Resolving the Spanning Puzzle in Macro-Finance Term Structure Models

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The views expressed here are those of the authors and do not necessarily represent the views of others in the Federal Reserve System.
Macro-finance term structure models

Yield curve analysis before Ang and Piazzesi

- **Macro**
  - Taylor rule connects short rate to macro variables
  - Long-term rates and risk premia ad hoc or ignored
- **Finance**
  - Affine no-arb models capture entire yield curve (Duffie-Kan)
  - Latent factors lack economic interpretation

Ang and Piazzesi (2003) and onward

- Combine Taylor rule and affine no-arbitrage model
- “Macro-Finance Term Structure Models” (MTSMs)
- Ability to analyze macro-yield interactions
  - Responses of yield curve and risk premia to macro shocks
  - Effects of monetary policy on yields and premia
MTSM literature

- Reduced-form MTSMs

- Structural MTSMs
A new road-block: what we call the “spanning puzzle”

- MTSMs generally imply *spanning*
  - Macro variation completely captured by yields
  - Regression of macro on yields should have high $R^2$

- This conflicts with evidence on unspanned macro risks

- Serious challenge for entire macro-finance literature
  - Gürkaynak and Wright (2012): “*thorny issue with the use of macroeconomic variables in affine term structure models*”
  - Kim (2009): “*may undermine the validity of the models that use inflation as a state variable*”
  - Duffee (2012a): “*important conceptual difficulty with macro-finance models*”
Joslin, Priebsch, Singleton (JPS, 2014)

- JPS critique: “current generation of MTSMs [...] enforce[s] strong and counterfactual restrictions on how the macroeconomy affects yields”

- JPS find that empirically, unspanned macro risks play large role for term premia

- Therefore, JPS develop new type of MTSM with unspanned macro factors as a “large step toward bringing MTSMs in line with the historical evidence”

- New trend: models with unspanned/hidden factors
Are economic activity measures really unspanned?
Open questions about unspanned macro risk

- What is the right way to model macro-finance interactions?
- Are unspanned models the only solution to the spanning puzzle?
- Are all macro variables unspanned?
- How does a monetary policy rule fit in macro-finance interaction?
- Do macro variables have robust predictive power for excess bonds returns/future yields?
- What is unspanned macro risk?
This paper

Contributions

- Salvage spanned macro-finance term structure models
- Critically assess the role of unspanned macro risk

Results

- Macro variables closely linked to monetary policy ("policy factors") display little evidence of unspanned macro risk
- Conventional spanned MTSMs with policy macro factors and small measurement errors are consistent with the data
- Knife-edge restrictions of unspanned MTSMs are rejected
Outline

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The Spanning Puzzle in Macro-Finance

Saving Spanned Macro-Finance Models

Reassessing Unspanned Macro-Finance Models

Conclusion
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Conventional MTSMs imply theoretical macro spanning

- Yields are affine in \( N \) risk factors:

\[
Y_t = A + BX_t
\]

- Risk factors \( X_t \) contain macro variables.
- Outside of knife-edge cases we have invertibility:

\[
X_t = (B_N)^{-1}(Y_t^{(N)} - A_N)
\]

- That is, macro factors are spanned by yields.
  - Yields completely capture macro information.
  - In theory, regression of yields on macro should give \( R^2 \approx 1 \).
Spanning puzzle: theoretical spanning vs. evidence of unspanned macro risks

- In fact, low explanatory power of yields for macro variables
  - Regressions of macro variables on contemporaneous yield curve (principal components)
  - “$R^2$ are on the wrong side of $1/2$” (Duffee, 2013b)
  - Duffee (2013a,b), JPS

- Also, macro variables help predict future yields/returns
  - Predictive regressions for excess bond returns using yields and macro variables
  - Some macro variables have (in-sample) predictive power
  - Cooper and Priestley (2009), Ludvigson and Ng (2009, 2010), JPS

- Finally, persistence of macro variables not fully captured by yields
  - Lags of macro variables matter when controlling for yields
  - Duffee (2013a,b)
Two solutions to the spanning puzzle

- JPS solution: Throw out past spanned models and adopt unspanned models
  - Premise: spanned models are invalid
  - Impose restrictions so that all macro factors are unspanned
  - No direct link from macro factors to yields

- Our new solution: Save spanned MTSMs—when constructed with appropriate policy-relevant macro variables
  - Document tight link between policy-relevant macro variables and the yield curve
  - Estimate spanned models with these policy factors
  - Show that these spanned models (with small measurement errors) are consistent with the regression evidence
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Are all macro factors unspanned?

<table>
<thead>
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<th></th>
<th>Macro-spanning</th>
<th>Returns</th>
<th>Policy rule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 PCs</td>
<td>level</td>
<td>slope</td>
</tr>
<tr>
<td><strong>Policy factors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp. gap</td>
<td>0.72</td>
<td>0.01</td>
<td>0.67</td>
</tr>
<tr>
<td>Output gap</td>
<td>0.57</td>
<td>0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>INF (JPS)</td>
<td>0.81</td>
<td>0.74</td>
<td>0.03</td>
</tr>
<tr>
<td>Core CPI (yoy)</td>
<td>0.81</td>
<td>0.67</td>
<td>0.04</td>
</tr>
<tr>
<td>Core PCE (yoy)</td>
<td>0.77</td>
<td>0.60</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Non-policy factors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRO (JPS)</td>
<td>0.28</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Real GDP (ma3)</td>
<td>0.14</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Real GDP (yoy)</td>
<td>0.20</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>IP (ma3)</td>
<td>0.32</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td>Payroll (ma3)</td>
<td>0.20</td>
<td>0.04</td>
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▶ Monthly observations from Jan-1985 to Dec-2007 (as in JPS)
▶ Unsmoothed Fama-Bliss Treasury yields – 3m, 6m, 2-10y
▶ Annual excess returns, averaged over 1-10y bonds
A spanned model with policy factors

- Risk factors $X_t = (P_t, M_t)$ are observable
  - Yield factors: First two/three PCs of yield curve
  - Macro factors: Unemp. gap, Core CPI

- Model specification
  - Gaussian VAR for $X_t$
  - Affine short rate
  - Essentially-affine, unrestricted risk prices
  $\Rightarrow$ Gaussian VAR for $X_t$ under $Q$-measure

- Estimation with Maximum Likelihood
  - Canonical form of Joslin, Le, Singleton (2013a)
  - $iid$ measurement errors, equal variance for all maturities

$\Rightarrow$ Models $SM(2, 2)$ and $SM(3, 2)$
Simulation study of spanning implications

- How empirically relevant is theoretical spanning?

- Simulate yield and macro data from estimated models
  - 1000 data sets of length $T = 276$
  - Simulate risk factors
  - Obtain fitted yields using affine loadings
  - Add small iid measurement error, SD $\sigma_e$
  - Obtain PCs of simulated yields

- Investigate spanning in simulated vs. actual data
  - Regress macro variables on yield PCs
  - Predict excess bond returns with yields and macro, test for joint significance of macro data
## Simulation results — Model $SM(2, 2)$

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>CPI</td>
<td>UGAP</td>
<td>$R^2$ Y</td>
</tr>
<tr>
<td><strong>Data</strong></td>
<td>0.81</td>
<td>0.73</td>
<td>0.33</td>
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Data vs. means (and SDs) across 1,000 simulations, four PCs
Simulation results — Model SM(2, 2)

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<td>CPI UGAP</td>
<td>$R^2$ Y $R^2$ Y+M</td>
</tr>
<tr>
<td>Data</td>
<td>0.81 0.73</td>
<td>0.33 0.39</td>
</tr>
<tr>
<td>$\sigma_e = \sigma_e^{MLE}$</td>
<td>0.66 0.66</td>
<td>0.29 0.34</td>
</tr>
<tr>
<td></td>
<td>(0.17) (0.16)</td>
<td>(0.12) (0.12)</td>
</tr>
<tr>
<td>$\sigma_e = 1bp$</td>
<td>0.96 0.77</td>
<td>0.32 0.35</td>
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<td>1.00</td>
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Data vs. means (and SDs) across 1,000 simulations, four PCs
Conclusions from simulation study

- Measurement error breaks theoretical spanning
- Only small amount of noise needed to match data
  - SD of yields is 1.75-2.00 %
  - SD of measurement errors (MLE) is 0.05-0.25 %
  - Even 1 bp leads to significant wedge
- Why include measurement errors?
  - Needed to avoid stochastic singularity of parsimonious factor model
  - Also needed to avoid macro spanning!

⇒ Spanned macro-finance models can be consistent with the data
Are spanned MTSMs consistent with reasonable policy rules?

- Role of monetary policy
  - Our resolution of the spanning puzzle focuses on monetary policy and the use policy factors
  - This only makes sense if spanned MTSMs imply reasonable monetary policy rules

- Some studies find implausible coefficients on measures of slack and inflation
  - Ang, Dong, Piazzesi (2007), Ang, Boivin, Dong (2011)

- Maybe the model-implied policy rule is not identified?
  - Joslin, Le, Singleton (2013b)

- These studies view short-rate equation of MTSMs as the policy rule
Orthogonality

- Yield factors are typically not orthogonal to macro factors
  - Coefficients in short-rate equation are not policy-rule coefficients
  - Macro variables are correlated with “policy shock”
  - Lack of identification

- Orthogonality is a fundamental premise of Taylor-rule and SVAR literature

- We can impose orthogonality on our MTSM
  - Rotate risk factors: $X_t^* = (P_t^*, M_t)$, $P_t^* \perp M_t$
  - Now short-rate equation is a policy rule
  - Affine loadings reveal yield responses
  - Straightforward to obtain variance decomposition
Model-implied policy rules

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>$cor(e_t, M_t)$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int. CPI UGAP CPI UGAP</td>
<td>0.00 0.00 0.77</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.42 1.51 1.30</td>
<td></td>
</tr>
<tr>
<td>$SM(2, 2)$ orthogonal</td>
<td>0.42 1.47 1.33</td>
<td>0.00 0.00 0.80</td>
</tr>
<tr>
<td>PC1, PC2</td>
<td>0.54 0.27 -0.05</td>
<td>0.61 0.50</td>
</tr>
</tbody>
</table>

- Orthogonal rotation uniquely identifies policy-rule coefficients
- Coefficients closely in line with OLS estimates
- Taylor principle satisfied
- In the paper: effects of shocks, macro determinants of yields

▶
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Unspanned macro-finance models

- Joslin, Priebsch, Singleton (2014) propose unspanned models as resolution to the spanning puzzle

- Unspanned macro factors
  - Yields only depend on yield factors
    \[ Y_t = A + B_P P_t + 0 \cdot M_t \]
  - No direct link from macro to yields
  - Macro factors only affect \( E_t(Y_{t+h}), \ h > 0 \)

- JPS justify this with regression-based and model-based evidence
  - Cross-sectional spanning regressions
  - Excess return regressions
  - Tests of spanning restrictions in JPS model
  - Term premium results for unspanned vs. spanned model
Evidence for unspanned macro models

- Cross-sectional spanning regressions
  - We find many variables to be almost spanned
  - For policy factors, 70-80% of variation is explained by yields

- Predictive power of macro variables
  - Evidence is shaky — very few macro variables work, significance (HAC), stability across sample periods
  - We only have *in-sample* evidence.
  - “Is there really information [...] not captured by the current yield curve? [...] Perhaps not.” Duffee (2013b)
  - There should be a high bar for inclusion of macro factors.

- Now consider direct tests of unspanned model
  - What does the $\chi^2$ test in JPS tell us?
  - Are the knife-edge restrictions reasonable?
  - Are the results of JPS robust?

- Is it justified to give up the direct macro-finance link?
Our unspanned MTSMs

- Risk factors
  - Two/three PCs of yields – \( USM(2, 2), USM(3, 2) \)
  - Same macro factors as before – \( CORECPI \) and \( UGAP \)
  - Also version with JPS macro vars – \( INF \) and \( GRO \)

- Estimation with Maximum Likelihood
  - JPS canonical form
  - Similar data as JPS
  - \( iid \) measurement errors on yields (same variance)
Testing spanned vs. unspanned

- JPS carry out a likelihood-ratio test of spanning
  - Zero restrictions on the VAR feedback matrix: all coefficients on macro variables are zero
  - Rejected with $\chi^2$-statistic of 1,189

- Is this the right comparison?
  - Exclusion of macro lags for yields and macro—known to be counterfactual
  - Restricted model is a yields-only model with added large macro forecast errors in likelihood function
  - No clear nesting of a spanned model by an unspanned model

- Unspanned model is nested by spanned model
  ⇒ Test knife-edge restrictions
Tests of knife-edge unspanned MTSM restrictions

<table>
<thead>
<tr>
<th></th>
<th>UGAP, CORECPI</th>
<th>GRO, INF</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM(2, 2)</td>
<td>22,292</td>
<td>23,697</td>
</tr>
<tr>
<td>USM(2, 2)</td>
<td>21,733</td>
<td>22,955</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>1,118</td>
<td>1,484</td>
</tr>
<tr>
<td>crit. val.</td>
<td>5.23</td>
<td>5.23</td>
</tr>
<tr>
<td>SM(3, 2)</td>
<td>21,298</td>
<td>22,737</td>
</tr>
<tr>
<td>USM(3, 2)</td>
<td>21,210</td>
<td>22,439</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>177</td>
<td>595</td>
</tr>
<tr>
<td>crit. val.</td>
<td>6.57</td>
<td>6.57</td>
</tr>
</tbody>
</table>

- Exclusion restrictions strongly rejected
- Spanned models have better cross-sectional fit
Term premia – unspanned model with $GRO$, $INF$
Term premia – unspanned model with *UGAP, CORECPI*

![Graph showing term premia over years with USM model projected on PCs and yields-only model.](image-url)
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- Evidence on unspanned macro risk
  - Policy factors are tightly linked to yield curve
  - Non-policy factors have substantial unspanned variation
  - Evidence on predicting excess returns is weak

- Spanned models
  - Should be specified with policy factors
  - Are consistent with the evidence of unspanned macro risk
  - Can be used for policy analysis

- Unspanned models
  - Knife-edge restrictions are rejected
  - Similar term premium implications of spanned and unspanned models when using policy factors