Is the Potential for International Diversification Disappearing? A Dynamic Copula Approach

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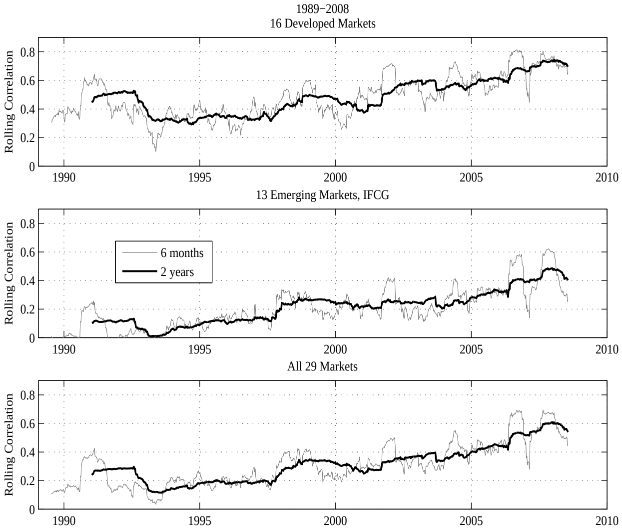
Motivation

Understanding the evolution of co-movements in international markets is crucial for asset pricing and portfolio selection

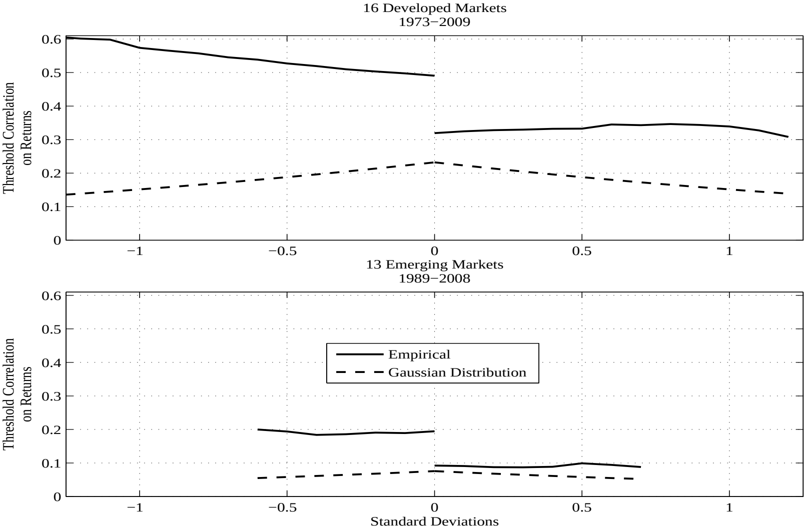
Research Questions

1. How and has cross-country dependence changed through time ?
   - Cross-country linear correlations have not increased (Bekaert, Hodrick, and Zhang (2009))
2. Is correlation a satisfactory dependence measure in international markets?
   - Correlations are higher in down markets (Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002))
3. How does the diversification benefit of emerging markets compare to developed countries?
   - Differences in the evolution of correlations?
   - Differences in tail dependence?

Average Linear Rolling Correlation on Weekly Returns



Average Threshold Correlations on Weekly Returns



Key Contributions

Our key contributions are

1. We develop a model which
   - can be estimated on a large set of countries
   - can accommodate for
     - dynamic dependence
     - a trend in correlation
     - positive tail dependence
     - univariate and multivariate asymmetries
2. We develop a diversification benefit measure that takes into account higher order moments

Key Results

Our key results are

1. Cross-country dependence has significantly increased over time
   - Dependence for emerging markets is still a lot lower than for developed countries
2. We find overwhelming evidence of non-normalities in dependence
   - Tail dependence is both positive and asymmetric for developed and emerging markets
3. We confirm with different panel regressions that
   - dependence is positively linked to volatility
   - although dependence is related to market integration, financial, and macro variables, the time trend is still significant and remains unexplained

Multivariate Model

We decompose the conditional multivariate log-likelihood function as

$$L = \sum_{t=1}^{T} \sum_{i=1}^{N} \underbrace{\log\left(f_{i,t}\left(R_{i,t}\right)\right)}_{\text{VOLATILITY MODEL FOR COUNTRY } i} + \sum_{t=1}^{T} \underbrace{\log\left(c_t\left(F_{1,t}\left(R_{1,t}\right), F_{2,t}\left(R_{2,t}\right), ..., F_{N,t}\left(R_{N,t}\right)\right)\right)}_{\text{DEPENDENCE MODEL FOR N COUNTRIES}}$$

where

T is the number of weeks in our sample

- developed markets 1973-2009
- emerging markets 1989-2008
- investable emerging market 1995-2009

N is the number of countries used in the estimation

- 16 developed markets
- 13 emerging markets
- 17 investable emerging markets

Volatility Model for Each Country

We decompose the conditional multivariate log-likelihood function as

$$L = \sum_{t=1}^{T} \sum_{i=1}^{N} \underbrace{\log\left(f_{i,t}\left(R_{i,t}\right)\right)}_{\text{VOLATILITY MODEL FOR COUNTRY } i} + \sum_{t=1}^{T} \underbrace{\log\left(c_t\left(F_{1,t}\left(R_{1,t}\right), F_{2,t}\left(R_{2,t}\right), ..., F_{N,t}\left(R_{N,t}\right)\right)\right)}_{\text{DEPENDENCE MODEL FOR N COUNTRIES}}$$

where

$f_{i,t}\left(R_{i,t}\right)$ is given by a AR-NGARCH model

$$\begin{aligned} R_{i,t} &= \mu_{i,t} + \sigma_{i,t} z_{i,t} \\ \sigma_{i,t}^2 &= \omega_i + \alpha_i \left(\varepsilon_{i,t-1} - \gamma_i \sigma_{i,t-1}\right)^2 + \beta_i \sigma_{i,t-1}^2 \end{aligned}$$

2 sources of univariate asymmetry

1. leverage effect $\Rightarrow \gamma_i$
2. residual asymmetry $\Rightarrow z_{i,t}$ comes from an asymmetric t distribution

Dependence Model

We decompose the conditional multivariate log-likelihood function as

$$L = \sum_{t=1}^{T} \sum_{i=1}^{N} \underbrace{\log\left(f_{i,t}\left(R_{i,t}\right)\right)}_{\text{VOLATILITY MODEL FOR COUNTRY } i} + \sum_{t=1}^{T} \underbrace{\log\left(c_t\left(F_{1,t}\left(R_{1,t}\right), F_{2,t}\left(R_{2,t}\right), \ldots, F_{N,t}\left(R_{N,t}\right)\right)\right)}_{\text{DEPENDENCE MODEL FOR N COUNTRIES}}$$

where

$c_t\left(F_{1,t}\left(R_{1,t}\right), \ldots\right)$ comes from a skewed t copula with

- Ψ_t a time-varying correlation matrix
- ν a degree-of-freedom parameter
- λ an asymmetry parameter

The Dynamic Asymmetric Copula Model

The copula correlation matrix is time-varying

At time t, it is given by a weighted average of 3 components

$$\Gamma_t = (1 - \beta_\Gamma - \alpha_\Gamma)\left[(1 - \varphi_\Gamma)\Omega + \varphi_\Gamma \Upsilon_t\right] + \beta_\Gamma \Gamma_{t-1} + \alpha_\Gamma z^*_{t-1} z^{*\top}_{t-1}$$

where

Υ_t captures a deterministic trend $\Upsilon_t = \frac{\delta^2 t^2}{1+\delta^2 t^2}$

Γ_{t-1} is the lagged correlation matrix

$z^*_{t-1} z^{*\top}_{t-1}$ is the cross-product of *copula shocks*

Estimation for Many Countries

Estimation on many countries is made possible by two improvements

1. We use a moment estimator for Ω

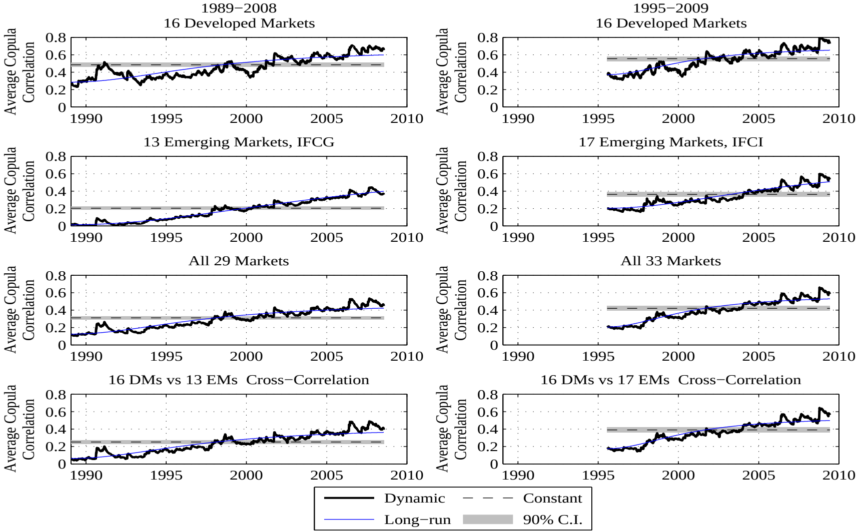
$$\hat{\Omega} = \frac{\frac{1}{T}\sum_{t=1}^{T} \bar{z}_t^* \bar{z}_t^{*\top} - \varphi_\Gamma \frac{1}{T}\sum_{t=1}^{T} \Upsilon_t}{1 - \varphi_\Gamma}$$

where $\frac{1}{T}\sum_{t=1}^{T} \bar{z}_t^* \bar{z}_t^{*\top}$ is the sample copula correlation

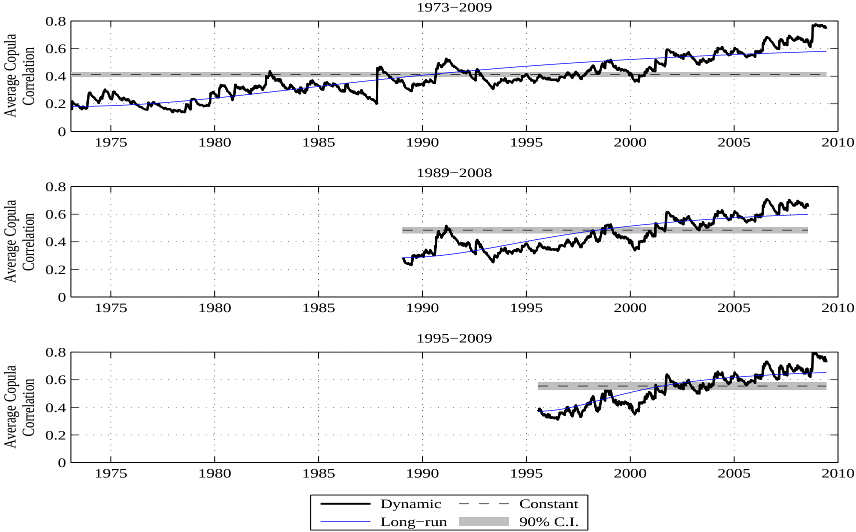
2. From Engle, Shephard and Sheppard (2008), we maximize the composite log-likelihood

$$CL(\theta) = \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j>i} \underbrace{\ln c_t(\eta_{i,t}, \eta_{j,t}; \theta)}_{\text{Bivariate log-likelihood for countries } i \text{ and } j}$$

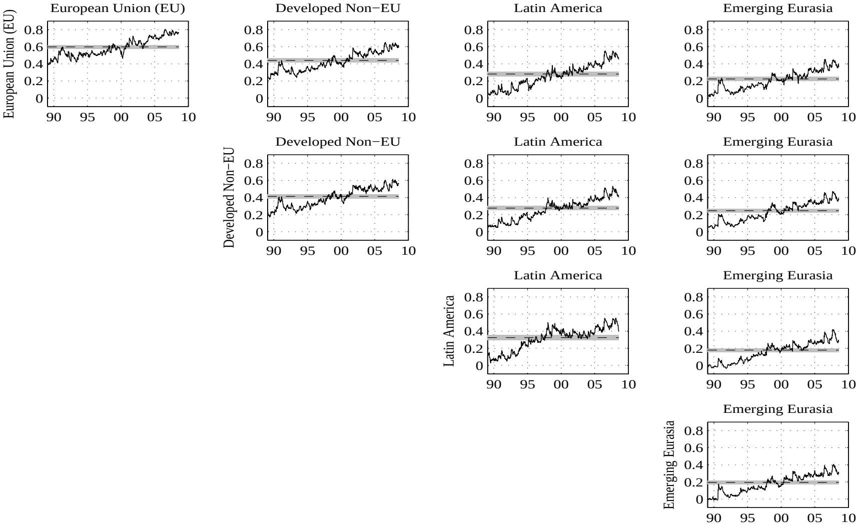
Evolution of Average Copula Correlation



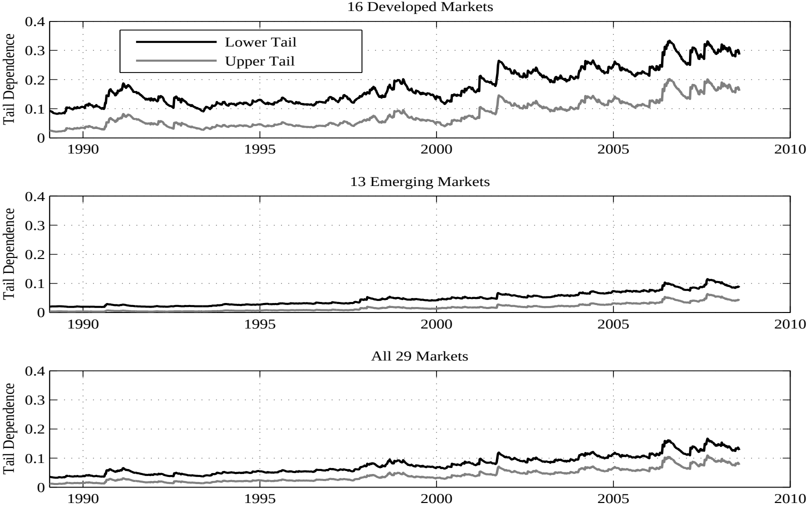
Evolution of Copula Correlation for Developed Markets



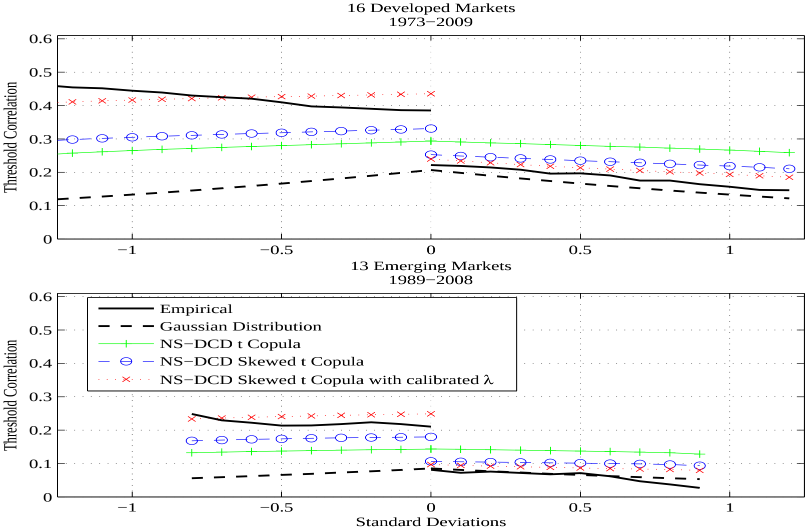
Evolution of Regional Copula Correlation



Evolution of Average Tail Dependence



Model Implied Threshold Correlation



A Conditional Diversification Benefit Measure

To take into account higher order moments in the portfolio return distribution, we construct a diversification benefit measure based on expected shortfall

$$ES_t^q(R_{i,t}) = -E\left[R_{i,t} | R_{i,t} \leq F_{i,t}^{-1}(q)\right]$$

Note that

$$\underbrace{\underline{ES}_t^q = VaR_t^q(w_t^\top R_t)}_{\text{Perfect diversification}} \leq \underbrace{ES_t^q(w_t^\top R_t)}_{\text{Portfolio expected shortfall}} \leq \underbrace{\overline{ES}_t^q = \sum_{i=1}^{N} w_{i,t} ES_t^q(R_{i,t})}_{\text{No diversification}}$$

We define

$$CDB_t(w_t, q) = \frac{\overline{ES}_t^q - ES_t^q(w_t^\top R_t)}{\overline{ES}_t^q - \underline{ES}_t^q}$$

A Conditional Diversification Benefit Measure

The conditional diversification benefit measure

1. lies between 0 and 1
2. does not depend of expected returns

The Special Case of Normality

If returns are multivariate normal and $q = 50\%$, then CDB_t reduces to

$$CDB_t(w_t, q) = 1 - \frac{\overbrace{\sqrt{w_t^\top \Sigma_t w_t}}^{\text{portfolio's volatility}}}{\underbrace{w_t^\top \sigma_t}_{\text{upper bound for portfolio's volatility}}}$$

Evolution of Diversification Benefit

Conclusion

1. We propose a new model capturing dynamic trending copula correlation, tail dependence, and multivariate asymmetries

2. We propose a conditional diversification benefit measure which takes into account higher order moments

We find that

1. Cross-country dependence has significantly increased over time

2. But dependence for emerging markets is still lower than for developed countries