Repo Specialness in the Transmission of Quantitative Easing*

Preliminary Draft

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When a central bank withholds bonds from repo markets, the repo specialness of sovereign bonds increases. Investors who cannot take advantage of the repo specialness of sovereign bonds substitute sovereign bonds with riskier assets, such as corporate bonds. The extra demand from this portfolio substitution lowers corporate financing costs. This magnifies the transmission of quantitative easing effects to the real economy. I quantify the magnitude of this repo specialness channel for quantitative easing transmission in the context of the Public Sector Purchase Program of the Eurosystem.

1. Introduction

This study demonstrates that the specialness premium of sovereign bonds in repurchase agreement (repo) markets can boost a central bank’s effort to lower corporate bond yields by purchasing sovereign bonds. I empirically estimate the magnitude of this channel for the eurozone sovereign bonds purchased by the Eurosystem through the Public Sector Purchase Program (PSPP). I demonstrate that corporate bond yields go down by 1.9 basis points when the repo

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specialness of European sovereign bonds rise by 10 basis points. In the fourth quarter of 2016, corporate bond yields in Germany may have been lowered by 6 basis points through the repo specialness effect.

Figure 1 provides an illustrative example of a repo transaction. A sovereign bond repo is a contract one firm (A) sells a sovereign bond to another firm (B), with a promise to re-purchase at a future date. In effect, firm B makes a cash loan to firm A, collateralized with a sovereign bond. The repo rate is the rate of return on the cash investment of firm B, indicated by the initial sale price and future re-purchase price of the bond. If the bond is scarce in the repo market, it will be costly for firm B to find an alternative counter-party that owns the bond. Consequently, firm B might be willing to lend cash at a repo rate lower than the risk-free rate. The downward deviation in the repo rate (1.5%) from the risk-free rate (2%) is the repo specialness premium (0.5%). By borrowing cash at a low repo rate and subsequently investing it at a higher risk-free rate, firm A earns extra revenue. In repo market terminology, firms A and B conduct a repo and a reverse repo, respectively. The repo specialness premium tends to be higher if the bonds are harder to locate in the market (Duffie, 1996; Vayanos and Weill, 2008; Corradin and Maddaloni, 2017; Jank and Mönch, 2018, 2019).

Figure 1: Illustrative Example of a Repo Transaction. This diagram illustrates an example of a repo transaction in which firm B executes a reverse repo on a bond from firm A. Firm A executes a repo on the bond to firm B. The risk-free rate is 2%. The repo rate is 1.5%. The downward deviation in the repo rate from the risk-free rate, which is 0.5% in this example, is the specialness premium.

This study considers the case of the PSPP through which the Eurosystem has been purchasing the debt instruments of eurozone governments and agencies in the secondary market since March 2015. After purchasing government bonds, the Federal Reserve (Fed) or Bank of Canada sells a significant portion of these bonds back to private firms, either through repos or securities lending agreements.¹ By contrast, the Eurosystem does not sell most of the bonds it purchases under the PSPP back to the market. For example, in November 2018, the Eurosystem was not willing to conduct more than 75 billion euros of securities in repo operations to private firms. At the same time, this accounted for less than 4% of the total value of bonds that it held for the

¹See section §C in the appendix for details.
Whether intended or not, this decision substantially reduced the availability of eurozone government bonds in the market and increased the repo specialness of these bonds (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018; Jank and Mönch, 2018, 2019). Indeed, Figure 2 shows that the average repo specialness premium of bunds increased steadily during the PSPP period. I demonstrate that increased repo specialness premium of securities purchased under the PSPP significantly lowered corporate financing costs. Reducing corporate financing costs is one of the most important goals of quantitative easing (QE) programs (see Praet (2015) and Bernanke (2011) for the objectives of the Eurosystem and the Federal Reserve respectively). Intentionally or not, the Eurosystem magnified the stimulus to the eurozone economy of its PSPP through an increase in repo specialness.

![Figure 2: Bund Repo Specialness Premium](image)

The datas are from BrokerTec. The figure plots the transaction volume-weighted average repo specialness premium of German federal government bonds. Repo specialness premium is defined as the GC Pooling ECB Extended Basket rate minus the repo rate. I removed data on quarter ends.

The magnification of the stimulus of the PSPP to the euro-zone economy is useful mainly because of the self-imposed rules of the Eurosystem that limit the size of the PSPP. For example, under the PSPP, the Eurosystem does not purchase more than 33% of the outstanding debts of any individual government. Some of the rules introduced by the Eurosystem at the beginning of the program became binding constraints and had to be relaxed to continue purchasing bonds

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2The number is from the European Central Bank website. See section E.3 for more information on the restrictive features of the securities lending facilities that the Eurosystem operated.

for the PSPP (Gros, 2016). Some economists have expressed concerns that the size limit could force the Eurosystem to end the program prematurely, even the inflation is not high enough (Gros, 2016). Nevertheless, it is politically challenging for the Eurosystem to relax these rules. A sizable expansion in the PSPP by amending these rules could invite the criticism that the PSPP violates the European Law on the prohibition of monetary financing (e.g., the direct financing of governments by the central bank) (Mersch, 2016; Boysen-Hogrefe, Fieldler, Jannsen, Kooths, and Reitz, 2016). Given these constraints on the PSPP’s size, it could be valuable to increase the stimulus of the program per unit of government bonds purchased. The repo specialness effect meets this objective, through lowering corporate financing costs.

My theoretical model works roughly as follows. Suppose a central bank purchases sovereign bonds without selling them on the repo market. With fewer bonds circulating in the repo market, the specialness premium of sovereign bonds will rise. Consequently, a dealer bank owning these sovereign bonds can earn more revenue in the repo market. Dealers will quote higher prices to their non-bank customers, such as insurance companies and pension funds (ICPFs), to reflect the opportunity to earn more revenue in the repo market (Duffie, 1996; Jordan and Jordan, 1997; Buraschi and Menini, 2002; Duffie, Garleanu, and Pedersen, 2002; Krishnamurthy, 2002; Vayanos and Weill, 2008; Bartolini, Hilton, Sundarensan, and Tonitti, 2011; D’Amico and Pancost, 2017; D’Amico, Fan, and Kitsul, 2018). However, ICPF cannot earn revenue from the repo specialness nearly as effectively as dealers can. For example, it is costlier for ICPF to search for and then negotiate with a counter-party to take the other side of the repo transaction (Hill, 2015b). They also cannot effectively manage the risks (e.g., counter-party credit risk) involved in the transaction (Hill, 2017). When sovereign bond yields become too low, ICPF substitute government bonds with corporate bonds or sub-sovereign bonds. Consequently, corporate bonds appreciate.

ICPF invest heavily in the long-term non-financial corporate bond market in Europe, so much so that their portfolio allocations can significantly affect these bond prices. For example, in the second quarter of 2016, ICPF accounted for 43% of the private holdings (e.g., excluding holdings by central banks) of the eurozone non-financial corporate debt with remaining maturities of more than one year.

To quantify the repo specialness effect, I use the granular instrumental variable (GIV) method of Gabaix and Koijen (2019). The repo data are from BrokerTec, a major inter-dealer electronic platform for the European repo market. Using principal component analysis (PCA), I extract the idiosyncratic components of the repo rates of sovereign bonds. The identification assumption is that these idiosyncratic parts of the repo rates are not correlated with the omitted variables, which affect the expected return from holding non-sovereign bonds in the cash market. I use the estimated idiosyncratic components to instrument the repo specialness.

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4The Article 123 of the Treaty on the Functioning of the European Union.
6See section E.2 for more details.
7The data source is the Statistical Data Warehouse of the European Central Bank.
premium of sovereign bonds. The impact of the repo specials of sovereign bonds on corporate bond yields differs across days and countries. The repo specials of sovereign bonds, particularly German bonds, were the highest during the fourth quarter of 2016. German corporate bond yields may have been lowered by 5.8 basis points through the repo specialness effect.

Section §2 explains the marginal contribution of this study to the literature. Section §3 describes the setting of my theoretical model. Section §4 solves for equilibrium and illustrates the theoretical intuition of my transmission channel of a QE. In section §5, I numerically calibrate my theoretical model. In section §6, I describe the data and my empirical strategy is detailed in section §7. In section §8, I present the results of my instrumental variable regression. In section §9, I discuss other channels through which the repo specialness premium of sovereign bonds can affect corporate financing costs. I also explore the implication of the repo specialness effect for fiscal policy and allocative efficiency in the bond and repo markets. I conclude in section §10.

2. Literature Review and Marginal Contribution

This study contributes to a growing strand of the theoretical literature on the effect of open market operations of central banks, also called QE programs, on asset prices. I demonstrate a new channel involving the repo specialness premium. To the best of my knowledge, the repo-specialness channel for increasing the impact of the central bank’s purchase of government bonds on the prices of other assets has not been identified in previous studies.

The seminal work of Wallace (1982) shows that open market operations cannot impact asset prices in traditional models with the representative household. Since the representative agent ultimately owns the central bank, a mere exchange of one asset for another on the balance sheet of the central bank does not impact the marginal consumption process of households (Woodford, 2012). Consequently, the representative household prices assets using the same pricing kernel. Since it is completely irrelevant whether central bank balance sheet contains reserves or other assets, open market operations have no impact on asset prices in the model of Wallace (1982).

Nevertheless, since the financial crisis, many central banks have initiated QE programs specifically to influence asset prices and make their monetary policies more accommodating. In response, scholars began studying the conditions of Wallace (1982) model that need to be relaxed for QE programs to influence asset prices. For example, asset prices can change if the purchased asset gives more non-pecuniary benefits than central bank deposits (Woodford, 2012). The convenience yield theory of U.S. Treasuries (Krishnamurthy and Vissing-Jorgensen, 2012a,b) supports this idea. The safety of cash flows from holding Treasuries is not the only reason investors like Treasuries. Treasuries are also useful as collateral for financial transactions. The Fed reserves are less valuable in financial transactions because only banks can access them. The Fed reduces the supply of assets that private investors can use to back financial transactions by purchasing Treasuries with reserves. Consequently, investors are willing to
pay higher prices to obtain assets that can back financial transactions. Krishnamurthy and Vissing-Jorgensen (2012b) call this the “asset scarcity channel.”

My study is different from the asset scarcity channel studies as scarcity of a purchased asset in the underlying cash market and in the repo market do not always go hand in hand. I use the term “cash market” to specifically refer to the government bond cash market — the sale and purchase of government bonds. QE programs inevitably make it more difficult for private investors to own assets. However, central banks can make it more or less difficult to borrow the same asset by adjusting the supply of the purchased asset to the repo market.

For the same reason, my study also departs from those on QE-induced scarcity of a class of assets (e.g., assets with long duration) to explain the asset pricing implications of QEs (Altavilla, Carboni, and Motto, 2015; Ferrero, Loberto, and Miccoli, 2017; Lenel, 2018). Most of these studies have focused on keeping certain assets out of circulation in the underlying cash market, not the repo market.

In particular, Ferdinandusse, Freier, and Ristiniemi (2017) notes that QE could make it more difficult for investors to locate counterparties to trade the asset purchased under the QE. An increase in search frictions moves asset prices a la Duffie, Garleanu, and Pedersen (2005). While Ferdinandusse, Freier, and Ristiniemi (2017) studied search frictions in the underlying cash market, I focus on search frictions in the repo market.

This study is also related to the empirical literature analyzing asset price movement in response to the PSPP (Altavilla, Carboni, and Motto, 2015; Falagiarda, McQuade, and Tirpak, 2015; Andrade, Breckenfelder, De Fiore, Karadi, and Tristani, 2016; Blattner and Joyce, 2016; De Santis, 2016; Arrata and Nguyen, 2017; Corradin and Maddaloni, 2017; De Santis and Holm-Hadulla, 2017; Ferrari, Guagliano, and Mazzacurati, 2017; Karadi, 2017; Lenke and Werner, 2017; Schlepper, Hofer, Riordan, and Schrimpf, 2017; Albertazzi, Becker, and Boucinha, 2018; Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018; Koijen, Koulischer, Nguyen, and Yogo, 2018; Jank and Mönhch, 2018; Pelizzon, Subrahmanyam, Tomio, and Uno, 2018; Jank and Mönhch, 2019). and the determination of European repo rates (Mancini, Ranaldo, and Wrampelmeier, 2015; Ebner, Fecht, and Schulz, 2016; Boissel, Derrien, Ors, and Thesmar, 2017; Corradin and Maddaloni, 2017; Nyborg, 2017; Ferrari, Guagliano, and Mazzacurati, 2017; Sangiorgi, 2017; Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018; Piquard and Salakhova, 2018; Brand, Ferrante, and Hubert, 2019; Nyborg, 2019; Nyborg and Rosler, 2019). In particular, Arrata, Nguyen, Rahmouni-Rousseau, and Vari (2018) shows that the PSPP contributed to lowering European repo rates by removing government bonds from the market. In addition, this paper is related to how regulations or central bank policies affect the repo market (Dunne, Fleming, and Zholos, 2011; Miglietta, Picillo, and Pietrunti, 2015; Fecht, Nyborg, Rocholl, and Woschitz, 2016; Fontaine, Garriott, and Gray, 2016; Corradin, Heider, and Hoerova, 2017; Fontaine, Hately, and Walton, 2017). Most of these studies have tackled the underlying cash market or the repo market separately, with the notable exception of D’Amico, Fan, and Kitsul (2018), which estimated the impact of the repo specialness of Treasuries on their yields throughout the Fed’s QE program. The increasing repo specialness during the implementation
Lastly, this study adds to the literature showing QE programs can lower corporate or household financing costs and lead to higher inflation. One of the mechanisms studied in the literature is the portfolio rebalancing channel (Vayanos and Vila, 2009; Meaning and Zhu, 2011; Krishnamurthy and Vissing-Jorgensen, 2012b; Carpenter, Demiralp, Ihrig, and Klee, 2013; Joyce, Liu, and Tonks, 2014; King, 2015; Andrade, Breckenfelder, De Fiore, Karadi, and Tristani, 2016; Jouvanseau, 2016; Bua and Dunne, 2017; Peydro, Polo, and Sette, 2017; Rios, Evans, and Shamloo, 2017; Albertazzi, Becker, and Boucinha, 2018; Bergant, Fiodra, and Schmitz, 2018; Boermans and Keshkov, 2018; Chadha and Hantzsche, 2018; Christensen and Krogstrup, 2018; Goldstein, Witmer, and Yang, 2018; Greenwood, Hanson, and Liao, 2018; Koijen, Koulischer, Nguyen, and Yogo, 2018; Awdjie, Everett, and Shin, 2019; Bottero, Minoiu, Peydro, Polo, Presbitero, and Stte, 2019; Zaghini, 2019). For example, in response to the QE-induced appreciation in government bonds, banks substitute holdings of government bonds with loans and other riskier assets (Gambetti and Musso, 2017; Albertazzi, Becker, and Boucinha, 2018; Paludkiewicz, 2018; Tischer, 2018). I demonstrate that the repo specialness premium plays the key role in prompting this portfolio substitution.

3. Model

I build a continuous-time infinite-period model with risk-neutral agents following Duffie, Garleanu, and Pedersen (2002, 2005). All exogenous variables are shown in blue. Table 5 summarizes the definitions of the parameters of the model. Table 6 summarizes the random variables. The proofs of all the lemmas and theorems are given in the appendix.

Following Duffie, Garleanu, and Pedersen (2002), I fix probability space \((\Omega, \mathcal{F}, P)\) and filtration \(\{\mathcal{F}_t : t \geq 0\}\), assumed to satisfy the usual conditions of Protter (1990). This filtration \(\mathcal{F}_t\) models the common information of agents at time \(t\).

There are two assets: sovereign bonds and corporate bonds. The main intuition would work with any financial asset (e.g., regional government bond) that non-banks consider substitutable for a sovereign bond. Further, consumption goods play the role of a numeraire.

There are four markets: over-the-counter sovereign bond repo market, sovereign bond dealer-to-dealer (D2D) cash market, sovereign bond dealer-to-customer (D2C) cash market, and corporate bond cash market. I model the repo market as over-the-counter market with search frictions. This approach is necessary because specialness premium can arise only if the collateral cannot circulate at infinite speed.

My theoretical model features the specific collateral (SC) repo market for sovereign bonds. The SC market and the general collateral (GC) market are the two main segments of a repo market. In the GC market, the collateral provider can fulfill its contractual obligation by delivering any security from a pre-approved list of securities. By contrast, in the SC market, the collateral provider must deliver the exact asset that its counterparty requires. SC repo
contracts are driven by the need to obtain specific collateral. In contrast, GC repos are driven by the need to borrow cash. Collateral only serves to secure the transaction.

Figure 3 shows the main players in my model. $N$ dealer banks act as intermediaries in the sovereign bond repo market and sovereign bond cash market. The ex-ante identical dealers are indexed by $i \in \{1, 2, ..., N\}$. Similarly, a continuum of non-banks indexed by $j$ is distributed along the interval $[0, m_n]$ with the Lebesgue measure $\phi_n$. Each non-bank can be in one of the four states explained in subsection 3.2. $k$ indexes a continuum of cash investors distributed along the interval $[0, m_c]$ with the Lebesgue measure $\phi_c$. For now, I assume that the probability space $(\Omega, \mathcal{F}, P)$ and investor’s measure space has a Fubini extension a la Sun (2006) (Duffie, 2017).

![Figure 3: Overview of the Model. D2D market stands for the dealer-to-dealer market.](image)

3.1. Assets

A sovereign bond is a perpetual risk-free bond with coupon rate $c$ per unit of time. A corporate bond is identical to a sovereign bond except it can default. The default time is the stopping time $\tau_\eta$ with the exogenous intensity $\eta$. Default shocks occur independently across a continuum of all corporate bonds in the economy. All agents in the economy can borrow or lend at the exogenous risk-free rate $r$. The repos mature stochastically with exogenous rate $\delta$. This assumption is needed to make the investor’s problem stationary, as in He and Xiong (2012).
3.2. Non-banks

The non-banks in my model can be most appropriately classified into insurance companies or pension funds in the real world. Figure 4 gives an overview of the life-cycle of a non-bank. Each non-bank in the “N” state decides between owning a sovereign bond or a corporate bond. For example, an insurance company regularly re-invests the cash received from its clients. It does not want to hold cash for a long time to match the long duration on its liability side.

A non-bank can buy a sovereign bond from one of N dealers. It can then generate extra revenue by supplying the purchased bond to the repo market. Alternatively, it can buy a corporate bond from corporate bond dealers. If the corporate bond defaults, the non-bank transitions back to the “N” state.

![Figure 4: Type Transition Diagram of Non-Bank](image)

The non-bank can change its investment decision — whether to purchase a sovereign bond or a corporate bond — in the Investment Grade (IG) state or the Owner Waiting (OW) state. For example, a non-bank in the IG state can sell its corporate bond and buy a sovereign bond. However, the type transition dynamics in Figure 4 follow a continuous-time Markov chain. In the steady-state equilibrium, the non-bank faces the same problem at any point in time. Hence, i will not change its investment decision in any of the three states — IG state, OW state, or N state.

Let $W_t$ denote the wealth that the non-bank invests in the risk-free bank account. $W_t$ is the only state variable in the non-bank’s problem. Let $V_\sigma (W_t)$ denote the value function of the non-bank at state $\sigma \in \{N, O, OW, IG\}$. Let $P$ and $P_{ig}$ denote the prices at which the non-bank can purchase a sovereign bond or a corporate bond from dealers, respectively. Equation (1) shows the non-bank’s problem in the N state.

$$V_N (W_t) = \max \{V_{OW} (W_t - P), V_{IG} (W_t - P_{ig})\}$$ (1)
3.2.1. Sovereign Bond Owning Non-Banks

The non-bank enters the “OW” state immediately after purchasing a sovereign bond from a dealer: it has a long position on a sovereign bond but has not conducted a repo on the bond yet. It immediately starts searching for dealers to execute a repo on the purchased bond.

OW non-banks and dealers meet each other through a pairwise independent random matching process as in Duffie, Garleanu, and Pedersen (2005). Let \( \lambda_o \) denote the endogenous search intensity chosen by the OW non-bank. From the perspective of an individual OW non-bank, its encounter with dealers is a Poisson shock process. The contact time is the first arrival time of the Poisson process with intensity \( \lambda_o \). Let stopping time \( \tau_D \) denote the contact time. Each dealer is equally likely to be the non-bank’s counterparty, conditional on the event that a given OW non-bank meets one of the dealers. The matching process is pairwise independent in the following sense (Duffie, 2017): for almost every OW non-bank, its matching process with dealers is independent of the matching process of almost every other OW non-bank. As in He and Milbradt (2014), all pairwise random matching processes in this model represent uninsurable uncertainty. Consequently, the Poisson shock processes modeling matching between non-banks and dealers are independent of all other random variables in the model.

The OW non-bank wants to make its search intensity \( \lambda_o \) high to monetize the repo specialness premium as soon as possible. Nevertheless, as in Duffie, Malamud, and Manso (2009), the non-bank incurs search cost \( K(\lambda_o) \) per unit of time. \( K(\cdot) \) is an exogenous function that is increasing and convex. Further, \( K(0) = 0 \) and \( K'(0) = 0 \). Hence, the non-bank chooses its search intensity that optimally trades-off the search cost and delay in monetizing repo specialness premium.

When the OW non-bank and dealer meet, they determine the specialness premium through Nash bargaining (Duffie, Garleanu, and Pedersen, 2005). The non-bank gives a unit of sovereign bond to the dealer. In exchange, the non-bank receives cash equivalent to \( P \) units of consumption numeraire. I assume a zero haircut. The OW non-bank transitions to the O state.

During the lifetime of this repo contract, the non-bank receives cash \( c + B \) per unit of time. As the owner of the bond, the non-bank is entitled to a “manufactured payment” \( c \) per unit of time. \( B \) is the specialness premium determined through Nash bargaining.

At rate \( \delta \), the repo matures and the non-bank transitions back to the OW state. The dealer returns the sovereign bond to the non-bank. Let the random variable \( \tilde{T} \) denote the duration of the repo contract. The non-bank returns cash equivalent to \( P \cdot e^{r\tilde{T}} \) units of consumption numeraire to the dealer and compensates the dealer for the interest foregone on investing this cash in the risk-free asset.

All the investors in my model — dealers, cash investors, and non-banks — have a time-discount rate equal to the risk-free rate \( r \). The OW non-bank’s problem can be formulated as (2) and (3). \( W_t \) is the bank account of the non-bank at time \( t \). \( DC_{t+s} \) is a semi-martingale consumption process that the non-bank optimizes. As the problem is stationary, \( \lambda_{o,t} \) is constant over time.

\[
V_{OW}(W_t) = \sup_{C, \lambda_o} \mathbb{E}_t \left[ \int_{s=0}^{\tau_D} e^{-rs} dC_{t+s} + e^{-r\tau_D} V_O(W_{t+\tau_D} + P) \right] \tag{2}
\]
Figure 5: The Schematics of Repo. The upper panel describes the life-cycle of a repo contract between a dealer and a cash investor. The lower panel describes the life-cycle of a repo contract between a dealer and a non-bank.
such that: $dW_t = rW_t dt - dC_t + dt - K(\lambda_{o,t}) dt \quad (3)$

A high repo specialness premium $B$ induces each sovereign bond owner to exert more search effort. Hence, larger quantities of sovereign bonds are supplied to repo dealers per unit of time. This modeling choice reflects the observation that as bonds became more special in the repo market, collaterals started circulating in the European repo market faster (see Jank and Mönch (2019) for empirical support). The problem of the non-bank with a repo position in the O state can be formulated similarly.

$$V_o(W_t) = \sup_C \mathbb{E}_t \left[ \int_{s=0}^{T_o} e^{-rs} dC_{t+s} + e^{-rT_o} V_{OW}(W_{t+T_o} - P \cdot e^{rT_o}) \right] \quad (4)$$

such that: $dW_t = rW_t dt - dC_t + (e + B) dt \quad (5)$

### 3.2.2. Corporate Bond Owning Non-banks

The problem of the non-bank owning a corporate bond can be formulated as follows.

$$V_{IG}(W_t) = \sup_C \mathbb{E}_t \left[ \int_{s=0}^{T_n} e^{-rs} dC_{t+s} + e^{-rT_n} V_{N}(W_{t+T_n} + (1 - A) \cdot \frac{c}{r}) \right] \quad (6)$$

such that: $dW_t = rW_t dt - dC_t + cd \quad (7)$

### 3.3. Cash Investors

The upper panel of Figure 5 describes the life-cycle of a cash investor. Examples of cash investors include hedge funds that want to obtain sovereign bonds for shorting.

The mass of cash investors is exogenously fixed at $m_c$. The cash investors in the cash investor waiting (CW) state want to obtain sovereign bonds for exogenous reasons that are not modeled here. Cash investors search for repo dealers through the pairwise independent random matching process and choose their search intensity $\lambda_c$ that balances the cost of delaying the possessing a sovereign bond and the search effort. The cash investor and dealer Nash bargain the specialness premium $A$, after which the cash investor transitions to the C state. Cash investors receive an exogenous flow benefit $b$ per unit of time until the repo matures; at which time, the investor transitions back to the CW state. The problem of a cash investor in the CW state can be formulated as equation (8) and equation (9).

$$V_{CW}(W_t) = \sup_{C, \lambda_c} \mathbb{E}_t \left[ \int_{s=0}^{T_D} e^{-rs} dC_{t+s} + e^{-rT_D} V_C(W_{t+T_D} - P) \right] \quad (8)$$

such that: $dW_t = rW_t dt - dC_t - K(\lambda_{c,t}) dt \quad (9)$

The problem of a cash investor in the C state can be stated similarly.

$$V_{C}(W_t) = \sup_C \mathbb{E}_t \left[ \int_{s=0}^{T_s} e^{-rs} dC_{t+s} + e^{-rT_s} V_{CW}(W_{t+T_s} + P \cdot e^{rT_s}) \right] \quad (10)$$
such that: \( dW_t = rW_t dt - dC_t + (b - c - A) dt \) \hspace{1cm} (11)

3.4. Dealers

Equations (12), (13), (14), (15), and (16) show the dealer’s problem. \( Q_{t}^{RR} \) and \( Q_{t}^{R} \) denote the mass of bonds that in reverse repos and repos, respectively, conducted by the dealer. The dealer also outright purchases \( x_t \) sovereign bonds from the cash market. If \( x_t \) is negative, the dealer outright sells bonds. \( \mu_t^{\sigma} \) denotes the mass of investors of type \( \sigma \in \{CW,C,OW,O,IG,N\} \) at time \( t \). Figure 6 illustrates the dealer’s activities in the repo market and cash market.

\[
V_D(W_t, Q_{t}^{RR}, Q_{t}^{R}) = \sup_{x,C} \mathbb{E}_t \int_{s=0}^{\infty} e^{-rs} dC_{t+s}
\] \hspace{1cm} (12)

such that

\[
dW_t = rW_t dt - dC_t + \frac{1}{N} \lambda_{c} \mu_t^{CW} P dt + AQ_t^{R} dt - \delta Q_t^{R} \mathbb{E}\left\{ e^{rT} \right\} P dt - \frac{1}{N} \lambda_{o} \mu_t^{OW} P dt - BQ_{t}^{RR} dt + \delta Q_{t}^{RR} \mathbb{E}\left\{ e^{rT} \right\} P dt + x_{t}cdt - Pdx_t \hspace{1cm} (13)
\]

\[
x_t + Q_{t}^{RR} \geq Q_{t}^{R} \hspace{1cm} (14)
\]

\[
dQ_{t}^{RR} = -\delta Q_{t}^{RR} dt + \frac{1}{N} \lambda_{o} \mu_t^{OW} dt \hspace{1cm} (15)
\]

\[
dQ_{t}^{R} = -\delta Q_{t}^{R} dt + \frac{1}{N} \lambda_{c} \mu_t^{CW} dt \hspace{1cm} (16)
\]

Equation (13) shows that the cash flow for the dealer comes from three sources: repo, reverse repo, and trade in the cash market. For example, the third, fourth and fifth terms show cash flow related to repo positions. Applying the exact law of large numbers (Sun, 2006), the mass of cash investors coming to this dealer to obtain sovereign bonds is \( \frac{1}{N} \lambda_{c} \mu_t^{CW} \) per unit of time almost surely. Each new repo position gives the dealer cash equivalent to \( P \) units of consumption numeraire. Hence, the third term \( \frac{1}{N} \lambda_{c} \mu_t^{CW} P \) is the cash collateral received by the dealer on new repo positions per unit of time. The fourth term \( AQ_t^{R} \) is the specialness premium collected by the dealer on outstanding repo positions per unit of time. The dealer has a continuum of outstanding repo positions, each of which matures at the rate \( \delta \) independently of one another. By the law of large numbers, the mass of repo positions maturing per unit of time is \( \delta Q_{t}^{R} \) almost surely. Upon maturity, the dealer returns the cash along with the interest accrued over the lifetime of the repo.

The cash flow from reverse repo positions can be explained similarly. The last two terms — \( x_{t}cdt - Pdx_t \) — come from trade in the cash market.

Equation (14) implies that in for the dealer to conduct a repo on a sovereign bond, the dealer must either purchase the bond or do a reverse-repo on the bond. This constraint is akin to the box constraint in Huh and Infante (2018). Using the exact law of large numbers, the
dealer’s repo reverse repo positions evolve according to equation (15) and equation (16) almost surely.

The dealers are not allowed to trade in corporate bonds in this model. As of 2017:Q2, the European monetary financial institutions held only 9.56% of the eurozone non-financial corporate liabilities with remaining maturities of no less than one year. On the other hand, ICPF s held 41.8% of the long-term euro-area non-financial corporate bonds during the same period.

Insurance companies might be willing to pay an extra premium for a long-dated bond that can relax their financing constraints and help meet capital regulations. Many insurance companies, particularly, life insurers, have very long-dated liabilities. By purchasing long-dated bonds, insurers can match the durations on their assets and liabilities. When their assets and liabilities have similar durations, insurers are less likely to incur substantial losses if interest rates fluctuate in the future. Consequently, under Solvency 2 capital regulation, insurance companies can hold less capital against potential future losses. Therefore, since raising equity capital is costly, insurance companies prefer holding long-dated bonds over short-dated bonds. They may be willing to pay a price higher than the simple present value of future cash flows from holding long-dated bonds. In comparison, bank dealers do not have such incentives to hold long-dated bonds. Thus, in an equilibrium, insurance companies are more likely to become marginal investors in the long-term bond market.

Figure 6: Schematics of the Dealer’s Problem
3.5. Agent Mass Dynamics

The value functions of IG and OW non-banks can be shown to be linearly separable in wealth (Duffie, Garleanu, and Pedersen, 2005). Hence, let \( V_{IG}(W_t) = J_{IG} + W_t \) and \( V_{OW}(W_t) = J_{OW} + W_t \). Non-banks in the N state buy a corporate bond, if \( J_{IG} > J_{OW} \) and a sovereign bond, if \( J_{IG} < J_{OW} \). Non-banks are indifferent between the two options, if \( J_{IG} = J_{OW} \). Let \( f_{IG} \) denote the mass of non-banks buying corporate bonds per unit of time. Similarly, let \( f_{OW} \) denote the mass rate non-banks buying sovereign bonds. Then, it must be the case that \( f_{IG} = 0 \) if \( J_{OW} > J_{IG} \) and \( f_{OW} = 0 \) if \( J_{OW} < J_{IG} \).

By the exact law of large numbers, the mass of investors of each type evolve according to (17), (18), (19), (20), (21) and (22) almost surely. Equation (23) denotes a restriction that the total mass of cash investors summed across all types should be \( m_c \). Equation (24) denotes a similar restriction for non-banks.

\[
\begin{align*}
\frac{d}{dt} \mu_t^O &= \lambda_o \mu_t^{OW} - \delta \mu_t^O \quad (17) \\
\frac{d}{dt} \mu_t^{OW} &= f_{OW} - \lambda_o \mu_t^{OW} \quad (18) \\
\frac{d}{dt} \mu_t^{IG} &= -\eta \mu_t^{IG} + f_{IG} \quad (19) \\
\frac{d}{dt} \mu_t^N &= \eta \mu_t^{IG} - f_{IG} - f_{OW} \\
\frac{d}{dt} \mu_t^{CW} &= \delta \mu_t^C - \lambda_c \mu_t^{CW} \quad (21) \\
\frac{d}{dt} \mu_t^C &= -\delta \mu_t^C + \lambda_c \mu_t^{CW} \quad (22) \\
\mu_t^C + \mu_t^{CW} &= m_c \quad (23) \\
\mu_t^N + \mu_t^{IG} + \mu_t^{OW} + \mu_t^O &= m_n \quad (24)
\end{align*}
\]

3.6. Nash Bargaining

Nash bargaining between a dealer and a customer (either a cash investor or a non-bank) determines the repo specialness premia \( A \) and \( B \). Let \( z \) and \( 1 - z \) denote the relative bargaining powers of a dealer and a customer where \( 0 < z < 1 \). See Duffie, Garleanu, and Pedersen (2007) for micro-foundations.

3.7. Cash Markets

I model the sovereign bond market as a two-tiered system comprising the D2D segment and the D2C segment. The D2D market is assumed to be competitive. In the D2C segment,
a non-bank buys a sovereign bond from dealers at a price equal to the D2D market price. This set-up reflects the actual structure of the European sovereign bond market (Ejsing and Sihvonen, 2009; Schlepper, Hofer, Riordan, and Schrimpf, 2017), especially the bund market (Huszar and Simon, 2017). I assume the corporate bond market is perfectly competitive, where only non-banks participate.

The outstanding quantity of sovereign bonds is exogenously fixed at $S_g$. $nc$ are the units of new corporate bonds issued per unit of time. As corporate bonds can default in my model, issuance of new corporate bonds is necessary for analyzing the steady-state equilibrium.

4. Equilibrium

I focus on a stationary equilibrium in which the mass of agents and market prices do not change over time. Superscript $i$ implies that the variable is associated with a dealer $i \in \{1, 2, ..., N\}$. When defining equilibrium, I do not consider the equilibrium consumption decisions of agents. This approach is without loss of generality because all agents are risk-neutral with the discount rate equal to the risk-free rate.

A continuum of ex-ante identical bond owners faces the same stationary problem. Let $\lambda_{o,j,t}$ denote the search intensity of non-bank $j$ at time $t$. Similarly, let $\lambda_{c,k,t}$ denote the search intensity of cash investor $k$ at time $t$. The constants $\lambda_o$ and $\lambda_c$ are such that $\lambda_{o,j,t} = \lambda_o$ for $\forall j, t$ and $\lambda_{c,k,t} = \lambda_c$ for $\forall k, t$.

**Definition 1.** Let $\mu = (\mu_{OW}, \mu_O, \mu_{CW}, \mu_C, \mu_{IG})$ and $V=(V_{OW}, V_O, V_{CW}, V_C, V_{IG}, V_N)$. An equilibrium consists of the mass of investors $\mu$, investor value functions $V$, market prices $(A, B, P, P_{ig})$, search intensities $(\lambda_o, \lambda_c)$, and non-bank bond purchase rate $(f_{IG}, f_{OW})$ such that:

1. Each OW non-bank chooses a search intensity $\lambda_o$ that solves the problem (2) and (3). Each CW cash investor chooses a search intensity $\lambda_c$ that solves the problem (8) and (9).
2. The mass of investors evolve according to (17), (18), (19), (20), (21) and (22) almost surely.
3. The non-bank bond purchase rate in the N state is incentive compatible. $f_{IG} = 0$ if $V_{OW} > V_{IG}$ and $f_{OW} = 0$ if $V_{OW} < V_{IG}$.
4. The investor’s value functions $V$ are defined by (1), (2), (3), (5), (6), (7), (8), (9), (10), (11), (12), (13), (14), (15) and (16).
5. Market clearing for the corporate bond market: $\mu^{IG}_t = \frac{nc}{\eta}$
6. Market clearing for the sovereign bond market: $\mu^O_t + \mu^{OW}_t + \sum_{i=1}^{N} x^i_t = S_g$

I assume that the total mass of non-banks $mn$ equals the sum of outstanding quantities of corporate bonds and sovereign bonds. That is, $mn = \frac{nc}{\eta} + S_g$. Hence, in equilibrium, the risk-neutral non-bank must be indifferent between owning a sovereign bond and owning a corporate bond. If the non-bank strictly prefers owning a corporate bond over a sovereign bond, then
there is excess demand. If the non-bank’s preference is the other way around, then there is an excess supply in the corporate bond market. This modeling assumption is mainly for simplicity. The same intuition holds even if I relax this assumption, but with much more complex algebra.

**Theorem 1. Unique stationary equilibrium.** There is a unique stationary equilibrium. The repo specialness premia \((A, B)\), the search intensities \((\lambda_o, \lambda_c)\) and the sovereign bond price \(P\) are determined by (25), (26), (27), (28) and (29). The corporate bond price \(P_c\) is determined by (30).

\[
b - c - A + K(\lambda_c) = K'(\lambda_c)(r + \lambda_c + \delta) \tag{25}
\]
\[
B + K(\lambda_o) = K'(\lambda_o)(r + \lambda_o + \delta) \tag{26}
\]
\[
A = z \{K(\lambda_c) + b - c\} + (1 - z)(rP - c) \tag{27}
\]
\[
B = -z \cdot K(\lambda_o) + (1 - z)(rP - c) \tag{28}
\]
\[
mc \frac{\lambda_c}{\delta + \lambda_c} = \left( m_n - \frac{n_c}{\eta} \right) \frac{\lambda_o}{\delta + \lambda_o} \tag{29}
\]
\[
P_c = \frac{c}{r} - (1 - \Lambda) \frac{c \cdot \eta}{r(r + \eta)} + \frac{B + K(\lambda_o)}{1 - z} \frac{r + \delta + z \cdot \lambda_o}{r + \delta + \lambda_o} \frac{1}{r + \eta} \tag{30}
\]

**Proof.** See section B.1.

Equations (27) and (28) are consistent with Duffie, Garleanu, and Pedersen (2002). Let \(\psi = rP - c\). It can be shown that \(\psi\) is positive in equilibrium. \(\psi\) can be interpreted as the premium that investors are willing to pay for purchasing a sovereign bond and it is increasing in the repo specialness premia \(A\) and \(B\). When the repo specialness premia are high, the sovereign bonds appreciate to reflect the potential opportunity for earning extra revenue from the repo market.

Equation (30) shows that the corporate bond price in the model can be decomposed into three parts. The first term is the present value of future coupon payments promised. The second term is the discount due to the credit risk of the corporate bond. The third term captures the portfolio rebalancing effect. The magnitude of this term increases in the repo specialness premium \(B\). If sovereign bonds are more special in the repo market, the portfolio rebalancing of non-banks causes corporate bonds to become more expensive. The magnitude of this term also increases in the search effort cost \(K(\lambda_o)\). Higher search frictions in the repo market imply that it is costlier for non-banks to monetize repo specialness premium. Thus, owning a sovereign bond becomes less attractive for non-banks.

### 4.1. The Impact of the PSPP

Koijen, Koulisher, Nguyen, and Yogo (2018) shows that most transactions under the PSPP took place between the Eurosystem and foreign institutions that did not have access to the central bank deposit facility. When foreign financial institutions sell sovereign bonds to the
Eurosystem, they are credited with unsecured deposits in European banks (Coeuré, 2017). However, they do not like parking large quantities of cash parked in unsecured deposits due to their risk management practices. Hence, they invest their cash in short-term European sovereign bonds or the GC repo market (Mersch, 2017). Foreign financial institutions that invest their cash get sovereign bonds in return. Hence, they can be mapped as cash investors in the model.

To understand how the PSPP impacts asset prices, I show how the solution to my model changes as \( m_c \) (mass of cash investors) increases and \( m_n \) (mass of non-banks) decreases.

**Theorem 2. The Impact of PSPP on Asset Prices.** Let \( \theta = \frac{m_n}{m_c} \). Then \( \frac{\partial}{\partial \theta} P < 0 \) and \( \frac{\partial}{\partial m} P_c < 0 \).

**Proof.** See section B.2.

The marginal impact of the PSPP on the repo market is to create excess demand for collateral. The equilibrium specialness premia \( A \) and \( B \) increase to equate demand and supply again. While the OW non-banks search for dealers more intensely, cash investors search for dealers less intensely. The PSPP can affect the revenue that dealers earn in the repo market in two ways. First, a higher specialness premia imply that dealers can earn more revenue for each repo transaction with cash investors. Second, because cash investors search for dealers less intensely, dealers enter into repo transactions less frequently. This reduces each dealer’s revenue. Nevertheless, for an arbitrary increasing and convex search cost function \( K(\cdot) \), I show that dealers that own sovereign bonds are expected to earn more revenue in the repo market. Consequently, dealers may be willing to pay more in the cash market to own sovereign bonds.

Since dealers charge non-banks more for sovereign bonds, without any change in the corporate bond price, non-banks will substitute sovereign bonds with corporate bonds. Corporate bonds appreciate to clear the market.

### 4.2. The Impact of the Securities Lending Facility

I now consider a hypothetical scenario where the Eurosystem operates a securities lending facility akin to the reverse repo facility of the Fed. That is, the Eurosystem conducts repo transactions on sovereign bonds with cash investors. In my model, the operation of this facility will reduce the value of \( m_c \) — the mass of cash investors trying to obtain sovereign bonds in the private market. Consequently, by theorem 2, both sovereign bonds and corporate bonds appreciate.

### 4.3. Welfare Analysis of the Securities Lending Facility

I take the cash market transactions under the PSPP as given. I show how welfare changes with respect to the quantity of sovereign bonds used for direct repo transactions between cash investors and the Eurosystem. Suppose cash investors of mass \( x \) obtain sovereign bonds directly
from the facility without incurring any search cost. The remaining cash investors of mass \( m_c - x \) obtain sovereign bonds from the private repo market.

As all investors in the model are risk-neutral, any monetary transfer between investors (e.g., repo specialness premium) is irrelevant for welfare analysis. I also assume that the central bank is risk-neutral. Hence, the specialness premium at which cash investors conduct reverse-repos in sovereign bonds from the securities lending facility is also irrelevant for welfare analysis.

For welfare analysis, allocative efficiency and search cost are relevant. Equation (31) shows my measure of social welfare as a function of \( x \). The first term is allocative efficiency; the more cash investors obtain sovereign bonds, the more flow of benefit \( b \) they get per unit of time. The second term is the search effort cost of OW non-banks per unit of time. The third term is the search effort cost of CW cash investors per unit of time. The mass of agents and search intensities vary with \( x \).

\[
W(x) = b \left\{ x + \mu^C(x) \right\} - \mu^{OW}(x) K(\lambda_o(x)) - \mu^{CW}(x) K(\lambda_c(x))
\]

**Theorem 3.** Suppose the mass of sovereign bond owners is greater than the mass of cash investors. \( \mu^{OW} + \mu^O > \mu^{CW} + \mu^C \). Social welfare \( W(x) \) increases in \( x \).

**Proof.** See section B.3.

The securities lending facility improves allocative efficiency. Since fewer cash investors try to obtain sovereign bonds from the private repo market, the repo specialness premia \( A \) and \( B \) decline. Consequently, each OW non-bank reduces its search effort while each CW cash investor exerts more search effort. With an arbitrary convex and increasing search cost function \( K(\cdot) \), the reduction in search cost of non-banks and improvement in allocative efficiency dominate the increase in search cost of cash investors.

## 5. Numerical Example

I match the risk-free rate \( r \) to the overnight index swap (OIS) rate with a 30-year tenor. Since the model contains perpetual bonds, I choose the OIS rate with the longest possible tenor. I match the mass of non-banks holding sovereign bonds \( \mu^O + \mu^{OW} \) to the Securities Holdings Statistics data.

I set the non-pecuniary benefit of obtaining sovereign bond \( b \) so that repo specialness premium in my model can be at least as high as the maximum specialness premium observed in the data. A high specialness premia observed in the data imply that the opportunity cost of not obtaining sovereign bonds is very high, at least for a subset of investors.

I allow non-banks and cash investors to have different cost functions. The non-bank search cost function is parameterized as \( K(\lambda) = k_o \cdot \lambda^\alpha \). The cash-investor search cost function is parameterized as \( K(\lambda) = k_c \cdot \lambda^\alpha \). All the other parameters are set to match the trading volume in BrokerTec, bid-ask spread in European repo\(^9\), 30-year German federal government bond

\(^9\)According to Hill (2015a), the bid-ask spread for sovereign bond term repo in Europe is around 7 basis points.
yield, and 30-year European investment grade corporate yield. Since there are no perpetual bonds, I use the Bloomberg yield curves at the longest remaining maturity possible. Table 5 summarizes the parameter values used for the numerical calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>0.962%</td>
<td>$c$</td>
<td>0.03</td>
<td>$\alpha$</td>
<td>2.5</td>
</tr>
<tr>
<td>$z$</td>
<td>0.1</td>
<td>$b$</td>
<td>4.5%</td>
<td>$\Lambda^{10}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$\mu_D$</td>
<td>10</td>
<td>$m_c$</td>
<td>450</td>
<td>$\eta$</td>
<td>0.065</td>
</tr>
<tr>
<td>$k_o$</td>
<td>$10^{-0.3}$</td>
<td>$k_c$</td>
<td>$5 \times 10^{-2.3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Parameter Values for the Numerical Calibration

The solutions to my model change as the PSPP withdraws sovereign bonds from circulation in the market. If the Eurosystem purchases $x$ units of sovereign bonds, $m_n$ decreases by $x$. At the same time, $m_c$ increases by $x$. Figure 7 shows this result, confirming theorem 2.

![Figure 7: The Impact of PSPP on Asset Prices.](image)

Figure 7: The Impact of PSPP on Asset Prices. The graph shows the change in the model solutions when the PSPP withdraws sovereign bonds from circulation in the private market. The midpoint on the extreme-left panel is the average of two repo specialness premia $A$ and $B$.

The sovereign bond price and corporate bond price in the model deviate completely from the expected value of future coupon payments because of search frictions in the repo market. To check this intuition, I show the change in the model solutions with respect to the constant $k_o$ that enters the search cost function of a non-bank. The ratio of the search cost parameter $k_c$ of a cash investor is fixed to that of a non-bank. Figure 8 shows the result.

Figure 8 shows that as the search cost asymptotically reduces to zero, the specialness premium $B$ converges to zero. Both the sovereign bond price and the corporate bond price converge

---

This value is in line with Heynderickx, Cariboni, Schoutens, and Smits (2016).
Figure 8: The Impact of Search Cost on Asset Prices. The graphs show the change in the model solutions with respect to the search cost parameters $k_o$ and $k_c$. The search cost functions of the non-bank and cash investor are $k_o \lambda_o^\alpha$ and $k_c \lambda_c^\alpha$, respectively. The ratio of $k_o$ to $k_c$ is kept constant at 20. The far-left panel shows the specialness premium $B$ at which the non-bank conducts a repo transaction on sovereign bonds with bank dealers. The remaining two panels show the impact of search cost on bond yields. The benchmark yields are what would be observed in the model without any search frictions. The benchmark sovereign bond yield is the risk-free rate $r$ and the benchmark corporate bond price is $c_a - (1 - A) \frac{\eta}{r(r + \eta)}$, the first two terms in (30). The vertical axis in the middle and right panels shows the model-implied yields minus the benchmark bond yields.

to the simple expected value of future coupon payments.

6. Data

I consider all repo transactions on BrokerTec that use the sovereign bonds of Austria, Belgium, Finland, Italy, Ireland, the Netherlands, Spain, and Portugal. French sovereign bonds are not included because of the unique institutional design of the French repo market. For example, most French repos are floating-rate while other European repos are fixed-rate. The prices of all euro-denominated corporate and government bonds (e.g., regional government bonds) on Bloomberg are considered. Only bonds with remaining maturity of more than one year are considered.

The sample period starts from March 1, 2013, because of my limited access to BrokerTec data, till March 31, 2019. The empirical strategy is applicable to the period before March 1, 2013 as well.

Only bonds with fixed or zero-coupon payments and bullet bonds (instead of amortizing bonds) are considered. The yield data are sourced from Bloomberg.

The repo transaction data are from BrokerTec, an electronic platform on which the majority
of interdealer European SC repo is traded (EMMI, 2017). I use only transactions with spot-
next (SN) tenor, the most commonly observed tenor on BrokerTec. SN tenor implies that the
collateral and cash are exchanged two business days after both parties have agreed on the terms
of the transaction. The collateral and cash are returned after another business day.

The specialness premium is defined as the GC pooling ECB extended basket rate minus the
repo rate. Table 2 summarizes data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Median</th>
<th>Standard Deviation</th>
<th>99th Percentile</th>
<th>1th Percentile</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repo Rate (%)</td>
<td>-0.324</td>
<td>0.277</td>
<td>0.209</td>
<td>-0.918</td>
<td>219,191</td>
</tr>
<tr>
<td>Yield-to-Maturity (%)</td>
<td>0.523</td>
<td>1.125</td>
<td>4.729</td>
<td>-0.576</td>
<td>3,893,418</td>
</tr>
<tr>
<td>GC Rate (%)</td>
<td>-0.217</td>
<td>0.225</td>
<td>0.339</td>
<td>-0.430</td>
<td>1,466</td>
</tr>
</tbody>
</table>

Table 2: **Summary Statistics** The repo rate data are from BrokerTec. The yield data are
from Bloomberg. The GC pooling ECB extended basket rate data is from Bloomberg.

### 7. Empirical Strategy

The goal of this empirical analysis is to measure the impact of the repo specialness premia of
sovereign bonds on the yields on all euro-denominated bonds. My strategy is based on the GIV

I start with (32), a model of the yield-to-maturity of a bond. $y_{icmt}$ denotes the yield-to-
maturity of bond $i$ on day $t$.

$c$ indexes the country in which the issuer of the bond is based. $m$ indexes the bucket for the
remaining maturity of the bond. $s_{icmt}$ is the repo specialness premium of bond $i$ on day $t$. $\overline{s}_{cmt}$ is the weighted average value of the repo specialness of all sovereign bonds of country $c$ and maturity bucket $m$ traded on BrokerTec. I use the transaction volumes reported by BrokerTec as weights. I have a time-invariant fixed-effect term $\theta_{icm}$ for each bond. For example, bonds with different credit ratings are likely to have different yields. $\omega_{icmt}$ is an unobserved error
term that can influence the yield.

$$ y_{icmt} = h \cdot \overline{s}_{cmt} + \beta \cdot s_{icmt} + \theta_{icm} + \omega_{icmt} \tag{32} $$

Theoretically, even if bond $i$ is not special in the repo market (e.g., $s_{icmt} = 0$), its price can
depend on the repo specialness premia of other bonds. One channel for this is through the
portfolio rebalancing of ICPFs. The average repo specialness of sovereign bonds can affect the
pricing kernel of financial institutions and the expected returns on non-sovereign bonds. The
first term in (32) captures this effect.

$\overline{s}_{cmt}$ is country-specific because the bond markets of different countries might have different
pricing kernels. For example, banks and insurance companies in Europe prefer to invest in
bonds whose issuer is based in their home country (Koijen, Koulischer, Nguyen, and Yogo, 2018). Thus, the pricing kernels of German financial institutions might be more important in pricing German bonds than the pricing kernels of French financial institutions. If German and French financial institutions have different kernels, then different kernels may be necessary to price German bonds versus French bonds.

The ordinary least squares (OLS) estimate of $h$ in (32) will not be consistent because $\bar{s}_{cmt}$ may be correlated with the unobserved factors $\omega_{icmt}$. Therefore, I need an instrument for $\bar{s}_{cmt}$.

### 7.1. Constructing an Instrumental Variable

Sovereign bonds with a different country of domicile and remaining time to maturity are likely to have different repo rates. The first step in the construction of an instrumental variable is to extract the common factors driving the repo rates of a diverse set of European sovereign bonds.

I first fix a list of remaining maturities to analyze and consider 31 evenly spaced values between 1 year and 21 years. For each pair of country $c$ and remaining maturity $\tau$, I construct the daily time-series of the repo rate of sovereign bonds. However, on most days, there will not be any bond that has the exact $\tau$ years to maturity, so I use nonparametric regression to estimate the value. I fill in the daily repo rates for each entry in Table 3. For example, the entry at the intersection of the row “Austria” and the column “1.67 years” is the repo rate of an Austrian government bond with 1.67 years to maturity.

I run a PCA on the time-series of 248 repo rates (31 maturities and 8 countries). Let $\hat{PC}_1t, \hat{PC}_2t$ and $\hat{PC}_3t$ denote the first three principal components that I estimate.

Suppose there is a factor model (33) for the repo rates of European sovereign bonds. $r_{kcmt}$ is the repo rate of a sovereign bond $k$ of country $c$ on day $t$. $\lambda_{cmf}$ denotes the loading of the repo rate $r_{kcmt}$ on the $f$th principal component. I assume that the idiosyncratic component $u_{icmt}$ is uncorrelated with the common factors. I stop at the first three principal components because they already explain 95% of the total variation in repo rates. See Section F of the Appendix for the result of the PCA.

$$r_{kcmt} = \sum_{f=1}^{3} \lambda_{cmf} \hat{PC}_ft + u_{kcmt}$$ (33)

To model the dependence of the loadings $\lambda_{cmf}$ on the remaining maturities, I use the maturity buckets from Table 3, assuming that there is a unique set of loadings for each pair of country of domicile and maturity bucket.

I run an OLS to estimate the residuals $u_{kcmt}$ and then construct $z_{cmt}$ as in (34) to instrument the country-maturity-bucket specific average repo specialness $\bar{s}_{cmt}$. $N_{cmt}$ is the number of sovereign bonds of country $c$ in maturity bucket $m$ that are traded on BrokerTec on day $t$. $z_{cmt}$ is the cross-sectional average of the idiosyncratic components of the repo rates of sovereign bonds of country $c$ and maturity bucket $m$. 
Table 3: **Estimation of the Repo Rates for Each Country and Remaining Maturity**

I impute the daily repo rate for a bond with a given country of domicile and remaining maturity. For each day of the sample period and for each country of domicile, I nonparametrically regress the repo rates on remaining maturities to impute the repo rate for a bond that has exactly a given years (e.g., 1.67 years) to maturity. I iterate this process for each pair of country and remaining maturity.

![Table showing countries and remaining maturities](image)

\[
z_{cmt} = \frac{1}{N_{cmt}} \sum_{k=1}^{N_{cmt}} \hat{u}_{kcmt}
\]  

(34)

The identifying assumption here is that these idiosyncratic components \( u_{kcmt} \) are not correlated with the unobserved factor \( \omega_{icmt} \), which could affect the expected returns on bonds in the cash market. \( u_{kcmt} \) can represent, for example, an idiosyncratic shock to the dealer bank’s ability to intermediate in a repo transaction in the bilateral segment.
Figure 9: \textbf{Example of Estimated Idiosyncratic Residuals and GIV.} The upper panel shows the time-series of the estimated idiosyncratic components of the repo rate of an Austrian sovereign bond. The ISIN of the bond is AT0000383864. The lower panel shows the time-series of the GIV that cross-sectionally averages the estimated residuals across all German sovereign bonds. See (33) and (34) for the definitions of an idiosyncratic residual and instrumental variable, respectively. The data is from BrokerTec.

My empirical specification is (35). $\theta_{icm}$ is the bond fixed-effect term. To mitigate reverse causality from the cash market to the repo market, I only consider bonds that have never been on special during the sample period. Thus, my specification does not have a term involving $s_{icmt}$. $X_{icmt}$ are control variables.

\begin{equation}
 y_{icmt} = h \cdot \sigma_{cnt} + \gamma \cdot X_{cnt} + \theta_{icm} + u_{icmt}
\end{equation}

8. \textbf{Empirical Result}

Table 4 shows the result. An increase in the repo specialness of sovereign bonds by ten basis points is associated with a decrease in yields on other bonds by 1.8 basis points.

Figure 10 shows the time-series of the average repo specialness of sovereign bonds issued by governments of core countries in the eurozone. Over the course of the PSPP, the average repo specials increased up to 30 basis points.

The largest impact is seen for German corporate bond yields during the fourth quarter of
Table 4: **The Effect of the Repo Specialness Premia of Sovereign Bonds on the Yield-to-Maturities of Bonds.** The table shows the two-stage least squares (2SLS) fixed-effect panel regression of the yield-to-maturities of bonds on the repo specialness premia of sovereign bonds whose issuer is based in the same country. The fixed-effect panel regression specification is
\[ y_{icmt} = h \cdot \bar{\pi}_{cmt} + \gamma \cdot X_{cmt} + \theta_{icm} + u_{icmt}. \]

$\bar{\pi}_{cmt}$ is the weighted average value of the repo specialness premia of sovereign bonds of country $c$ on day $t$. The weight is the transaction volume on BrokerTec. $h$ instrument $\bar{\pi}_{cmt}$ with the GIV, following Gabaix and Koijen (2019). $\theta_{icm}$ is the bond fixed-effect term.

The repo specialness is defined as the GC pooling ECB extended basket rate minus the repo rate at SN tenor. $y_{icmt}$ is the yield-to-maturity of bond $i$. The $t$-statistics are in parentheses. The standard errors are two-way clustered at the bond level and time level (Petersen, 2009). *, **, and *** indicate that the coefficient is statistically significant at 1%, 5%, and 10%, respectively. The sample period is from March 1, 2013 to March 31, 2019. There are 1,351 time clusters and 4,270 bond clusters. Anderson-Rubin Wald test for the weak instrument is performed. The null hypothesis is that the exclusion restriction holds and the instrumental variable is not significant in the first-stage regression. The null is rejected at the p-value of 2.35%.

<table>
<thead>
<tr>
<th>Dependent Variable: $y_{icmt}$ (Basis Points)</th>
<th>$\bar{\pi}_{cmt}$ (in Basis Points)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.189**</td>
</tr>
<tr>
<td></td>
<td>(-2.23)</td>
</tr>
<tr>
<td>$EONIA_t$</td>
<td>0.0357***</td>
</tr>
<tr>
<td></td>
<td>(12.25)</td>
</tr>
<tr>
<td>Controls</td>
<td>Principal Components</td>
</tr>
<tr>
<td>Cluster</td>
<td>Bond, Time</td>
</tr>
<tr>
<td>Observations</td>
<td>3,144,332</td>
</tr>
<tr>
<td>Frequency</td>
<td>Daily</td>
</tr>
<tr>
<td>Fixed Effect</td>
<td>Bond</td>
</tr>
</tbody>
</table>

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2016. German corporate bond yields, in particular, may have been lowered by up to 5.8 basis points through the repo specialness effect.

Figure 10: The Average Repo Specialness of Sovereign Bonds. The graph shows the average repo specialness of sovereign bonds issued by governments of core countries in the eurozone. The data are from BrokerTec. I removed data on quarter ends.

9. Discussion

9.1. The Repo Specialness Channel in Other Settings

I present evidence of the transmission of the repo specialness premia of government bonds to corporate bond yields in this study. Nevertheless, there is anecdotal evidence that in response to the recent depreciation in the euro-area government bond yields, European insurance companies increased the purchase of sub-sovereign and agency (SSA) bonds (AXA, 2016). Some ICPFs started investing in risky properties (O’Donnell and Cohn, 2016). Such portfolio rebalancing would cause these asset prices (e.g., SSA bonds) to appreciate.

The transmission can happen not only through ICPFs but also through any financial institution that faces frictions in their use of the repo market. For example, the new European regulation\textsuperscript{11} on money market funds does not allow funds to conduct repo transactions on more than 10% of their assets.

Another example is small banks. Small banks that are not members of central clearing parties (CCPs) participate in the repo market indirectly through large bank dealers. Small banks do not have a large flow of repo transactions that justify the fixed cost of becoming members of CCPs (ICMA, 2015). Hence, like the non-banks in my model, they incur extra costs, such

as search costs or relationship costs. As the repo specialness premia cause government bond yields to become even lower, small banks are likely to substitute government bonds with risky assets, such as loans to firms. In fact, inducing the banking sector to substitute government bonds with riskier corporate loans is an important function that the Eurosystem had in mind in the context of the PSPP (Cœuré, 2015). The repo specialness premia on government bonds amplify this function.

Once the yields on investment-grade corporate bonds decline, there can be knock-on effects on firms that do not have access to the capital market. For example, firms with access to the capital market can substitute bank loans with corporate bonds. As a result, banks will have more lending capacity available and can use that capacity to give loans to firms that do not have access to the bond market. Grosse-Rueschkamp, Steffen, and Streitz (2017) identified this effect for the case of the Corporate Sector Purchase Program (CSPP).

9.2. Fiscal Implications

An increase in the repo specialness of government bonds can help the government issue its bonds more cheaply in the primary market. This effect is likely to have been particularly relevant to the governments of the two largest euro-area economies — France and Germany — during the PSPP period. Of the government bonds of the seven largest eurozone economies, the French and German bonds were the most special in the repo market (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018). Moreover, both the French and German governments rely on primary dealers to issue their bonds. Primary dealers are banks that can actively conduct repo transactions on their government bonds once acquired. Therefore, primary dealers could be willing to bid higher prices for the purchase of government bonds that are on special in the repo market. Their aggressive bids could have helped lower the financing costs of the French and German governments.

9.3. The Allocative Efficiency

An increase in the repo specialness of government bonds through the scarcity of available bonds can have an adverse impact on the allocative efficiency of both the bond market and the repo market. Specialness reflects heightened search frictions in the repo market. That is, collateral providers and cash investors are not matched to each other efficiently. The inefficiency arises because the bonds stay with the less efficient users of these bonds (collateral provider) instead of the more efficient users (cash investors).

Besides, dealers in the bond market often deliver bonds to their customers by reversing-in the requested bonds from the repo market (ICMA, 2015). Larger search frictions in the repo market imply that dealers may not be able to obtain the bonds promptly. Thus, search frictions

\footnote{According to the website of Agence France Trésor, “The principal method of issuing French government securities since 1985 has been” bid price auctions in which primary dealers participate. According to the website of the German Finance Agency, more than 90% of the bunds are auctioned to primary dealers.}
in the repo market, signaled by large specialness, can also interfere with the matching of buyers and sellers of the bond.

10. Conclusion

The repo specialness of sovereign bonds can magnify the impact of a central bank’s sovereign bond purchases on corporate bond prices. The specific case of the PSPP of Europe motivated this study. Unlike the Bank of Canada or the Federal Reserve System of the United States, the Eurosystem does not sell most of the bonds that it purchases under the PSPP back on the market. The repo specialness premium causes sovereign bonds to become more expensive in the cash market. Consequently, non-banks substitute the expensive sovereign bonds with investment-grade corporate bonds, thereby, allowing firms to finance themselves more cheaply. I estimate that an increase in the repo specials of sovereign bonds by 10 basis points lowers corporate bond yields by 1.8 basis points. In particular, German corporate bonds during the fourth quarter of 2016 may have been lowered by 5.8 basis points through the repo specialness effect.

References


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Praet, P. (2015, June). The APP Impact on the Economy and Bond Markets. Intervention by Peter Praet, Member of the Executive Board of the ECB, at the annual dinner of the ECB’s Bond Market Contact Group, Frankfurt am Main.


A. List of Parameters and Variables in the Theoretical Model

Table 5: The list of parameters used in the model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Meaning</th>
<th>Exogenous?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>The rate at which repo matures</td>
<td>Y</td>
</tr>
<tr>
<td>$\eta$</td>
<td>The rate at which a unit of corporate bond expires</td>
<td>Y</td>
</tr>
<tr>
<td>$r$</td>
<td>The risk-free rate</td>
<td>Y</td>
</tr>
<tr>
<td>$\lambda_o$</td>
<td>Search intensity of OW non-bank</td>
<td>N</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Search intensity of CW cash investor</td>
<td>N</td>
</tr>
<tr>
<td>$A$</td>
<td>The specialness premium at which cash investors reverse repo government bonds</td>
<td>N</td>
</tr>
<tr>
<td>$B$</td>
<td>The specialness premium at which OW non-banks repo government bonds</td>
<td>N</td>
</tr>
<tr>
<td>$m_c$</td>
<td>The mass of cash investors</td>
<td>Y</td>
</tr>
<tr>
<td>$m_n$</td>
<td>The mass of non-banks</td>
<td>Y</td>
</tr>
<tr>
<td>$b$</td>
<td>The benefit of obtaining government bond for cash investors per unit of time</td>
<td>Y</td>
</tr>
<tr>
<td>$c$</td>
<td>The coupon rate of a government bond and a corporate bond</td>
<td>Y</td>
</tr>
<tr>
<td>$P_{ig}$</td>
<td>The corporate bond price</td>
<td>N</td>
</tr>
<tr>
<td>$P$</td>
<td>The government bond price</td>
<td>N</td>
</tr>
<tr>
<td>$z$</td>
<td>Dealer’s bargaining power</td>
<td>Y</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of dealers</td>
<td>Y</td>
</tr>
<tr>
<td>$S_g$</td>
<td>The outstanding quantity of government bonds</td>
<td>Y</td>
</tr>
<tr>
<td>$n_c$</td>
<td>The quantity of new corporate bonds issued per unit of time</td>
<td>Y</td>
</tr>
<tr>
<td>$f_{IG}$</td>
<td>The mass rate of N non-banks that buy corporate bonds</td>
<td>N</td>
</tr>
<tr>
<td>$f_{OW}$</td>
<td>The mass rate of N non-banks that buy government bonds</td>
<td>N</td>
</tr>
</tbody>
</table>

Table 6: The list of random variables used in this model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_\delta$</td>
<td>Stopping time at which repo matures</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>Stopping time at which a customer meets a dealer</td>
</tr>
<tr>
<td>$\tau_q$</td>
<td>Stopping time at which a corporate bond defaults</td>
</tr>
</tbody>
</table>
Table 6: The list of random variables used in this model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_t^\sigma )</td>
<td>The mass of investors of type ( \sigma \in { OW, O, CW, C, IG } ) at time ( t ).</td>
</tr>
<tr>
<td>( x_t )</td>
<td>The number of units of government bonds that dealer owns at time ( t ).</td>
</tr>
<tr>
<td>( Q_t^{RR} )</td>
<td>The number of units of government bonds that dealer reverse repos at time ( t ).</td>
</tr>
<tr>
<td>( Q_t^R )</td>
<td>The number of units of government bonds that dealer repos at time ( t ).</td>
</tr>
</tbody>
</table>

B. Proofs

B.1. Theorem 1

Lemma 1. Dealer’s value function is given as (36).

\[
V_D(W_t, Q_t^{RR}, Q_t^R) = W_t + Q_t^R \frac{A + c - rP}{r + \delta} + Q_t^{RR} \frac{r - P - c - B}{r + \delta} + (A + c - rP) \frac{\lambda c \mu_{t}^{CW}}{N \delta (r + \delta)} + (-c - B + rP) \frac{\lambda_o \mu_{t}^{OW}}{N \delta (r + \delta)} \tag{36}
\]

Proof. I assume that dealers have an incentive to intermediate both sides of the repo market. I check this incentive compatibility later. Then the dealer’s problem is isomorphic to (37). Cash collateral exchanged as part of repo and reverse repo transactions is completely irrelevant. Dealer can always undo that exchange of collateral by borrowing or lending at the rate \( r \).

\[
V_D\left( W_t, Q_t^{RR}, Q_t^R \right) = \sup_{x,C} \mathbb{E}_t \int_0^\infty e^{-rs} dC_{t+s} \tag{37}
\]

s.t.

\[
dW_t = rW_t dt - dC_t + AQ_t^{RR} dt - BQ_t^{RR} dt + x_t c dt - P dx_t
\]

\[
x_t + Q_t^{RR} \geq Q_t^R
\]

\[
dQ_t^{RR} = -\delta Q_t^{RR} dt + \frac{1}{N} \lambda_o \mu_{t}^{OW} dt
\]

\[
dQ_t^R = -\delta Q_t^R dt + \frac{1}{N} \lambda_c \mu_{t}^{CW} dt
\]

Because the dealer’s time discount rate is equal to the risk free rate \( r \), it is without loss of generality to assume that \( W_t = 0 \) for all \( t \). Then the value function is linearly separable in \( W_t \). Define a new value function \( J_D(Q_t^{RR}, Q_t^R) \) such that \( V_D(W_t, Q_t^{RR}, Q_t^R) = W_t + J_D(Q_t^{RR}, Q_t^R) \). Duffie, Garleanu, and Pedersen (2005) also made this assumption. Then the dealer’s problem can be reformulated as (38).

\[
J_D\left( Q_t^{RR}, Q_t^R \right) = \sup_x \mathbb{E}_t \int_0^\infty e^{-rs} \left\{ AQ_t^{R} dt - BQ_t^{RR} dt + x_{t+s} c dt - P dx_t \right\} \tag{38}
\]
s.t. 

\[ x_t + Q_t^{RR} \geq Q_t^R \]

\[ dQ_t^{RR} = -\delta Q_t^{RR} dt + \frac{1}{N} \lambda_o \mu_{t}^{OW} dt \]

\[ dQ_t^R = -\delta Q_t^R dt + \frac{1}{N} \lambda_c \mu_t^{CW} dt \]

If the government bond price \( P \) is below \( \xi \), dealers would purchase government bonds in the inter-dealer cash market as much as possible. Thus, the government bond inter-dealer cash market cannot clear.

Suppose \( P = \xi \). Suppose a dealer obtains the government bond from the inter-dealer cash market and subsequently repo the same bond to cash investors. The opportunity cost of using cash to purchase government bonds is zero. As long as \( b \) is positive and the dealer has non-zero bargaining power vis-a-vis cash investors, dealers can profit from this transaction. Hence, the dealer will not refuse to transact with cash investors. \( \theta^R = 1 \).

The feasibility constraint implies that to obtain a government bond for repo position vis-a-vis cash investors, the dealer can purchase it or reverse-in. If \( B \) is zero, then non-banks would exert zero search effort, and there would not be any flow of bonds to repo dealers. In equilibrium, dealers would have to go to the cash market to obtain government bonds to be delivered to cash investors. If \( B \) is positive, then any dealer would choose to go to the cash market to obtain a government bond instead of reverse repoing it. Dealer would not want to incur the cost of \( B \) per unit of time. In either case, dealers would go to the cash market to obtain government bonds. Therefore, in equilibrium, \( x_t = Q_t^R > 0 \) for all dealers. Nevertheless, I already mentioned that for the government bond cash market to clear, dealers as a whole must demand zero government bonds. Hence, the market cannot clear.

Thus, the government bond cash market can clear only when \( P > \xi = \int_{0}^{\infty} e^{-rt} cd t \). Because government bonds are inflated, dealers would want to minimize their position in the cash market \( x_t \). Thus, it must be the case that \( x_t + Q_t^{RR} = Q_t^R \). Thus, \( dx_t = dQ_t^R - dQ_t^{RR} = -\delta Q_t^R dt + \frac{1}{N} \lambda_c \mu_t^{CW} dt + \delta Q_t^{RR} dt - \frac{1}{N} \lambda_o \mu_t^{OW} dt \).

The Hamilton-Jacobi-Bellman equation can be formulated as (39). \( \mathcal{D} \) is a symbol for generator.

\[
\mathcal{D} J_D \left( Q^{RR}, Q^R \right) - r \cdot J \left( Q^{RR}, Q^R \right) + A Q^R - BQ^{RR} + c \left( Q^{RR} - Q^R \right) - P \left( -\delta Q_t^R + \frac{1}{N} \lambda_c \mu_t^{CW} + \delta Q_t^{RR} - \frac{1}{N} \lambda_o \mu_t^{OW} \right) = 0 \tag{39}
\]

Conjecture that the value function is linear in state variables.

\[
J_D \left( Q^{RR}, Q^R \right) = H_0 + H^{RR} Q^{RR} + H^R Q^R \tag{40}
\]
Substitute the conjecture (40) into the HJB equation (39). Solve for the values of $H_0$, $H^{RR}$, and $H^R$ such that the equation holds for any value of $Q^{RR}$ and $Q^R$.

Value functions of all the other investors are also linear in $W_t$. Thus, define value functions $J_\sigma$ such that $V_\sigma(W_t) = W_t + J_\sigma$ for $\sigma \in \{OW, O, CW, W, N, IG\}$.

**Lemma 2.** Value functions of non-banks that own government bonds are given as (41) and (42). The optimal search intensity $\lambda_o$ maximizes $J_{OW}$. The first-order condition for $\lambda_o$ is (43).

\[
J_{OW} = \frac{c}{r} + \frac{-(r+\delta)K(\lambda_o) + \lambda_o B}{r(r+\lambda_o + \delta)} \tag{41}
\]

\[
J_O = -P + \frac{c}{r} + \frac{-\delta K(\lambda_o) + (r + \lambda_o) B}{r(r+\lambda_o + \delta)} \tag{42}
\]

\[
B + K(\lambda_o) = K'(\lambda_o)(r + \lambda_o + \delta) \tag{43}
\]

**Proof.** Without loss of generality, assume zero bank-account process. Then non-bank’s problem (2), (3), (4) and (5) can be simplified.

\[
J_{OW} = \sup_{\lambda_o} E \left[ \int_0^{\tilde{T}} e^{-rt} \{ -K(\lambda_o) + c \} dt + e^{-r\cdot\tilde{T}} (J_o + P) \right] = \sup_{\lambda_o} \frac{c - K(\lambda_o) + \lambda_o (J_o + P)}{r + \lambda_o} \tag{44}
\]

\[
J_o = E \left[ \int_0^{\tau_D} e^{-rt} (c + B) dt + e^{-r\cdot\tau_D} (J_{OW} - P \cdot e^{r\cdot\tau_D}) \right]
\]

\[
J_o + P = \frac{J_{OW} \delta + c + B}{r + \delta} \tag{45}
\]

For each given value of $\lambda_o$, (44) and (45) can be solved jointly to get (41) and (42). Differentiate the right-hand side of (41) with respect to $\lambda_o$ to obtain the first-order condition (43).

For each given value of $\lambda_o$, (44) and (45) can be solved jointly to get (41) and (42). Differentiate the right-hand side of (41) with respect to $\lambda_o$ to obtain the first-order condition (43).

A non-bank gets to choose the search intensity $\lambda_o$ only in the OW state. A non-bank in the OW state faces the exact same stationary problem at every point in time. Hence, it is without loss of generality to assume that the non-bank chooses one value of $\lambda_o$ that it will stick to forever.

**Lemma 3.** Value functions of cash-investors $J_C$ and $J_{CW}$ are given as (46) and (47) respectively. The optimal search intensity $\lambda_c$ satisfies the first-order condition (48).

\[
J_{CW} = \frac{-(r+\delta)K(\lambda_c) + \lambda_c(b - c - A)}{r(r+\lambda_c + \delta)} \tag{46}
\]

\[
J_C = P + \frac{-\delta K(\lambda_c) + (\lambda_c + r)(b - c - A)}{r(r+\lambda_c + \delta)} \tag{47}
\]

\[
b - c - A + K(\lambda_c) = K'(\lambda_c)(r + \lambda_c + \delta) \tag{48}
\]
Proof.

\[
J_{CW} = \sup_{\lambda_C} E \left[ -\int_0^{T_D} e^{-rt} K(\lambda_C)dt + e^{-rT_D} (J_C - P) \right]
\]

\[
J_C = E \left[ \int_0^{T_S} e^{-rt}(b - c - A)dt + e^{-rT_S} (J_{CW} + P \cdot e^{rT_S}) \right]
\]

The remaining proof is similar to lemma 2. \(\square\)

**Lemma 4.** Nash bargaining between dealers and customers leads to specialness premia \(A\) and \(B\) as (49) and (50).

\[
A = z \cdot \{K(\lambda_c) + b\} + (1 - z) \cdot rP - c \quad (49)
\]

\[
B = -z \cdot K(\lambda_o) + (1 - z) \cdot (-c + rP) \quad (50)
\]

Proof. Suppose a dealer repos a government bond to a cash investor. Dealer's \(Q_R\) increases by 1. Lemma 1 implies that the dealer's value function increases by \(\frac{A + c - rP}{r + \delta}\). At the same time, the cash investor transitions from the CW state to the C state. Hence, the trading gain for the cash investor is (51).

\[
J_C - J_{CW} - P = \frac{K(\lambda_c) + b - c - A}{r + \lambda_c + \delta} \quad (51)
\]

According to the asymmetric Nash bargaining theory, \(A\) is a solution to (52).

\[
\sup_A \left\{ \frac{A + c - rP}{r + \delta} \right\}^z \left\{ \frac{K(\lambda_c) + b - c - A}{r + \lambda_c + \delta} \right\}^{1-z} \quad (52)
\]

Similarly, \(B\) is a solution to (53).

\[
\sup_B \left\{ \frac{-c - B + rP}{r + \delta} \right\}^z \left\{ \frac{K(\lambda_o) + B}{r + \lambda_o + \delta} \right\}^{1-z} \quad (53)
\]

In a steady-state equilibrium, the outstanding quantity of corporate bonds is \(\frac{n_c}{\eta}\). Each non-bank holds either a government bond or a corporate bond. Hence, non-bank's demand for government bonds is \(m_n - \frac{n_c}{\eta}\). Let \(x\) denote each dealer's demand for government bonds. Then the market-clearing condition for the government bond cash market is (54).

\[
Nx + m_n - \frac{n_c}{\eta} = S_g \quad (54)
\]

Equations (15) and (16) imply that the steady-state values of \(Q^R_t\) and \(Q^{RR}_t\) are \(\frac{1}{N^\delta} \lambda_{\mu}^{OW}\) and \(\frac{1}{N^\delta} \lambda_{\mu}^{CW}\), respectively. Throughout the remainder of this paper, the subscript * indicates that the value is from the steady-state equilibrium. In the steady-state, dealer’s demand for government bonds \(x^*_s\) is (55).

\[
x^*_s = Q^*_s - Q^{RR}_s = \frac{1}{N^\delta} \lambda_{c\mu}^{CW} - \frac{1}{N^\delta} \lambda_{o\mu}^{OW} \quad (55)
\]
To get steady-state investor masses, let all time-derivatives be zero for ordinary differential equations of agent mass dynamics in subsection 3.5.

\[ \mu^C W = m_c \delta \frac{\delta}{\delta + \lambda_c} \]  
(56)

\[ \mu^O W = \left( m_n - \frac{n_c}{\eta} \right) \delta \frac{\delta}{\delta + \lambda_o} \]  
(57)

Substitute (55), (56) and (57) into the market clearing condition (54). Also, use the relation \( m_n = \frac{n_c}{\eta} + S_g \).

What remains is the corporate bond price. Non-banks are risk-neutral, and their mass is larger than the outstanding quantity of corporate bonds. Thus, the non-bank should be indifferent between owning a government bond and owning a corporate bond.

\[ J_N = J_{IG} - P_c = J_{OW} - P \]  
(58)

Solve for \( J_{IG} \) using (6) and (7).

\[ J_{IG} = E \left[ \int_0^{\tau_n} e^{-rt} e^{r \cdot t} dt + \frac{c \cdot \eta}{r} \left\{ J_{IG} - P_c + (1 - \Lambda) \frac{c \cdot \eta}{r^2} \right\} \right] \]

\[ J_{IG} = \frac{c \cdot \eta}{r} - P_c \cdot \frac{\eta}{r} + (A - 1) \frac{c \cdot \eta}{r^2} \]  
(59)

Substitute (41) and (59) into the indifference condition (58). Solve (50) for \( P \). Then substitute the equation into (60). Solve for the corporate bond price \( P_c \).

\[ \frac{c \cdot \eta}{r} - P_c \cdot \frac{\eta}{r} + (A - 1) \frac{c \cdot \eta}{r^2} - P_c = \frac{c}{r} - \frac{(r + \delta)K(\lambda_o) + \lambda_o B}{r(r + \lambda_o + \delta)} - P \]  
(60)

### B.2. Theorem 2

Define a new parameter \( m_o = m_n - \frac{n_c}{\eta} \). Solve (27) and (28) for \( A \) and \( B \), respectively. Substitute resulting equations into (25) and (26), respectively. Combine the resulting two equations to substitute out \( P \). Then I get (61). Equation (61) together with (62) characterize equilibrium.

\[ K'(\lambda_c)(r + \lambda_c + \delta) + K'(\lambda_o)(r + \lambda_o + \delta) = (1 - z) \left\{ b + K(\lambda_c) + K(\lambda_o) \right\} \]  
(61)

\[ m_c \frac{\lambda_c}{\delta + \lambda_c} = m_o \frac{\lambda_o}{\delta + \lambda_o} \]  
(62)

As I vary \( \theta \), the value of \( n_c \) or \( \eta \) does not change. Hence, to see how my solution changes with
respect to \( \theta \), I can just vary \( m_o/m_c \). Let \( \hat{\theta} = m_o/m_c \). Then I can do implicit differentiation of (61) with respect to \( \hat{\theta} \).

\[
K''(\lambda_c)\frac{\partial \lambda_c}{\partial \theta}(r+\lambda_c+\delta)+K'(\lambda_c)\frac{\partial \lambda_c}{\partial \theta}+K''(\lambda_o)\frac{\partial \lambda_o}{\partial \theta}(r+\lambda_o+\delta)+K'(\lambda_o)\frac{\partial \lambda_o}{\partial \theta} = (1-z) \left\{ K'(\lambda_c)\frac{\partial \lambda_c}{\partial \theta} + K'(\lambda_o)\frac{\partial \lambda_o}{\partial \theta} \right\}
\]

\[
\{K''(\lambda_c)(r+\lambda_c+\delta)+z \cdot K'(\lambda_c)\} \frac{\partial \lambda_c}{\partial \theta} + \{K''(\lambda_o)(r+\lambda_o+\delta)+z \cdot K'(\lambda_o)\} \frac{\partial \lambda_o}{\partial \theta} = 0
\]  

(63)

Similarly, do implicit differentiation of (62) with respect to \( \hat{\theta} \).

\[
\lambda_c(\delta + \lambda_o) = \hat{\theta} \lambda_o(\delta + \lambda_c)
\]

\[
- \hat{\theta} \frac{\lambda_o \delta}{\lambda_c} \frac{\partial \lambda_c}{\partial \theta} + \frac{\lambda_c \delta}{\lambda_o} \frac{\partial \lambda_o}{\partial \theta} = -\lambda_o(\delta + \lambda_c)
\]  

(64)

Combine (63) and (64) in matrix form.

\[
\begin{bmatrix}
K''(\lambda_c)(r+\delta+\lambda_c)+z \cdot K'(\lambda_c) & K''(\lambda_o)(r+\delta+\lambda_o)+z \cdot K'(\lambda_o) \\
-\hat{\theta} \frac{\lambda_o \delta}{\lambda_c} & -\hat{\theta} \frac{\lambda_c \delta}{\lambda_o}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial \lambda_c}{\partial \theta} \\
\frac{\partial \lambda_o}{\partial \theta}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
-\lambda_o(\delta + \lambda_c)
\end{bmatrix}
\]

(65)

Let \( A = \begin{bmatrix}
K''(\lambda_c)(r+\delta+\lambda_c)+z \cdot K'(\lambda_c) & K''(\lambda_o)(r+\delta+\lambda_o)+z \cdot K'(\lambda_o) \\
-\hat{\theta} \frac{\lambda_o \delta}{\lambda_c} & -\hat{\theta} \frac{\lambda_c \delta}{\lambda_o}
\end{bmatrix} \).

\[
\begin{bmatrix}
\frac{\partial \lambda_c}{\partial \theta} \\
\frac{\partial \lambda_o}{\partial \theta}
\end{bmatrix}
= \frac{1}{\text{det}(A)}
\begin{bmatrix}
\lambda_o \{K''(\lambda_o)(r+\delta+\lambda_o)+z \cdot K'(\lambda_o)\}(\delta + \lambda_c) \\
-\lambda_o \{K''(\lambda_c)(r+\delta+\lambda_c)+z \cdot K'(\lambda_c)\}(\delta + \lambda_c)
\end{bmatrix}
\]

Because the search cost function is assumed to be convex and increasing, \( K''(\lambda_c) > 0 \), \( K'(\lambda_c) > 0 \), \( K''(\lambda_o) > 0 \) and \( K'(\lambda_o) > 0 \). Thus, \( \text{det}(A) \) is positive. Then inspection of the left-hand side of (65) allows me to conclude that \( \frac{\partial \lambda_c}{\partial \theta} > 0 \) and \( \frac{\partial \lambda_o}{\partial \theta} < 0 \).

Now perform implicit differentiation of (26).

\[
K'(\lambda_o)\frac{\partial \lambda_o}{\partial \theta} + \frac{\partial B}{\partial \theta} = K''(\lambda_o)\frac{\partial \lambda_o}{\partial \theta}(r+\lambda_o+\delta)+K'(\lambda_o)\frac{\partial \lambda_o}{\partial \theta}
\]

\[
\frac{\partial B}{\partial \theta} = K''(\lambda_o)(r+\lambda_o+\delta)\frac{\partial \lambda_o}{\partial \theta} < 0
\]

Do implicit differentiation of (28).

\[
(1-z) \cdot r \cdot \frac{\partial P}{\partial \theta} = z \cdot K'(\lambda_o)\frac{\partial \lambda_o}{\partial \theta} + \frac{\partial B}{\partial \theta} < 0
\]

Hence, \( \frac{\partial P}{\partial \theta} \) is negative. To see how the corporate bond price \( P_c \) varies with respect to \( \hat{\theta} \), substitute (26) into (30).
\[ P_c = \frac{c}{r} - (1 - \Lambda) \frac{c \cdot \eta}{r(r + \eta)} + \frac{K'(\lambda_o) r + \delta + \lambda \cdot \lambda_o}{1 - z} \frac{1}{r + \eta} \]  
(66)

\[ \frac{\partial}{\partial \theta} P_c = \frac{K''(\lambda_o)(r + \delta + \lambda \cdot \lambda_o) + K'(\lambda_o) \delta \lambda_o}{(1 - z)(r + \eta)} \frac{\partial \lambda_o}{\partial \theta} < 0 \]

### B.3. Theorem 3

Again, let \( m_o = m_n - \frac{n_c}{\eta} \).

\[ \mu^{CW}(x) = (m_c - x) \frac{\delta}{\delta + \lambda_c(x)} \]  
(67)

\[ \mu^{OW}(x) = m_o \frac{\delta}{\delta + \lambda_o(x)} \]  
(68)

Substitute (67) and (68) into the equation for social welfare.

\[ W(x) = xb - (m_c - x) \frac{\delta \cdot K(\lambda_o(x))}{\delta + \lambda_c(x)} + (m_c - x) \frac{\lambda_c(x) \cdot b}{\delta + \lambda_c(x)} - m_o \frac{\delta \cdot K(\lambda_o(x))}{\delta + \lambda_o(x)} \]  
(69)

#### Lemma 5. \( \frac{\partial}{\partial x} \lambda_c(x) > 0 \) and \( \frac{\partial}{\partial x} \lambda_o(x) < 0 \).

**Proof.** For a given value of \( x \), an equilibrium can be characterized with two equations (70) and (71).

\[ K'(\lambda_c)(r + \lambda_c + \delta) + K'(\lambda_o)(r + \lambda_o + \delta) = (1 - z) \{ b + K(\lambda_c) + K(\lambda_o) \} \]  
(70)

\[ (m_c - x) \frac{\lambda_c}{\delta + \lambda_c} = m_o \frac{\lambda_o}{\delta + \lambda_o} \]  
(71)

Do implicit differentiation of (70) and (71) with respect to \( x \).

\[ \begin{bmatrix} m_o \frac{\delta}{(\delta + \lambda_o)^2} & -(m_c - x) \frac{\delta}{(\delta + \lambda_c)^2} \\ K''(\lambda_o)(r + \lambda_o + \delta) + z \cdot K'(\lambda_o) & K''(\lambda_c)(r + \lambda_c + \delta) + z \cdot K'(\lambda_c) \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda_o}{\partial \theta} \\ \frac{\partial \lambda_c}{\partial \theta} \end{bmatrix} = \begin{bmatrix} -\frac{\lambda_o}{\delta + \lambda_o} \\ 0 \end{bmatrix} \]

Let \( \mathcal{A} = \begin{bmatrix} m_o \frac{\delta}{(\delta + \lambda_o)^2} & -(m_c - x) \frac{\delta}{(\delta + \lambda_c)^2} \\ K''(\lambda_o)(r + \lambda_o + \delta) + z \cdot K'(\lambda_o) & K''(\lambda_c)(r + \lambda_c + \delta) + z \cdot K'(\lambda_c) \end{bmatrix} \). Then \( \det(\mathcal{A}) > 0 \).

\[ \begin{bmatrix} \frac{\partial \lambda_o}{\partial \theta} \\ \frac{\partial \lambda_c}{\partial \theta} \end{bmatrix} = \frac{1}{\det(\mathcal{A})} \begin{bmatrix} K''(\lambda_c)(r + \lambda_c + \delta) + z \cdot K'(\lambda_c) & (m_c - x) \frac{\delta}{(\delta + \lambda_c)^2} \\ -K''(\lambda_o)(r + \lambda_o + \delta) - z \cdot K'(\lambda_o) & m_o \frac{\delta}{(\delta + \lambda_o)^2} \end{bmatrix} \begin{bmatrix} -\frac{\lambda_o}{\delta + \lambda_o} \\ 0 \end{bmatrix} \]

\[ \begin{bmatrix} \frac{\partial \lambda_o}{\partial \theta} \\ \frac{\partial \lambda_c}{\partial \theta} \end{bmatrix} = \frac{1}{\det(\mathcal{A})} \begin{bmatrix} -\{ K''(\lambda_c)(r + \lambda_c + \delta) + z \cdot K'(\lambda_c) \} \frac{\lambda_c}{\delta + \lambda_c} \\ \{ K''(\lambda_o)(r + \lambda_o + \delta) + z \cdot K'(\lambda_o) \} \frac{\lambda_o}{\delta + \lambda_o} \end{bmatrix} \]
Differentiate $W(x)$ in (69) with respect to $x$.

\[
\frac{dW(x)}{dx} = \frac{\delta}{\delta + \lambda_c} + (m_c - x) \cdot \delta \cdot \frac{-(\delta + \lambda_c)K'(\lambda_c) + b + K(\lambda_c)}{\delta + \lambda_c^2} \frac{1}{\delta + \lambda_c} \frac{\lambda_c}{\det \, \delta + \lambda_c} \phi_o \\
- m_o \delta \frac{\delta + \lambda_o)K'(\lambda_o) - K(\lambda_o)}{(\delta + \lambda_o)^2} \frac{1}{\delta + \lambda_0} \frac{-\lambda_c}{\phi_c} \quad (72)
\]

\(\phi_o\) and \(\phi_c\) are defined as (73) and (74), respectively.

\[
\phi_o = K''(\lambda_o) (r + \lambda_o + \delta) + K'(\lambda_o) \cdot z \\
\phi_c = K''(\lambda_c) (r + \lambda_c + \delta) + K'(\lambda_c) \cdot z 
\]

Because I assume \(m_o > m_c - x\), (71) implies that \(\lambda_o < \lambda_c\). Since there are more government bond owners than cash investors, each government bond owner searches for dealers less intensively than each cash investor.

Equation (72) implies that \(dW(x)/dx > 0\) if and only if (75) holds.

\[
b + K(\lambda_c) + \frac{\lambda_c}{\det \, \delta} \left[ (m_c - x) \phi_o \frac{rK'(\lambda_c) + A}{\delta + \lambda_c^2} + m_o \phi_c \frac{B - rK'(\lambda_o)}{(\delta + \lambda_o)^2} \right] > 0 
\]

Define a new function \(f(\cdot)\) as (76).

\[
f(\lambda) = K'(\lambda) (\lambda + \delta) - K(\lambda) 
\]

Due to the assumptions that I make about the search cost function, \(f(0) = 0\).

\[
f'(\lambda) = K''(\lambda) (\lambda + \delta) > 0
\]

Thus, for \(\forall \lambda \geq 0\), \(f(\lambda) \geq 0\). Because \(\lambda_o \geq 0\), \(K'(\lambda_o)(\lambda_o + \delta) \geq K(\lambda_o)\). Combine this inequality with (26) to get \(B \geq rK'(\lambda_o)\). Hence, \(m_o \phi_c \frac{B - rK'(\lambda_o)}{(\delta + \lambda_o)^2}\) in (75) is positive.

I already showed that \(\det(\delta) > 0\). Hence, (75) holds.
C. The Supply of Bonds by the Central Banks in the United States and Canada

The Federal Reserve Bank of New York initiated the Fixed-Rate Overnight Reverse Repo (RRP) facility in September 2013. A wide range of financial institutions including non-banks can reverse repo Treasuries from the New York Fed. Each institution can borrow Treasuries up to 30 billion dollars on each day. Unlike its counterparts in Europe, the RRF facility does not have any limit on the overall size of operation, at least from December 2015. The New York Fed is willing to lend all bonds on its System Open Market Account (SOMA) portfolio to the extent possible. The impact on the effective size of collateral available in the private repo market can be limited to the extent that non-primary-dealer counter-parties cannot rehypothecate Treasuries from the RRP facility (Protter, 2018). Nevertheless, after reverse-repoing Treasuries from the New York Fed, these non-banks are less likely to be seeking extra collateral from the private market. Consequently, the net effect is likely to be increased availability of collateral in the private market (Protter, 2018).

The Fed RRP facility is much more accessible than the securities lending facility of the Eurosystem. For example, unlike the RRP facility, the Eurosystem cannot repo more than 50 billion euros of its PSPP-eligible bonds through its securities lending facility (Marraffino, 2017). Marraffino (2016) notes that the introduction of a facility akin to the RRP facility may considerably alleviate any problem associated with the reduced supply of collateral in the European repo market.

In addition, the Federal Reserve has been operating a securities lending program since 1969. Through this program, primary dealers can borrow specific securities by positing any Treasury general collateral. Investors used to borrow on-the-run securities by posting other Treasury general collateral. Because on-the-run securities are far more special than off-the-run securities in the repo market, this program is likely to have reduced the repo specialness premium of on-the-run Treasuries (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018).

From the market user’s point of view, the securities lending program of the Federal Reserve is more attractive than the European counterparts (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018). The minimum lending fee of the Fed facility was 5 basis points as of 2016 (Fleming, Keane, Schurmeier, and Weiss, 2016). On the contrary, most national central banks of the Eurosystem have been repoing government bonds through their respective securities lending facilities at a repo rate at least 10 basis points lower than the prevailing market GC repo rate (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018). Because the securities lending fee is economically equivalent to the repo specialness premium, Arrata, Nguyen, Rahmouni-

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14According to the New York Fed website, a transaction needs “to be collateralized with Treasury bills, notes, bonds and inflation-indexed securities.”
Rousseau, and Vari (2018) conclude that the Fed facility offered a pricing that is more favorable than the European facilities did.

Although the Bank of Canada did not explicitly implement QE, it has been consistently purchasing debt instruments of the federal government of Canada. As of 2015, the majority of the asset portfolio of the Bank of Canada, which is worth around 94 billion Canadian dollars, consists of bonds and bills of the government of Canada (Patterson, 2015). At the same time, the majority of repo transactions in Canada is collateralized by the federal government bonds. Consequently, the Bank of Canada is aware that its sizable purchase of federal government bonds can reduce the float size in the private market and negatively impact the repo market liquidity.

The Bank of Canada took several measures to address potential dry-up of liquidity in the repo market. For example, the Bank repos federal government bonds whenever the general collateral (GC) repo market rate is below the target rate of the Bank. This measure contrasts with the policy of the ECB. The ECB did not take action when the German GC repo rate became substantially lower than the deposit facility rate.

In addition, the Bank of Canada has a securities lending facility through which primary dealers can borrow up to 50 percent of the bonds held by the Bank of Canada. In return for borrowing federal government bonds, primary dealers can post a wide range of assets, including provincial government bond, municipal government bond, banker’s acceptances, commercial papers, and covered bonds. Primary dealers can gain access to federal government bonds, the most widely used collateral in the Canadian repo market, in exchange for other less widely used assets. Therefore, this facility can mitigate any problem associated with the reduced supply of collateral in the repo market.

D. Implementation Details of the PSPP

The information is from the following EU documents:


\[16\] https://www.bankofcanada.ca/markets/market-operations-liquidity-provision/framework-market-operations-liquidity-provision/

\[17\] https://www.bankofcanada.ca/2015/10/securities-lending-program/


Table 7: Timeline of the implementation details of the PSPP

<table>
<thead>
<tr>
<th>Period</th>
<th>APP monthly net purchase (billion euros)</th>
<th>The share of purchase quantities allocated to the debts of international organizations and multilateral banks</th>
<th>Minimum remaining maturity (years)</th>
<th>Maximum remaining maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 9, 2015 ~ April 18, 2016</td>
<td>60</td>
<td>12%</td>
<td>2</td>
<td>30 years and 364 days</td>
</tr>
<tr>
<td>April 19, 2016 ~ January 12, 2017</td>
<td>80</td>
<td>12%</td>
<td>2</td>
<td>30 years and 364 days</td>
</tr>
<tr>
<td>January 13, 2017 ~ March 31, 2017</td>
<td>60</td>
<td>12%</td>
<td>1</td>
<td>30 years and 364 days</td>
</tr>
</tbody>
</table>

Table 8: Timeline of the implementation details of the PSPP: eligibility of local and regional government debt, issuer limit, restriction on yield-to-maturity, and issue (ISIN) limit

<table>
<thead>
<tr>
<th>Period</th>
<th>Eligibility of debt instruments of regional and local governments</th>
<th>Issuer limit</th>
<th>Restriction on yield-to-maturity</th>
<th>Issue (ISIN) limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 9, 2015 ~ November 9, 2015</td>
<td>N</td>
<td>33%</td>
<td>Yes, except inflation linked bonds(^{18})</td>
<td>25%</td>
</tr>
</tbody>
</table>

\(^{18}\)See Schlepper, Hofer, Riordan, and Schrimpf (2017).
<table>
<thead>
<tr>
<th>Date Range</th>
<th>Condition</th>
<th>Exception</th>
<th>Additional Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 10, 2015 ~ December 31, 2015</td>
<td>N</td>
<td>33%</td>
<td>Yes, except inflation linked bonds</td>
</tr>
<tr>
<td>January 1, 2016 ~ February 16, 2016</td>
<td>Yes technically. Nevertheless, anecdotal evidence suggest that the Eurosystem did not start purchasing during this period</td>
<td>33%</td>
<td>Yes, except inflation linked bonds</td>
</tr>
<tr>
<td>February 17, 2016 ~ April 18, 2016</td>
<td>Yes, but not municipal bonds&lt;sup&gt;19&lt;/sup&gt;</td>
<td>33%</td>
<td>Yes, except inflation linked bonds</td>
</tr>
<tr>
<td>April 19, 2016 ~ January 12, 2017</td>
<td>Yes, but not municipality bonds</td>
<td>33% but 50% for international organizations and multilateral development banks</td>
<td>Yes, except inflation linked bonds</td>
</tr>
<tr>
<td>January 13, 2017 ~ March 31, 2017</td>
<td>Yes, but not municipality bonds</td>
<td>33% but 50% for international organizations and multilateral banks</td>
<td>Bonds with yields-to-maturity below the deposit facility rate can be purchased to the extent necessary to meet capital key based allocation rule</td>
</tr>
</tbody>
</table>

<sup>19</sup> It is because municipality bonds are not rated (UniCredit, 2016). To be eligible for the PSPP, securities must have credit ratings.
E. Other Institutional Details

E.1. The Political Cost of the PSPP

QE programs in Europe has been controversial legally because such program can be interpreted as monetary financing: the purchase of bonds of member states by the Eurosystem (Nyborg, 2017). The Treaty of the Functioning of the European Union (TFEU) explicitly prohibits monetary financing. In particular, Germany has been adamantly opposed to monetary financing because it dislikes the fiscal support of peripheral member countries (Brunnermeier, James, and Landau, 2016). Hence, the PSPP could be carried out only after the European Court of Justice confirmed the legality of such operations in 2015, subject to adherence to several conditions (Nyborg, 2017). For example, the PSPP never purchases bonds in the primary market to circumvent potential legal challenge (Nyborg, 2017). Nevertheless, right-wing politicians in Germany continued to challenge the legality of the PSPP. On August 15, 2017, the Federal Constitutional Court of Germany stated that the PSPP may be violating the mandate of the Eurosystem.

E.2. Why Insurance Companies and Pension Funds Cannot Easily Monetize Repo Specialness Premia

The European repo market is mostly intermediated by banks (Duffie, 2018). Since only banks can join electronic platforms such as BrokerTec, the only feasible way for insurers or pension funds to repo their bonds is through bilateral repo with banks. Nevertheless, recent regulatory drives have made banks increasingly reluctant to intermediate repo and reverse-repo transactions (Hill, 2017). One important factor is the Basel III Leverage Ratio requirement that forces banks to maintain leverage ratio - the ratio of Tier 1 capital to the total asset - above 3% (Hill, 2017). The Leverage Ratio requirement forces banks to economize the usage of their balance sheets (Hill, 2017). Consequently, banks are strongly discouraged from remaining in the repo intermediation business, which heavily expands the size of their balance sheet while bringing them a small profit (Hill, 2017).

There were signs that banks were beginning to retreat from the repo intermediation business in 2015 (Hill, 2015b). One exception is when banks can net repo and reverse repo transactions (Marraffino, 2017). By doing so, banks can avoid expanding their balance sheets. Banks often net repos and reverse repo by registering them at Central Counterparties (CCPs). Nevertheless, for any transaction to be registered at CCPs, both parties to the transaction need to be members of the CCP (Marraffino, 2017). Because membership is costly, not all financial institutions belong to CCP. Only financial institutions that have a constant flow of two way business (repos and reverse repos) have an incentive to become members of CCPs (ICMA, 2015). For this reason, CCP membership is mostly for banks (Hill, 2017). Insurers and pension funds without CCP membership is, thus, not favored clients of the repo desks of banks. To persuade the repo desk the profitability of transaction, insurers and pension funds need to accept unfavorable
rates (Marraffino, 2017), or bring other profitable businesses on the side (Hill, 2015b).

In principle, insurers and pension funds can search for counterparties in the bilateral repo market. Nevertheless, doing so requires an ability to assess the credit risk of the counterparty, comply with legal requirements, and negotiate with a wide range of non-bank participants, which most buy-side firms do not possess (Hill, 2017). Thus, many buy-side firms are heavily reliant on bank-based intermediation (Hill, 2017).

E.3. The Restrictive Operation of the Securities Lending Facility

Although the European Central Bank did allow individuals to borrow government bonds through the securities lending facility, many structural factors prevented the facility from effectively addressing collateral scarcity in the repo market. The European Central Bank and National Central Banks in the Eurosystem operate their own securities lending facilities in a decentralized manner.

Before December 2016, in order to obtain specific collateral, firms had to bring another PSPP-eligible securities. Bringing cash was not acceptable. The securities lending facilities were said to have been operated in a cash-neutral manner. The operation of lending facilities in a cash-neutral manner does not change the overall quantity of collateral circulating in the market (Hill, 2017).

The Eurosystem allowed firms to reverse-repo bonds while posting cash collateral at the securities lending facility. Nevertheless, even this cash-collateral option could not fully mitigate the collateral scarcity problem due to the following features.

To obtain bunds, firms had to go to the security lending facility of the Bundesbank. Nevertheless, the Bundesbank facility had the following restrictive features as of the end of 2016.

- The firm needed to have “a credit line with the Bundesbank” (Hill, 2017).

- The lending operation was to be done with the European Master Agreement (EMA). Nevertheless, the norm in the private market is to use either the GMRA for repo and the GMSLA for securities lending (Hill, 2017).

- There was no option to automatically roll-over repo contract with the Bundesbank. The firm had to deliver the bond to the Bundesbank and then re-start the contract (Hill, 2017).

- The securities lending facilities in the Eurosystem were not allowed to lent more than 50 billion euros of securities with cash collateral option. This quote is distributed across facilities run by different national central banks. Thus, the Bundesbank facility was not allowed to lend more than 12 billion euros with cash collateral option (McGuire, Graham-Taylor, and Cairns, 2017). This number is far smaller than the size of the PSPP. The Eurosystem purchased 304 billion euros of German securities under the PSPP by December 2016.
• Each transaction could not be larger than 200 million euros (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018).

• Firms could reverse-in bunds only at a rate significantly less favorable than the market prevailing rates (Arrata, Nguyen, Rahmouni-Rousseau, and Vari, 2018).

These restrictive features might have been the reason why the market did not utilize the cash collateral option to the fullest extent. The data on the ECB website shows that the total quantity of securities lent with cash collateral option has been consistently lower than the 50 billion euros cap each month.

E.4. The Portfolio Allocation Decisions of Life Insurance Companies

It is true that life insurers face strict regulation on their choice of investment strategies. Nevertheless, there is anecdotal evidence that European life insurers recently substituted for ultra low-yielding government bonds with alternative higher yielding assets. See Section C of ESRB (2016). Because a significant fraction of European insurers have sold products with guaranteed returns (IAIS, 2016), they need to make sure they generate sufficiently high returns from their investments to remain solvent.

The Securities Holdings Statistics data shows that the holdings of euro-denominated investment grade corporate bonds by the insurance sector declined recently. Nevertheless, much of that decline is due to the redemption of bonds that insurance companies purchased a long time ago (Davies and Hetland, 2017). If one looks at how insurance companies invest cash that they get from new underwritten policies, there has been a shift from government bonds to investment-grade corporate bonds (Davies and Hetland, 2017).
F. The Principal Component Analysis of Repo Rates

I run a principal component analysis on the sample correlation matrix of the repo rates of sovereign bonds that are traded on BrokerTec. Figures 11, 12 and 13 show the estimated loadings of the first three principal components. Table 9 shows the fraction of the total variance that the first four principal components explain.

I exclude French government bonds from analysis because the French repo market is different from the rest of the European repo market in many different ways. For example, the French repos are predominantly floating repos while repos in other countries are mostly fixed-rate repos.

Figure 11: The Loadings of the First Principal Component I run a principal component analysis on the correlation matrix of the repo rates of sovereign bonds that are traded on BrokerTec. Cross-sectionally, I consider the repo rate of sovereign bonds of a given country of domicile (Austria, Belgium, Germany, Spain, Finland, Ireland, Italy, the Netherlands, and Portugal) and a given remaining maturity in years (1 year, 1.2 year, ..., 15 years). If there is no sovereign bond that exactly matches the remaining maturity that I want to analyze, I impute the value with nonparametric regression. The sample period is from March 1, 2013 to March 31, 2019. The horizontal axis shows the remaining maturity of sovereign bond under analysis. The vertical axis is the loading of the first principal component. For example, the loading of the first principal component on the Austrian government bond with 1 year to maturity is 0.041.
Figure 12: **The Loadings of the Second Principal Component** I run a principal component analysis on the correlation matrix of the repo rates of sovereign bonds that are traded on BrokerTec. Cross-sectionally, I consider the repo rate of sovereign bonds of a given country of domicile (Austria, Belgium, Germany, Spain, Finland, Ireland, Italy, the Netherlands, and Portugal) and a given remaining maturity in years (1 year, 1.2 year, ..., 15 years). If there is no sovereign bond that exactly matches the remaining maturity that I want to analyze, I impute the value with nonparametric regression. The sample period is from March 1, 2013 to March 31, 2019. The horizontal axis shows the remaining maturity of sovereign bond under analysis. The vertical axis is the loading of the second principal component. For example, the loading of the second principal component on the Austrian government bond with 1 year to maturity is -0.030.

<table>
<thead>
<tr>
<th>PC</th>
<th>Proportion of the Total Variance Explained (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>89.24</td>
</tr>
<tr>
<td>PC2</td>
<td>4.66</td>
</tr>
<tr>
<td>PC3</td>
<td>1.14</td>
</tr>
<tr>
<td>PC4</td>
<td>0.95</td>
</tr>
</tbody>
</table>

**Table 9: The Proportion of the Total Variance Explained by Principal Components**
Figure 13: The Loadings of the Third Principal Component I run a principal component analysis on the correlation matrix of the repo rates of sovereign bonds that are traded on BrokerTec. Cross-sectionally, I consider the repo rate of sovereign bonds of a given country of domicile (Austria, Belgium, Germany, Spain, Finland, Ireland, Italy, the Netherlands, and Portugal) and a given remaining maturity in years (1 year, 1.2 year, ..., 15 years). If there is no sovereign bond that exactly matches the remaining maturity that I want to analyze, I impute the value with nonparametric regression. The sample period is from March 1, 2013 to March 31, 2019. The horizontal axis shows the remaining maturity of sovereign bond under analysis. The vertical axis is the loading of the third principal component. For example, the loading of the third principal component on the Austrian government bond with 1 year to maturity is 0.002.