In Search of the Armington Elasticity

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Abstract

The elasticity of substitution between goods from different countries—the Armington elasticity—is important for many questions in international macroeconomics, but its magnitude is subject to debate. We argue that the wide range of estimates arises because the "macro" elasticity between home and import goods is in fact smaller than the "micro" elasticity between foreign sources of imports. We model this difference using a nested CES preference structure with heterogeneous firms. We employ a unique source of highly disaggregate U.S. production data, matched to Harmonized System trade data, to estimate the micro and macro Armington elasticities using a cross-country approach. While the median estimate of the micro elasticity between foreign countries is 4.4, the macro elasticity between home and imported goods is not significantly different from 1. We also explore a time-series approach that aggregates across goods and countries. This time-series approach only identifies the macro elasticity and, contrary to common belief, need not result in aggregation bias. We conclude with several applications of our results, including to the measurement of trade costs during the recent financial crisis.

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1 Introduction

The elasticity of substitution between goods produced in different countries has long been one of the key parameters in international economics. Because it governs the strength of the relative demand response to relative international prices, this elasticity is central to understanding many features of the global economy. These include the role of international prices in trade balance adjustment, the optimal extent of international portfolio diversification, the effects of regional trade agreements, and the welfare benefits of expanding world trade.

Since at least the 1940s, economists have used both aggregate and disaggregate trade data in attempts to estimate the responsiveness of demand to international prices. Periodic comprehensive surveys by Cheng (1959), Leamer and Stern (1970), Magee (1975), Goldstein and Khan (1985), Shiells, Stern and Deardorf (1986) and Marquez (2002), among others, document the growth over time in the supply of econometric studies on larger and increasingly detailed data sets. Yet despite an ever-expanding body of empirical study, there remains substantial uncertainty about the appropriate elasticity values to apply to different research and policy questions. Our view, which we document in this paper, is that the uncertainty stems from confusion over the precise elasticity being measured.

The starting point for our analysis is a general-equilibrium trade model based on work of Melitz (2003) and Chaney (2008) that is flexible enough to encompass the main alternative estimation approaches adopted in the recent empirical literature. A key foundation for that literature is the Armington (1969) preference model, in which goods are differentiated according to their country of origin, with a constant substitution elasticity between national output aggregates. This paper likewise builds on that foundation, but crucially, our model allows the Armington substitution elasticity between *domestic and foreign suppliers* to differ from that between *alternative foreign suppliers*, using a nested CES preference structure.

A recurring theme of the empirical studies is that trade elasticities estimated from disaggregate data appear to be higher than those based on aggregate data.[1] Goldstein and Khan (1985) survey a large body of research on empirical aggregate import equations

[1] See, among other references, Houthakker and Magee (1969), Hummels (2001), McDaniel and Balistreri (2003), Broda and Weinstein (2006), and Imbs and Méjean (2009). The table in Appendix A offers a partial overview of published findings.

and endorse a much earlier judgment by Harberger (1957) that for a typical country the price elasticity of import demand "lies in or above the range of -0.5 to -1.0...." It is fair to say that this consensus has not changed much in the ensuing quarter-century. Thus, more recent macro studies such as Heathcote and Perri (2002) and Bergin (2006) estimate substitution elasticities around unity.

In apparently sharp contrast, studies of individual product groups such as Feenstra (1994), Lai and Trefler (2002), Broda and Weinstein (2006), Romalis (2007), and Imbs and Méjean (2009) tend to identify much stronger price responses. One factor behind this regularity is often thought to be the classical aggregation bias first identified by Orcutt (1950), but we emphasize here a second factor: the difference between the substitution elasticities that disaggregate and aggregate studies have tended to measure. For international macroeconomists, who work with aggregate import data, "the" Armington elasticity typically refers to substitution between home production and imports, or what we will refer to as the "macro" elasticity. But for international trade researchers who work with disaggregate data, "the" Armington elasticity instead governs substitution between different import suppliers, or the "micro" elasticity. Combined with a new disaggregate data set that we develop, our framework allows us to identify both of these elasticities separately. We show that, empirically, the substitution elasticity between home goods and imports tends to be much smaller than that between similar imports from different exporting countries. This finding helps reconcile the relatively high Armington elasticities estimated on disaggregate data with the low elasticities typically found in more aggregative macro-level studies.

The rest of this paper is organized as follows. Section 2 sets out the model from which we derive our basic import equations. On the supply side, the model allows for endogenous entry into exporting, and thus for an extensive margin in trade. On the demand side, the model allows for *home substitution bias*: the elasticity of substitution between domestic and foreign suppliers may differ from that between pairs of foreign suppliers.

Section 3 develops the disaggregate (by good and country) import demand equation implied by the model. Endogeneity of the terms in this equation, along with measurement error due to the use of unit-values rather than ideal price indexes in the estimation, introduce statistical biases which can be significant in magnitude. We illustrate these biases through OLS estimation using U.S. data. The U.S. dataset matches data on the cross-section of United States imports and exports for each good with product-level data on U.S. production, and therefore implied apparent consumption. The U.S. production data are obtained from *Current Industrial Reports*, and our estimation is the first time that such data has been matched to the highly-disaggregate (Harmonized System) level for imports.

Because the OLS estimation is subject to various biases, we draw on Feenstra (1994) to propose a cross-country, GMM estimation strategy that corrects for these biases while also accounting for the simultaneity implied by our model's supply responses. Notably, this estimation strategy yields estimates of both the micro and macro elasticities, though identifying the latter requires pooling across two or more goods. We find considerable heterogeneity across goods in the micro elasticities, with a median estimate of 4.4. These micro elasticities in general are estimated to be above the macro elasticities, which are not significantly different from unity, in line with the conventional macro findings.

Identification of the micro and macro elasticities in the cross-country estimation relies on heteroscedasticity in the data. Because this is an untested assumption, in section 4 we confirm that our model is capable of generating consistent GMM estimates in simulated data that closely match what we find in our U.S. dataset. We also confirm the large biases found in the OLS estimates of the micro elasticity, though interestingly, the macro elasticity estimated from OLS performs much better. The lack of substantial bias in the macro elasticities, even when obtained from disaggregate data, can be understood from the properties of aggregate import demand, which we turn to next.

In section 5 we assess the common belief that trade elasticities estimated from aggregate data are necessarily biased and lower than those estimated from disaggregate data. We first aggregate across countries, which in our framework poses no problem at all. The aggregated equation allows for consistent estimation of the macro elasticity. We then further aggregate across goods and find that this result is preserved, provided that the macro elasticity of home-foreign substitution is uniform across goods. That is, the import demand equations that aggregate across countries and goods can still give accurate estimates of the macro elasticity, as we demonstrate using simulated data. This result is at first sight surprising because the nested CES utility function that we assume, not being weakly separable in imports and domestic consumption, does not obviously allow one to treat imports as a separate "good" with a separate price index. In analogy to the "latent separability" concept of Blundell and Robin (2000), however, we show that our utility function allows for consistent aggregation across goods and countries even when conventional weak separability does not hold.

In section 6 we consider two applications of a our framework. In the first application, we allow the macro elasticity to differ across goods, and consider the implications of an (exogenous) change in the exchange rate. We derive a formula similar to Imbs and Méjean (2009), whereby a devaluation affects aggregate imports according to an *expenditure-weighted average* of the Armington elasticities. However, in contrast to Imbs and Méjean (2009) who estimate the micro elasticities only, we believe that the relevant elasticities in this formula are those at the macro level, which we find are low: therefore, we are closer to the "elasticity pessimists" on this score.

Our second application shows how the nested CES structure affects the use of the gravity equation to infer trade costs, as done by Jacks, Meissner and Novy (2009a,b), for example, for the recent financial crisis. The nested CES structure provides a new explanation for the declining share of trade during the crisis: this decline can occur due to higher prices or reduced variety of *domestic* goods, which draws expenditure away from imports when the "macro" elasticity is less than unity, rather than being explained by increases in trade costs.

Section 7 summarizes our results and points towards one new direction for "elasticity optimism" that we propose to explore in future work. Various technical results are gathered in the appendices.

2 An Illustrative Model

2.1 Preferences and Prices

There are J countries in the world and a fixed number G of different goods. Each country produces a range of distinct varieties of each good $g \in \{1, ..., G\}$, the set of varieties produced to be determined endogenously within our model.

In the classical Armington (1969) model, goods are differentiated not only by inherent differences in their characteristics, but also by their place of production. In country j, the representative consumer has a comprehensive consumption index given by

$$C^j = \left[\sum_{g=1}^{G} \left(\alpha_g^j\right)^{\frac{1}{\eta}} \left(C_g^j\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where the weights $\left\{\alpha_g^j\right\}_{g=1}^{G}$ are random preference shocks, with $\sum_g \alpha_g^j = 1$, and η is the elasticity of substitution between different goods. Since each good is produced in all countries, the Armington assumption does not become relevant until one defines good-specific consumption sub-indexes $\left\{C_g^j\right\}_{g=1}^G$.

A general Armington setup differentiates products not only by their domestic or foreign origin, but also by the specific foreign country of origin. Define β_g^j as a random preference weight that country-j residents attach to domestically produced units of good g. We assume that

$$C_g^j = \left[\left(\beta_g^j\right)^{\frac{1}{\omega_g}} \left(C_g^{jj}\right)^{\frac{\omega_g - 1}{\omega_g}} + \left(1 - \beta_g^j\right)^{\frac{1}{\omega_g}} \left(C_g^{Fj}\right)^{\frac{\omega_g - 1}{\omega_g}}\right]^{\frac{\omega_g}{\omega_g - 1}}, \tag{2}$$

where C_g^{jj} denotes the consumption index of varieties of good g produced at home, C_g^{Fj} denotes the consumption aggregate of varieties of good g produced abroad, and ω_g is the substitution elasticity between home and foreign varieties of good g.

In turn, the country j foreign consumption index C_g^{Fj} depends on consumption from all possible sources of imports $i \neq j$, with random country-of-origin weights $\left\{\kappa_g^{ij}\right\}_{i \neq j}, \sum_{i \neq j} \kappa_g^{ij} = 1$:

$$C_g^{Fj} = \left[\sum_{i=1, i \neq j}^{J} \left(\kappa_g^{ij}\right)^{\frac{1}{\sigma_g}} \left(C_g^{ij}\right)^{\frac{\sigma_g - 1}{\sigma_g}}\right]^{\frac{\sigma_g}{\sigma_g - 1}}. \tag{3}$$

Here, σ_g is the elasticity of substitution between baskets of good g varieties originating in different potential exporters to country j.

We assume that the same elasticity σ_g that applies between different countries that export good g to country j also applies between the varieties of the particular good g that country i firms export to country j.[2] To that end, we assume that country j can import a measure N_g^{ij} of distinct varieties of good g from country i. (It will itself produce a measure N_g^{jj} of varieties for home consumption.) In our model, each set of measure N_g^{ij} is determined endogenously by country-pair-specific fixed costs of trade and other factors to be described in detail below. Because σ_g also denotes the elasticity of substitution between different varieties φ of good g produced by a particular country i, then for all $i \in \{1, ..., J\}$,

$$C_g^{ij} = \left[\int_{N_g^{ij}} \left(c_g^{ij}(\varphi)^{\frac{\sigma_g - 1}{\sigma_g}} \mathrm{d}\varphi\right)\right]^{\frac{\sigma_g}{\sigma_g - 1}}, \tag{4}$$

where the notation indicates that integration is done over a set of varieties that we indicate

[2] Alternatively, we could assume a distinct elasticity of substitution between varieties in the lower-level aggregation (4), as do Imbs and Méjean (2009). However, such a lower-level elasticity is not identified from the country-level data that we and they employ.

by its measure, N_g^{ij}.

The preceding preference setup defines a structure of canonical cost-of-living indexes and sub-indexes. The comprehensive consumer price index (CPI) for country j is

$$P^j = \left[\sum_{g=1}^{G} \alpha_g^j \left(P_g^j\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

Corresponding to the consumption aggregator (2) for country j residents is a price index P_g^{jj} for varieties of good g produced at home and an index P_g^{Fj} for the aggregate of imported varieties. For example, the price index for imported goods P_g^{Fj} is given by

$$P_g^{Fj} = \left[\sum_{\substack{i=1\\i\neq j}}^{J} \kappa_g^{ij} \left(P_g^{ij}\right)^{1-\sigma_g}\right]^{\frac{1}{1-\sigma_g}} \tag{5}$$

Let us assume that when good g is shipped from i to j, only a fraction $1/\tau_g^{ij} \leq 1$ arrives in j. Thus, the model makes a distinction between c.i.f and f.o.b. prices. If p_g^i denotes the f.o.b. price of a variety of good g produced in country i, the (c.i.f.) price faced by country j consumers who import the good from country i is $\tau_g^{ij} p_g^i$. If P_g^{ij} denotes the price index for varieties that country j imports from i, then the good-by-good components of the country j CPI, $\left\{P_g^j\right\}_{g=1}^{G}$ are given by

$$\begin{aligned}
P_g^j &= \left\{\beta_g^j \left(P_g^{jj}\right)^{1-\omega_g} + \left(1-\beta_g^j\right)\left(P_g^{Fj}\right)^{1-\omega_g}\right\}^{\frac{1}{1-\omega_g}} \\
&= \left\{\beta_g^j \left(P_g^{jj}\right)^{1-\omega_g} + \left(1-\beta_g^j\right)\left[\sum_{i=1, i\neq j}^{J} \kappa_g^{ij}\left(P_g^{ij}\right)^{1-\sigma_g}\right]^{\frac{1-\omega_g}{1-\sigma_g}}\right\}^{\frac{1}{1-\omega_g}} \\
&= \left\{\beta_g^j \left[\int_{N_g^{jj}} p_g^j(\varphi)^{1-\sigma_g}\, \mathrm{d}\varphi\right]^{\frac{1-\omega_g}{1-\sigma_g}}\right. \\
&\quad \left. + \left(1-\beta_g^j\right)\left[\sum_{i=1, i\neq j}^{J} \kappa_g^{ij} \int_{N_g^{ij}} \left(\tau_g^{ij} p_g^i(\varphi)\right)^{1-\sigma_g} \mathrm{d}\varphi\right]^{\frac{1-\omega_g}{1-\sigma_g}}\right\}^{\frac{1}{1-\omega_g}}.
\end{aligned}$$

2.2 Productivity and Production

Recall that in each country i and for each good g, N_g^{ij} represents the measure of goods exported to country j. Let φ denote a producer-specific productivity factor. In our model, N_g^{ij} will be the size of an interval of producer-specific productivity factors and firms can be indexed by φ in that interval. For a firm φ in i that exports the amount $y_g^{ij}(\varphi)$ to country j, the unit labor requirement is

$$\ell_g^{ij}(\varphi) = \frac{y_g^{ij}(\varphi)}{A_g A^i \varphi} + f_g^{ij},$$

where A_g is a global good-specific productivity shock, A^i is a country-specific productivity shock, and f_g^{ij} is a fixed cost of exporting g from i to j.

The distribution of producer-specific productivity factors φ among varieties follows the cumulative distribution function $H_g^i(\varphi)$. With a continuum of firms the law of large numbers applies and the measure of potential varieties produced at a firm-specific productivity exceeding φ is $1-\ H_g^i(\varphi)$. We will determine an endogenous *cutoff productivity level* $\hat{\varphi}_g^{ij}$ below which country i producers of varieties of g will find it unprofitable to ship to j's market. Under this notation, if the distribution of productivity levels is unbounded from above, country i producers with $\varphi \in \left[\hat{\varphi}_g^{ij}, \infty\right)$ export to j and the measure of varieties of g exported from i to j is given by $N_g^{ij} = 1-\ H_g^i\left(\hat{\varphi}_g^{ij}\right)$.

Let W^i be country i's wage denominated in some global numeraire. Then the price of a variety of good g "exported" to the same country i in which it is produced (its f.o.b. price) is

$$p_g^i(\varphi) = \frac{\sigma_g}{\sigma_g - 1}\left(\frac{W^i}{A_g A^i \varphi}\right). \tag{6}$$

In the presence of trade costs, as we have seen, higher (c.i.f.) prices $\tau_g^{ij} p_g^i(\varphi)$will prevail in the countries j that import this product from i.

Exporter revenues less variable costs on shipments of g from i to j are given by $\pi_g^{ij}(\varphi) = p_g^i(\varphi)\, y_g^{ij}(\varphi)/\sigma_g$. Invoking the standard demand functions implied by CES utility, we therefore define the cutoff productivity level for exports from i to j by:

$$\begin{aligned} \pi_g^{ij}\left(\hat{\varphi}_g^{ij}\right) &= \tau_g^{ij} p_g^i\left(\hat{\varphi}_g^{ij}\right) \kappa_g^{ij} \left[\frac{\tau_g^{ij} p_g^i\left(\hat{\varphi}_g^{ij}\right)}{P_g^{Fj}}\right]^{-\sigma_g} \left(1-\beta_g^j\right) \left(\frac{P_g^{Fj}}{P_g^j}\right)^{-\omega_g} \alpha_g^j \left(\frac{P_g^j}{P^j}\right)^{-\eta} C_j \\ &= W^i f_g^{ij}. \end{aligned} \tag{7}$$

The first line above follows because, due to shipping costs, exporter production $y_g^{ij}(\varphi)$ must equal τ_g^{ij} times the number of units that actually end up being consumed by importers in country j. Equation (6) allows one to solve condition (7) explicitly for $\hat{\varphi}_g^{ij}$ as a function of variables exogenous to the firm. The cutoff productivity $\hat{\varphi}_g^{jj}$ for country j "imports" from ("exports" to) itself is found by replacing the product $\kappa_g^{ij}(1-\beta_g^j)$ by β_g^j in equation (7), setting $i = j$ (where $\tau_g^{jj} = 1$), and replacing P_g^{Fj} by $P_g^{jj} = \left[\int_{N_g^{jj}} p_g^j(\varphi)^{1-\sigma_g}\,\mathrm{d}\varphi\right]^{\frac{1}{1-\sigma_g}}$. Notice that P_g^{ij}, the price index for varieties of g imported by j from i, is given by

$$\begin{aligned} P_g^{ij} &= \left[\int_{N_g^{ij}} \left(\tau_g^{ij} p_g^i(\varphi)\right)^{1-\sigma_g} \mathrm{d}\varphi\right]^{\frac{1}{1-\sigma_g}} \\ &= \left[\int_{\hat{\varphi}_g^{ij}}^{\infty} \left(\tau_g^{ij} p_g^i(\varphi)\right)^{1-\sigma_g} \mathrm{d}H_g^i \varphi\right]^{\frac{1}{1-\sigma_g}} \\ &= \left(N_g^{ij} \mathbb{E}\left\{\left(\tau_g^{ij} p_g^i(\varphi)\right)^{1-\sigma_g} | \varphi \geq \hat{\varphi}_g^{ij}\right\}\right)^{\frac{1}{1-\sigma_g}}. \end{aligned} \tag{8}$$

If the labor supply in each country j, L^j, is fixed, imposing labor-market clearing conditions for each country yields the equilibrium allocation. In an appendix we show how to solve for this equilibrium under the assumption that the distribution of variety-specific productivity shocks is Pareto:

$$H_g^i(\varphi) = 1 - \varphi^{-\gamma_g}. \tag{9}$$

Under this specification, the price index for varieties of g imported by j from i becomes:

$$P_g^{ij} = \left(\frac{\sigma_g}{\sigma_g - 1}\right)\left[\frac{\gamma_g}{\gamma_g - (\sigma_g - 1)}\right]\frac{\tau_g^{ij} W^i}{A^i A^g}(N_g^{ij})^{\frac{-[\gamma_g - (\sigma_g - 1)]}{\gamma_g(\sigma_g - 1)}}, \tag{10}$$

where the standard assumption that $\gamma_g > (\sigma_g - 1)$ is needed for this price index to be well defined.

3 Estimating the Armington Elasticities

3.1 Import Demand

The assumptions on preferences imply that we can express country j's imports of good g from country $i \neq j$ (covering all varieties N_g^{ij}) as:

$$V_g^{ij} = \alpha_g^j \kappa_g^{ij} (1 - \beta_g^j) \left(\frac{P_g^{ij}}{P_g^{Fj}} \right)^{1-\sigma_g} \left(\frac{P_g^{Fj}}{P_g^j} \right)^{1-\omega_g} \left(\frac{P_g^j}{P^j} \right)^{1-\eta} P^j C^j. \tag{11}$$

Spending on good g from home supply is:

$$V_g^{jj} = \alpha_g^i \beta_g^j \left(\frac{P_g^{jj}}{P_g^j} \right)^{1-\omega_g} \left(\frac{P_g^j}{P^j} \right)^{1-\eta} P^j C^j. \tag{12}$$

Dividing (11) and (12) we obtain imports from country i relative to home demand,

$$\frac{V_g^{ij}}{V_g^{jj}} = \kappa_g^{ij} \left(\frac{1 - \beta_g^j}{\beta_g^j} \right) \left(\frac{P_g^{ij}}{P_g^{jj}} \right)^{1-\sigma_g} \left(\frac{P_g^{Fj}}{P_g^{jj}} \right)^{\sigma_g - \omega_g}. \tag{13}$$

Notice that this import demand equation includes the multilateral import price index P_g^{Fj} on the right, from which the elasticity ω_g is identified, whereas the elasticity σ_g is estimated from the relative bilateral import price.

This import demand equation differs from the form in which it would be estimated, however, because the CES price index, P_g^{ij}, is rarely if ever measured in practice. As it is specified in (8), P_g^{ij} will fall whenever there is an expansion in the set of varieties N_g^{ij}, because such an expansion provides a utility gain for consumers and therefore lowers the "true" price index. This negative relationship between P_g^{ij} and N_g^{ij} can be seen from (10), for example. Price indexes used in practice, such as the Laspeyres import and export prices used by the Bureau of Labor Statistics (BLS), do not make such a correction for variety. The same is true for unit-values, which we shall use in our empirical application and which are in fact *adversely* affected by changes in variety.

The unit-value for good g sold from country i to j is defined as a consumption-weighted average of prices:

$$UV_g^{ij} = \int_{\hat{\varphi}_{g,0}^{ij}}^{\infty} \tau_g^{ij} p_g^i(\varphi) \left[\frac{c_g^{ij}(\varphi)}{\int_{\hat{\varphi}_g^{ij}}^{\infty} c_g^{ij}(\varphi)\, dH_g^i(\varphi)} \right] dH_g^i(\varphi). \tag{14}$$

To simplify this expression, we make use of $c_g^{ij}(\varphi_1) = c_g^{ij}(\varphi_2)(\varphi_1/\varphi_2)^{\sigma_g}$ to evaluate the integral appearing in the denominator as:

$$\begin{aligned}
\int_{\hat{\varphi}_g^{ij}}^{\infty} c_g^{ij}(\varphi)\, dH_g^i(\varphi) &= \int_{\hat{\varphi}_g^{ij}}^{\infty} c_g^{ij}\left(\hat{\varphi}_g^{ij}\right) \left(\frac{\varphi}{\hat{\varphi}_g^{ij}} \right)^{\sigma_g} dH_g^i(\varphi) \\
&= c_g^{ij}\left(\hat{\varphi}_g^{ij}\right) \int_{\hat{\varphi}_g^{ij}}^{\infty} \left(\frac{\varphi}{\hat{\varphi}_g^{ij}} \right)^{\sigma_g} \gamma \varphi^{-\gamma-1} d\varphi \\
&= \frac{\gamma_g}{(\gamma_g - \sigma_g)} c_g^{ij}\left(\hat{\varphi}_g^{ij}\right) \left[1 - H_g^i\left(\hat{\varphi}_g^{ij}\right)\right], \text{ for } \gamma_g > \sigma_g.
\end{aligned}$$

This expression illustrates a general property of integrating any power function of φ using the Pareto distribution: the result is the *initial value* of the function, $c_g^{ij}\left(\hat{\varphi}_g^{ij}\right)$, times the hazard rate $\left[1 - H_g^i\left(\hat{\varphi}_g^{ij}\right)\right]$, times a factor of proportionality. Applying this rule to the rest of the integral in (14), the initial values and hazard rates cancel and we readily obtain:

$$UV_g^{ij} = \frac{(\gamma - \sigma)}{(\gamma - \sigma - 1)} \tau_g^{ij} p_g^i\left(\hat{\varphi}_g^{ij}\right).$$

Taking the ratio of unit-values in the current and base periods, we are left with:

$$\frac{UV_g^{ij}}{UV_{g,0}^{ij}} = \frac{\tau_g^{ij} p_g^i\left(\hat{\varphi}_g^{ij}\right)}{\tau_{g,0}^{ij} p_g^i\left(\hat{\varphi}_{g,0}^{ij}\right)} = \left(\frac{\tau_g^{ij} W^i / A_g A^i}{\tau_{g,0}^{ij} W_0^i / A_{g,0} A_0^i} \right) \left(\frac{N_g^{ij}}{N_{g,0}^{ij}} \right)^{1/\gamma_g}, \tag{15}$$

where the second equality makes use of the prices in (6) and $N_g^{ij} = 1 - H_g\left(\hat{\varphi}_g^{ij}\right) = \left(\hat{\varphi}_g^{ij}\right)^{-\gamma_g}$. It is apparent from this expression that the unit-value is *positively* associated with an increase in product variety N_g^{ij}, in contrast to the CES price index.[3] Another way to state this result is that product variety N_g^{ij} is the *measurement error* in the unit-value as compared to relative wages. The reason for this is that an expansion of demand in country

[3]It can be shown that a Laspeyres price index, such as used by BLS, lies in between the CES and unit-value cases and is not related to product variety at all: it equals the final expression in (15) but without the term involving N_g^{ij}.

i for the goods from j will lead to entry in country j, thereby driving *up* the average price as less efficient firms enter. The rate at which the average price rising depends on the inverse of the Pareto parameter, $1/\gamma_g$, which appears in (15).

In order to estimate import demand we also need to aggregate the bilateral prices into the overall import price index, denoted by P_g^{Fj}, which is a CES function of the underlying bilateral prices P_g^{ij}. The ratio of CES import price indexes can be measured by the exact index due to Sato (1976) and Vartia (1976):

$$\frac{P_g^{Fj}}{P_{g,0}^{Fj}} = \prod_{i=1, i \neq j}^{J} \left(\frac{P_g^{ij}}{P_{g,0}^{ij}} \right)^{w_g^{ij}} \tag{16}$$

with

$$w_g^{ij} \equiv \frac{\left(\frac{s_g^{ij} - s_{g,0}^{ij}}{\ln s_g^{ij} - \ln s_{g,0}^{ij}} \right)}{\sum_{i=1, i \neq j}^{J} \left(\frac{s_g^{ij} - s_{g,0}^{ij}}{\ln s_g^{ij} - \ln s_{g,0}^{ij}} \right)}. \tag{17}$$

The numerator in (17) is the "logarithmic mean" of the import shares s_g^{ij} and $s_{g,0}^{ij}$, and lies in-between these two shares, while the denominator ensures that the weights w_g^{ij} sum to unity. The special formula for these weights in (17) is needed for the geometric mean in (16) to precisely measure the ratio of the CES functions, $P_g^{Fj}/P_{g,0}^{Fj}$. But in practice the Sato-Vartia formula will give very similar results to using other index number formulae.[4]

If we use the unit-values $UV_g^{ij}/UV_{g,0}^{ij}$ instead of the CES prices on the right of (16), then we obtain a multilateral *unit-value index* as we shall use in our empirical work:

$$\frac{UV_g^{Fj}}{UV_{g,0}^{Fj}} \equiv \prod_{i=1, i \neq j}^{J} \left(\frac{UV_g^{ij}}{UV_{g,0}^{ij}} \right)^{w_g^{ij}}.$$

Because the unit-value index does not properly correct for variety, it will differ from the

[4] For example, the Bureau of Economic Analysis takes the disaggregate Laspeyres indexes from BLS, and then aggregates them to the level of total imports using a Fisher Ideal Formula. The Fisher gives extremely similar results to the Sato-Vartia formula, which we use instead for consistency with the CES framework.

CES multilateral index by an aggregate of variety terms:

$$\frac{UV_g^{Fj}}{UV_{g,0}^{Fj}} = \left(\frac{P_g^{Fj}}{P_{g,0}^{Fj}}\right)\left(\frac{N_g^{Fj}}{N_{g,0}^{Fj}}\right)^{\frac{1}{(\sigma_g-1)}} \text{ where, } \frac{N_g^{Fj}}{N_{g,0}^{Fj}} = \prod_{i=1,i\neq j}^{J}\left(\frac{N_g^{ij}}{N_{g,0}^{ij}}\right)^{w_g^{ij}}.$$

as is obtained by using (10) and (15) with (16).

We can now specify the import demand equation (13) in a form that we shall estimate. Taking logs and using Δ to denote the difference with respect to period "0", we obtain

$$\Delta\ln\left(\frac{V_g^{ij}}{V_g^{jj}}\right) = -(\sigma_g-1)\Delta\ln\left(\frac{UV_g^{ij}}{UV_g^{jj}}\right) + (\sigma_g-\omega_g)\Delta\ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right) + \varepsilon_g^{ij}, \tag{18}$$

with the error term,

$$\varepsilon_g^{ij} \equiv \Delta\ln\kappa_g^{ij} + \Delta\ln\frac{(1-\beta_g^j)}{\beta_g^j} + \Delta\ln\left(\frac{N_g^{ij}}{N_g^{jj}}\right) - \frac{(\sigma_g-\omega_g)}{(\sigma_g-1)}\Delta\ln\left(\frac{N_g^{Fj}}{N_g^{jj}}\right), \tag{19}$$

which reflects exogenous taste shocks and endogenous changes to product variety at several levels of aggregation.

We have every reason to expect that this error term is correlated with the relative prices that appear on the right of (18). For example, a taste shock towards goods from country i (a rise in κ_g^{ij}) would raise imports (V_g^{ij}/V_g^{jj}) but would also tend to raise the unit-value (UV_g^{ij}/UV_g^{jj}), because wages in country i would increase. This correlation will tend to create a bias toward zero in the price elasticity $-(\sigma_g-1)$ appearing in (18). A further bias occurs because the unit-values measures the true price indexes with error, so that the error term incorporates relative variety, which is itself changing endogenously.

For all these reasons, conventional estimation of import demand can be expected to perform poorly. We demonstrate that in the next section, where we estimate the import demand in (18) using OLS on disaggregate U.S. data. Following that, we turn to an alternative estimation method from Feenstra (1994) that allows us to recover consistent parameter estimates.

3.2 Data and OLS Estimation

We investigate U.S. import supply for multiple foreign countries at the most disaggregate level possible. The import data at 10-digit Harmonized System (HS) level are readily available, along with the associated unit values, but it is difficult to match these imports to the associated U.S. supply. We make use of a unique data source called *Current Industrial Reports* (CIR), which is published by the U.S. Bureau of the Census and reports imports, exports and U.S. production at a disaggregate "product code" level. Recent years are available online,[5] and past years were obtained from an online archive, so the dataset spans 1992-2007. The data are in readable PDF or similar format, so we laboriously transcribed these to machine-readable datasets.

Limitations of the CIR data are that: (i) it is only a subset of U.S. manufacturing industries; (ii) the list of industries changes over time, especially with the shift from SIC (Standard Industrial Classification) to NAICS (North American Industry Classification System) in 1997; (iii) not all industries include import, export and U.S. domestic supply data for both values and quantities (as needed to compute unit-values); (iv) while a concordance from HS to the "product codes" used to track industries in CIR is provided, a given HS is sometimes associated with more than one product code. In the latter case, we needed to aggregate U.S. shipments across multiple product codes to obtain a correspondence to the import and export data.

After this aggregation procedure, the resulting dataset has 191 goods, by which we mean an SIC-based product code (up to 7 digits) or a NAICS-based product code (up to 10 digits). Of these, 80 goods are based on a single 10-digit Harmonized System commodity, and another 42 goods are based on two or three 10-digit HS commodities. So the majority of the dataset is at a highly disaggregate level: this is the first time that U.S. production data have been matched to imports and exports at such a disaggregate level (although there are earlier efforts at higher levels of aggregation, such as Reinert and Roland-Holst 1992). Since we have the matching exports and imports for these 191 goods, we can also compute U.S. apparent consumption, and consumption from U.S. supply, as appears in the denominator of the dependent variable in (18). When estimating this equation we pool across goods when they share some common HS commodities: this happens frequently for a SIC and NAICS-based product pair spanning 1992-1996 and 1997-2007, but sometimes a product code in one period will correspond to two codes in the other period, or there may be no correspondence over time. So after this pooling we end up with 113 slightly broader "goods" used in the estimation rather than 191. Each of these 113 goods is available for at most 16 years, but very often less than that.

[5] http://www.census.gov/manufacturing/cir/index.html.

Using this dataset, we estimate (18) using panel OLS over all export sources i for each good; standard errors are obtained using a panel bootstrap.[6] In Figure 1 we report the kernel density of estimates for σ_g obtained over the 113 goods. The median estimate for σ_g is at 1.06, and the medians of the lower and upper 95% confidence bounds are $[0.70, 1.34]$. Approximately 90% of the estimates of σ_g are below 1.5, which indicates rather low estimates for this micro Armington elasticity. In contrast, the median estimate from Broda and Weinstein (2008) of 3.7. Since we are estimating these parameters over much the same disaggregate data as Broda and Weinstein, these difference strongly suggest that our OLS estimates are downward biased, as we shall confirm using consistent estimates in the next section. In Figure 2 we report the kernel density of estimates for ω_g. The median for ω_g is at 0.86, with the medians of the lower and upper 95% confidence bounds are $[0.19, 1.37]$. In this case we do not have a comparable set of estimates for comparison, so we are uncertain of the bias in these OLS estimates. This will also be explored in the next section.

3.3 Estimation with Moment Conditions

We now turn to estimation of import demand using the general approach of Feenstra (1994) which is a generalized method of moments (GMM) technique. The challenge with estimating (18) is that the variety and taste shocks are likely to be correlated with the prices. To overcome this, we make use of the supply-side pricing equation by (15), differenced with respect to the base period "0" and country j:

$$\Delta \ln \left(\frac{UV_g^{ij}}{UV_g^{jj}} \right) = \Delta \ln \left(\tau_g^{ij} \frac{W^i/A^i}{W^j/A^j} \right) + \frac{\mu_g^{ij}}{\gamma_g}, \tag{20}$$

where $\mu_g^{ij} \equiv \Delta \ln \left(N_g^{ij}/N_g^{jj} \right)$ denotes changes to relative variety sold to country j, and $\mu_g^{ij}/\gamma_g$is the measurement error in the relative unit-value. Recall that τ_g^{ij} are the trade costs (with $\tau_g^{jj} \equiv 1$) while A^i reflects technology shocks in country i (the shock to sector g cancels out), both of which are stochastic. Wages are endogenously determined by the full-employment conditions (see the appendix), and do not admit a closed-form solution.

[6] Notice that the multilateral unit-value index (UV_g^{Fj}/UV_g^{jj}) appearing in (18) does not vary over exporting countries i, but only varies over time, so that accurate standard errors would need to cluster the error term. But the requisite clustering is by year, and cannot be performed in STATA due to insufficient degrees of freedom. Using the panel bootstrap gives similar results to robust standard errors in the panel.

So for the purpose of estimation, we need to make some assumption about how these wages are stochastically related to the various shocks.

To motivate our assumption, substitute (20) into (18) and solve for relative wages as:

$$\Delta \ln\left(\tau_g^{ij}\frac{W^i/A^i}{W^j/A^j}\right) = \frac{-\Delta \ln(V_g^{ij}/V_g^{jj})}{(\sigma_g - 1)} + \left[\frac{\varepsilon_g^{ij}}{(\sigma_g - 1)} - \frac{\mu_g^{ij}}{\gamma_g} + \frac{(\sigma_g - \omega_g)}{(\sigma_g - 1)}\Delta \ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\right].$$

The term in brackets are shocks to the relative demand for imports from country i, due to changes in tastes, variety, or the the multilateral import price. If there was no response at all in the demand shares (V_g^{ij}/V_g^{jj}), then the relative import price – and the relative wage adjusted for the trade costs and productivity – would rise by the full amount of the term in brackets. But we expect that V_g^{ij}/V_g^{jj} will increase with a positive shock to demand, thereby *dampening* the response of the relative wage. The amount of dampening could very well depend on the source of the shock, however. Accordingly, we will suppose that for every exporter i, a linear projection of the adjusted relative wage on the demand shock takes the form:

$$\Delta \ln\left(\tau_g^{ij}\frac{W^i/A^i}{W^j/A^j}\right) = \rho_{1g}\frac{\varepsilon_g^{ij}}{(\sigma_g - 1)} - \rho_{2g}\frac{\mu_g^{ij}}{\gamma_g} + \rho_{3g}\frac{(\sigma_g - \omega_g)}{(\sigma_g - 1)}\Delta \ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right) + \delta_g^{ij}, \qquad (21)$$

where ρ_{1g}, ρ_{2g}, and ρ_{3g} denote the correlation of the adjusted relative wage to a demand shock, variety shock, or change in the multilateral relative import price, respectively, and we expect that $0 < \rho_{1g}, \rho_{2g}, \rho_{3g} < 1$. The term δ_g^{ij} is a residual that certainly depends on the trade costs and productivity shocks, but is uncorrelated with the other shocks on the right of (21) by construction.

As it is stated, equation (21) is essentially without loss of generality, but it becomes effective once we impose assumptions on the residual. The residual's non-correlation with the right-hand side variables can be expressed as

$$E\left(\sum_{i\neq j}\varepsilon_g^{ij}\delta_g^{ij}\right) = E\left(\sum_{i\neq j}\mu_g^{ij}\delta_g^{ij}\right) = E\left(\sum_{i\neq j}\Delta \ln(UV_g^{Fj}/UV_g^{jj})\delta_g^{ij}\right) = 0,$$

where the summations are over all partner countries. These three moment conditions can be achieved by choice of ρ_{1g}, ρ_{2g}, and ρ_{3g}, in the usual OLS fashion. We will, however, make the stronger assumption that the first moment condition holds *for each* partner country:

Assumption 1: $E\left[\varepsilon_g^{ij}\delta_g^{ij}\right] = 0$ for $i, j = 1, \ldots, J$, $i \neq j$.

This assumption means that the demand shock ε_g^{ij} and the residual supply shocks δ_g^{ij} are uncorrelated for each partner country. Since we are deriving the supply side from the Melitz model, Assumption 1 effectively constrains the nature of the solution for relative wages in (21), and therefore for the relative import price in (20). The same assumption was made in Feenstra (1994) in a simpler system that involved only imports, without domestic demand or the Melitz model, using an *ad hoc* supply curve.

In addition to Assumption 1, we need a second assumption, namely:

Assumption 2: $E\left(\varepsilon_g^{ij}\mu_g^{ij}\right)$ does not vary over $i, j = 1, \ldots, J$, $i \neq j$.

Noting that the error term ε_g^{ki} in (19) incorporates the taste shocks as well as measurement error in relative unit-value, $\mu_g^{ki} \equiv \Delta \ln\left(N_g^{kj}/N_g^{ij}\right)$, then Assumption 2 essentially says that the variance of measurement error and its covariance with taste shocks is constant across countries. This assumption was also made by Feenstra (1994, Appendix), but it takes on more force in the Melitz model where the measurement error is endogenously determined as relative variety, $\mu_g^{ki} \equiv \Delta \ln\left(N_g^{kj}/N_g^{ij}\right)$.

We now proceed as in Feenstra (1994), by isolating the error terms in (18) and (21), while replacing the adjusted relative wage by the relative unit-value in (21), using (20):

$$
\begin{aligned}
\varepsilon_g^{ij} &= \Delta \ln\left(\frac{V_g^{ij}}{V_g^{ij}}\right) + (\sigma_g - 1)\Delta \ln\left(\frac{UV_g^{ij}}{UV_g^{jj}}\right) - (\sigma_g - \omega_g)\Delta \ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right) \\
\delta_g^{ij} &+ \left(1 - \rho_{2g}\right)\frac{\mu_g^{ij}}{\gamma_g} = \Delta \ln\left(\frac{UV_g^{ij}}{UV_g^{jj}}\right) - \left[\rho_{1g}\frac{\varepsilon_g^{ij}}{(\sigma_g - 1)} + \rho_{3g}\frac{(\sigma_g - \omega_g)}{(\sigma_g - 1)}\Delta \ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\right] \\
&= (1 - \rho_{1g})\Delta \ln\left(\frac{UV_g^{ij}}{UV_g^{jj}}\right) - \frac{\rho_{1g}}{(\sigma_g - 1)}\Delta \ln\left(\frac{V_g^{ij}}{V_g^{ij}}\right) + \frac{\left(\rho_{1g} - \rho_{3g}\right)(\sigma_g - \omega_g)}{(\sigma_g - 1)}\Delta \ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right).
\end{aligned}
$$

where the second line follows by substituting for ε_g^{ij}. Multiplying these two equations together and dividing by $\left(1 - \rho_{1g}\right)(\sigma_g - 1)$, we obtain:

$$
Y_g^{ij} = \sum_{n=1}^{4} \theta_{ng} X_{ng}^{ij} + \theta_{5g} X_{5g}^{j} + u_g^{ij},
$$

where

$$
\begin{aligned}
Y_g^{ij} &= [\Delta\ln(UV_g^{ij}/UV_g^{jj})]^2, \quad X_{1g}^{ij} = [\Delta\ln(V_g^{ij}/V_g^{jj})]^2, \\
X_{2g}^{ij} &= [\Delta\ln(UV_g^{ij}/UV_g^{jj})][\Delta\ln(V_g^{ij}/V_g^{jj})], \quad X_{3g}^{ij} = [\Delta\ln(UV_g^{Fj}/UV_g^{jj})][\Delta\ln(UV_g^{ij}/UV_g^{jj})], \\
X_{4g}^{ij} &= [\Delta\ln(UV_g^{Fj}/UV_g^{jj})][\Delta\ln(V_g^{ij}/V_g^{jj})], \;\; X_{5g}^{j} = [\Delta\ln(UV_g^{Fj}/UV_g^{jj})]^2,
\end{aligned}
$$

with,

$$
\theta_{1g} = \frac{\rho_{1g}}{(\sigma_g-1)^2(1-\rho_{1g})}, \theta_{2g} = \frac{(2\rho_{1g}-1)}{(\sigma_g-1)(1-\rho_{1g})}
$$

$$
\theta_{3g} = \frac{(\sigma_g - \omega_g)(1+\rho_{3g}-2\rho_{1g})}{(\sigma_g-1)(1-\rho_{1g})}, \theta_{4g} = \frac{(\sigma_g - \omega_g)(\rho_{3g}-2\rho_{1g})}{(\sigma_g-1)^2(1-\rho_{1g})}, \theta_{5g} = \frac{(\rho_{1g}-\rho_{3g})(\sigma_g - \omega_g)^2}{(\sigma_g-1)^2(1-\rho_{1g})}. \tag{22}
$$

and the error term,

$$
u_g^{ki} = \frac{\varepsilon_g^{ki}\left[\delta_g^{ki} + (1-\rho_{2g})\,\mu_g^{ki}\right]}{(\sigma_g-1)(1-\rho_{1g})}. \tag{23}
$$

We have not made explicit that all the variables differ across time (or across random draws within the simulations), but we now let $\bar{X}_{kg}^{ij} \equiv \frac{1}{T}\sum_{t=1}^{T} X_{kgt}^{ij}$ denote the mean value for any variable, and take the probability limit as $T \to \infty$. We average the variables over time, and consider estimating the following equation as a linear regression across partner countries $i = 1, \ldots, J, i \neq j$:

$$
\bar{Y}_g^{ij} = \sum_{n=1}^{4} \theta_{ng}\bar{X}_{ng}^{ij} + \theta_{5g}\bar{X}_{5g}^{j} + \bar{u}_g^{ij}, i = 1, \ldots, J, i \neq j. \tag{24}
$$

Assumptions 1 and 2 implies that $\text{plim}_{T\to\infty}\ \bar{u}_g^{ij}$ is a constant, so that by adding a further constant to the estimation of (24) and subtracting that from $\bar{u}_g^{ij}$, we can presume that $\text{plim}_{T\to\infty}\ \bar{u}_g^{ij} = 0$. Then the orthogonality conditions $\text{plim}_{T\to\infty}\sum_{i\neq j}\bar{X}_g^{ij}\bar{u}_g^{ij} = 0$ needed for consistency will hold. However, these orthogonality conditions are not sufficient for consistency: we also need a rank condition on the matrix composed of $\bar{X}_{ng}^{ij}$, $n = 1, \ldots, 5$ to ensure that the coefficients θ_{ng} are identified.

Estimating the "Micro" Elasticities

In order to understand this rank condition, it is useful to begin with the simpler system analyzed by Feenstra (1994), which is derived from the import demand equations, with domestic demand or prices. That system is simply:

$$\bar{Y}_g^{ik} = \theta_{1g}\bar{X}_{1g}^{ik} + \theta_{2g}\bar{X}_{2g}^{ik} + \bar{u}_g^{ik},\ \ k = 1, \ldots, J, k \neq i \text{ and } k \neq j, \tag{25}$$

where variables are defined as above, but now using an importing country i differenced with respect to another importing country k, and not differenced with respect to the home country j. This alternative differencing strategy eliminates the variables X_{3g}^{ij}, X_{4g}^{ij}, and X_{5g}^{j}, which arise due to the "multilateral resistance" terms contained in equations (18) and (21).

Feenstra argues that the rank condition needed to obtain consistent estimates from OLS or WLS on (25) is that $\text{plim}_{T\to\infty}\bar{X}_{1g}^{ij}$ and $\text{plim}_{T\to\infty}\bar{X}_{2g}^{ij}$ are not co-linear, which is satisfied provided that there is sufficient heteroscedasticity in the errors terms in relative demand and supply. Specifically, $\text{plim}_{T\to\infty}\bar{X}_{1g}^{ij}$ and $\text{plim}_{T\to\infty}\bar{X}_{2g}^{ij}$ are not co-linear provided that there exist importing countries i, j and k such that:

$$\frac{\left(\sigma_{g\varepsilon}^{ij}\right)^2}{\left(\sigma_{g\varepsilon}^{ik}\right)^2} \neq \frac{\left(\sigma_{g\delta}^{ij}\right)^2}{\left(\sigma_{g\delta}^{ik}\right)^2}.$$

That is, there must be heteroscedasticity in the demand and supply-side shocks.[7]

We will follow Feenstra (1994) and obtain the estimates for $\hat{\theta}_{1g}$ and $\hat{\theta}_{2g}$ by applying OLS to (25), for each of our 113 products. Given these estimates, a quadratic equation is solved to obtain the "micro" Armington elasticities $\hat{\sigma}_g$, along with $\hat{\rho}_{1g}$. This quadratic equation provides estimates satisfying $\hat{\sigma}_g > 1$ and $0 \leq \hat{\rho}_{1g} < 1$ provided that $\hat{\theta}_{1g} > 0$ (see Feenstra, 1994). Otherwise, the solution to the quadratic equations may be outside these bounds, as occurred in 17 out of the 113 goods in our dataset. Following Broda and Weinstein (2006), in those cases we apply a grid search satisfying $1.05 < \hat{\sigma}_g < 100.05$ and $0 \leq \hat{\rho}_{1g} < 1$ to obtain estimates within these bounds the minize the sum of sqaured residuals in (25).

After running (25) with OLS or the grid search, greater efficiency of the estimates can obtained by weighting equation (25) using the inverse of the variance of the error term obtained from the OLS estimates. The re-weighted estimates for σ_g over 96 goods are

[7] Notice that the heteroskedasticity condition conflicts with Assumption 2, which states that the measurement error – equal to the endogenous change in product variety – is homoskedastic. So Assumptions 1 and 2 needed to apply the GMM approach to estimating the micro Armington elasticities are strong.

graphed in Figure 3 (the graph is very similar for the unweighted estimates). The median estimate is 4.05. This compares to the median estimate of 3.7 from Broda and Weinstein (2006), computed over some 10,000 HS categories of imports, so our much more limited sample of 97 goods is similar in this respect. Comparing the density of estimates in Figure 3 and Figure 1 makes it clear that the OLS estimates for σ_g are strongly downward biased. To obtain confidence intervals on the OLS estimates we perform a panel bootstrap of the entire dataset. That is, we randomly re-draw observations by exporting country and re-estimate (25). Ignoring cases where the grid search is used, the confidence intervals are also graphed in Figure 3. The medians of the lower and upper 95% confidence bounds are $[2.01, 12.91]$.

The grid search process gives some larger σ_g estimates. For all the 113 industries including those with grid search estimates, the median estimate for σ_g is 4.42 and medians of the lower and upper 95% confidence bounds are $[2.14, 95.15]$. The median of the upper 95% confidence bound is very large because there are about 50 industries that have the upper limit (100.05) of the grid search range as their upper 95% confidence bound. That is, more than one-half of the industries needed the grid search for *some* re-sampling of countries, and this technique often yields the upper-bound of 100.05 for σ_g.

Estimating the "Macro" Elasticities

Now we return to the estimation of the "macro" Armington elasticities ω_g, using the more complex equation (24). What is new in (24) are the variables X_{3g}^{ij}, X_{4g}^{ij} and X_{5g}^{j}, all of which depend on the multilateral unit-value, and their coefficients depend on a new supply-side parameter ρ_{3g} as well as on the macro Armington elasticity ω_g. The variable $\bar{X}_{5g}^{j}$ is a constant term since it does not vary with i. We have already argued above that measurement error in the unit-values creates a constant term in (24), so we cannot rely on this additional constant term to reveal information about the parameters within θ_{5g}. Without that information, we have two new coefficients θ_{3g} and θ_{4g}, from which to calculate the two new parameters ω_g, and ρ_{3g}. It follows that the system is just identified provided that the matrix of right-hand side variables in (24) has full rank.

In the appendix we argue that the conditions needed to ensure that this rank condition holds are more difficult to satisfy than the for the simpler system (25). Specifically, we require that the new terms appearing in (24), $\text{plim}_{T\to\infty}\bar{X}_{3g}^{ij}$ and $\text{plim}_{T\to\infty}\bar{X}_{4g}^{ij}$, are not colinear with each other or with the constant term. We show that these new terms depend on the covariances:

$$\text{plim}_{T\to\infty}\left[\Delta\ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\varepsilon_g^{ij}\right], \text{ and } \text{plim}_{T\to\infty}\left[\Delta\ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\delta_g^{ij}\right].$$

To avoid colinearity with the constant in (24), we need these terms to vary across exporting countries i. But we should not necessarily expect to find this type of heteroscedasticity in these terms because the the multilateral relative unit-value which appears in each *does not vary* across exporting countries.

For this reason, we cannot rely on equation (24) to estimate ω_g and ρ_{2g} for each good g. Instead, we will constrain these parameters to be constant across a set of goods, relying on *cross-good* variation in the covariances $\text{plim}_{T\to\infty}\left[\Delta\ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\varepsilon_g^{ij}\right]$ and $\text{plim}_{T\to\infty}\left[\Delta\ln\left(\frac{UV_g^{Fj}}{UV_g^{jj}}\right)\delta_g^{ij}\right]$ to identify them. In order to pool across goods we are therefore assuming that:

Assumption 3: $\omega_g = \omega$ and $\rho_{3g} = \rho_3$ for $g = 1, \ldots, G$.

Thus, within a set of goods $g = 1, \ldots, G$ we will suppose that the macro Armington elasticity is constant. Using this assumption, we can then take the estimates of the micro elasticities $\hat{\sigma}_g$, which are not constrained to be constant across goods, and substitute these along with $\hat{\rho}_{1g}$ into (24). We then run NLS on (24) over the goods $g = 1, \ldots, G$ to obtain estimates of the new parameters ω and ρ_3. This "two step" estimation procedure is possible because relative demand between two importing countries—used to obtain (25)—is separable from home country demand and does not rely on ω and ρ_3, Therefore, we can substitute the first-step estimates of $\hat{\sigma}_g$and $\hat{\rho}_{1g}$ into (22) and therefore (24) to obtain $\hat{\omega}$ and $\hat{\rho}_3$. Standard errors are again obtained by bootstrapping the entire system.

Using the 113 goods in our dataset, we divide these into 10 sectors as shown in Table 1, where we report the estimates of ω and their confidence intervals (in parentheses). We report two sets of estimates: the first uses the unweighted estimates of σ_g and ρ_{1g}, which are substituted into (24) to obtain NLS estimates of ω and ρ_3 for each sector; and the second set of estimates uses the more efficient, weighted estimates of σ_g and ρ_{1g} to again obtain estimates of ω and ρ_3.[8]

[8] When we estimate ω and ρ_3, grid search estimates of σ_g and ρ_{1g} are included.

Table 1: Estimates of Omega

Sector	Number of Goods	Unweighted Estimate	Confidence Interval(95%)	NO. of goods sigma>omega	p-value	Weighted Estimate	Confidence Interval (95%)	NO. of goods sigma>omega	p-value
Food Products	6	3.545	(-16.131 11.256)	4	0.243	1.057	(-12.714 53.145)	6	.231
Apparel Products	13	0.764	(0.652 1.214)	13	0.000	0.704	(0.525 1.402)	13	.000
Chemicals & Rubber	12	0.659	(-3.451 6.453)	12	0.002	-1.721	(-1.475 9.844)	12	.002
Primary Metals	23	0.991	(0.161 2.276)	23	0.028	1.357	(-0.434 2.328)	22	.040
Fabricated Metals	9	0.775	(0.067 1.999)	9	0.050	0.810	(0.116 5.976)	9	.051
Farm Machinery	7	-0.052	(-2.295 11.364)	7	0.012	0.775	(-6.835 8.587)	7	.002
Construction Machinery	8	1.462	(-0.664 1.901)	8	0.049	1.509	(-0.953 2.003)	7	.052
Other Machinery	5	-1.198	(-2.188 55.594)	5	0.062	2.018	(0.581 2.177)	5	.082
Computer and Electronics	19	1.051	(0.313 1.389)	19	0.010	1.107	(0.448 1.369)	19	.008
Electrical Equipment	11	0.940	(0.604 1.143)	11	0.011	1.232	(-3.905 2.049)	11	.014

This table reports estimate of ω obtaining by running NLS on equation (24), where the number of observations equals the number of number of goods shown. The estimates of σ_g and ρ_{1g} used in (24) are obtained from first-stage regressions of (25), either unweighted or weighted. Reported in parentheses are the 95% confidence intervals obtained by bootstrapping the entire system.

In the first sector shown, food products, we find an unweighted point estimate for ω that is substantially greater than unity. But that sector is estimated over a rather small number of goods (six), and has an extremely large confidence interval, so that our estimation technique basically fails. In several other cases – such chemicals & rubber and farm machinery – we also find extremely large confidence intervals. But for products such as apparel, and computer and electronics, we find estimates for ω that are reasonably tight: around 0.75 and 1, respectively, with confidence intervals between about 0.5 and 1.5. In these cases, it appears that the estimates are identified with some degree of precision. Fabricated metals has a similar point estimate to apparel – around 0.75 – but larger confidence intervals, and electrical equipment has a similar point estimate to computer and electronics – around 1 – but a larger confidence interval in the weighted estimate.

In no case can we reject the hypothesis that the macro Armington elasticity is unity. Furthermore, regardless of the large standard errors obtained, we are able to *decisively reject* the hypothesis that the macro Armington elasticity exceeds the micro elasticity. This result is seen in two ways. First, we report in Table 1 the number of products in each sector for which the estimated macro elasticity $\hat{\omega}$ is less than the estimated micro elasticities $\hat{\sigma}_g$. Those simple counts are compared to the total number of products in each sector in the first column, and it can be seen that in nearly all cases, $\hat{\omega} < \hat{\sigma}_g$.

Second, we formally test the null hypothesis that $\hat{\omega} \geq \hat{\sigma}_g$ by comparing these estimates on each draw of the bootstrap for the entire system (where we randomly draw the exporting countries used in each bootstrap). The number of instances where $\hat{\omega} \geq \hat{\sigma}_g$, relative to the total draws in the bootstrap, is the p-value for rejecting this null hypothesis. As can be seen from the table, these p-values are extemely low for all sectors except food products. With the exception of that sector, we conclude that the macro Armington elasticity is below the micro elasticity.

4 Simulation Results

We simulate a small-scale model to demonstrate that the shocks to productivity, preferences, and to a lesser extent trade costs can generate the observed downward bias in OLS estimates but still yield an identified system for GMM estimation.

[To be completed]

5 Aggregate Import Demand

Earlier sections have developed a model of international trade flows and estimated the implied import demand equations using disaggregate United States data. Much previous literature focuses on estimation of aggregate import demand equations, in an attempt to ascertain directly the average relationship between aggregate measures of international competitiveness and aggregate imports. (That relationship is governed by the parameter that we have called ω). Such aggregate equations figure prominently in the macro-models used for forecasting and analysis by central banks and other macroeconomic policy makers. While an aggregate approach can be useful in clarifying the essential transmission channels of policy actions, as well as in diagnosing certain model misspecifications, aggregation comes with pitfalls that have been well recognized at least since the time of Orcutt's (1950) classic critique. In this section, we explore the performance of aggregate import demand equations within our framework in order to clarify the conditions under which estimates derived from aggregate data will be accurate.

Equation (11) gives country j's spending on imports of good g from country i. Summing over all trade partners $i \neq j$ yields country j spending on imports of good g from all foreign sources, denoted V_g^{Fj}:

$$
\begin{aligned}
V_g^{Fj} &= \sum_{i \neq j} V_g^{ij} \\
&= \alpha_g^j \left(1 - \beta_g^j\right) \left(\frac{P_g^{Fj}}{P_g^j}\right)^{1-\omega_g} \left(\frac{P_g^j}{P^j}\right)^{1-\eta} P^j C^j \left[\sum_{i \neq j} \kappa_g^{ij} \left(\frac{P_g^{ij}}{P_g^{Fj}}\right)^{1-\sigma_g}\right] \\
&= \alpha_g^j \left(1 - \beta_g^j\right) \left(\frac{P_g^{Fj}}{P_g^j}\right)^{1-\omega_g} \left(\frac{P_g^j}{P^j}\right)^{1-\eta} P^j C^j. \qquad (26)
\end{aligned}
$$

The last line follows from definition (5). Combining the foregoing expression with the demand V_g^{jj} as computed from equation (12), we see that

$$
\frac{V_g^{Fj}}{V_g^{jj}} = \frac{1 - \beta_g^j}{\beta_g^j} \left(\frac{P_g^{Fj}}{P_g^{jj}}\right)^{1-\omega_g},
$$

so that the home-foreign Armington elasticity ω_g can be identified from a multilaterally aggregated equation for imports of good g. In this aggregate specification with home bias, one estimates the Armington elasticity of substitution between home and foreign varieties of a good, which one would expect to be less than the elasticity of substitution σ_g between varieties of a particular foreign good.[9]

The next step is to sum these imports across all available goods g. Observe that the parameter σ_g — the possibly good-specific substitution elasticity between varieties of g purchased from the same source (domestic or foreign) — enters the preceding equations only through its role in constructing the price indexes. *Given* those indexes, the parameter σ_g does not appear in the aggregate demand for imports of good g. While we will *not* assume that σ_g is the same across goods, we will make the assumption that $\omega_g = \omega$ is invariant across different goods.[10]

From equation (26), total country j expenditure on imports is:

$$V^{Fj} = \sum_g V_g^{Fj} = \sum_g \left[\alpha_g^j \left(1 - \beta_g^j\right) \left(\frac{P_g^j}{P^j} \right)^{\omega - \eta} \left(\frac{P_g^{Fj}}{P^j} \right)^{1-\omega} \right] P^j C^j.$$

Define

$$\bar{P}^{Fj} \equiv \left[\sum_g \alpha_g^j \left(\frac{P_g^j}{P^j} \right)^{\omega - \eta} \left(P_g^{Fj} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}}. \tag{27}$$

For the case $\omega = \eta$, consumers substitute between domestic and foreign varieties just as readily as between different goods. In that case, therefore, the utility function can be written as a weakly separable function of import consumption and domestic-product consumption, and $\bar{P}^{Fj}$ is simply a standard CES price-index of the good-specific foreign price indexes P_g^{Fj}:

$$\bar{P}^{Fj} = \left[\sum_g \alpha_g^j \left(P_g^{Fj} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}.$$

[9] This estimating equation is closely related to those that Reinert and Roland-Holst (1992) and Blonigen and Wilson (1999) use. They match Unites States consumption to import data as we do, but at a higher aggregation level than in our data. At the same time, their estimation method aggregates across different foreign suppliers to the United States. It is notable, therefore, that their estimated Armington elasticities (for substitution between home production and imports) are very similar to ours estimated above.

[10] This estimating equation is closely related to those that Reinert and Roland-Holst (1992) and Blonigen and Wilson (1999) use. They match Unites States consumption to import data as we do, but at a higher aggregation level than in our data. At the same time, their estimation method aggregates across different foreign suppliers to the United States. It is notable, therefore, that their estimated Armington elasticities (for substitution between home production and imports) are very similar to ours estimated above.

Even in the case $\omega \neq \eta$, however, definition (27) shows that the weight on P_g^{Fj} in the overall index $\bar{P}^{Fj}$ depends on the variation in country j's comprehensive price index for good g relative to its overall CPI. The reason we define the index $\bar{P}^{Fj}$ is that, with one added assumption, the aggregate import share is simply a function of the latter index and the overall CPI. The added assumption is that $\beta_g^j = \beta^j$ for all g: in words, β^j is a uniform (across goods) country j demand shock in favor of domestic products. Then, whether or not $\omega = \eta$, $V^{Fj} = \left(1 - \beta^j\right)\left(\frac{\bar{P}^{Fj}}{P^j}\right)^{1-\omega} P^j C^j$.

Likewise, define V^{Hj} to be total country j spending on home-produced goods:

$$V^{Hj} \equiv \sum_g V_g^{jj} = \beta^j \sum_g \alpha_g^j \left(\frac{P_g^j}{P^j}\right)^{\omega-\eta} \left(\frac{P_g^{jj}}{P^j}\right)^{1-\omega} P^j C^j.$$

Furthermore, define the home index $\bar{P}^{Hj}$ (in analogy to) $\bar{P}^{Fj}$ as

$$\bar{P}^{Hj} = \left[\sum_g \alpha_g^j \left(\frac{P_g^j}{P^j}\right)^{\omega-\eta} \left(P_g^{jj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

which depends on domestic prices only in the weakly separable case $\omega = \eta$. Whether or not $\omega = \eta$, we can write

$$V^{Hj} = \beta^j \left(\frac{\bar{P}^{Hj}}{P^j}\right)^{1-\omega} P^j C^j.$$

The result is the key equation:

$$\frac{V^{Fj}}{V^{Hj}} = \frac{1-\beta^j}{\beta^j} \left(\frac{\bar{P}^{Fj}}{\bar{P}^{Hj}}\right)^{1-\omega}, \tag{28}$$

which shows that aggregate imports relative to domestic demand are a simple log-linear function of their relative price, with elasticity ω.

To understand the properties of this aggregate demand equation, define the comprehensive CPI P^j as

$$P^j \equiv \left[\sum_g \alpha_g^j \left(P_g^j\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},$$

where $\left(P_g^j\right)^{1-\omega} = \beta^j \left(P_g^{jj}\right)^{1-\omega} + \left(1 - \beta^j\right)\left(P_g^{Fj}\right)^{1-\omega}$. Then it turns out that we can express P^j implicitly in the form

$$P^j = \left[\beta^j \left(\bar{P}^{Hj}\right)^{1-\omega} + \left(1-\beta^j\right)\left(\bar{P}^{Fj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}}, \tag{29}$$

as we demonstrate in the appendix. This representation directly shows the CPI's relationship to the "domestic" and "foreign" price indexes $\bar{P}^{Hj}$ and $\bar{P}^{Fj}$. This description of import demand is precisely what would come out of the hypothetical consumer problem

$$\max_{D,M}\left[\left(\beta^j\right)^{\frac{1}{\omega}} D^{\frac{\omega-1}{\omega}} + \left(1-\beta^j\right)^{\frac{1}{\omega}} M^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

subject to $\bar{P}^H D + \bar{P}^F M = PC$, where D stands for aggregate real domestic consumption and M stands for aggregate real imports.[11] In this sense, the model of import demand admits exact aggregation across goods, with ω as the substitution elasticity between aggregate imports and domestic consumption. Because $\bar{P}^H$ and $\bar{P}^F$ both depend on *all* prices, however, the aggregation is less straightforward than it would be in the case of weakly separable utility ($\omega = \eta$). Instead, aggregation is possible because of the property of *latent separability* analyzed by Blundell and Robin (2000).[12]

By taking logs of equation (28), we obtain the log-linear regression equation

$$\log\left(\frac{V^{Fj}}{V^{Hj}}\right) = \mu + (1-\omega)\log\left(\frac{\bar{P}^{Fj}}{\bar{P}^{Hj}}\right) + u^j, \tag{30}$$

where, $\mu \equiv \mathbb{E}\log\left[\left(1-\beta^j\right)/\beta^j\right]$ and $u^j \equiv \log\left[\left(1-\beta^j\right)/\beta^j\right] - \mu$. This specification can potentially yield an estimate of the Armington elasticity ω directly, but in practise we will use unit-values rather than the "true" price indexes used in (30). To make this conversion, we start from the disaggregate import equation (18), with ω_g =ω and $\beta_g^j = \beta^j$. We first take the weighted average over exporting countries i using the weights w_g^{ij}, shown in (17).

[11] This formulation is the starting point for many empirical studies, for example, Reinert and Roland-Holst (1992), Blonigen and Wilson (1999), and Broda and Weinstein (2006). Our analysis shows the exact form of the price indexes under which their approach valid.

[12] Whereas weak separability requires that the utility or expenditure function is partitioned into *mutually exclusive* sets of goods, the more general concept of latent separability allows the set of goods to be overlapping: some goods can appear in many of the sub-groups. To see how this concept applies in our case, consider the aggregate $\bar{P}^{Fj}$ defined in (27). It is a summation over the import price indexes P_g^{Fj}, which depend on the import prices of good g from all source countries. But in addition, the "weights" $\alpha_g^j\left(\frac{P_g^j}{P^j}\right)^{\omega-\eta}$ appear in the formula. When $\omega \neq \eta$ these weights depend on the prices for all imported and domestic goods.

This results in;

$$\Delta \ln \left(\frac{V_g^{Fj}}{V_g^{jj}} \right) = (1-\omega)\Delta \ln \left(\frac{UV_g^{Fj}}{UV_g^{jj}} \right) + \varepsilon_g^{Fj},$$

with the error term,

$$\varepsilon_g^{Fj} \equiv \sum_{i \neq j} w_g^{ij} \Delta \ln \kappa_g^{ij} + \Delta \ln \frac{(1-\beta^j)}{\beta^j} - \frac{(1-\omega)}{(\sigma_g - 1)} \Delta \ln \left(\frac{N_g^{Fj}}{N_g^{jj}} \right).$$

Recall that the difference shown in these equations are with respect to a base period "0". Without loss of generality, we can absorb these initial values into a constant term, and rewrite import demand in levels as:

$$\ln \left(\frac{V_g^{Fj}}{V_g^{jj}} \right) = \mu + (1-\omega) \ln \left(\frac{UV_g^{Fj}}{UV_g^{jj}} \right) + \varepsilon_g^{Fj}.$$

Next, we aggregate across goods. For this purpose, define the weights υ_g^{Fj} as:

$$\upsilon_g^{Fj} \equiv \frac{\left(\frac{V_g^{Fj} - V_g^{jj}}{\ln V_g^{Fj} - \ln V_g^{jj}} \right)}{\left(\frac{V^{Fj} - V^{jj}}{\ln V^{Fj} - \ln V^{jj}} \right)},$$

where $V^{Fj} \equiv \sum_g V_g^{Fj}$ and $V^{Hj} \equiv \sum_g V_g^{Hj}$. This slightly unusual formula for the weights corresponds to the second formula investigated by Sato and Vartia, or what Diewert (1978) refers to as the Sato-Vartia II weights. These weights are unusual in that they do not sum to unity across the goods g, and we denote their sum by $\upsilon^{Fj} \equiv \sum_g \upsilon_g^{Fj}$. Their advantage is that they allow for consistent aggregation, as analyzed by Diewert (1978) and which we now show.

Taking the weighted sum of import demand over goods using υ_g^{Fj}, it is immediate that the aggregate import demand equation becomes:

$$\ln \left(\frac{V^{Fj}}{V^{jj}} \right) = \upsilon^{Fj} \mu + (1-\omega) \ln \left(\frac{UV^{Fj}}{UV^{Hj}} \right) + \varepsilon^{Fj}, \tag{31}$$

where import and domestic unit-value indexes are,

$$UV^{Fj} \equiv \prod_{g=1}^{G} \left(UV_g^{Fj}\right)^{\upsilon_g^{Fj}},$$

$$UV^{Hj} \equiv \prod_{g=1}^{G} \left(UV_g^{jj}\right)^{\upsilon_g^{Fj}},$$

and with the error term,

$$\varepsilon^{Fj} \equiv \upsilon^{Fj} \Delta \ln \frac{(1-\beta^j)}{\beta^j} + \sum_{g=1}^{G} \upsilon_g^{Fj} \left[\sum_{i \neq j} \upsilon_g^{ij} \Delta \ln \kappa_g^{ij} - \frac{(1-\omega)}{(\sigma_g - 1)} \Delta \ln \left(\frac{N_g^{Fj}}{N_g^{jj}} \right) \right] . \qquad (32)$$

An important feature of the error term in (32) is that the change in relative varieties $\Delta \ln(N_g^{Fj}/N_g^{jj})$ vanishes when $\omega = 1$. That is, the potential bias caused by correlation of the aggregate unit-value indexes on the right of (31) with the change in varieties is not an issue when $\omega = 1$, and we expect this bias to be minimal for ω close to unity. Provided also that the idiosyncratic shocks to demand are not significantly correlated with the unit-value indexes, the macro-level import demand equation then becomes a powerful tool for estimating the macro-level elasticity ω. [Plan to demonstrate this with simulation and estimation]

6 Applications

6.1 Impact of a Devaluation

Imbs and Méjean (2009) argue that there are grounds for "elasticity optimism" regarding the responsiveness of imports to a change in the terms of trade. To make this argument, they contrast two approaches to the estimation of σ_g: first, estimating this elasticity separately for 56 sectors using a modification of the GMM method in Feenstra (1994); and second, pooling the data across all sectors and estimating a *single* elasticity. They show that if the sectoral estimates are weighted by their shares in expenditure and summed, then we obtain a theoretically consistent aggregate elasticity. That aggregate elasticity of substitution is found to be significantly larger than the single estimate obtained by pooling the data. Therefore, they conclude, a pooled estimate that ignores heterogeneity across sectors is downward biased and gives too "pessimistic" a view of the impact of a devaluation on the value of imports.[13]

We have repeated the exercise of Imbs and Méjean on our own data by pooling across the industries and estimating a single value for σ. In the unweighted version of equation (24), we fail to obtain an estimate satisfying $\hat{\theta}_1 > 0$, as needed to ensure that $\hat{\sigma} > 1$ and $0 \leq \hat{\rho}_1 < 1$.[14] In the weighted estimation of (24), we obtain $\hat{\sigma} = 2.84$, which is indeed below its median estimate across all sectors of 4.4. So it appears that Imbs and Méjean are correct that pooling across sectors in the estimation of (24) gives a downward bias to the value of $\hat{\sigma}$.

However, the more important message of our paper is that the aggregate elasticity they compute by taking a weighted average of the sectoral estimates *does not indicate the impact of a devaluation on aggregate imports*. The reason for this is that the data which Imbs and Méjean (2009) use in their estimation is for imports only, without any matching domestic production data. Therefore, they are estimating the micro Armington elasticity. But in order to understand the impact of exchange rate changes on imports, as shown in (28), we need to use the macro Armington elasticity, about which they have no information. Therefore, their results cannot be interpreted as supporting either "elasticity optimism" or "elasticity pessimism," at least in regard to the impact of a devaluation on imports.[15]

When the macro Armington elasticity ω_g differs across goods g, then the impact of a devaluation cannot be obtained from the simple aggregate demand equation (28). Rather, we should instead compute the total derivatives of imports while adding up across sectors. This will yield a weighted average formula that is broadly similar to that found in Imbs and Méjean (2009), but now using the macro Armington elasticities ω_g rather than the micro elasticities σ_g. For simplicity, we omit the home country j superscript in this calculation.

Total imports of the home country are given by

$$V^F = \sum_{g=1}^{G} V_g^F = \sum_{g=1}^{G} P_g^F C_g^F = \sum_{g=1}^{G} \left[P_g^F \alpha_g (1 - \beta_g) \left(\frac{P_g^F}{P_g} \right)^{-\omega_g} \left(\frac{P_g}{P} \right)^{-\eta} C \right].$$

[13] We note that this result is not the same as the aggregation bias actually suggested by Orcutt (1950). Orcutt was concerned with the case, typical in the macroeconomic literature on trade equations, in which disaggregate sectoral data are not used, but instead, a single equation for aggregate imports is estimated as in the last section's summation over all trade partners and goods.

[14] When we apply a grid search over values $\hat{\sigma}_g > 1$ and $0 \leq \hat{\rho}_{1g} < 1$, following Broda and Weinstein, then in the pooled dataset we obtain $\hat{\sigma}_g = 49.05$. But we view this value as uninformative since the initial estimation fails to give us $\hat{\sigma}_g > 1$.

[15] As we note in the concluding section, the impact of a devluation on *exports* will depend on both the "micro" and "macro" elasticities found in foreign countries. So Imbs and Méjean (2009) are providing some optimism regarding the elasticity of exports with respect to the real exchange rates.

Assume that $P_g^F = EP_g^{F*}$, with P_g^{F*} fixed, implying full immediate pass-through from the exchange rate to import prices. Then for a given level of consumption C,

$$\begin{aligned}
\frac{d\ln V^F}{d\ln E} &= \frac{E}{V^F}\frac{dV^F}{dE} \\
&= 1+\left(\frac{E^2}{V^F}\right)\frac{d}{dE}\sum_{g=1}^{G}\left[P_g^{F*}\alpha_g(1-\beta_g)\left(\frac{P_g^F}{P_g}\right)^{-\omega_g}\left(\frac{P_g}{P}\right)^{-\eta}C\right] \\
&= 1+\sum_{g=1}^{G}\left(\frac{E^2}{V^F}\right)\frac{d}{dE}\left[P_g^{F*}\alpha_g(1-\beta_g)\left(\frac{P_g^F}{P_g}\right)^{-\omega_g}\left(\frac{P_g}{P}\right)^{-\eta}C\right].
\end{aligned}$$

In the appendix we simplify this equation to obtain:

$$\frac{d\ln V^F}{d\ln E} = 1-\sum_{g=1}^{G}(1-m_g)w_g^F\omega_g - \eta\sum_{g=1}^{G}w_g^F\left(m_g-m\right), \tag{33}$$

where $w_g^F \equiv V_g^F/V^F$ is the share of good g in total imports, $m_g \equiv V_g^F/V_g$ is the import share of good g, and $m \equiv V^F/V$ is the share of imports (of all goods) in total consumption spending.

The intuition for (33) is that the first term of unity is the valuation effect, which the other effects must offset for a devaluation to reduce the value of imports. The second term reflects the impact of the rise in E on $\left(P_g^F/P_g\right)^{-\omega_g}$: this negative effect is smaller when a bigger share of good g is imported, because the percent rise in P_g will then be closer to that in P_g^F. The third and last term reflects the impact on $(P_g/P)^{-\eta}$: the negative influence of demand for good g is larger when good g has a higher than average import share (P_g will then rise relative to P). An alternative way to group the preceding terms would be as

$$\frac{d\ln V^F}{d\ln E} = 1-\sum_{g=1}^{G}w_g^F\left[\omega_g - m_g\left(\omega_g-\eta\right)\right] + m\eta.$$

Given estimates of η and data on import shares, it is straightforward to calculate the preceding devaluation elasticity. We see from this formula that if goods with higher macro Armington elasticities – or more precisely a higher value of $[\omega_g - m_g(\omega_g-\eta)]$ – also have a higher share of imports w_g^F, then they will contribute more towards obtaining a negative value for this devaluation elasticity. In this respect we agree with Imbs and Méjean (2009);

but contrary to them, the Armington elasticities appearing in the formula are the macro and not the micro elasticities.

6.2 Measurement of Trade Costs

Since the work of Dekle, Eaton, and Kortum (2007) it has been known that the Eaton and Kortum (2002) model, or its close cousin, the gravity equation, can be used to predict changes in trade flows. That approach has been taken by a number of authors to explain the contraction in trade during the recent financial crisis: for example, Eaton, Kortum, Neiman and Romalis (2010) used an extended version of the Dekle, Eaton, and Kortum model, while Jacks, Meissner and Novy (2009a,b) use the simple gravity equation. The latter authors use the gravity equation to measure the change in *trade costs* implied by a contraction in trade, as also done by Head and Ries (2001), for example, for the expansion of trade due to NAFTA. We will argue that this measurement of trade costs is no longer accurate when preferences take the nested CES form used here, and that quite different explanations can arise for any contraction or expansion in trade.

Let us start with the disaggregate imports demand equation in (13). The term P_g^{ij} appearing there refers to an aggregate of the prices of individual varieties exported from country i to country j, inclusive of trade costs. Let us rewrite that term as $P_g^{ij} = P_g^{ii}\tau_g^{ij}$,where P_g^{ii} denotes the price index of f.o.b. prices in country i, and τ_g^{ij} denotes an aggregate of the trade costs from country i to country j. We then express imports from country i relative to home demand as:

$$\frac{V_g^{ij}}{V_g^{jj}} = \kappa_g^{ij}\left(\frac{1-\beta_g^j}{\beta_g^j}\right)\left(\frac{\tau_g^{ij}P_g^{ii}}{P_g^{jj}}\right)^{1-\sigma_g}\left(\frac{P_g^{Fj}}{P_g^{jj}}\right)^{\sigma_g-\omega_g}. \tag{34}$$

It follows that we can construct the following measure of trade costs:

$$\left[\left(\frac{V_g^{ij}}{V_g^{jj}}\right)\left(\frac{V_g^{ji}}{V_g^{ii}}\right)\right]^{\frac{1}{1-\sigma_g}} = \left[\kappa_g^{ij}\kappa_g^{ji}\left(\frac{1-\beta_g^j}{\beta_g^j}\right)\left(\frac{1-\beta_g^i}{\beta_g^i}\right)\right]^{\frac{1}{1-\sigma_g}}\left(\tau_g^{ij}\tau_g^{ji}\right)\left[\left(\frac{P_g^{Fj}}{P_g^{jj}}\right)\left(\frac{P_g^{Fi}}{P_g^{ii}}\right)\right]^{\frac{\sigma_g-\omega_g}{1-\sigma_g}}. \tag{35}$$

The middle term in this equation, $\left(\tau_g^{ij}\tau_g^{ji}\right)$, measures trade costs between country i and country j. These are adjusted by the taste parameters appearing first on the right, which are often ignored.[16] Our focus is on the last bracketed term on the right. Notice that if the macro elasticity equals the micro elasticity, $\omega_g = \sigma_g$, then this final bracketed term vanishes. In that case, the product of the trade *ratios* on the left-hand side of (35) becomes an estimate of the product of the trade *costs* on the right, again adjusted by the micro Armington elasticity. In other words, movements in the trade ratios on the left are used to infer changes in trade costs on the right. This is the approach taken by Jacks, Meissner and Novy (2009a,b).

However, when the macro elasticity *does not* equal the micro elasticity, then it is evident that this simple approach to measure the change in trade costs is untenable. In the case we have found where $\omega_g < \sigma_g$, then either a rise in domestic prices $P_g^{jj}P_g^{ii}$, or a fall in import prices $P_g^{Fj}P_g^{Fi}$, would have the effect of lowering the trade ratios on the left, even if there is no change in trade costs. A rise in domestic prices could come, for example, from a credit crunch that constrains the variety of domestic goods (making the loans needed to obtain these goods unavailable): in that case, the rise in the "true" CES indexes P_g^{jj} or P_g^{ii} would lead to a fall in the trade ratios, which would be mistakenly interpreted as a rise in trade costs if $\omega_g = \sigma_g$ was assumed.[17] We are not arguing that such changes in domestic activity are necessarily the most important explanation for changes in trade flows. Rather, our point is that the simple mapping between the bilateral trade ratios and bilateral trade costs, exploited by Jacks, Meissner and Novy (2009a,b) and other authors, no longer holds true in the nested CES model.

7 Conclusions

In this paper we develop a new data set of highly disaggregated concorded domestic production and import data for the United States. These data allow us to simultaneously estimate at the product level both the substitution elasticity between different foreign im-

[16] Note that these taste parameters cancel out if they are constant and we take the ratio of (**??**) over two time periods.

[17] Of course, we should also consider the potential fall in traded varieties due to credit constraints, in which case it is the changes in the import prices P_g^{Fj} and P_g^{Fi} (these are inclusive of trade costs) relative to the domestic prices P_g^{jj} and P_g^{ii} that determines the trade ratios.

port sources and the substitution elasticity between domestic and foreign import sources. These two elasticities are conceptually quite distinct, except within the two-country models that predominate in macroeconomic discussion. They are also empirically quite distinct, despite the tendency in some recent literature to conflate them. We find overwhelming evidence in our data that the former elasticity—which we call the "micro" Armington elasticity—is much larger than the latter elasticity—the "macro" Armington elasticity. Our median estimate of the micro elasticity across individual industries is 4.4, whereas the macro elasticities tend to be in the neighborhood of unity regardless of sector.

Interestingly, values around unity are also common in the various studies of substitution between domestic and imported goods carried out over decades by researchers who used datasets that were more highly aggregated than ours. In contrast to these earlier works, ours is the first to estimate the micro and macro elasticities simultaneously at the disaggregate level for a number of products. We also frame the analysis within a theoretical general-equilibrium trade model, based on Chaney (2008), as a guide to both econometric specification and simulation analysis of alternative estimation approaches. Furthermore, our econometric methodology, based on Feenstra (1994), corrects for potential biases in OLS estimation, including the errors introduced by reliance on unit-value price indexes rather than the exact indexes implied by theory.

Given these contrasts with earlier, more aggregative studies, why do we reach a similar conclusion regarding the size of macro Armington elasticities? Our theoretical analysis of our model's import equations suggests an answer. In our setup, aggregation over trade partners is never a problem, whereas aggregation across goods is not a big problem if macro elasticities mostly cluster around unity. Finally, for such macro elasticities, we show that even the downward estimation bias due to mismeasured price indexes will be small in the aggregate trade equation, making simple OLS estimation potentially quite informative. In contrast, we document that for estimating micro Armington elasticities, unit-value price measures lead to substantial downward bias.

The empirical findings raise the question of why substitution between home goods and imports should be more difficult than substitution between different foreign supply sources. Blonigen and Wilson (1999) documented several factors influencing the size of macro Armington elasticities across sectors, but to our knowledge there has been no corresponding study of micro elasticities. Anderson, de Palma, and Thisse (1992), shows that a CES indirect utility function for the aggregate consumer can be derived from certain discrete choice models with random utility. In this framework, a relatively smaller elasticity for the macro Armington elasticity is obtained if the variance of the random utility component between home and foreign goods in general is greater than the variance of the random utility component between two foreign varieties.[18] One theoretical answer might come from the theory of discrete choice under uncertainty. Given our strong empirical findings, the question certainly deserves more study.[19]

We close by emphasizing that while the macro Armington elasticity, which we have labeled ω, is the prime determinant of aggregate *import* response to a terms of trade change, the overall *trade balance* sensitivity may depend powerfully on the micro elasticity governing substitution between alternative foreign suppliers. Once one moves beyond the unrealistic assumption of a two-country world, it is evident that the *export* response to a terms of trade change depends not only on ω, but also on the foreign-foreign substitution elasticities that we labeled σ above.

As an example, suppose that the Korean won depreciates against all trading-partner currencies. Three things will happen. First, Korean residents will switch consumption from imports to domestic import-competing firms with elasticity ω. Second, consumers and firms outside Korea will switch from domestic goods competing with Korean exports to Korean exports with elasticity ω. But third, consumers and firms outside Korea will switch their demand from Korea's export competitors to Korea with elasticity σ. (For example, United States residents will import more ships and steel from Korea, less from China.) Thus, the overall effect of currency depreciation on Korea's net exports depends on both σ and ω. Because σ is apparently quite a bit larger than ω, there may be grounds for some degree of "elasticity optimism" after all.

[18] As shown by Feenstra (2004, Appendix B) who builds on Anderson, de Palma and Thisse (1992) to consider the nested CES case.

[19] Another possibility is related trade policy (which is also considered by Blonigen and Wilson 1999). Local content requirements and balanced-trade restrictions will obviously cause an asymmetry between domestic-foreign and foreign-foreign substitution. Those practices were definitively banned under the Uruguay Round of 1994, however, which became effective shortly after the start of our data sample.

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A Equations used in the Simulation

[To be added]

B Rank Condition in GMM estimation

Consider equation (24) as a regression run over countries $i = 1, \ldots, J, i \neq j$, and consider the third and fourth variables on the right, $\theta_{ng}\bar{X}^{ij}_{ng}$, $n = 3, 4$. Using the definition of these variables and their coefficients, we can substitute from (18) to readily compute the difference between them as:

$$\theta_{3g}\bar{X}^{ij}_{3g} - \theta_{4g}\bar{X}^{ij}_{4g} = \frac{(\sigma_g - \omega_g)}{(\sigma_g - 1)(1 - \rho_{1g})}\bar{X}^{ij}_{3g} + (\sigma_g - \omega_g)\theta_{4g}\bar{X}^{j}_{5g} + \theta_{4g}\left[\overline{\Delta \ln(\frac{UV^{Fj}_g}{UV^{jj}_g})\varepsilon^{ij}_g}\right].$$

We see that a linear combination of the vectors $\bar{X}^{ij}_{3g}$, $\bar{X}^{ij}_{4g}$ and $\bar{X}^{j}_{5g}$ equal the vector $\theta_{4g}\left[\overline{\Delta \ln(UV^{Fj}_g/UV^{jj}_g)\varepsilon^{ij}_g}\right]$, $i = 1, \ldots, J, i \neq j$. Since the variable $\bar{X}^{j}_{5g}$ does not vary over i, it acts like a constant term in equation (24) and its coefficient could not be distinguished from another constant included for measurement error in the unit-values. Therefore, the coefficients of $\bar{X}^{ij}_{3g}$ and $\bar{X}^{ij}_{4g}$ are all that identify ω_g and ρ_{3g} in the regression (24). If $\underset{T\to\infty}{plim}\left[\overline{\Delta \ln(UV^{Fj}_g/UV^{jj}_g)\varepsilon^{ij}_g}\right]$ itself is constant over $i = 1, \ldots, J, i \neq j$, then the variables $\bar{X}^{ij}_{3g}$ and $\bar{X}^{ij}_{4g}$ are co-linear with a constant term and we fail to identify ω_g and ρ_{3g} from this regression.

The condition that $\underset{T\to\infty}{plim}\left[\overline{\Delta \ln(UV^{Fj}_g/UV^{jj}_g)\varepsilon^{ij}_g}\right]$ is constant over $i = 1, \ldots, J, i \neq j$, can be interpreted by using the above equation for ε^{ij}_g, together with the definition of $\Delta \ln UV^{Fj}_g$ as the weighted average of the unit-values from each foreign country using the logarithmic mean of their import shares defined in (17). Multiply the above equation for ε^{ij}_g by the shares w^{ij}_g and sum over $= 1, \ldots, J, i \neq j$. We find that $\sum_{i=1,i\neq j}^{J}\Delta \ln(V^{ij}_g/V^{jj}_g)w^{ij}_g = 0$ from the definition of w^{ij}_g in (17), and also $\sum_{i=1,i\neq j}^{J}\Delta \ln(UV^{ij}_g/UV^{jj}_g)w^{ij}_g = \Delta \ln(UV^{Fj}_g/UV^{jj}_g)$. It follow that:

$$\sum_{i=1,i\neq j}^{J}\varepsilon^{ij}_g w^{ij}_g = (\omega_g - 1)\,\Delta \ln(UV^{Fj}_g/UV^{jj}_g).$$

Therefore, $\underset{T\to\infty}{plim}\left[\overline{\Delta \ln(UV^{Fj}_g/UV^{jj}_g)\varepsilon^{ij}_g}\right] = \underset{T\to\infty}{plim}\sum_{i=1,i\neq j}^{J}\overline{\varepsilon^{ij}_g\varepsilon^{kj}_g w^{ij}_g}\,/\,(\omega_g - 1)$. It is evident that this term will depend on heteroskedasticity in the demand shocks along, along with the relative size of foreign countries as measured by their imports shares into country j. If the demand shocks are homoskedastic and if foreign countries are of symmetric size, then $\underset{T\to\infty}{plim}\sum_{i=1,i\neq j}^{J}\overline{\varepsilon^{ij}_g\varepsilon^{kj}_g w^{ij}_g}$ is constant over $k = 1, \ldots, J, k \neq j$, so that ω_g is not identified from data for a single good. By the same argument, if demand shocks and/or foreign country size differ *across goods*, then that variation can identify ω by running (24) across goods under Assumption 3.

C Derivation of Equation (29)

Observe that,

$$
\begin{aligned}
& \left[\beta^j \left(\bar{P}^{Hj}\right)^{1-\omega} + (1-\beta^j)\left(\bar{P}^{Fj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} \\
= & \left\{\sum_g \alpha_g^j \left(\frac{P_g^j}{P^j}\right)^{\omega-\eta} \left[\beta^j \left(P_g^{jj}\right)^{1-\omega} + (1-\beta^j)\left(P_g^{Fj}\right)^{1-\omega}\right]\right\}^{\frac{1}{1-\omega}} \\
= & \left(P^j\right)^{\frac{\eta-\omega}{1-\omega}} \left[\sum_g \alpha_g^j \left(P_g^j\right)^{1-\eta}\right]^{\frac{1}{1-\omega}} \\
= & \left(P^j\right)^{\frac{\eta-\omega}{1-\omega}} \left(P^j\right)^{\frac{1-\eta}{1-\omega}} = P^j,
\end{aligned}
$$

where the second equality above follows from

$$
P_g^j = \left[\beta^j \left(P_g^{jj}\right)^{1-\omega} + \left(1-\beta^j\right)\left(P_g^{Fj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} .
$$

If, contrary to what the main text assumes, β_g^j actually differs across goods, define

$$
\check{P}^{Hj} \equiv \left[\sum_g \alpha_g^j \beta_g^j \left(\frac{P_g^j}{P^j}\right)^{\omega-\eta} \left(P_g^{jj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}},\ \check{P}^{Fj} \equiv \left[\sum_g \alpha_g^j \left(1-\beta_g^j\right) \left(\frac{P_g^j}{P^j}\right)^{\omega-\eta} \left(P_g^{Fj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} .
$$

Then

$$
P^j = \left[\left(\check{P}^{Hj}\right)^{1-\omega} + \left(\check{P}^{Fj}\right)^{1-\omega}\right]^{\frac{1}{1-\omega}} .
$$

D Derivation of Equation (33)

As derived in the test,

$$\frac{d\ln V^F}{d\ln E} = 1 + \sum_{g=1}^{G} \left(\frac{E^2}{V^F}\right) \frac{d}{dE} \left[P_g^{F*} \alpha_g (1-\beta_g) \left(\frac{P_g^F}{P_g}\right)^{-\omega_g} \left(\frac{P_g}{P}\right)^{-\eta} C \right].$$

Let's analyze the last sum term by term. Observe that we can write a generic term in the summation as

$$\begin{aligned} & \left(\frac{E P_g^F C_g^F}{V^F P_g^{F*} C_g^F}\right) \frac{d}{dE} \left[P_g^{F*} \alpha_g (1-\beta_g) \left(\frac{P_g^F}{P_g}\right)^{-\omega_g} \left(\frac{P_g}{P}\right)^{-\eta} C \right] \\ = \; & w_g^F \frac{d}{d\ln E} \ln \left[P_g^{F*} \alpha_g (1-\beta_g) \left(\frac{P_g^F}{P_g}\right)^{-\omega_g} \left(\frac{P_g}{P}\right)^{-\eta} C \right], \end{aligned}$$

where

$$w_g^F \equiv V_g^F / V^F.$$

So we compute

$$\frac{d}{d\ln E} \left[-\omega_g \ln P_g^F + (\omega_g - \eta) \ln P_g - \eta \ln P_g + \eta \ln P + \text{ constants} \right].$$

The result is

$$-\omega_g + (\omega_g - \eta) m_g + \eta \sum_g m_g w_g$$

where

$$m_g \equiv \frac{V_g^F}{V_g}, \; w_g \equiv \frac{V_g}{V}.$$

Thus each term in the summation above is given by

$$\begin{aligned} & \left(\frac{E^2}{V^F}\right) \frac{d}{dE} \left[P_g^{F*} \alpha_g (1-\beta_g) \left(\frac{P_g^F}{P_g}\right)^{-\omega_g} \left(\frac{P_g}{P}\right)^{-\eta} C \right] \\ = \; & w_g^F \left[-\omega_g + (\omega_g - \eta) m_g + \eta \sum_{g=1}^{G} m_g w_g \right] \end{aligned}$$

and so

$$\frac{d\ln V^F}{d\ln E} = 1 + \sum_{g=1}^{G} w_g^F \left[-\omega_g + (\omega_g - \eta) m_g + \eta \sum_{g=1}^{G} m_g w_g \right].$$

A first simplification is to note that, because $\sum_{g=1}^{G} w_g^F = 1$, the last equation becomes

$$\begin{aligned}
\frac{d\ln V^F}{d\ln E} &= 1 - \sum_{g=1}^{G} w_g^F \omega_g + \sum_{g=1}^{G} w_g^F m_g (\omega_g - \eta) + \eta \sum_{g=1}^{G} m_g w_g \\
&= 1 - \sum_{g=1}^{G} (1 - m_g) w_g^F \omega_g + \eta \sum_{g=1}^{G} m_g \left(w_g - w_g^F \right).
\end{aligned}$$

Note further that

$$m_g w_g = \frac{V_g^F}{V_g} \frac{V_g}{V} = \frac{V_g^F}{V^F} \frac{V^F}{V} = w_g^F m,$$

where

$$m \equiv V^F / V$$

is the share of imports (of all goods) in total consumption spending. Thus, we rewrite the derivative above in the final form $\frac{d\ln V^F}{d\ln E} = 1 - \sum_{g=1}^{G} (1 - m_g) w_g^F \omega_g - \eta \sum_{g=1}^{G} w_g^F \left(m_g - m \right).$

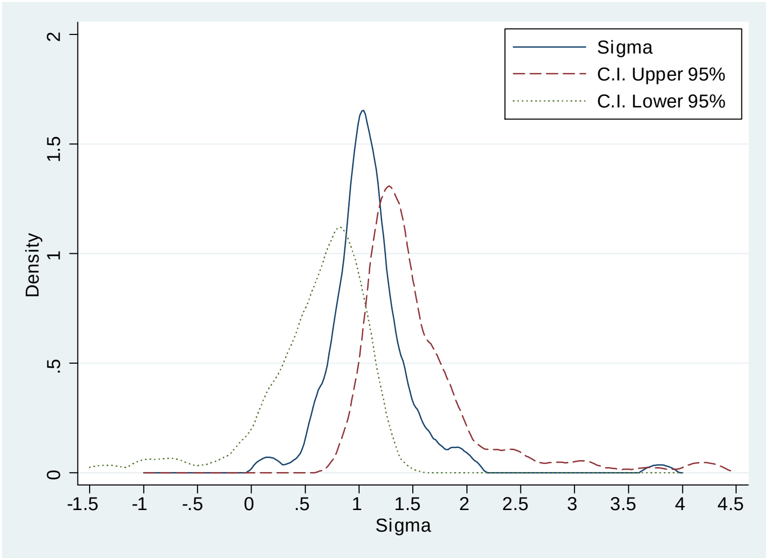

Figure 1: OLS results for Sigma

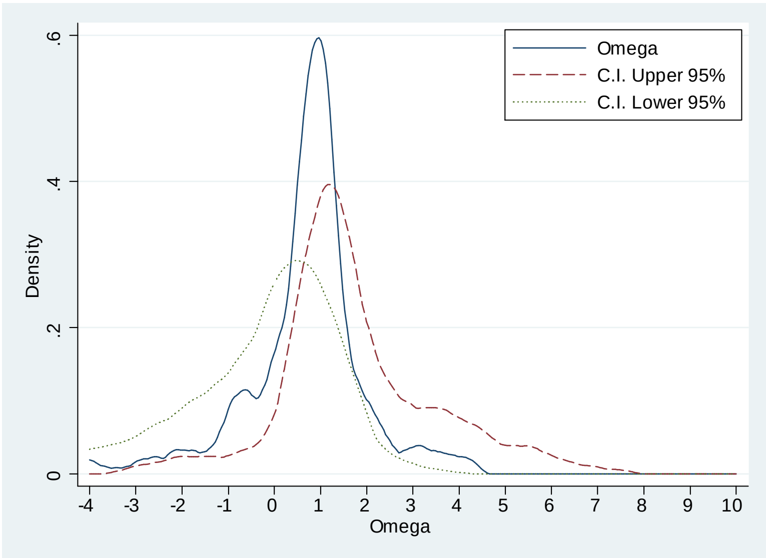

Figure 2: OLS results for omega

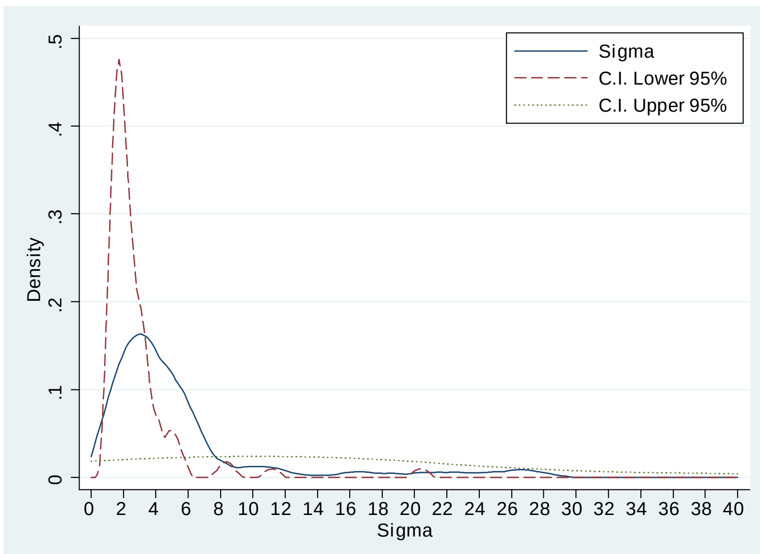

Figure 3: GMM results of Sigma