Global Real Rates: A Secular Approach

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The Questions We Address:

- Why have global real interest rates declined so much?

- Propose a simple empirical approach using the world budget constraint and historical data.
  1. Gives us insights regarding the forces behind low frequency movements in real rates.
  2. Allows us to forecast future global real rates.
Global Interest Rates (10-year nominal yields)

Ex-ante real yields on U.S. Treasury Securities constructed using median expected price changes from the University of Michigan’s Survey of Consumers. Source: FRED.
The figure reports the annualized ex-post real 3-month interest rate for the U.S. since 1871.

Facts and Possible Stories


- Secular Stagnation:


- Deleveraging after the crisis (Eggertson Krugman (2012); Guerrieri and Lorenzoni (2011); Lo and Rogoff (2015))

- Technological progress is capital-augmenting (decline in price of investment goods), Sajedi & Thwaites (2016)
Theoretical framework

- Law of accumulation of wealth for the world (closed economy):

\[ \tilde{W}_{t+1} = \tilde{R}_{t+1}(\tilde{W}_t - C_t) \]

- \( \tilde{W}_t \): Total Private wealth: financial wealth (incl. gov. debt) as well as housing, non incorporated businesses, land, + human wealth; \( \tilde{R}_{t+1} \) gross return on total private wealth; \( C_t \) world private consumption. No Ricardian equivalence.

- Accounting identity.
Theoretical Framework

- Most models deliver a stationary $C/\bar{W}$. Details unimportant.

- Log-linearize around the steady-state consumption-wealth ratio and derive the world’s intertemporal budget constraint (Campbell (1986) or Lettau and Ludvigson (2001)):

$$\ln \frac{C_t}{\bar{W}_t} \approx \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left( \bar{r}_{t+s} - \Delta \ln C_{t+s} \right)$$

- Today’s aggregate consumption to wealth ratio is low if:
  - Expected future rates of return on wealth are low
  - Expected future aggregate consumption growth is high
Theoretical Framework: Two Adjustments

- **Private wealth vs. human wealth.** $\bar{W} = W + H$. $H$ unobserved.

\[ \ln C_t/W_t \leq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_s^w (r_{t+s}^w - \Delta \ln C_{t+s}) + \varepsilon_t \]

with $\varepsilon_t \propto \mathbb{E}_t \sum_{s=1}^{\infty} \rho_s^w (r_{t+s}^h - r_{t+s}^w) - (\ln W_t - \ln H_t)$. Interpretation.

- **Safe and risky returns.** write $r^w = r^f + r^p_w$.
  - proxy $r^p_w$ with $r^p_w = \nu r^p$ where $r^p$ is equity excess return
  - estimate $\nu$ from the data to maximize fit.

- **Present value relation:**

\[ \ln C_t/W_t \leq \mathbb{E}_t \sum_s \rho_s^w r_{t+s}^f + \nu \mathbb{E}_t \sum_s \rho_s^w r^p_{t+s} - \mathbb{E}_t \sum_s \rho_s^w \Delta \ln C_{t+s} + \varepsilon_t \]

\[ \equiv cW_t^f + cW_t^{rp} + cW_t^c + \varepsilon_t \]
Identify co-movements of $\ln C/W$ and components.

- **Productivity Growth**
  Euler equation: $E_t r^w_{t+1} = \rho + \sigma E_t g_{t+1}$

  $\ln C_t/W_t \simeq \sigma E_t \sum_s \rho^s_w g_{t+s} + \nu E_t \sum_s \rho^s_w r p_{t+s} - F_t \sum_s \rho^s_w g_{t+s} + \varepsilon_t$

  $\equiv c w^f_t + c w^{r_p}_t + c w^c_t + \varepsilon_t$

  $c w^f$ and $c w^c$ negatively correlated. Impact on $C/W$ depends on $\sigma - 1$. IES close to 1: no impact on $C/W$.

- **Demographics**
  Write $\Delta \ln C_t = \Delta \ln c_t + n_t$.
  $C/W$ depends on direct effect ($c w^c$) and indirect effect ($c w^f$).
  Literature suggests $n < 0$ increases savings: $corr(c w^f, c w^c) < 0$. 

Identification II

- **Deleveraging**: Shock to $\rho$.
  - *Outside the ELB*:
    \[
    \ln \frac{C_t}{W_t} \simeq \mathbb{E}_t \sum_s \rho_w \rho_{t+s} + \nu \mathbb{E}_t \sum_s \rho_w r_{p_{t+s}} - 0 \\
    \equiv cw_t^f + cw_t^{rp} + cw_t^c
    \]

- *At the ELB*:
  \[
  \mathbb{E}_t r_{t+1}^f = 0 \quad \text{and} \quad \mathbb{E}_t \Delta \ln C_{t+1} = -\rho_{t+1} \\
  \ln \frac{C_t}{W_t} \simeq 0 + \nu \mathbb{E}_t \sum_s \rho_w r_{p_{t+s}} + \mathbb{E}_t \sum_s \rho_w \rho_{t+s} \\
  \equiv cw_t^f + cw_t^{rp} + cw_t^c
  \]

Either way, low $\rho_{t+s}$ lowers $C/W$
Positive comovement with $cw_t^f$ and $cw_t^c$.  

Demand for Safe Assets

Separate IES from CRRA and shock CRRA. Epstein-Zin preferences:

\[
U_t = \left\{ (1 - \beta)C_t^{1-\sigma} + \beta \left( \mathbb{E}_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}}
\]

Risk-free rate and risk premium: \((\theta_t = (1 - \gamma_t)/(1 - \sigma))\)

\[
r_{t+1}^f = \rho - \frac{1 - \theta_t}{2} \sigma_{r,t}^2
\]

\[
\mathbb{E}_t r_{t+1}^w - r_{t+1}^f = (1 - \theta_t)\sigma_{r,t}^2.
\]

\[
\ln C_t/W_t \simeq -\frac{1}{2} \mathbb{E}_t \sum_s \rho_w^s (1 - \theta_{t+s}) \sigma_{r,t+s}^2 + \mathbb{E}_t \sum_s \rho_w^s (1 - \theta_{t+s}) \sigma_{r,t+s}^2 + 0
\]

\[
\equiv cw_t^f + cw_t^{rp} + cw_t^c
\]

\(cw^f\) negatively correlated with \(cw^{rp}\) and \(C/W\)
Data

- World is an aggregate of the United States, the United Kingdom, Germany and France.

- Historical data on private wealth, population and private consumption for the period 1870-2011 for the United States, and 1920-2011 for the United Kingdom, Germany and France from Piketty et al. (2014) and Jordà et al. (2016).
  - Risk-free return: ex-post real return on three-months Treasuries minus CPI inflation.
  - Real return on risky assets: total equity return for each country minus CPI inflation.
‘Global’ Consumption & Wealth per capita, 1920-2011

The figure reports the ratio of aggregate annual private consumption expenditures to private wealth (land, housing, financial assets) for the U.S., U.K., Germany and France. Source: Jordà et al (2016), Piketty & Zucman (2014). Mean: $C/W = 0.2$. 

‘Global’ Consumption/Wealth Ratio, 1920-2011
The figure reports the ratio of aggregate annual private consumption expenditures to private wealth (land, housing, financial assets) for the U.S. Source: Jordà et al (2016), Piketty & Zucman (2014).
Empirical Framework

▶ Assume $\rho_w = 0.96$ (Equivalently, asset income/output $\approx 0.2$)

▶ Construct $c\omega^i$ using a reduced form VAR($p$)

▶ Estimate $\nu$ to maximize fit, Find $\hat{\nu} = 0.37$. (interpretation)
The figure decomposes the fluctuations in \( \ln(C/W) \) around its mean into a risk-free component \( (cw^f) \), an excess return component \( (cw^{rp}) \) and a consumption growth component \( (cw^c) \).
The figure decomposes \( \ln(C/W) \) into a risk-free component (\( cw^f \)), an excess return component (\( cw^{rp} \)) and a consumption growth component (\( cw^c \)).
The figure decomposes $\ln(C/W)$ into a risk-free component ($cw^f$), an excess return component ($cw^{rp}$) and a consumption growth component ($cw^c$).
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## Unconditional Variance Dec.

<table>
<thead>
<tr>
<th>#</th>
<th>percent</th>
<th>U.S.</th>
<th>G4</th>
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<tbody>
<tr>
<td>1</td>
<td>$\beta_{rf}$</td>
<td>1.364</td>
<td>1.406</td>
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<tr>
<td>2</td>
<td>$\beta_{rp}$</td>
<td>0.005</td>
<td>0.025</td>
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<tr>
<td>3</td>
<td>$\beta_{c}$</td>
<td>-0.329</td>
<td>-0.336</td>
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<tr>
<td></td>
<td>of which:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$\beta_{cp}$</td>
<td>0.056</td>
<td>-0.168</td>
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<tr>
<td>4</td>
<td>$\beta_{n}$</td>
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<td>-0.168</td>
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<tr>
<td>5</td>
<td>Total</td>
<td>1.041</td>
<td>1.094</td>
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<tr>
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<td>(lines 1+2+3)</td>
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### Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>$\ln C/W$</th>
<th>$cw^f$</th>
<th>$cw^{rp}$</th>
<th>$cw^c$</th>
<th>$cw^n$</th>
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</thead>
<tbody>
<tr>
<td>$\ln C/W$</td>
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<td>0.901</td>
<td>0.121</td>
<td>-0.659</td>
<td>-0.872</td>
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<td>$cw^f$</td>
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<td>1.000</td>
<td>-0.054</td>
<td>-0.658</td>
<td>-0.923</td>
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<td>$cw^{rp}$</td>
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<td>1.000</td>
<td>0.0538</td>
<td>1.000</td>
<td>0.734</td>
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<tr>
<td>$cw^c$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
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</tr>
<tr>
<td>$cw^n$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
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Results & Interpretation

▸ Very good fit of the decomposition

▸ Most of the movements in $C/W$ reflect expected movements in the future risk-free rate

▸ Productivity and demographic shocks: small contribution overall.


▸ Demand for Safe Assets: negative correlation between risk premium and risk free component. Some contribution.
Interpretation

- Most of the action is in the joint dynamics of the consumption wealth ratio and the return component, particularly the risk free rate.

- Plausible interpretation:
  - ‘Irrational exuberance’ in asset prices (in the 1920s and in the 1990-2000s) leads to fast growing financial wealth and fast declining consumption-wealth ratios.
  - Large financial crises (in 1929 and in 2008) lead to deleveraging (increased savings and low consumption) for an extended time (low consumption wealth ratios) and to low real rates.
  - Therefore low consumption wealth ratios tend to be associated with low real rate components.

- This is consistent with debt overhang effects (Reinhart and Rogoff (2014)) and a global financial boom/bust cycle (Miranda-Agrippino & Rey (2015)).

- Demand for safe assets seem to play some role (Caballero et al (2016))
Interpretation

- Deleveraging post crisis leads to increased demand for ‘safe’ assets and low risk free rate. Should also be associated with some negative comovements between risk free rate and risk premium. A bit of that.

- Technological slowdown or demographic factors: Return component and consumption growth component should exhibit negative comovements.

- But most of the action is in the joint dynamics of the consumption wealth ratio and the risk free rate. How predictive is this relation?
Predicting Global Real Risk-free Rates

- Predictive power of the consumption-wealth ratio:

\[ y_{t+k} = \alpha + \beta cw_t + \epsilon_{t+k} \]

- \( y_{t+k} \) denotes the variable we are trying to forecast at horizon \( k \) and \( cw_t \) is the consumption-wealth ratio at the beginning of period \( t \).

- Candidates are: real risk free rates, equity premium, consumption growth per capita, population growth, term premium.

- Strong predictive power for long run real rates. (Adj.\( R^2 \) is 0.43 on a 10 year horizon).
The figure forecasts the 10-year average future short risk-free rate using \( \ln(C/W) \). Graph includes 2 standard deviation bands.

2011-2021 forecast: \(-1.3\%\)
The figure forecasts the 10-year average future term premium using $\ln(C/W)$. Graph includes 2 standard deviation bands.

**2011-2021 forecast:** 1.22%
Conclusions

- We use a very general almost a-theoretical framework to understand determinants of long run real rates.

- Empirical evidence favors global financial boom/bust cycle.

- Euphoria pre-crisis leads to rapid increase in wealth (1920s, 1990s). This is followed by deleveraging post crisis (1929, 2008) and increased demand for ‘safe’ assets.

- Hence low consumption-wealth ratios are associated with lower future real rates.

- Little evidence for technological slowdown or demography factors (?)

- Predictive power: How long will the real rates stay low? Into next decade! Unless major macroeconomic policy changes.