Forecast Errors

Learning

More on Pro

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References

Expecting the Fed Anna Cieslak and Pavol Povala

Discussion Kenneth J. Singleton

Stanford University and NBER

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Mis-Forecasting the Fed Affects Risk Premiums in Bond Markets

- Point 1: Macro information other than current bond yields predict future yields and excess returns. A puzzle (?)
- Point 2: Forecast errors based on survey data are forecastable using yield and macro information ⇒? investors make systematic errors in forecasting the effects of monetary policy on interest rates.
- Point 3: Transformations of lagged forecast errors predict excess returns \Rightarrow ? expectational errors resolve the yield-forecast puzzle.



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Point 1: Macro Information Forecasts Risk Premiums Over and Above Information in the Yield Curve

- "empirical evidence suggests that variables other than current bond yields have predictive power for ... future yields."
- "surprising given that yields today reflect market's conditional expectations of short rates ... and therefore, the current yield curve should contain all information useful for forecasting."



Point 1: Macro Information Forecasts Risk Premiums Over and Above Information in the Yield Curve

- "empirical evidence suggests that variables other than current bond yields have predictive power for ... future yields."
- "surprising given that yields today reflect market's conditional expectations of short rates ... and therefore, the current yield curve should contain all information useful for forecasting."
- Rationale? Arbitrage-free dynamic term structure model:

$$r_t = \rho_{0X} + \rho_{1X} \cdot X_t.$$

• X_t follows an affine process $\Rightarrow y_t^{(n)} = A(n) + B(n)X_t$.



Macro Information Should Forecast Yields Over and Above the Current Yield Curve

• The risks that impinge on traded securities are described by the *R*-dimensional Gaussian (data-generating) process

$$Z_t = K_{0Z}^{\mathbb{P}} + K_{1Z}^{\mathbb{P}} Z_{t-1} + \sqrt{\Sigma_Z} \eta_t^{\mathbb{P}}, \ \eta_t^{\mathbb{P}} \sim N(0, I).$$

• Agents value nominal payoffs using the pricing kernel

$$\mathcal{M}_{Z,t+1} = e^{-r_t - \frac{1}{2}\Lambda'_{Zt}\Lambda_{Zt} - \Lambda'_{Zt}\eta^{\mathbb{P}}_{t+1}}.$$

- $\Lambda_Z(Z_t)$ is the \mathcal{R} -dimensional vector of market prices of risks.
- η_t represents all aggregate risks that are relevant for pricing nominal cash flows (debt and equity).



Dimension Reduction for Pricing Bonds

- Suppose that bond yields only depend on $\mathcal{N}(<\mathcal{R})$ risk factors: $y_t^{(n)} = A(n) + B(n) \cdot \mathcal{P}_t$.
- $\mathcal{R}\searrow\mathcal{N}$ could arise because
 - the economy-wide risks underlying \mathcal{M}_Z impinge on bond yields only through the \mathcal{N} portfolios of risks X.
 - $\hbox{ ocrtain risks in $\eta^{\mathbb{P}}_t$ (e.g., cashflow risks in equity markets) are largely inconsequential for the pricing of Treasury bonds. }$
- A natural interpretation of the literature is that it proceeds by constructing a bond-market specific pricing kernel $\mathcal{M}_{\mathcal{P},t+1}$.
- The priced risks in bond markets are taken to be the first \mathcal{N} PCs, \mathcal{P}_t , of bond yields, consistent with historical evidence.



A Bond-Market Specific Pricing Kernel

• Construct $\mathcal{M}_{\mathcal{P},t+1}$ by projecting $\mathcal{M}_{Z,t+1}$ onto (\mathcal{P}_{t+1}, Z_t) :

$$\mathcal{M}_{\mathcal{P},t+1} \equiv Proj\left[\mathcal{M}_{Z,t+1}\middle|\mathcal{P}_{t+1}, Z_t\right] = e^{-r_t - \frac{1}{2}\Lambda'_{\mathcal{P}_t}\Lambda_{\mathcal{P}_t} - \Lambda'_{\mathcal{P}_t}\epsilon^{\mathbb{P}}_{\mathcal{P},t+1}}.$$

• The market prices of $\mathcal{P}_t(\Lambda_{\mathcal{P}t})$ are affine functions of the full set of \mathcal{R} risks Z_t , and Z_t follows the VAR

$$\begin{bmatrix} \mathcal{P}_{t} \\ M_{t} \end{bmatrix} = \begin{bmatrix} K_{0\mathcal{P}}^{\mathbb{P}} \\ K_{0\mathcal{P}}^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} K_{\mathcal{P}\mathcal{P}}^{\mathbb{P}} & K_{\mathcal{P}M}^{\mathbb{P}} \\ K_{M\mathcal{P}}^{\mathbb{P}} & K_{MM}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ M_{t-1} \end{bmatrix} + \sqrt{\Sigma_{Z}} \epsilon_{Zt}^{\mathbb{P}}.$$

- M_t and $E_t[M_{t+1}]$ are unspanned by $\mathcal{P}_t.$ /
- M_t has predictive content for risk premiums.



Points 2 and 3: Market Participants Make Systematic Errors When Forecasting Future Yields

• Decomposition of 1-year excess return on a 2-year zero bond:

$$rx_{t,t+1}^{(2)} = \underbrace{\left[f_t^{(2)} - E_t^s(y_{t+1}^{(1)})\right]}_{\text{risk premium}} + \underbrace{\left[y_{t+1}^{(1)} - E_t^s(y_{t+1}^{(1)})\right]}_{\text{expectations error}}$$
$$= \tilde{RP}_t^{(2)} + \tilde{FE}_{t+1}^{(1)}$$

- Three ingredients to C-P's argument:
 - There are systematic forecast errors.
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 - So Risk premiums $(\tilde{RP}_t^{(2)})$ are accurately measured so that we can quantify the effects of expectational errors on excess returns.



Forecast Errors Have Predictive Content? A First Look

- Yes- Piazzesi and Schneider (2009); confirmed by C-P.
- Linear projections of excess returns over one-year horizon:

$$rx_{t+1}^{(i)} = \alpha + \beta_{PC1}PC1_t + \beta_{PC2}PC2_t + \beta_{PC3}PC3_t + ??? + \epsilon_{t+1}$$

Conditioning	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$	$r\bar{x}$
PCs	0.20	0.17	0.19	0.18	0.18
$PCs + (g,\pi)$	0.33	0.32	0.35	0.35	0.34
PCs + 1QFEs	0.21	0.19	0.20	0.19	0.19
PCs+1YFEs	0.42	0.40	0.39	0.36	0.38

Table: Adjusted R^2 's for the sample period 1985:1 - 2011:12



How Should we Interpret These "Errors"?

- Departures from rationality/myopia?
- Learning in the presence of parameter uncertainty?
 - Piazzesi and Schneider (2009) and Collin-Dufresne, Johannes, and Lochstoer (2013) develop equilibrium models with learning that could rationalize these findings.
- How do we accommodate investor heterogeneity and the impact of their diversity of views in price formation?
 - Nimark (2009) and Buraschi and Whelan (2012)
- Throughout the C-P analysis, all projections- including extraction of cyclical components- are full-sample and, as such, they potentially embed substantial look-ahead bias.



Learning Using A DTSM

- Suppose bond markets are arbitrage free and that investors learn from the history of bond yields using a 3-factor Gaussian dynamic term structure model estimated by FIML.
- Bond yields are affine in \mathcal{P}_t , the first 3 PCs of yields:

$$y_t^{(n)} = A_{\mathcal{P}}(n) + B_{\mathcal{P}}(n) \cdot \mathcal{P}_t,$$

where \mathcal{P}_t follows a Gaussian VAR (macro info irrelevant!)

- Training period is 12/1952 5/1984. CRSP Treasury Yields
- DTSM is re-estimated every month rolling through time to compute optimal ahead forecasts of $y_t^{(n)}$.
- These forecasts are then compared to the 1-Q and 1-Y ahead survey forecasts of $y_t^{(n)}.$



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Survey and DTSM Forecast Errors: 1 Quarter Ahead





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Survey and DTSM Forecast Errors: 1 Year Ahead





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RMSEs For Survey and DTSM Forecasts

	1Y	2Y	3Y	5Y
Survey 1Q	0.0071	0.0072	0.0072	0.0067
DTSM 1Q	0.0075	0.0071	0.0069	0.0064
Survey 4Q	0.0163	0.0156	0.0150	0.0133
DTSM 4Q	0.0177	0.0168	0.0157	0.0138



Summary So Far...

- Reproduced survey forecasts with a "simple" learning rule based on an arbitrage-free DTSM.
- Learning induces forecast errors that have predictive content (as seen by the econometrician) for excess returns.
- No (direct) information about monetary policy used in these excersizes. Learning from the shape of the yield curve.



Summary So Far...

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- No (direct) information about monetary policy used in these excersizes. Learning from the shape of the yield curve.
- C-P present circumstantial evidence that it is related to policy.
 - Persistence in policy rate changes with creation of the FRB (see Mankiw and Miron (1986)).
 - Large literature associating the shape of the yield curve with FRB policy actions.
 - Yield curve responses to macroeconomic announcements (e.g., Fleming and Remolona (1999)).



Projections with 1Q Errors: 1985:1-2011:12

	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$	$r\bar{x}$
â	-0.00794	-0.01366	-0.02286	-0.02974	-0.01855
	(-0.87833)	(-0.76862)	(-0.90258)	(-0.93245)	(-0.88959)
$\widehat{\beta}_{PC1}$	$\underset{(2.76537)}{0.30784}$	$\underset{(2.76004)}{0.60109}$	$\underset{(2.54991)}{0.84722}$	$\underset{(2.50614)}{1.04490}$	$\underset{(2.61260)}{0.70026}$
\widehat{eta}_{PC2}	-0.86161	-1.86824	-3.01541	-3.99830	-2.43589
	(-1.50268)	(-1.58496)	(-1.71266)	(-1.80146)	(-1.70521)
\widehat{eta}_{PC3}	-0.10894	-0.87536	-2.73301	-4.89237	-2.15242
	(-0.04902)	(-0.18199)	(-0.38289)	(-0.55060)	(-0.37479)
$\widehat{\beta}_{PC1,1QFE}$	-0.02024 (-0.35469)	-0.01910 (-0.17081)	$\underset{(0.11511)}{0.01612}$	$\underset{(0.68969)}{0.10455}$	$\underset{(0.18155)}{0.02033}$
$\widehat{eta}_{PC2,1QFE}$	-0.18974 (-0.26236)	-0.19770 (-0.13415)	-0.19925 (-0.08887)	$\underset{(0.04915)}{0.14204}$	-0.11116 (-0.06078)
$\widehat{eta}_{PCf3,1QFE}$	-1.75841	-3.55001	-4.89270	-6.05732	-4.06461
	(-1.37996)	(-1.49389)	(-1.42122)	$_{(-1.46051)}$	(-1.46476)
\widehat{eta}_{CPI}	-0.79799	-1.72112	-2.47441	-3.20485	-2.04959
	(-1.74997)	(-1.99392)	(-1.92537)	(-2.04158)	(-1.97692)
\widehat{eta}_{Out}	-0.71639	-1.20889	-1.32738	-1.37031	-1.15574
	(-2.26024)	(-2.11168)	(-1.64342)	(-1.45437)	(-1.75743)
Adj. R^2	0.3299	0.3006	0.2832	0.2693	0.2857

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Projections with 1Y Errors: 1985:1-2011:12

	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$	\bar{rx}
â	$\underset{(0.07021)}{0.00063}$	$\underset{(0.26778)}{0.00495}$	$0.00562 \\ (0.20999)$	$\underset{(0.18371)}{0.00624}$	$\underset{(0.19856)}{0.00436}$
$\widehat{\beta}_{PC1}$	$\underset{(1.84144)}{0.19212}$	$\underset{(1.64486)}{0.35095}$	$\underset{(1.39813)}{0.45867}$	$\underset{(1.32957)}{0.55709}$	$\underset{(1.47024)}{0.38971}$
\widehat{eta}_{PC2}	$\underset{(0.14174)}{0.08520}$	$\underset{(0.09737)}{0.11776}$	-0.10640 (-0.05808)	-0.41146 (-0.17537)	$\begin{array}{c} -0.07873 \\ \scriptscriptstyle (-0.05265) \end{array}$
\widehat{eta}_{PC3}	$\underset{(0.89982)}{1.59508}$	$\underset{(0.33400)}{1.20994}$	$-0.00590 \\ (-0.00114)$	$\substack{-3.55232 \\ (-0.55727)}$	$-0.18830 \\ (-0.04467)$
$\hat{\beta}_{PC1,1YFE}$	$\underset{(0.82056)}{0.05366}$	$\underset{(1.20592)}{0.16668}$	$\underset{(1.35856)}{0.28056}$	$\underset{(1.70279)}{0.44944}$	$\underset{(1.41499)}{0.23759}$
$\hat{\beta}_{PC2,1YFE}$	-1.24842 (-2.43789)	-2.53575 (-2.41335)	-3.67791 (-2.30711)	-4.47813 (-2.19722)	-2.98505 (-2.30301)
$\hat{\beta}_{PC3,1YFE}$	$-3.93340 \\ _{(-3.44951)}$	-6.81624 (-3.40272)	-9.19680 (-3.38602)	$-9.73382 \\ (-3.03785)$	-7.42007 (-3.29762)
$\widehat{\beta}_{CPI}$	$-0.35526 \\ (-0.83888)$	-0.82774 (-0.99865)	-1.13137 (-0.91016)	$-1.59403 \\ (-1.02995)$	$-0.97710 \\ (-0.97208)$
\widehat{eta}_{Out}	-0.39742 (-1.54385)	-0.60247 (-1.27149)	-0.46847 (-0.69237)	$-0.37184 \\ (-0.45321)$	-0.46005 (-0.82865)
Adj. R^2	0.4493	0.4198	0.3975	0.3782	0.4003



What Should We Make of This?

- Errors in forecasting "slope" and "curvature" of the yield curve have predictive content for excess returns in Treasury markets- they drive out date-t macro information.
- But first: Who are these forecasters? Are they the marginal investors in US Treasury bonds?
- If these forecasters' views are relevant for pricing setting (in some direct way), then why the systematic errors?
- Preliminary evidence suggests that learning is playing a role (not modeled in C-P).
- Is it learning about the FRB? Or are investors confused about the future course of economic growth (which is influenced by the FRB and many other factors)?



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