The dark side of low(er) interest rates

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Abstract

Retirement benefits in industrialized countries have come under great pressure. How should one conduct monetary policy in this scenario? Do low interest rates necessarily stimulate demand? The paper builds a New Keynesian overlapping generations model. The "standard" monetary transmission obtains whenever pensions are sufficiently generous. Below a critical level of government-provided pensions, however, low interest rates can show their darker side: lower interest rates may turn into the cause of a recession rather than supporting economic activity. This is more likely if households do not want to substitute intertemporally, or if they cannot, for example, because the young are borrowing-constrained and the constraints do not ease sufficiently. Using OECD data, we find some evidence showing that the strength of monetary policy depends on the generosity of the retirement system. Consistent with our model, countries having less generous systems display less responsive monetary policy.

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1 Introduction

Conventional monetary policy to date rests on two implications of New Keynesian monetary theory that emerge from a representative household paradigm: first, in order to anchor economic fluctuations, the central bank has to move nominal rates disproportionately with inflation (the "Taylor principle," Taylor, 1993). Second, lowering nominal rates will stimulate economic activity (for example, Woodford, 2003). This is precisely the recipe that industrialized economies' central banks followed in the Great Recession: reduce rates, persistently. Indeed, forward guidance promised to keep real rates low for long. At the same time, a central tenet of modern macroeconomics is that monetary and fiscal policy cannot be analyzed in isolation. The current paper asks: what happens to the monetary transmission mechanism if the government retracts from social security?

This matters because the viability of Social Security is a central concern of households and policymakers in industrialized countries (for the U.S., see Luttmer and Samwick, forthcoming; for the European Union, see European Commission, 2015). The current paper stays within the New Keynesian class of theories, in which nominal rigidities render aggregate supply demand-determined and the medium-term real interest rate under the control of the central bank. To this theory, the paper adds a three-period overlapping generations structure, as has recently been applied to the New Keynesian environment by Eggertsson and Mehrotra (2014) and Sheedy (2014). Inside debt allows applying the cash-less limit of Woodford (1998) so as to delineate the implications of interest-rate policy most clearly. The paper, then, revisits the case for persistently lowering interest rates to stimulate demand.

The questions that this paper asks are simple: how does monetary policy transmission depend on the retirement social security system? "Substantially," we find. Do low nominal interest rates necessarily stimulate aggregate demand? Our answer is "no." The intuition for both of this is simple. Consider a household that has to save for retirement. To the extent that interest rates (or, more generally, returns on assets still to be bought) are low for long, the household has to curtail current spending in order to meet its goals for retirement consumption. If the economy is not able to match the household's increased willingness to save by providing additional credit, a monetary "easing" may be less stimulating than if social security is plentiful. Indeed, low interest rates may, instead, *reduce* aggregate activity, rather than stimulating it. The key to this are the middle-aged. As interest rates fall, future consumption becomes more expensive. If the elasticity of intertemporal substitution is sufficiently low, and if savings are sufficiently important for old-age consumption, the middle-aged respond by saving more. If, then, the young generation does not increase consumption because it cannot (due to borrowing constraints) or does not want to (due to a low intertemporal elasticity of substitution also for the young), the middle-aged's desire to save can only be in line with equilibrium if aggregate activity falls.

Through this mechanism, then, low interest rates can generate persistent *reductions* in aggregate demand.[1]

We also derive novel implications for equilibrium uniqueness. The current best practice of central banks is to raise nominal interest rates sufficiently much in response to inflation so that the real interest rate rises. If private provision for retirement (through nominal savings) finances a sufficiently large part of old-age consumption, we demonstrate that obeying this "so-called" Taylor principle may no longer ensure determinacy of equilibrium but give rise to fluctuations driven by sunspots. An economy with a less generous public pension system may be more prone to nominal and real indeterminacy.

How can this be? Suppose that non-fundamental beliefs form about persistently lower inflation. Suppose that the central bank engineers a persistently lower real rate of interest in response. If private saving are sufficiently important for old-age consumption, and households sufficiently unwilling to substitute over time, middle-aged households will want to work more (and consume less) since saving for retirement is hard to provide for. If the middle-aged are an important provider of labor input, economy-wide wages may *fall* if the central bank follows the Taylor principle, validating the expectation of low inflation.

Instead, in some cases, even constant interest rates, $\phi_\Pi = 0$, would ensure determinacy, a sharp reversal of Sargent and Wallace (1975), and conventional wisdom. How can this be? When retirement savings are important, inflationary beliefs can be self-regulating amid constant interest rates. Suppose that, in the example above, nominal interest rate were to be held constant. Then, the real interest rate would rise. The wealth effect on labor supply being important for the middle-aged, the middle-aged would want to work less. This raises the wage and marginal costs, in turn contradicting the beliefs of high inflation. Self-fulfilling expectations are ruled out. We show that such a low response to inflation is not fool-proof, however, for wages would need to fall by just the right amount to keep inflation expectations and real activity anchored.

What will always keep the equilibrium determinate is a rather strong response of the central bank to inflation, potentially then envisioned by the Taylor principle. Indeed, in this sense, we conclude from our analysis that central banks may need to redouble their focus on inflation once self-provisioning for retirement attains an important role.

Next to providing a channel through which low rates may be contractionary, the current paper also provides a novel explanation as to why some countries may have recovered faster from the

[1] There is an extensive literature on the size of the intertemporal elasticity of substitution. The case for a low value is made by the classic contribution of Hall (1988), in the micro consumption literature, for example, by Attanasio and Weber (1995), and in macro regressions by Yogo (2004), among others. More recently, Best et al. (2017) estimate an intertemporal elasticity of substitution of 0.1. That said, we do not wish to take a stand. Even if the intertemporal elasticity of substitution is larger than unity, our propositions suggest that increased reliance on private provision for old age may reduce the effectiveness of monetary policy.

recession than others. Namely, it shows that the very same low-interest monetary policies can have very different effects in different countries. In particular, we highlight the financial system and retirement social security as key drivers of the response.[2]

In the last part of the paper, we provide some empirical support for the main prediction in our model. In particular, we look at the effects of monetary policy in OECD countries and rank them based on the quality of the retirement system. Consistent with our model, we find a negative relation between the strength of monetary policy and a generosity index.[3] That is, consumption is very responsive to monetary policy shocks in countries with strong retirement systems, such as Portugal. In contrast, U.S. has one of the weakest retirement systems and its monetary policy is comparatively less effective than its peers.

The current paper proceeds as follows. Next, we review the literature. Section 2 spells out the three-period OLG model that we use to analytically show the key points of the paper. The results are discussed in Section 3. A preliminary calibration in Section 4 suggests that episodes with low rates may, indeed, be contractionary if not accompanied by a well-functioning financial system or a sufficiently generous social security. Section 7 concludes.

Related literature

Our paper explores the implications of the pension system for the design of conventional monetary policy. We show that both the transmission of monetary shocks and the local determinacy properties of the policy rule can depend on the generosity of government-provided pensions. These results, to the best of our knowledge, are new to the literature.

The three-period setup of the paper draws extensively on Eggertsson and Mehrotra (2014) and the working version of Sheedy (2014). The focus differs, however. Eggertsson and Mehrotra (2014) focus on the effect of demographic aging on the natural rate of interest. If the central bank cannot anchor real rates at the natural rate, secular stagnation may result. Several other papers study the role of demography for the natural rate or monetary transmission. Cooley and Henriksen (2018), Gagnon et al. (2016), Carvalho et al. (2016) all study the role of demographic change on the natural rate. Kantur (2013) and Fujiwara and Teranishi (2008) study the effect of demographic change on both the steady-state natural rate and transmission of TFP and monetary policy shocks in two-period perpetual youth or OLG models. Wong (2016) studies the transmission of interest-rate shocks both in the data and a partial-equilibrium OLG setting for the US economy, emphasizing the role of housing and refinancing. She finds that consumption

[2] Of course, there are many candidate explanations for the divergence between Europe and the US, say. We see our paper as suggesting a complementary explanation to those that already exist.

[3] We rank countries based on the strength of monetary policy using the maximal IRF of consumption to a monetary shock. The weakest response receives the lowest rank and the other way around. Similarly, a country with the best retirement system is ranked first in our procedure.

by the young is more responsive to an interest rate change than consumption of the old, in line with the main mechanism in our paper. Bean (2004) provides a verbal discussion of demography and monetary policy. Among other dimensions, he emphasizes that aging society may mean that the wealth effect becomes relatively more important. An older population might also flatten the short-run Phillips curve through increased labor supply by the old, assuming that the latter have more elastic labor supply, an effect we currently do not yet assess. Relative to all the above, we wish to focus on the interaction of monetary and fiscal policy. That is, we are not interested in demographic change itself, but in the effect of social insurance on monetary transmission. This change in focus is important. Second, we model credit relationships across the generations so as to most clearly separate fiscal and monetary policies.

Previous work has used the OLG setup to study implications of alternative monetary implementation or operations. Most closely related in terms of setup to our paper is the working paper version of Sheedy (2014).[4] He studies the case for nominal GDP targeting. Azariadis et al. (2016) study an OLG setup to focus on optimal monetary policy under flexible prices with and without a zero lower bound on interest rats. Contrary to all these papers, we highlight the role that the public pension system has for monetary transmission. Sterk and Tenreyro (2016) study aggregate effects of alternative monetary operations (open open market operations vs. helicopter drops) in a perpetual youth model. In their setting, fiscal policy matters for it distributes wind-fall gains or losses from such operations. In our paper, instead, we deliberately abstract from monetary policy working through fiscal transfers, and rather analyze how the social security system affects monetary transmission.

Our paper links to earlier work in the New Keynesian paradigm two other ways. First, to the literature on forward guidance. McKay et al. (2016) highlight that borrowing constraints can mean that forward guidance is less powerful. Our paper is also related to the literature on the cost channel of monetary policy, for example, Ravenna and Walsh (2006). What this literature shares with us is stressing that lower nominal rates may, for some time, lead to lower marginal costs and inflation. In our case, however, this links works directly through the wage. That is, we do not alter the New Keynesian Phillips curve. Last, there is a literature on limited asset market participation, that also finds that the Taylor principle can be reversed, Bilbiie (2008).

Two other papers have, in parallel to our work, have highlighted that the conventional monetary transmission mechanism can break down, and low interest rates become contractionary. Brunnermeier and Koby (2018) focus on financial intermediation. Then, low interest rates may squeeze banks' intermediation margins, lowering credit supply. Beaudry and Portier (2018) provide a new Keynesian model in which households have liquidity demand but supply may be less than perfectly elastic. From this, like us, for some regions of the parameter space, they get contractionary effects of a conventional monetary-policy easing.

[4] The published version resorts to a model of infinitely lived borrowers and savers.

2 A three-period OLG model

In order to derive analytical results, we consider a three-period overlapping generations model, that builds on Eggertsson and Mehrotra (2014) and Sheedy (2014). We first describe the general setup of the model. Then, we examine the effect of interest-rate changes by the central bank on aggregate activity, and highlight the determinants thereof.

2.1 Model setup

Time is discrete, indexed by t, and runs forever, $t = 0, 1, 2, \ldots$. A period should be thought of as a longer span of time. In every period, there are three generations of households, each of which with a unit mass: The young (y), the middle-aged (m), and the old (o). Households receive income and they can save and borrow at the risk-free nominal interest rate R_t (gross). We look at the cashless limit-assumption Woodford (1998). The central bank sets this interest rate. Monetary policy does not affect government financing. Inside debt is the only asset that can be used for saving.

2.1.1 Preferences

Households maximize their life-time utility by choosing consumption, labor supply, and savings subject to the period budget constraints which are described in detail further below. Life-time utility for an individual born in period t is given by

$$E_t \left\{ u\big(c_t^y, h_t^y; \xi^y\big) + \beta u\big(c_{t+1}^m, h_{t+1}^m; \xi^m\big) + \beta^2 u\big(c_{t+2}^o, h_{t+2}^o; \xi^o\big) \right\}. \tag{1}$$

Here c_t^y is consumption when young, c_t^m consumption when middle-aged, and c_t^o old-age consumption. h_t^y, h_t^m, and h_t^o are hours worked by the respective age groups. ξ^y, ξ^m, ξ^o mark a vector parameters of the period utility function. $0 < \beta < 1$ is the common time discount factor. For the period utility function, we entertain additively separable preferences and GHH preferences. By way of example, here we spell these out for the middle-aged only. For the other generations period utility functions are analogous. For additively separable preferences,

$$u(c^m, h^m; \xi^m) = \frac{(c^m - \overline{c})^{1-\sigma} - 1}{1-\sigma} - \psi^m \frac{h^{1+\nu^m}}{1+\nu^m}.$$

For GHH preferences

$$u(c^m, h^m; \xi^m) = \frac{(c^m - \overline{c}^m - \psi^m \frac{h^{1+\nu^m}}{1+\nu^m})^{1-\sigma} - 1}{1-\sigma}.$$

In each case, $\sigma > 0$. In addition, $\psi^m > 0, \nu^m \geq 0$. $\overline{c}^m$ marks a minimum consumption threshold that households have to satisfy. This threshold can reflect, depending on age, for example, committed expenses such as minimum food consumption, out-of-pocket health expenditures, or high expenses from living in a retirement home. Age-dependent parameters are collected in $\xi^m := (\bar{c}^m, \psi^m, \nu^m)$.

2.1.2 Incomes and endowments

Consumers purchase homogenous goods. Some of them are produced in the same period, others originate from endowments. Endowments are perfectly substitutable with produced goods. They are tradable between households, but not over time. Neither produced goods nor endowments are storable. Each generation is endowed with home production $\omega^y \geq 0$, $\omega^m \geq 0$ and $\omega^o \geq 0$, respectively. Each generation pays tax τ_t^y, τ_t^m, and τ_t^o, respectively, to a social security system. Taxes are lump-sum and can be positive or negative. Throughout the paper, we will assume that endowments, taxes, and parameters are such that households borrow when young, save when middle-aged, and dissave in old age. Marking by d_t^y the real value of borrowing by the young, the *period budget constraint for the young* is

$$c_t^y = d_t^y + \omega^y - \tau_t^y + w_t h_t^y.$$

On the income side, the young borrow, have an endowment ω^y, pay social security taxes τ_t^y, and derive labor labor income (with w_t being the competitive real wage). Borrowing may be unconstrained, or there may be a borrowing limit $\bar{d}_t$, such that $d_t^y \leq \bar{d}_t$ always. Whenever that is the case, we specify the borrowing limit as the paper progresses.

Next, we turn to the *period budget constraint for the middle-aged.* Having borrowed when young, households start saving for retirement when middle-aged. Saving by the middle-aged being marked by b_t^m, their budget constraint is

$$c_t^m + b_t^m + d_{t-1}^y R_{t-1}/\Pi_t = \omega^m - \tau_t^m + w_t h_t^m + \Gamma_t.$$

On the expenditure side, there are consumption in middle age and saving for retirement. In addition, the middle-aged repay the debts incurred when they were young. R_{t-1}/Π_t is the real interest rate due on this (with R_{t-1} being the gross nominal interest and Π_t gross inflation). On the income side, middle-aged households have endowment ω^m, pay social security taxes τ_t^m, and receive income from selling their labor-hours, h_t^m, on a perfectly competitive labor market. Last, each middle-aged household is endowed with equity of a portfolio of one-period lived firms, which provide dividends Γ_t.

The *budget constraint of the old* in turn is

$$c_t^o = \omega^o - \tau_t^o + w_t h_t^o + b_{t-1}^m R_{t-1}/\Pi_t$$

The old consume their endowment after taxes, any income they may have from working, and their savings. For the mechanics of the model, the relative size of endowment and savings will matter.

2.1.3 Production

There is a representative producer of final homogenous consumption good. Production of final goods follows the Dixit-Stiglitz production function

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \ \theta > 1.$$

Here y_t is aggregate output of produced final goods and $y_t(j)$ is the final goods producer's demand for intermediate good j. Letting $P_t(j)$ mark the price of variety j, and P_t the aggregate price level, the final producer's demand function for good j is given by

$$y_t(j) = (P_t(j)/P_t)^{-\theta} y_t.$$

The aggregate price level being

$$P_t = \left(\int_0^1 P_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}.$$

Intermediate goods are produced by a unit mass of intermediate goods firms. Intermediate goods producers are one-period-lived firms owned only by the middle-aged. Intermediate goods firms and the goods that they produce are indexed by $j \in (0, 1)$. Firm j's production function is

$$y_t(j) = z h_t(j),$$

where z marks labor productivity and $h_t(j)$ are the work hours hired by firm j. The firms sell their output in a monopolistically competitive market, subject to quadratic price adjustment costs analogous to Rotemberg (1982). Anticipating the production function of the representative final-goods producer, the problem of intermediate goods producer j, then, is to

$$\max_{P_t(j)} \left(\frac{P_t(j)}{P_t} \right)^{1-\theta} y_t - \left(\frac{P_t(j)}{P_t} \right)^{-\theta} y_t \frac{w_t}{z_t} - \frac{\phi_p}{2} y_t \left(\frac{P_t(j)}{P_{t-1}} - \overline{\Pi} \right)^2 - \frac{\phi_p}{2} y_t \beta E_t \left\{ \left(\frac{P_{t+1}}{P_t(j)} - \overline{\Pi} \right)^2 \right\}.$$

Parameter $\phi_p > 0$ indexes the price adjustment costs. In order to facilitate accounting, we assume that price adjustment costs have to be paid in the current period only, in relation to what the price $P_t(j)$ implies for movements relative to last period's aggregate price level (the first term involving ϕ_p) and, in expectation, to movements relative to tomorrow's aggregate price level (the second term involving ϕ_p). $\overline{\Pi}$ marks the level of gross inflation to which prices are implicitly indexed. In equilibrium all firms will set the same price.
Aggregate profits are given by

$$\Gamma_t = \int_0^1 \left[\frac{P_t(j) y_t(j)}{P_t} - w_t h_t(j) - \frac{\phi_p}{2} y_t \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\Pi} \right)^2 - \frac{\phi_p}{2} y_t \beta E_t \left\{ \left(\frac{P_{t+1}(j)}{P_t(j)} - \overline{\Pi} \right)^2 \right\} \right] dj.$$

Note that profits are reduced by price adjustment costs. These costs – according to the accounting adopted here – involve both squared deviations of today's price from last period's price and expected costs of future changes.

2.1.4 Monetary policy

The central bank controls the nominal rate of interest using a Taylor-type rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\phi_R} \cdot \left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\phi_\Pi (1-\phi_R)} \cdot \exp\{e_t\}, \ \phi_R \in [0,1), \phi_\Pi \geq 0. \tag{2}$$

This links deviations of the current gross interest R_t from its steady-state value R to past interest rates and deviations of inflation from target. e_t are monetary shocks, that follow

$$e_t = \rho_e e_{t-1} + u_t^e$$

With u_t^e being a white noise shocks and $\rho_e \in [0, 1)$.

2.1.5 Fiscal policy

The social security system runs a balanced budget

$$\tau_t^y + \tau_t^m + \tau_t^o = 0. \tag{3}$$

2.1.6 Equilibrium and market clearing

In equilibrium, all firms will set the same price, such that $P_t(j) = P_t$ for all firms j. This means that all firms face the same demand $y_t(j) = y_t$, and will recruit the same amount of labor $h_t(j) = h_t$. Market clearing for final goods, then, means that

$$y_t + (\omega^y + \omega^m + \omega^o) = (c_t^y + c_t^m + c_t^o) + y_t \frac{\phi_p}{2} \left[\left(\Pi_t - \overline{\Pi} \right)^2 + \beta E_t \left\{ \left(\Pi_{t+1} - \overline{\Pi} \right)^2 \right\} \right],$$

that is, output and the endowments of the three age groups are used for consumption by each of them and price-adjustment costs. The labor market clears if

$$h_t = h_t^y + h_t^m + h_t^o.$$

Bond-market clearing requires

$$d_t^y = b_t^m$$

All the non-linear equilibrium conditions are summarized in Appendix A.

3 Analytical results

This section derives analytical results as to monetary policy transmission in the New Keynesian model, when accounting for retirement. We start with the case that the borrowing constraint for the young is binding, always. This most easily illustrates one channel: the role of the savings behavior of the middle-aged. Thereafter, we discuss the case without borrowing constraints.

3.1 Simplifying assumptions

In order to derive tractable results, throughout this section, we will assume that only the middle-aged supply labor. More formally, $\psi^y \to \infty$, and $\psi^o \to \infty$, such that $h_y = 0$ and $h_o = 0$. The young and the old households' income, therefore, does not depend on the response of the wage. Neither do the households adjust their labor supply. We will also abstract from fluctuations in the social security system, so that $\tilde{\tau}_t^y = \tilde{\tau}_t^m = \tilde{\tau}_t^o = 0$.

3.2 Binding borrowing constraint

By assumption, the equilibria will be such that the borrowing constraint for the young is binding always. For now, we will treat $\tilde{\overline{d}}_t$ as exogenous. Since the borrowing constraint is binding $\gamma_t^y > 0$ in (21) and consumption of the young is governed by their budget constraint (28) combined with the borrowing constraint (31).

$$\tilde{c}_t^y = \tilde{\overline{d}}_t^y \tag{4}$$

For the old, consumption is given by (36). Substituting bond-market clearing eq: bond market clearing, linear and borrowing constraint (31), consumption by the old is

$$\tilde{c}_t^o = \frac{R}{\Pi}\tilde{b}_{t-1}^m + b^m \frac{R}{\Pi}[\widehat{R}_{t-1} - \widehat{\Pi}_t]. \tag{5}$$

The households relevant for aggregate dynamics are then the middle-aged. Next, we derive an expression for consumption of the middle-aged. The consumption Euler equation of the middle-aged is (34). Combining this with (5), the budget constraint for the old, we have that

$$\widetilde{c}_t^m = \frac{c^m - c_h^m}{c^o - \overline{c}^o}\left[\frac{R}{\Pi}\right]\widetilde{\widetilde{d}}_t - (c^m - c_h^m)\left[\frac{1}{\sigma} - \frac{b^m \frac{R}{\Pi}}{c^o - \overline{c}^o}\right] E_t\left\{\widehat{R}_t - \widehat{\Pi}_{t+1}\right\} + \psi(h^m)^\nu \tilde{h}_t^m \cdot \mathbb{I}_{\text{GHH}}. \tag{6}$$

Here, $\mathbb{I}_{\text{GHH}}$ is the indicator function for GHH preferences. For the case of additively separable preferences, and keeping the borrowing constraint fixed, equation (6) allows us to characterize the response of middle-aged consumption to a change in the *ex-ante* real interest rate. Namely, consumption of the middle-aged will show a "conventional response" to lower real interest rates (that is, consumption of the middle-aged will rise) whenever

$$A := \left[\frac{1}{\sigma} - \frac{b^m \frac{R}{\Pi}}{c^o - \overline{c}^o}\right] > 0. \tag{7}$$

Term "A" will appear repeatedly. The first part $1/\sigma$ captures the substitution effect. This is unambiguously positive. See Footnote 1 for a discussion of the size of the elasticity. The last term shows the combined income and wealth effects. This term, again, will be unambiguously positive. What is important is that its size depends on the importance of private saving in effective retirement consumption. Keeping the borrowing constraint fixed and hours worked fixed, middle-aged consumption will rise with lower real interest rates as long as households are sufficiently willing to substitute intertemporally ($1/\sigma$ is large enough) and as long as the combined income and wealth effect of the interest change will be sufficiently small. The latter will be the case if savings are a small-enough part of old-age consumption. Putting it differently: lower pension will make consumption less interest elastic. What is more, a conventional response of the economy to low interest rates is by no means guaranteed.
To see this more clearly, combine goods market clearing (40), with consumption by the young (5) and consumption by the old (4). Then, output evolves as

$$\widetilde{y}_t - \psi^m \cdot (h^m)^{\nu^m} \cdot \tilde{h}_t^m \cdot \mathbb{I}_{\text{GHH}} = \widetilde{\widetilde{d}}_t + \widetilde{c}_t^m + \frac{R}{\Pi}\widetilde{b}_{t-1}^m + b^m \frac{R}{\Pi}\left[\widehat{R}_{t-1} - \widehat{\Pi}_t\right].$$

Combining this with the behavior of the middle-aged, equation (6), and bond market clearing

(41),

$$\widetilde{y}_t - \psi^m \cdot (h^m)^{\nu^m} \cdot \tilde{h}^m_t \cdot \mathbb{I}_{\text{GHH}} \quad = \quad \left[1 + \tfrac{c^m - c^m_h}{c^o - \overline{c}^o} \tfrac{R}{\Pi}\right] \widetilde{\widetilde{d}}_t + \tfrac{R}{\Pi} \widetilde{\widetilde{d}}_{t-1} \\ - (c^m - c^m_h) \left[\tfrac{1}{\sigma} - \tfrac{b_m \frac{R}{\Pi}}{c^o - \overline{c}^o}\right] E_t \left\{\widehat{R}_t - \widehat{\Pi}_{t+1}\right\} + b^m \tfrac{R}{\Pi} \left[\widehat{R}_{t-1} - \widehat{\Pi}_t\right] \tag{8}$$

Above, for better readability, we have defined $c^m_h := \overline{c}^m + \psi^m \frac{(h^m)^{1+\nu^m}}{1+\nu^m}$. Next, from the production function (38) and labor-market clearing (42), $\tilde{h}^m_t = \frac{1}{z} \tilde{y}_t$.

Next, the Phillips curve (27) in steady state implies $w = (\theta - 1)/\theta z$. For the case of GHH preferences, in addition, the labor supply first-order condition of the middle-aged (24) in steady state is $w = \psi^m (h^m)^{\nu^m}$. Combining this with the above equation, we can simplify to get the following aggregate IS curve.

$$\widetilde{y}_t \cdot \left[1 - \mathbb{I}_{\text{GHH}} \tfrac{\theta - 1}{\theta}\right] \quad = \quad \left[1 + \tfrac{c^m - c^m_h}{c^o - \overline{c}^o} \tfrac{R}{\Pi}\right] \widetilde{\widetilde{d}}_t + \tfrac{R}{\Pi} \widetilde{\widetilde{d}}_{t-1} \\ - (c^m - c^m_h) \underbrace{\left[\frac{1}{\sigma} - \frac{b_m \frac{R}{\Pi}}{c^o - \overline{c}^o}\right]}_{A} E_t \left\{\widehat{R}_t - \widehat{\Pi}_{t+1}\right\} + b^m \tfrac{R}{\Pi} \left[\widehat{R}_{t-1} - \widehat{\Pi}_t\right]. \tag{9}$$

Equation (9) suggests that, depending on precisely on term "A" highlighted in (7), the aggregate stimulus derived off of a reduction in the real rate of interest all else equal is decreasing in the extent to which households rely on own savings (b_m) for old-age consumption.

For a given response of the real rate of interest, this effect on output will be strongest with little market power of firms and GHH preferences. Note that, for GHH preferences, the term $\left[1 - \mathbb{I}_{\text{GHH}} \frac{\theta-1}{\theta}\right]$ approaches zero as $\theta \to \infty$ (the limit of perfect competition).

Next, we highlight the implications of private savings for retirement for for the size and sign of the response of output and inflation to a monetary easing, and for the uniqueness of that response. We wish to do so analytically and, therefore, we focus on two limiting cases: perfectly rigid prices and perfectly elastic labor supply.

3.2.1 A special case: perfectly rigid prices

We first follow Werning (2015) and highlight the implications of a change in interest rates, under the assumption that the central bank perfectly controls the *real rate*. The case of perfectly fixed prices is a useful baseline, for we can abstract from the supply side (the NKPC (37)) altogether. More formally, let prices be perfectly rigid ($\phi_p \to \infty$). In that case, $\Pi_t = 1$ for all t, and the linearized inflation has $\widehat{\Pi}_t = 0$ in all periods. The central bank, by steering the nominal interest rate, thus directly steers aggregate demand. Supply being demand-determined, the central bank also directly steers aggregate activity.

With fixed prices, equation (9) implies

$$
\begin{aligned}
\widetilde{y}_t \cdot \left[1 - \mathbb{I}_{\text{GHH}}\tfrac{\theta-1}{\theta}\right] \quad = \quad & \left[1 + \tfrac{c^m - c^m_h}{c^o - \overline{c}^o}\tfrac{R}{\Pi}\right]\widetilde{\overline{d}}_t + \tfrac{R}{\Pi}\widetilde{\overline{d}}_{t-1} \\
& - (c^m - c^m_h)\left[\tfrac{1}{\sigma} - \tfrac{b_m \frac{R}{\Pi}}{c^o - \overline{c}^o}\right]\widehat{R}_t + b^m \tfrac{R}{\Pi}\widehat{R}_{t-1}
\end{aligned} \tag{10}
$$

This gives rise to the following proposition.

Proposition 1. *Consider the three-period OLG model described above. Suppose that prices are perfectly rigid, $\phi_p \to \infty$, $\phi_R \in (0,1)$, and the borrowing constraint of the young always binds. Further, suppose that borrowing constraints are constant $\widetilde{\overline{d}}_t = 0$ and that that the economy initially is in its steady state. Consider the effect of a one-time monetary policy shock e_t in $t = 0$, and no shocks afterwards. Then, up to first order, equilibrium output evolves according to*

$$
\widetilde{y}_0 \quad = -(c^m - c^m_h)\left[\tfrac{1}{\sigma} - \tfrac{b^m R}{c^o - \overline{c}^o}\right] \cdot \theta^{\mathbb{I}_{GHH}} \cdot \hat{R}_0, \tag{11}
$$

$$
\widetilde{y}_t \;= \phi_R^{t-1}\left[-(c^m - c^m_h)\left[\tfrac{1}{\sigma} - \tfrac{b^m R}{c^o - \overline{c}^o}\right]\phi_R + b^m R\right] \cdot \theta^{\mathbb{I}_{GHH}} \cdot \hat{R}_0, \; t = 1, 2, ... \tag{12}
$$

Proof: Direct application of combining equation (10) with the linearized Taylor rule, (39).

We summarize the implications of Proposition 1 in the following corollary:

Corollary 1. *Consider the assumptions of Proposition 1. Consider a "monetary easing" $\widehat{R}_0 < 0$. Up to first order, the following is true:*

a) $\frac{d}{d(b^m R)}\frac{d\tilde{y}_t}{d\widehat{R}_0} > 0$*. That is a monetary easing will stimulate output the less, the more private saving $b_m R$ there is for old age.*

b) On impact, in $t = 0$, a monetary easing raises output if $A > 0$ ("A" is defined in (7)). Otherwise, output will fall (or, in the knife-edge case, stay constant).

c) Suppose that "A<0." Then, a monetary easing will be the more contractionary *for output $\widetilde{y}_t$ in $t = 1, 2, ...$ the more* persistent *the easing is (the larger ϕ_R).*

d) If the initial response of output to a monetary easing is negative, so will the responses of all future periods. If the initial response of output is expansionary, output may rise or fall in future periods.

Proof: all of these are direct implications of equation (12).

Corollary 1 highlights that more reliance of households on their own savings for retirement may reduce the expansionary effect of a monetary easing on output. Up to the point, indeed, that a "monetary easing" becomes contractionary altogether.

3.2.2 Another special case: perfectly elastic labor supply

What we have not discussed yet is the effect of private provision for retirement on the ability of monetary policy to anchor the price level and economic activity. Any indeterminacy in the current model will result from nominal indeterminacy. In order to discuss this, we have to bring back the Phillips curve and have prices less than perfectly rigid. In this section, let prices be less than perfectly rigid $0 < \phi_p < \infty$. For tractability, assume that labor supply is perfectly elastic ($\nu^m \to 0$).

To see the implications for local determinacy, recall the linearized New Keynesian Phillips curve in equation (37), where the wage $\widehat{w}_t$ is the driving term. Combining the labor-supply first-order condition and the production function for intermediate goods, we have that

$$\widehat{w}_t = \frac{\nu}{y}\tilde{y}_t + \frac{\sigma \cdot (1 - \mathbb{I}_{\text{GHH}})}{c^m - \overline{c}^m}\tilde{c}_t^m. \tag{13}$$

We will focus here on the case $\nu \to 0$. Clearly, for GHH preferences, this amounts to a constant wage and thereby $\widehat{\Pi}_t = 0$ for all t. That is, for GHH preferences, the results of Section 3.2.2 apply.

Here we will, therefore, only focus on additively separable preferences. In the above, substitute for $\tilde{c}_t^m$ and $\tilde{y}_t$ from (6) and(9), respectively. With this, inflation under additively separable preferences can be shown to evolve according to

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} - \frac{\theta - 1}{\phi_p}\sigma \left[\frac{1}{\sigma} - \frac{b^m \frac{R}{\Pi}}{c^o - \overline{c}^o} \right] [\widehat{R}_t - E_t \widehat{\Pi}_{t+1}]. \tag{14}$$

The standard New Keynesian model has that – as long as $\phi_\Pi > 1$ (the so-called Taylor principle) – inflation and output are uniquely determined. Self-fulfilling expectations cannot form. The key to this is the interaction of the central bank's reaction and the response of marginal costs. Any non-fundamental belief of higher inflation would meet with a central bank that raises the real rate of interest. Consumers substitute so as to save more and consume less, the wage falls – and with it marginal costs. This invalidates non-fundamental inflationary beliefs. Key to this is that the response of marginal costs to a monetary tightening is unambiguously negative. What equation (14) shows, is that this again cannot be taken for granted. If $A < 0$, that is if the wealth/income effect of an interest-rate change is strong enough, there could be scope for indeterminacy even if the central bank obeys the Taylor principle. By way of nominal rigidities, this indeterminacy will be both nominal and real.

Conditions for determinacy

The following proposition spells out the conditions for determinacy in more detail. The main message, we believe, is the following: in order to anchor inflation expectations when the government recedes from providing pensions, the central bank under the conditions spelled out here, may be asked for a stronger response to inflation.

Proposition 2. *Consider the three-period OLG model with perfectly elastic labor supply, $\nu \to 0$ and additively separable preferences. Suppose that $\phi_R = 0$, and that the borrowing constraint of the young always binds. Further, suppose that borrowing constraints are constant $\tilde{\tilde{\bar{d}}}_t = 0$. Then, the determinacy properties of the model are as follows*

a) If $A > 0$, any response of $\phi_\Pi > 1$ (the Taylor principle) will ensure determinacy. Smaller responses may ensure determinacy as well.

b) If $A < 0$ and $\beta + \kappa\sigma A > 0$, there is determinacy for any $\phi_\Pi > 0$, except if

$$1 + \frac{1-\beta}{-\kappa\sigma A} < \phi_\Pi < 1 + \frac{1-\beta}{-\kappa\sigma A} + \frac{2(\beta + \kappa\sigma A)}{-\kappa\sigma A}.$$

Since $\kappa > 0, \beta > 0$ and, by assumption for the case, $A < 0$, the lower threshold is larger than unity. There, thus, exist intermediate response coefficients for which the Taylor principle holds, but that do not ensure determinacy. In addition, a weak enough response of the central bank (setting $0 < \phi_\Pi < 1$) will ensure determinacy.

c) If $A < 0$ and $\beta + \kappa\sigma A < 0$, then there will be determinacy for any $\phi_\Pi > 0$, except if

$$-1 + \frac{1+\beta}{-\kappa\sigma A} < \phi_\Pi < -1 + \frac{1+\beta}{-\kappa\sigma A} + 2 \cdot \frac{\beta + \kappa\sigma A}{\kappa\sigma A}.$$

The upper limit is larger than unity. The Taylor principle need not ensure determinacy.

Proof: See Appendix D.

Item a) of the proposition states that whenever retirement savings are not sufficiently large to make the income effect of a cut in the real interest rate dominate the intertemporal substitution effect (that is, if $A > 0$), the Taylor principle continues to hold: whenever the central bank is committed to raising the real rate of interest in response to inflation exceeding the target level, the central bank uniquely anchors both inflation and real activity.

If the income effect is sufficiently strong, however, so that $A < 0$, this no longer is the case. The logic is simple. Whenever retirement savings form a large-enough part of retirement consumption, so that $A < 0$, the real wage may rise with higher real interest rates, rather than fall as in the conventional New Keynesian model. The reason is that real rates mean an *positive* wealth effect for the middle-aged. Suppose that households form non-fundamental beliefs of mean-reverting, but perhaps persistently high inflation. Suppose that the central bank raises the nominal interest rate sufficiently so that the real interest rate rises (the Taylor principle). When real rates are high, middle-aged households will want to work less (and consume more) since saving for retirement is provided for easily. The wage rises. Thus, what can happen if the central bank reacts to a non-fundamental belief of inflation by raising the real rate of interest, is that precisely this validates these very beliefs. In other words, the Taylor principle may no longer guarantee a unique stable rational expectations equilibrium. A similar logic applies to case c).

What b) shows is that there, still are two ways to anchor inflation expectations. On the one hand, any response to inflation that is *small* enough will rule out sunspots. Any reaction of the central bank with ϕ_Π *smaller (!)* than unity will ensure determinacy, including the case of constant interest rates, $\phi_\Pi = 0$, a sharp reversal of the conventional wisdom; for example, Sargent and Wallace (1975). How can this be? When retirement savings are important ($A < 0$, inflationary beliefs can be self-regulating amid constant interest rates. Suppose that inflationary beliefs formed and that the nominal interest rate were to be held constant. Then, the real interest rate would fall. The wealth effect being important, the middle-aged would want to work more, reducing the wage and marginal costs, in turn contradicting the beliefs of high inflation. If the wage falls just enough, self-fulfilling expectations would be ruled out.

In contrast to this, the central could also more strongly respond to inflation than the Taylor principle suggests. For sure, as $\phi_\Pi \to \infty$, the central bank would anchor inflation expectations and real activity. It is important to note, however, that the underlying rationale is rather different from that of the conventional New Keynesian model. In the conventional New Keynesian model, the central bank raises the real rate of interest so as to ensure that wages fall. Now, as the central bank embarks on a strong response to inflation, it makes sure that whenever non-fundamental inflation expectations form, the wage would *rise* without bounds. That is, there will be no *bounded* level of these expectations. The reason is simple: if such beliefs formed, the central bank would strongly raise the real rate of interest, wages would strongly rise, and so would marginal costs.

Indeed, item c) shows that whereas a non-response to inflation is not fool-proof, a strong response to inflation is. Even if retirement savings are important, a non-response by the central bank does not always anchor inflation expectations and real activity (the range given in item c) could cover 0). What will always work to ensure determinacy is a strong-enough response of the central bank to inflation, in all of the scenarios given above. It is in this sense, that we conclude from the above that central banks may need to redouble their focus on inflation once self-provisioning for retirement attains an important role.[5] Figure 1 illustrates this intuition

[5] The indeterminacy that we have stressed here differs from that emphasized in the OLG literature **[add reference]**. The literature emphasizes, in real economies, that indeterminacy can arise in models with endogenous capital accumulation. Then, the logic is as follows. Suppose households believe that future returns to savings (the rental rate of capital) are low. If the income effect dominates the substitution effect, then households will wish to save more, that is, accumulate more capital. Due to decreasing returns to scale, however, returns fall as households accumulate capital. That is, their non-fundamental beliefs of low returns can validate themselves. In our model, we do not have capital. The indeterminacy would vanish entirely if prices are flexible (the limit

graphically.

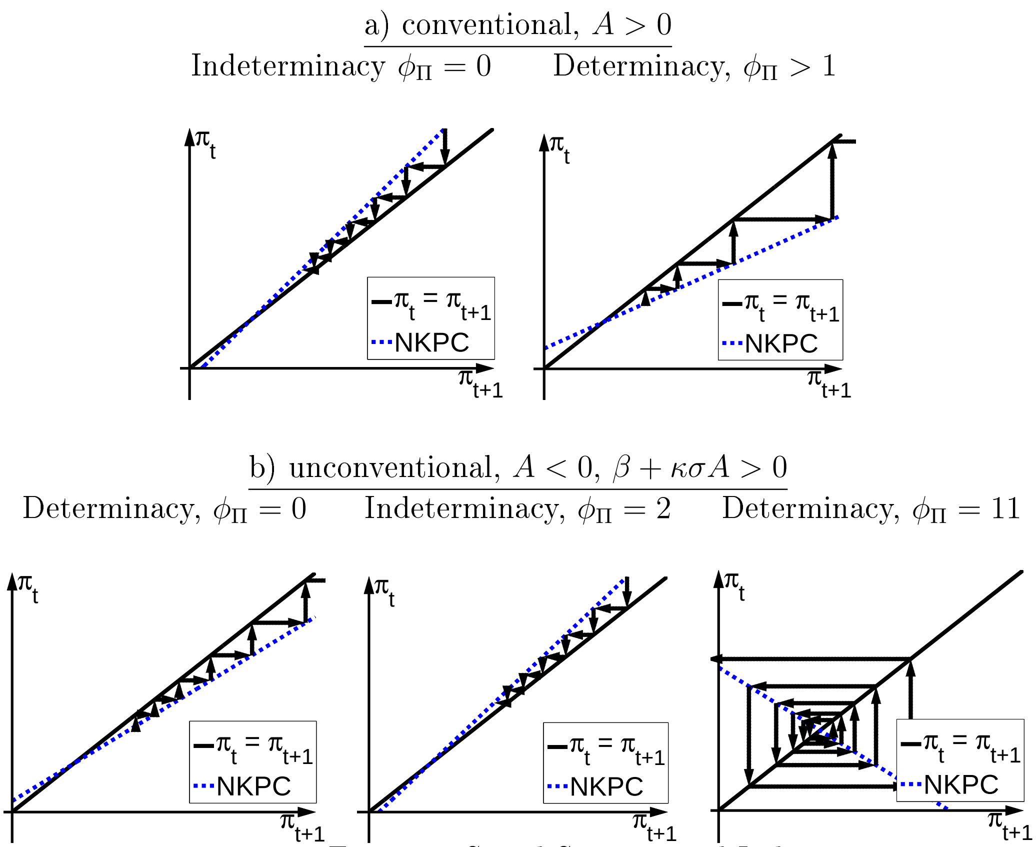

Figure 1: Social Security and Indeterminacy

Response of inflation and output

The next proposition summarizes the response of inflation and output to a persistent monetary easing, provided that there is determinacy.

Proposition 3. *Consider the same assumptions as in Proposition 2. In addition, assume that there is a monetary shock e_t that follows an AR(1) process with autocorrelation $\rho \in (0,1)$. Suppose that the economy initially is in its steady state and consider a one-time monetary shock in period 0. Then, the fundamental solution for inflation is given by*

$$\widehat{\Pi}_t = a\epsilon_t^m,$$

with $a = -\kappa\sigma A/(1-\beta\rho+\kappa\sigma A(\phi_\Pi-\rho))$. The fundamental solution for output is :

$$\widetilde{y}_t = -\left(c^m - c_h^m\right)A\left(a\phi_\Pi + 1 - a\rho\right)\epsilon_t^m + \overline{d}\frac{R}{\Pi}\left(a\phi_\Pi\epsilon_{t-1}^m + \epsilon_{t-1}^m - a\epsilon_t^m\right).$$

In addition, we have the following

$\phi_p \to 0$, that is, $\kappa \to \infty$).

a) ***Response of inflation****. If $A > 0$ and the Taylor principle holds, the response of inflation will be conventional ($a < 0$).*

b) If $A < 0$ and $\beta + \kappa\sigma A > 0$, responses of ϕ_Π below the lower threshold given in Proposition 2 item b) will show an unconventional response of inflation, $a > 0$, whereas response coefficients of ϕ_Π larger than the upper threshold ensure a conventional response.

c) If $A < 0$ and $\beta + \kappa\sigma A < 0$, , responses of ϕ_Π below the lower threshold given in Proposition 2 item c) will show an unconventional response of inflation, $a > 0$, whereas response coefficients of ϕ_Π larger than the upper threshold ensure a conventional response.

d) ***Impact response of output****. Consider the case the $\rho = 0$. $c^m - c_h^m > \overline{d}\frac{R}{\Pi}\kappa\sigma$ (such that either price rigidities are large enough, or private provision for retirement not too large). Suppose that there is determinacy. In this case, if $A > 0$, the impact response of output will be conventional (output will fall with a positive monetary shock). In the same case, if $A < 0$, the impact response of output will be conventional only if $\phi_\Pi > \frac{1}{\kappa\sigma(-A)}$, that is, if the response to inflation is strong enough. Otherwise, the response of output is unconventional.*

Proof: See Appendix D.

3.3 Slack borrowing constraint

So far, we have proceeded under the assumption that the borrowing constraints of the young bind. This section discusses the case when borrowing constraints on the young are slack.

3.3.1 Simplifying assumptions

In order to be able to derive tractable analytical results, we continue to focus on the case in which only the middle-aged supply working hours. In addition, we set $\beta = 1$. We focus on a parameterization in which the effective endowment of young and old households is identical, namely, $\omega^y - \tau^y - \overline{c}^y = \omega^o - \tau^o - \overline{c}^o$. We also continue to focus on a zero-inflation steady state and on the case that taxes are constant.

3.3.2 Steady state

With these assumption, the steady state can be characterized as follows. In steady state, $\Pi = 1$, $R = 1$. In addition

$$c^i = c_h^i + \frac{\sum \omega^i - \sum c_h^i + y}{3}, \text{with } i \in \{y, m, o\}.$$

In terms of borrowing and lending, respectively, the steady state is characterized by

$$d^y = b^m = c^y - (\omega^y - \tau^y) = \frac{1}{3}\left[(\omega_m - \tau^m + y - c_h^m) - (\omega_y - \tau^y - c_h^y)\right].$$

The term in square brackets is the effective income of the middle-aged minus the effective income of the young.

3.3.3 Linearized dynamics

Combining the consumption Euler equations of the young and middle-aged, the respective versions of equation (32), with the budget constraints of the young and old, equations (31) and (36), gives the following law of motion for saving:

$$\tilde{b}^m_t = \frac{c^y - \overline{c}^y}{c^o - \overline{c}^o}\left[E_t \tilde{b}^m_{t+1}\right] - (c^y - \overline{c}^y)\left[\frac{1}{\sigma} - \frac{b^m}{c^o - \overline{c}^o}\right] E_t\{\widehat{R}_{t+1} - \widehat{\Pi}_{t+2}\} - \frac{1}{\sigma}(c^y - \overline{c}^y)\left[\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right]. \tag{15}$$

3.3.4 A special case: perfectly rigid prices

As in the sections with borrowing constraints, we focus first on the case of constant prices. This gives rise to the following proposition.

Proposition 4. *Consider the three-period OLG model described above. Suppose that prices are perfectly rigid, $\phi_p \to \infty$, $\phi_R \in (0,1)$, and the borrowing constraint of the young never binds. Suppose that the economy initially is in its steady state. Consider the effect of a one-time monetary policy shock e_t in $t = 0$, and no shocks afterwards. Then, up to first order, saving evolves according to*

$$\widetilde{b}^m_t \;=\; -\tfrac{1}{1-\phi_R}(c^y - \overline{c}^y)\left[\tfrac{1}{\sigma}(1+\phi_R) - \tfrac{b^m}{c_o - \overline{c}^o}\phi_R\right]\widehat{R}_t. \tag{16}$$

In addition, we have that

$$\tilde{y}_0 = -\theta^{\mathbb{I}_{GHH}}(c^y - \overline{c}^y)\left[\frac{1}{\sigma}(3+\phi_R) - \frac{b^m}{c_o - \overline{c}^o}(1+\phi_R)\right]\frac{1}{1-\phi_R}\widehat{R}_0 \tag{17}$$

and

$$\tilde{y}_0 = -\theta^{\mathbb{I}_{GHH}}(c^y - \overline{c}^y)\left[\frac{1}{\sigma}(1+4\phi_R+\phi_R^2) - \frac{b^m}{c_o - \overline{c}^o}(1+\phi_R+\phi_R^2)\right]\frac{1}{1-\phi_R}\widehat{R}_t,\;\; t \geq 1 \tag{18}$$

Proof: See Appendix D.

We summarize the implications of Proposition 4 in the following corollary:

Corollary 2. *Consider the assumptions of Proposition 4. Consider a "monetary easing" $\widehat{R}_0 < 0$. Up to first order, the following is true:*

a) $\frac{d}{d(b^m)}\frac{d\tilde{y}_t}{d\widehat{R}_0} > 0$. *That is a monetary easing will stimulate output the less, the more of a role private saving, b^m, plays in financing old-age consumption.*

b) On impact, a monetary easing has an expansionary effect on output if an amended form of the inequality in equation (7) holds, namely if

$$\frac{1}{\sigma}(3+\phi_R) - \frac{b^m}{c_o - \overline{c}^o}(1+\phi_R) > 0.$$

Otherwise, output will fall (or, in the knife-edge case, stay constant).

c) For subsequent periods, the easing will be expansionary if

$$\frac{1}{\sigma}(1+4\phi_R+\phi_R^2) - \frac{b^m}{c_o - \overline{c}^o}(1+\phi_R+\phi_R^2) > 0.$$

d) Suppose that the steady state with and without borrowing constraints on the young is the same. Then, one can compare the current corollary with Corollary 1. We have that

i) whenever the impact response is expansionary with borrowing constraints, the impact response will be expansionary without a borrowing constraint on the young as well. The reverse is not true.

ii) the same is true for the response of future output.

In this sense, a non-conventional effect of a conventional monetary easing is more likely when borrowing constraints on the young bind.

Proof: Direct implications of Proposition 4 and Corollary 1.

Corollary 4 highlights that more reliance of households on their own savings for retirement may reduce the expansionary effect of a monetary easing on output. Up to the point, indeed, that a "monetary easing" becomes contractionary altogether. That is, as for the case with a binding borrowing constraint on the young, we have that a monetary "easing" can be contractionary. As item d) of the corollary shows, however, such an outcome is less likely when the borrowing constraints of the young do not bind.

3.3.5 A special case: perfectly elastic labor supply

To be done.

4 Simulation results

Next, we provide simulation results based on a rough calibration of the model. The idea of this section is to illustrate that the mechanism highlighted above is borne out in a reasonably straightforward exercise.

4.1 Parameters

Table 1 parameters reports the parameters that we choose. $\beta = 0.995$ matches a real rate of 1

Table 1: Parameters of 3-period OLG model

Par.	value	target	Par.	value	target
β	0.995	real rate of 1 percent	ϕ_p	367.4	Calvo stickiness of 0.85
ν	0.5	Frisch elasticity of 2	ϕ_R	0.85	interest rate persistence
ϵ	11	Markup of 10%	ϕ_Π	1.2	moderate response to inflation
$\bar{c}$	0	arbitrary choice.	$\overline{\Pi}$	1	zero-inflation steady state
χ	4.1744	$h = 1$ (scales disut. of work)	$\bar{d}$	0.2627	borrowing constraint hardly binds.
σ	2	standard value	z	1	steady-state output $y = 1$.
ω_y	0.2	some value			
ω_m	0	middle-aged inc. endog.			
ω_o	0.2	roughly 40% of old age c			

Notes: Parameters chosen for the calibration of the 3-period OLG model.

percent, $\nu = 0.5$ gives a Frisch elasticity of 2. A demand elasticity of $\epsilon = 11$ matches a markup of 10 percent, a conventional choice. In order to illustrate the mechanism most clearly, for now we set $\bar{c} = 0$. $\chi = 4.1744$ reconciles the model with a target for hours worked of 1. $\sigma = 2$ is a standard value for risk aversion in the DSGE literature, and in macro and public finance more generally. It fits with the estimates for the intertemporal elasticity of substitution (of about 0.5) that the micro consumption literature obtains, for example, Attanasio and Weber (1995). Endowments are such that the endowment $\omega_o = 0.2$ accounts for about 40% of old-age consumption. These choice is not innocuous, compare condition (**??**). ω_y, the endowment of the young, is set to the same value.

The parameter determining price rigidities, ϕ_p, is chosen such that the slope of the New Keynesian Phillips curve coincides with the slope from a Calvo-type nominal rigidities with price stickiness parameter 0.85. The parameters of the Taylor rule, $\phi_R = 0.85$ and $\phi_\pi = 1.2$ are conventional choices. Without loss of generality, we set the inflation target to zero ($\overline{\Pi} = 1$). The borrowing constraint, if it exists, is set to 99 percent of the amount of borrowing that would emerge absent the constraint. Steady-state output is normalized to unity, from which the value for z follows.

In the current section, we do not discuss productivity shocks, or shocks to the threshold level of consumption so that $z_t = z$ and $\bar{c}_t = 0$ always.

Table 2 reports the steady state absent the borrowing constraint. Since we picked the borrowing limit to be close to what would be borrowed absent a constraint, the steady state in the calibration with the borrowing constraint is virtually identical.

Table 2: Steady State 3-period OLG model

Variable	value	description	Variable	value	description
c^y	0.4651	consumption young	R	1.0117	nominal rate (gross)
c^m	0.4666	consumption middle-aged	Π	1	inflation (gross)
c^o	0.4682	consumption old	w	0.9091	wage rate
d^y	0.2651	borrowing young	h	1	hours worked
b^m	0.2651	saving middle-aged	y	1	variable output

Notes: steady state for the calibrated three-period OLG model. The steady state reported here refers to the case absent a borrowing constraint for the young. The steady state for the case with a borrowing constraint is similar.

4.2 Determinacy

[Add charts here]

4.3 The effect of a monetary easing

Next, we look at the effect of a monetary easing, that is, of what typically would be considered an "expansionary" monetary policy shock. This shocks is a one-time surprise reduction in nominal interest rates, $\epsilon_t^m < 0$. We look at the effect of an identical shock in two economies: the economy with borrowing constraints and the economy without. We start with the case of the model-variant in which the borrowing constraint for the young does not bind.

Figure 2 shows the impulse responses to a monetary easing if the borrowing constraint for the young is not binding. In that case, the response of the economy is conventional: a monetary easing generates an increase output. The responses for the aggregate economy are very similar to the responses commonly known from the representative-agent New Keynesian model. Namely, as the nominal interest rate falls, so does the real interest rate. This stimulates consumption demand and aggregate activity (output and hours worked rise). An increase in the workload raises wages. Thereby, inflation rises as well. The second row of the figure shows the consumption responses by the young, middle-aged, and old. What is important to note is that the increase in aggregate demand stems both from the young and the middle-aged with the young's consumption increase supported by an increase in borrowing (d_t^y, bottom right). On impact, the consumption by the old declines because of his savings are in nominal terms. Indeed, the very first period of the shock apart, there are few distributional effects of that easing. Beyond the very first period, consumption rises for *all* households, as the rise in income for the middle-aged means the middle-aged seek to increase the absolute amount of savings, which in turn lifts the consumption of the young – who borrow more at lower rates.

While the increase in aggregate consumption is conventional, it is important to recall the

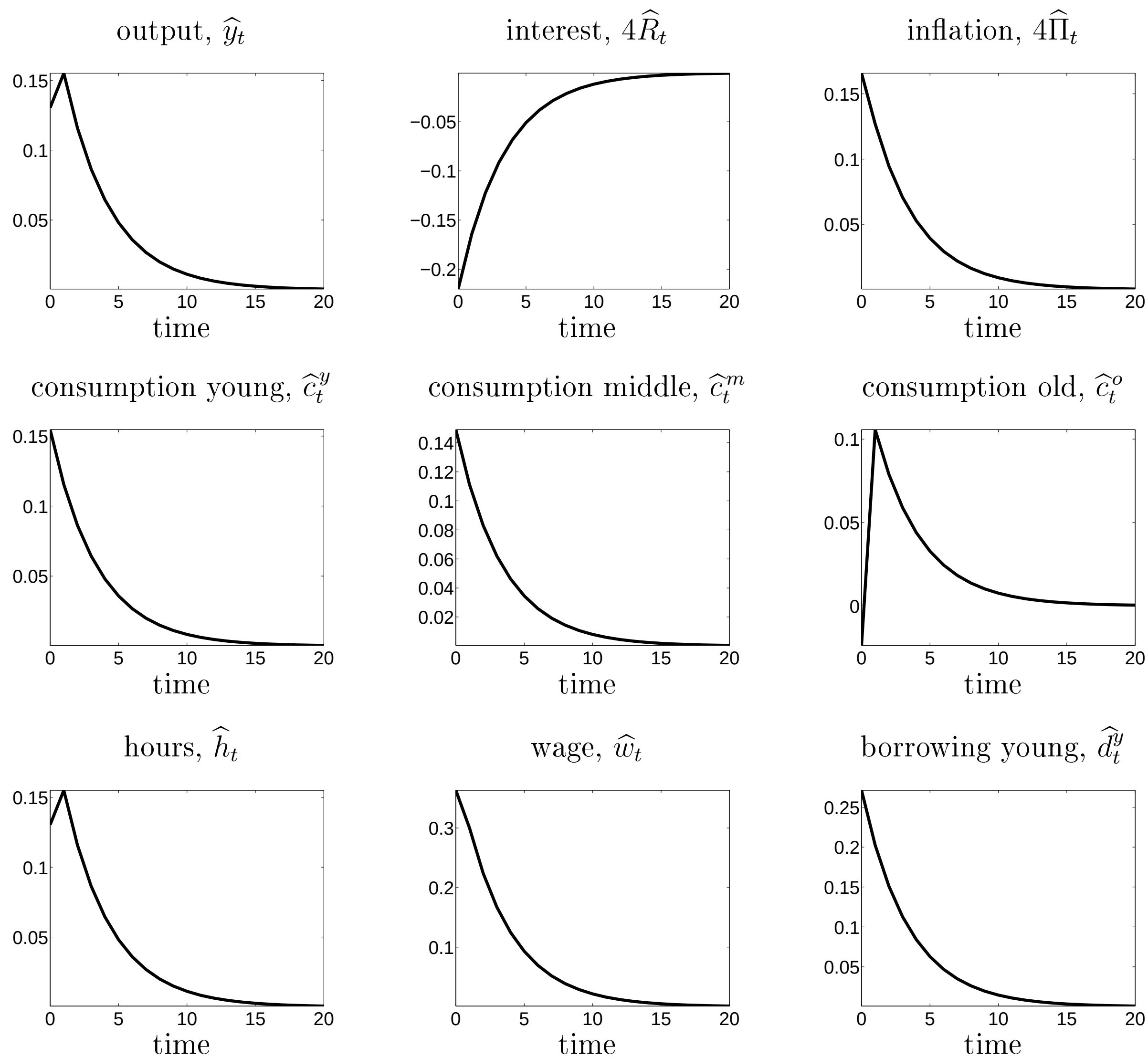

Figure 2: **No borrowing constraint: Impulse response to a 25 basis point monetary easing (1 pp. when annualized)**. For the three-period OLG model. All data are in log deviation from the non-stochastic steady state. The graphs show percent changes of the respective variable. Responses of interest rates and inflation rates are annualized.

channels through which this increase comes about. Namely, under our calibration condition (7) is violated. This means that the *direct effect* of the monetary easing on consumption by the middle-aged is to reduce that consumption. The reason is that the middle-aged are not particularly willing to substitute intertemporally ($1/\sigma = 0.5$). At the same time, for given savings, the future value of the savings is reduced by the fall in real rates. What makes consumption of the middle-aged rise in response to the monetary easing are general-equilibrium effects. And these heavily depend on the response by the *young*. The young face a lower real interest rate. They seek to move consumption over time, increasing consumption when young and saving. Absent borrowing constraints, the additional consumption by the young will be financed by lending. Indeed, the first-round consumption increase by the young is larger than the first-round fall in consumption by the middle-aged. In sum, therefore, aggregate demand rises. This lifts hours worked and incomes of the middle-aged, and thereby allows the middle-aged to save more without cutting back middle-aged consumption.

Matters are quite different if the borrowing constraint *is binding*. This case is examined in Figure 3.

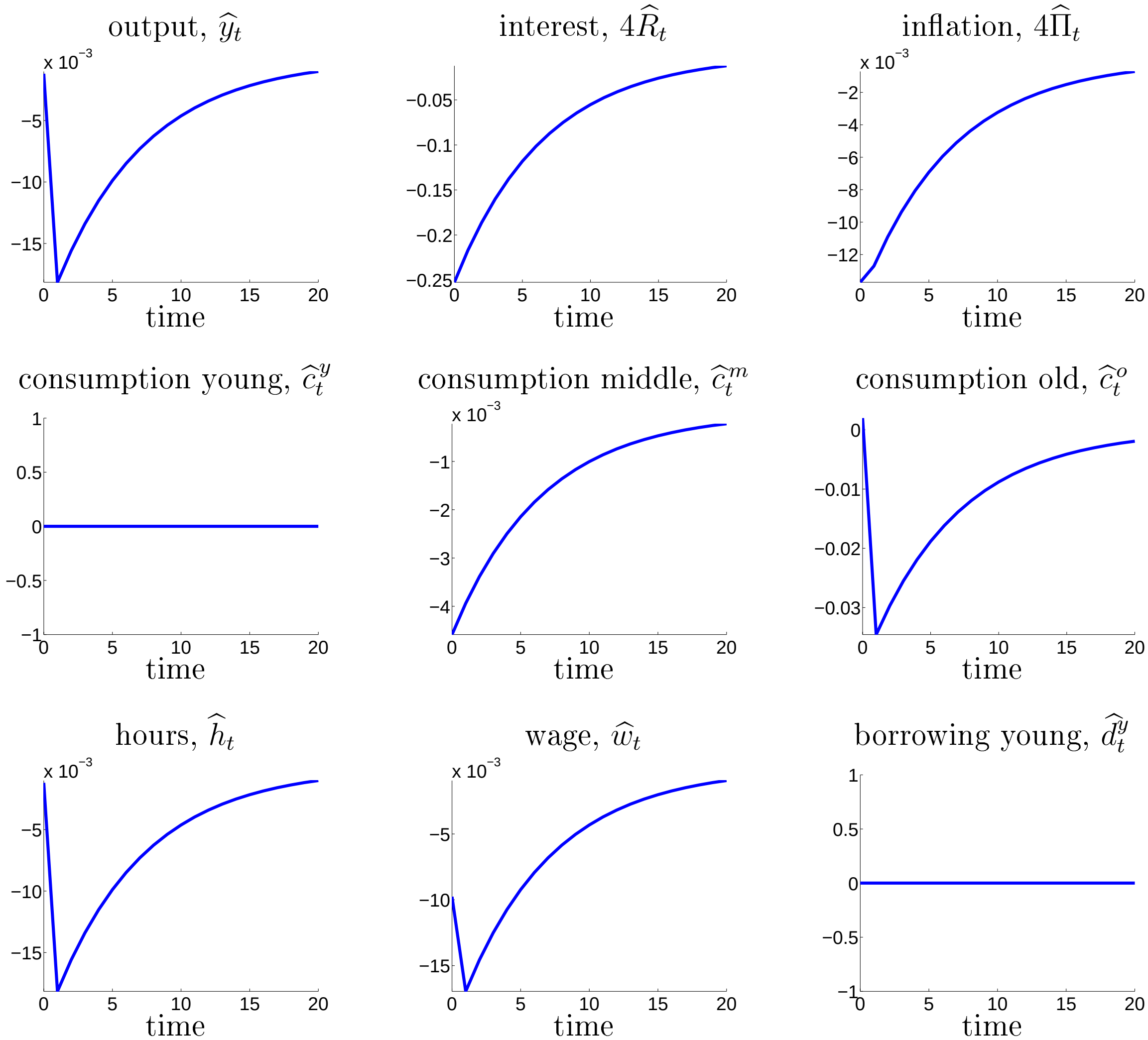

Figure 3: **Borrowing constraint binds: Impulse response to a 25 basis point monetary easing (1 pp. when annualized)**. For the three-period OLG model. All data are in log deviation from the non-stochastic steady state. The graphs show percent changes of the respective variable. Responses of interest rates and inflation rates are annualized.

If the borrowing constraint of the young binds, the increase desire to save that the middle-aged have will not be met by additional demand for goods by the young and an increase in labor income for the middle-aged. Therefore, middle-aged consumption falls. Output falls and labor income falls. As a result, the middle-aged household's desire to save will be brought in line with aggregate demand for savings vehicles through a reduction in aggregate income. With borrowing constraints on the side of the young, a reduction in interest rates, which *all else equal* makes it harder to save for retirement, therefore leads to a persistent fall both in old-age consumption and in consumption by the middle-aged. Rather than stimulating output and inflation, therefore, an interest rate cut depresses the economy. Quantitatively, the effects are on the order 0.02 percent. This may seem small but the broader point is that monetary easing fails to stimulate the economy. As the following section shows, however, once one allows for borrowing constraints to *endogenously* depend on income, the contraction can become quantitatively sizable.

4.4 More realistic borrowing constraints

The results above were based on a constant borrowing constraint. Instead, state-dependent borrowing constraints may be considered more realistic. In the following, we consider two such cases, both of which have been considered in the literature (following Mendoza (2010) and Schmitt-Grohe and Uribe (2017)). As we will see, these alternative constraints exacerbate the recessionary effect of the interest rate cut if the borrowing constraint is binding.

In addition, we will abstract from a wealth effect on labor supply.[6] Otherwise, for some of the specifications of the endogenous borrowing constraints below, we have experienced problems with indeterminacy. We have not studied those difficulties in sufficient detail yet, hence the simulations here seek to avoid those issues. Figure 4 presents the response of this economy for the baseline of a constant borrowing constraint as a solid blue line. As can be witnessed, the baseline economy evolves in virtually the same way as in Figure 3. In this sense, abstracting from the wealth effect on labor supply is rather inconsequential.

In this section, we consider collateral constraints that depend on flows rather than stocks. The reason is that our simple model does not have capital or other stock variables. As regards the constraints, the first case that we consider is one in which the young generation's borrowing capacity directly varies with the state of the economy. In particular, we assume that the borrowing constraint is

$$\overline{d}_t \equiv \overline{d} \cdot (y_t/y), \tag{19}$$

[6] We do so by changing the utility function to

$$E_t \left\{ \frac{(c_t^y)^{1-\sigma}}{1-\sigma} + \beta \left(\frac{(c_{t+1}^m)^{1-\sigma}}{1-\sigma} - \chi \frac{1}{1+\nu} (h_{t+1}^m)^{1+\nu} \lambda_{t+1}^m \right) + \beta^2 \frac{(c_{t+2}^o - \overline{c}_{t+2})^{1-\sigma}}{1-\sigma} \right\},$$

where the choices of the individual household are c_t^y, c_t^m, c_t^o, h_t^m, in the respective periods. λ_t^m is the marginal utility of the aggregate consumption of the group and beyond the individual household's control. We adjust parameter χ to retain the same steady-state in the unconstrained economy as described in Table 2.

where the steady-state value of the constraint $\overline{d}$ remains at the value specified above and where the last term makes the borrowing constraint (in percentage terms) move in lockstep with the percentage deviation of GDP from its own steady-state value. Broadly speaking, this constraint captures the notion that during good times, borrowing is easier.

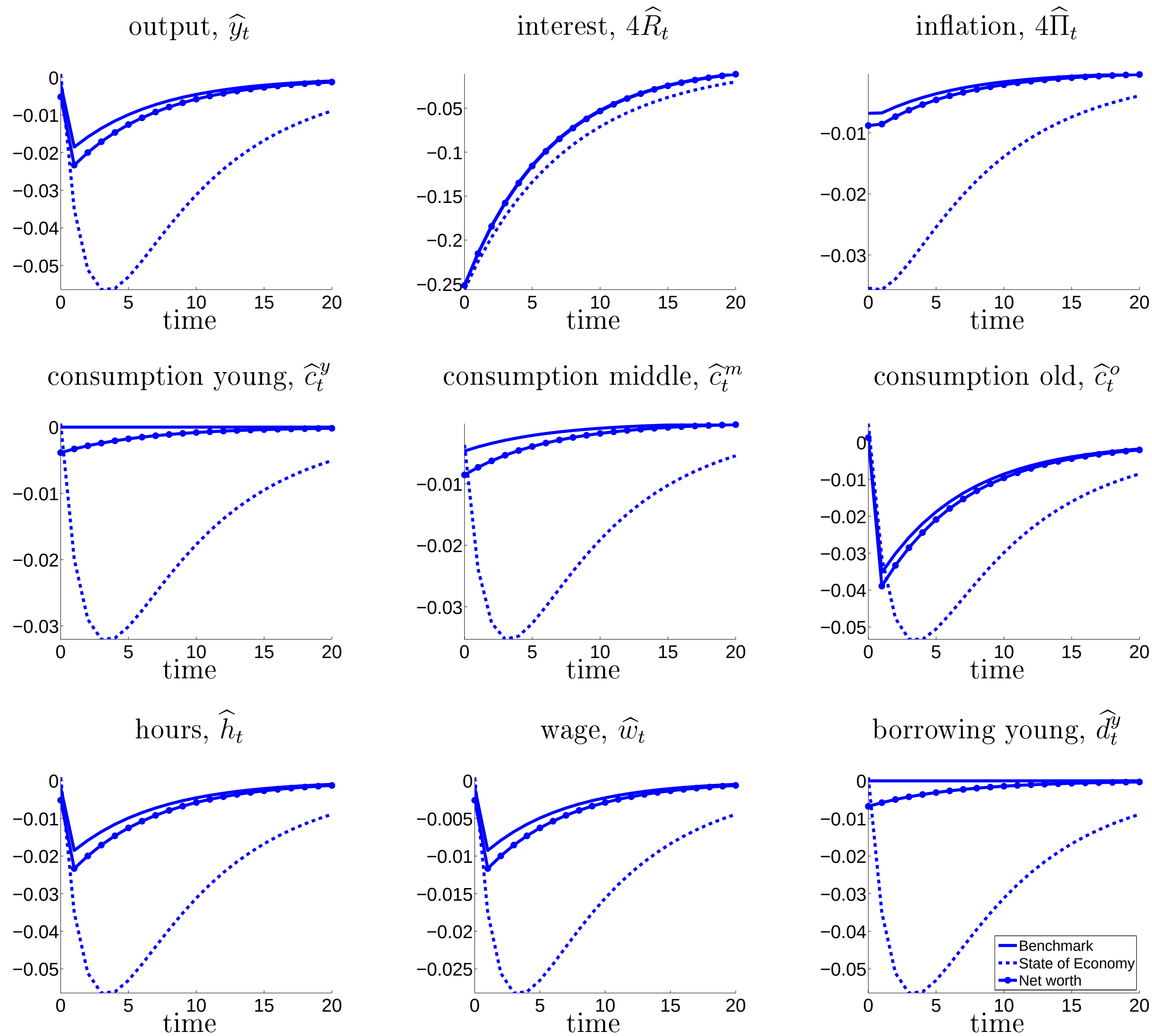

Figure 4: **Borrowing constraint binds: Comparing different formulations of the constraint**. Impulse response to a 25 basis point monetary easing (1 pp. when annualized). For the three-period OLG model. All data are in log deviation from the non-stochastic steady state. The graphs show percent changes of the respective variable. Responses of interest rates and inflation rates are annualized. Blue solid: baseline economy with constant borrowing constraint. Blue dashed: the borrowing constraint moves one to one with GDP as in (19).

Figure 4 presents the response to a a monetary easing under the new constraint (19) as a dashed line (the solid line corresponds to the benchmark model). As expected, if the borrowing limit depends on the state of the economy, the *recessionary* impulse of a monetary *easing* is exacerbated notably. Output falls by about a factor of 3 more than in the baseline with a fixed constraint. In addition, the recession now is more persistent. Note that as the economy enters into the recession, the collateral constraint becomes tighter, forcing young households to cut consumption, which in turn exacerbates the recession even more. This feedback effect resembles the Fisherian debt deflation result found in models that display stock collateral constraints Mendoza (2010). In other words, not only is there scope for low-interest policies to be *recessionary*, but also would there seem to be scope for low-interest policy to exhibit these effects in a rather persistent manner.

We also entertain a different form of the borrowing constraint. For this, we allow borrowing conditions to depend entirely of the prospects of the young only, rather than on the state of the economy as a whole. Suppose, in particular, that borrowing is a function of future networth (income net of debt obligations):

$$\overline{d}_t \equiv \overline{d} E_t \frac{n^m_{t+1}}{n^m}, \tag{20}$$

where $n^m_t = \omega_m + h_t w_t + \Gamma_t - d^y_{t-1} R_{t-1}/\Pi_t$ is the networth of the middle-aged generation. Here, networth is the middle aged's income (endowment + earnings + dividends) net of debt that the middle-aged took up when young.[7] The circled lines in Figure 4 show the corresponding impulse responses. Also with this constraint, the recession is deeper than in the benchmark model. However, this constraint ameliorates the impact of the shock compared to the business-cycle condition constraint.

In sum, we have shown that low interest rates can indeed lead to persistent recessions. In this section, we have shown further that the resulting recessions can be quantitatively significant if the borrowing constraints interact with economic activity.

Naturally, though, there is some tension between the frequency typically assumed in a three-period OLG model and the business-cycle frequency that the New Keynesian structure is designed for. We address this in the next section, where we study the effect of low interest rates in a many-generation life-cycle model.

5 Some Evidence

One clear prediction from our model is that monetary policy becomes less effective as savings for retirement become more important. Do we observe this in the data? In this section, we provide some evidence toward an affirmative answer.

As a first step, we need data, which shows the strength of the retirement system across countries.

[7] We normalize the borrowing constraint by the steady-state values to make it comparable with the other cases.

To this end, we collect data from the OECD's "Pensions at a Glance" database. We use the net replacement rate, which according to the OECD, equals the ratio of the pension entitlement to lifetime average earnings, both calculated after taxes. We think this measure is informative about the adequacy of the retirement income. Based on this index, we build a ranking of the generosity of the retirement system for several countries. The country with the weakest system is ranked first (USA), while the country with the most generous retirement package goes last (Netherlands).

In the second step, we review the literature for the effects of monetary policy in the countries in our sample. From these studies, we rank the impact of a 25 bps contractionary monetary shock on consumption. For each country, we compute the largest impact on the impulse response of consumption to the shock. We rank countries from those with the weakest response (Austria) to the strongest response (Portugal). Figure 5 shows the rankings.

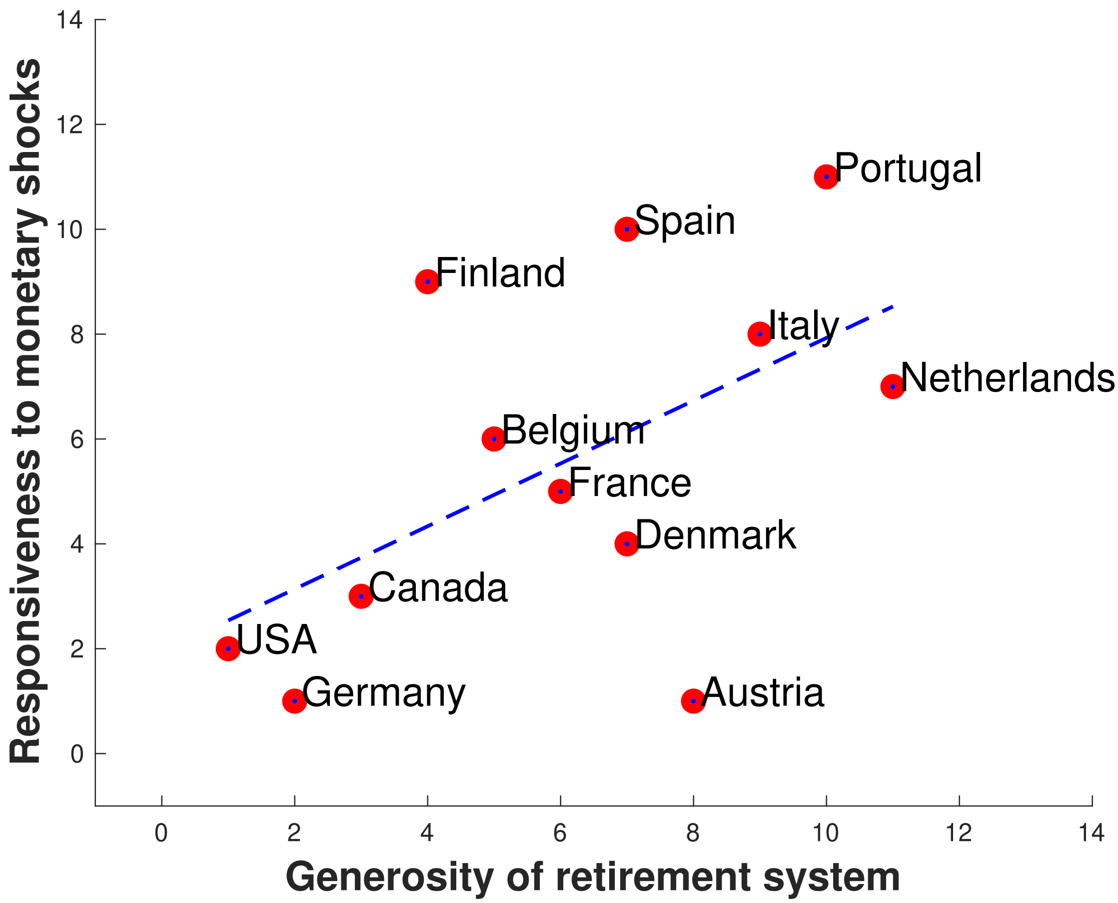

Figure 5: **Monetary Responsiveness and Retirement System**. Generosity is ranked from weakest (= 1, USA) to strongest (= 11, Netherlands). Monetary response is ranked from weakest (= 1, Austria) to strongest (= 11, Portugal).

There are 12 countries for which we have data on both the retirement system and monetary policy (appendix E provides details on the data sources and robustness checks). The "robust" correlation between the generosity index and the consumption index is 0.29. This value is strongly influenced by Ireland, which has a weak retirement system and a very responsive consumption. Once we remove this country the correlation jumps to 0.63. It worth highlighting two extreme cases. On one hand, U.S. has second less generous retirement system and the second least responsive consumption to monetary policy in our sample. In contrast, Portugal has the most responsive consumption and ranks second in the retirement index. As a robustness check (see appendix), we computed the response of consumption to a monetary shock using a Cholesky decomposition. We arrived to similar results.

6 Setup with many periods

We keep the setting with many periods as close to the exposition in Section 2 as possible.

6.1 Preferences

Households live for $K = n_y + n_m + n_o$ periods, where n_y, n_m, and n_o mark the number of periods spent young, middle-aged, and old, respectively.
Lifetime utility of a household born in period t (at age $k = 1$) is given by

$$E_t \left\{ \sum_{k=1}^{K} \beta^{k-1} u(c_{k,t}, h_{k,t}; \zeta_k) \right\},$$

with felicity function u as given earlier. $c_{k,t}, h_{k,t}$ marking consumption and hours worked of the household when aged k in period t.

6.2 Incomes and endowments

$$c_{k,t} + b_{k,t} = b_{k-1,t-1} R_{t-1}/\Pi_t + \omega_k - \tau_k + w_t z_k h_{k,t} + \Gamma_t / n_m \cdot \mathbb{I}(n_y < k \leq n_y + n_m).$$

Here, ω_k is the endowment, τ_k are taxes paid by the household. w_t is the real wage per efficiency unit of labor. z_k is labor productivity of a household of age k. Γ_t are the profits of firms, which accrue in equal measure to each middle-aged generation. $b_{0,t-1} \equiv 0$, and $b_{K,t} geq 0$.

6.3 Production

The firm sector is changed marginally relative to Section 2.1.3. In order to facilitate matching price-setting frequencies, we switch to a Calvo setup. There is a unit mass of infinitely lived firms. Firms behave risk-neutral. Firms are not traded. Rather, their profits are distributed lump-sum, and in equal measure, to all middle-aged workers. Let α be the price rigidity. Firms now hire different types of workers in a competitive labor market. Different types of workers are perfect substitutes. Firms produce according to

$$y_t(j) = \sum_{k=1}^{K} z_k h_{k,t}(j).$$

6.4 Monetary and Fiscal Policy

Monetary policy is as above. Fiscal policy remains balanced-budget, the government's budget being

$$\sum_{k=1}^{K} \tau_k = 0,$$

with τ_k being positive for the young and middle-aged and negative for the old.

6.5 Equilibrium and market clearing

In equilibrium, the labor market clears if

$$y_t \cdot \Delta_t = \sum_{k=1}^{K} z_k h_{k,t}.$$

The goods market clears if

$$y_t + \sum_{k=1}^{K} \omega_k = \sum_{k=1}^{K} c_{k,t}.$$

The bond market clears if

$$\sum_{k=1}^{k} b_{k,t} = 0.$$

Profits are given by

$$\Gamma_t = y_t - w_t \sum_{k=1}^{K} z_k h_{k,t}.$$

Price dispersion evolves as

$$\Delta_t = \alpha \Delta_{t-1} (1/\Pi_t)^(- \epsilon) + (1-\alpha)(p_t^r)^{-\epsilon}.$$

With the relative price evolving as

$$1 = \alpha (1/\Pi_t)^{1-\epsilon} + (1-\alpha)(p_t^r)^{1-\epsilon}.$$

7 Conclusions

To be completed

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A Equilibrium conditions – nonlinear

This appendix summarizes the equilibrium conditions in the three-period OLG model. We mark $u_c(\cdot, \cdot; \cdot)$ the partial derivative of period utility with respect to consumption and by $u_h(\cdot, \cdot; \cdot)$ the partial derivative with respect to hours worked.

A.1 Young households

The consumption Euler equation of the young is

$$u_c(c_t^y, h_t^y; \xi^y) = \beta E_t \left\{ u_c(c_{t+1}^m, h_{t+1}^m; \xi^m) \frac{R_t}{\Pi_{t+1}} \right\} + \gamma_t^y, \tag{21}$$

where γ_t^y is the Lagrange multiplier on the borrowing constraint of the young, with $\gamma_t^y \geq 0$ and complementary slackness condition

$$\gamma_t^y \cdot (d_t^y - \overline{d}_t) = 0.$$

The borrowing constraint is

$$d_t^y \leq \overline{d}_t.$$

The labor-supply first-order condition is

$$w_t = -u_h(c_t^y, h_t^y; \xi^y) / u_c(c_t^y, h_t^y; \xi^y). \tag{22}$$

The budget constraint is

$$c_t^y = d_t^y + \omega^y - \tau_t^y + w_t h_t^y.$$

A.2 Middle-aged households

The consumption Euler equation of the middle-aged is

$$u_c(c_t^m, h_t^m; \xi^m) = \beta E_t \left\{ u_c(c_{t+1}^o, h_{t+1}^o; \xi^o) \frac{R_t}{\Pi_{t+1}} \right\}. \tag{23}$$

The labor-supply first-order condition is

$$w_t = -u_h(c_t^m, h_t^m; \xi^m) / u_c(c_t^m, h_t^m; \xi^m). \tag{24}$$

The budget constraint is

$$c_t^m + b_t^m + d_t^y R_{t-1} / \Pi_t = \omega^y + w_t h_t^m + \Gamma_t,$$

A.3 Old households

Old households consume their wealth in its entirety, their budget constraint being

$$c_t^o = \omega^o - \tau_t^o + w_t h_t^o + b_{t-1}^m R_{t-1} / \Pi_t. \tag{25}$$

$$w_t = -u_h(c_t^o, h_t^o; \xi^o)/u_c(c_t^o, h_t^o; \xi^o). \tag{26}$$

A.4 Firms

All firms face the same demand and produce the same

$$y_t(j) = y_t = zh_t, \forall j \in [0,1],$$

Imposing symmetry on the price setting decisions, we have the Phillips curve

$$\Pi_t(\Pi_t - \overline{\Pi}) = \beta E_t \left\{ \Pi_{t+1}(\Pi_{t+1} - \overline{\Pi}) \right\} + \frac{\epsilon}{\phi} \left[\frac{w_t}{z_t} - \frac{\epsilon - 1}{\epsilon} \right]. \tag{27}$$

Since in equilibrium all intermediate goods producers set the same price, all will face the same demand for their good. In equilibrium, therefore, aggregate profits are given by

$$\Gamma_t = y_t \left[1 - \frac{w_t}{z_t} - \frac{\phi_p}{2} \left(\Pi_t - \overline{\Pi} \right)^2 - \frac{\phi_p}{2} \beta E_t \left(\Pi_{t+1} - \overline{\Pi} \right)^2 \right].$$

A.5 Market clearing

Labor market clearing requires

$$h_t^y + h_t^m + h_t^o = h_t.$$

The market for final goods clears if means

$$y_t + (\omega^y + \omega^m + \omega^o) = c_t^y + c_t^m + c_t^o + \frac{\phi_p}{2} y_t \left[\left(\Pi_t - \overline{\Pi} \right)^2 + \beta E_t \left(\Pi_{t+1} - \overline{\Pi} \right)^2 \right].$$

Bond-market clearing requires

$$b_t^m = d_t^y.$$

B Steady state

[To be added]

C Linearized 3-period economy

In the following, we will look at dynamics around the deterministic state. In terms of notation, all variables without a time subscript henceforth mark the steady state. For example, x would mark the steady state of a variable x_t. Let $\widehat{x}_t : (x_t - x)/x$ mark percent deviations of variable x_t from its steady state. Let $\widetilde{x}_t := x_t - x$ mark deviations in levels of x_t from its steady state. For the linearization we shall assume that the borrowing constraint on the young either is binding always, or it never binds.

C.1 Young households

The household budget constraint for the young is

$$\widetilde{c}_t^y = \widetilde{d}_t^y + w\widetilde{h}_t^y + h^y w \cdot \widehat{w}_t - \widetilde{\tau}_t{}^y. \tag{28}$$

The labor supply first-order condition is

$$\widehat{w}_t = \left[\frac{u_{hh}^y}{u_h^y} - \frac{u_{ch}^y}{u_c^y}\right]\tilde{h}_t^y + \left[\frac{u_{ch}^y}{u_h^y} - \frac{u_{cc}^y}{u_c^y}\right]\tilde{c}_t^y, \tag{29}$$

For the young, if the borrowing constraint never binds, the consumption Euler equation implies

$$\tilde{c}_t^y + \frac{u_{ch}^y}{u_{cc}^y}\tilde{h}_t^y = \beta\frac{R}{\Pi}E_t\left[\frac{u_{cc}^m}{u_{cc}^y}\tilde{c}_{t+1}^m + \frac{u_{ch}^m}{u_{cc}^y}\tilde{h}_{t+1}^m\right] - \left(-\frac{u_c^y}{u_{cc}^y}\right)E_t\{\widehat{R}_t - \widehat{\Pi}_{t+1}\}. \tag{30}$$

Here $u_{ch}^y := \partial^2 u(c^y, h^y; \xi^y)/(\partial c^y \partial h^y)$, and analogously for the other terms and (below) for the other generations.
Instead, if the borrowing constraint always binds, the Euler equation does not hold with equality, and

$$\tilde{d}_t^y = \tilde{\bar{d}}_t^y \tag{31}$$

C.2 Middle-aged households

For the middle-aged, the consumption Euler equation is

$$\tilde{c}_t^m + \frac{u_{ch}^m}{u_{cc}^m}\tilde{h}_t = \beta\frac{R}{\Pi}E_t\left[\frac{u_{cc}^o}{u_{cc}^m}\tilde{c}_{t+1}^o + \frac{u_{ch}^o}{u_{cc}^m}\tilde{h}_{t+1}^o\right] - \left(-\frac{u_c^m}{u_{cc}^m}\right)E_t\{\widehat{R}_t - \widehat{\Pi}_{t+1}\} \tag{32}$$

The labor supply first-order condition is

$$\widehat{w}_t = \left[\frac{u_{hh}^m}{u_h^m} - \frac{u_{ch}^m}{u_c^m}\right]\tilde{h}_t^m + \left[\frac{u_{ch}^m}{u_h^m} - \frac{u_{cc}^m}{u_c^m}\right]\tilde{c}_t^m. \tag{33}$$

And the budget constraint for the middle-aged is

$$\widetilde{c}_t^m + \widetilde{b}_t^m + \frac{R}{\Pi}\widetilde{d}_{t-1}^y + d^y\frac{R}{\Pi}[\widehat{R}_{t-1} - \widehat{\Pi}_t] = +w\tilde{h}_t^m + h^m w \cdot \hat{w}_t - \tilde{\tau}_t{}^m + \tilde{\Gamma}_t \tag{34}$$

C.3 Old households

For the old, he labor supply first-order condition is

$$\widehat{w}_t = \left[\frac{u_{hh}^o}{u_h^o} - \frac{u_{ch}^o}{u_c^o}\right]\tilde{h}_t^o + \left[\frac{u_{ch}^o}{u_h^o} - \frac{u_{cc}^o}{u_c^o}\right]\tilde{c}_t^o. \tag{35}$$

And the budget constraint for the old is

$$\tilde{c}_t^o = \frac{R}{\Pi}\tilde{b}_{t-1}^m + b^m\frac{R}{\Pi}[\widehat{R}_{t-1} - \widehat{\Pi}_t] + w\tilde{h}_t^o + h^o w \cdot \hat{w}_t - \tilde{\tau}_t{}^o. \tag{36}$$

C.4 Firms

The New Keynesian Phillips curve can be linearized to yield

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} + \frac{\theta - 1}{\phi_p} \widehat{w}_t. \tag{37}$$

Labor-market clearing and the production function for final goods imply

$$\tilde{y}_t = z\tilde{h}_t. \tag{38}$$

Up to a first-order approximation profits are given by

$$\widetilde{\Gamma}_t = \tilde{y}_t - wh\widehat{w}_t - w\tilde{h}_t.$$

C.4.1 Government

The interest rate rule is

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + (1 - \phi_R)\phi_\Pi \widehat{\Pi}_t + e_t. \tag{39}$$

The fiscal balance implies

$$\tilde{\tau}_t^y + \tilde{\tau}_t^m + \tilde{\tau}_t^o = 0.$$

C.4.2 Market clearing

Goods market clearing means

$$\tilde{y}_t = \tilde{c}_t^y + \tilde{c}_t^m + \tilde{c}_t^o. \tag{40}$$

Asset-market clearing means

$$\tilde{d}_t^y = \tilde{b}_t^m \tag{41}$$

Labor-market clearing means

$$\tilde{h}_t = \tilde{h}_t^y + \tilde{h}_t^m + \tilde{h}_t^o. \tag{42}$$

C.5 Euler equations and labor-supply FOC spelled out

For completeness, we spell out the Euler equations and labor-supply first-order conditions for the case of additively separable and GHH preferences. We spell out the conditions for the middle-aged; analogous conditions hold for the other age groups.

C.5.1 CRRA preferences

For CRRA preferences,

$$u(c^m, h^m; \xi^m) = \frac{(c^m - \overline{c}^m)^{1-\sigma} - 1}{1 - \sigma} - \psi^m \frac{(h^m)^{1+\nu^m}}{1 + \nu^m},$$

so that the linearized consumption Euler equation simplifies to

$$\tilde{c}_t^m = \frac{c^m - \overline{c}^m}{c^o - \overline{c}^o} E_t \tilde{c}_{t+1}^o - \frac{1}{\sigma}(c^m - \overline{c}^m) E_t\{\widehat{R}_t - E_t \widehat{\Pi}_{t+1}\}, \tag{43}$$

And the labor supply FOC is given by

$$\widehat{w}_t = \frac{\nu^m}{h^m}\tilde{h}_t^m + \frac{\sigma}{c^m - \overline{c}^m}\tilde{c}_t^m. \tag{44}$$

C.5.2 GHH preferences

For GHH preferences

$$u(c^m, h^m; \xi^m) = \frac{(c^m - \overline{c}^m - \psi^m \frac{h^{1+\nu^m}}{1+\nu^m})^{1-\sigma} - 1}{1-\sigma},$$

so that the linearized consumption Euler equation simplifies to

$$\tilde{c}_t^m - \psi^m (h^m)^{\nu^m} \tilde{h}_t^m = \frac{c^m - c_h^m}{c^o - c_h^o} E_t \left\{ \tilde{c}_{t+1}^o - \psi^m (h^o)^{\nu^m} \tilde{h}_{t+1}^o \right\} - \frac{1}{\sigma}\left(c^m - c_h^m\right) E_t\{\widehat{R}_t - \widehat{\Pi}_{t+1}\}, \tag{45}$$

where for better readability, we have defined term $c_h^m := \overline{c}^m + \psi^m \frac{(h^m)^{1+\nu^m}}{1+\nu^m}$, and analogously for term c_h^o. The labor supply first order condition is given by

$$\widehat{w}_t = \frac{\nu^m}{h^m}\tilde{h}_t^m. \tag{46}$$

D Proofs

This appendix collects the proofs to the propositions.

D.1 Proof of Proposition 2

Due to perfectly elastic labor supply, there is no feedback from the IS equation (9) to the Phillips curve (14). Rather output is determined residually from the monetary response and inflation.

For checking determinacy, the equations that are relevant, therefore, are the NKPC and the systematic part of Taylor rule (39). Combining these:

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} - \frac{\theta - 1}{\phi_p}\sigma A[\phi_\Pi \widehat{\Pi}_t - E_t \widehat{\Pi}_{t+1}].$$

Rearranging to bring into Blanchard-Kahn form:

$$E_t[\hat{\Pi}_{t+1}] = \frac{1 + \kappa\sigma A \phi_\Pi}{\beta + \kappa\sigma A}\hat{\Pi}_t.$$

There will be determinacy if and only if

$$\left| \frac{1 + \kappa\sigma A \phi_\Pi}{\beta + \kappa\sigma A} \right| > 1. \tag{47}$$

<u>Case A>0</u>

If $A > 0$, both denominator and numerator are positive (recall, we look at $\phi_\Pi \geq 0$). Determinacy requires

$$1 + \kappa\sigma A\phi_\Pi > \beta + \kappa\sigma A,$$

or

$$\phi_\Pi > 1 - \frac{1-\beta}{\kappa\sigma A}.$$

This proves item a) of the proposition.

Case $A < 0$, $\beta + \kappa\sigma A > 0$

In that case (47) is satisfied if

$$|1 + \kappa\sigma A\phi_\Pi| > \beta + \kappa\sigma A. \tag{48}$$

If $1 + \kappa\sigma A\phi_\Pi > 0$, (48) gives

$$1 + \kappa\sigma A\phi_\Pi > \beta + \kappa\sigma A,$$

or (bearing in mind that for this case $A < 0$),

$$\phi_\Pi < 1 + \frac{\beta - 1}{\kappa\sigma A}$$

Rearranging the last term, this gives the lower cutoff in item b) of the proposition.
If $1 + \kappa\sigma A\phi_\Pi < 0$, (48) gives

$$-(1 + \kappa\sigma A\phi_\Pi) > \beta + \kappa\sigma A,$$

Rearranging, we get

$$1 + \kappa\sigma A\phi_\Pi < -\beta - \kappa\sigma A,$$

or (bearing in mind again that $A < 0$ for this case)

$$\phi_\Pi > -1 + \frac{1+\beta}{-\kappa\sigma A}.$$

Rearranging the last term, this gives the upper cutoff in item b) of the proposition. The claim that the lower threshold is larger than unity is verified easily (bearing in mind the conditions $A < 0$ $\beta + \kappa\sigma A > 0$).

Case $A < 0$, $\beta + \kappa\sigma A < 0$

In that case (47) is satisfied if either $1 + \kappa\sigma A\phi_\Pi > 0$

$$|1 + \kappa\sigma A\phi_\Pi| < \beta + \kappa\sigma A. \tag{49}$$

If $1 + \kappa\sigma A\phi_\Pi > 0$, (49) gives

$$1 + \kappa\sigma A\phi_\Pi < \beta + \kappa\sigma A,$$

or (bearing in mind that for this case $A < 0$),

$$\phi_\Pi > 1 + \frac{\beta - 1}{\kappa\sigma A}$$

Rearranging the last term, this gives the upper cutoff in item c) of the proposition. If $1 +$

$\kappa\sigma A\phi_\Pi < 0$, (49) gives

$$-(1+\kappa\sigma A\phi_\Pi) < \beta + \kappa\sigma A,$$

Rearranging, we get

$$1+\kappa\sigma A\phi_\Pi > -\beta - \kappa\sigma A,$$

or (bearing in mind again that $A < 0$ for this case)

$$\phi_\Pi < -1 + \frac{1+\beta}{-\kappa\sigma A}.$$

Rearranging the last term, this gives the lower cutoff in item c) of the proposition. The claim that the upper threshold is larger than unity is verified easily (bearing in mind the conditions $A < 0$ $\beta + \kappa\sigma A < 0$). qed.

D.2 Proof of Proposition 3

The relevant equations are the New Keynesian Phillips curve

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} - \kappa\sigma A[\widehat{R}_t - E_t\widehat{\Pi}_{t+1}],$$

where $\kappa := \frac{\theta-1}{\phi_p}$. And IS equation (9), which gives

$$\widetilde{y}_t \;\; = \;\; -\left(c^m - c^m_h\right) A \left\{\widehat{R}_t - E_t\widehat{\Pi}_{t+1}\right\} + b^m \tfrac{R}{\Pi}\left[\widehat{R}_{t-1} - \widehat{\Pi}_t\right].$$

and Taylor rule (39).

There is no feedback from the Phillips curve to the IS equation, so that the Phillips curve and Taylor rule combined, namely,

$$\widehat{\Pi}_t = \beta E_t \widehat{\Pi}_{t+1} - \frac{\theta-1}{\phi_p}\sigma A[\phi_\Pi \widehat{\Pi}_t + e_t - E_t\widehat{\Pi}_{t+1}],$$

can be solved for inflation irrespective of the IS equation. Using the method of undetermined coefficients with guess $\widehat{\Pi}_t = a e_t$, we get (assuming parameters are such that the ratio is well-defined)

$$a = -\frac{\kappa\sigma A}{1-\beta\rho_e + \kappa\sigma A(\phi_\Pi - \rho)}. \tag{50}$$

Using this in the IS equation above we have that

$$\widetilde{y}_t = -\left(c^m - c^m_h\right) A \left[\phi_\Pi a e_t + e_t - a\rho_e e_t\right] + b^m \frac{R}{\Pi}\left[\left(\phi_\Pi a e_{t-1} + e_{t-1}\right) - a e_t\right].$$

With $b^m = \overline{d}$ (by bond-market clearing) the law of motion for output given in the proposition follows.

Item a)

A "conventional" response of inflation obtains if $a < 0$ (so that inflation falls if the nominal rate is raised. If $A > 0$ and $\phi_\Pi > 1$, it is easy to see that both the numerator and the denominator in (50) are positive, so that $a < 0$ unambiguously.

Item b)

If $A < 0$ than the numerator in (50) is unambiguously negative. A conventional response of inflation to a monetary shocks then requires that the denominator be negative as well.
Suppose $\beta + \kappa\sigma A > 0$. The relevant conditions for determinacy are given by case b) of Proposition 2. Denote by $\underline{\phi}_\Pi := 1 + \frac{1-\beta}{-\kappa\sigma A}$ the lower threshold for determinacy given in that proposition and by $\overline{\phi}_\Pi = 1 + \frac{1-\beta}{-\kappa\sigma A} + \frac{2(\beta+\kappa\sigma A)}{-\kappa\sigma A}$ the upper threshold. Note that we can, alternatively, write this as $\overline{\phi}_\Pi = -1 + \frac{1+\beta}{-\kappa\sigma A}$.
Suppose that $\phi_\Pi > \overline{\phi}_\Pi$. Since $A < 0$ by assumption, for any $\phi_\Pi > \overline{\phi}_\Pi$

$$\begin{aligned}
1 - \beta\rho_e + \kappa\sigma A(\phi_\Pi - \rho_e) &< 1 - \beta\rho_e + \kappa\sigma A(\overline{\phi_\Pi} - \rho) \\
&= 1 - \beta\rho_e + \kappa\sigma A(-1 - \rho_e + \tfrac{1+\beta}{-\kappa\sigma A}) \\
&= 1 - \beta\rho_e + \kappa\sigma A(-1 - \rho_e) - 1 - \beta \\
&= -\beta(1 + \rho_e) - \kappa\sigma A(1 + \rho_e) \\
&= -(1 + \rho_e)[\beta + \kappa\sigma A] \\
&< 0.
\end{aligned}$$

Where the last step follows from the conditioning assumption $\beta + \kappa\sigma A > 0$ in item b). This proves that any response to inflation stronger than the upper limit will yield a conventional response of inflation (inflation will fall with a monetary tightening).
Suppose that $\phi_\Pi < \underline{\phi}_\Pi$. Then we have that

$$\begin{aligned}
1 - \beta\rho_e + \kappa\sigma A(\phi_\Pi - \rho_e) &> 1 - \beta\rho_e + \kappa\sigma A(\underline{\phi_\Pi} - \rho_e) \\
&= 1 - \beta\rho_e + \kappa\sigma A(1 - \rho_e + \tfrac{1-\beta}{-\kappa\sigma A}) \\
&= 1 - \beta\rho_e + \kappa\sigma A(1 - \rho_e) - 1 + \beta \\
&= \beta(1 - \rho_e) + \kappa\sigma A(1 - \rho_e) \\
&= (1 + \rho_e)[\beta + \kappa\sigma A]. \\
&> 0.
\end{aligned}$$

Where the last step follows from the conditioning assumption $\beta + \kappa\sigma A > 0$ in item b). This proves that any response to inflation weaker than the lower limit imn item b) implies $a > 0$, and therefore an conventional response of inflation (inflation will rise with a monetary tightening).
Item c)

The proof for item c) proceeds as with the proof for item b). The only differences are the bounds and the conditioning assumption $\beta + \kappa\sigma A < 0$.

Item d)

Under determinacy, the impact response of output to a unit monetary shock ($\epsilon_t^m = 1$) is

$$\tilde{y}_0 = \left[-(c^m - c_h^m)A(a\phi_\Pi + 1) - \overline{d}\frac{R}{\Pi}a \right].$$

For $\rho = 0$, $a = -\frac{\kappa\sigma A}{1+\kappa\sigma A\phi_\Pi}$. A conventional response of output on impact is $\tilde{y}_0 < 0$. So that there is a conventional response of output on impact if $-(c^m - c_h^m)A(a\phi_\Pi + 1) - \overline{d}\frac{R}{\Pi}a < 0$.

Rewriting, using the definition for a:

$$-\left(c^m - c_h^m\right)A\frac{1}{1+\kappa\sigma A\phi_\Pi} + \overline{d}\frac{R}{\Pi}\frac{\kappa\sigma A}{1+\kappa\sigma A\phi_\Pi} < 0. \tag{51}$$

For $A > 0$, (51) reduces to

$$-(c^m - c_h^m) + \overline{d}\frac{R}{\Pi}\kappa\sigma < 0,$$

which is true as long as

$$c^m - c_h^m > \overline{d}\frac{R}{\Pi}\kappa\sigma \tag{52}$$

For $A < 0$, (51) reduces to the same condition (52) only if $1 + \kappa\sigma A\phi_\Pi < 0$, which then means $\phi_\Pi > \frac{1}{\kappa\sigma(-A)}$. Else, if (52) holds, $\tilde{y}_0 > 0$. qed.

D.3 Proof of Proposition 4

Under constant prices, equation (15) reads as

$$\tilde{b}_t^m = \frac{c^y - \overline{c}^y}{c^o - \overline{c}^o}E_t\left\{\tilde{b}_{t+1}^m\right\} - (c^y - \overline{c}^y)\left[\frac{1}{\sigma} - \frac{b^m}{c^o - \overline{c}^o}\right]E_t\{\widehat{R}_{t+1}\} - \frac{1}{\sigma}(c^y - \overline{c}^y)\widehat{R}_t.$$

In the steady-state $c^i - c_h^i = c^j - c_h^j$ for any $i, j \in \{y, m, o\}$. In addition, since only the middle-aged supply labor, $c^y - c_h^y = c^y - \overline{c}^y$ and $c^o - c_h^o = c^o - \overline{c}^o$. This means that the above further simplifies to

$$\tilde{b}_t^m = E_t\{\tilde{b}_{t+1}^m\} - (c^y - \overline{c}^y)\left[\frac{1}{\sigma} - \frac{b^m}{c^o - \overline{c}^o}\right]E_t\{\widehat{R}_{t+1}\} - \frac{1}{\sigma}(c^y - \overline{c}^y)\widehat{R}_t.$$

For $\widehat{R}_t = \phi_R^t\widehat{R}_0$, equation (16) follows from iterated substitution.
Next, derive equations (17) and (18). Use goods-market clearing condition (40). Substitute for young-age consumption from the budget constraint for the young (28) (which gives $\tilde{c}_t^y = \tilde{b}_t$). Substitute for old-age consumption, from the budget constraint for the old (36) (which gives $\tilde{c}_t^o = \tilde{b}_{t-1} + \tilde{b}_t^m(\widehat{R}_{t-1} - \widehat{\Pi}_t)$). Substitute for middle-aged consumption from Euler (53). Having substituted $\tilde{h}_t^m$ using the labor supply FOC and the production function, and the budget constraint of the old, we have that

$$\tilde{c}_t^m = \frac{\theta-1}{\theta}\mathbb{I}(GHH)\tilde{y}_t + \tilde{b}_t^m + b^m\widehat{R}_t - \frac{1}{\sigma}\left(c^m - c_h^m\right)\widehat{R}_t. \tag{53}$$

Adding up and using the goods market clearing condition, we have that

$$\tilde{y}_t = \tilde{b}_t^m + \frac{\theta-1}{\theta}\mathbb{I}(GHH)\tilde{y}_t + \tilde{b}_t^m + b^m\widehat{R}_t - \frac{1}{\sigma}\left(c^m - c_h^m\right)\widehat{R}_t + \tilde{b}_{t-1}^m + b^m\widehat{R}_{t-1}.$$

Or, using the steady-state relations,

$$\tilde{y}_t\cdot\left[1 - \frac{\theta-1}{\theta}\mathbb{I}(GHH)\right] = 2\tilde{b}_t^m - (c^y - \overline{c}^y)\left[\frac{1}{\sigma} - \frac{b^m}{c^o - \overline{c}^o}\right]\widehat{R}_t + \tilde{b}_{t-1}^m + b^m\widehat{R}_{t-1}.$$

Derive expressions for the terms dated t and $t-1$ separately. Using (16), we have that

$$2\tilde{b}_t^m - (c^y - \overline{c}^y)\left[\frac{1}{\sigma} - \frac{b^m}{c^o - \overline{c}^o}\right]\widehat{R}_t = -\frac{c^y - \overline{c}^y}{1-\phi_R}\left[\frac{1}{\sigma}(3+\phi_R) - \frac{b^m}{c^y - \overline{c}^y}(1+\phi_R)\right]\widehat{R}_t$$

This gives (17).
Next, observe that from (16)

$$\tilde{b}_{t-1}^m + b^m \widehat{R}_{t-1} = -\frac{1}{1-\phi}(c^y - \overline{c}^y)\left[\frac{1}{\sigma}(1+\phi) - \frac{b_m}{c^y - \overline{c}^y}\right]\widehat{R}_{t-1}$$

Combining both yields

$$\begin{aligned}\tilde{y}_t \cdot \left[1 - \tfrac{\theta-1}{\theta}\mathbb{I}(GHH)\right] \quad &= \quad -\tfrac{c^y - \overline{c}^y}{1-\phi_R}\left[\tfrac{1}{\sigma}(3+\phi_R) - \tfrac{b^m}{c^y - \overline{c}^y}(1+\phi_R)\right]\widehat{R}_t \\ &\quad -\tfrac{c^y - \overline{c}^y}{1-\phi_R}\left[\tfrac{1}{\sigma}(1+\phi_R) - \tfrac{b_m}{c^y - \overline{c}^y}\right]\widehat{R}_{t-1}\end{aligned}$$

Or, for $t = 1, 2, \ldots$

$$\tilde{y}_t \cdot \left[1 - \tfrac{\theta-1}{\theta}\mathbb{I}(GHH)\right] \quad = \quad -\tfrac{c^y - \overline{c}^y}{1-\phi_R}\left[\tfrac{1}{\sigma}(1+4\phi_R+\phi_R^2) - \tfrac{b^m}{c^y - \overline{c}^y}(1+\phi_R+\phi_R^2)\right]\widehat{R}_{t-1}.$$

This is (18).

E Data

The countries for which we have data on the generosity of the pension system and the response of consumption to monetary policy are: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Portugal, Spain, and United States. As mentioned in the main text, data for the retirement system comes from the OECD's "Pensions at a Glance" database. Our analysis is based on the net replacement rate, which equals the ratio of the pension entitlement to lifetime average earnings, both calculated after taxes.
We survey the literature on the response of consumption to monetary policy. Then we rank the countries from those with the weakest to the strongest response, which corresponds to our index of monetary policy effectiveness. The data for Austria, Belgium, France, Finland, Germany, Ireland, Italy, Netherlands, Portugal, and Spain comes from Corsetti et al. (2018). For the other countries, the sources are: Canada – Ji (2017); Denmark – Pedersen and Ravn (2013); U.S. – Christiano et al. (2005)
As a separate check, rather than relying on other studies, we compute the response of consumption in Australia, Canada, US, and some EU countries to a tightening in monetary policy. For the EU countries, the shock is estimated using a VAR with four lags, Euro Area wide data (Fagan et al. (2001)) on GDP growth, inflation, and short-term interest rates for the period 1995.Q1 - 2011.Q4, and a Cholesky factorization.[8] With the structural shocks in hand, we use data for Austria, Belgium, Denmark, France, Germany, Italy, Portugal, and Spain, and compute the IRF of consumption to the a contractionary monetary shock. For the non-EU countries, the VAR has consumption growth, inflation, and short-term interest rates for the

[8]We stop the sample in 2011 to avoid the quantitative easing episode.

period 1984.Q1 - 2008.Q1. The correlation between the generosity index and monetary policy is -0.57.