		Autoregressive parameters		Variances	
β	0.99	ξ_P	0.90	ρ_{ε^a}	0.88	$\sigma^2_{\varepsilon^a}$	0.103
τ	0.025	ξ_W	0.76	ρ_{ε^b}	0.69	$\sigma^2_{\varepsilon^b}$	0.177
α	0.35	γ_P	0.68	ρ_g	0.0^I	σ^2_g	0.000^I
s_c	0.60	σ_c	0.63	ρ_{ε^L}	0.62	$\sigma^2_{\varepsilon^L}$	0.009
s_I	0.22	h	0.68	ρ_{λ_p}	-0.15	$\sigma^2_{\lambda_p}$	0.034
$1/\psi$	0.2	φ	4.40	ρ_{ε^K}	0.59	$\sigma^2_{\varepsilon^K}$	1.049
γ_ω	1.0	σ_L	10.44	ρ_π	0.975^I	$\sigma^2_{\bar{\pi}}$	0.001
λ_P	0.2	r_π	1.71	ρ_{ε^R}	0.0^I	$\sigma^2_{\varepsilon^R}$	0.023
λ_W	0.7	r_r	0.76	ρ_Y	0.72	σ^2_Y	0.079
		$r_{\Delta\pi^e}$	0.36	ρ_I	0.90	σ^2_I	0.756
		$r_{\Delta y}$	0.21	ρ_W	0.29	σ^2_W	0.130

Note: I = imposed

Table 2
Historical variance decomposition

		C	I	Y	L	π	W	R
t = 0	productivity shock	0.00	0.00	0.00	0.37	0.00	0.00	0.00
	inflation objective shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	preference shock	0.86	0.00	0.44	0.37	0.00	0.01	0.30
	labour supply shock	0.00	0.00	0.00	0.05	0.01	0.33	0.00
	Investment shock	0.02	0.50	0.10	0.09	0.00	0.00	0.08
	price markup shock	0.00	0.02	0.01	0.00	0.98	0.25	0.07
	interest rate shock	0.12	0.04	0.14	0.11	0.00	0.00	0.54
	observation error	0.00	0.44	0.30	0.00	0.00	0.41	0.00
	total forecast variance	0.35	1.71	0.23	0.28	0.06	0.32	0.02
t = 4	productivity shock	0.01	0.01	0.02	0.13	0.02	0.00	0.01
	inflation objective shock	0.00	0.01	0.01	0.01	0.02	0.00	0.02
	preference shock	0.80	0.00	0.44	0.38	0.05	0.06	0.49
	labour supply shock	0.01	0.03	0.04	0.16	0.06	0.69	0.02
	Investment shock	0.02	0.63	0.14	0.12	0.02	0.02	0.18
	price markup shock	0.03	0.06	0.09	0.05	0.79	0.12	0.08
	interest rate shock	0.12	0.08	0.19	0.16	0.04	0.03	0.20
	observation error	0.00	0.19	0.08	0.00	0.00	0.09	0.00
	total forecast variance	2.78	13.86	1.75	1.88	0.10	1.61	0.07
t = 10	productivity shock	0.03	0.05	0.06	0.08	0.02	0.01	0.01
	inflation objective shock	0.00	0.01	0.01	0.01	0.05	0.01	0.05
	preference shock	0.72	0.00	0.33	0.29	0.09	0.10	0.46
	labour supply shock	0.06	0.11	0.14	0.31	0.06	0.59	0.02
	Investment shock	0.02	0.52	0.13	0.10	0.04	0.05	0.21
	price markup shock	0.06	0.09	0.12	0.08	0.67	0.13	0.07
	interest rate shock	0.12	0.08	0.17	0.14	0.07	0.06	0.18
	observation error	0.00	0.14	0.05	0.00	0.00	0.05	0.00
	total forecast variance	3.97	24.63	3.09	3.07	0.12	2.65	0.09
t = 100	productivity shock	0.06	0.07	0.10	0.08	0.03	0.04	0.01
	inflation objective shock	0.01	0.01	0.01	0.01	0.11	0.02	0.13
	preference shock	0.60	0.00	0.27	0.25	0.09	0.10	0.41
	labour supply shock	0.13	0.17	0.22	0.37	0.07	0.52	0.03
	Investment shock	0.04	0.45	0.11	0.09	0.04	0.06	0.19
	price markup shock	0.05	0.09	0.11	0.08	0.59	0.14	0.07
	interest rate shock	0.10	0.08	0.14	0.12	0.07	0.08	0.16
	observation error	0.00	0.12	0.04	0.00	0.00	0.04	0.00
	total forecast variance	4.75	31.56	3.79	3.57	0.14	3.19	0.10

Table 3

Weights in the Quadratic Welfare Function

Labour Term	1.0	
Consumption Term	0.08	
Price Inflation	106.5	(31.7 – 74.8)
Change in Price Inflation	116.3	(41.5 – 74.8)
Wage Inflation	74.8	
Covariance Term	-51.4	

Note: The weights are based on equation (42) and the estimated parameters.

Table 4

Variance decomposition with optimal monetary policy under commitment

		C	I	Y	L	π	W	R
t = 0	productivity shock	0.09	0.04	0.13	0.14	0.00	0.00	0.12
	inflation objective shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	preference shock	0.21	0.16	0.02	0.01	0.00	0.00	0.40
	labour supply shock	0.57	0.25	0.83	0.80	0.00	0.39	0.23
	Investment shock	0.11	0.49	0.02	0.01	0.00	0.00	0.02
	price markup shock	0.02	0.02	0.00	0.04	0.99	0.44	0.22
	interest rate shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	observation error	0.00	0.05	0.01	0.00	0.00	0.17	0.00
	total forecast variance	0.31	1.11	0.16	0.26	0.06	0.18	0.04
t = 4	productivity shock	0.09	0.06	0.11	0.02	0.02	0.02	0.08
	inflation objective shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	preference shock	0.08	0.18	0.01	0.00	0.00	0.00	0.40
	labour supply shock	0.68	0.33	0.77	0.94	0.03	0.62	0.43
	Investment shock	0.13	0.30	0.00	0.00	0.00	0.00	0.05
	price markup shock	0.02	0.07	0.05	0.03	0.95	0.22	0.05
	interest rate shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	observation error	0.00	0.06	0.06	0.00	0.00	0.15	0.00
	total forecast variance	3.42	14.48	2.54	2.95	0.08	0.86	0.27
t = 10	productivity shock	0.10	0.09	0.14	0.01	0.02	0.06	0.07
	inflation objective shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	preference shock	0.07	0.16	0.00	0.00	0.00	0.00	0.38
	labour supply shock	0.65	0.36	0.72	0.91	0.04	0.54	0.44
	Investment shock	0.12	0.24	0.00	0.00	0.00	0.00	0.06
	price markup shock	0.05	0.12	0.10	0.07	0.94	0.29	0.05
	interest rate shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	observation error	0.00	0.03	0.04	0.00	0.00	0.11	0.00
	total forecast variance	4.95	23.73	4.09	4.44	0.09	1.22	0.29
t = 100	productivity shock	0.13	0.11	0.16	0.02	0.02	0.12	0.07
	inflation objective shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	preference shock	0.07	0.15	0.00	0.00	0.00	0.00	0.38
	labour supply shock	0.63	0.35	0.70	0.91	0.04	0.45	0.44
	Investment shock	0.12	0.23	0.00	0.00	0.00	0.00	0.06
	price markup shock	0.05	0.13	0.11	0.07	0.93	0.33	0.06
	interest rate shock	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	observation error	0.00	0.03	0.03	0.00	0.00	0.09	0.00
	total forecast variance	5.52	26.47	4.35	4.56	0.09	1.47	0.30