

Simple monetary policy rules and exchange rate uncertainty

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Abstract

We analyze the performance and robustness of some common simple rules for monetary policy in a new-Keynesian open economy model under different assumptions about the determination of the exchange rate. Adding the exchange rate to an optimized Taylor rule gives only slight improvements in terms of the volatility of important variables in the economy. Furthermore, although the rules including the exchange rate (and in particular, the real exchange rate) perform slightly better than the Taylor rule on average, they sometimes lead to very poor outcomes. Thus, the Taylor rule seems more robust to model uncertainty in the open economy.

Keywords: Open economy; Exchange rate determination; Model uncertainty; Robustness of policy rules

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1 Introduction

In open economies, the exchange rate is an important element of the transmission of monetary policy. As stressed by Svensson (2000), the exchange rate allows for several channels in addition to the standard aggregate demand and expectations channels in closed economies: (*i*) the real exchange rate affects the relative price between domestic and foreign goods, and thus contributes to the aggregate demand channel; (*ii*) the exchange rate affects consumer prices directly via the domestic currency price of imports; and (*iii*) the exchange rate affects the price of imported intermediate goods, and thus the pricing decisions of domestic firms. It therefore seems natural to include the exchange rate as an indicator for monetary policy. The recent years have also seen a surge in research concerning monetary policy in open economies, and, in particular, the performance of simple policy rules.¹

However, movements in the exchange rate are not very well understood in practice. In particular, the parity conditions typically used in theoretical analyses—uncovered interest rate parity (UIP) and purchasing power parity (PPP)—do not find much support in empirical studies.² Furthermore, the equilibrium real exchange rate is not easily observed by central banks. Given the high degree of uncertainty regarding exchange rate determination, a challenge for monetary policymakers is to design policy strategies that are reasonably robust to different specifications of the exchange rate model.³

The main objective of this paper, therefore, is to study the role of the exchange rate as an indicator for monetary policy when there is uncertainty about the exchange rate model. This is done in three steps: we first analyze a “baseline model” to see whether including the exchange rate in an optimized Taylor rule leads to any improvement in terms of a standard intertemporal loss function for the central bank. Second, we perform the same analysis in a variety of model configurations, to see how sensitive the results are to the exact specification of the model. Third, we examine the robustness of the different policy rules to model uncertainty by analyzing the outcome when using the optimized policy rules from the baseline model in the alternative model specifications.

¹A non-exhaustive list includes Ball (1999), Svensson (2000), Taylor (1999), McCallum and Nelson (1999a), Obstfeld and Rogoff (2000), Leitemo (1999), Batini et al. (2000), Dennis (2000), Corsetti and Pesenti (2000), Monacelli (1999) and Ghironi (1999).

²See, e.g., the survey by Froot and Thaler (1990). On the other hand, McCallum (1994) and Chinn and Meredith (2000) argue that the empirical rejection of UIP is due to a failure to properly account for other relationships in the economy, such as the behavior of monetary policy.

³See McCallum (1988, 1999) about the robustness of policy rules.

Our representation of the economy allows for several modifications of the model determining the exchange rate and its influence on the domestic economy. These modifications are introduced in three broad categories. First, we allow for longer-term departures from PPP than what is due to nominal rigidities by considering varying degrees of exchange rate pass-through onto import prices and thus to CPI inflation. The departure from PPP is motivated by the overwhelming evidence of pricing-to-market and incomplete exchange rate pass-through.⁴

Second, we study different departures from the rational expectations UIP condition, in the form of non-rational exchange rate expectations and varying behavior of the risk premium on foreign exchange. The first departure from rational expectations UIP is also motivated by empirical evidence. Although most evidence from surveys of expectation formation in the foreign exchange market point to expectations not being rationally formed (e.g., Frankel and Froot, 1987), rational expectations remain the workhorse assumption about expectation formation in structural and theoretical policy models. We extend such analyses by considering exchange rate expectations being partly formed according to adaptive, equilibrium (or regressive), and distributed-lag schemes. The second departure from UIP considers the behavior of the foreign exchange risk premium, by allowing for varying degrees of risk premium persistence.

Third, we analyze the consequences of uncertainty concerning the equilibrium real exchange rate. Measuring the equilibrium real exchange rate is a daunting task for policymakers, similar to measuring the equilibrium real interest rate or the natural level of output. For this reason, rules that respond to the deviation of the real exchange rate from its equilibrium (or steady-state) level may not be very useful in practice. In some versions of the model, we try to capture such uncertainty by forcing the central bank to respond to a noisy measure of the real exchange rate.

The analysis is performed in an empirically oriented new-Keynesian open economy model, incorporating inertia and forward-looking behavior in the determination of both output and inflation. The economy is assumed to be sufficiently small so that the rest of the world can be taken as exogenous. In the baseline model, capital mobility is near-perfect as the risk premium is treated as an exogenous autoregressive process. Capital mobility constitutes the linkage between the domestic and foreign economies, hence the foreign interest rate affects the domestic economy via the price of the domestic currency. The analysis will concentrate on the performance

⁴See, e.g., Naug and Nymoen (1996) or Alexius and Vredin (1999) for empirical evidence, or Goldberg and Knetter (1997) for a survey.

of relatively simple monetary policy rules, such as the Taylor (1993) rule, as a first attempt in understanding the mechanisms at play.⁵

Our first set of results indicates that the gains from extending an optimized Taylor rule to include a measure of the exchange rate (the nominal or real rate of depreciation or the level of the real exchange rate) are small in most specifications of the model. In fact, the outcome from an optimized Taylor rule is often very close to the globally optimal outcome under commitment. Thus, the output gap and annual CPI inflation seem almost sufficient as indicators for monetary policy also in the open economy.

Interestingly, the optimized coefficients on the exchange rate variables are often negative, implying that a nominal or real depreciation is countered by easing monetary policy. This is due to a conflict between the direct effects of the exchange rate on CPI inflation and the indirect effects on aggregate demand and domestic inflation: a positive response to the exchange rate decreases the volatility of CPI inflation, but increases the volatility of the interest rate and output, since tighter policy leads to more exchange rate depreciation which leads to even tighter policy, etc. In most of our specifications this effect is reduced by responding negatively to the rate of depreciation. This mechanism holds true also for intermediate degrees of exchange rate pass-through and most degrees of rationality in the foreign exchange market. When the risk premium is very persistent or expectations are very non-rational, however, the optimal policy coefficient on the exchange rate is positive.

The fact that the optimal response to the exchange rate differs across model specifications not only in magnitude but also in sign makes the exchange rate rules more sensitive to model uncertainty: if the central bank optimizes its rule in the wrong model, the outcome using the exchange rate rules is sometimes very poor. In this sense, the Taylor rule is more robust to model uncertainty.

The remainder of the paper is outlined as follows. The next Section presents the model framework, and discusses the monetary transmission mechanism, as well as the policy rules and objectives of the monetary authorities. Section 3 briefly discusses our methodology and the calibration of the model. Section 4 analyzes the performance of policy rules in different specifications of the model and the robustness of policy rules to model uncertainty. Finally, Section 5 contain some concluding

⁵Future work could instead concentrate on forecast rules, which may be a more accurate description of the actual strategies of central banks. See, e.g., Batini and Nelson (2000), Batini et al. (2000) and Leitimo (2000a,b).

remarks, and Appendices A–C contain some technical details and additional figures.

2 A model of a small open economy

The model we use is a small-scale macro model, similar to those of Batini and Haldane (1999), Svensson (2000), Batini and Nelson (2000) and Leitemo (2000a). It can be viewed as an open-economy version of the models developed by Rotemberg and Woodford (1997), McCallum and Nelson (1999b) and others, although it is designed primarily to match the data and not to provide solid microfoundations. The model is quarterly, all variables are measured as (log) deviations from long-run averages, and interest rates are measured as annualized rates while inflation rates are measured on a quarterly basis.

2.1 The monetary transmission mechanism

In the model monetary policy affects the open economy through several transmission channels. Policy influences nominal variables, but due to nominal rigidities, it also has important temporary effect upon real variables. First, the central bank is able to influence the real interest rate by setting the nominal interest rate. Monetary policy works through this “interest rate channel” by affecting consumption demand through the familiar intertemporal substitution effect. As the interest rate is increased, the trade-off between consumption today and tomorrow is affected, making consumption today in terms of consumption tomorrow more costly, leading to a reduction in current domestic demand. Moreover, the interest rate affects the user cost of capital, influencing investment demand.

Second, and specific to the open economy, monetary policy influences the price of domestic goods in terms of foreign goods by affecting the price of domestic currency. The exchange rate affects the open economy in different ways, making it convenient to distinguish between a direct channel and an indirect channel. The “direct exchange rate channel” affects the price of imported goods in terms of domestic currency units, which influences the consumer price level. The “indirect exchange rate channel” influences demand by affecting the price of domestic goods in terms of foreign goods. Furthermore, the exchange rate affects producer real wages in the tradable sector that influences production decisions, and it may also influence wage setting: a depreciated exchange rate reduces consumer real wages as imported goods become relatively more expensive.

2.2 Output and inflation

Aggregate output is determined in the short term by demand and is forward-looking, but with considerable inertia.⁶ Aggregate demand is influenced through intertemporal substitution effects by the real interest rate and through intratemporal price effects induced by changes in the real exchange rate. Furthermore, output is predetermined one period (cf. Svensson and Woodford, 1999), and there are explicit control lags in the monetary transmission mechanism, so output depends on the previous period's real interest rate, real exchange rate, and foreign output gap. Thus, the output gap is given by

$$y_{t+1} = \beta_y \left[\varphi_y y_{t+2|t} + (1 - \varphi_y) y_t \right] - \beta_r \left(i_t - 4\pi_{t+1|t}^d \right) + \beta_q q_t + \beta_{yf} y_t^f + u_{t+1}^y, \quad (1)$$

where $\beta_y \leq 1$; i_t is the (annualized) quarterly interest rate, set by the central bank; $\pi_t^d \equiv p_t^d - p_{t-1}^d$ is the quarterly rate of domestic inflation; q_t is the real exchange rate (defined in terms of the domestic price level, see below); y_t^f is the foreign output gap; and $0 \leq \varphi_y \leq 1$ measures the degree of forward-looking.⁷ Throughout, the notation $x_{t+1|t}$ denotes $E_t x_{t+1}$, i.e., the rational expectation of the variable x in period $t+1$, given information available in period t . The variable $i_t - 4\pi_{t+1|t}^d$ is thus the quarterly ex-ante real interest rate, in annualized terms. Finally, the disturbance term u_{t+1}^y follows the stationary auto-regressive process

$$u_{t+1}^y = \rho_y u_t^y + \varepsilon_{t+1}^y, \quad (2)$$

where $0 \leq \rho_y < 1$ and ε_{t+1}^y is a white noise shock with variance σ_y^2 .

Domestic inflation follows an expectations-augmented Phillips curve, and so is influenced by the tightness in product and factor markets via aggregate output and the real exchange rate, and is also predetermined one period,⁸

$$\pi_{t+1}^d = \varphi_\pi \pi_{t+2|t}^d + (1 - \varphi_\pi) \pi_t^d + \gamma_y y_{t+1|t} + \gamma_q q_{t+1|t} + u_{t+1}^\pi, \quad (3)$$

⁶Such inertia could come from, e.g., habit formation (Estrella and Fuhrer, 1998, 1999; Fuhrer, 2000), costs of adjustment (Pesaran, 1987; Kennan, 1979; Sargent, 1978), or rule-of-thumb behavior (Amato and Laubach, 2000).

⁷A special case sets $\beta_y = 1$, but the formulation in (1) also allows for a purely backward-looking output gap with a coefficient on lagged output below unity (i.e., $\varphi_y = 0, \beta_y < 0$) as in, e.g., Rudebusch (2000a) or Batini and Haldane (1999).

⁸This is similar to the open-economy Phillips curve specification of Walsh (1999), but with inertia, and can thus be seen as an open-economy application of the wage contracting model of Buiter and Jewitt (1981) and Fuhrer and Moore (1995), along the lines of Batini and Haldane (1999). See also footnote 6.

where $0 \leq \varphi_\pi \leq 1$ measures the degree of forward-looking in pricing/wage setting decisions. Again, the disturbance term u_{t+1}^π follows the stationary process

$$u_{t+1}^\pi = \rho_\pi u_t^\pi + \varepsilon_{t+1}^\pi, \quad (4)$$

where $0 \leq \rho_\pi < 1$ and the shock ε_{t+1}^π is white noise with constant variance σ_π^2 .

Although in most versions of the model the pass-through of exchange rate movements to import prices is instantaneous, some versions will allow for slow exchange rate pass-through, so import prices adjust gradually to movements in foreign prices according to

$$\begin{aligned} p_t^M &= p_{t-1}^M + \kappa (p_t^f + s_t - p_{t-1}^M) \\ &= (1 - \kappa) p_{t-1}^M + \kappa (p_t^f + s_t), \end{aligned} \quad (5)$$

where $\kappa \leq 1$, p_t^f is the foreign price level, and s_t is the nominal exchange rate.⁹ *Imported inflation* then follows

$$\pi_t^M = (1 - \kappa) \pi_{t-1}^M + \kappa (\pi_t^f + \Delta s_t), \quad (6)$$

and *aggregate CPI inflation* is given by

$$\pi_t = (1 - \eta) \pi_t^d + \eta \pi_t^M, \quad (7)$$

where $0 < \eta < 1$ is the weight of imported goods in aggregate consumption.

Foreign output and *inflation* are assumed to follow the stationary autoregressive processes

$$y_{t+1}^f = \rho_{yf} y_t^f + \varepsilon_{t+1}^{yf}, \quad (8)$$

$$\pi_{t+1}^f = \rho_{\pi f} \pi_t^f + \varepsilon_{t+1}^{\pi f}, \quad (9)$$

where $0 \leq \rho_{yf}, \rho_{\pi f} < 1$, and the shocks $\varepsilon_{t+1}^{yf}, \varepsilon_{t+1}^{\pi f}$ are white noise with variances $\sigma_{yf}^2, \sigma_{\pi f}^2$. The *foreign nominal interest rate* follows the simple Taylor-type rule¹⁰

$$i_t^f = g_{\pi f} \pi_t^f + g_{yf} y_t^f. \quad (10)$$

⁹A value of $\kappa = 1$ represents instantaneous pass-through, and $\kappa = 0$ no pass-through. See Adolfson (2000) for a more detailed analysis of incomplete pass-through in a similar model, and, e.g., Naug and Nymoen (1996) for some empirical evidence.

¹⁰If $\rho_{\pi f} = \rho_{yf} \equiv \rho_{if}$, this specification means that the foreign interest rate also follows a stationary AR-process:

$$i_{t+1}^f = \rho_{if} i_t^f + \varepsilon_{t+1}^{if},$$

where $\varepsilon_{t+1}^{if} = g_{yf} \varepsilon_{t+1}^{yf} + g_{\pi f} \varepsilon_{t+1}^{\pi f}$.

2.3 Exchange rate determination

The model uncertainty that is the focus of the paper is primarily related to the determination of the exchange rate. As mentioned in the Introduction, many empirical studies have failed to find support for uncovered interest rate parity, and several explanations for these failures have been suggested, such as persistent movements in the exchange rate risk premium or non-rational exchange rate expectations. In order to allow for different deviations from UIP, the *nominal exchange rate* is assumed to satisfy the adjusted interest rate parity condition

$$s_t = \hat{s}_{t+1,t} + \frac{1}{4} [i_t^f - i_t] + u_t^s, \quad (11)$$

where $\hat{s}_{t+1,t}$ is the possibly non-rational expectation of the exchange rate in period $t + 1$, given information in period t , and u_t^s is a risk premium, which follows the stationary process

$$u_{t+1}^s = \rho_s u_t^s + \varepsilon_{t+1}^s, \quad (12)$$

where $0 \leq \rho_s < 1$ and ε_{t+1}^s is white noise with variance σ_s^2 . To analyze the effects of persistent movements in the risk premium, we will allow for varying values of the persistence parameter ρ_s , holding the variance σ_s^2 fixed.

Several studies reject the hypothesis that exchange rate expectations are rational, e.g., MacDonald (1990), Cavaglia et al. (1993), Ito (1990) and Froot and Frankel (1989).¹¹ Using survey data, Frankel and Froot (1987) test the validity of alternative expectation formation mechanisms on the foreign exchange market: adaptive expectations, equilibrium (or regressive) expectations, and distributed-lag expectations. Their results indicate that expectations at the 3-month, 6-month and 12-month horizon can be explained by all three models. We therefore incorporate each of these three alternatives to rational expectations in our model, and allow for different weights on each mechanism. Thus, the expected exchange rate in equation (11) is given by

$$\hat{s}_{t+1,t} = \vartheta_R s_{t+1|t} + \vartheta_A s_{t+1,t}^A + \vartheta_E s_{t+1,t}^E + \vartheta_D s_{t+1,t}^D, \quad (13)$$

where $s_{t+1|t}$ represents rational expectations, $s_{t+1,t}^A$ represents adaptive expectations, $s_{t+1,t}^E$ represents equilibrium expectations, and $s_{t+1,t}^D$ represents distributed-lag ex-

¹¹On the other hand, Liu and Maddala (1992) and Osterberg (2000), using cointegration techniques, cannot reject the hypothesis that expectations are rational. Furthermore, Lewis (1989) notes that if agents do not use the correct model, evidence of biased expectations do not necessarily imply non-rational expectations.

peptations, and where $\vartheta_R + \vartheta_A + \vartheta_E + \vartheta_D = 1$.¹²

Under *adaptive expectations*, agents update their exchange rate expectations slowly in the direction of the observed exchange rate, so their expectations are given by

$$s_{t+1,t}^A = (1 - \xi_E) s_t + \xi_A s_{t,t-1}^A, \quad (14)$$

where $0 < \xi_A < 1$ measures the rate of updating. Under *equilibrium (regressive) expectations*, the nominal exchange rate is expected to converge to its equilibrium rate s_t^* , determined by the PPP condition $q_t^* = 0$, so

$$s_{t+1,t}^E = (1 - \xi_E) s_t + \xi_E s_t^*, \quad (15)$$

where

$$s_t^* = p_t^d - p_t^f, \quad (16)$$

and ξ_E measures the rate of expectations convergence to the equilibrium exchange rate. Finally, under *distributed-lag expectations* the exchange rate is expected to move in the opposite direction to the previous period's movement, so a depreciation is expected to be followed by an appreciation and vice versa. Thus,

$$s_{t+1,t}^D = (1 - \xi_D) s_t + \xi_D s_{t-1}, \quad (17)$$

where ξ_D measures the sensitivity of exchange rate expectations to past movements in the exchange rate.

The *real exchange rate* is defined in terms of domestic prices as

$$q_t = s_t + p_t^f - p_t^d. \quad (18)$$

However, instead of directly observing the real exchange rate's deviation from its equilibrium value, in some versions of the model we assume that the central bank only observes the noisy variable

$$\hat{q}_t = q_t + u_t^q, \quad (19)$$

where the measurement error u_t^q follows the stationary AR(1) process

$$u_{t+1}^q = \rho_q u_t^q + \varepsilon_{t+1}^q, \quad (20)$$

¹²Although this formulation allows for complicated combinations of all four expectations mechanisms, in the analysis we concentrate on combinations of rational expectations and one of the three alternatives.

where $0 \leq \rho_q < 1$ and ε_t^q is an i.i.d. shock with mean zero and constant variance σ_q^2 . This specification is intended to capture the uncertainty involved in measuring the equilibrium real exchange rate, uncertainty that poses difficult problems for policymakers who want to respond to the real exchange rate's deviation from equilibrium.¹³

2.4 Monetary policy rules

Monetary policy is conducted by a central bank which follows a simple Taylor-type rule when setting its interest rate, under perfect commitment. Thus, the interest rate instrument of the central bank is set as a linear function of the deviations of the current output gap and the annual inflation rate from their zero targets, and possibly an exchange rate variable.

As a benchmark we use the standard Taylor (1993) rule (denoted “ T ”), where the interest rate depends only on output and inflation;

$$T : \quad i_t = f_\pi \bar{\pi}_t + f_y y_t, \quad (21)$$

where $\bar{\pi}_t = \sum_{\tau=0}^4 \pi_{t-\tau}$ is the four-quarter CPI inflation rate. The coefficients of this policy rule are chosen by the central bank to minimize its intertemporal loss function defined below.¹⁴

We then analyze three types of exchange rate rules, when the optimized Taylor rule is extended to include an exchange rate variable. The first rule (denoted “ ΔS ”) includes the change in the nominal exchange rate,¹⁵

$$\Delta S : \quad i_t = f_\pi \bar{\pi}_t + f_y y_t + f_{\Delta s} \Delta s_t; \quad (22)$$

the second rule (denoted “ Q ”) includes the level of the real exchange rate (possibly observed with an error),

$$Q : \quad i_t = f_\pi \bar{\pi}_t + f_y y_t + f_q \hat{q}_t; \quad (23)$$

¹³This way of modeling data uncertainty is similar to that used by Orphanides (1998) and Rudebusch (2000b) when analyzing the effects of output gap uncertainty.

¹⁴Note that we do not include a lagged interest rate in the policy rules: when optimizing the Taylor rule, the coefficient on the lagged interest rate is typically very close to zero. See more below.

¹⁵Our numerical algorithm is not very reliable when including non-stationary state variables, such as the level of the nominal exchange rate. We therefore confine the analysis to policy rules including stationary variables.

and the third rule (denoted “ ΔQ ”) includes the change in the real exchange rate,

$$\Delta Q : \quad i_t = f_\pi \bar{\pi}_t + f_y y_t + f_{\Delta q} \Delta \hat{q}_t. \quad (24)$$

In these three exchange rate rules, the optimized coefficients from the Taylor rule (21) are taken as given, and the coefficient on the respective exchange rate variable is optimized. Thus, the value of the exchange rate coefficient indicates whether there are any extra gains from adding the exchange rate variable to the optimized Taylor rule.¹⁶

2.5 Central bank preferences

The policy rules will be evaluated using the intertemporal loss function

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau}, \quad (25)$$

where the period loss function is of the standard quadratic form

$$L_t = \bar{\pi}_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2, \quad (26)$$

and $0 < \delta < 1$ is society’s discount factor. The parameters λ and ν measure the weights on stabilizing output and the interest rate relative to stabilizing inflation.

It can be shown (see Rudebusch and Svensson, 1999) that as the discount factor δ approaches unity, the value of the intertemporal loss function (25) becomes proportional to the unconditional expected value of the period loss function (26), i.e.,

$$EL = \text{Var}(\bar{\pi}_t) + \lambda \text{Var}(y_t) + \nu \text{Var}(\Delta i_t). \quad (27)$$

Thus, for each policy rule we calculate the resulting dynamics of the economy and the unconditional variances of the goal variables $(\bar{\pi}_t, y_t, \Delta i_t)$, and evaluate the weighted variances in (27). For comparison, we normalize the value of the loss function in each model with that resulting from the globally optimal outcome, when the central bank optimizes its objective function under commitment, using a non-restricted rule. This will give an idea of the quantitative differences in welfare resulting from the different policy rules.

¹⁶If we optimize all coefficients in the exchange rate rules, the results are very similar, and the main conclusions remain unaltered. However, such an approach does not provide a clear answer to the question whether there are any extra gains from responding separately to the exchange rate in an open economy.

To solve the model, we rewrite it on the state-space form

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + B i_t + \varepsilon_{t+1}, \quad (28)$$

where x_{1t} is a vector of predetermined state variables, x_{2t} is a vector of forward-looking state variables, and ε_{t+1} is a vector of disturbances to the predetermined variables. The period loss function (26) can then be expressed as

$$L_t = z_t' K z_t, \quad (29)$$

where K is a matrix of preference parameters and z_t is a vector of potential goal variables (and other variables of interest) that can be constructed from the state variables and the interest rate as

$$z_t = C_x x_t + C_i i_t. \quad (30)$$

Appendix B.1 shows how to set up the system; Appendix B.2 demonstrates how to calculate the dynamics of the system under a given simple (or optimized) policy rule, following Söderlind (1999); and Appendix B.3 shows how to calculate the unconditional variances of state and goal variables under the different rules.

3 Methodology and calibration

3.1 Methodology

Our primary interest lies in uncertainty about the determination of the exchange rate and its effects on the economy. For this reason the model allows for variations in the exchange rate model in several different dimensions:

1. The degree of exchange rate pass-through to import prices: the parameter κ in equation (6);
2. The persistence of the risk premium: the parameter ρ_s in equation (12);
3. The volatility of the real exchange rate measurement error: the variance of ε_t^q , σ_q^2 , in equation (20); and
4. The expectations formation mechanism (rational, adaptive, equilibrium or distributed-lag): the weights $\vartheta_R, \vartheta_A, \vartheta_E, \vartheta_D$ in equation (13).

In a purely backward-looking model, it would be fairly straightforward to define the stochastic properties of any uncertain parameters and explicitly solve for the optimal policy rule.¹⁷ However, when the model contains forward-looking elements, it

¹⁷See, e.g., Söderström (2000a,b) or Sack (2000) for such analyses of closed economies.

is (to our knowledge) not possible to calculate the optimal rule under multiplicative parameter uncertainty. Therefore, we will investigate the effects of uncertainty in a less stringent fashion. First, we vary the parameters in consideration and analyze how the policy rules and the resulting dynamics differ across parameter configurations. Second, we assume that the central bank is ignorant about the true behavior of the exchange rate and optimizes its policy rule under a baseline configuration of parameters, and we analyze the outcome when the actual configuration turns out to be different. The first exercise will indicate how the different rules perform under varying assumptions about the economy, while the second exercise—along the lines of Rudebusch (2000a)—will give an indication as to what types of policy rules are more robust to variations in the parameters, and thus more attractive when the true state of the economy is unknown.

Since the model cannot be solved analytically, we will use numerical methods, developed by Backus and Driffill (1986), Currie and Levine (1993) and others, as described by Söderlind (1999). Thus we begin by choosing a set of parameter values.

3.2 Model calibration and the propagation of shocks

To calibrate the model, we choose parameter values that we deem reasonable to loosely match the dynamic behavior of the model with the stylized facts of small open economies. In the output equation (1) the degree of forward-looking is set to 0.3, the parameter β_y on the lagged and future output gap is set 0.9, and the elasticities with respect to the real interest rate, the real exchange rate and the foreign output gap are 0.15, 0.05 and 0.12, respectively. In the determination of domestic inflation in equation (3) the degree of forward-looking is slightly higher, at 0.5, and the elasticities with respect to output and the real exchange rate are, respectively, 0.05 and 0.01. The weight of imported goods in the CPI basket is set to 0.35, which is close to the actual weights in Norway and Sweden.

Foreign output and inflation are both assigned the AR(1)-parameter 0.8, and the foreign central bank's Taylor rule has coefficients 0.5 on the output gap and 1.5 on inflation. The domestic central bank's preference parameters are set such that it gives equal weight to stabilizing the output gap and CPI inflation (so $\lambda = 1$), but also has some preference for interest rate smoothing ($\nu = 0.25$), and the discount factor δ is set close to unity, at 0.99. All disturbance terms have AR(1)-coefficients 0.3, and their variances are taken from a structural vector auto-regression on the Norwegian economy, so $\sigma_y^2 = 0.656$, $\sigma_\pi^2 = 0.389$, $\sigma_{y_f}^2 = 0.083$, and $\sigma_{\pi_f}^2 = 0.022$ (see Leitemo and Røisland, 2000, for details). While the persistence parameter of the

Table 1: Fixed parameter values

Output		Inflation		Foreign economy		Exchange rate		Preferences	
β_y	0.9	φ_π	0.5	ρ_{yf}	0.8	σ_s^2	0.844	δ	0.99
φ_y	0.3	γ_y	0.05	$\rho_{\pi f}$	0.8	ρ_q	0.3	λ	1
β_r	0.15	γ_q	0.01	g_{yf}	0.5			ν	0.25
β_q	0.05	η	0.35	$g_{\pi f}$	1.5				
β_{yf}	0.12	ρ_π	0.3	σ_{yf}^2	0.083				
ρ_y	0.3	σ_π^2	0.389	$\sigma_{\pi f}^2$	0.022				
σ_y^2	0.656								

Table 2: Alternative model configurations

Configuration	Parameter involved	Baseline value	Range
1. Exchange rate pass-through	κ	1	[0.1, 1]
2. Persistence of risk premium	ρ_s	0.3	[0, 1]
3. Variance of measurement error	σ_q^2	0	[0, 3]
4. Adaptive expectations	ϑ_A	0	[0, 1]
	ξ_A	–	0.1
5. Equilibrium expectations	ϑ_E	0	[0, 1]
	ξ_E	–	0.1
6. Distributed-lag expectations	ϑ_D	0	[0, 1]
	ξ_D	–	0.1

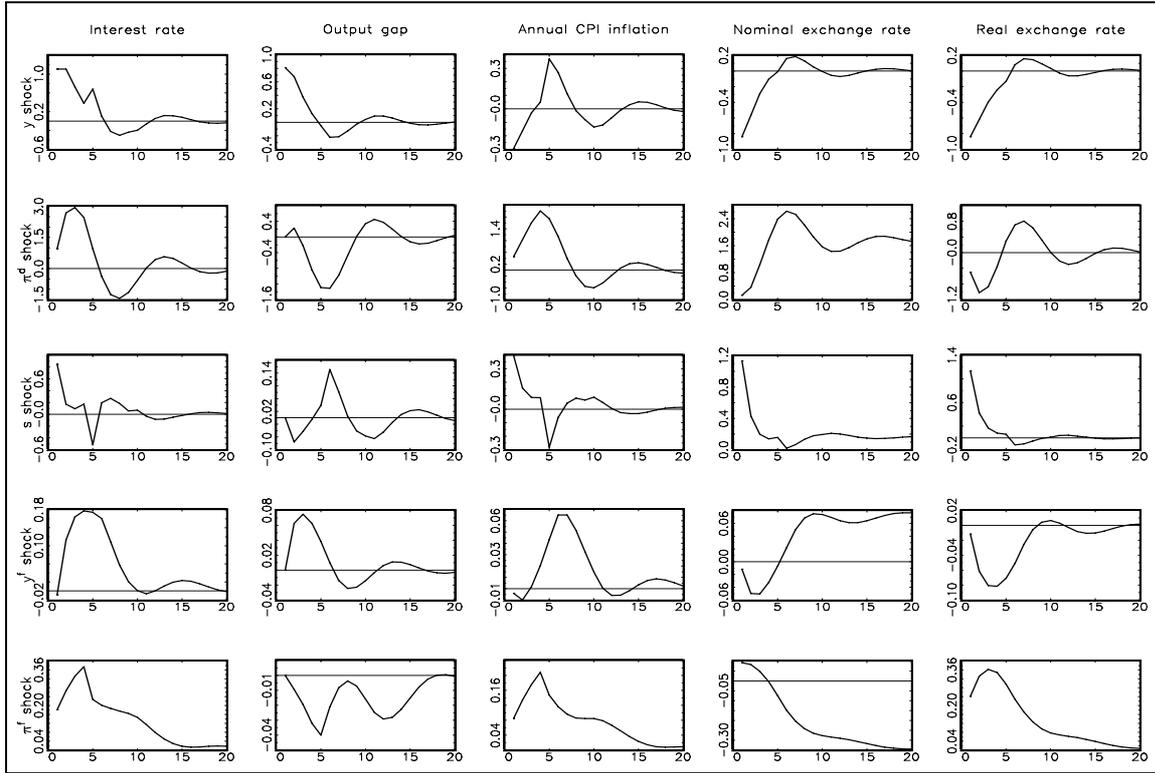
Under non-rational expectations (configurations 4–6), $\vartheta_R = 1 - \vartheta_A - \vartheta_E - \vartheta_D$.

risk premium will vary between model configurations (see below), the variance of the risk premium shock is always fixed at 0.844, also from the VAR-model. Finally, the persistence parameter of the real exchange rate measurement error is set to 0.3, while its variance is allowed to vary.

Table 1 summarizes these parameter values, which are kept fixed throughout the analysis.

In the baseline model, the parameters relating to the exchange rate are set to their conventional values, so there is instantaneous exchange rate pass-through ($\kappa = 1$), the risk premium is not very persistent ($\rho_s = 0.3$), there is no error when observing the real exchange rate ($\sigma_q^2 = 0$), and expectations are rational ($\vartheta_R = 1, \vartheta_A = \vartheta_E = \vartheta_D = 0$). We then analyze each departure from the baseline in turn, varying the rate of exchange rate pass-through, the persistence of the risk premium, the variance of the measurement error, and the weights on rational and non-rational expectations.

Figure 1: Impulse responses under optimized Taylor rule in baseline model



One standard deviation shocks, policy given by rule T in Table 3.

Under non-rational expectations, the parameters describing the rate of updating or convergence of the non-rational expectations (ξ_A, ξ_E, ξ_D) are all set to 0.1, which is slightly larger than the empirical findings of Frankel and Froot (1987). Table 2 presents the values of the exchange rate parameters in the baseline model, and their range of variation in the alternative configurations.

In order to get an overview of the transmission mechanism in the model, it is useful to examine the impulse responses of the baseline model under the optimized Taylor rule, in particular in response to three shocks: an output shock, a domestic inflation shock and a shock to the risk premium. The impulse responses to a one standard deviation shock in each disturbance are shown in Figure 1.

A shock to the *output gap* in the first row produces a level of output that is above the natural level for four quarters before slightly undershooting the natural level for the following five to eight quarters and then converging towards the equilibrium (zero) level. CPI inflation first falls (reacting to an initial nominal exchange rate appreciation) and then increases above the target level (due to the positive output gap). As a result of the output and inflation movements, the nominal interest rate is quickly raised and then follows a slightly jagged pattern back to zero. Since the

output disturbance has small effects on domestic inflation (not shown), and the long-run effects on the domestic price level are zero, the nominal and real exchange rates follow each other closely, and the nominal exchange rate settles at the same level as before the shock. (The long-run value of the exchange rate is given by the differential between the domestic and foreign price levels.) The initial exchange rate appreciation is driven by future expected interest rate differentials. In the following periods, the exchange rate gradually depreciates back to its equilibrium level.

A shock to *domestic inflation* in the second row has a hump-shaped effect on annual CPI inflation, and thus on the nominal interest rate. The interest rate increase, together with the appreciated real exchange rate (since the domestic price level rises), drives output down to a minimum level after five to six quarters. Output is persistently below its equilibrium for about ten quarters before the output gap is closed. Inflation is gradually forced back to target, with a slight undershooting. Interestingly, there is no initial nominal exchange rate appreciation: since the inflation disturbance is expected to be offset only gradually, the long-run price differential increases, resulting in an exchange rate depreciation towards a new higher equilibrium level.

A shock to the *risk premium* in the third row produces a depreciation of the nominal exchange rate, which has an immediate effect on CPI inflation and hence the interest rate is increased. The nominal exchange rate then quickly appreciates, rapidly reducing CPI inflation and the interest rate. Since domestic prices are sticky, the nominal exchange rate depreciation is translated into a real exchange rate depreciation, which has an expansive effect on output, after an initial fall.

4 Optimized simple rules and exchange rate model uncertainty

We now turn to the main objective of the paper: the performance and robustness of different policy rules in the various versions of our model. The analysis proceeds in three steps: we begin by discussing the optimized policy rules for the baseline model; we then demonstrate how these optimized rules vary as the model changes; and we end by analyzing how well the optimized rules for the baseline model perform when the true model is different.

In the open economy, there are seemingly good reasons to suspect that there is a role for the exchange rate as an indicator for monetary policy, and thus that it should be included in the central bank's policy rule. As described in previous sections, the

exchange rate enters as an important forcing variable for all endogenous variables in the model, and is perhaps the most central element of the monetary transmission mechanism. At the same time, the exchange rate is a highly endogenous variable: it is the price of foreign currency that equilibrates the demand and supply of foreign and domestic currency in a market with small transaction and price adjustment costs—making it highly responsive to the forces determining supply and demand. If monetary policy is already responding to these underlying forces to an appropriate degree, there is no role for the exchange rate as an extra indicator. (See also Taylor, 2000.) The question thus is whether the inflation and output gaps are sufficient indicators for the state of the small open economy.

4.1 Optimized policy rules in the baseline model

Table 3 shows the four optimized policy rules in the baseline model,¹⁸ and Table 4 shows the resulting unconditional variances of some important variables. We first note that the optimized coefficients in the Taylor rule (T) are fairly large and virtually identical: 2.14 on the output gap and 2.13 on annual CPI inflation. The T rule also does very well in comparison with the optimal rule under commitment (rule C): the loss is only around 12% higher. Including the exchange rate variables in the policy rule yields only small improvements relative to the T rule, and the biggest gain comes in the form of a more stable output gap due to a more stable real exchange rate. Interestingly, the optimized coefficients on the exchange rate variables are negative, so the central bank lowers the interest rate in the face of a nominal or real exchange rate depreciation.

That extending the Taylor rule with an exchange rate variables gives little improvement in terms of the volatility of the economy is a common result in the literature (see Taylor, 2000, for an overview). However, the result that the optimized exchange rate coefficients are negative is less common, and may seem counterintuitive. It therefore warrants some further consideration.

First note that in the baseline model with rational expectations, the solution for

¹⁸The optimized coefficients are found using the Constrained Optimization (CO) routines in Gauss. As mentioned earlier, when including also a lagged interest rate, its optimized coefficient is very small, around 0.05. This is partly due to the low degree of forward-looking behavior in the model. As shown by Woodford (1999), optimal policy in a forward-looking model (under commitment) displays a large degree of interest rate inertia, so including a lagged interest rate in a suboptimal rule often is beneficial. But this result hinges crucially on the degree of forward-looking behavior: with the low degree of forward-looking in our model, there are almost no gains from introducing more inertia in the policy rule. Furthermore, since our policy rule includes the annual inflation rate, there is already some inertia in the rule.

Table 3: Optimized policy rules in baseline model

Rule	Coefficient on				
	y_t	$\bar{\pi}_t$	Δs_t	\hat{q}_t	$\Delta \hat{q}_t$
T	2.14	2.13	–	–	–
ΔS	2.14	2.13	–0.15	–	–
Q	2.14	2.13	–	–0.29	–
ΔQ	2.14	2.13	–	–	–0.32

Output and inflation coefficients optimized for Taylor rule, exchange rate coefficients optimized for each rule.

Table 4: Unconditional variances of important variables and value of loss function in baseline model

Rule	Variance of						Loss	Relative loss	
	y_t	$\bar{\pi}_t$	Δs_t	\hat{q}_t	$\Delta \hat{q}_t$	i_t			Δi_t
C	6.46	13.45	5.17	6.87	4.79	32.52	11.74	22.84	100.00
T	7.48	14.38	4.76	7.80	4.66	35.98	14.56	25.50	111.63
ΔS	7.25	14.63	4.83	7.66	4.68	35.42	14.20	25.43	111.31
Q	7.10	14.40	5.15	7.60	4.98	37.12	15.24	25.31	110.79
ΔQ	6.98	14.60	4.91	7.39	4.68	35.25	14.34	25.16	110.15

Loss calculated as $EL = \text{Var}(\bar{\pi}_t) + \lambda \text{Var}(y_t) + \nu \text{Var}(\Delta i_t)$, where $\lambda = 1, \nu = 0.25$. Relative loss expressed as percent of loss from optimal policy under commitment (rule C).

the nominal exchange rate is given by iterating on equation (11), with $\hat{s}_{t+1,t} = s_{t+1|t}$, resulting in

$$s_t = -\frac{1}{4} \sum_{j=0}^{\infty} [i_{t+j|t} - i_{t+j|t}^f] + \sum_{j=0}^{\infty} u_{t+s|t}^s + \lim_{j \rightarrow \infty} (p_{t+j|t} - p_{t+j|t}^f), \quad (31)$$

where we have used the definition of the real exchange rate and the long-run PPP condition $\lim_{j \rightarrow \infty} q_{t+j|t} = 0$. Thus, the current exchange rate depends on the sum of expected future interest rate differentials corrected for a risk-premium and the equilibrium price level differential.¹⁹ Although an upward revision of expected future interest rate differentials leads to an instantaneous exchange-rate appreciation, a current positive interest rate differential means that the exchange rate is expected to depreciate:

$$\Delta s_{t+1|t} = \frac{1}{4} [i_t - i_t^f] - u_t^s. \quad (32)$$

Hence, a monetary policy rule that induces a higher interest rate (differential) will increase the expected rate of depreciation.

¹⁹This price level effect may be quite substantial, depending on the type of shock hitting the economy. In Figure 1 shocks to domestic inflation have the strongest long-run nominal exchange rate response, as the loss function only provides incentives for bringing the inflation rate back to its pre-shock value, leading to base drift in the price level.

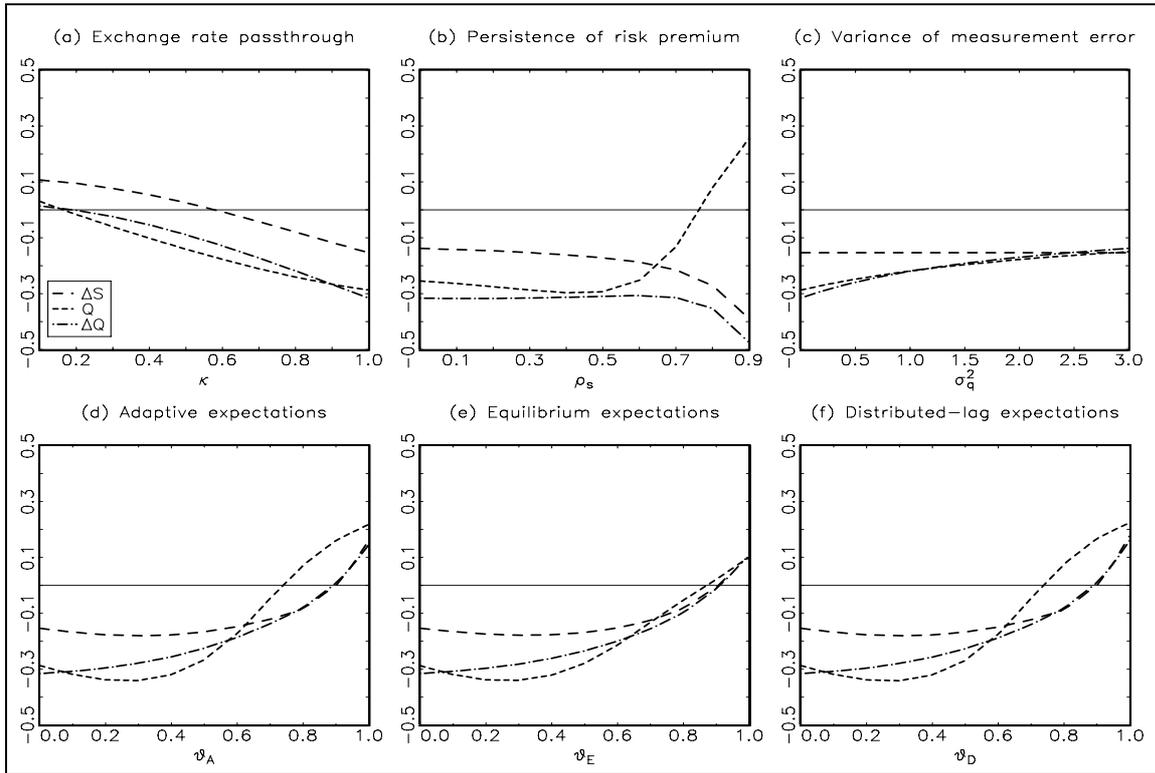
For these reasons, forward-looking behavior in the exchange rate market introduces a potential conflict between the direct exchange rate channel (affecting CPI inflation via imported inflation) and the other channels of monetary policy. A depreciation caused by a high interest rate differential feeds directly into CPI inflation, which, induces an even higher interest rate differential, leading to an even greater rate of depreciation, etc. As a result, when the central bank responds separately to the exchange rate, it may become more volatile, leading to larger volatility in the output gap.

However, this effect is reduced by letting monetary policy respond negatively to the rate of exchange rate depreciation. An expected interest rate differential then first produces an immediate appreciation, leading to an increased interest rate. But the following depreciation implies that the interest rate is lowered and remains below that implied by the Taylor rule during the time of depreciation. The expected future sum of interest rate differentials may therefore well be smaller than under the Taylor rule and the exchange rate therefore closer to its equilibrium rate. As a consequence, the exchange rate and the interest rate are both more stable, leading to less output volatility.

When it comes to the real exchange rate rule (Q), the intuition is rather different, however. It seems that the main advantage with a negative response to the level of the real exchange rate is in the face of foreign shocks. After a foreign inflationary disturbance the central bank diminishes the long-run price differential by accommodating some of the inflationary effects on the domestic economy when the real exchange rate depreciates, leading to more price level base drift in the domestic economy. The domestic price level settles down closer to the foreign price level, and the nominal exchange rate is stabilized closer to the initial level. Again, the central bank stabilizes the nominal (and real) exchange rate and thus also the output gap.

The conflict between the direct and indirect exchange rate channels seems to be present also in other studies. Svensson (2000) compares a Taylor rule including domestic inflation with one including CPI inflation, and thus with a positive coefficient (of 0.45) on the change in the real exchange rate. In his model, the latter rule leads to a lower variance in CPI inflation but higher variance in output. Likewise, Taylor (1999) includes the current and lagged real exchange rates in a policy rule for Germany, France and Italy (with coefficients 0.25 and -0.15 , respectively). In France and Italy this lowers the variances of both output and inflation, but in Germany the variance in output increases. Thus, there seems to be a trade-off in both Svensson's model and in Taylor's model for Germany: a positive response to

Figure 2: Exchange rate coefficients in different model configurations



Optimized exchange rate coefficients, given output and inflation coefficients.

the real exchange rate increases the variance in output and decreases the variance in inflation, just as in our model.²⁰ Whether this trade-off makes the central bank prefer a positive or negative response to the exchange rate naturally depends both on parameter values (which determine the sensitivity of the variances) and on the central bank's preferences. In our baseline specification, the central bank prefers a negative response, allowing for a higher variance of inflation, but a lower variance in output (and in the interest rate).

4.2 Optimized policy rules in the different models

The optimized policy rule coefficients are of course sensitive to the exact specification of the model. Figure 2 shows how the exchange rate coefficients vary in the different model configurations.²¹ The long-dashed lines represent the coefficient in the ΔS rule, the short-dashed lines represent the Q rule, and the dashed-dotted lines represent the coefficient in the ΔQ rule. (When applicable, a solid line represents

²⁰When we force the central bank to respond positively to changes in the real exchange rate, this is exactly what happens.

²¹The coefficients on output and inflation are shown in Figure C.1 in Appendix C.

the T rule.)

As the speed of *exchange rate pass-through* (κ) falls in panel (a), all exchange rate coefficients increase, and with a very slow pass-through the ΔS coefficient is positive while the coefficients on the real exchange rate are close to zero.²² As the direct exchange rate channel becomes more sluggish (when κ falls) the conflict between this channel and the other channels of monetary policy becomes less important. There is therefore less of a need for a negative response to the exchange rate variables, and the central bank instead tightens policy when the nominal exchange rate depreciates to avoid the indirect inflationary effects.²³

Increasing the *persistence of the risk premium* (ρ_s) in panel (b) the coefficients on the change in the nominal and real exchange rate fall further, whereas that on the level of the real exchange rate increases and eventually becomes positive. When the risk premium becomes more persistent, its direct effects on inflation (via exchange rate depreciation) become larger and more long-lived. Thus, the motivation for offsetting such shocks is stronger, and the optimized coefficients in the ΔS and ΔQ rules become more negative. At the same time, since the nominal exchange rate is more affected by shocks to the risk premium than by foreign shocks, the motivation for accommodating foreign shocks becomes less important. Thus, the coefficient in the Q rule instead increases and turns positive.

Varying the *variance of the measurement error in the real exchange rate* in panel (c) only affects the coefficients on the real exchange rate, and to a small extent. As the variance increases, the coefficients on both the level and the change of the real exchange rate become smaller, and as the variance increases indefinitely, the optimal response to the real exchange rate approaches zero. This result is in line with the results of Orphanides (1998) and Rudebusch (2000b) concerning output gap uncertainty: the optimal response to a noisy indicator becomes smaller as the amount of noise increases.²⁴

Introducing *non-rational expectations* in panels (d)–(f) initially has little effect on the exchange rate coefficients, but as the weights on non-rational expectations become large, the exchange rate coefficients increase and, again, eventually become

²²Note that also at a value of κ around 0.2–0.25, the coefficients on the real exchange rate are very close to zero. Naug and Nymoen (1996) find that the rate of exchange rate pass-through is around 0.28 per quarter, so with that (possibly more reasonable) parameterization, there is even less reason to use the real exchange rate as a monetary policy indicator.

²³Figure C.1 in Appendix C shows that the optimized coefficient on inflation decreases in the speed of pass-through, much for the same reason.

²⁴See also Svensson and Woodford (2000) and Swanson (2000) for analyses of the optimal response to noisy indicators.

positive. As a simple explanation of the slow effect on the exchange rate coefficients, note that allowing for only one type of non-rational expectations at a time, the three expectations mechanisms can be written on the form

$$s_t = \vartheta_R s_{t+1|t} + (1 - \vartheta_R) [\xi_j \tilde{s}_t^j + (1 - \xi_j) s_t] - \frac{1}{4} [i_t - i_t^f] + u_t^s, \quad (33)$$

where $j = A, E, D$, and

$$\tilde{s}_t^A = s_{t,t-1}^A, \quad \tilde{s}_t^E = s_t^*, \quad \tilde{s}_t^D = s_{t-1}. \quad (34)$$

We can then express the exchange rate as

$$s_t = \Theta s_{t+1|t} + (1 - \Theta) \tilde{s}_t^j - \frac{\Theta}{4\vartheta_R} [i_t - i_t^f] + \frac{\Theta}{\vartheta_R} u_t^s, \quad (35)$$

where

$$\Theta = \frac{\vartheta_R}{\vartheta_R + \xi_j (1 - \vartheta_R)}. \quad (36)$$

The coefficient Θ can be seen as the weight on the forward-looking component, after adjusting for the rate of updating or convergence of expectations, ξ_j . As ϑ_R is decreased from its baseline value of 1, Θ initially falls very slowly, since ξ_j is small. As a consequence, the optimal exchange rate coefficients are not very sensitive to small degrees of non-rationality, but as the weights on non-rational expectations increase towards unity (and $\vartheta_R \rightarrow 0$), the effect on the exchange rate coefficients becomes large.

Appendix A describes in detail the implications for the exchange rate of combining non-rational and rational expectations. The main insight is that as long as the weight on non-rational expectations is not too large, the implications of rational expectations dominate those of the other expectations schemes and the model properties are kept by and large. A moderate weight on adaptive or distributed-lag expectations introduces a positive autoregressive component in the exchange rate process, without changing the fact that the exchange rate reacts to the entire expected sum of future interest rate differentials. Thus, the initial appreciation of the exchange rate to an upward revision of the future interest rate differentials are exacerbated through time: the expected future movement in the exchange rate is affected not only by the interest rate differential but it also moves in the same direction as the *current* movement. An initial appreciation is followed by more appreciation before the movement turns into a depreciation due to the interest rate differential. The rate of depreciation is then accelerated. Thus the conflict between the direct exchange rate channel and the other channels may indeed be *intensified* when expectations

are not fully non-rational (thus the exchange rate coefficients may initially become more negative). In the fully non-rational case, however, there is no conflict as the exchange rate is given by

$$s_t = \tilde{s}_t^j - \frac{1}{4\xi_j} [i_t - i_t^f] + \frac{1}{\xi_j} u_t^s, \quad (37)$$

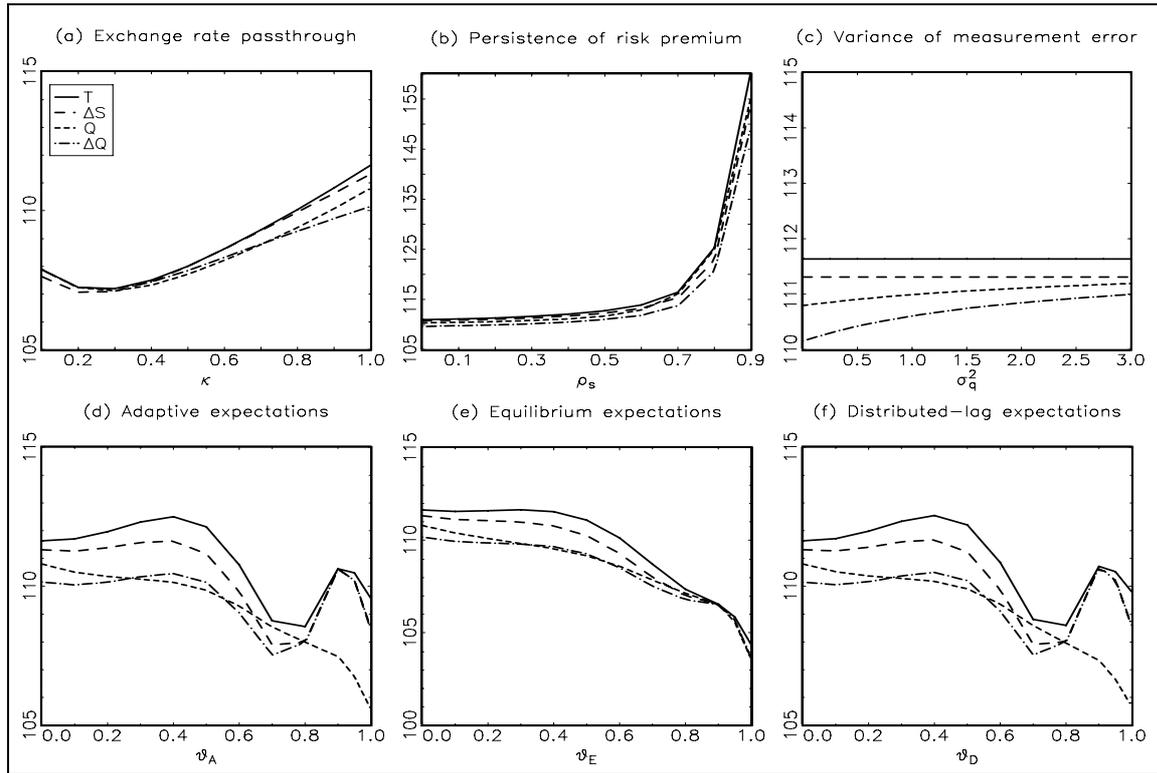
so only the current interest rate differential and risk premium matter for the exchange rate. As a consequence, the optimized exchange rate coefficients are positive.²⁵

The largest effects on the exchange rate coefficients thus seem to be due to changes in the rate of exchange rate pass-through, the persistence of the risk premium (for large values), and the weight on non-rational expectations (again for large weights). The degree of exchange rate pass-through clearly plays an important role in the model, since it determines the direct effects of exchange rate movements on CPI inflation, and the importance of the conflict between the direct exchange rate channel and the other transmission channels. The persistence of the risk premium determines the effects of risk premium shocks on the economy. And the weights on non-rational expectations determine the degree of forward-looking in the foreign exchange market, and thus the effects of interest rate changes on the exchange rate. That these three alterations of the model have large effects on optimal policy should therefore be no surprise. The error when measuring the real exchange rate, on the other hand, has fairly small effects on the optimized policy rules. Our specification of the measurement error is fairly simple, however, so this issue may warrant further research.

Figure 3 shows the loss resulting from each policy rule when the model is altered. Typically, the Q and ΔQ rules have lower loss than the ΔS rule, unless the weight on non-rational expectations is large, and the ΔQ rule typically performs better than the Q rule. (By definition, the exchange rate rules always weakly dominate the T rule.) Still, the differences are fairly small, and also the loss relative to the optimal rule under commitment is small, unless the persistence of the risk premium is close to one. Again, the gains from including the exchange rate in the optimized Taylor rules are fairly small in most parameterizations of the model.

²⁵Figure C.2 in Appendix C shows that the optimized exchange rate coefficients under non-rational expectations are positive for all degrees of exchange rate pass-through.

Figure 3: Value of loss function with optimized rules in different model configurations



Value of loss function as percent of loss from optimal policy under commitment.

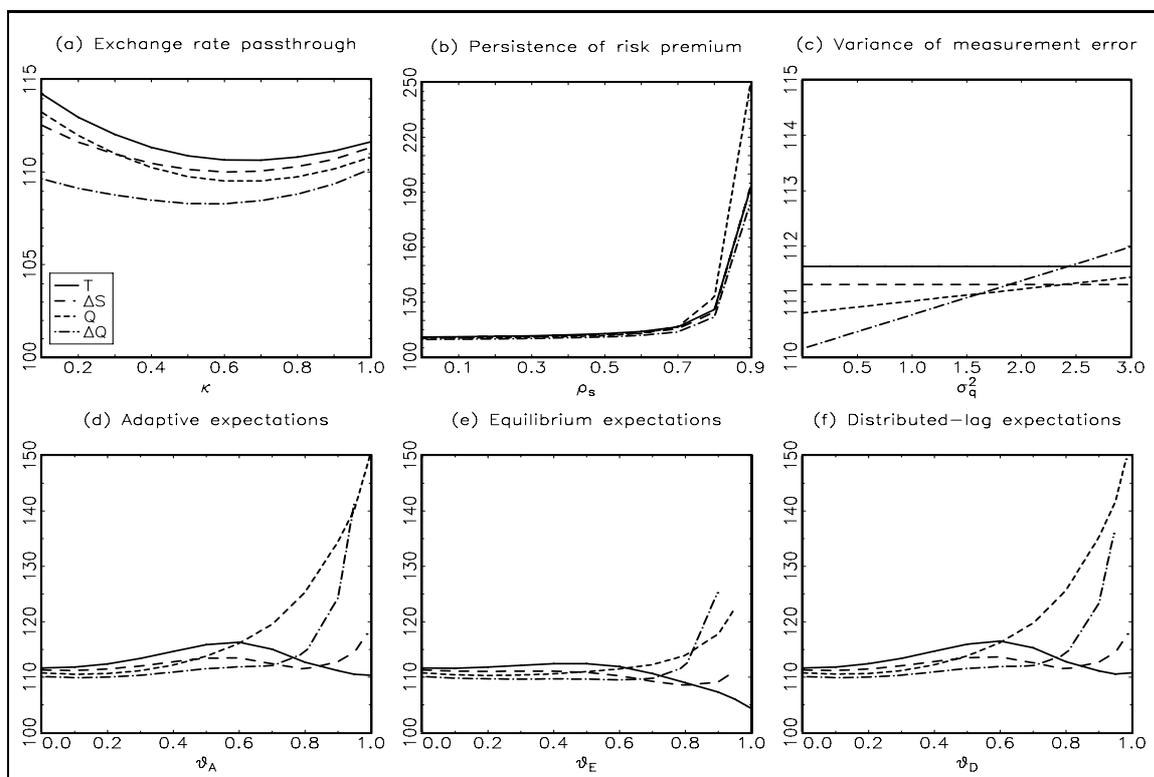
4.3 Robustness of policy rules in different models

Having discussed the optimized policy rules when the central bank knows the true model, we now turn to the case of pure model uncertainty. In this section we assume that the central bank is unaware of the true model, but optimizes its policy rule for the baseline model (which could be seen as the most likely model). We then calculate the outcome if the true model turns out to be different from the baseline. This way we hope to say something about the risks facing policymakers and about the robustness of the alternative policy rules.²⁶

Figure 4 shows the loss resulting from using the optimized baseline rules of Table 3 in the different model configurations. (The loss is expressed as percent of the loss from the optimal rule under commitment in each model.) Again we note that the variation in loss is fairly small, except in some extreme parameterizations. There are, however, some differences in the relative performance of the policy rules.

²⁶Note that this modeling strategy does not put the central bank on the same footing as the other agents in the model, since these know the true model while the central bank does not. We thus look at the robustness of policy rules in the sense of McCallum (1988, 1999) and Rudebusch (2000a) rather than in the sense of robust control theory (e.g., Hansen and Sargent, 2000), where all agents in the model share the same doubts about the true model specification.

Figure 4: Value of loss function using baseline policy rules in different model configurations



Value of loss function as percent of loss from optimal policy under commitment.

Varying the degree of exchange rate pass-through in panel (a) has no important effects on the outcome using the baseline rule. The ΔQ rule performs slightly better than the Q rule and the ΔS rule, and the Taylor rule gives the worst outcome out of the four rules. (Now, of course, the T rule may well perform better than the other rules, since the rules are optimized in one model and evaluated in another.)

Increasing the persistence of the risk premium increases the loss under all rules, but particularly so for the Q rule. When the risk premium is very persistent, all policy rules perform considerably worse than the optimal rule under commitment. This is particularly true for the Q rule: when $\rho_s = 0.9$ the Q rule leads to a loss which is 2.5 times higher than the optimal rule under commitment, and some 30% higher than the other simple rules.

Increasing the variance of the measurement error worsens the performance of both real exchange rate rules, and this time the ΔQ rule is the most sensitive. Although the real exchange rate rules perform better than the other rules when the measurement is small, their performance deteriorates as the amount of noise increases, although the differences are fairly small overall.

Under non-rational expectations, there is large variation in the outcomes, especially when the weight on non-rational expectations becomes large. Again, for moderate degrees of non-rationality, there is little difference across the rules, and the ranking is the same as before, but for large degrees of non-rationality, the real exchange rate rules perform considerably worse than the ΔS and T rules. (A missing value in the figures indicates that the policy rule in question leads to an unstable system.)

On average, the real exchange rate rules seem to perform better than the other rules. But at the same time they seem less robust to model uncertainty, since they lead to very bad outcomes in some configurations of the model. The nominal exchange rate rule and, in particular, the Taylor rule seem more robust to model uncertainty, but perform worse than the real exchange rate rules on average.

Comparing Figure 4 with the baseline rules in Table 3 and the optimized coefficients in Figure 2, we see that the greatest risks with extending the Taylor rule to include the exchange rate variables lie in responding to shocks in the wrong direction: the baseline rules imply a negative response to the exchange rate variables, but as the model is changed, the optimized coefficients in Figure 2 often become positive. Therefore, when the central bank uses the baseline rules in a configuration that warrants a positive response (e.g., when the risk premium is very persistent or expectations are highly non-rational), the resulting outcome can be very poor.

Thus, the fact that different models not only imply different *magnitude* in the response to the exchange rate variables, but also different *sign* makes the exchange rate rules less robust to model uncertainty.

5 Concluding remarks

As mentioned in the Introduction, the exchange rate is an important part of the monetary transmission mechanism in an open economy. Therefore, it may seem natural to include some exchange rate variable in the central bank's policy rule, in order to better stabilize the economy.

At the same time, the model determining the exchange rate and its effects on the economy is inherently uncertain. This paper therefore analyzes the gains from including the exchange rate in an optimized Taylor rule, and how these gains vary with the specification of the exchange rate model. We also ask how robust different policy rules are to model uncertainty.

We find that including the exchange rate in an optimized Taylor rule gives little improvement in terms of decreased volatility in important variables. This is true in most configurations of the exchange rate model. Furthermore, the policy rules that include the exchange rate (and in particular the real exchange rate) are more sensitive to model uncertainty. This is partly because the optimal response to the exchange rate variables differs across models, not only in magnitude, but also in sign. Applying a rule optimized for the wrong model can therefore lead to very poor outcomes. The standard Taylor rule, although it performs slightly worse on average, avoids such bad outcomes, and thus is more robust to model uncertainty.

This paper has concentrated on uncertainty concerning the exchange rate model. Future work could extend the analysis by including other types of uncertainty, e.g., concerning the equilibrium real interest rate or the natural level of output. Another extension would be to use robust control techniques (see Hansen and Sargent, 2000) in order to design robust rules for monetary policy in an open economy. Since the exchange rate process is perhaps the least understood part of the monetary transmission mechanism, we believe that the analysis of robustness to exchange rate model uncertainty is central to monetary policy research in open economies.

A Exchange rate dynamics under non-rational expectations

A.1 Adaptive expectations

Under *adaptive expectations*, the equation system to be solved is given by

$$s_t = \vartheta_R s_{t+1|t} + (1 - \vartheta_R) s_{t+1,t}^A - \frac{1}{4} [i_t - i_t^f] + u_t^s, \quad (\text{A1})$$

$$s_{t+1,t}^A = (1 - \xi_A) s_t + \xi_A s_{t,t-1}^A. \quad (\text{A2})$$

Equation (A2) may be written as

$$s_{t+1,t}^A = (1 - \xi_A) s_t + \xi_A L s_{t+1,t}^A, \quad (\text{A3})$$

where L is the lag operator. Isolating for the next-period expected exchange rate yields

$$s_{t+1,t}^A = \frac{1 - \xi_A}{1 - \xi_A L} s_t, \quad (\text{A4})$$

and substituting the expected rate into equation (A1) yields

$$s_t = \vartheta_R s_{t+1|t} + (1 - \vartheta_R) \frac{1 - \xi_A}{1 - \xi_A L} s_t - \frac{1}{4} [i_t - i_t^f] + u_t^s, \quad (\text{A5})$$

$$(1 - \xi_A L) s_t = (1 - \xi_A L) \vartheta_R s_{t+1|t} + (1 - \vartheta_R) (1 - \xi_A) s_t - \frac{1}{4} (1 - \xi_A L) [i_t - i_t^f] + (1 - \xi_A L) u_t^s, \quad (\text{A6})$$

$$(\vartheta_R + \xi_A - \vartheta_R \xi_A) s_t = \xi_A s_{t-1} + \vartheta_R s_{t+1|t} - \xi_A \vartheta_R s_{t|t-1} - \frac{1}{4} [i_t - i_t^f] + \frac{1}{4} \xi_A [i_{t-1} - i_{t-1}^f] + u_t^s - \xi_A u_{t-1}^s \quad (\text{A7})$$

$$s_t = \Theta_1^A s_{t-1} + \Theta_2^A s_{t+1|t} + \Theta_3^A s_{t|t-1} + \omega_t^A, \quad (\text{A8})$$

where

$$\Theta_1^A = \frac{\xi_A}{\vartheta_R + (1 - \vartheta_R) \xi_A}, \quad (\text{A9})$$

$$\Theta_2^A = \frac{\vartheta_R}{\vartheta_R + (1 - \vartheta_R) \xi_A}, \quad (\text{A10})$$

$$\Theta_3^A = \frac{-\vartheta_R \xi_A}{\vartheta_R + (1 - \vartheta_R) \xi_A}, \quad (\text{A11})$$

$$\omega_t^A = \frac{1}{\vartheta_R + (1 - \vartheta_R) \xi_A} \times \left\{ -\frac{1}{4} [i_t - i_t^f] + \frac{1}{4} \xi_A [i_{t-1} - i_{t-1}^f] + u_t^s - \xi_A u_{t-1}^s \right\}. \quad (\text{A12})$$

The characteristic equation associated with (A8) is given by

$$\Theta_2^A \mu^2 - (1 - \Theta_3^A) \mu + \Theta_1^A = 0 \quad (\text{A13})$$

which solves for the backward and forward roots respectively keeping in mind the restriction on the Θ^A 's,

$$\mu_B = \frac{(\Theta_1^A + \Theta_2^A) - \sqrt{(\Theta_2^A - \Theta_1^A)^2}}{2\Theta_2^A} \quad (\text{A14})$$

$$\mu_F = \frac{(\Theta_1^A + \Theta_2^A) + \sqrt{(\Theta_2^A - \Theta_1^A)^2}}{2\Theta_2^A}. \quad (\text{A15})$$

The solution to equation (A7) can now be written as

$$s_t = \mu_B s_{t-1} + (1 - \Theta_2^A \mu_B)^{-1} \sum_{i=0}^{\infty} (\mu_F)^{-i} \omega_{t+i|t}^A \quad (\text{A16})$$

$$\begin{aligned} &+ \frac{(1 - \Theta_2^A - \Theta_1^A)}{\Theta_2^A \mu_F (1 - \Theta_2^A \mu_B)} \sum_{i=0}^{\infty} (\mu_F)^{-i} \omega_{t+i|t-1}^A \\ &+ (1 - \mu_B) \lim_{j \rightarrow \infty} (p_{t+j|t} - p_{t+j|t}^f). \end{aligned} \quad (\text{A17})$$

In order to explain the slow effects of changes in ϑ_A on the optimized exchange rate coefficients, first note that the forward root, μ_F , equals unity for $\vartheta_R \geq \xi_A$, which means that the expected future sum of the risk-premium corrected interest rate differentials remain undiscounted when the degree of adaptive expectations is not too large. Thus, there is a relatively strong feedback to the exchange rate from a future persistent interest rate movement. This implies that the initial reaction to exchange rate may be quite substantial as in the pure rational expectations case. As the weight on adaptive expectations increases, the persistence (μ_B) in the exchange rate process increases and the initial reaction to the exchange rate is exacerbated.

The rationally expected rate of depreciation can be expressed as in the rational expectations case, assuming $\vartheta_R \geq \xi_A$, as

$$\begin{aligned} \Delta s_{t+1|t} &= \mu_B \Delta s_t + (1 - \Theta_2^A \mu_B)^{-1} \left\{ \frac{1}{4} [i_t - i_t^f] - u_t^s \right\} \\ &+ \frac{(1 - \Theta_2^A - \Theta_1^A)}{\Theta_2^A (1 - \Theta_2^A \mu_B)} \left\{ \frac{1}{4} [i_{t-1} - i_{t-1}^f] - u_{t-1}^s \right\} \\ &+ (1 - \mu_B) \lim_{j \rightarrow \infty} (\pi_{t+j} - \pi_{t+j}^f). \end{aligned} \quad (\text{A18})$$

For conventional parameter values, the expected next-period rate of depreciation is determined mainly by the *current* interest rate differential. However, as the rate

of persistence increases, the direction of the current period exchange-rate movement has an increasingly stronger influence upon the future movement. An initial exchange rate appreciation, e.g., due to an upward revision of future interest rate differentials, produces expectations of further appreciations. This effect gradually dies out as the persistence and the rate of movement indicated by the interest rate differentials dominate and pull in the opposite direction. The persistence now exacerbates the depreciation. In this situation a given interest rate differential exerts a greater influence on inflation through the direct exchange rate channel than in the rational expectations case. The inflation coefficient is therefore even more inappropriate and the optimal exchange-rate coefficient stays negative for very large weights on adaptive expectations. In the limit, however, when $\vartheta_A = 1$, the exchange rate process is given by

$$s_t = s_{t-1} - \frac{1}{\xi_A} \left\{ \frac{1}{4} [i_t - i_t^f] + u_t^s \right\} + \frac{1}{4} [i_{t-1} - i_{t-1}^f] + u_{t-1}^s, \quad (\text{A19})$$

which implies that there is no response of the exchange rate to the future expected interest rate differential and a positive current interest rate differential implies a gradual *appreciation*, under the condition that the interest rate differential in the previous period does not deviate too much from the current one (the second term will dominate the third). There is hence no conflict between the interest rate channel and exchange rate channels; an interest rate increase implies a contractionary policy through all channels.

A.2 Distributed-lag expectations

Under distributed-lag expectations, agents form next-period exchange rate expectations as a weighted average between the exchange rate in the current and previous periods. When expectations are formed using a combination of distributed-lag and rational expectations, the UIP condition is written as

$$s_t = \vartheta_R s_{t+1|t} + (1 - \vartheta_R) [(1 - \xi_D) s_t + \xi_D s_{t-1}] - \frac{1}{4} [i_t - i_t^f] + u_t^s, \quad (\text{A20})$$

which may be rearranged as

$$s_t = \Theta_1^D s_{t+1|t} + (1 - \Theta_1^D) s_{t-1} + \omega_t^D, \quad (\text{A21})$$

where

$$\Theta_1^D = \frac{\vartheta_R}{\vartheta_R + (1 - \vartheta_R)\xi_D}, \quad (\text{A22})$$

$$\omega_t^D = \frac{1}{\vartheta_R + (1 - \vartheta_R)\xi_D} \left\{ -\frac{1}{4} [i_t - i_t^f] + u_t^s \right\}. \quad (\text{A23})$$

The characteristic equation associated with (A21) is

$$\Theta_1^D \mu^2 - \mu + (1 - \Theta_1^D) = 0 \quad (\text{A24})$$

with characteristic roots

$$\mu_B = \frac{1/2 - \sqrt{(\Theta_1^D - 1/2)^2}}{\Theta_1^D} \mu^F = \frac{1/2 + \sqrt{(\Theta_1^D - 1/2)^2}}{\Theta_1^D}, \quad (\text{A25})$$

and the solution to the exchange rate is given by

$$\begin{aligned} s_t = & \mu^B s_{t-1} + \frac{1}{\vartheta_R \mu^F} \sum_{j=0}^{\infty} (\mu^F)^{-j} \left\{ -\frac{1}{4} [i_{t+j|t} - i_{t+j|t}^f] + u_{t+j|t}^s \right\} \\ & + (1 - \mu^B) \lim_{j \rightarrow \infty} (p_{t+j|t} - p_{t+j|t}^f). \end{aligned} \quad (\text{A26})$$

The forward root, μ^F , will be equal to unity if $\vartheta_R/(1 - \vartheta_R) \geq \xi_D$, or equivalently, if $\vartheta_R \geq \xi_D/(1 + \xi_D)$. Then the current exchange rate depends on the expected future undiscounted sum of risk-premium corrected interest rate differentials if the weight on distributed-lag expectations is not too large. A reduction in the weight on rational expectations produces an even stronger feedback from the interest rate differentials. As $\vartheta_R/(1 - \vartheta_R) \rightarrow \xi_D$ the degree of persistence increases and the initial reaction is exacerbated. Under the assumption that $\vartheta_R/(1 - \vartheta_R) \geq \xi_D$, the (rationally) expected movement in the exchange rate is given by

$$\begin{aligned} \Delta s_{t+1|t} = & \mu^B \Delta s_t + \frac{1}{\vartheta_R \mu^F} \left\{ \frac{1}{4} [i_t - i_t^f] - u_{t+1|t}^s \right\} \\ & + (1 - \mu^B) \lim_{j \rightarrow \infty} (\pi_{t+j|t} - \pi_{t+j|t}^f). \end{aligned} \quad (\text{A27})$$

We note that the current interest rate differential has a strong influence on the expected exchange rate movement. In the valid range of ϑ_R , the interest rate differential exerts an increasingly stronger influence as ϑ_R decreases. In order to reduce the inflationary effect caused by the exchange rate movements through the direct exchange rate channel, it may be welfare-enhancing to allow interest rates to be reduced if the exchange rate is depreciating.

However, as ϑ_R is getting closer to zero, the backward root rapidly approaches unity and $\vartheta_R \mu^F \rightarrow \xi_D$, the influence from the interest rate differential “flattens out” and the present direction of the exchange rate has a stronger influence on the expected future development. This has similar effects as in the adaptive expectations case. Say the initial revision of expected future interest rate differentials produces a strong appreciation. Then, a larger backward root produces rational expectations of a further appreciation, as the first term outweighs the effect from the interest rate

differential. There may then be stronger incentive to reduce the initial appreciation and the exchange rate coefficient eventually turns positive. In the limit, when $\vartheta_D = 1$, the exchange rate process is given from equation (A26) by

$$s_t = s_{t-1} - \frac{1}{\xi_D} \left\{ \frac{1}{4} [i_t - i_t^f] - u_t^s \right\}. \quad (\text{A28})$$

In equilibrium, the distributed-lag-expected rate of depreciation must equal the risk-premium corrected interest rate differential as before. If the next-period expected exchange rate is influenced to a large degree by the present rate (ξ_D small), the exchange rate becomes rather sensitive to the interest rate differential. The reason is that as the current exchange rate moves in order to satisfy the equilibrium conditions, the expected next-period expectations move in the same direction and the expected future movement is only moderately affected. There is hence no depreciating effect from a positive interest rate differential, on the contrary, a positive interest rate differential leads to a gradual appreciation of the exchange rate.

A.3 Equilibrium (regressive) expectations

If agents have equilibrium exchange rate expectations, the expected next-period exchange rate is a weighted average of the current level and the purchasing power parity rate, $s_t^* = p_t - p_t^f$. By allowing a combination of equilibrium and rational expectations, the uncovered interest parity condition may be written as

$$s_t = \vartheta_R s_{t+1|t} + (1 - \vartheta_R) \left[(1 - \xi_E) s_t + \xi_E (p_t - p_t^f) \right] - \frac{1}{4} [i_t - i_t^f] + u_t^s. \quad (\text{A29})$$

After isolating for the current exchange rate, the UIP condition may be stated as

$$s_t = \Theta_1^E s_{t+1|t} + (1 - \Theta_1^E) (p_t - p_t^f) - \omega_t^E, \quad (\text{A30})$$

where

$$\Theta_1^E = \frac{\vartheta_R}{\vartheta_R + (1 - \vartheta_R)\xi_E} \quad (\text{A31})$$

$$\omega_t^E = \frac{1}{\vartheta_R + (1 - \vartheta_R)\xi_E} \left\{ \frac{1}{4} [i_t - i_t^f] - u_t^s \right\}. \quad (\text{A32})$$

The solution to equation (A30) is given by

$$\begin{aligned} s_t = & (1 - \Theta_1^E) \sum_{j=0}^{\infty} (\Theta_1^E)^j (p_{t+j|t} - p_{t+j|t}^f) - \sum_{j=0}^{\infty} (\Theta_1^E)^j \omega_{t+j|t}^E \\ & + \lim_{j \rightarrow \infty} (\Theta_1^E)^j (p_{t+j|t} - p_{t+j|t}^f). \end{aligned} \quad (\text{A33})$$

A similar story as the one given under the other expectations schemes can explain the development of the optimal exchange rate coefficients. According to equation (A33), a combination of future expected interest rate differentials and price level differentials has an influence upon the current exchange rate. The future differentials are however discounted more heavily when the weight on equilibrium expectations increases (Θ_1^E decreases) and the current and near-future price level differentials have a stronger influence on the current exchange rate. A shock to domestic prices will lead agents to revise their price differentials upwards which makes the exchange rate depreciate as a result of investors expecting a future appreciation since an unchanged interest rate differentials does not offset the change in the rate of return differentials between domestic and foreign assets induced by the shock to the price level differential. Since the exchange rate depreciation provides a positive inflationary impulse, i.e., feeding back to the future price level differentials, the exchange rate may react quite strongly if the weight attached to equilibrium expectations is not too large.

The expected change is given by

$$\begin{aligned} \Delta s_{t+1|t} = & -(1 - \Theta_1^E)(p_t - p_t^f) + \omega_t^E \\ & + (1 - \Theta_1^E) \sum_{j=0}^{\infty} (\Theta_1^E)^j \left[(1 - \Theta_1^E) (p_{t+1+j|t} - p_{t+1+j|t}^f) - \omega_{t+1+j|t}^E \right] \\ & + \lim_{j \rightarrow \infty} (\Theta_1^E)^j (\pi_{t+j|t} - \pi_{t+j|t}^f). \end{aligned} \quad (\text{A34})$$

We observe that the expected future rate of depreciation remains a positive function of the current interest rate differential in addition to the discounted sum of future price level differentials. On the other side, the current price level differential have a negative influence together with the discounted sum of future interest rate differentials. If the weight attached to equilibrium expectations is not too large, positive influence will dominate the negative influence which implies that a positive interest rate differential causes an expected depreciation. As in the other cases, the direct exchange rate channel will pull in the opposite direction as the other exchange rate channels.

When agents expectations are fully described as equilibrium expectations, the exchange rate development is

$$s_t = p_t - p_t^f - \frac{1}{4\xi_E} [i_t - i_t^f] + u_t^s, \quad (\text{A35})$$

$$q_t = -\frac{1}{4\xi_E} [i_t - i_t^f] + u_t^s. \quad (\text{A36})$$

The risk-premium corrected interest rate differential now determines the gap between the equilibrium and the current real exchange rate. As in the distributed-lag case, the current rate influences the next-period exchange rate to a strong degree, and the interest rate differential has a strong impact on the exchange rate deviation. Thus, also in this case the exchange rate market implies no conflict between the exchange rate channels, and the optimal exchange rate coefficient is positive.

B Model appendix

B.1 Setting up the model

To solve the model, we first formulate it on state-space form. First, define the real variable

$$q_{t+1}^A \equiv s_{t+1,t}^A + p_{t+1}^f - p_{t+1}^d, \quad (\text{B1})$$

and use in (14) to get

$$\begin{aligned} q_{t+1}^A - p_{t+1}^f + p_{t+1}^d &= s_t + \xi_A [q_t^A - p_t^f + p_t^d - q_t + p_t^f - p_t^d] \\ &= q_t - p_t^f + p_t^d + \xi_A [q_t^A - q_t], \end{aligned} \quad (\text{B2})$$

yielding the state variable

$$\begin{aligned} q_{t+1}^A &= (1 - \xi_A) q_t + \pi_{t+1}^f - \pi_{t+1}^d + \xi_A q_t^A \\ &= (1 - \xi_A) q_t + \xi_A q_t^A + \rho_{\pi f} \pi_t^f + \varepsilon_{t+1}^{\pi f} - \pi_{t+1|t}^d - \varepsilon_{t+1}^{\pi}. \end{aligned} \quad (\text{B3})$$

Also,

$$s_{t+1,t}^A = (1 - \xi_A) q_t + \xi_A q_t^A + p_t^d - p_t^f. \quad (\text{B4})$$

Use (18), (13), (15), (17), and (B4) in (11) to eliminate the nominal exchange rate:

$$\begin{aligned} q_t + p_t^d - p_t^f &= \vartheta_R [q_{t+1|t} + p_{t+1|t}^d - p_{t+1|t}^f] + \vartheta_A [(1 - \xi_A) q_t + \xi_A q_t^A + p_t^d - p_t^f] \\ &+ \vartheta_E [\xi_E (p_t^d - p_t^f) + (1 - \xi_E) (q_t + p_t^d - p_t^f)] \\ &+ \vartheta_D [(1 - \xi_D) (q_t + p_t^d - p_t^f) + \xi_D (q_{t-1} + p_{t-1}^d - p_{t-1}^f)] + \frac{1}{4} [i_t^f - i_t] + u_t^s, \end{aligned} \quad (\text{B5})$$

and rearrange to get

$$\begin{aligned}
\Omega_q q_t &= \vartheta_R q_{t+1|t} + \vartheta_D \xi_D q_{t-1} + \vartheta_A \xi_A q_t^A + \vartheta_R [p_{t+1|t}^d - p_{t+1|t}^f] \\
&+ (\vartheta_D \xi_D - \vartheta_R) [p_t^d - p_t^f] + \vartheta_D \xi_D [p_{t-1}^d - p_{t-1}^f] \\
&+ \frac{1}{4} [i_t^f - i_t] + u_t^s,
\end{aligned} \tag{B6}$$

where

$$\begin{aligned}
\Omega_q &\equiv 1 - \vartheta_A (1 - \xi_A) - \vartheta_E (1 - \xi_E) - \vartheta_D (1 - \xi_D) \\
&= \vartheta_R + \vartheta_A \xi_A + \vartheta_E \xi_E + \vartheta_D \xi_D.
\end{aligned} \tag{B7}$$

Now use

$$p_t^d = \pi_t^d + p_{t-1}^d \tag{B8}$$

$$p_t^f = \pi_t^f + p_{t-1}^f \tag{B9}$$

$$\begin{aligned}
p_{t+1|t}^d &= \pi_{t+1|t}^d + p_t^d \\
&= \pi_{t+1|t}^d + \pi_t^d + p_{t-1}^d
\end{aligned} \tag{B10}$$

$$\begin{aligned}
p_{t+1|t}^f &= \pi_{t+1|t}^f + p_t^f \\
&= (1 + \rho_{\pi_f}) \pi_t^f + p_{t-1}^f
\end{aligned} \tag{B11}$$

to eliminate the non-stationary price levels and solve for $q_{t+1|t}$:

$$\begin{aligned}
\vartheta_R q_{t+1|t} &= \Omega_q q_t - \vartheta_A \xi_A q_t^A - \vartheta_D \xi_D q_{t-1} - \vartheta_R \pi_{t+1|t}^d + \vartheta_D \xi_D \pi_t^d \\
&+ (\vartheta_R \rho_{\pi_f} - \vartheta_D \xi_D) \pi_t^f + \frac{1}{4} (i_t - i_t^f) - u_t^s.
\end{aligned} \tag{B12}$$

Likewise, eliminate s_t, p_t^d, p_t^f from (6):

$$\begin{aligned}
\pi_t^M &= (1 - \kappa) \pi_{t-1}^M + \kappa (\pi_t^f + q_t - q_{t-1} + \pi_t^d - \pi_t^f) \\
&= (1 - \kappa) \pi_{t-1}^M + \kappa (q_t - q_{t-1} + \pi_t^d)
\end{aligned} \tag{B13}$$

Finally, lead (10) one period and combine with (8) and (9) to get

$$\begin{aligned}
i_{t+1}^f &= g_{yf} y_{t+1}^f + g_{\pi_f} \bar{\pi}_{t+1}^f \\
&= g_{yf} \rho_{yf} y_t^f + g_{\pi_f} [(1 + \rho_{\pi_f}) \pi_t^f + \pi_{t-1}^f + \pi_{t-2}^f] + g_{yf} \varepsilon_{t+1}^{yf} + g_{\pi_f} \varepsilon_{t+1}^{\pi_f}.
\end{aligned} \tag{B14}$$

Then the equations for the 21 predetermined state variables are given by (2), (4), (12), (20), (B13), (8), (9), (B15), (B3), (19), the additional equations

$$y_{t+1} = y_{t+1|t} + \varepsilon_{t+1}^y, \quad (\text{B15})$$

$$\pi_{t+1}^d = \pi_{t+1|t}^d + \varepsilon_{t+1}^\pi, \quad (\text{B16})$$

and identities for $\pi_t^d, \pi_{t-1}^d, \pi_{t-2}^d, \pi_{t-1}^M, \pi_{t-2}^M, \pi_t^f, \pi_{t-1}^f, \varepsilon_{t+1}^q, q_t$ and i_t .

The equations for the 3 forward-looking variables are given by (B12), (1) and (3) after taking expectations at t and solving for the forward-looking variables $y_{t+2|t}, \pi_{t+2|t}$:

$$\begin{aligned} \beta_y \varphi_y y_{t+2|t} &= y_{t+1|t} - \beta_y (1 - \varphi_y) y_t + \beta_r i_t - 4\beta_r \pi_{t+1|t}^d - \beta_q q_t \\ &\quad - \beta_{yf} y_t^f - \rho_y u_t^y, \end{aligned} \quad (\text{B17})$$

$$\varphi_\pi \pi_{t+2|t}^d = \pi_{t+1|t}^d - (1 - \varphi_\pi) \pi_t^d - \gamma_y y_{t+1|t} - \gamma_q q_{t+1|t} - \rho_\pi u_t^\pi. \quad (\text{B18})$$

Using the predetermined state variables and the forward-looking variables, we can calculate CPI inflation, the change in the nominal exchange rate and the change in the observed real exchange rate as

$$\begin{aligned} \pi_t &= (1 - \eta) \pi_t^d + \eta \pi_t^M \\ &= (1 - \eta) \pi_t^d + \eta \left[(1 - \kappa) \pi_{t-1}^M + \kappa (q_t - q_{t-1} + \pi_t^d) \right], \end{aligned} \quad (\text{B19})$$

$$\Delta s_t = q_t - q_{t-1} + \pi_t^d - \pi_t^f, \quad (\text{B20})$$

$$\Delta \hat{q}_t = q_t + u_t^q - \hat{q}_{t-1}. \quad (\text{B21})$$

The annual domestic and CPI inflation rates are given by

$$\bar{\pi}_t^d = \sum_{\tau=0}^{\tau=3} \pi_{t-\tau}^d \quad (\text{B22})$$

$$\pi_t = (1 - \eta) \pi_t^d + \eta \left[(1 - \kappa) \pi_{t-1}^M + \kappa (q_t - q_{t-1} + \pi_t^d) \right] \quad (\text{B23})$$

Then we can define 10 potential goal variables (or variables to be included in the policy rules) by equations (B19)–(B23), (B3), identities for y_t, π_t^d , and i_t , and the change in the interest rate, Δi_t .

To set up the system, define an $(n_1 \times 1)$ vector ($n_1 = 21$) of backward-looking (predetermined) state variables as

$$\begin{aligned} x_{1t} &\equiv \left\{ u_t^y, u_t^\pi, u_t^s, u_t^q, y_t, \pi_t^d, \pi_{t-1}^d, \pi_{t-2}^d, \pi_{t-3}^d, \pi_{t-1}^M, \pi_{t-2}^M, \pi_{t-3}^M, \right. \\ &\quad \left. y_t^f, \pi_t^f, \pi_{t-1}^f, \pi_{t-2}^f, i_t^f, q_t^A, \hat{q}_{t-1}, q_{t-1}, i_{t-1} \right\}' , \end{aligned} \quad (\text{B24})$$

an $(n_2 \times 1)$ vector ($n_2 = 3$) of forward-looking state variables as

$$x_{2t} \equiv \left\{ y_{t+1|t}, \pi_{t+1|t}^d, q_t \right\}', \quad (\text{B25})$$

an $(n_1 \times 1)$ vector of disturbances to the predetermined variables as

$$\begin{aligned} \varepsilon_{1t} \equiv & \left\{ \varepsilon_t^y, \varepsilon_t^\pi, \varepsilon_t^s, \varepsilon_t^s, \varepsilon_t^y, \varepsilon_t^\pi, \mathbf{0}'_{1 \times 6}, \varepsilon_t^{yf}, \varepsilon_t^{\pi f}, \mathbf{0}'_{1 \times 2}, \right. \\ & \left. g_{yf} \varepsilon_t^{yf} + g_{\pi f} \varepsilon_t^{\pi f}, \varepsilon_t^{\pi f} - \varepsilon_t^\pi, \mathbf{0}'_{3 \times 1} \right\}', \end{aligned} \quad (\text{B26})$$

and an $(n_z \times 1)$ vector ($n_z = 10$) of goal variables as

$$z_t \equiv \left\{ y_t, \pi_t^d, \bar{\pi}_t^d, \pi_t, \bar{\pi}_t, \Delta s_t, \hat{q}_t, \Delta \hat{q}_t, i_t, \Delta i_t \right\}'. \quad (\text{B27})$$

Then the model can be written on state-space form as

$$A_0 \begin{bmatrix} x_{1t+1} \\ \mathbf{E}_t x_{2t+1} \end{bmatrix} = A_1 x_t + B_1 i_t + \varepsilon_{t+1}, \quad (\text{B28})$$

or

$$\begin{bmatrix} x_{1t+1} \\ \mathbf{E}_t x_{2t+1} \end{bmatrix} = A x_t + B i_t + \varepsilon_{t+1}, \quad (\text{B29})$$

where

$$x_t \equiv \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}, \quad \varepsilon_{t+1} \equiv \begin{bmatrix} \varepsilon_{1t+1} \\ \mathbf{0}_{n_2 \times 1} \end{bmatrix}, \quad (\text{B30})$$

and $A \equiv A_0^{-1} A_1$, $B \equiv A_0^{-1} B_1$.²⁷

Defining ι_j as a row vector of suitable length with 1 in the j th position and zero elsewhere, the matrices A_0, A_1, B_1 are given by

$$A_0 = \begin{bmatrix} I_{n_1} & \mathbf{0}_{n_1 \times n_2} & & & \\ & \beta_y \varphi_y & 0 & 0 & \\ \mathbf{0}_{n_2 \times n_1} & 0 & \varphi_\pi & \gamma_q & \\ & 0 & 0 & \vartheta_R & \end{bmatrix}, \quad (\text{B31})$$

$$\begin{aligned} A_1 = & \left\{ \rho_y \iota'_1, \rho_\pi \iota'_2, \rho_s \iota'_3, \rho_q \iota'_4, \iota'_{22}, \iota'_{23}, \iota'_6, \iota'_7, \iota'_8, A'_{1,10}, \iota'_{10}, \iota'_{11}, \rho_{yf} \iota'_{13}, \rho_{\pi f} \iota'_{14}, \right. \\ & \left. \iota'_{14}, \iota'_{15}, A'_{1,17}, A'_{1,18}, \iota'_4 + \iota'_{24}, \iota'_{24}, \mathbf{0}'_{1 \times n}, A'_{1,22}, A'_{1,23}, A'_{1,24} \right\}', \end{aligned} \quad (\text{B32})$$

²⁷Note that $A_0^{-1} \varepsilon_t = \varepsilon_t$, since A_0 is block diagonal with an identity matrix as the block $(1 : n_1, 1 : n_1)$ and elements $(n_1 + 1 : n)$ of ε_t are all zero.

where

$$A_{1,10} = \kappa (\iota_6 - \iota_{20} + \iota_{24}) + (1 - \kappa) \iota_{10}, \quad (\text{B33})$$

$$A_{1,17} = g_{yf} \rho_{yf} \iota_{13} + g_{\pi f} [(1 + \rho_{\pi f}) \iota_{14} + \iota_{15} + \iota_{16}], \quad (\text{B34})$$

$$A_{1,18} = \rho_{\pi f} \iota_{14} + \xi_A \iota_{18} - \iota_{23} + (1 - \xi_A) \iota_{24} \quad (\text{B35})$$

$$A_{1,22} = -\rho_y \iota_1 - \beta_y (1 - \varphi_y) \iota_5 - \beta_{yf} \iota_{13} + \iota_{22} - 4\beta_r \iota_{23} - \beta_q \iota_{24}, \quad (\text{B36})$$

$$A_{1,23} = -\rho_\pi \iota_2 - (1 - \varphi_\pi) \iota_6 - \gamma_y \iota_{22} + \iota_{23}, \quad (\text{B37})$$

$$\begin{aligned} A_{1,24} &= -\iota_3 + \vartheta_D \xi_D \iota_6 + (\vartheta_R \rho_{\pi f} - \vartheta_D \xi_D) \iota_{14} - \frac{1}{4} \iota_{17} - \vartheta_A \xi_A \iota_{18} \\ &\quad - \vartheta_D \xi_D \iota_{20} - \vartheta_R \iota_{23} + \Omega_q \iota_{24}, \end{aligned} \quad (\text{B38})$$

and

$$B_1 = \{\mathbf{0}'_{20 \times 1}, 1, \beta_r, 0, 1/4\}'. \quad (\text{B39})$$

Likewise, the goal variables can be written as

$$z_t = C_x x_t + C_i i_t, \quad (\text{B40})$$

where

$$\begin{aligned} C_x &= \{l'_5, l'_6, (l'_6 + l'_7 + l'_8 + l'_9), C'_{x,4}, C'_{x,5}, C'_{x,6}, \\ &\quad l'_4 + l'_{24}, l'_4 + l'_{24} - l'_{19}, \mathbf{0}'_{1 \times n}, -l'_{21}\}', \end{aligned} \quad (\text{B41})$$

$$C_i = \{\mathbf{0}'_{8 \times 1}, 1, 1\}', \quad (\text{B42})$$

where

$$C_{x,4} = (1 - \eta + \eta\kappa) \iota_6 + \eta(1 - \kappa) \iota_{10} + \eta\kappa (\iota_{24} - \iota_{20}), \quad (\text{B43})$$

$$\begin{aligned} C_{x,5} &= (1 - \eta + \eta\kappa) \iota_6 + (1 - \eta) [\iota_7 + \iota_8 + \iota_9] \\ &\quad + \eta [(2 - \kappa) \iota_{10} + \iota_{11} + \iota_{12}] + \eta\kappa (\iota_{24} - \iota_{20}), \end{aligned} \quad (\text{B44})$$

$$C_{x,6} = \iota_6 - \iota_{14} - \iota_{20} + \iota_{24}. \quad (\text{B45})$$

The covariance matrix of the disturbance vector ε_{1t} is given by

$$\begin{aligned} \Sigma_{\varepsilon 1} &= \left\{ \sigma_y^2 [\iota_1 + \iota_5]', \sigma_\pi^2 [\iota_2 + \iota_6 - \iota_{18}]', \sigma_s^2 \iota'_3, \sigma_q^2 \iota'_4, \sigma_y^2 [\iota_1 + \iota_5]', \right. \\ &\quad \sigma_\pi^2 [\iota_2 + \iota_6 - \iota_{18}]', \mathbf{0}'_{6 \times n1}, \sigma_{yf}^2 [\iota_{13} + g_{yf} \iota_{17}]', \\ &\quad \left. \sigma_{\pi f}^2 [\iota_{14} + g_{yf} \iota_{17} + \iota_{18}]', \mathbf{0}'_{2 \times n1}, \Sigma'_{\varepsilon 1,17}, \Sigma'_{\varepsilon 1,18}, \mathbf{0}'_{3 \times n1} \right\}'. \end{aligned} \quad (\text{B46})$$

where

$$\Sigma_{\varepsilon 1,17} = g_{yf}\sigma_{yf}^2\iota_{13} + g_{\pi f}\sigma_{\pi f}^2[\iota_{14} + \iota_{18}] + (g_{yf}^2\sigma_{yf}^2 + g_{\pi f}^2\sigma_{\pi f}^2)\iota_{17}, \quad (\text{B47})$$

$$\Sigma_{\varepsilon 1,18} = \sigma_{\pi f}^2[\iota_{14} - \iota_2 - \iota_6] + g_{\pi f}\sigma_{\pi f}^2\iota_{17} + (\sigma_{\pi}^2 + \sigma_{\pi f}^2)\iota_{18}. \quad (\text{B48})$$

B.2 Solving the model

The central bank is assumed to minimize the intertemporal loss function

$$J_0 = \sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau}, \quad (\text{B49})$$

where the period loss function is given by

$$\begin{aligned} L_t &= \bar{\pi}_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 \\ &= z_t' K z_t, \end{aligned} \quad (\text{B50})$$

where K is a diagonal matrix of preference parameters. Using (B40), the period loss function can be written on the standard form as

$$\begin{aligned} L_t &= z_t' K z_t \\ &= \begin{bmatrix} x_t' & i_t' \end{bmatrix} \begin{bmatrix} C_x' \\ C_i' \end{bmatrix} K \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix} \\ &= x_t' C_x' K C_x x_t + x_t' C_x' K C_i i_t + i_t' C_i' K C_x x_t + i_t' C_i' K C_i i_t \\ &\equiv x_t' Q x_t + x_t' U i_t + i_t' U' x_t + i_t' R i_t, \end{aligned} \quad (\text{B51})$$

where

$$Q \equiv C_x' K C_x \quad (\text{B52})$$

$$U \equiv C_x' K C_i \quad (\text{B53})$$

$$R \equiv C_i' K C_i. \quad (\text{B54})$$

In the benchmark case when the central bank minimizes (B49) under *commitment*, it can be shown that the solution is given by²⁸

$$\underline{k_{1t+1}} \equiv \begin{bmatrix} x_{1t+1} \\ \zeta_{2t+1} \end{bmatrix} = M_c k_{1t} + \begin{bmatrix} \varepsilon_{1t+1} \\ \mathbf{0}_{n2 \times 1} \end{bmatrix}, \quad (\text{B55})$$

²⁸Söderlind (1999) shows how to calculate optimal policy rules and the associated dynamics in forward-looking rational expectations models. Throughout, the numerical solutions are calculated using Söderlind's Gauss routines.

$$k_{2t} \equiv \begin{bmatrix} x_{2t} \\ i_t \\ \zeta_{1t} \end{bmatrix} = N_c k_{1t}, \quad (\text{B56})$$

where ζ_{jt} is the vector of Lagrange multipliers associated with x_{jt} .

When the central bank optimizes its objective function under *discretion*, the optimal policy rule is given by

$$i_t = F_d x_{1t}, \quad (\text{B57})$$

leading to the dynamics

$$x_{1t+1} = M_d x_{1t} + \varepsilon_{1t+1}, \quad (\text{B58})$$

$$x_{2t} = N_d x_{1t}, \quad (\text{B59})$$

where $M_d = A_{11} + A_{12}N_d + B_1F_d$.

An arbitrary *simple rule* is given by

$$i_t = F_j x_t, \quad (\text{B60})$$

for $j = T, \Delta S, Q, \Delta Q$. To construct such a rule, first express it in terms of the goal variables as

$$i_t = F_j^z z_t, \quad (\text{B61})$$

where

$$F_j^z = \{f_{j,y}, \mathbf{0}_{1 \times 3}, f_{j,\pi}, f_{j,\Delta s}, f_{j,q}, f_{j,\Delta q}, \mathbf{0}_{1 \times 2}\}. \quad (\text{B62})$$

Since $F_j^z C_i = 0$ for all j , $F_j^z z_t = F_j^z C_x x_t$, and thus

$$F_j = F_j^z C_x. \quad (\text{B63})$$

Given this rule, the dynamics of the system is

$$x_{1t+1} = M_j x_{1t} + \varepsilon_{1t+1} \quad (\text{B64})$$

$$x_{2t} = N_j x_{1t}, \quad (\text{B65})$$

and the value of the loss function is

$$J_0 = x'_{10} V x_{10} + \frac{\delta}{1 - \delta} \text{tr}(V \Sigma_{\varepsilon 1}), \quad (\text{B66})$$

where Σ_{ε_1} is the covariance matrix of ε_{1t+1} and where V_s is determined by

$$V_s = P' \begin{bmatrix} Q & U \\ U' & R \end{bmatrix} P + \delta M_j V_{s+1} M_j, \quad (\text{B67})$$

where

$$P = \begin{bmatrix} I_{n1} \\ N_j \\ F_j \begin{bmatrix} I_{n1} \\ N_j \end{bmatrix} \end{bmatrix}. \quad (\text{B68})$$

Thus, an optimized simple rule can be found by minimizing J_0 in equation (B66).

B.3 Calculating unconditional variances

In the case of *commitment*, the covariance matrix of k_{1t+1} satisfies

$$\Sigma_{k1} = M_c \Sigma_{k1} M_c' + \Sigma_{\varepsilon k1}, \quad (\text{B69})$$

where

$$\Sigma_{\varepsilon k1} = \begin{bmatrix} \Sigma_{\varepsilon_1} & \mathbf{0}_{n1 \times n2} \\ \mathbf{0}_{n2 \times n1} & \mathbf{0}_{n2 \times n2} \end{bmatrix}. \quad (\text{B70})$$

Thus, Σ_{k1} is given by

$$\text{vec}(\Sigma_{k1}) = [I_{n^2} - M_c \otimes M_c]^{-1} \text{vec}(\Sigma_{\varepsilon k1}). \quad (\text{B71})$$

Since

$$\begin{aligned} k_{2t+1} &= N_c k_{1t+1} \\ &= N_c (M_c k_{1t} + \varepsilon_{k1t+1}), \end{aligned} \quad (\text{B72})$$

stacking k_{1t+1} and k_{2t+1} , we get

$$k_{t+1} = H_c k_{1t}, \quad (\text{B73})$$

where

$$k_{t+1} = \begin{bmatrix} k_{1t+1} \\ k_{2t+1} \end{bmatrix}, \quad H_c = \begin{bmatrix} I_n \\ N_c \end{bmatrix}. \quad (\text{B74})$$

Thus the covariance matrix of k_{t+1} is given by

$$\Sigma_k = H_c \Sigma_{k1} H_c'. \quad (\text{B75})$$

Finally, since the goal variables are given by

$$z_t = C_x x_t + C_i i_t \quad (\text{B76})$$

$$\begin{aligned} &= \begin{bmatrix} C_x & C_i \end{bmatrix} \begin{bmatrix} x_t \\ i_t \end{bmatrix} \\ &= C_c \begin{bmatrix} x_t \\ i_t \end{bmatrix}, \end{aligned} \quad (\text{B77})$$

its covariance matrix is

$$\Sigma_z = C_c \Sigma_{xi} C_c', \quad (\text{B78})$$

where Σ_{xi} is the covariance matrix of the stacked vector $\{x_t', i_t'\}'$, picked out from the matrix Σ_k .

Under *discretion*, the covariance matrix of x_{1t} is given by

$$\text{vec} \Sigma_{x1} = [I_{n1^2} - M_d \otimes M_d]^{-1} \text{vec} \Sigma_{\varepsilon 1}, \quad (\text{B79})$$

and since the goal variables are given by (partitioning C_x conformably with x_{1t}, x_{2t})

$$\begin{aligned} z_t &= C_x x_t + C_i i_t \\ &= \begin{bmatrix} C_{x1} & C_{x2} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + C_i F_d x_{1t} \\ &= \begin{bmatrix} C_{x1} & C_{x2} \end{bmatrix} \begin{bmatrix} x_{1t} \\ N_d x_{1t} \end{bmatrix} + C_i F_d x_{1t} \\ &= C_d x_{1t}, \end{aligned} \quad (\text{B80})$$

where $C_d \equiv [C_{x1} + C_{x2} N_d + C_i F_d]$, its covariance matrix is

$$\Sigma_z = C_d \Sigma_{x1} C_d'. \quad (\text{B81})$$

Under a *simple rule*, the covariance matrix of x_{1t} is similarly given by

$$\text{vec} \Sigma_{x1} = [I_{n1^2} - M_j \otimes M_j]^{-1} \text{vec} \Sigma_{\varepsilon 1}, \quad (\text{B82})$$

and since the goal variables are given by (partitioning also F_j)

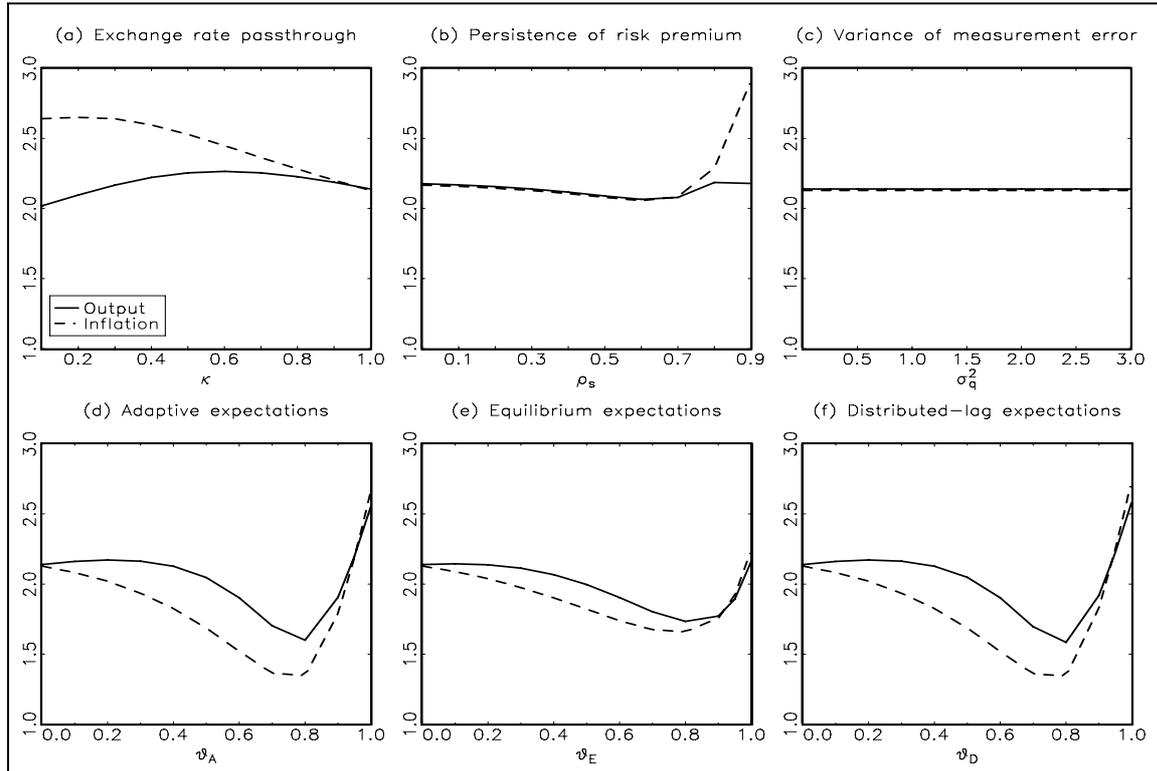
$$\begin{aligned} z_t &= \begin{bmatrix} C_{x1} & C_{x2} \end{bmatrix} \begin{bmatrix} x_{1t} \\ N_j x_{1t} \end{bmatrix} + C_i \begin{bmatrix} F_{j1} & F_{j2} \end{bmatrix} \begin{bmatrix} x_{1t} \\ N_j x_{1t} \end{bmatrix} \\ &= C_j x_{1t}, \end{aligned} \quad (\text{B83})$$

where $C_j \equiv [C_{x1} + C_i F_{j1} + C_{x2} N_j + C_i F_{j2} N_j]$, its covariance matrix is

$$\Sigma_z = C_j \Sigma_{x1} C_j'. \quad (\text{B84})$$

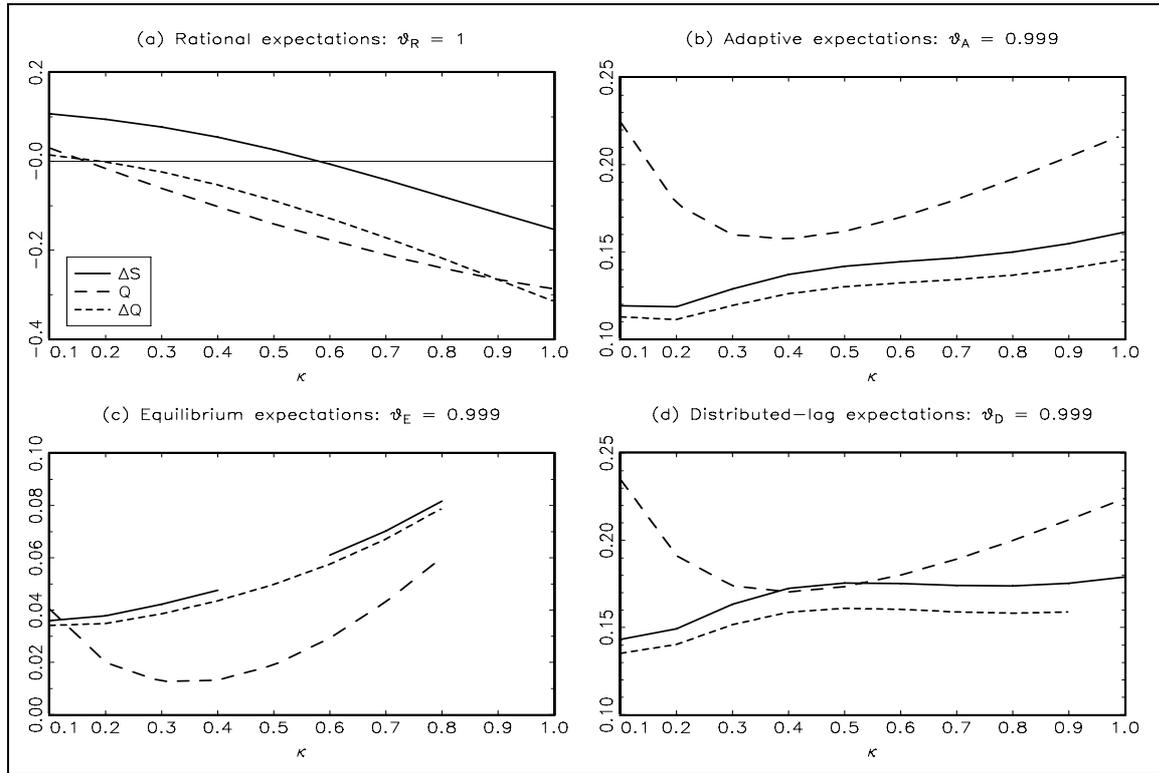
C Additional figures

Figure C.1: Optimized output and inflation coefficients in different model configurations



Optimized coefficients on inflation and output.

Figure C.2: Exchange rate coefficients when varying exchange rate pass-through under fully rational and non-rational expectations



Optimized exchange rate coefficients, given output and inflation coefficients.

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