The Empirical Implications of the Interest-Rate Lower Bound

Christopher Gust, David López-Salido, and Matthew E. Smith
Federal Reserve Board

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Disclaimer

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Motivation

- The ZLB can limit the corrective response of monetary policy to adverse shocks. A key question is how much did the ZLB constrain policy during the last recession.

- We address this question by estimating a nonlinear version of a New Keynesian model widely used in monetary economics.

- Our methodology can be applied in other contexts in which occasionally binding constraints have important economic consequences (e.g., financial constraints).
Preview of Main Results

- Our estimates suggest that 20% of the drop in U.S. GDP during the last recession was due to the ZLB.

- The credibility of this finding depends upon the empirical plausibility of our model. We find that:
  
  - The model provides a reasonably good fit of the dynamics for output, inflation, and the nominal interest rate.
  
  - The model matches up well with evidence from financial markets and professional forecasters regarding the expected duration of the ongoing ZLB spell. Matches this evidence even though it was not used to estimate the model.
Key Features of the Model

- A nonlinear New Keynesian model featuring:
  - Representative household with habits for consumption.
  - Monopolistically-competitive firms that face adjustment costs in changing prices.
  - Monetary policy operates according to an interest rate rule that is constrained by the lower bound.
  - The sources of uncertainty are a \textit{unit root} shock to technology, and \textit{AR(1)} shock to the intertemporal allocation of demand, and an \textit{iid} monetary policy surprise.
Nonlinear Equilibrium Conditions

- An intertemporal Euler equation:

\[ \lambda_t = \frac{\beta}{\eta_t} R_t E_t \{ \lambda_{t+1} \pi_{t+1}^{-1} \}, \quad \text{where} \quad \lambda_t = [c_t - \gamma c_{t-1}]^{-1} \]

- A price-setting relationship:

\[
\left( \frac{\pi_t}{\bar{\pi}} - 1 \right) \frac{\pi_t}{\bar{\pi}} = \frac{\beta}{\eta_t} E_t \left\{ \frac{\lambda_{t+1} y_{t+1}}{\lambda_t y_t} \left( \frac{\pi_{t+1}}{\bar{\pi}} - 1 \right) \frac{\pi_{t+1}}{\bar{\pi}} \right\} - \frac{\epsilon - 1}{\phi} + \frac{\epsilon}{\phi} \frac{1}{\lambda_t} \\

y_t = c_t + \frac{\phi}{2} \left( \frac{\pi_t}{\bar{\pi}} - 1 \right)^2 y_t \quad y_t = Z_t h_t
\]

- An interest rate rule for the desired or notional interest rate:

\[
R_t^* = \left( \frac{\bar{\pi} G}{\beta} \right)^{1-\rho_R} (R_{t-1}^*)^{\rho_R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\gamma_\pi} \left( \frac{y_t}{y_{t-1}} \right)^{\gamma_y} \exp(\epsilon_{R,t})
\]

- Lower bound constraint:

\[ R_t = \max[1, R_t^*] \]
We could have imposed the ZLB but log-linearized the remaining equilibrium conditions (i.e., constrained linear). We found this solution method inaccurate at the lower bound.

Instead, we use a projection method: collocation along with Chebychev polynomials.

We do not approximate the decision rules with polynomials directly. Our approximation takes a piecewise approach to account for the kink associated with the lower bound.

We find a large improvement in accuracy from this approach relative to the constrained linear approach.
Estimation Procedure

- We use the particle filter within a Metropolis-Hastings (MH) algorithm.

- As observables, we use U.S. data on GDP growth, inflation, and the nominal interest rate from 1983Q1-2011Q4.

- We follow Smith (2011) and introduce a surrogate into the MH algorithm. The surrogate is used to prescreen proposed values to avoid expensive likelihood evaluations. For our surrogate, we use linearization with the Kalman filter.
### Posterior Distribution of Selected Parameters

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<th>stdev</th>
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The Path of Estimated Shocks
What pushed the economy to the ZLB?
Contribution of the ZLB to the Great Recession

- Output Level
- Inflation
- Nominal Interest Rate
- Notional Rate

Data
Fitted
No ZLB Counter
95% Bands

2008 2009 2010 2011 2012
94
95
96
97
98
99
100
101

−8
−6
−4
−2
0
2
4
6
Annual Percent
Annual Percent
Annual Percent
Annual Percent

Notional Rate
Nominal Interest Rate
Inflation
Output Level

Data
Fitted
No ZLB Counter
95% Bands
Distribution of the Probability of Hitting the ZLB

The CDF of the $\text{Prob}(R_t = \bar{R})$

- 50% of samples outside ZLB
- Average: 3.1%

Right Tail of Population Histogram of $\text{Prob}(R_t = \bar{R})$

- 97.5th Percentile
- 99.5th Percentile

Right Tail of Sample Histogram of $\text{Prob}(R_t = \bar{R})$

- 97.5th Percentile
- 99.5th Percentile

50% of samples outside ZLB

Average: 3.1%
Distribution of the Duration of a ZLB Spell

- Estimated CDF of Duration of ZLB Spells
  - \( \Pr(\text{Duration}<4) = 78\% \)
  - \( \Pr(\text{Duration}>12) = 2.6\% \)

- Right Tail of Histogram of Duration of ZLB Spells
  - 97.5th Percentile
  - 99.5th Percentile
Expected Duration of U.S. Lower Bound Spell

Financial Market Federal Funds–Rate Expectations

Blue Chip 3–month Treasury Bill Expectations
Conclusions

- The ZLB constraint was a significant factor in exacerbating the last recession; accounted for 20% of the drop in output.

- Duration of the ZLB spell implied by the model in line with financial markets and Blue-Chip expectations: the protracted nature of the ongoing ZLB spell was largely unexpected.

- Ongoing work: Extend the model to incorporate endogenous capital and financial frictions to better account for the nature of the shocks that push the economy to the ZLB and their effects on the composition of aggregate demand.

- Apply our methodology to the Great Depression and Japan.
Thank you for your attention.
Figure 1: Smoothed Estimates of Model Objects
Figure 2: The Dynamic Path of Alternative Initial Conditions
Figure 3: The Effects of a Discount Rate Shock
Figure 4: The Effects of a Productivity Shock
Figure 5: The Path of the Estimated Shocks

Discount Rate Shock

Productivity Innovation

Monetary Policy Innovation
Figure 11: The Dynamic of Short and Long Lower Bound Spells
Figure B.1: Comparison of Model Solutions (2009:Q2 Initial Conditions)