Financial Intermediaries and the Yield Curve*

Andrés Schneider§

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Abstract

I study the yield curve dynamics in a general equilibrium model with financial intermediaries facing financing constraints. The economy features a positive real term premium in equilibrium, stemming from the fact that constraints may occasionally bind. A flat yield curve and loose financial conditions are associated with lower future credit growth because there is a higher probability the economy hits financing constraints in the near term. I show this mechanism 1) rationalizes why a flattening of the yield curve precedes recessions; and 2) rationalizes why the term structure of distributions of future real outcomes are negatively skewed when financial conditions are tight.

JEL classification: E44, G12, G21, G24.

Keywords: Yield Curve, Financial Intermediaries, Term Premium, Financing Constraints

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§Federal Reserve Board; andres.m.schneider@frb.gov.

1 Introduction

Financial intermediaries are key players in a variety of financial markets, particularly in long-term assets. Fluctuations in long-term asset values can lead to tighter (or looser) financing constraints, affecting intermediaries' marginal valuations. Long-term yields are therefore affected, because long-term yields are the forecast of marginal valuations. The direct link among financing constraints, marginal valuations, and long-term yields suggests the yield curve contains information about the health of the financial system, and viceversa.

In this paper I propose a general equilibrium model to study the connection between financial intermediaries and the yield curve. The model is able to rationalize the salient puzzling properties of the U.S. yield curve, where the central piece in the analysis is the intermediaries' financing constraints. In particular, the model features an average upward sloping real yield curve and highly volatile long-term yields, purely driven by the fact that financial intermediaries face occasionally binding constraints. Indeed, if intermediaries were always unconstrained, then the yield curve would be flat (i.e., long term yields equal to short term yields) and constant.

The mechanism is grounded in two main elements: intermediaries operate with leverage in equilibrium and they face financing constraints. These two elements have been extensively studied in the macro-finance literature but in this paper I focus the analysis on the yield curve.[1] To obtain leverage in equilibrium, I follow Brunnermeier and Sannikov (2014), among others, and I assume intermediaries are more efficient in handling risky assets. That is, financial intermediaries issue short-term deposits to

[1] Recent literature, reviewed below, has departed from the representative agent analysis of the yield curve, but without stressing the role of financing constraints—a salient characteristic of intermediaries.

savers to fund positions in long-term risky assets and take advantage of their relatively better investment technology. However, intermediaries' positions in long-term assets can be constrained in certain states of the world due to agency problems as in Gertler and Kiyotaki (2015). As a consequence, if intermediaries hit their constraints, they are forced to sell risky assets to less efficient savers, so prices decline, the aggregate price of risk increases, financial intermediaries wealth deteriorates even further, which force intermediaries to reallocate their portfolios, and so on. This well-know feedback mechanism has important implications for the yield curve, which I detail next.

The presence of occasionally binding constrains implies the economy features a bimodal distribution: it spends the vast majority of time in a "normal regime", where constraints are slack, risk premia are low, the real interest rate is low, and volatility of asset prices is moderate. When negative aggregate shocks occur, the economy can enter in a "crisis regime". Here intermediaries reallocate their portfolios and wealth is transferred to inefficient savers. This inefficiency pushes the consumption level persistently below the trend growth and therefore real interest rate persistently increases as agents perceive the "crisis regime" as transitory—consumption level will recover its trend in the future. But this occurs precisely when the price of risk spikes, implying that real bond prices go down in value in states in which the marginal investor value those resources the most—a "crisis regime". Thus, real bonds carry an endogenously time varying term premium and the yield curve is upward sloping on average, due to the fact there is always a non-zero probability the economy can hit financing constraints.

Besides accounting for the salient properties of the yield curve (positive term premium and highly volatile long-term yields), I show the mechanism relating financial intermediary wealth and the yield curve is able to rationalize interesting macroeconomic phenomena. These exercises are useful in the sense that they show the mechanism in the model is consistent with evidence beyond the scope of yield curve, therefore providing external validation of the key economic forces in the model.

First, there is ample reduced-form evidence indicating that a flattening of yield curve (i.e., long-term yields equal or lower than short-term yields) is associated with lower future economic activity. I rationalize this evidence through the lens of the model. The model predicts that the key connection between the slope of the yield curve and future economic activity is the quantity of credit intermediated in the economy. Put differently, in the model a flattening of the yield curve indicates credit conditions are slack (the economy has experienced a credit boom) and anticipates intermediaries could be constrained in the near future, should a sufficiently negative aggregate shock materialize. Intuitively, the yield curve flattens after opportunities to exploit the term spread have been exhausted, credit levels are high, and constraints are slack. However, intermediaries are leveraged and vulnerable to negative aggregate shocks. If negative shocks materialize, financial intermediaries could hit financing constraints and their ability to intermediate credit becomes unpaired: credit contracts and growth follows. I argue this mechanism is, at least partially, a reason for why a flattening of the yield curve precedes recessions: yield curve flattening anticipates lower credit growth, and thus lower economic activity.

Second, recent literature have stressed the role of financial conditions in driving the distribution of real variables in the near future (Adrian, Boyarchenko and Giannone, 2019; Giglio, Kelly and Pruitt, 2016). More precisely, when financial conditions deteriorate, the forecasted conditional distribution of GDP growth becomes more negatively skewed, with a lower mean and higher variance. Moreover, this distribution changes with the forecasted horizon: there is a term structure of conditional distributions that changes over the forecasted horizon. This object is intimately related with the yield curve, because long-term yields are conditional expectations of future variables (a point estimate), while the forecasted distribution includes computing the entire distribution of future realizations. To rationalize the evidence, I compute the time evolution of the conditional probability density function of consumption growth and intermediaries wealth across the horizon. This is the model's theoretical counterpart of the estimated conditional distributions in, for example, Adrian et al., 2019. I show the model captures the evidence relatively well: conditional on a state in which intermediaries are constrained (tight financial conditions), the term structure of conditional distributions of growth exhibit a negative skewness. The skewness in the term structure of conditional distributions becomes zero (or slightly positive) when conditioning to a state in which intermediaries are unconstrained (loose financial conditions).

Related literature. This paper relates to a strand of literature that has departed from the representative agent analysis of the yield curve. In this line, part of the literature has stressed the role of certain agents (arbitrageurs, intermediaries, etc) in explaining the yield curve dynamics, typically in a partial equilibrium setup (Vayanos and Vila 2009; Greenwood and Vayanos 2014; Haddad and Sraer 2018). Relative to this literature, the contribution of this paper is to use a general equilibrium framework to emphasize the importance of (potentially) constrained institutions in driving the yield curve dynamics.[2]

The general equilibrium framework I build on has been extensively studied in the

[2]Some other papers have studied the yield curve in a general equilibrium setup with heterogeneous agents (e.g., Wang 1996; Schneider 2018) but without financing constraints.

macro-finance literature, particularly after the Great Recession (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Gertler and Kiyotaki, 2015, among many others). Relative to this literature, the contribution of this paper is to shift the focus away from stocks (or "capital") to the yield curve dynamics. In particular, I show that financing constraints play a crucial role in producing an endogenously time-varying real term premium. Additional, I show the connection between the yield curve and financial intermediaries' wealth is important to understand why a flattening of the yield curve anticipates recessions, and also to understand why tight financial constraints imply a negatively skewed distribution of future economic outcomes.

2 Model

I present a general equilibrium model along the lines ofHe and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) and focus on the pricing implications for the yield curve. For simplicity, I abstract from inflation and highlight the real forces driving the yield curve dynamics. The model can be extended to include an exogenous inflation process following the seminal work of Cox, Ingersoll and Ross (1985), but the data suggest that persistent shocks to inflation explain a very small fraction of yields' variance (Duffee (2018)).

Time is continuous and denoted by $t > 0$. Aggregate output, denoted by y_t, evolves as

$$\frac{dy_t}{y_t} = \mu dt + \sigma dW_t,$$

where $\mu > 0$, $\sigma > 0$ are constants, and W_t is a standard Brownian motion in a complete probability space (Ω, F, P). The economy is populated by a continuum of savers (denoted by s) and a continuum of financiers (denoted by f). The main source of heterogeneity between f and s is that the former have a comparative advantage in operating risky assets over the latter—which implies f and s engage in borrowing and lending in equilibrium.

Agents can trade two classes of assets, namely long-term risky assets and short-term risk free deposits. First, there is a long-term asset in exogenous fixed supply, whose ex-dividend price I denote by q_t. This asset pays a dividend y_t per unit if held by f, but ωy_t, $\omega < 1$, if held by s. That is, it is more costly for savers than financiers to operate this risky asset.[3] The total return on investing in the dividend paying security consists of the dividend yield plus the capital gains. For financiers, this is

$$dR_{f,t} = \frac{y_t}{q_t}dt + \frac{dq_t}{q_t},$$

while for savers the total return is

$$dR_{s,t} = \frac{\omega y_t}{q_t}dt + \frac{dq_t}{q_t},\ \omega < 1.$$

Second, the short-term deposit account is in zero net supply and it yields a risk-free interest rate per unit of time, denoted by r_t. Finally, agents can also trade zero-coupon bonds of all maturities, which are also in zero net supply. I denote as $P_t^{(\tau)}$ the price of a bond that pays a unit of consumption in period $\tau + t$, and there is a continuum of bonds

[3]This assumption is equivalent to assume savers have to pay a cost to operate risky assets (Gertler and Kiyotaki (2015)). For tractability, I model savers' relative disadvantage as a wedge in the dividend yield (Brunnermeier and Sannikov (2014)).

$\left\{ P_t^{(\tau)} \right\}_{\tau \in (0,T]}$. Provided agents can trade shares the long-term asset, zero-coupons are redundant in the construction of the equilibrium, but they are useful to characterize the economy's equilibrium yield curve.

Savers choose how much to consume and save in order to maximize their expected discounted utility. They can allocate portfolios between risk-free deposits issued by financiers or risky assets. Their problem can be written as

$$U_t = \max_{c_t, \theta_{s,t}, \left\{ \theta_{s,t}^{(\tau)} \right\}} E_t \left[\int_t^\infty e^{-\rho(u-t)} \frac{c_u^{1-\gamma}}{1-\gamma} du \right],$$

subject to

$$dn_{s,t} = \left[n_{s,t} r_t - c_t + q_t \theta_{s,t} \left(E_t \left[dR_{s,t} \right] - r_t \right) + T_t \right] dt + \int_0^T P_t^{(\tau)} \theta_{s,t}^{(\tau)} \left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} d\tau - r_t dt \right) + q_t \theta_{s,t} \sigma_{q,t} dW_t, \tag{1}$$

where $n_{s,t}$ is the savers' net worth, $\theta_{s,t}$ is the holding of risky asset, c_t the consumption flow, and T_t the net transfers received from financiers' profits.

Financiers are in charge of managing a financial intermediary firm. They operate this firm by issuing deposits to savers as well as using their own wealth, $n_{f,t}$, but they face financing constrains (detailed below). To avoid financiers growing out of their constraints[4], I assume they pay dividends to savers with a Poisson probability λ. After paying dividends, new financiers receive a fraction $\overline{x}$ of total wealth to start the financial firm. Then, financiers' problem is to maximize the value of the firm (i.e., the expected

[4]Recall financiers possess a technological advantage over savers, a force that pushes financiers to absorb all the wealth in the economy.

discounted value of firms' wealth), that is

$$V_{f,t} = \max_{\theta_{f,t},\left\{\theta_{f,t}^{(\tau)}\right\}} E_t\left[\int_t^{\infty} \frac{m_u}{m_t}\lambda e^{-\lambda(u-t)} n_{f,u} du\right] \tag{2}$$

subject to

$$dn_{f,t} = \left[r_t n_{f,t} + q_t\theta_{f,t}\left(E_t\left[dR_{f,t}\right] - r_t\right)\right] dt + \int_0^T P_t^{(\tau)}\theta_{f,t}^{(\tau)}\left(\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} d\tau - r_t dt\right) + \theta_{f,t} q_t \sigma_{q,t} dW_t,\ n_{f,t} > 0, \tag{3}$$

where $m_t = e^{-\rho} c_t^{-\gamma}$ is savers' marginal utility and $\theta_{f,t}$ financiers' holdings of the risky asset. Financiers also face a financing constraint that limits their ability to issue deposits. Specifically, I follow Gertler and Kiyotaki (2015).and assume the value of the financial intermediary firm has to be greater than a fraction of the assets the firm holds

$$V_{f,t} \geq \kappa\left(\theta_{f,t} q_t + \int_0^T P_t^{(\tau)}\theta_{s,t}^{(\tau)} d\tau\right). \tag{4}$$

I next define a competitive equilibrium.

Definition 1 (Competitive equilbrium) *A competitive equilibrium is a set of aggregate stochastic processes: prices q_t, r_t, policy functions for savers $(\theta_{s,t}, c_t)$, policy function for financiers' $\theta_{f,t}$, the value of the financiers firm $V_{f,t}$, such that*

1. *Given prices, $\left(\theta_{s,t}, \left\{\theta_{s,t}^{(\tau)}\right\}, c_t\right)$ solves savers' problem*
2. *Given prices, $\left(\theta_{f,t}, \left\{\theta_{f,t}^{(\tau)}\right\}, V_{f,t}\right)$ solves financiers' problem*

3. Markets clear (long-term asset and consumption good)

$$\begin{aligned}\theta_{s,t} + \theta_{f,t} &= 1, \\ \theta_{s,t}^{(\tau)} + \theta_{f,t}^{(\tau)} &= 0, \ \forall \tau, \\ c_t &= \omega \theta_{s,t} y_t + \theta_{f,t} y_t.\end{aligned}$$

Before turning to the solution of the model, it is useful to characterize agents' optimization problems with their first order conditions. For savers,

$$r_t = -E_t \left[\frac{dm_t}{m_t} \right],$$

and

$$E_t \left[dR_{s,t} \right] - r_t dt \geq -E_t \left[\frac{dm_t}{m_t} dR_{s,t} \right]$$

with equality if households are holding long-term assets (i.e., $\theta_{s,t} > 0$). The optimality conditions for financiers requiere a few more steps, and it is useful to first write financiers' problem in a recursive way. First, notice that due to the linearity of financiers' objective function and constraints, the value function can be written as[5]

$$V_{f,t} = \psi_t n_{f,t}, \tag{5}$$

where $\psi_t \geq 1$ is an endogenous Ito process whose drift $\mu_{\psi,t}$ and diffusion $\sigma_{\psi,t}$ are solved

[5]See Gertler and Kiyotaki (2015).

in equilibrium. Then the financing constraint can be written as

$$\begin{aligned}
\psi_t n_{f,t} &\geq \kappa \theta_{f,t} q_t, \\
\psi_t &\geq \kappa \frac{\theta_{f,t} q_t}{n_{f,t}} + \int_0^T \frac{P_t^{(\tau)} \theta_{f,t}^{(\tau)}}{n_{f,t}} d\tau \equiv \alpha_{f,t} + \int_0^T \alpha_{f,t}^{(\tau)} d\tau,
\end{aligned}$$

where $\alpha_{f,t}$ is the endogenous financiers' portfolio share in the risky asset. The financiers' problem can be written in a recursive way as (i.e., the Hamilton-Jacobi-Bellman, HJB)

$$0 = \max_{\theta_{f,t}, \left\{\theta_{f,t}^{(\tau)}\right\}} \lambda \left(n_{f,t} - V_{f,t}\right) m_t dt + E_t \left[d\left(m_t V_{f,t}\right)\right] + \chi_t \left(V_{f,t} - \kappa \theta_{f,t} q_t\right) dt. \tag{6}$$

where χ_t is the Lagrange multiplier associated with the financing constraint. Using (5) in (6), the first order conditions for financiers can be written as

$$E_t \left[dR_{f,t}\right] - r_t \geq -E_t \left[\left(\frac{dm_t}{m_t} + \frac{d\psi_t}{\psi_t}\right) dR_{f,t}\right]$$

with equality if $\chi_t = 0$. Put differently, financiers are the marginal investors in long-term risky assets if their constraints are not binding. If financing constraints are binding, then their holdings in risky assets are pinned down by such constraints (i.e., $\psi_t = \alpha_{f,t}$), and savers are the marginal investors in risky assets.

Finally, I characterize the yield curve in the economy, which consists of the endogenous price vector $\left\{P_t^{(\tau)}\right\}_{\tau \geq 0}$. Yields can then be obtained simply as $y_t^{(\tau)} = -\log P_t^{(\tau)} / \tau$. The zero coupon bonds are risky assets in the sense that they convey a premium in equilibrium (i.e., there are endogenous fluctuations in the interest rate and in the price of risk). As with the long-term risky asset, savers are the marginal investor zero coupon bond when financiers are constrained. When financiers are unconstrained, their risk bearing capacity is high enough and they are the marginal investor. That is, by no-arbitrage, the expected excess return of zero-coupon bonds is

$$E_t\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right]-r_t dt=\begin{cases}-cov_t\left(\frac{dm_t}{m_t},\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right) & \text{if } \chi_t>0,\\ -cov_t\left(\left(\frac{dm_t}{m_t}+\frac{d\psi_t}{\psi_t}\right),\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right) & \text{if } \chi_t=0.\end{cases}$$

3 Model Solution

I use the homogeneity property of objective functions and constraints to solve the equilibrium in a recursive fashion, using a single endogenous state variable,

$$x_t=\frac{n_{f,t}}{q_t}\in[0,1]\,. \tag{7}$$

The endogenous state variable x_t follows an Ito process with drift $x\mu_{x,t}$ and diffusion $x\sigma_{x,t}$.The objective is to characterize the equilibrium with the optimality conditions for savers and financiers as a function of x.. That is, I solve for financiers marginal value, $\psi(x_t)$, rescaled risky asset $p(x_t)=q_t/y_t$, short-term risk free rate $r(x_t)$, zero-coupon bonds $\{P(x_t,\tau)\}_{\tau\geq 0}$, and financiers' portfolio share $\alpha_f(x_t)$. Then the equilibrium can be characterized by a system of non-linear ordinary differential equations on the state variable x_t. In particular, I use the financiers' HJB equation together with the asset pricing condition for savers when financiers are constrained and the asset pricing condition for financiers when they are unconstrained. Thus, a critical element for the solution is to find the value $x_t^*\in[0,1]$ at which financiers become constrained. The next proposition shows the system of equations.

Proposition 2 *The Markov equilibrium is characterized by the following system of ordinary differential equations. In the unconstrained region (i.e., $x > x^*$) the system is*

$$
\begin{aligned}
0 &= \frac{1}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) + \left(\sigma_\psi(x) - \gamma\sigma_c(x)\right)\left(\sigma_p(x) + \sigma\right), \\
0 &= \frac{\lambda(1-\psi(x))}{\psi(x)} + \mu_\psi(x) - \gamma\sigma_c(x)\sigma_\psi(x), \\
0 &= -\frac{P'_\tau(x,\tau)}{P(x,\tau)} + \mu_P(x,\tau) + \frac{1}{2}\sigma_P(x,\tau)^2 - r(x) + \left(\sigma_\psi(x) - \gamma\sigma_c(x)\right)\sigma_P(x,\tau), \ P(x,0) = 1\ \forall x.
\end{aligned}
$$

In the constrained region (i.e., $x \leq x^*$) the system is

$$
\begin{aligned}
0 &= \frac{\omega}{p(x)} + \mu_p(x) + \mu + \sigma_p(x)\sigma - r(x) - \gamma\sigma_c(x)\left(\sigma_p(x) + \sigma\right) \\
0 &= \frac{\lambda(1-\psi(x))}{\psi(x)} + \frac{\psi(x)}{\kappa}\left(\frac{(1-\omega)}{p_t} + \sigma_\psi(x)\left(\sigma_p(x) + \sigma\right)\right) + \mu_\psi(x) - \sigma_\psi(x)\gamma\sigma_c(x) \\
0 &= -\frac{P'_\tau(x,\tau)}{P(x,\tau)} + \mu_P(x,\tau) + \frac{1}{2}\sigma_P(x,\tau)^2 - r(x) - \gamma\sigma_c(x)\sigma_P(x,\tau), \ P(x,0) = 1\ \forall x
\end{aligned}
$$

where for a given function $z(x)$, the diffussion $\sigma_z(x)$ and drift $\mu_z(x)$ are in geometric form, i.e.,

$$
\begin{aligned}
\frac{dz_t}{z_t} &= \mu_{z,t}(x_t)\,dt + \sigma_{z,t}(x_t)\,dW_t \\
&= \left[\frac{z'_x}{z} x\mu_x(x) + \frac{1}{2}\frac{z'_x}{z}\left(x\sigma_x(x)\right)^2\right]dt + \frac{z'_x}{z}x\sigma_x(x)\,dW_t
\end{aligned}
$$

The point x^* is such that $\forall x > x^*$, $\frac{\psi(x)}{\kappa} > \alpha(x)$ (constraint is slack) and $\forall x < x^*$ $\frac{\psi(x)}{\kappa} = \alpha_f(x)$ (constraint is binding)

Proof. See appendix.

4 Results

Calibration. I calibrate the model at an annual frequency and solve it numerically. Table 1 shows the parameters. As I highlight below, risk aversion, γ, plays a critical role in the model solution. I use γ=5, which is a standard value in the asset pricing literature. Lower values of risk aversion alleviate the non-linearity produced by the occasionally binding constraints. I use σ=0.034, a number that is in line with the volatility of productivity in the US and close to the value used in He and Krishnamurthy (2019).[6]

The remaining parameters are associated with technology and constraint of the financial intermediary firms. I calibrate κ=0.4 to target an average leverage of 3 (He and Krishnamurthy (2019)). I set λ=0.08, which gives an expected payout rate of the intermediary as in Gertler and Kiyotaki (2015). I set ω=0.85, which implies asset prices can drop at the most 50% across the state space (i.e., when savers hold the entire wealth in the economy, the price-dividend ratio is 50% lower than if intermediaries hold the entire wealth in the economy). This is a conservative assumption for the lower bound of changes in the price dividend ratio. Lastly, I set $\overline{x} = 0.2$ to stabilize the wealth of intermediaries below 0.5.

Solution and mechanism. Figures 1 and 2 show the solution of the key endogenous variables. Both figures display the endogenous variables in the Markov equilibrium (i.e., endogenous variables as a function of the state variable x). The red dashed line in all panels represents the point at which the financing constraint binds.

The invariant distribution, displayed in lower left panel of Figure 1, shows the econ-

[6]Previous papers have used a much larger value for σ. For example, He and Krishnamurthy (2013), in a similar setup, uses σ=0.09; Brunnermeier and Sannikov (2014), also in a similar setup but with endogenous production, uses σ=0.1.

omy has two modes. It spends the majority of the time in a normal regime in which constraints are slack (i.e., to the right of the red dashed line), and some time in a crisis regime, where constraints are binding. Normal times are characterized by low volatility, low rates, and moderate leverage. As it is common in these types of models, leverage is counter-cyclical: the lower the intermediaries wealth is (i.e., lower x), the higher the leverage.

If the economy is in the normal regime and a sufficiently negative aggregate shock occurs, financial intermediaries reallocate their portfolios, the price of risky assets declines, and the price of risk increases. Financing constraints may bind (depending on the magnitude of the shock) and trigger the well-known financial accelerator mechanism, in which lower valuations deteriorate intermediaries' wealth even further. Figure 2 shows how total wealth (which is equal to the price of the risky asset, q_t/y_t) declines with x, while financiers' marginal utility increases.

A central element in the yield curve dynamics is the behavior of the short-term interest rate, r. Notice that when the economy enters in a crisis regime, the price of risk spikes, and the real interest rate increases. This is because wealth is transferred to savers, who are inefficient in handling risky assets, which means the level of aggregate dividends (and consumption) declines. Because the inefficiencies caused by the misallocation of risky assets are temporary, savers expect consumption level to increase in the future, which increases the real interest rate. Put differently, the dynamics for consumption level are similar to a random walk with drift, where deviations from the trend are persistent. When consumption is below the trend, is expected to mean-revert in the future. In the model, the trend is endogenously driven by financial intermediaries wealth dynamics.

The yield curve. Intuitively, investors require a premium to hold an asset whose value persistently declines in states in which the price of risk is high. This is precisely what drives the real term premium in the economy: real bond prices decline (i.e. real rate persistently increases) in states in which the price of risk is high. Figure 3 shows the average yield curve in the economy. Simply put, in the stochastic steady state—where expected short rates are constant—long-terms yields are driven by the term premium, causing the yield curve to be upward sloping on average. The left panel of Figure 3 illustrates the dynamics of yields at different horizons across the state space. The mechanism through which financial intermediaries reduce their positions in risky assets by selling those to less efficient savers, is noticeable only in short maturity rates—long-term yields are less sensitive to the missallocation of wealth in the economy. Put differently, current fluctuations in financiers' wealth has a lower incidence in driving longer maturity bonds, a feature that can be appreciated in the left panel of Figure 3. The panel shows the yield of bonds at 1, 10, and 30 maturity and also displays the yield of a very long-term bond. As the horizon of the bond increases, the yields become less sensitive the current financial conditions: x_t has a smaller impact on yields dynamics. This result, driven by the persistence and stationarity of x, shows that even very long term rates can display substantial volatility.

Figure 4 shows the yield curve for different for different levels of x. The circles in the Figure represents the average real yields reported in Backus, Boyarchenko and Chernov (2018). When x is high, the yield curve is flat, mainly because term premiums and real rates are low. Intuitively, a high x is a state in which intermediaries are relatively well capitalized, all term premium opportunities have been exhausted, and financing constraints are slack. When x is low, however, the economy is in crisis times, constraints are binding and real yields are high. In this state, the short term rate is expected to mean revert, and this force pushes down long-term rates. Thus, the gray line shows a downward sloping yield curve.

The role of risk aversion. Risk aversion plays a crucial role in the equilibrium dynamics. Figure 5 shows that using a γ=2, the yield curve is almost flat, the price of risk is low and does not move much, and real interest rates are less volatile. Financing constraints, however, bind at a similar endogenous level to the baseline calibration of γ=5. That is, the red dashed line, which is the point at which financing constraints bind in the baseline calibration, is close to the black dashed line. Notice that the lower level of risk aversion implies that the invariant distribution is not bimodal as in the baseline, but instead shows a single mode. The economy spends the vast majority of the time in a constrained region.

When savers' are less risk-averse, the real rate has to fluctuate much less to clear the deposit markets. Also, the price of risk demanded by intermediaries is lower, because they discount prices with saver's marginal utility. This feature implies intermediaries' balance sheets are less volatile with a lower level of γ (i.e., their liability side fluctuates much less), which in turn affects the volatility of asset prices and the yield curve dynamics—intermediaries' wealth and asset prices are endogenously determined.

Credit cycle and the yield curve. A relatively well-known empirical regularity is that a flattening of the yield curve (i.e., long-term rates at an equal or lower level than short term rates) is associated with lower future economic growth. The model predicts that a critical element linking yields and growth is the credit cycle. That is, a plausible explanation for why the yield curve precedes recessions is that the yield curve fluctuates, at least partially, with aggregate credit. The strong association between credit cycles and recession suggests credit fluctuates.

That is, a flattening of the yield curve in states in which constraints are slack indicates that opportunities to exploit the term premium have been exhausted, intermediaries wealth is high, and therefore the economy is vulnerable to negative aggregate shocks. If those shocks occur, asset prices will adjust and term premium (and long term rates) will increase.

Figure 6 shows the relationship between credit growth and the slope of the yield curve both in the data and in the model. In the data, the correlation is close to -0.5, and a bit more negative if a more recent sample is considered. In the model, this correlation is -0.4. This relationship indicates that a flattening of the yield curve (red line going down) is associated with the peak of the credit cycle. Through the lens of the model, this indicates a state in which financial intermediaries are well capitalized and their constraints are slack, they have expanded their leveraged balance sheets and thus are vulnerable to negative aggregate shocks.

Figure 6 shows the fluctuations in the slope of the yield curve in the data (bottom panel) and in the model (upper panel). The correlation in the data and in the model is negative: a reduction in the slope of the yield curve is associated with the peak of the credit cycle—which anticipates a future contraction in credit and a steepening of the yield curve. The model relates fluctuations in intermediaries' wealth with the yield curve dynamics.

Term structure of conditional distributions. Recent literature has been stressing the role of financing conditions in forecasting the distribution of future real variables—e.g., Adrian et al. (2019). The term structure of distributions is related to the yield curve because the former is a forecast of the conditional distribution of a random variable at a certain point in the future, while the latter is the expected value of a given payoff at certain point in the future.[7] I show the key economic forces driving the yield curve, elaborated above, is also consistent with the evidence about the term structure of conditional distributions of future outcomes.

To compute the term-structure of distributions, consider a process z_t in the model, that follows an Ito process

$$dz_t = \mu_{z,t}dt + \sigma_{z,t}dW_t,$$

where $\mu_{z,t} = \mu_z(x_t)$ and $\sigma_{z,t} = \sigma_z(x_t)$ are the drift and diffusion. Next, I define the function $f(x_s|x_t = x^*, s)$ as the conditional distribution of x at each point in time $s > t$, starting from a point x^*. The evolution of the density over time can be described by the following partial differential equation

$$\frac{\partial f(z(x)|x^*,t)}{\partial t} = -\frac{\partial}{\partial x}\left[f(z(x)|x^*,t)\,\mu(x)\right] + \frac{1}{2}\frac{\partial^2}{\partial x^2}\left[f(z(x)|x^*)\,\sigma(x)^2\right],$$

which is also known as the forward Kolmogorov equation (or Fokker-Planck equation).

Figure 7 shows the forecasted conditional distributions for log consumption growth (top two panels) and the state variable x (bottom two panels) at different horizons. The blue line represents the forecasted density conditional on current financial conditions being loose. More precisely, the forecast is conditional on $x_t = x^*$ where x^* is 10% above the point at which financing constraints bind. The distribution in red represents the

[7] In technical terms, these two objects are the forward Kolmogorov equation and the backward Kolmodorov equation.

forecasted density conditional on current financial conditions being tight. I assume x_t is 10% below the point at which financing constraints bind. The red dashed line in all four panels represents the point in the state space at which financing constraints bind.

In line with the evidence reported in Adrian et al. (2019), the top two panels indicate that, conditional on the economy facing tight financial conditions, the distribution of future growth is negatively skewed. Also, the conditional distribution fluctuates across the horizon, and the relatively smaller skewness of the conditionally constrained distribution persists. The main source of the asymmetry is that economic outcomes are quite different in the constrained and unconstrained regions. For example, in the constrained region the economy is more leveraged (thus more sensitive to shocks), the real rate is much more volatile, and the price of risk moves faster. And these conditions may persist, because it takes time for intermediaries wealth to be rebuilt. Put differently, x. is a persistence process.

The bottom two panels display the forecasted conditional distributions for the state variable x. The intuition is similar to that of consumption growth. Tight financial conditions are persitant and can trigger quite volatile and unstable outcomes. Simply put, the model rationalizes the data with two main elements: tighter financial conditions are persistent outcomes and they lead to quite different economic outcomes than those implied by the economy functioning in an unconstrained region. As in the yield curve, the key elements are the bimodal nature of the economy together with the persistent dynamics of intermediaries' wealth.

5 Conclusion

In this paper I study the yield curve dynamics in a general equilibrium model with financially constrained intermediaries. The economy features an endogenously time varying real term premium that implies an upward sloping real yield curve and highly volatile long-term yields, consistent with the data. This feature is purely driven by the fact that financing constraints may occasionally bind.

Intuitively, financial intermediaries hold long-term assets, and therefore fluctuations in the valuation of long-term assets can alleviate (or not) the extent to which intermediaries are financially constrained. These constraints affect marginal valuations, not only of the intermediaries, but in general equilibrium they could affect other agents' marginal valuations as well. Because long-term yields are forecasts of marginal valuations, they turn out to be a useful indicator of the health of the intermediaries' wealth.

I show the key economic mechanism connecting intermediaries' wealth and the yield curve can rationalize interesting macroeconomic phenomena such as why a flattening of the yield curve precedes recessions, and why the forecasted distributions of growth is negatively skewed when financial conditions are tight.

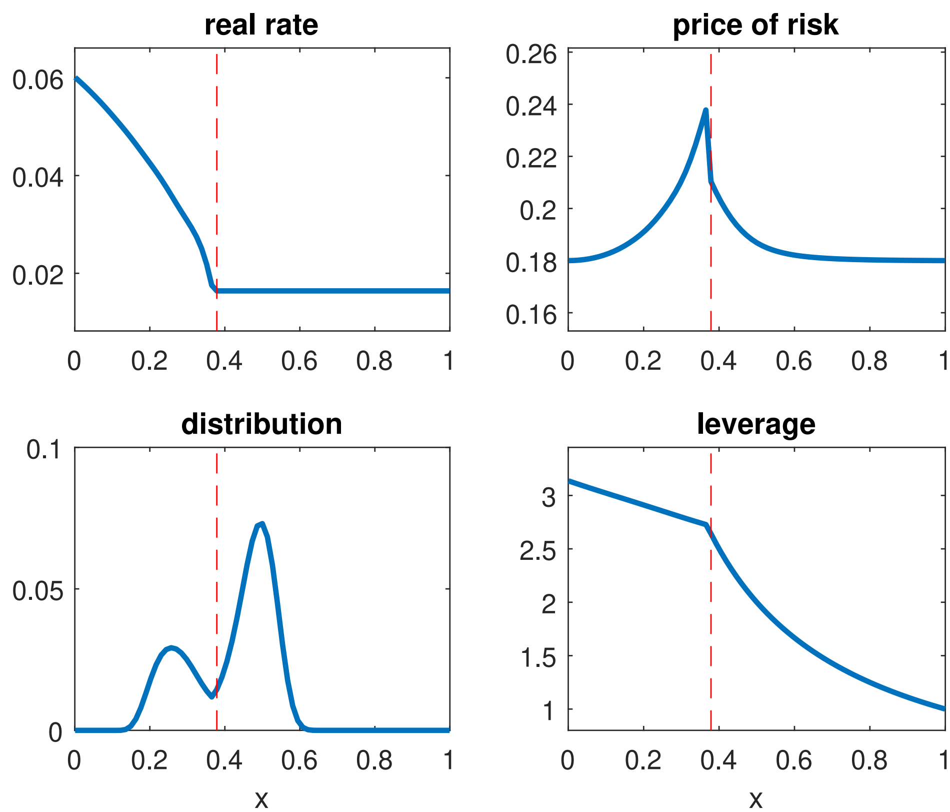
TABLE 1. Calibration

PARAMETERS

	Value	Description
γ	5	risk aversion
σ	0.034	volatility y_t
λ	0.08	dividend payout
ω	0.85	managment cost
κ	0.4	fraction divertible assets
$\overline{x}$	0.2	initial capital

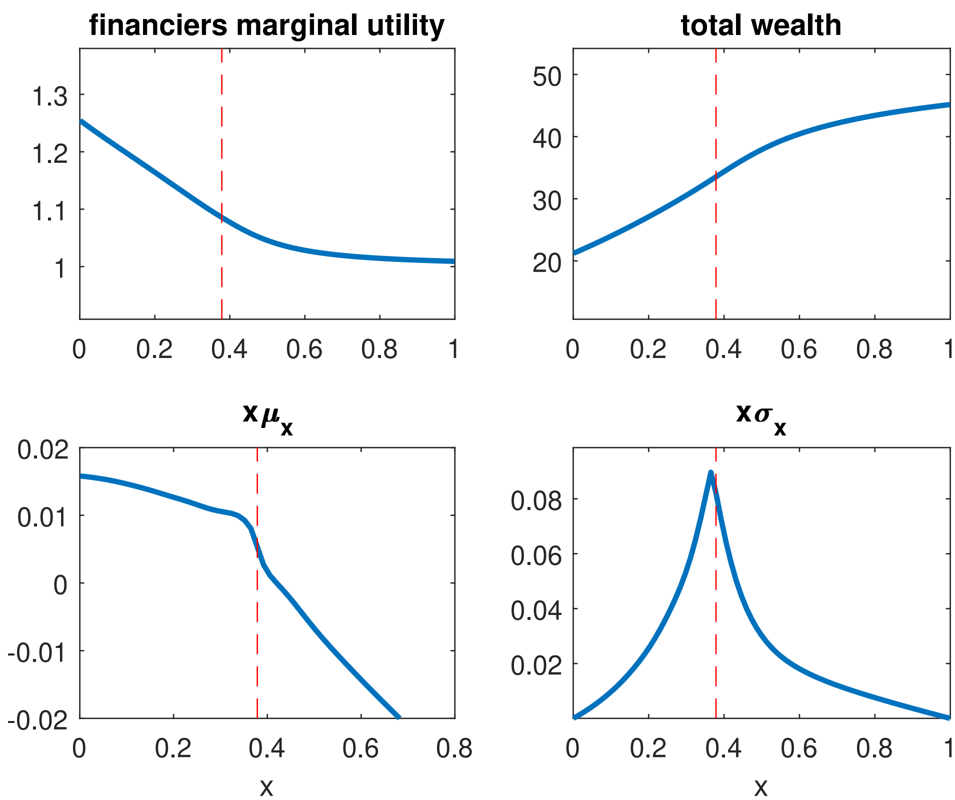
NOTES: This table shows the calibration of the model at an annual frequency.

FIGURE 1. Model solution I



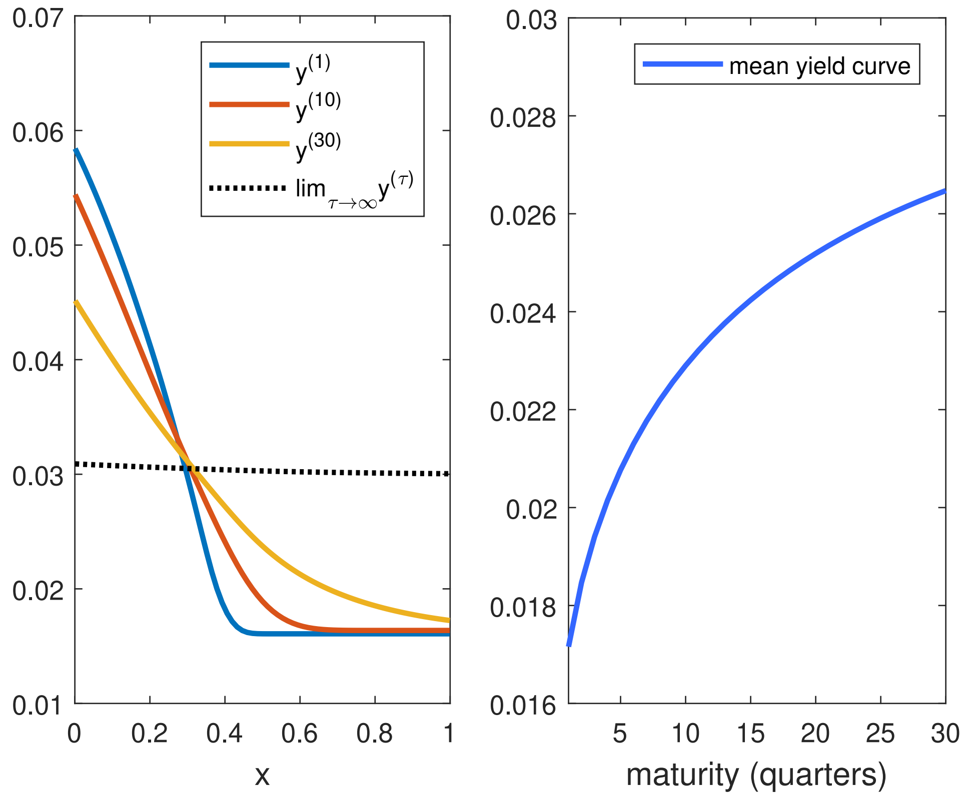
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 2. Model solution II



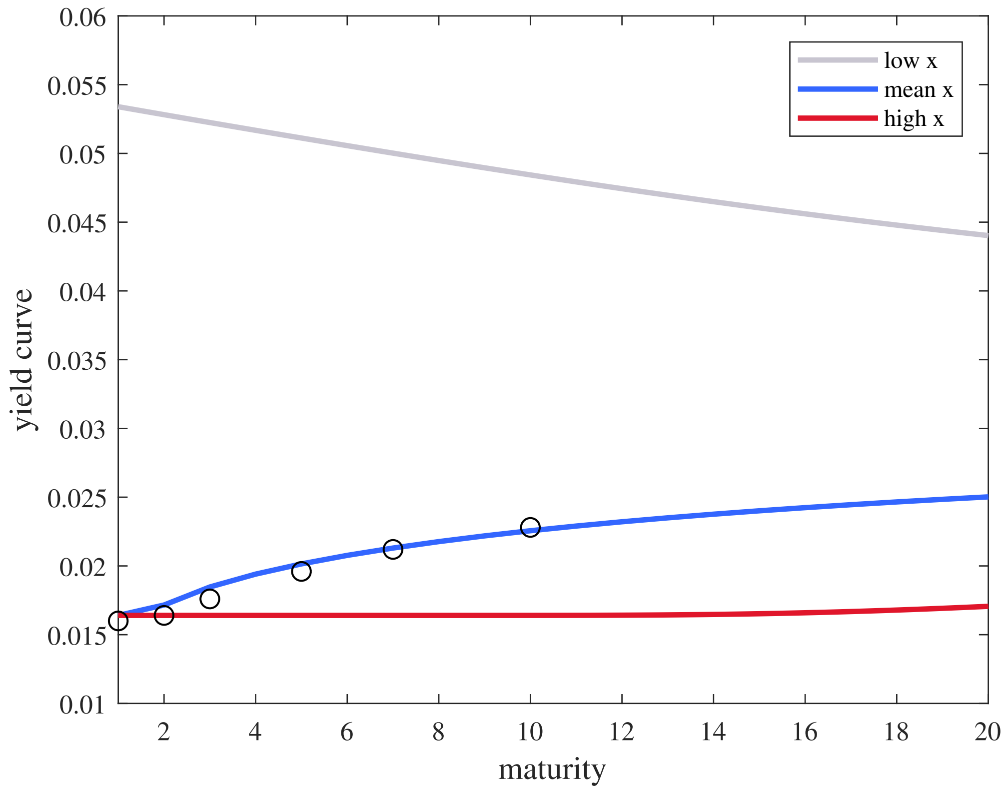
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 3. Model solution: Yields



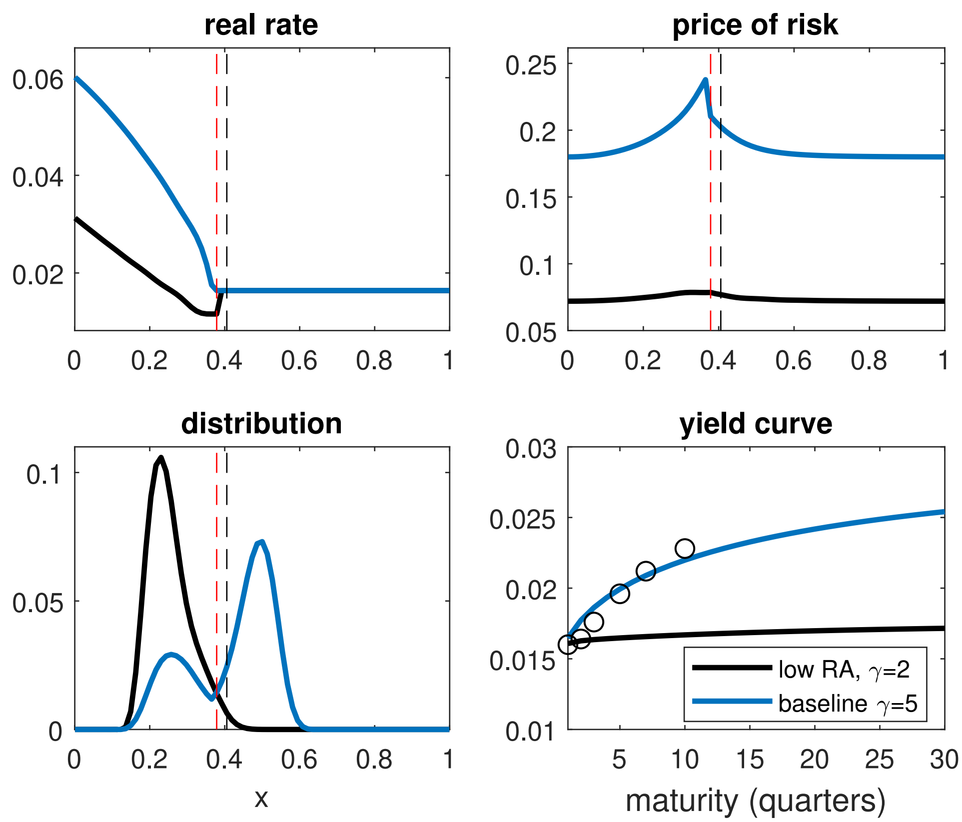
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 4. Yield Curve



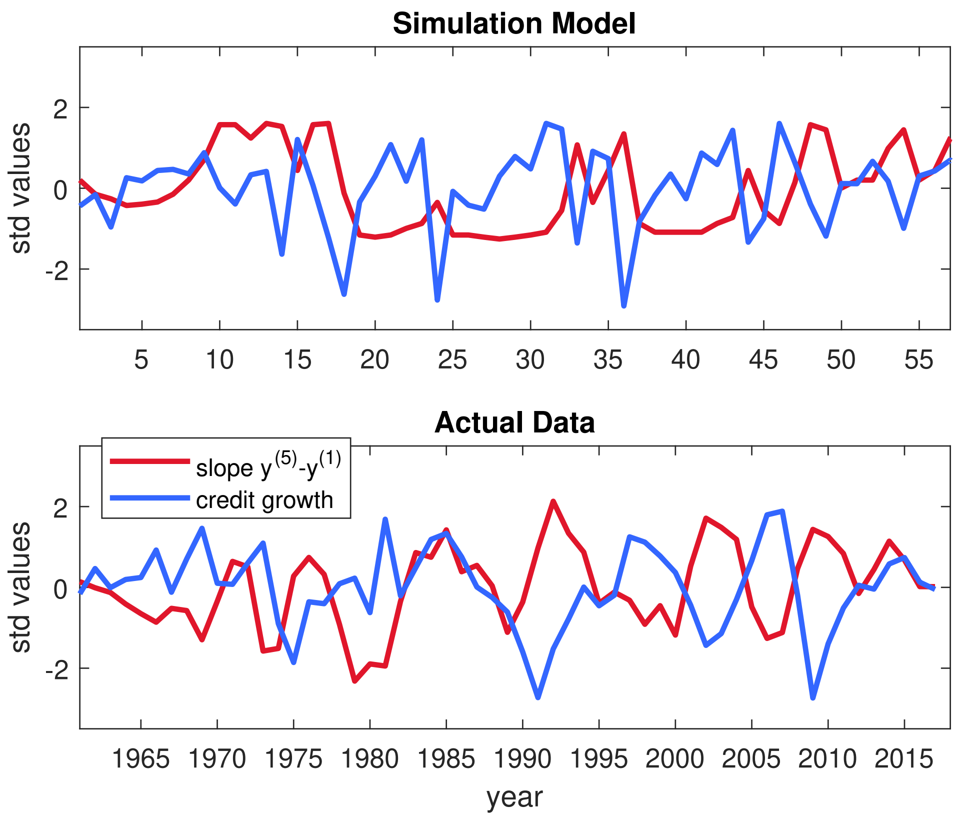
NOTES: This figure shows the model solution, with the calibrated parameters from Table 1 .

FIGURE 5. The role of risk aversion



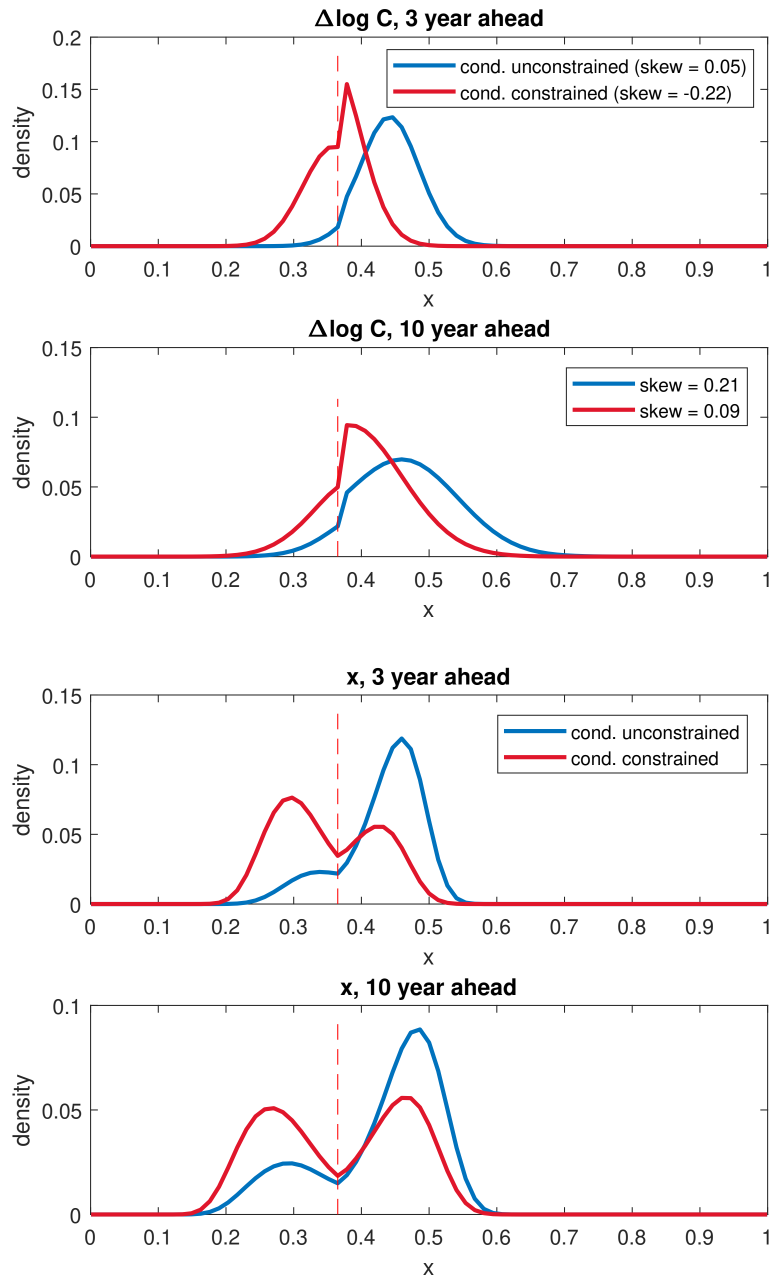
NOTES: This figure compares the model's solution for a lower risk aversion (γ=2) and a the baseline risk aversion (γ=5).

FIGURE 6. Credit and yields over the cycle



NOTES: The upper panel shows the simulation of the slope $y^{(5)} - y^{(1)}$ and credit growth b_f from the model. The bottom panel shows slope $y^{(5)} - y^{(1)}$ and credit growth in the data. The correlation between the series is -0.45 in the data and -0.41 in the model.

FIGURE 7. Term structure of conditional distributions for real variables



NOTES: This figure shows the conditional distributions of consumption growth 3 and 10 years ahead (top two panels), and the conditional distributions of the endogenous state variable x 3 and 10 years ahead (bottom two panels). The distributions are conditional on the current state of the economy being an unconstrained one (blue lines) and a constrained one (red lines). This is, following Adrian et al. (2019), the distributions of growth conditional on financial conditions being loose or tight. The red dashed line is the point in the state space at which constraints binds.

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7 Appendix

[In progress]