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## Cost-Benefit Analysis of Leaning Against the Wind: Are Costs Larger Also with Less Effective Macroprudential Policy?

by Lars E.O. Svensson



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**Prepared by Lars E.O. Svensson**[1]

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**Abstract**

"Leaning against the wind" (LAW) with a higher monetary policy interest rate may have benefits in terms of lower real debt growth and associated lower probability of a financial crisis but has costs in terms of higher unemployment and lower inflation, importantly including a higher cost of a crisis when the economy is weaker. For existing empirical estimates, costs exceed benefits by a substantial margin, even if monetary policy is nonneutral and permanently affects real debt. Somewhat surprisingly, less effective macroprudential policy and generally a credit boom, with resulting higher probability, severity, or duration of a crisis, increases costs of LAW more than benefits, thus further strengthening the strong case against LAW.

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Author's E-Mail Address: Leosven@gmail.com, LSvensson@imf.org.

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# 1 Introduction

By "leaning against the wind" (of asset prices and credit booms) I here mean a monetary policy with a somewhat higher policy interest rate than what is justified by just stabilizing inflation around an inflation target and unemployment around its estimated long-run sustainable rate without taking any effects on financial stability into account. Leaning against the wind has obvious costs in terms of a weaker economy with higher unemployment and lower inflation. It has been justified as a way of reducing the probability and severity of a future financial crisis (Bank for International Settlements (2014), Olsen (2015), Sveriges Riksbank (2013)). A somewhat worse macro outcome in the near future is then considered to be an acceptable cost to be traded off against a better expected macro outcome further into the future. But a crisis can come any time, and the cost of a crisis is higher if initially the economy is weaker. If the unemployment rate is higher when a crisis occurs, the unemployment rate during the crisis will be higher, which increases the cost of a crisis. Leaning against the wind thus not only has cost in terms of a weaker economy if no crisis occurs; it has an additional cost in terms of a higher cost of a crisis *if* a crisis occurs. The present paper shows that, with this additional cost of leaning against the wind, for existing empirical estimates, the cost of leaning against the wind can be shown to exceed, by a substantial margin, the benefit from a lower probability of a crisis.

Furthermore, empirically the channel through which a higher policy rate might reduce the probability of a crisis is through lower real debt growth. According to existing empirical estimates, the probability of a crisis is positively correlated with the growth rate of real debt during the previous few years (Schularick and Taylor (2012)). If a higher policy rate reduces real debt growth, it might therefore reduce the probability of a crisis. However, there are three important limitations of this channel.

First, if monetary policy is neutral in the long run, it cannot affect real debt in the long run. Therefore, even if a higher policy rate would reduce real debt growth and thereby the probability of a crisis for a few years, if there is no permanent effect on the real debt level, a lower real debt growth and probability of a crisis will be followed by a higher debt growth and probability, and the average and accumulated debt growth and probability would not be affected over a longer period. The probability of a crisis would just be shifted between different periods.

Second, as discussed in Svensson (2013a), the effect on real debt of a higher policy rate is likely to be small and could be of either sign. The stock of nominal debt, in particular the stock of mortgages, has considerable inertia. A higher interest rate may reduce the growth rate of housing prices and, at given loan-to-value ratios, reduce the growth rate of new mortgages. But only a fraction of the stock of mortgages is turned over each year. Furthermore, even if a higher policy rate slows down the rate of growth of nominal mortgages, it also slows down the rate of growth of the price level. Thus, both the numerator and the denominator of real debt are affected in the same direction by the policy rate, making the effect on the ratio smaller. And if the price level is affected more or quicker than the stock of debt, real debt will rise rather than fall. Indeed, the "stock" effect may dominate over the "flow" effect for several years or longer. The effect on the debt-to-GDP ratio of a higher policy rate is even more likely to be small or of the opposite sign, because then not only the price level but also real GDP enter in the denominator, and the growth of both are slowed down by a higher policy rate. Several recent papers have indeed found empirical evidence supporting the notion that a higher policy rate increases rather than decreases the debt-to-GDP ratio (Alpanda and Zubairy (2014), Gelain, Lansing, and Natvik (2015), and Robstad (2014)).

Third, the empirical relation between previous real debt growth and the probability of a crisis is of course a reduced-form and correlation result. The underlying determinants of the probability of a financial crisis are the nature and magnitude of the shocks to the financial system and the resilience of the system. The former depend on, among other things, possible overvaluation and riskiness of assets. The latter depends on such things as the strength of balance sheets and thereby the resilience of borrowers and lenders, the quality of assets, the amount of loss-absorbing capital, the degree of liquidity and of maturity transformation, the quality of lending standards, the debt-servicing capacity of borrowers, the amount of risk-taking and speculation, and so on. The extent to which higher real debt growth increases the probability of a crisis depends on to what extent it is "bad" credit growth that is related to things such as lower lending standards, higher loan-to-value ratios, speculation, overvaluation of assets, and so on, rather than "good" credit growth related to financial deepening and developments that does not weaken but rather strengthens the financial system. With better data on the underlying determinants of the nature and magnitude of shocks and the resilience of the system, it should be possible to assess the probability of a crisis without relying on aggregate real debt growth. Given the list of underlying determinants of the probability of a crisis, it is also rather clear that the policy rate is unlikely to have any systematic impact on most or any of them, and that micro- and macroprudential policy is much more likely to have such an impact.[1]

[1] International Monetary Fund (2015) discusses the transmission channels from the policy rate to the probability of

In this paper, I will take into account the first limitation, the implication of long-run neutrality of monetary policy, but I will also consider the result of non-neutrality and possible permanent effects on real debt of monetary policy. As for the second and third limitations, I will simply take existing empirical estimates as given, in particular those of the Riksbank in Ekholm (2013) and Sveriges Riksbank (2014a) and of Schularick and Taylor (2012), to see what follows from them. Thus, I arguably stack the cards somewhat in favor of leaning against the wind.[2]

The existing small literature that has tried to quantify the costs and benefits of leaning against the wind has mainly considered a two-period setup where a higher policy rate has a cost in terms of higher unemployment in the first period and a benefit in terms of a lower probability of a crisis in the second period (Kocherlakota (2014), Svensson (2014, 2015), Ajello, Laubach, Lopez-Salido, and Nakata (2015), and International Monetary Fund (2015)).[3] By assumption there is no possibility of a crisis in the first period, and by assumption a crisis in the second period would start from an initial situation when unemployment equals its long-run sustainable rate and the unemployment gap thus is zero.

This two-period framework is an over-simplification. By disregarding the possibility of a crisis in the first period and by assuming that a crisis in the second period occurs when the unemployment gap initially is zero, it disregards that a crisis could come any time and that leaning against the wind increases the cost of a crisis by causing it to start from a higher unemployment rate. Thus it *understates the cost* of leaning against the wind. Furthermore, by assuming that there is only one period for which the probability of a crisis can be affected, it disregards the consequences of the long-run neutrality of monetary policy and the resulting property that then the probability of a crisis is shifted between periods but the sum of the probabilities remains the same. Thus it

a crisis and documents its complexity, uncertainty of direction, and variation over time. Dagher, Dell'Ariccia, Laeven, Ratnovisk, and Tong (2015) shows that more but still relatively moderate bank capital relative to risk-weighted assets would likely have had a dramatic effect in reducing the frequency of banking crises in the advanced countries since 1970.

[2] Another possible benefit of a higher policy rate might be a smaller increase in the unemployment rate in a crisis. According to the empirical results of Flodén (2014), for OECD countries, a higher household debt-to-income ratio before the recent financial crisis is associated with a somewhat lower increase in unemployment during the crisis. If a higher policy rate reduces the debt-to-income or debt-to-GDP ratios, a higher policy rate might this way reduce the cost of the crisis. However, according to Flodén (2014), the impact of the initial debt-to-income rate on the crisis increase in the unemployment rate is very small (and not significant for the OECD countries for which housing prices fell during the crisis). Furthermore, as noted, the effect of the policy rate on the debt-to-income ratio is apparently quite small, often not statistically significant from zero, and, according to both theoretical and empirical analysis, a higher policy rate probably *increases* rather than decreases the debt-to-GDP ratio. This means that there is hardly theoretical or empirical support for the idea that this channel would provide any benefit from leaning against the wind. Nevertheless, the empirical importance of this possible channel is examined in appendix D.

Clouse (2013) provides a theoretical analysis of optimal policy in a two-period model where the policy rate affects the severity of a possible crisis.

[3] Leaning against the wind has been discussed in more general terms by, for instance, Bernanke (2015), Evans (2014), International Monetary Fund (2015), Laséen, Pescatori, and Turunen (2015), Smets (2013), Stein (2013, 2014), Svensson (2013b), Williams (2015), Woodford (2012), and Yellen (2014).

*overstates the benefit* of leaning against the wind.

Given these simplifications of the two-period model, Svensson (2014, 2015) and International Monetary Fund (2015) nevertheless show that, given existing empirical estimates and reasonable assumptions, the cost of a higher unemployment rate the next few years because of a higher policy rate is many times larger than the benefit of leaning against the wind in terms of an expected lower future unemployment rate due to a lower probability of a crisis. Ajello, Laubach, Lopez-Salido, and Nakata (2015) furthermore shows that a tiny amount of leaning against the wind may be justified, corresponding to a few basis points increase in the policy rate, but that extreme assumptions are needed to justify more significant leaning against the wind. In particular, the net benefit of such a tiny amount of leaning against the wind is completely insignificant.[4]

An exception to this two-period framework is the dynamic approach and analysis of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015) in a quarterly model, where the probability of a crisis varies over quarters and the cost and benefit of leaning against the wind are accumulated over time. The present paper follows that approach and uses a multi-period quarterly model.

The preliminary results of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015), summarized in International Monetary Fund (2015, box 7, p. 41), indicate that the cost dominates over the benefit during the first few years but that the cost is about equal to the benefit over a longer period. However, the cost of a crisis is still assumed to be fixed and independent of the initial state of the economy. It is as if a crisis is assumed to result in a 5 percent unemployment gap regardless of whether the initial unemployment gap is zero or 3 percent. Furthermore, it is assumed that monetary policy has a permanent effect on real debt and thus is non-neutral in the long run. If the cost of a crisis is depends on the initial state of the economy or if monetary policy is neutral in the long run, the cost exceeds the benefit.

The new elements in the present paper are (i) to take into account that the cost of a crisis depends in the initial state of the economy, which in turn depends on the amount of leaning against the wind that has preceded the crisis, (ii) to derive the effect on the policy rate on the probability of a crisis, taking into account that this probability depends both on the probability of a crisis start and the duration of a crisis, (iii) to derive the expected marginal cost and marginal benefit of leaning against the wind, in order to assess whether leaning against or with the wind is justified, (iv) to take into account and assess the role of monetary neutrality, (v) to assess whether more or less effective macroprudential policy affects the case for leaning against the wind, in the context

[4] The early and innovative contribution of Kocherlakota (2014), expressing the value of reducing the probability of a crisis to zero in terms of an unemployment-gap equivalent, is discussed in appendix E.

of examining how a higher probability or severity of a crisis affects the marginal cost and benefit of leaning against the wind. The last element thus challenges the common argument that leaning against the wind is justified as a last resort, if macroprudential policy is ineffective.[5]

The main result of this paper is then that the cost of leaning against the wind, for existing empirical estimates, exceeds the benefit by a substantial margin. If anything, a positive probability of a crisis justifies a modest leaning *with* the wind rather than against. This result is quite robust and holds for a variety of alternative assumptions, including if monetary policy is non-neutral and has a long-run effect on real debt. Furthermore, somewhat surprisingly, a less effective macroprudential policy is likely to increase the cost of leaning against the wind more than the benefit, thus strengthening the case against leaning against leaning against the wind.

Why is the cost of leaning against the wind normally so much larger than the benefit? We can understand this by representing a crisis by a fixed *increase* in the unemployment rate from its non-crisis level and, in particular, by preliminarily assuming that the probability of a future crisis is given and not affected by the policy rate. With a given positive probability of a crisis, the expected unemployment gap (taking into account the probability of a crisis increase in the unemployment rate) is larger than the non-crisis unemployment gap. If the future non-crisis unemployment gap is zero, the expected future unemployment gap is positive. The optimal policy, the policy that minimizes the expected future squared unemployment gap, is to set the expected future unemployment gap equal to zero. This requires the future non-crisis unemployment gap to be somewhat negative, more precisely such that the probability-weighted future negative non-crisis unemployment gap in absolute value equals the probability-weighted future positive crisis unemployment gap.

Thus, if the probability of a crisis is given, the optimal policy is actually to *lower* the policy rate and lean *with* the wind. There is thus an initial incentive to lean with the wind. If the probability of crisis is not given but depends on and decreases with a higher policy rate, there is an incentive to increase the policy rate from its lower level and thereby reduce the probability of a crisis. For the incentive to increase the policy rate to dominate over the initial incentive to lower the policy rate, so the net incentive is to lean against the wind, the effect of the policy rate on the probability of a crisis must be sufficiently large. However, for existing empirical estimates, the effect is much to small, so the net incentive is a modest leaning with the wind.

Why would a less effective macroprudential policy increase the cost of leaning against the wind more than the benefit? The incentive to lean with the wind is stronger if the probability of a

[5] This common argument is challenged by Williams (2015): "[M]onetary policy is poorly suited for dealing with financial stability concerns, even as a last resort."

crisis is higher. With a higher probability of a crisis, everything else equal the expected future unemployment gap is larger. In order to make it zero, the non-crisis unemployment gap must become more negative and the policy rate has to be lowered more. This is also the case if a crisis is deeper and involves a larger increase in the unemployment rate.

Therefore, if a less effective macroprudential policy, for instance by resulting in a credit boom, leads to a higher probability of a crisis or a deeper crisis, the less effective macroprudential policy actually strengthens the case *against* leaning against the wind, counter to the common view that less effective macroprudential policy strengthens the case *for* leaning against the wind. Even if a credit boom and higher probability of a crisis might increase the effect of credit growth and the policy rate on the probability of a crisis, empirically the increase in the effect is too small to significantly increase the benefit of leaning against the wind.

The paper is outlined as follows: Section 2 examines the effect of leaning against the wind on the expected future unemployment rate, taking the possibility of a crisis into account. This is a generalization of the previous two-period analysis in Svensson (2014, 2015). Section 3 examines the effect of leaning against the wind on expected future quadratic losses, demonstrates the importance of the assumption that the cost of a crisis is larger when the economy is weaker, and contrasts with the case when the cost of a crisis is fixed and independent of the state of the economy. Section 4 derives the corresponding marginal cost and benefit of leaning against the wind, to assess whether the optimal policy is to lean against or with the wind. The sensitivity of the results to the initial state of the economy, to the magnitude of the policy-rate effect on the expected non-crisis unemployment rate, and to the probability of a crisis is also reported. Section 5 examines the common argument that leaning against the wind is justified if there is a less effective macroprudential policy. Section 6 provides additional sensitivity analysis by examining whether monetary non-neutrality with a permanent effect on real debt changes the results. Sections 2–6 uses estimates from Schularick and Taylor (2012) of the effect of real debt growth on the probability of crisis with data for 14 countries for 1870–2008. Section 7 shows that recent IMF staff estimates in International Monetary Fund (2015) with the Laeven and Valencia (2012) data for 35 advanced countries for 1970-2012 give similar results. Section 8 summarizes the conclusions. Appendices A-J provide further details, sensitivity analysis, and extensions.

## 2 The effect on expected future unemployment of leaning against the wind

This section examines the effect of leaning against the wind, that is, a somewhat higher policy rate, on the expected future unemployment rate in an economy, taking the possibility of a crisis into account. This is in line with the approach in Svensson (2014, 2015), but extends it from a two-period framework to a multi-period quarterly framework.

Let $u_t$ denote the unemployment rate in quarter $t$. Assume that, in each quarter $t$, there are two possible states in the economy, non-crisis and crisis. In a crisis, the unemployment rate is higher by a fixed magnitude, the *crisis increase in the unemployment rate*, $\Delta u > 0$.[6] This crisis increase in the unemployment rate should more generally be interpreted as the unemployment increase after possible policy actions, including policy-rate cuts after the crisis has occurred, to moderate the cost of the crisis.[7] Let $u_t^{\mathrm{n}}$ and $u_t^{\mathrm{c}}$ denote the quarter-$t$ *non-crisis and crisis unemployment rates*, respectively. They then satisfy

$$u_t^{\mathrm{c}} = u_t^{\mathrm{n}} + \Delta u > u_t^{\mathrm{n}}. \tag{2.1}$$

Let $q_t$ denote the probability of a crisis *starting* in (the beginning of) quarter $t$, meaning that the unemployment rate increases by $\Delta u$ and equals the crisis unemployment rate, $u_t^{\mathrm{c}}$, during quarter $t$. Assume that a crisis has a fixed duration of $n$ quarters, so if a crisis starts in (the beginning of) quarter $t$ it ends in (the beginning of) quarter $t+n$. Thus, if a crisis starts in quarter $t$, the unemployment rate equals the crisis unemployment rate for the $n$ quarters $t$, $t+1$, ..., $t+n-1$.

Let $p_t$ denote the probability of the economy *being* in a crisis in quarter $t$. If a crisis lasts $n$ quarters, the probability of being in a crisis (approximately) equals the probability that a crisis started in any of the last $n$ quarters, including the current quarter $t$, that is, in any of the quarters $t-n+1$, $t-n+2$, ..., $t$. Then the probability of a being in a crisis in quarter $t$ satisfies

$$p_t = \sum_{\tau=0}^{n-1} q_{t-\tau}. \tag{2.2}$$

In the rest of the paper, I will refer to $p_t$ as the *probability of a crisis* in quarter $t$ and to $q_t$ as the *probability of a crisis start* in quarter $t$.[8]

[6] For simplicity, the crisis increase in the unemployment rate is taken to be deterministic. As shown in appendix G, the analysis can easily be generalized to include the case where the crisis increase is random with a fixed mean, $\Delta u$, and a fixed variance, $\sigma_{\Delta u}^2$, but this would not affect the results.

[7] If a crisis occurs in quarter $t$, the increase $\Delta u$ in the unemployment rate will in reality not occur within the quarter but over the next few quarters. For simplicity, the increase is nevertheless assumed to occur within the quarter.

[8] I am grateful to Stefan Laséen and David Vestin for alerting me to the fact that equation (2.2) is a linear

It follows that the quarter-$t$ unemployment rate, $u_t$, will equal the non-crisis unemployment rate, $u_t^{\mathrm{n}}$, with probability $1-p_t$ and the crisis unemployment rate with probability $p_t$. The unemployment rate in quarter $t \geq 1$ that is expected in quarter 1, the *expected unemployment rate*, is then given by

$$\mathrm{E}_1 u_t = (1-p_t)\mathrm{E}_1 u_t^{\mathrm{n}} + p_t \mathrm{E}_1 u_t^{\mathrm{c}} = (1-p_t)\mathrm{E}_1 u_t^{\mathrm{n}} + p_t(\mathrm{E}_1 u_t^{\mathrm{n}} + \Delta u) = \mathrm{E}_1 u_t^{\mathrm{n}} + p_t \Delta u, \tag{2.3}$$

where $\mathrm{E}_1$ denotes the expectations held in quarter 1. The expected future unemployment rate equals the *expected non-crisis unemployment rate*, $\mathrm{E}_1 u_t^{\mathrm{n}}$, plus the increase in the *expected* unemployment rate due to the possibility of a crisis, $p_t \Delta u$, the *crisis increase in the expected unemployment rate*.

What is then the effect of a higher policy rate on the expected future unemployment rates? Let $\bar{i}_1$ denote a constant policy rate during quarters 1–4, so the policy rate in quarter $t$, $i_t$, satisfies $i_t = \bar{i}_1$ for $1 \leq t \leq 4$. Consider the effect on the expected future unemployment rate of increasing the policy rate during quarters 1–4. By (2.3), it is given by the derivative

$$\frac{d\mathrm{E}_1 u_t}{d\bar{i}_1} = \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + \Delta u \frac{dp_t}{d\bar{i}_1}. \tag{2.4}$$

It consists of the effect on the expected non-crisis unemployment rate, $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$, and the effect on the crisis increase in the expected unemployment rate, $\Delta u \, dp_t/d\bar{i}_1$.[9] Let us examine these in turn.

## 2.1 The effect of the policy rate on the expected non-crisis unemployment rate

The effect on the policy rate on the expected non-crisis unemployment rate is just the standard impulse response of the unemployment rate to an increase in the policy rate. As an example and benchmark, I use the impulse response in the Riksbank's main model, the DSGE model Ramses, shown in Figure 2.1.[10] The grey line shows an increase in the policy rate of 1 percentage points during quarters 1–4 ($\Delta\bar{i}_1 = 1$ percentage point) and then a return to the baseline level. The red line shows the corresponding deviation of the unemployment rate from the baseline level ($\Delta\mathrm{E}_1 u_t^{\mathrm{n}}$). The unemployment rate increases above the baseline level to about 0.5 percentage points in quarter 6 and then slowly falls back towards the baseline level. Under the assumption of approximate linearity,

approximation to the probability of a crisis. A more thorough treatment is to model the dynamics of the probability of a crisis as a Markov process, as discussed in appendix A. For the parameter range used here, the linear approximation slightly exaggerates the probability of a crisis but simplifies the derivation of the effect of the policy rate on the probability of a crisis.

[9] Here I am abstracting from the possible effect of the policy rate on the crisis increase in the unemployment rate, $d\Delta u/di_t$. It is examined separately in appendix D, where it is shown that the effect can be of either sign but is so very small that it can be disregarded.

[10] The figure shows the impulse response in Ramses of the unemployment rate that was reported by Riksbank deputy governor Karolina Ekholm in Ekholm (2013). It is the same response as the one reported to alternative policy-rate paths for quarters 1–12 in Sveriges Riksbank (2014b).

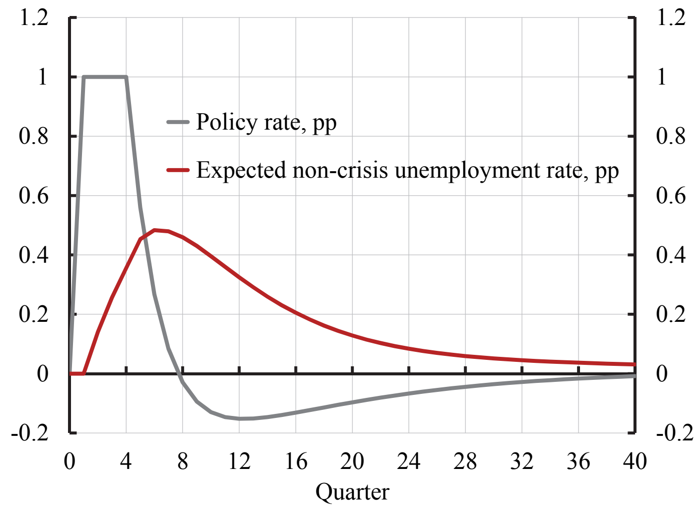


Figure 2.1: The effect on the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Sveriges Riksbank.)

I can take this effect on the expected future non-crisis unemployment rates as the derivative with respect to the policy rate $\bar{i}_1$ of the expected future non-crisis unemployment rate,

$$\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} = \frac{\Delta \mathrm{E}_1 u_t^{\mathrm{n}}}{\Delta \bar{i}_1} = \Delta \mathrm{E}_1 u_t^{\mathrm{n}} \quad \text{for } t \geq 1, \tag{2.5}$$

where $\Delta \mathrm{E}_1 u_t^{\mathrm{n}}$ is given by figure 2.1.

Thus, we have determined the first term in (2.4). It remains to determine the second term, that is, the product of the crisis increase in the unemployment rate and the effect on the probability of a crisis of the policy rate. As a benchmark crisis increase in the unemployment rate, I will use the same assumption as in a crisis scenario discussed in Sveriges Riksbank (2013), that the crisis increase in the unemployment rate is 5 percentage points ($\Delta u = 5$ percentage points). It remains to determine $dp_t/d\bar{i}_1$, the effect of the policy rate on the probability of a crisis in quarter $t \geq 1$.

## 2.2 The effect of the policy rate on the probability of a crisis

In order to determine the effect of the policy rate on the probability of a crisis, $p_t$, I will use that the probability of a crisis depends on the probability of a crisis start, $q_t$, in the $n$ quarters before and including quarter $t$ according to (2.2), that the probability of a crisis start may depend on real debt growth, and that real debt growth may depend on the policy rate.

### 2.2.1 The effect of real debt growth on the probability of a crisis start

According to Schularick and Taylor (2012), the probability of a crisis start depends on the growth rate of real debt during the previous few years. Schularick and Taylor use annual data for 14 developed countries for 1870–2008 and estimate the annual probability of a crisis as a function of annual debt growth lagged 1–5 years. I use their estimates of the coefficients in their main logit regression, Schularick and Taylor (2012, table 3, column (5)), in a quarterly variant of their equation,

$$q_t = \frac{1}{4}\frac{\exp(X_t)}{1+\exp(X_t)},$$

where

$$X_t = -3.89 - \underset{(2.110)}{0.398}\, g_{t-4} + \underset{(2.631)}{7.138^{***}}\, g_{t-8} + \underset{(2.948)}{0.888}\, g_{t-12} + \underset{(1.378)}{0.203}\, g_{t-16} + \underset{(1.640)}{1.867}\, g_{t-20}, \tag{2.6}$$

numbers within parenthesis are robust standard errors,[11]

$$g_t \equiv \left(\sum\nolimits_{\tau=0}^{3} d_{t-\tau}/4\right)/\left(\sum\nolimits_{\tau=0}^{3} d_{t-4-\tau}/4\right) - 1, \tag{2.7}$$

and $d_t$ is the level of real debt in quarter $t$.[12] That is, $g_t$ is the annual growth rate of the average annual real debt level. Schularick and Taylor (2012, p. 1046) report a marginal effect on the annual probability of a crisis start over all lags equal to 0.30, implying the summary result that 5 percent lower real debt in 5 years reduces the probability of a crisis by about 0.3 percentage points per year. That is, it reduces the quarterly probability $q_t$ by 7.5 basis points.[13] [14]

However, we notice that the coefficients in (2.6) are not uniform, so the summary result strictly only applies for *uniform* annual real debt growth during 5 years. If real debt growth fluctuates, the dynamics of the probability of a crisis start is more complicated, as in the dynamic approach

[11] One, two, and three stars denote significance at the 10, 5, and 1 percent level, respectively. The five lags are jointly significant at the 1 percent level.

[12] More precisely, what I call real debt is in Schularick and Taylor (2012) total bank loans, defined as the end-of-year amount of outstanding domestic currency lending by domestic banks to domestic households and nonfinancial corporations (excluding lending within the financial system).

[13] The linear regression in Schularick and Taylor (2012, table 3, column (1)) implies a corresponding somewhat higher marginal effect of 0.4. This explains the summary result that I have used in Svensson (2014, 2015): 5 percent lower real debt in 5 years reduces the annual probability of a crisis start by about 0.4 percentage points. In figure 2.2, real debt decreases by 0.25 percent in 5 years. Then the summary result implies that the annual probability of a crisis decreases by about $0.25 \cdot 0.4/5 = 0.02$ percentage points, which is the summary result that I have used in Svensson (2014, 2015).

[14] A full 1 percentage point reduction of the annual real debt growth for 5 years actually reduces the annual probability of a crisis start by 0.288 percentage points rather than 0.30 percentage points, because of the curvature of the logistic function. A smaller reduction of the real debt growth of 0.1 percentage points per year reduces the probability of crisis start by 0.03 percentage points per year, corresponding to the marginal effect equal to 0.30.

Given the sum of the coefficients in (2.6), 9.698, the marginal effect of 0.30 is consistent with a probability of a crisis start equal to 3.2 percent per year, that is, 0.8 percent per quarter.

The constant in (2.6), $-3.89$, is chosen so as to be consistent with this probability and a steady real debt growth rate of 5 percent per year. See appendix B for details.

of Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015). In particular, we see that annual real debt growth lagged 2 years, $g_{t-8}$, has by far the largest coefficient in (2.6). Thus, annual real growth lagged two years is the major determinant of the probability of a crisis start.[15]

### 2.2.2 The effect of the policy rate on real debt, real debt growth, the probability of a crisis start, and the probability of a crisis

Given the effect on the probability of crisis start of real debt growth in (2.6), it remains to determine the effect of the policy rate on real debt growth.

As an example and benchmark, I use the Sveriges Riksbank (2014a) estimate of the effect on the *level* of real household debt, $d_t$, of a 1 percentage point higher policy rate during 4 quarters, shown as the red line in figure 2.2.[16] Real debt falls relative to the baseline level by 1 percentage in two years and then rises back and reaches the baseline level again in about 8 years.[17] Because monetary policy is neutral, there is no long-run effect on real debt.

We can interpret the red line as showing the derivative of real debt $d_t$ with respect to the policy rate $\bar{i}_1$, $d(d_t)/d\bar{i}_1$ for $t \geq 1$, where furthermore $d(d_t)/d\bar{i}_1 \approx 0$ for $t \geq 32$.

The yellow line in figure 2.2, shows the resulting effect on real debt growth $g_t$, the annual growth rate of the average annual real debt level defined by (2.7). Because the real debt *level* first falls and then rises back to the baseline level, real debt *growth* will first fall below the baseline growth rate and then rise above the baseline growth rate. Thus, lower real debt growth rates are followed by higher real debt growth rates. Importantly, because there is no effect of the policy rate on real debt in the longer run, there is no effect on the average growth rate over a longer period.

We can interpret the yellow line as showing the derivative of the annual real debt growth $g_t$ with respect to the policy rate $\bar{i}_1$, $dg_t/d\bar{i}_1$ for $t \geq 1$, where furthermore

$$\sum\nolimits_{t=1}^{40} \frac{dg_t}{d\bar{i}_1} \approx 0.$$

The blue line in figure 2.2 shows the resulting dynamics of the probability of a crisis start for each quarter, $q_t$, that follows from (2.6). Because annual real debt growth lagged two years

[15] Schularick and Taylor (2012, table 7, column (22)) reports the result of a model specification that adds debt to GDP as an explanatory variable. The coefficient is significantly different from zero, but as discussed in detail in appendix H, it is so small that it has a very small impact on the probability of a crisis start and the probability of a crisis. I therefore disregard that effect here.

[16] The Schularick and Taylor (2012) estimates refer loans to both households and nonfinancial corporations, whereas the estimates in Sveriges Riksbank (2014a) refer to loans to households only. I assume that this difference does not affect the conclusions.

[17] As discussed in Svensson (2014, 2015), there is a wide 90 percent probability band around the red line, and the effect is not significantly different from zero and could be of either sign.

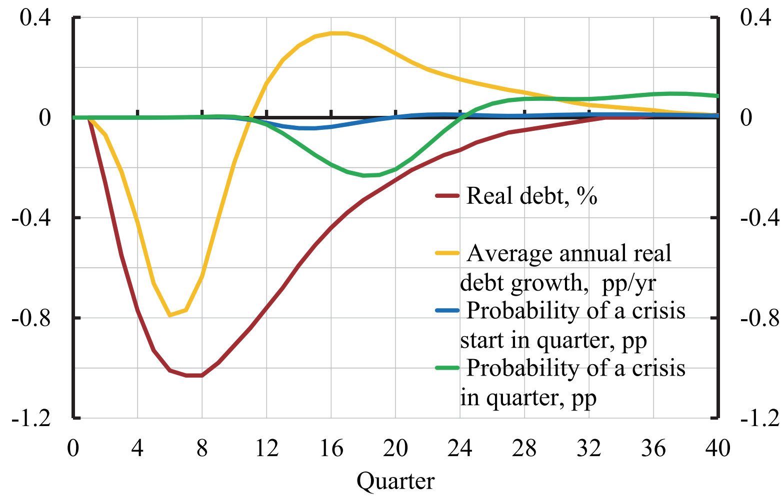


Figure 2.2: The effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

is the main determinant of the probability of a crisis start and the annual real debt growth falls below the baseline and has a negative peak (of about $-0.8$ percentage points per year) in quarter 6, the probability of a crisis will fall below the baseline and have a negative peak (at $-0.04$ percentage points) about two years later, in quarters 14 and 15. Furthermore, annual real debt growth rises above the baseline in quarter 12, which causes the probability of a crisis start to rise above the baseline and have a *positive* peak (of 0.013 percentage points, barely visible) about 2 years later. Thus, these results imply that an increase in the policy rate actually, after about five years, *increases* the probability of a crisis start above the baseline. The increase in the policy rate shifts the probability of a crisis start between quarters, first reducing it and then increasing it. But importantly, because the average effect over time on real debt growth is zero, the average effect over time on the probability of a crisis start is also zero.

We can hence interpret the blue line as showing the derivative $dq_t/d\bar{i}_1$ for $t \geq 1$, with

$$\sum\nolimits_{t=1}^{40} \frac{dq_t}{d\bar{i}_1} \approx 0.$$

The green line in figure 2.2 shows the dynamics of the probability of a crisis, $p_t$. According to (2.2), that probability depends on the sum of all the probabilities of a crisis start, $q_t$, during the last $n$ quarters, the duration of a crisis. I assume that the benchmark duration of a crisis is $n = 8$

quarters, so that a crisis implies that the unemployment rate is 5 percentage points higher during the 8 quarters, corresponding to 10 point-years of higher unemployment. Thus, the green line shows an 8-quarter moving sum of the blue line. It has a negative peak of about $-0.23$ percentage points in quarter 18 and then rises back to zero and turns positive from quarter 25. It is still positive in quarter 40 but will eventually fall to zero.[18]

The green line can be interpreted as showing the derivative of the probability of a crisis with respect to the policy rate, $dp_t/d\bar{i}_1$ for $t \geq 1$. Furthermore,

$$\sum\nolimits_{t=1}^{40} \frac{dp_t}{d\bar{i}_1} \approx 0. \tag{2.8}$$

Thus, the higher policy rate reduces the probability somewhat after 3 years and increases it after 6 years, but without any accumulated and average effect over the 40 quarters.

## 2.3 The effect of the policy rate on the expected future unemployment rate

Given the effect of the policy rate on the probability of a crisis $dp_t/d\bar{i}_1$ from figure 2.2, the assumption that the crisis increase in the unemployment rate $\Delta u$ is 5 percentage points from Sveriges Riksbank (2013), and the effect of the policy rate on the non-crisis expected unemployment rate $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$ from figure 2.1, we can compute the effect of the policy rate on the expected unemployment rate $d\mathrm{E}_1 u_t/d\bar{i}_1$ according to (2.4). It is shown in figure 2.3.

The red line shows the effect on the expected non-crisis unemployment rate, the same line as in figure 2.1. The blue line shows the effect on the expected unemployment rate. It hardly differs from the effect on the non-crisis unemployment rate. The reason is that the effect on the crisis increase in the expected unemployment rate, $\Delta u \, dp_t/d\bar{i}_1$, is very small compared to the effect on the expected non-crisis unemployment rate. It is shown as the green line, in *basis* points, measured along the right vertical axis. As we have noticed in figure 2.2, the largest effect on the probability occurs in quarter 18, when $dp_{18}/d\bar{i}_1$ is $-0.23$ percentage points. This means that the term $\Delta u \, dp_t/d\bar{i}_1 = -0.0023 \cdot 5 = -0.0116$ percentage points $= -1.16$ basis points, is quite small compared to the effect on the expected non-crisis unemployment rate in quarter 18, $d\mathrm{E}_1 u_{18}^{\mathrm{n}}/d\bar{i}_1 = 0.16$ percentage points $= 16$ basis points. And from quarter 25 the effect of the policy rate on the probability of a crisis continues to be very small, but positive.

Furthermore, because the accumulated and average effect on the probability of a crisis over the

[18] Note that the Schularick and Taylor estimates in (2.6) have a relatively large coefficient (although not significant) on the annual real growth rate lagged 5 years, meaning that the probability of a crisis start and the probability of a crisis are still affected by the higher real debt growth 5-6 years earlier.

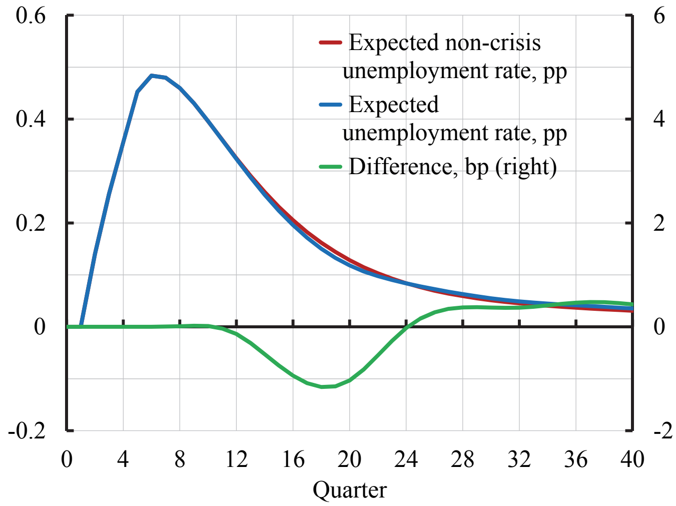


Figure 2.3: The effect on the expected unemployment rate and the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

40 quarters is approximately zero, the accumulated effect on the expected unemployment rate is approximately equal to the effect on the expected non-crisis unemployment rate,

$$\sum\nolimits_{t=1}^{40} \frac{d\mathrm{E}_1 u_t}{d\bar{i}_1} = \sum\nolimits_{t=1}^{40} \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + \Delta u \sum\nolimits_{t=1}^{40} \frac{dp_t}{d\bar{i}_1} \approx \sum\nolimits_{t=1}^{40} \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}.$$

In figure 2.3, the accumulated effect on the expected non-crisis unemployment rate is 6.9 point-quarters, whereas the accumulated effect on the expected crisis increase in the unemployment rate is only $-0.03$ point-quarters. The area under the red and the blue curves are approximately equal for a horizon of 40 quarters.

In summary, the effect of the policy rate on the expected future unemployment rate is the sum of the effect on the expected non-crisis unemployment rate and the effect on the crisis increase in the expected unemployment rate, the product of the probability of a crisis and the crisis increase in the unemployment rate. The latter effect is very small, because a higher policy rate has only a modest decreasing effect on the probability of a crisis for a few years. Furthermore, after a few years the effect is a small increase. Because, by the long-run neutrality of monetary policy, the accumulated effect on the probability of a crisis is approximately zero, there is no accumulated effect of the policy rate on expected crisis increase in the unemployment rate.[19]

[19] This zero long-run effect is strictly true only under the assumption of the probability being a linear function of debt growth. But the effects of nonlinearities, for instance from a logistic model of the probability of a crisis, will be of second order under these small changes and will hardly change the conclusions. Furthermore, the logistic function

According to these results, it is simply not true that a higher unemployment rate in the near future can be traded for a lower expected unemployment rate further into the future. Instead, leaning against the wind increases the expected unemployment rate both in the near future and further into the future.

## 3 The effect on expected future quadratic losses of leaning against the wind

In order to assess whether leaning against the wind is justified or not, it is not sufficient to only look at the expected future unemployment rate. The marginal welfare loss from a higher unemployment rate is larger the more the initial unemployment rate exceeds its desirable level, something that is captured by a quadratic loss function. In this section I therefore examine whether or not leaning against the wind is justified when gains and losses are measured by a quadratic loss function. For simplicity, the quadratic loss function has only unemployment as an argument, instead of both inflation and unemployment. However, such a simple loss function can be seen as an indirect loss function resulting from the minimization of a loss function of both inflation and unemployment.

More precisely, let $u_t^*$ denote the *benchmark unemployment rate*. This benchmark unemployment rate should be interpreted as the unemployment rate resulting from the minimization of a quadratic loss function of inflation and unemployment subject to a Phillips curve, as shown in some detail in appendix C. Furthermore, this minimization is undertaken under the assumption that the possibility of a crisis is disregarded and thus that the probability of a crisis is set to zero, $p_t \equiv 0$ for $t \geq 1$. Thus, the benchmark unemployment rate can be seen as the optimal unemployment rate under flexible inflation targeting, when the possibility of a financial crisis is disregarded. It is assumed to depend on exogenous shocks (see appendix C for details).

Let $\tilde{u}_t$ denote the *unemployment gap*, the gap between the unemployment rate and the benchmark unemployment rate,

$$\tilde{u}_t \equiv u_t - u_t^*, \tag{3.1}$$

and let $\tilde{u}_t^{\mathrm{n}} \equiv u_t^{\mathrm{n}} - u_t^*$ and $\tilde{u}_t^{\mathrm{c}} \equiv u_t^{\mathrm{c}} - u_t^*$ denote the non-crisis and crisis unemployment gaps,

(2.6) is slightly convex in the range of the relevant real debt growth rates (see figure 5.1 below), meaning that any increased variability in real debt growth rates caused by the higher policy rate will *increase* the average probability of a crisis, but very slightly so.

respectively. Introduce the expected intertemporal loss,

$$\mathrm{E}_1 \sum_{t=1}^{\infty} \delta^{t-1} L_t = \sum_{t=1}^{\infty} \delta^{t-1} \mathrm{E}_1 L_t, \tag{3.2}$$

where $\delta$ denotes a discount factor and satisfies $0 < \delta < 1$ and the quarter-$t$ loss function, $L_t$, is a simple quadratic loss function of the unemployment gap,

$$L_t = (\tilde{u}_t)^2. \tag{3.3}$$

Thus, (3.3) can be seen as an indirect loss function resulting from the minimization of a quadratic loss function of inflation and unemployment in quarter $t$, when the possibility of a financial crisis is disregarded.[20]

Let me next examine the expected quarter-$t$ loss, $\mathrm{E}_1 L_t$, when the possibility of a financial crisis is taken into account. It can be expressed as

$$\mathrm{E}_1 L_t = \mathrm{E}_t(\tilde{u}_t)^2 = (1 - p_t)\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2 + p_t \mathrm{E}_1(\tilde{u}_t^{\mathrm{c}})^2 = (1 - p_t)\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2 + p_t \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}} + \Delta u)^2, \tag{3.4}$$

where I have used that

$$\tilde{u}_t^{\mathrm{c}} = \tilde{u}_t^{\mathrm{n}} + \Delta u. \tag{3.5}$$

Thus, the expected quarter-$t$ loss can be seen as the probability-weighted expected loss in a non-crisis, $(1 - p_t)\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2$, plus the probability-weighted expected loss in a crisis, $p_t \mathrm{E}_1(\tilde{u}_t^{\mathrm{c}})^2$. [21]

Furthermore, because the expected square of a random variable equals the square of the expected random variables plus its variance,[22] we have

$$\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2 = (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}},$$
$$\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}} + \Delta u)^2 = (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} + \Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}},$$

where $\mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}}$ denotes the variance of $\tilde{u}_t^{\mathrm{n}}$ conditional on information available in quarter 1. Then I can write the quarter-$t$ expected loss (3.4) as [23]

$$\mathrm{E}_1 L_t = (1 - p_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + p_t(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} + \Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}}. \tag{3.6}$$

[20] Stein (2013) also uses a loss function in terms of unemployment only.

[21] Here, the fixed crisis increase in the unemployment rate, $\Delta u$, in the expected crisis loss, $\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}} + \Delta u)^2$, should be interpreted as the crisis increase in the unemployment rate after possible policy actions during the crisis to moderate the crisis cost, as in section 2. More generally, since a crisis has many different costs, $\Delta u$ represents the unemployment-increase *equivalent* of these crisis costs.

[22] For a random variable $X$, we have $\mathrm{E}(X)^2 = \mathrm{E}[\mathrm{E}X + (X - \mathrm{E}X)]^2 = (\mathrm{E}X)^2 + \mathrm{E}(X - \mathrm{E}X)^2 = (\mathrm{E}X)^2 + \mathrm{Var}\, X$.

[23] As noted in footnote 6, the crisis increase in the unemployment rate could be random instead of deterministic. As shown in appendix G, this can easily be incorporated but would not affect the results.

Consider the initial situation when there is no crisis in quarter 1 and where the expected future unemployment gaps are zero,

$$\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0 \quad \text{for } t \geq 1. \tag{3.7}$$

That is, the expected future unemployment rates are equal to the expected benchmark unemployment rates, and the situation is optimal if the probability of a crisis in future quarters is assumed to equal zero. Under that assumption, the quarter-$t$ expected loss is just $(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \mathrm{Var}_1\tilde{u}_t^{\mathrm{n}}$, which is minimized if $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$.

However, the actual probability of a future crisis is not zero. Let $\bar{p}_t$ for $t \geq 1$ denote the actual probability of a crisis in quarter $t$, conditional on the initial situation (3.7) and the corresponding current and expected future policy rates. I will call it the *benchmark probability of a crisis* in quarter $t$. By adding and subtracting $(1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2$ from (3.6), the expected quarter-$t$ loss when the probability of a crisis is taken into account can be rewritten as

$$\begin{aligned}\mathrm{E}_1 L_t - \mathrm{Var}_1\tilde{u}_t^{\mathrm{n}} &= [(1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2] - (\bar{p}_t - p_t)[(\Delta u)^2 + 2\Delta u \mathrm{E}_1\tilde{u}_t^{\mathrm{n}}] \\ &\equiv C_t - B_t,\end{aligned} \tag{3.8}$$

where I have used that the crisis loss increase satisfies

$$(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2 - (\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 = (\Delta u)^2 + 2\Delta u \mathrm{E}_1\tilde{u}_t^{\mathrm{n}}. \tag{3.9}$$

Also, under the assumption of a linear relation between the policy rate and the expected non-crisis unemployment gap together with additive shocks, the conditional variance $\mathrm{Var}_1\tilde{u}_t^{\mathrm{n}}$ is independent of policy. Therefore I have moved it to the left side, and it is sufficient for our purpose to examine the terms on the right side of (3.8).

The expression (3.8) allows us to assess the effect of a higher policy rate on the expected future losses. A higher policy rate will increase the expected future unemployment gap, $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}$, above zero and possibly reduce the probability of a crisis in future quarters, $p_t$, below the benchmark probability of a crisis, $\bar{p}_t$. In particular, I will refer to the first term in (3.8),

$$C_t \equiv (1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2 \equiv C_t^{\mathrm{n}} + C_t^{\mathrm{c}}, \tag{3.10}$$

as the *cost* of deviating from a zero unemployment gap. It consists of the sum of the probability-weighted expected loss in a non-crisis, $C_t^{\mathrm{n}}$, and the probability weighted loss in a crisis, $C_t^{\mathrm{c}}$, when the benchmark probability of a non-crisis and crisis is used. Furthermore, I will refer to the second term in (3.8),

$$B_t \equiv (\bar{p}_t - p_t)[(\Delta u)^2 + 2\Delta u \mathrm{E}_1\tilde{u}_t^{\mathrm{n}}], \tag{3.11}$$

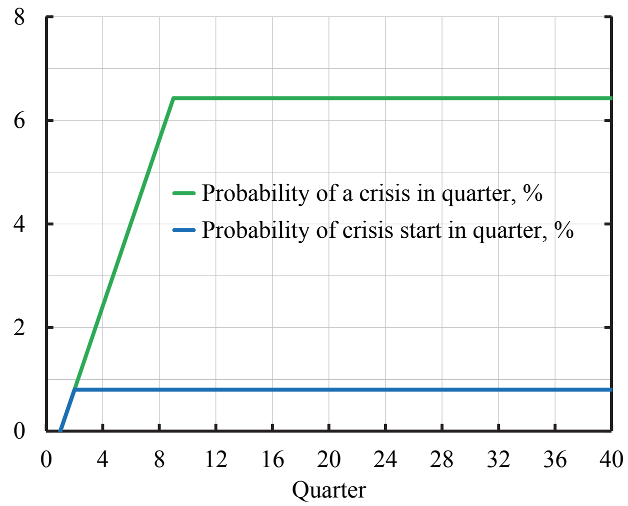


Figure 3.1: The benchmark probability of a crisis start and the benchmark probability of a crisis, conditional on no crisis in quarter 1.

as the *benefit* of deviating from a zero unemployment gap. It consists of the reduction in the probability of a crisis from the benchmark probability, $\bar{p}_t - p_t$, multiplied by the loss increase in a crisis, (3.9).

## 3.1 The benchmark probability of a crisis

Before looking more closely at this expression for the cost and benefit, let me specify the estimate of the benchmark probability of a crisis. The sum of the coefficients in (2.6) and the reported marginal effect of 0.30 by Schularick and Taylor (2012) is consistent with a constant annual probability of a crisis start equal to 3.2 percent.[24] This corresponds to a crisis start on average every 31 years. A constant annual probability of a crisis start of 3.2 percent implies a corresponding constant probability of a crisis start in a given quarter, denoted $q$, equal to $3.2/4 = 0.8$ percent. I will use this as my benchmark probability of a crisis start. Furthermore, as mentioned, I have assumed that a crisis lasts 8 quarters ($n = 8$).

Conditional on no crisis in quarter 1, for a given $q$ and $n$, the benchmark probability of a crisis in quarter $t$ is then, according to (2.2),

$$\bar{p}_t = \begin{cases} 0 & \text{for } t = 1, \\ (t-1)q > 0 & \text{for } 2 \le t \le n, \\ nq > 0 & \text{for } t \ge n+1. \end{cases} \tag{3.12}$$

[24] See appendix B for details.

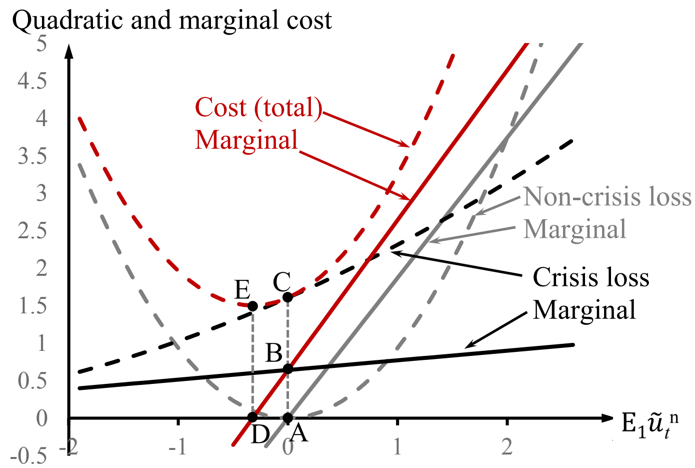


Figure 3.2: The probability-weighted quadratic (dashed line) and marginal (solid) expected non-crisis loss (grey), expected crisis loss (black), and (total) cost (red), as a function of the expected non-crisis unemployment gap (under the assumption that the benchmark probability of a crisis is 6.4 percent and the crisis increase in the unemployment rate is 5 percent).

Thus, $\bar{p}_t$ rises linearly from 0 in quarter 1 to its steady-state value $p \equiv nq$ in quarter $n+1$. With $n = 8$ quarters and $q = 0.8$ percent, $\bar{p}_t$ rises linearly from 0 in quarter 1 to $p = 6.4$ percent in quarter 9 and then stays at 6.4 percent, as shown in figure 3.1.[25]

## 3.2 The cost of deviating from a zero expected non-crisis unemployment gap

Let me next examine in some detail the quarter-$t$ cost and benefit (3.8) of deviating from a zero expected non-crisis unemployment gap. To best understand what determines the cost and benefit for a particular quarter, it is practical to examine how they depend on the expected non-crisis unemployment gap.

Let me do this for quarters $t \geq 9$, when the benchmark probability of a crisis is constant and equal to the steady-state level, $\bar{p}_t = p = 6.4$ percent. Let me start with the cost, $C_t$, given by (3.10) for $\bar{p}_t = p$, the sum of the probability-weighted expected loss in a non-crisis ($C_t^{\rm n}$) and a crisis ($C_t^{\rm c}$).

In figure 3.2, the grey dashed line shows the probability-weighted non-crisis expected loss,

$$C_t^{\rm n} = (1 - \bar{p}_t)(\mathrm{E}_1 \tilde{u}_t^{\rm n})^2 = 0.936(\mathrm{E}_1 \tilde{u}_t^{\rm n})^2,$$

[25] As mentioned in footnote 8, (3.12) is a linear approximation to a Markov process for the probability of a crisis. As shown in appendix A and figure A.1, for the relevant Markov process, the benchmark probability of a crisis can be shown to rise from zero in quarter 1 to 6.2 percent in quarter 9 and then converges to 6.0 percent in quarter 16.

as a function of the expected non-crisis unemployment gap, $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}$. It has a minimum for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$, corresponding to point A. The grey solid line shows the corresponding probability-weighted *marginal* non-crisis loss (with respect to an increase in the expected non-crisis unemployment gap),

$$\frac{dC_t^{\mathrm{n}}}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = \frac{d[(1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2]}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = 0.936 \cdot 2\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}.$$

It is zero where the probability-weighted non-crisis loss has a minimum, for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$, and has a positive slope of 1.872.

Under the assumption that the probability of a crisis is zero, the non-crisis loss is the only loss that matters, and the optimal policy is to set the expected non-crisis unemployment gap equal to zero. But if the probability of a crisis is positive, the probability-weighted crisis loss has to be taken into account.

The black dashed line shows the probability-weighted crisis loss,

$$C_t^{\mathrm{c}} = \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2 = 0.064(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + 5)^2,$$

where I have used that the crisis increase in the unemployment rate is assumed to be 5 percent. The probability-weighted crisis loss has a minimum for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = -5$ percentage points, and is upward-sloping for the range of expected non-crisis unemployment gaps shown in the figure. For $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$, the probability-weighted crisis loss is $0.064 \cdot 5^2 = 1.61$, corresponding to point C in the figure. The black solid line shows the corresponding probability-weighted marginal crisis loss,

$$\frac{dC_t^{\mathrm{c}}}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = \frac{d[\bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2]}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = 0.064 \cdot 2(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + 5) = 0.128\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + 0.64.$$

The probability-weighted marginal crisis loss is zero for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = -5$ and positive and equal to $0.064 \cdot 2(5) = 0.64$ for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$, and it has a positive slope of 0.128.

That the marginal crisis loss is positive for $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0$ reflects the realistic and crucial assumption that the cost of crisis is larger if the economy is weaker.

The red dashed line shows the cost of deviating from a zero expected non-crisis unemployment gap, the sum of the probability-weighted expected non-crisis and crisis losses,

$$C_t = (1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2 = 0.936(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + 0.064(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + 5)^2,$$

that is, the vertical sum of the grey and black dashed lines. The red solid line shows the corresponding marginal cost (with respect to increasing the expected non-crisis unemployment gap),

$$\frac{dC_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = \frac{d[(1-\bar{p}_t)(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \Delta u)^2]}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = 2(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + \bar{p}_t\Delta u) = 2(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + 0.32). \tag{3.13}$$

For $\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} = 0$, the total loss is $0.064 \cdot 5^2 = 1.61$, corresponding to point C, and the marginal loss is 0.64, corresponding to point B. It is obvious from the figure that this is not a minimum for the cost.

The minimum for the cost occurs where the marginal cost is zero, for which $\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} = -0.32$ percentage points, corresponding to point D. Then the cost is 1.50, corresponding to point E. The gain, the reduction in total loss from point C to point E is $0.11 = 0.32^2$, thus equivalent to the negative of the loss of increasing the unemployment rate by 0.32 percentage points from its optimal level.

Thus, if the probability of a crisis is zero, it is optimal to set the expected non-crisis unemployment gap equal to zero. If the probability of a crisis is positive, it is optimal to *reduce* the expected non-crisis unemployment gap below zero. That is, it is optimal to *lower* the policy rate and thus lean *with* the wind. More precisely, as long as the probability of a crisis is taken as given, without taking into account the possible benefit from a reduced probability of a crisis, it is optimal to lean with the wind.

We can see this in a alternative way. We can rewrite the expected quarter-$t$ loss as the sum of the squared expected unemployment gap and the conditional variance of the unemployment gap, the first equality in (3.14),[26]

$$\mathrm{E}_1 L_t = \mathrm{E}_1 (\tilde{u}_t)^2 = (\mathrm{E}_1 \tilde{u}_t)^2 + \mathrm{Var}_1 \tilde{u}_t = (\mathrm{E}_1 \tilde{u}_t)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}} + p_t(1-p_t)(\Delta u)^2. \tag{3.14}$$

The second equality in (3.14) uses that the conditional variance of the unemployment gap ($\mathrm{Var}_1 \tilde{u}_t$) is the sum of the conditional variance the non-crisis unemployment gap ($\mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}}$) and the variance of a binomial distribution $[p_t(1-p_t)(\Delta u)^2]$ because the unemployment gap is the sum of the non-crisis unemployment gap and a binomial random variable that takes the value $\Delta u$ with probability $p_t$ and the value 0 with probability $1 - p_t$.[27]

Under the assumption that the probability of a crisis is given by the benchmark probability of a crisis, $p_t = \bar{p}_t$, the variance terms in (3.14) are exogenous and independent of policy.[28] Then the marginal loss with respect to an increase in the expected unemployment gap satisfies,

$$\frac{d\mathrm{E}_1 L_t}{d\mathrm{E}_1 \tilde{u}_t} = \frac{d(\mathrm{E}_1 \tilde{u}_t)^2}{d\mathrm{E}_1 \tilde{u}_t} = 2\mathrm{E}_1 \tilde{u}_t,$$

[26] Kocherlakota (2014) and Stein (2014) use such a decomposition of the loss function.

[27] The conditional covariance between the non-crisis unemployment gap and a crisis start is assumed to be zero.

[28] If the conditional variance terms are independent of policy, Certainty Equivalence holds, and it is sufficient to focus on the conditional means of the relevant variables. When the conditional variance terms depend on policy, as when the probability of a crisis depends on the policy rate, Certainty Equivalence no longer holds and optimal policy also has to take into account the effect on the conditional variance terms.

and the optimal policy is to set the marginal loss and thereby the expected unemployment gap equal to zero,

$$\mathrm{E}_1 \tilde{u}_t = \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} + \bar{p}_t \Delta u = 0.$$

This implies setting the expected non-crisis unemployment gap equal to the minus the probability-weighted crisis increase in the unemployment rate,

$$\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} = -\,\bar{p}_t \Delta u < 0.$$

Once seen, this is completely obvious. If there is a given positive probability of a crisis, the expected unemployment gap is greater than the expected non-crisis unemployment gap. It is optimal to set the expected unemployment gap equal to zero; hence it is optimal to set the expected non-crisis unemployment gap below zero. Leaning *with* the wind is the obvious policy in this case. The optimal amount of leaning with the wind is a non-crisis unemployment gap equal to $-\,\bar{p}_t \Delta u = -\,0.064 \cdot 5 = -\,0.32$ percentage points instead of zero, a rather modest amount of leaning with the wind.

### 3.3 The possible benefit of leaning against the wind

The discussion in section 3.2 above shows that for a given probability of a crisis, the optimal policy is to lean somewhat *with* the wind. However, an increase in the policy rate may reduce the probability of a crisis and this way reduce the expected loss increase in a crisis, as the benefit $B_t$ given by (3.11) shows. Thus, the increase in the expected non-crisis unemployment gap from the increase in the policy rate is here accompanied by a change in the probability of a crisis.

The probability of a crisis can then be seen as an implicit function of the expected non-crisis unemployment gap, $p_t = p_t(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})$, with the linear approximation

$$p_t(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}) - \bar{p}_t = \frac{dp_t}{d\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}} \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}, \text{ for } t \geq 1, \tag{3.15}$$

where $p_t(0) = \bar{p}_t$ and the implicit derivative $dp_t / d\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}$ is given by[29]

$$\frac{dp_t}{d\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}} \equiv \frac{dp_t / d\bar{i}_1}{d\mathrm{E}_1 u_t^{\mathrm{n}} / d\bar{i}_1}. \tag{3.16}$$

This makes it possible to specify the benefit from deviating from a zero unemployment gap as a

[29] Because the derivative $d\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} / d\bar{i}_1$ by figure 2.1 is positive for $1 < t \leq 40$, the implicit derivative $dp_t / d\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}$ in (3.16) is well-defined for $1 < t \leq 40$.

function of the expected non-crisis unemployment gap and rewrite it as

$$B_t \equiv [\bar{p}_t - p_t(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})][(\Delta u)^2 + 2\Delta u\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}] \;=\; (-\frac{dp_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}})\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}[(\Delta u)^2 + 2\Delta u\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}] \tag{3.17}$$

$$= 0.0086\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}(25 + 10\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}).$$

Here the number $-0.0086$ is the average of the derivative (3.16) over quarters 12-24, where the derivatives $dp_t/d\bar{i}_1$ and $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$ are given by the green line in figure 2.2 and the red line in figure 2.1, respectively. This is clearly an overestimate of the magnitude of the average reduction of the probability of a crisis per unemployment rate increase, something which exaggerates the benefit and stacks the cards in favor of leaning against the wind.[30]

In figure 3.3, the green dashed line shows the benefit $B_t$ of deviating from a zero expected non-crisis unemployment gap. It is by (3.17) equal to zero for a zero expected non-crisis unemployment gap. Furthermore, it is quadratic and, for a negative derivative $dp_t/d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}$, convex and increasing in the expected non-crisis unemployment gap. It is convex because the crisis loss increase (3.9) is increasing in the expected non-crisis unemployment gap, making the benefit increase more than linearly.

The red dashed line in figure 3.3 is the cost $C_t$ of deviating from a zero expected non-crisis unemployment gap, the same line as in figure 3.2. The blue dashed line is the net cost, $C_t - B_t$, the difference between the red and the green dashed lines.

The green solid line is the marginal benefit of increasing the expected non-crisis loss, given by

$$\frac{dB_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = (-\frac{dp_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}})[(\Delta u)^2 + 4\Delta u\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}] = 0.0086\,(25 + 20\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}) \tag{3.18}$$

It is linear and increasing in the expected non-crisis unemployment gap when the derivative of the probability of a crisis with respect to the expected non-crisis unemployment gap is negative. The blue solid line is the net marginal cost, the difference between the red and green solid lines.

We see that the net cost has a minimum at point G, between the points E and C, where the marginal cost and marginal benefit are equal (point F) and where the net marginal cost thus equals zero, at point H between points D (corresponding to an expected non-crisis unemployment gap equal to $-0.32$ percentage point) and A (corresponding to an expected non-crisis unemployment gap equal to zero). We see that in this case, the optimal policy is rather close to point D and still involves some modest leaning with the wind, not against. It corresponds to an expected non-crisis unemployment gap of $-0.23$ percentage points.

[30] Figure F.1 in appendix F shows the negative of the derivative (3.16) for each quarter.

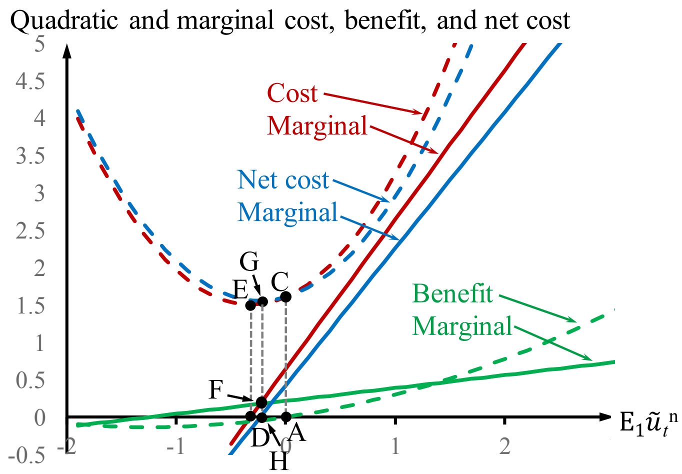


Figure 3.3: The probability-weighted quadratic (dashed line) and marginal (solid) cost (red), benefit (green), and net cost (blue), as a function of the expected non-crisis unemployment gap (under the assumption that the benchmark probability of a crisis is 6.4 percent and the crisis increase in the unemployment rate is 5 percent).

Consistent with this, we see that the net marginal cost for a zero expected non-crisis unemployment gap is positive. This means that in the initial situation, where the expected future non-crisis unemployment gaps are zero, the marginal cost of leaning against the wind exceeds the marginal benefits, meaning that the optimal policy is a small leaning *with* the wind, not against. This happens in spite of the exaggerated estimate that I have used of the reduction of the probability of a crisis per expected non-crisis unemployment gap increase

In summary, we see that there is a strong tendency towards some leaning *with* the wind rather than against. Only if the policy rate has a sufficiently strong negative effect on the probability of a crisis can leaning *against* the wind be justified.

### 3.4 The alternative assumption of a fixed cost of a crisis

As mentioned, an important assumption in this paper is the realistic assumption that a crisis is more costly if the economy is weak. This is represented by the assumption that a crisis implies an *increase* in the unemployment gap by $\Delta u$,

$$\tilde{u}_t^{\mathrm{c}} = \tilde{u}_t^{\mathrm{n}} + \Delta u, \tag{3.5 revisited}$$

so the expected crisis unemployment gap is higher if the non-crisis unemployment gap is higher,

$$\mathrm{E}_1 \tilde{u}_t^{\mathrm{c}} = \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} + \Delta u.$$

In this subsection, let me briefly examine the consequences of the unrealistic assumption that that a crisis means that the expected unemployment gap does not *increase* by $\Delta u$ but *reaches* $\Delta u$, regardless of what the expected non-crisis unemployment gap is. That is, the expected crisis unemployment gap simply satisfies

$$\mathrm{E}_1 \tilde{u}_t^{\mathrm{c}} = \Delta u,$$

regardless of $\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}$. This means that the cost of a crisis is fixed and independent of the non-crisis unemployment gap, in line with the assumption made in Svensson (2014, 2015), Ajello, Laubach, Lopez-Salido, and Nakata (2015), and Diaz Kalan, Laséen, Vestin, and Zdzienicka (2015).

We first note that it is practical in this case to make the innocuous assumption that the crisis unemployment gap is random with mean $\Delta u$ and conditional variance equal to that of the non-crisis unemployment gap. Then the expected loss in a crisis satisfies

$$\mathrm{E}_1 (\tilde{u}_t^{\mathrm{c}})^2 = (\Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{c}} = (\Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}},$$

and is independent of the expected non-crisis unemployment gap, $\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}$.

The expected quarter-$t$ loss then satisfies

$$\begin{aligned}
\mathrm{E}_1 L_t &= (1 - p_t)\mathrm{E}_1 (\tilde{u}_t^{\mathrm{n}})^2 + p_t[(\Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}}] \\
&= (1 - p_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + p_t(\Delta u)^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}},
\end{aligned}$$

where I have used that $\mathrm{E}_1 (\tilde{u}_t^{\mathrm{n}})^2 = (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}}$. Furthermore, by adding and subtracting $(1 - \bar{p}_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\Delta u)^2$, the expected quarter-$t$ loss can be rewritten

$$\begin{aligned}
\mathrm{E}_1 L_t - \mathrm{Var}_1 \tilde{u}_t^{\mathrm{n}} &= [(1 - \bar{p}_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\Delta u)^2] - (\bar{p}_t - p_t)[(\Delta u)^2 - (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2] \\
&\equiv C_t - B_t,
\end{aligned}$$

where the cost (at the benchmark probability of a crisis) and the benefit (from a reduction in probability of a crisis) now satisfy

$$\begin{aligned}
C_t &\equiv (1 - \bar{p}_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\Delta u)^2, \\
B_t &\equiv (\bar{p}_t - p_t)[(\Delta u)^2 - (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2].
\end{aligned}$$

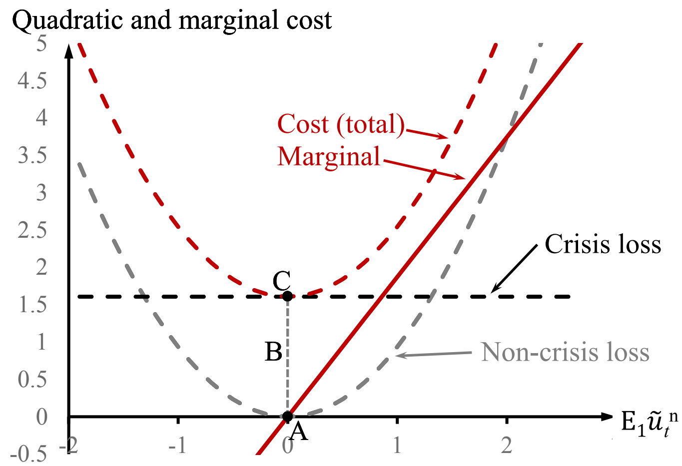


Figure 3.4: For a fixed cost of a crisis, the probability-weighted non-crisis loss (grey dashed line), probability-weighted fixed crisis loss (black dashed), and the (total) cost (red dashed) and marginal cost (red solid), as a function of the expected non-crisis unemployment gap (under the assumption that the benchmark probability of a crisis is 6.4 percent).

We note that the assumption of a fixed cost of a crisis has the strange implication that the crisis loss increase,

$$(\Delta u)^2 - (\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2, \tag{3.19}$$

is *decreasing* in the magnitude of the expected non-crisis unemployment gap. In particular, if the expected non-crisis unemployment gap is larger than $\Delta u$, the crisis loss increase is negative; that is, it is better to have a crisis than a non-crisis. In contrast, under our main assumption that a crisis increases the unemployment gap by $\Delta u$, the crisis loss increase is given by (3.9), which is increasing in the expected non-crisis unemployment gap.

In figure 3.4 (the analogue of figure 3.2) the grey dashed line shows the benchmark-probability-weighted non-crisis loss, and the black dashed line shows the benchmark-probability-weighted fixed crisis loss, which is independent of the expected non-crisis unemployment gap. The red dashed and solid lines show the corresponding cost and marginal cost of deviating from a zero expected non-crisis unemployment gap.

The cost, benefit, and net benefit are shown in figure 3.5 (the analogue of figure 3.3). The red dashed line shows the cost,

$$C_t \equiv (1 - \bar{p}_t)(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + \bar{p}_t(\Delta u)^2 = 0.936\,(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + 0.064 \cdot 25 = 0.936\,(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 + 1.6.$$

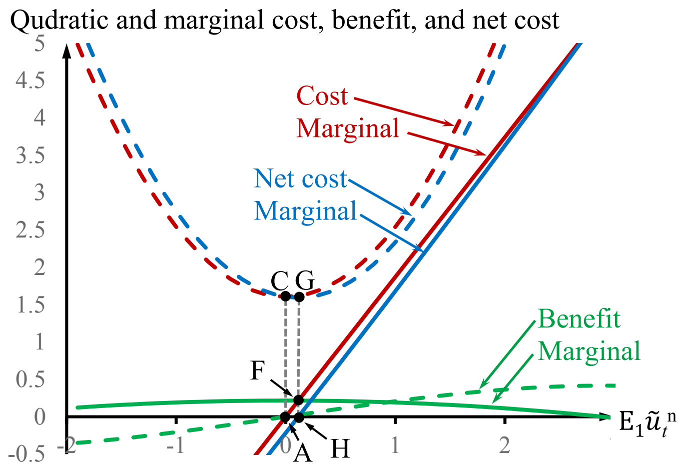


Figure 3.5: For a fixed cost of a crisis, the cost (red dashed line), the marginal cost (red solid), the benefit (green dashed), the marginal benefit (green solid), the net cost (blue dashed), and the net marginal cost (blue solid) as a function of the expected non-crisis unemployment gap (under the assumption that the benchmark probability of a crisis is 6.4 percent).

The red solid line shows the marginal cost of increasing the expected non-crisis unemployment gap,

$$\frac{dC_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = (1-\bar{p}_t)2\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 1.872\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}.$$

The cost has a minimum ( point C) for $\mathrm{E}_t\tilde{u}_t^{\mathrm{n}} = 0$ (point A). Because the crisis loss is independent of the expected non-crisis unemployment gap, a positive constant probability of a crisis does not induce any leaning with the wind, in contrast to when the crisis loss is increasing in the expected non-crisis unemployment gap.

The green dashed line shows the benefit as a function of the expected non-crisis unemployment gap,

$$B_t \equiv (\bar{p}_t - p_t)[(\Delta u)^2 - (\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2] = 0.0086\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}[25 - (\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2] = 0.215\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} - 0.0086\,(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^3,$$

where I use the linear approximation (3.15). The green solid line shows the marginal benefit of increasing the expected non-crisis unemployment gap,

$$\frac{dB_t}{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}} = 0.215 - 0.0258\,(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2.$$

The blue dashed line shows the net cost, $C_t - B_t$, the difference between the red and the blue lines. The blue solid line shows the net marginal cost of increasing the expected non-crisis unemployment gap. Because, for a zero expected non-crisis unemployment gap, the marginal cost is zero but the marginal benefit is positive, the net cost has a minimum at point G for a positive expected non-crisis unemployment gap at point H, for which the marginal cost and marginal benefit are equal, point F. That is, some leaning against the wind is optimal. However, it is quite small, corresponding to a 0.12 percentage point expected non-crisis unemployment gap, with a completely insignificant improvement in the net cost. This is consistent with the results of Ajello, Laubach, Lopez-Salido, and Nakata (2015). The small impact of the possibility of affecting the probability of a crisis is illustrated by the fact that the benefit is so small relative to the cost in figure 3.5 and that the cost (the red dashed line) and the net cost (the blue dashed line) are so similar.

The case of a fixed cost of a crisis is further examined in appendix J. There it is shown in more detail that the optimal leaning against the wind is very small and that the net gain from leaning against the wind is completely insignificant. In particular, it is shown that this result holds even under the assumption of monetary non-neutrality and a permanent effect on real debt and positive effect on accumulated marginal benefit. Thus, even then any significant leaning of the wind is not justified.

# 4 The marginal cost, marginal benefit, and net marginal cost of leaning against the wind

Let me now return to the main assumption that the cost of a crisis is not fixed but larger if the economy is initially weaker, represented by the assumption that a crisis implies that the unemployment gap increases by $\Delta u$.

The discussion in section 3 focused on the cost and benefit of leaning against wind in a given quarter, as a function of the expected non-crisis unemployment gap in that quarter. However, increasing the policy rate $\bar{i}_1$ in quarter 1–4 increases the expected crisis and non-crisis unemployment gaps in all quarters 2–40 and reduces the probability of a crisis mainly in quarters 12–24. Assessing the cost and benefit from increasing the policy rate thus requires that all the costs and benefits in all relevant quarters are compared, in particular when assessing how robust the results are to changes in the relevant parameters.

Thus, let me consider the initial situation when the expected non-crisis unemployment gap is equal to zero for all quarters, (3.7), and examine whether increasing the policy rate increases or reduces the intertemporal loss, when the impact in all future quarters are taken into account. This means to examine the derivative of the intertemporal loss function with respect to the policy rate during quarters 1–4, the marginal expected loss from increasing the policy rate,

$$\frac{d}{d\bar{i}_1}\mathrm{E}_1\sum_{t=1}^{\infty}\delta^{t-1}L_t = \sum_{t=1}^{\infty}\delta^{t-1}\frac{d\mathrm{E}_1 L_t}{d\bar{i}_1}. \tag{4.1}$$

If this marginal expected loss from increasing the policy rate is *negative*, it is optimal to *raise* the policy rate and *increase* the expected future unemployment gaps above zero, and thus lean *against* the wind. If the marginal expected loss is *positive*, it is optimal to *lower* the policy rate and *reduce* the expected future unemployment gaps below zero, and thus lean *with* the wind.

The marginal expected loss is equal to the discounted sum of the derivatives of expected future quarterly losses, the future quarter-$t$ marginal expected losses. Let me examine the marginal expected loss for a given quarter $t$, starting from the expression (3.4) for the expected quarter-$t$ loss and taking the derivative with respect to the policy rate,

$$\frac{d\mathrm{E}_1 L_t}{d\bar{i}_1} = 2(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + p_t\Delta u)\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + [(\Delta u)^2 + 2\Delta u\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}]\frac{dp_t}{d\bar{i}_1}, \tag{4.2}$$

where I have used (3.9). I have also assumed sufficient linearity, such that the derivatives $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$ and $dp_t/d\bar{i}_1$ are independent of the non-crisis unemployment gap.

In order to examine this more closely, let me identify the left side of (4.2), the marginal expected loss from a policy-rate increase $d\mathrm{E}_1 L_t/d\bar{i}_1$, with the quarter-$t$ *net marginal cost*, $\mathrm{NMC}_t$, of leaning against the wind. The first term on the right side of (4.2) can be identified with the quarter-$t$ *marginal cost* of increasing the policy rate, $\mathrm{MC}_t$. It consists of the marginal cost of increasing the expected non-crisis unemployment gap (3.13) multiplied by the effect of the policy rate on the expected unemployment rate, $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$. The second term can be identified with the quarter-$t$ *marginal benefit* of increasing the policy rate, $\mathrm{MB}_t$. It consists of the crisis loss increase, $(\Delta u)^2 + 2\Delta u\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}]$, multiplied by the negative of the effect of the policy rate on the probability of a crisis, $-\,dp_t/d\bar{i}_1$. Thus we can write

$$\mathrm{NMC}_t = \mathrm{MC}_t - \mathrm{MB}_t, \tag{4.3}$$

where

$$\mathrm{MC}_t \equiv 2[\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + p_t\Delta u]\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}, \tag{4.4}$$

$$\mathrm{MB}_t \equiv [(\Delta u)^2 + 2\Delta u\,\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}](-\frac{dp_t}{d\bar{i}_1}). \tag{4.5}$$

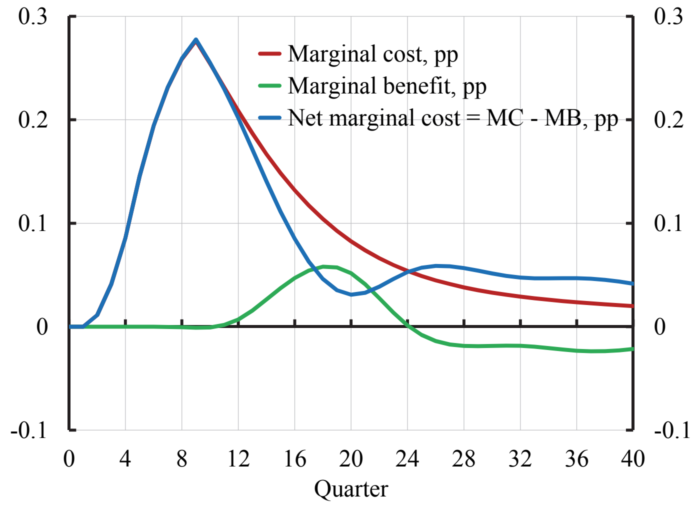


Figure 4.1: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap equals zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

When the expected non-crisis unemployment gap is zero, (3.7), we have

$$\mathrm{MC}_t = 2\bar{p}_t \Delta u \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}, \tag{4.6}$$

$$\mathrm{MB}_t = (\Delta u)^2 (-\frac{dp_t}{d\bar{i}_1}), \tag{4.7}$$

where the probability of a crisis equals the benchmark probability of a crisis, (3.12).

Given this and (3.7), the marginal cost, the marginal benefit, and the net marginal benefit in are shown for each quarter 1–40 in figure 4.1. The red line in figure 4.1 shows the marginal cost, (4.6). From quarter 9, when $p_t$ is constant, it is proportional to $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$ (the red line in figure 2.1) and positive. For quarter 1–8, the marginal cost is affected by the fact that $p_t$ is increasing, giving it a sharper and later peak than $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$. The green line in figure 4.1 shows the marginal benefit, (4.7). It is proportional to $-dp_t/d\bar{i}_1$ (the green line in figure 2.2). The blue line shows the net marginal cost, (4.3), the difference between the red and the green lines in the figure.

Importantly, from (2.8) we know that accumulated effects of the policy rate on the probability of a crisis is approximately zero. This means that the undiscounted sum of the marginal benefits (4.7) is approximately zero,

$$\sum_{t=1}^{40} \mathrm{MB}_t \approx 0. \tag{4.8}$$

This implies that the undiscounted sum of the net marginal costs is approximately equal to the undiscounted sum of the marginal costs,

$$\sum_{t=1}^{40} \mathrm{NMC}_t = \sum_{t=1}^{40} \mathrm{MC}_t - \sum_{t=1}^{40} \mathrm{MB}_t \approx \sum_{t=1}^{40} \mathrm{MC}_t > 0. \tag{4.9}$$

Discounting the sums will not affect this result much, so it is clear that, for (3.7), the intertemporal expected loss is increasing in the policy rate. This means that leaning *with* the wind is indeed justified, not leaning against.[31] [32]

Furthermore, at a closer look, the assumption about monetary neutrality and the resulting negative marginal benefit in later years do not seem essential for rejecting leaning against the wind. From figure 4.1, it is apparent that if the marginal benefit would be zero instead of negative beyond quarter 24, the conclusions would not change. This is also the case if we would disregard the positive marginal cost beyond quarter 24 and only consider marginal cost and benefit up to quarter 24. The role of monetary neutrality and non-neutrality is further examined in section 6.

### 4.1 The sensitivity to the initial state of the economy

The above examination is for an initial situation of a zero expected non-crisis unemployment gap, (3.7). Some advocacy for leaning against the wind seems to recommend it more or less regardless of the initial state of the economy (for instance, Bank for International Settlements (2014)). But an initial positive expected non-crisis unemployment gap – an initially weaker economy – dramatically strengthens the case against leaning against the wind.

In figure 4.2, the dashed lines show the marginal cost, marginal benefit, and net marginal benefit of leaning against the wind when the expected non-crisis unemployment gap is positive and equal to a modest 0.25 percentage point for all quarters, whereas the solid lines show these variables when the expected non-crisis unemployment gap equals zero (as in figure 4.1). With a non-zero expected unemployment gap, the marginal cost is given by (4.4) rather than (4.6). This modest expected non-crisis unemployment gap has a substantial impact on the marginal cost (because in (4.4) the term $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 0.25$ percentage points is of a similar order of magnitude as the term $p_t\Delta u$ (which rises from 0 in quarter 1 to 0.32 percentage points in quarter 9). The marginal benefit is given by (4.5) rather than (4.7), but the expected non-crisis unemployment gap has a quite small impact on it (because in (4.5) the term $2\Delta u\, \mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = 2.5$ is small relative to $(\Delta u)^2 = 25$). The net

[31] We can look more closely at quarter 18, when the marginal benefit is the largest. Because for that quarter, $dp_{18}/d\bar{\imath}_1 = -0.23$ percentage points, and we have $\Delta u = 5$ percentage points, by (4.7), $\mathrm{MB}_{18} = 0.0023 \cdot 5^2 = -0.058$. But for quarter 18, $d\mathrm{E}_1 u_{18}^n/d\bar{\imath}_1 = 0.16$ and, by (4.7), $\mathrm{MC}_{18} = 2 \cdot 0.064 \cdot 5 \cdot 0.16 = 0.10$, still larger than $\mathrm{MB}_{18}$.

[32] A particular constrained-optimal policy is examined in appendix I.

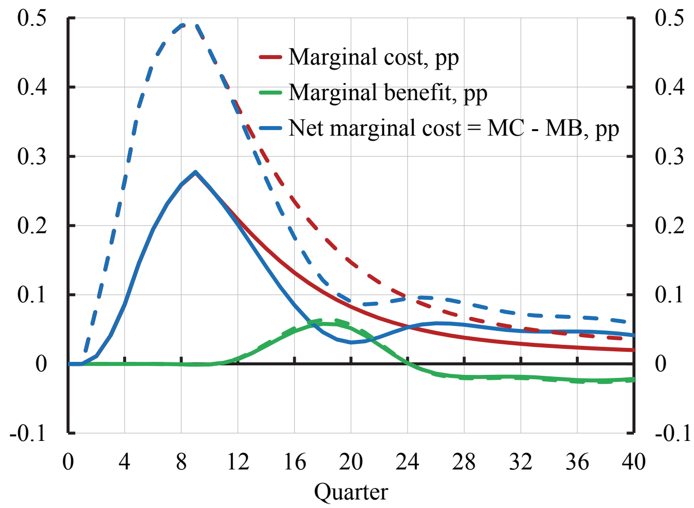


Figure 4.2: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is positive and equals 0.25 percentage points for all quarters. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

marginal cost therefore shifts up substantially and the case against leaning against the wind gets even stronger.

Assuming that initially the expected non-crisis unemployment gap is positive and the same for all future quarters allows us to examine the impact on the marginal cost and benefit at all quarters. A more realistic initial situation is arguably when the expected unemployment gap is positive for the first few quarters and approaches zero in later quarters. From figure 4.2 we realize that we still get a substantial increase in the marginal cost of leaning against the wind if, for instance, the expected non-crisis unemployment gap is positive and equal to 0.25 percentage points for only the first 8 quarters and then falls and equals zero from quarter 12 and onwards, in which case the dashed lines for the marginal cost and net marginal cost are equal to the solid lines from quarter 12 onwards.

Because leaning against the wind is not justified with a zero initial expected unemployment gap, it is of course even less justified for an initial positive expected unemployment gap. The marginal and net marginal cost of leaning increase substantially with a higher initial expected unemployment gap.

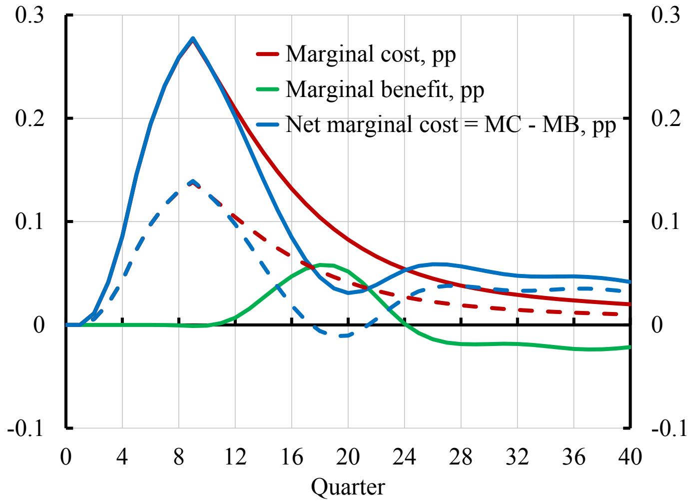


Figure 4.3: The effect of a reduction in the policy-rate effect on the expected non-crisis unemployment rate from the benchmark (solid lines) by a half (dashed lines) on the marginal cost and the net marginal cost of leaning against the wind, when the expected unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

## 4.2 The sensitivity to the effect of the policy rate on the non-crisis unemployment rate

The marginal cost and net marginal cost of leaning against the wind depend by (4.4) on the initial expected unemployment gap ($\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}$), the probability of a crisis ($p_t$), the crisis increase in the unemployment gap ($\Delta u$), and the effect of the policy rate on the expected non-crisis unemployment rate ($d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$). The sensitivity to the initial expected unemployment gap has been examined in section 4.1. The sensitivity to the probability of a crisis and the crisis increase in the unemployment gap will be examined in section 5. Here I look at the sensitivity to the effect of the policy rate increase on the expected non-crisis unemployment rate.

As a benchmark, I have used the Riksbank estimate of the effect shown in figure 2.1. In figure 4.3, the dashed lines show the marginal cost and net marginal cost when benchmark policy-rate effect on the expected non-crisis unemployment gap is reduced to a half of the Riksbank estimate, whereas the solid lines show the benchmark case. The marginal cost shifts down by a half, the marginal benefit (4.7) is not affected, and the net marginal cost shifts down with the marginal cost but remains substantially positive, except in quarters 18–21 where it is slightly negative. Clearly, leaning against the wind is still not justified.

### 4.3 A preliminary note on the sensitivity to the probability of a crisis

We will look more closely at the sensitivity to the probability of a crisis in section 5, but we can already here make a preliminary note about the sensitivity to the probability of a crisis. From (4.6) we have seen that the marginal cost is two times the product of the probability of a crisis ($p_t$), the crisis increase in unemployment rate ($\Delta u$), and the effect of the policy rate on the expected non-crisis unemployment rate ($d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$). This means that figure 4.3 can alternatively be seen as the result of the *full* (instead of the half) benchmark effect of the policy rate on the expected non-crisis unemployment rate in figure 4.3 and the *half* (instead of the full) probability of crisis in figure 3.1. Thus, even if the probability of a crisis start in a particular quarter was only 0.4 percent (instead of 0.8 percent), so the probability of crisis from quarter 9 onward was only 3.2 percent (instead of 6.4 percent), in which case the marginal cost would be half of the benchmark case, leaning against the wind would still not be justified. We realize that for an even slower rise of the probability of a crisis than half of that in figure 3.1, there is still a substantial margin for marginal cost to dominate over marginal benefit.

The above note is under the simplifying assumption that a lower probability of a crisis does not affect the effect of the policy rate on the probability of a crisis ($-\,dp_t/d\bar{i}_1$) and thereby not the marginal benefit. However, when we look more closely at this in section 5, we will see that, under the logistic relation between the probability of a crisis start and real debt growth, the effect on the probability of a crisis and the marginal benefit varies slightly with the probability of a crisis and is actually slightly lower with a lower probability of a crisis.

## 5 Does less effective macroprudential policy strengthen the case for leaning against the wind?

A common view is that macroprudential policy should provide the first line of defense of financial stability but that monetary policy may have a role as a second line of defense, in case macroprudential policy is not sufficiently effective. In line with this view, one might ask whether less effective macroprudential policy might strengthen the case for leaning against the wind. Let me examine this issue in the present framework.

What would a less effective macroprudential policy imply in the present framework? Such macroprudential policy would in general imply less resilience of the financial system to shocks, for instance through weaker balance sheets with less loss-absorbing capital. Less effective macroprudential policy may result in more credit growth and credit booms with more "bad" credit growth due to lower credit standards. All together this might increase the probability of a crisis start, $q_t$. It might also increase the severity of a crisis, in the sense of implying a large crisis increase in the unemployment rate, $\Delta u$, or a longer duration of a crisis, $n$.

Consequently, in order to assess whether less effective macroprudential policy provides a case for leaning against the wind, I examine how the marginal cost, marginal benefit, and thus the net marginal cost of leaning against the wind shifts, if the probability of crisis start is higher due to higher credit growth, the increase in the unemployment rate is larger, or the duration of a crisis is longer. This way I also conduct some further sensitivity analysis of the results.

### 5.1 A higher probability of a crisis start due to higher credit growth

Let me first examine the consequences of a higher probability of a crisis start. So far I have assumed an annual probability of a crisis start of 3.21 percent (corresponding to a crisis start on average every 31 years), which for the estimates in (2.6) is consistent with a steady annual growth rate of real debt of 5 percent. This corresponds to point A in figure 5.1, which shows how the annual probability of a crisis start depends on the steady annual real debt growth during the previous five years. Let me now consider an increase in the annual probability of a crisis start by 1 percentage points to 4.21 percent (corresponding to a crisis start on average every 24 years). This is consistent with an annual steady growth rate of 7.9 percent, corresponding to point B in the figure. Thus, we might think of less effective macroprudential policy resulting in a credit boom with higher real debt growth, which in turn increases the probability of a crisis start.

In figure 5.2, dashed lines show the marginal cost, marginal benefit, and net marginal cost from leaning against the wind for the higher annual probability 4.21 percent of a crisis start, to be compared with the solid lines for the benchmark case of an annual probability of 3.21 percent. A higher probability of a crisis start $q_t$ leads by (2.2) to a higher probability of a crisis $p_t$. We see in (4.6) that the marginal cost of leaning against the wind is proportional to $p_t$. A higher $p_t$ thus shifts up the marginal cost from the solid to the dashed red line in the figure.

What is the effect on the marginal benefit? A higher steady growth rate of real debt will increase the marginal effect of steady real debt growth on the probability of a crisis start, because the logistic function with the estimates in (2.6) is convex for growth rates in this range (see figure 5.1).[33] This

[33] For the benchmark steady annual growth rate of 5 percent, the marginal effect on the annual probability of a crisis start is 0.30. For a steady annual growth rate of 7.9 percent, the marginal effect is 0.39. See appendix B for details.

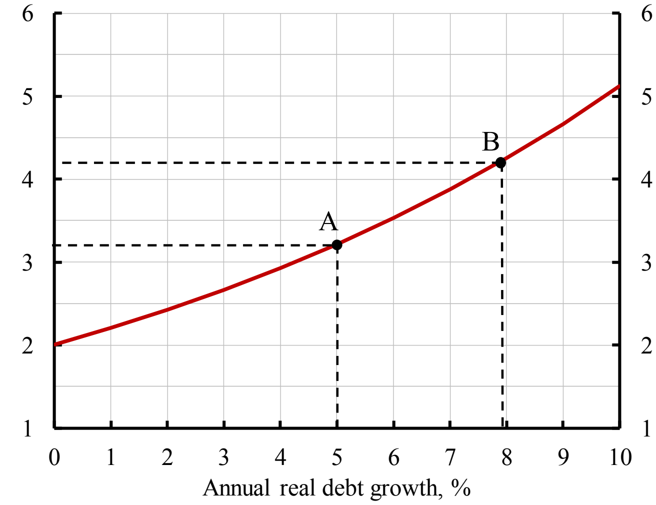


Figure 5.1: The annual probability of a crisis start (percent) as a function of real debt growth during the previous five years. (Source: Schularick and Taylor (2012) and own calculations.)

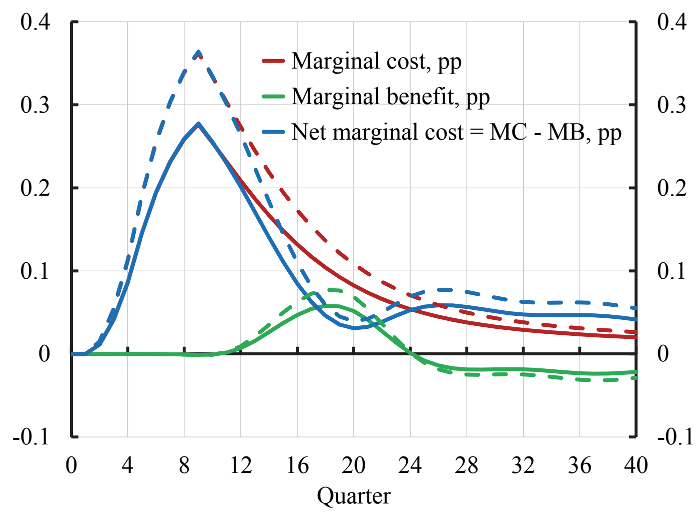


Figure 5.2: The effect of an increase in the annual probability of a crisis start from 3.21 percent (solid lines) to 4.21 percent (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

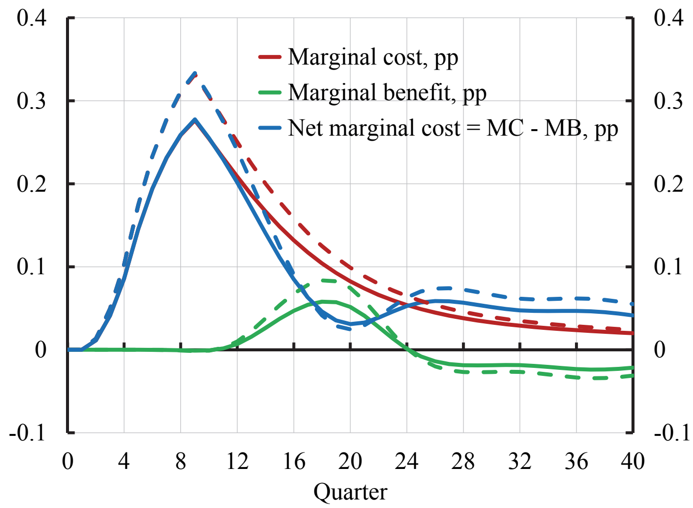


Figure 5.3: The effect of an increase in the crisis increase in the unemployment rate from 5 (solid lines) to 6 percentage points (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

will increase the magnitude of the effect of the policy rate on the probability of a crisis start and on the probability of a crisis, $-dp_t/d\bar{i}_1$. We see in (4.7) that the marginal benefit is proportional to $-dp_t/d\bar{i}_1$, so this will increase the magnitude of the marginal benefit and shift it from the solid to the dashed green line in the figure. We see in figure 5.2 that the net effect on the net marginal cost is a significant increase in the net marginal cost, from the solid to the dashed blue line.

It follows that the discounted net marginal cost increases. Thus, less effective macroprudential policy, to the extent that it leads to higher real debt growth and a higher probability of a crisis start, does not strengthen the case for leaning against the wind; it strengthens the case *against* leaning against the wind.

We may interpret this result more generally. In this framework, a credit boom with higher real debt growth and a higher probability or severity of a crisis, whether caused by an ineffective macroprudential policy or anything else, would, everything else equal, tend to strengthen the case *against* leaning against the wind, because it tends to increase the marginal cost of increasing the policy rate more than it increases the marginal benefit.[34]

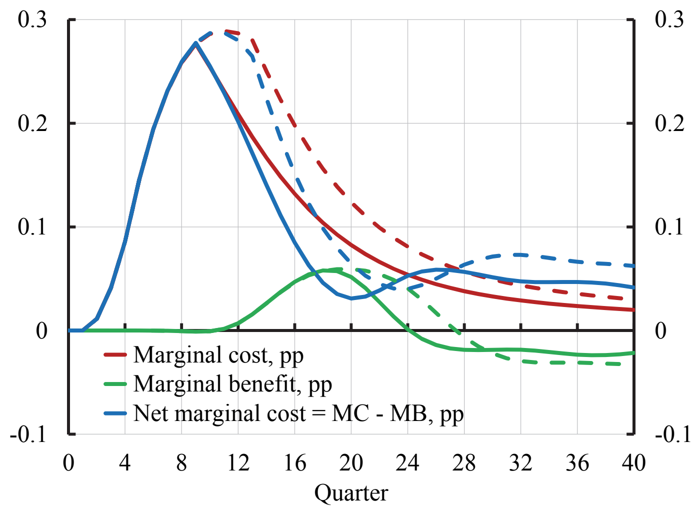


Figure 5.4: The effect of an increase in crisis duration from 8 (solid lines) to 12 quarters (dashed lines) on the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

### 5.2 A larger crisis increase in the unemployment rate

In figure 5.3, the dashed lines show the marginal cost, marginal benefit, and net marginal cost for a larger the crisis increase $\Delta u$ in the unemployment rate of 6 percentage points, to be compared with the solid lines for the benchmark crisis increase in the unemployment rate of 5 percentage points. We see in (4.6) and (4.7) that the marginal cost is linear in $\Delta u$ and the marginal benefit is quadratic in $\Delta u$. Thus, the magnitudes of the marginal cost and marginal benefit increase with $\Delta u$. We see that the net effect is an increase of the net marginal cost except around quarter 19 where the marginal benefit increases slightly more than the marginal cost.

It follows that, also in this case, the sum of discounted net marginal costs increases. Less effective macroprudential policy, to the extent that it implies a larger crisis increase in the unemployment rate, again strengthens the case *against* leaning against the wind.

### 5.3 A longer duration of a crisis

Finally, in figure 5.4, dashed lines show the marginal cost, marginal benefit, and net marginal cost for a longer crisis duration of $n = 12$ quarters, to be compared with the solid lines for the benchmark crisis duration of 8 quarters. A longer duration means by (2.2) that the probability of a crisis, for the linear approximation used here, is the sum over a few more previous quarterly probabilities

[34] Helpful discussions with Helge Berger alerted me to this more general interpretation of the results.

of a crisis start, implying a shift to the right of the marginal cost and marginal benefit of leaning against the wind.[35] As a result, the net marginal cost shifts up, except around quarter 24, where the marginal benefit increases slightly more than the marginal cost. The sum of discounted net marginal costs increases. Thus, to the extent that less effective macroprudential policy increases the crisis duration, it again strengthens the case against leaning against the wind.

Overall, for this section's intuitive assumptions about the consequence of a less effective macroprudential policy, such a less effective policy consistently further strengthens the already strong case against leaning against the wind. The presumption that a less effective macroprudential policy would strengthen the case for leaning against the wind does not stand up to scrutiny.

# 6 Non-neutral monetary policy: a permanent effect on real debt

Monetary neutrality implies that monetary policy has no effect on real debt in the long run, and therefore no effect on average and accumulated real debt growth over a longer period. Thus there is no effect on average and accumulated probabilities of a crisis over a longer period. One might think that, if monetary policy would be non-neutral and would have a permanent effect on the real debt level, this might strengthen the case for leaning against the wind.

Thus, in order to examine this, assume that the effect of the policy rate on real debt is permanent. More precisely, suppose that real debt permanently stays down at its maximum deviation from the baseline in figure 2.2 ($-1.03$ percent), from quarter 8 onwards, as shown in figure 6.1. As seen in the figure, there is a large and persistent, but not permanent, reduction in the probability of a crisis. As seen in figure 6.2, the marginal benefit is larger and more persistent. Nevertheless, this marginal benefit is not sufficient to prevent the net marginal cost from being positive and the discounted sum of the net marginal cost to be positive and large. Thus leaning against the wind still has a large positive net marginal cost, and leaning *with* the wind remains a better policy.

## 6.1 How much larger an effect on the probability of a crisis start is needed to justify leaning against the wind?

Under the assumption of a permanent effect on real debt of a 1 percentage point higher policy rate during quarters 1–4, the accumulated marginal benefit in figure 6.2 is positive and equal to 0.64.

[35] For the linear approximation (2.2), the probability of a crisis increases from zero in quarter 1 to 9.6 percent in quarter 13 and then stays at 9.6 percent. For the relevant Markov process discussed in appendix A, the probability of a crisis increases from zero in quarter 1 to 9.1 percent in quarter 13 and then converges to 8.8 percent in quarter 20.

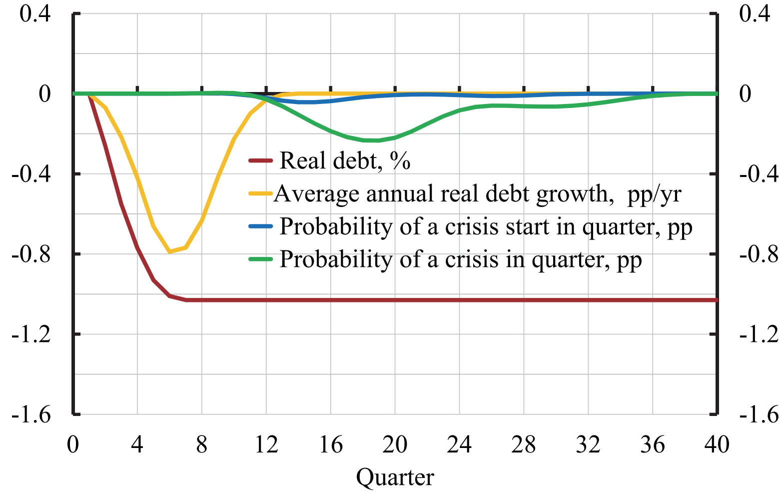


Figure 6.1: For a permanent effect on real debt, the effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

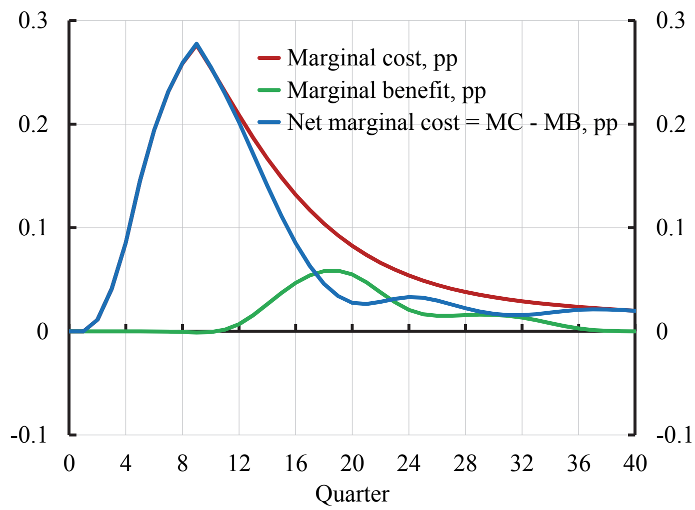


Figure 6.2: For a permanent effect on real debt, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

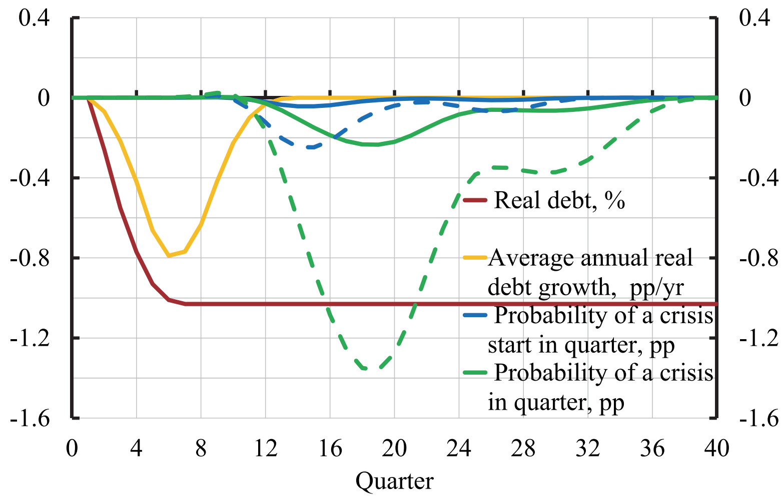


Figure 6.3: For a 5.8 times larger effect on the probability of a crisis start than the benchmark and a permanent effect on real debt, the effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

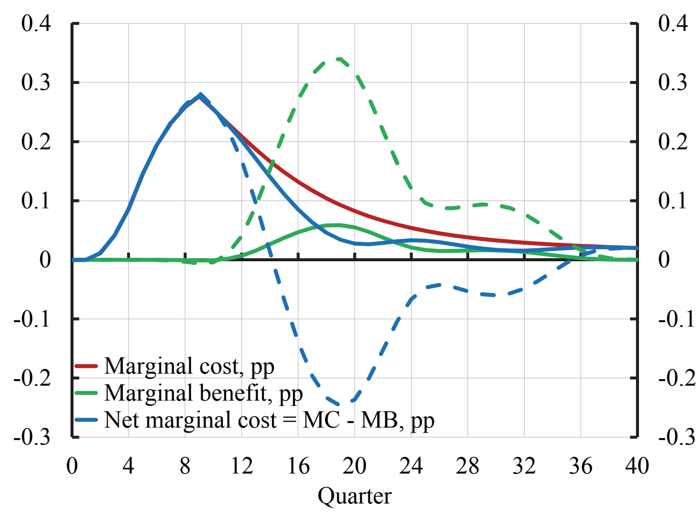


Figure 6.4: For a 5.8 times larger effect on the probability of crisis start than the benchmark and a permanent effect on real debt, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

The accumulated marginal cost is 3.71, about 5.8 times larger. We then realize that, for leaning against the wind to be justified, the effect on the policy rate on the probability of a crisis must be more than 5.8 times larger than the estimates in (2.6). If the largest coefficient, 7.138, on the two-year lag of the annual growth rate, would be two standard deviations larger, it would be 12.4, that is 1.74 times larger than 7.138. This is very far from 5.8 times larger. The dashed lines in figures 6.3 and 6.4 show the case when the effect on the probability of a crisis is 5.8 larger and the accumulated marginal cost and marginal benefit are equal.

Clearly, even under the extreme assumption of a large permanent effect on real debt, we need an extreme assumption on the effect of the policy rate on the probability of a crisis for the accumulated net marginal cost not being positive but zero.

# 7 Results for a dataset of Laeven and Valencia (2012)

So far I have used the estimates in Schularick and Taylor (2012) for their dataset covering 14 developed countries for 1870–2008. During the work on International Monetary Fund (2015), IMF staff used a dataset of Laeven and Valencia (2012) to estimate the quarterly probability of a crisis start for banking crises in 35 advanced countries 1970–2011.[36] The equation and estimates are

$$q_t = \frac{\exp(X_t)}{1 + \exp(X_t)},$$

where[37]

$$X_t = -\underset{(1.008)}{5.630^{***}} - \underset{(3.171)}{5.650^{*}}\, g_t + \underset{(3.580)}{4.210}\, g_{t-4} + \underset{(5.408)}{12.342^{**}}\, g_{t-8} - \underset{(3.591)}{5.259}\, g_{t-12}. \tag{7.1}$$

As in the Schularick and Taylor (2012) estimates, the annual growth rate of the average annual debt lagged two years is the major determinant of the probability of a crisis start, $q_t$. For 5 percent steady real debt growth, the annual probability of a crisis start is 1.89 percent, approximately equal to the frequency of crises starts in the sample. It implies a crisis start on average every 53 years. The corresponding constant quarterly probability of a crisis start, $q$, is thus about 0.47 percent. The coefficients in (7.1) sum to 5.64, implying that the marginal effect on the annual probability of a crisis start over all lags is equal to 0.11, implying the summary result that 1 percentage point lower steady real debt growth reduces the annual probability of a crisis by about 0.1 percentage points.

[36] I am grateful to Damiano Sandri for several discussions about the estimates.
[37] One, two, and three stars denote significance at the 10, 5, and 1 percent level, respectively.

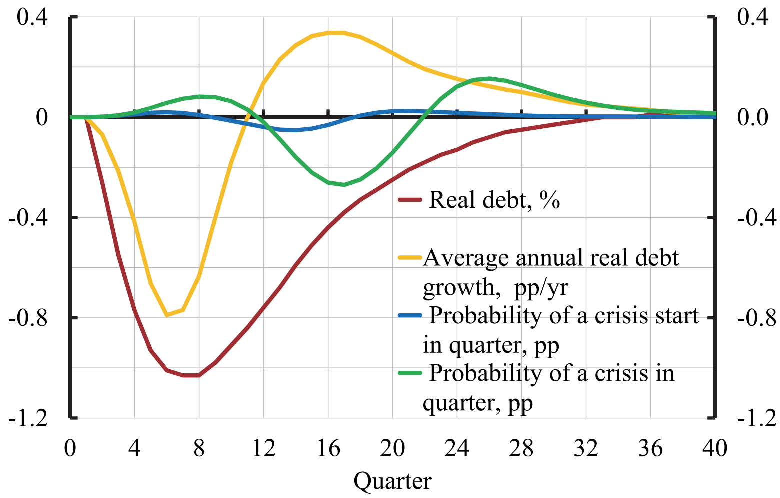


Figure 7.1: The effect on real debt, the average annual real debt growth, the probability of a crisis start in quarter, and the probability of being in a crisis in quarter of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

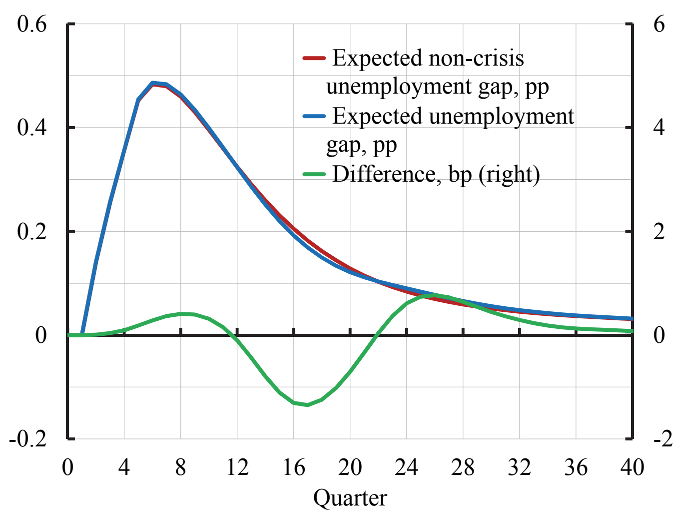


Figure 7.2: The effect on the expected unemployment rate and the expected non-crisis unemployment rate of a 1 percentage point higher policy rate during quarters 1–4; deviations from baseline. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

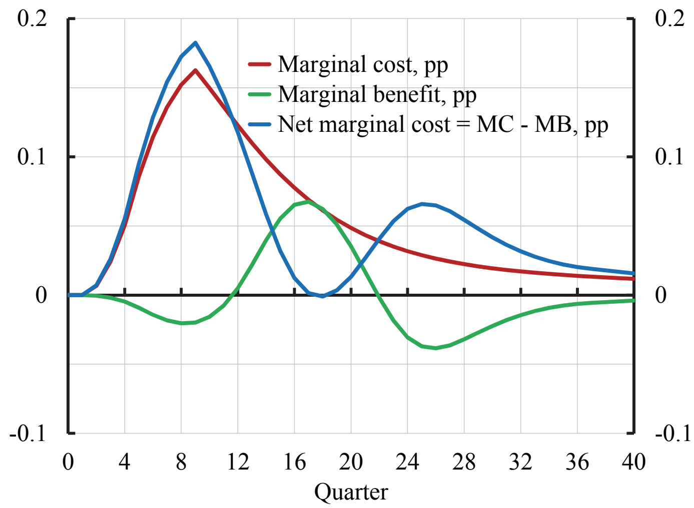


Figure 7.3: The marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap equals zero. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

Figure 7.1 shows the resulting effect of the policy rate on the probability of a crisis start and of a crisis. Comparing with the previous figure 2.2 for the Schularick and Taylor (2012) estimates, we see that now the effect on the probability of a crisis start ($dq_t/d\bar{i}_1$, the blue line) fluctuates more. As a result, the effect on the probability of a crisis ($dp_t/d\bar{i}_1$, the green line) also fluctuates more: first it increases relative to the baseline before it falls to a negative peak of $-0.27$ percentage point in quarter 17, after which it increases and reaches a positive peak of 0.15 percentage points in quarter 26, after which it finally falls to zero in quarter 40. It lacks the long positive tail that shows in figure 2.2. The accumulated effect on the probability of a crisis is of course still approximately zero, (2.8).

Figure 7.2 shows the resulting effect on the expected future unemployment rate, which is still very similar to the effect on the expected future non-crisis rate. Clearly, a higher policy rate increases the expected unemployment rate at all horizons. The effect on the expected unemployment rate does not provide any support for leaning against the wind.

Figure 7.3 shows the corresponding marginal cost, marginal benefit, and net marginal cost according to (4.3)-(4.7). The marginal cost depends on $p_t$, which in this case, by (3.12) increases from 0 in quarter 1 to 1.89 percent in quarter 9, after which it stays at 1.89 percent. As a result, the marginal cost has a lower peak and is smaller than in the previous figure 4.1. Compared with the previous figure, the marginal benefit now fluctuates more. In quarters 17-19, it is equal to the marginal cost, making the net marginal cost equal to zero for those quarters. However, around quarter 8 and, in particular, around quarter 26, the marginal benefit is negative and adds to the net marginal cost. The sum of the marginal benefits over the 40 quarters is $-0.02$ percentage points and thus very close to zero (so the sum of the net marginal costs over the 40 quarters is approximately equal to the sum of the marginal costs, as in (4.9)). Again, the net marginal cost of leaning against the wind is clearly positive, implying that leaning *with* the wind, not against, is justified.

## 8 Conclusions

The conclusions from this analysis are quite strong: For existing empirical estimates and reasonable assumptions, the marginal cost of leaning against the wind is much higher than the marginal benefit. Thus, leaning against the wind is not justified. If anything, a modest leaning *with* the wind, in the sense of a somewhat lower policy rate, is justified.

The main component of the marginal cost of leaning against the wind is the marginal cost of increasing the crisis unemployment gap. Leaning against the wind increases both the non-crisis and the crisis unemployment gaps. Even if the initial non-crisis unemployment gap is zero, in which case the marginal cost of increasing the non-crisis unemployment gap is zero, the crisis unemployment gap is not zero, and the marginal cost of increasing the crisis unemployment gap is positive.

The main component of the marginal benefit is the reduction in the expected cost of a crisis due to a possibly lower probability of a crisis from a higher policy rate. For existing empirical estimates and channels, this possible effect of the policy rate on the probability of a crisis is too small to match the marginal cost of a higher policy rate.

The main empirical channel through which the policy rate might reduce the probability of a crisis is via an effect on the growth rate of real debt. As discussed in section 1, this channel is subject to several limitations in that it represents a reduced-form and correlation result, may not be statistically significant, is likely to be small, and may be of either sign. Nevertheless, for the sake of the argument, and in order to implicitly stack the cards in favor of leaning against the wind, the channel is taken for granted in this paper.

Even so, if monetary policy is neutral in the long run, there is no effect on the accumulated real debt growth over the longer run. A possible lower real debt growth rate and a lower probability of a crisis for a few years is then followed by a higher growth rate and a higher probability of a crisis in later years. The probability of a crisis is shifted between periods, but there is no effect on the average and accumulated probability of a crisis over the longer run. Then neither is there any effect on the average and accumulated marginal benefit over the longer run.

But even if monetary policy would be non-neutral and able to reduce the real debt level and thereby the accumulated debt growth in the longer run, so the accumulated marginal benefit would be positive in the longer run, empirically the marginal benefit is still too small to match the marginal cost. For leaning against the wind to be justified, the effect of the policy rate on the probability of a crisis must be so large as to be completely unrealistic.

It is sometimes argued that leaning against the wind is justified if macroprudential policy is less effective. But if macroprudential is less effective and this results in a crisis being more likely, being deeper, or having a longer duration, the marginal cost of a crisis increases more than the marginal benefit, making the case against leaning against the wind even stronger. Similarly, and more generally, if the economy is in a credit boom that implies a higher probability or severity of a crisis, again the case against leaning against the wind becomes stronger rather than weaker.

The sensitivity analysis presented shows that the results are robust to a several alternative assumptions, including using an alternative dataset of Laeven and Valencia (2012) with more recent data and more countries than the benchmark Schularick and Taylor (2012) dataset.

Furthermore, some results for the unrealistic assumption of the cost of a crisis being fixed and independent of the initial state of the economy are reported. Then, if the initial non-crisis unemployment gap is zero, the marginal cost of leaning against the wind is zero, whereas the marginal benefit is small but positive. Then *some* leaning against the wind is optimal. But, for existing empirical estimates, the optimal leaning against the wind is extremely small and correspond to only a few basis points higher policy rate and non-crisis unemployment gap (this is in line with the results of Ajello, Laubach, Lopez-Salido, and Nakata (2015)). The net gain from leaning against the wind is completely insignificant. This is the case even under the assumption of monetary non-neutrality and a permanent effect on real debt and positive effect on accumulated marginal benefit. Thus, even then any significant leaning of the wind is not justified.

A possible objection to the analysis in this paper is that "leaning against the wind" need not be an unanticipated temporary increase in the policy rate as represented here but instead a different policy regime, where the policy rate systematically responds to indicators of financial instability and where this systematic response is incorporated into agents' expectations and changes their behavior. However, if leaning against the wind is a good and robust policy, it should be beneficial also when it is done as a temporary increase in the policy rate, and the policy should pass the test conducted in this paper. Indeed, if a new systematic policy of leaning against the wind would be introduced, it would not be immediately credible but for several years rather correspond to a surprise increase in the policy rate. Furthermore, any examination of a systematic policy of leaning against the wind would require a complicated model and the results would be heavily model-dependent and not very robust. A simple and minimalist approach as the one taken in this paper should be a much more reliable and robust examination of the cost and benefit of leaning against the wind. Its simplicity also has the advantage that anyone can easily reproduced it with other assumptions and estimates than the ones I have used, especially if new relevant empirical estimates would be found. Finally, the substantial margin by which the cost of leaning against the wind exceeds the benefit in the simple and robust cost-benefit analysis done here makes it rather unlikely that the outcome for a systematic such policy would be much different.

In summary, no support for leaning against the wind is found, even under assumptions strongly biased in favor of it. A minimal policy conclusion would be that no leaning against the wind should be undertaken without support from a thorough cost-benefit analysis. Given the strong case found against leaning against the wind, it seems that the burden of proof should be on its advocates. So far, it seems extremely unlikely that such a cost-benefit analysis would lend any support to leaning against the wind.

Another policy conclusion is that, when it comes to reducing the probability or severity of a financial crisis, so far there seems to be no choice but to use other policies than monetary policy, such as micro- and macroprudential policy, housing policy, or fiscal policy, depending on the nature of the problem. For instance, results of Dagher, Dell'Ariccia, Laeven, Ratnovisk, and Tong (2015) indicate that 15–22 percent bank capital relative to risk-weighted assets would have been enough to avoid 85 percent of the historical banking crises in the OECD countries since 1970; thus, sufficient capital may lead to a dramatic reduction in the probability of a crisis.

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# Appendix

## A A Markov process for crisis and non-crisis states

Consider the situation when the probability of a crisis start is $q$ and the duration of a crisis is $n$ quarters. We can model this as a Markov process with $n+1$ states, where state 1 corresponds to a non-crisis and state $j$ for $2 \leq j \leq n+1$ corresponds to a crisis in its $(j-1)$th quarter.[38]

Let the $(n+1) \times (n+1)$ transition matrix be $P = [P_{ij}]$, where $P_{ij} = \Pr(j|i)$ is the probability of a transition from state $i$ in quarter $t$ to state $j$ in quarter $t+1$. The transition probabilities will be zero except for $P_{11} = 1-q$, $P_{12} = q$, $P_{i,i+1} = 1$ for $2 \leq i \leq n$, and $P_{n+1,1} + 1$. As an illustration, for $n = 3$ the $4 \times 4$ transition matrix is

$$P = \begin{bmatrix} 1-q & q & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Let the row vector $\pi_t = (\pi_{ti})_{i=1}^{n+1}$ denote the probability distribution in quarter $t$, and let $\pi_1 = (1, 0, ..., 0)$, corresponding to a non-crisis in quarter 1. Then the probability distribution in quarter $t \geq 1$, conditional on a non-crisis in quarter 1, is given by

$$\pi_t = \pi_1 P^t,$$

and the probability of crisis in quarter $t$, $p_t$, is given by

$$p_t = 1 - \pi_{t1} \quad \text{for } t \geq 1. \tag{A.1}$$

Figure A.1 shows the result for the linear approximation (2.2) (as in figure 3.1) and the Markov process (A.1), when $q = 0.8$ percent and $n = 8$ quarters. The probability of a crisis converges to 6.4 percent for the linear approximation and to 6.0 percent for the Markov process. The linear approximation thus exaggerates the probability of a crisis somewhat.

The main advantage with the linear approximation (2.2), is that the effect of the policy rate on the probability of a crisis is easy to calculate. Given the effect on the probability of a crisis start, $dq_t/d\bar{i}_1$ for $t \geq 1$, from figure 2.2, it simply satisfies

$$\frac{dp_t}{d\bar{i}_1} = \sum_{\tau=0}^{n-1} \frac{dq_t}{d\bar{i}_1}. \tag{A.2}$$

[38] I am grateful for helpful discussion with Stefan Laséen and David Vestin on the Markov process of crisis and non-crisis states.

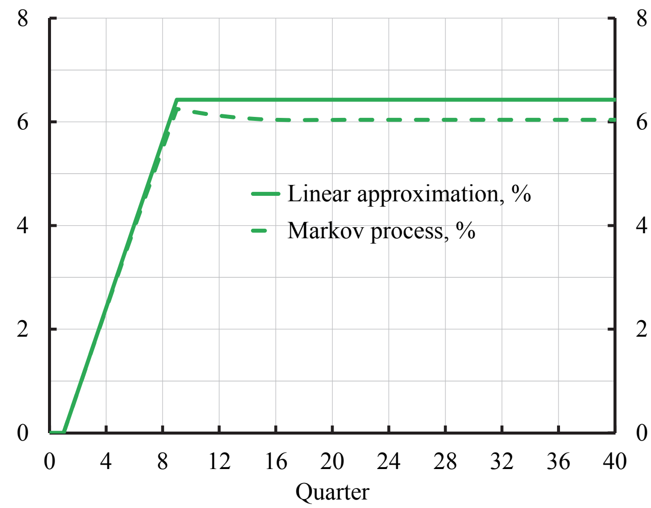


Figure A.1: The probability of a crisis in quarter by the linear approximation (2.2) and by the Markov process (A.1) for $q = 0.8$ percent and $n = 8$ quarters.

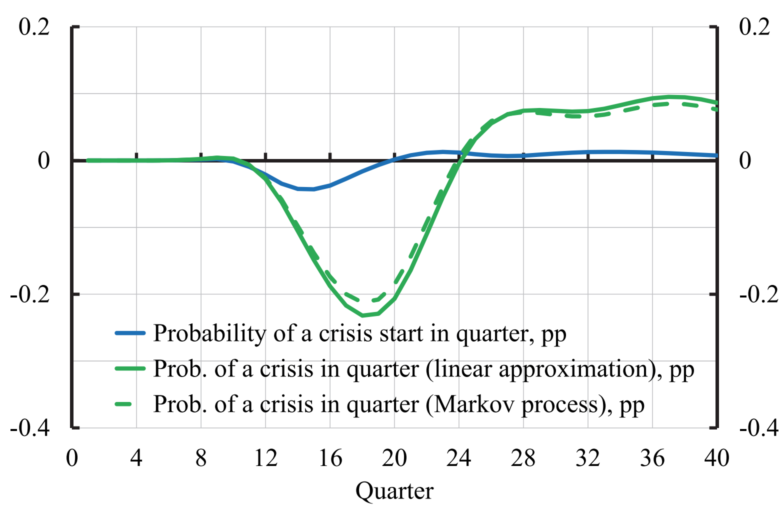


Figure A.2: The effect on the probability of a crisis for the linear approximation (A.2) and the Markov process (A.3) (for $q = 0.8$ percent and $n = 8$ quarters) of a 1 percentage points higher policy rate during quarters 1–4.

For the Markov process, the calculation is a bit more complicated. Let $P_t = [P_{t,ij}]$ for $t \geq 1$, denote the transition matrix from states in quarter $t$ to states in quarter $t+1$, where $P_{t-1,11} = 1 - q_t$, $P_{t-1,12} = q_t$ and $P_{t-1,ij} = P_{ij}$ for $(i,j) \neq (1,1),(1,2)$. Furthermore, we can write

$$\pi_t = \pi_{t-1} P_{t-1} \text{ for } t \geq 2.$$

Then the effect of the policy rate on the probability distribution satisfies

$$\frac{d\pi_t}{d\bar{i}_1} = \frac{d\pi_{t-1}}{d\bar{i}_1} P_{t-1} + \pi_{t-1} \frac{dP_{t-1}}{d\bar{i}_1} \quad \text{for } t \geq 2,$$

where $dP_{t-1,11}/d\bar{i}_1 = -\, dq_t/d\bar{i}_1$, $dP_{t-1,12}/d\bar{i}_1 = dq_t/d\bar{i}_1$, and $dP_{t-1,ij}/d\bar{i}_1 = 0$ for $(i,j) \neq (1,1),(1,2)$. Then

$$\frac{dp_t}{d\bar{i}_1} = -\frac{d\pi_{t1}}{d\bar{i}_1} \text{ for } t \geq 2. \tag{A.3}$$

Figure A.2 shows the effect of a higher policy rate on the probability of a crisis, for the linear approximation (A.2) (as in figure 2.2) and the Markov process (A.2). The linear approximation exaggerates somewhat the effect on the probability of a crisis and thus the marginal benefit of leaning against the wind.

# B The logistic function

Consider the logistic function

$$q = \frac{\exp(a + bg)}{1 + \exp(a + bg)} = \frac{1}{1 + \exp[-\,(a + bg)]}, \tag{B.1}$$

where $q$ here is the *annual* probability of a crisis start, $g$ is a the steady annual growth rate of real debt and $a$ and $b$ are constants. In a logit regression of crises starts on current and lagged annual growth rates of real debt, $b$ corresponds to the sum of the coefficients on the lagged annual growth rates.

The derivative of $q$ with respect to $g$, the marginal effect of steady real debt growth on the probability $q$, satisfies

$$\frac{dq}{dg} = bq(1 - q). \tag{B.2}$$

The sum of coefficients in (2.6) is $b = 9.698$. Given the $dq/dg = 0.30$ reported by Schularick and Taylor (2012), it follows from (B.2) that $q = 0.032$. (To be precise, the values used are $dq/dg = 0.3016$ and $q = 0.0321$.) Given $b$ and $q$, if $g = 0.05$ it follows from (B.1) that $a = -\,3.890$.

Section 5.1 examines the case when $g = 0.079$. Then $q = 0.0421$, and $dq/dg = 0.3914$.

## C The simple loss function

Assume a quadratic loss function of inflation and unemployment,

$$L_t^*(\pi_t, u_t) \equiv \pi_t^2 + \lambda(u_t - \bar{u})^2, \tag{C.1}$$

where $\pi_t$ denotes the gap between the inflation rate and and a fixed inflation target in quarter $t$, and $u_t - \bar{u}$ is the gap between the unemployment rate $u_t$ in quarter $t$ and the long-run sustainable unemployment rate $\bar{u}$. Assume a simple Phillips curve,

$$\pi_t = z_t - \gamma(u_t - \bar{u}), \tag{C.2}$$

where the intercept, $z_t$, is a stochastic process representing cost-push shocks that cause a tradeoff between achieving an inflation rate equal to the inflation target and an unemployment rate equal to the long-run sustainable rate. A positive (negative) $z_t$ implies that a zero inflation gap requires a positive (negative) unemployment tap.

By combining (C.1) and (C.2), the loss function incorporating the Phillips curve can be written

$$\begin{aligned} L_t^0[(u_t - \bar{u}); z_t] \equiv L_t^*[z_t - \gamma(u_t - \bar{u}), u_t] &= [z_t - \gamma(u_t - \bar{u})]^2 + \lambda(u_t - \bar{u})^2 \\ &= (\gamma^2 + \lambda)(u_t - \bar{u})^2 - 2\gamma z_t(u_t - \bar{u}) + z_t^2 \\ &= (\gamma^2 + \lambda)\{(u_t - \bar{u})^2 - 2[u^*(z_t) - \bar{u}](u_t - \bar{u}) + (1 + \lambda/\gamma^2)[u^*(z_t) - \bar{u}]^2\} \\ &= (\gamma^2 + \lambda)\{[u_t - u^*(z_t)]^2 + (\lambda/\gamma^2)[u^*(z_t) - \bar{u}]^2\}, \end{aligned} \tag{C.3}$$

where

$$u^*(z_t) - \bar{u} \equiv \frac{\gamma z_t}{\gamma^2 + \lambda}. \tag{C.4}$$

It follows from (C.3) that $u^*(z_t)$, given by (C.4), is the unemployment rate that for given $z_t$ minimizes the loss function (C.1) subject to the Phillips curve (C.2). Furthermore, it is clear that choosing $u_t$ to minimize the *simple* quadratic loss function

$$L_t(u_t; u_t^*) \equiv (u_t - u_t^*)^2, \tag{C.5}$$

where $u_t^* \equiv u^*(z_t)$ is equivalent to choosing $u_t$ to minimize the loss function $L_t^0(u_t; z_t)$ incorporating the Phillips curve. I call $u_t^*$ the *benchmark unemployment rate.*

A crisis is considered to be a negative demand shock that, given possible policy actions during the crisis to reduce its costs, increases the unemployment rate by the fixed amount $\Delta u > 0$. The demand shock is assumed to be independent of the shock $z_t$ and thus independent of the benchmark unemployment rate.

# D The effect of the policy rate on the crisis increase in the unemployment rate

A possible benefit of a higher policy rate might be a smaller increase in the unemployment rate in a crisis. According to Flodén (2014), for the OECD countries, a lower household debt-to-income ratio in 2007 is associated with a lower increase in the unemployment rate during 2007-2012. More precisely, a 1 percentage point lower debt-to-income ratio is associated with a 0.02 percentage point lower increase in the unemployment rate. This is a small effect. It is statistically significant for the sample of all OECD countries but not for the sample of OECD countries during which housing prices fell.

*If* a higher policy rate would lower the debt-to-income ratio, a higher policy rate might through this channel reduce the severity of a crisis, by reducing the crisis increase in the unemployment rate, $d\Delta u/d\bar{i}_1 < 0$. Taking such a possibility into account, the effect of the policy rate on the expected unemployment rate, (2.4), will by (2.3) have a third term.

$$\frac{d\mathrm{E}_1 u_t}{d\bar{i}_1} = \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + \Delta u \frac{dp_t}{d\bar{i}_1} + p_t \frac{d\Delta u}{d\bar{i}_1}. \tag{D.1}$$

Furthermore, the effect of a higher policy rate on the expected quadratic loss, (4.2), will by have an additional term,

$$\frac{d\mathrm{E}_1 L_t}{d\bar{i}_1} = 2[\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} + p_t \Delta u] \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + [(\Delta u)^2 + 2\Delta u \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}] \frac{dp_t}{d\bar{i}_1} + 2p_t[\Delta u + \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}] \frac{d\Delta u}{d\bar{i}_1}.$$

This leads to an additional term in the marginal benefit, (4.5),

$$\mathrm{MB}_t \equiv [(\Delta u)^2 + 2\Delta u \, \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}](-\frac{dp_t}{d\bar{i}_1}) + 2p_t[\Delta u + \mathrm{E}_1 \tilde{u}_t^{\mathrm{n}}](-\frac{d\Delta u}{d\bar{i}_1}).$$

For (3.7), the marginal benefit is then given by

$$\mathrm{MB}_t \equiv (\Delta u)^2(-\frac{dp_t}{d\bar{i}_1}) + 2p_t \Delta u(-\frac{d\Delta u}{d\bar{i}_1}). \tag{D.2}$$

I wrote "*if* a higher policy rate would lower the debt-to-income ratio," because, as discussed in section 1, it his highly uncertain what the direction is of any effect of the policy rate on the debt-to-income ratio.

As an example and benchmark, I nevertheless use the Sveriges Riksbank (2014a) estimate of the effect on the Swedish household debt-to-income ratio of a 1 percentage point higher policy rate during four quarters, the point estimate of which is shown as the red line in figure D.1. It shows

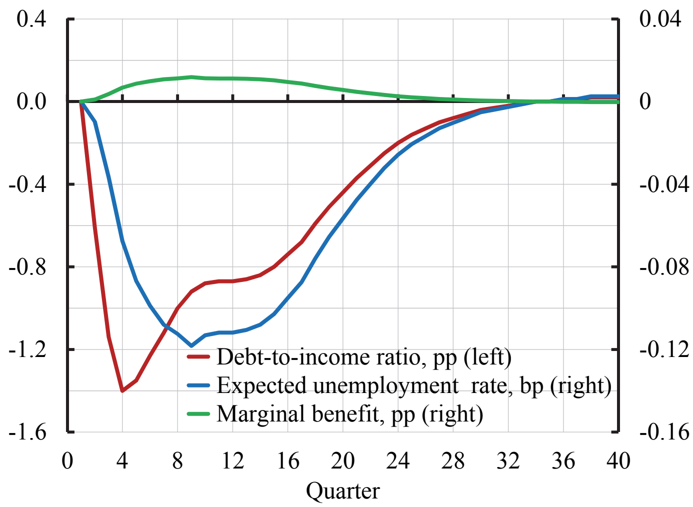


Figure D.1: The effect on the debt-to-income ratio and, via the effect on the crisis increase in the unemployment rate, on the expected unemployment rate and the marginal benefit of leaning against the wind; deviations from baseline. (Source: Flodén (2014), Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

the debt-to-income ratio falling below baseline from a policy-rate increase. However, as discussed in Svensson (2014, 2015), the 90-percent uncertainty band around the estimate is very wide, and the estimate is not statistically significant and could be of the opposite sign. Nevertheless, for the sake of the argument, I take it as given and examine the resulting effects.

We see that the largest effect, a negative peak of $-1.4$ percentage points, occurs already after 4 quarters. Given the estimate of Flodén (2014), this would imply that the crisis increase in the unemployment rate would be $-\,d\Delta u/d\bar{i}_1 = 0.02 \cdot 1.4 = 0.28$ percentage points lower, that is, the crisis increase would be 4.72 percentage points rather than 5 percentage points. However, after 5 years, in quarter 20, the debt-to-income ratio is only 0.44 percentage points below the baseline, meaning that the increase in the crisis unemployment rate would be $-\,d\Delta u/d\bar{i}_1 = 0.088$ percentage points less.[39]

Furthermore, to get the effect on the expected unemployment rate, the third term in (D.1), these numbers should be multiplied by the probability $p_t$ of a crisis in quarter $t$, (3.12). This results in very small effects, shown as the blue line in figure D.1. The largest effect is a negative peak in quarter 9, when the probability of a crisis has increased to 6.4 percent. The peak is $-\,0.12$ *basis* points $= -\,0.0012$ percentage points, which is only a tenth of the largest difference in figure 2.3,

[39] This is the summary results I have used in Svensson (2014, 2015).

which difference is already very small. Clearly, the effect through this channel on the expected future unemployment rate can be disregarded.

In order to calculate the additional third term in the marginal benefit (D.2), we simply have to multiply this third term in the expected unemployment term (D.1) by $2\Delta u = 10$ percentage point and switch the sign to get the green line in figure D.1. It thus has a positive peak at 0.012 percentage points for quarter 11, which is small relative to the peak of the marginal benefit in figure 4.1.

It seems clear that the conclusions of this paper are not affected by disregarding this (doubtful) effect of the policy rate on the crisis increase in the unemployment rate.

# E Kocherlakota on the value of eliminating the possibility of a crisis

An early and innovative cost-benefit analysis of leaning against the wind is provided by Kocherlakota (2014). He assesses the value of eliminating the probability of a crisis, in the sense of reducing the probability of a crisis to zero. He assumes that the crisis increase in the unemployment rate is 4 percentage points, that the expectation in 2014 of the 2017 unemployment rate is equal to a natural unemployment rate of 5 percent, and that a crisis would therefore imply that the unemployment rate would reach 9 percent. As an estimate of the upper limit of the probability of a crisis, he then uses the probability of the 2017 unemployment rate exceeding 9 percent that can be inferred from the 2014Q1 Survey of Professional Forecasters. This probability is 0.29 percent. It is considered an upper limit because the unemployment rate could exceed 9 percent for other reasons than a crisis. We immediately notice that this probability is much smaller than the probability of crisis beyond quarter 9 of 6.4 percent used here. Given Kocherlakota's estimate, the expected loss increase of a crisis is $0.0029 \cdot 4^2 = 0.0464 = 0.22^2$. That is, eliminating the possibility of a crisis is worth an expected non-crisis unemployment gap from zero to only 0.22 percentage points.

With a crisis increase in the unemployment rate of 5 percentage points, as assumed in the present paper, the expected loss from a crisis would be $0.0029 \cdot 25 = 0.0725 = 0.27^2$, in which case it is worth an increase in the expected non-crisis unemployment gap from zero to 0.27 percentage points.

For the benchmark assumptions in the present paper, the steady-state probability of a crisis is 6.4 percent, so the expected loss equals $0.064 \cdot 25 = 1.6 = 1.26^2$ under the benchmark assumptions.

Thus, reducing the probability of a crisis from 6.4 percent to zero is under the benchmark assumptions worth an increase in the non-crisis unemployment gap from zero to 1.26 percentage points, a substantial increase.

For the IMF staff estimates discussed in section 7, the steady-state probability of a crisis is 3.78 percent, implying that the expected loss due to the possibility of a crisis is $0.0378 \cdot 25 = 0.945 = 0.97^2$. Then eliminating the possibility of a crisis is worth an increase in the expected non-crisis unemployment gap of 0.97 percentage points, still a substantial increase.

These numbers are much higher than the estimate in Kocherlakota (2014). The main difference is that the estimates of a probability of a crisis that follow from Schularick and Taylor (2012) or the IMF staff estimates, 6.4 and 3.8 percent, respectively, are much higher than the estimate from Survey of Professional Forecasters, 0.29 percent. However, I am not sure that the forecasts in the Survey of Professional Forecasters take the possibility of a crisis into account. If they don't, they can obviously not be used to infer the professional forecasters' estimate of a probability of a crisis.

# F The reduction of the probability of a crisis per expected non-crisis unemployment gap increase for each quarter

Figure F.1 shows, for each future quarter, the reduction in the probability of a crisis per increase in the expected non-crisis unemployment gap, the negative of the ratio (3.16), for the two datasets, Schularick and Taylor (2012) and Laeven and Valencia (2012). The derivative $dp_t/d\bar{i}_1$ are given by the green lines in figures 2.2 and 7.1, respectively. The derivative $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{i}_1$ is given by the red line in figure 2.1. Taking the average over quarters 12-24, as is done in section 3.4 for the Schularick and Taylor (2012) case, clearly exaggerates the estimate of the magnitude of the average reduction of the probability of a crisis per unemployment rate increase, something that exaggerates the benefit and stacks the cards in favor of leaning against the wind.

# G The case of a random crisis increase in the unemployment rate

In the benchmark case, the crisis increase in the unemployment rate is taken to be deterministic and given by $\Delta u > 0$ (and assumed to equal 5 percentage points in the benchmark case). Alternatively, the crisis increase in the unemployment rate could be a random variable $\widetilde{\Delta u}$ with mean $\Delta u$ and

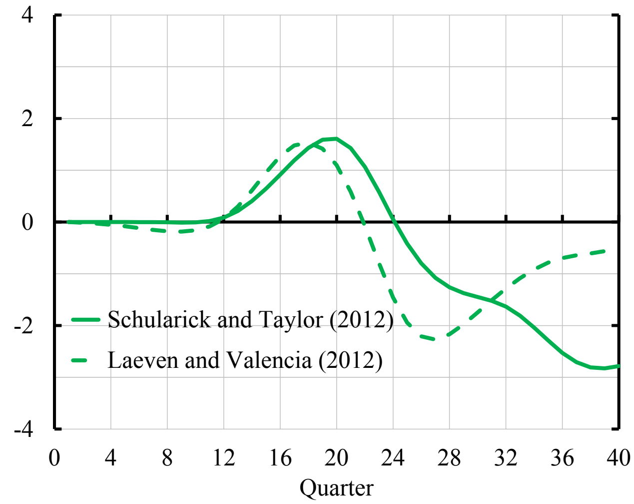


Figure F.1: The percentage-point reduction in the probability of a crisis per percentage-point increase in the expected non-crisis unemployment gap for the datasets of Schularick and Taylor (2012) and Laeven and Valencia (2012).

variance $\sigma^2_{\Delta u}$. In that case, we have

$$\mathrm{E}_1(\tilde{u}^{\mathrm{n}}_t + \widetilde{\Delta u})^2 = \mathrm{E}_1[\mathrm{E}_1\tilde{u}^{\mathrm{n}}_t + \Delta u + (\tilde{u}^{\mathrm{n}}_t - \mathrm{E}_1\tilde{u}^{\mathrm{n}}_t) + (\widetilde{\Delta u} - \Delta u)]^2 = (\mathrm{E}_1\tilde{u}^{\mathrm{n}}_t + \Delta u)^2 + \mathrm{Var}_1\tilde{u}^{\mathrm{n}}_t + \sigma^2_{\Delta u},$$

so the expected quarter-$t$ loss satisfies

$$\mathrm{E}_1 L_t = (1-p_t)\mathrm{E}_1(\tilde{u}^{\mathrm{n}}_t)^2 + p_t\mathrm{E}_1(\tilde{u}^{\mathrm{n}}_t + \widetilde{\Delta u})^2 = (1-p_t)\mathrm{E}_1(\tilde{u}^{\mathrm{n}}_t)^2 + p_t[\mathrm{E}_1(\tilde{u}^{\mathrm{n}}_t + \Delta u)^2 + \sigma^2_{\Delta u}] + \mathrm{Var}_1\tilde{u}^{\mathrm{n}}_t,$$

where the covariance between the crisis increase in the unemployment rate and the non-crisis unemployment rate is taken to be zero. Compared with (3.6), the additional term $p_t\sigma^2_{\Delta u}$ enters on the right side. This will not affect the results.

# H The debt-to-GDP term in Schularick and Taylor (2012, table 7, column 22)

Schularick and Taylor (2012, table 7, column 22) contains a logit regression of the annual probability of a crisis start, where the log of debt-to-GDP ratio is added as an explanatory variable. The coefficient is 1.1 with a standard error of 0.624 and is significant at the 10 percent level. The estimates of the coefficients of the lagged real debt growth rates and of the sum of these coefficients do not change much: the sum is 9.984 rather than 9.698. We can represent this variant as

$$q_t = \frac{1}{4}\frac{\exp(X_t + \gamma h_t)}{1+\exp(X_t + \gamma h_t)},$$

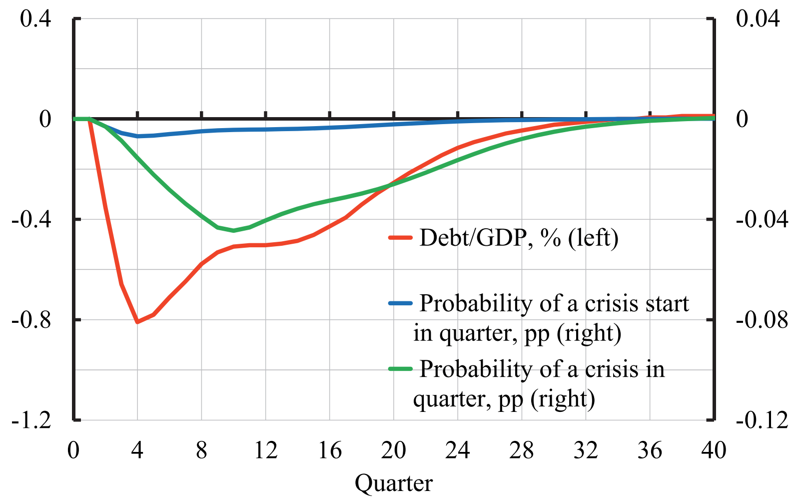


Figure H.1: The effect on the debt-to-income ratio and the separate effect on the probability of a crisis start and of a crisis. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

where $q_t$ is a quarterly probability, $\gamma = 1.1$ and $h_t$ is the log of the debt-to-GDP ratio. The derivative of $q_t$ with respect to $h_t$ is

$$\frac{dq_t}{dh_t} = \frac{1}{4}\gamma 4q_t(1 - 4q_t) = \gamma q_t(1 - 4q_t).$$

With $q_t = 0.008$ and $\gamma = 1.1$, $dq_t/dh_t = 1.1 \cdot 0.008 \cdot 0.968 = 0.0085$. That is, 1 percentage point lower debt-to-GDP ratio lowers the probability of a crisis start by 0.0085 percentage points.

Figure H.1 shows the Riksbank's estimate of the effect of the policy rate on the debt to GDP ratio (expressed in the percentage change) and the resulting separate effect on the probabilities of a crisis start and of a crisis in each quarter. We see that the effect is very small. In figure H.2, the dashed blue and green lines shows the total effect on the probability if a crisis start and the probability of a crisis, when this additional effect via a lower debt-to-GDP ratio is taken into account. We see that the total effect is only marginally larger than the effect via real debt growth only.

# I A constrained-optimal policy

What is the optimal policy for the intertemporal loss function (3.2)? We can simplify this question a bit by just asking, given the relations presented in section 2 and 3, what is the optimal policy rate $\bar{i}_1$ and the optimal non-crisis unemployment gap, $\tilde{u}_t \equiv u_t^{\mathrm{n}} - u_t^*$? That is, if policy is constrained

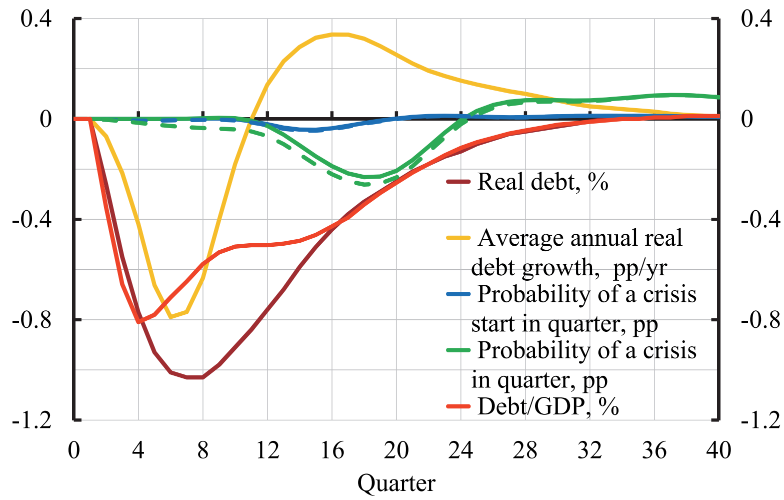


Figure H.2: The estimates of the policy-rate effect on the unemployment rate, real debt and the debt-to-GDP ratio. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

to choose the same constant policy rate $\bar{i}_1$ for quarters 1–4 and to return the policy rate to its baseline level after that, what level of the policy rate $\bar{i}_1$ is then optimal. Thus, we are considering a particular constrained optimal policy, not an unconstrained policy.

This means that we are looking for the policy rate that sets the derivative of the intertemporal loss function (3.2) equal to zero,

$$\frac{d}{d\bar{i}_1}\mathrm{E}_1 \sum\nolimits_{t=0}^{\infty} \delta^{t-1} L_t = 0. \tag{I.1}$$

We can write (I.1) as

$$\sum\nolimits_{t=0}^{\infty} \delta^{t-1}\mathrm{NMC}_t = \sum\nolimits_{t=0}^{\infty} \delta^{t-1}[\mathrm{MC}_t - \mathrm{MB}_t] = 0, \tag{I.2}$$

and use that

$$\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}} = \frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}\bar{i}_1 \quad \text{for } t \geq 1 \tag{I.3}$$

and where I have normalized the policy rate such that $\bar{i}_1 = 0$ for (3.7). That is, I assume that we start from an equilibrium where (3.7) holds and consider deviating from that.

We can write, by (4.4) and (4.5),

$$\begin{aligned}
\mathrm{MC}_t &= 2p_t\Delta u\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + 2\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} \;=\; 2p_t\Delta u\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} + 2(\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1})^2\,\bar{i}_1 \\
&\equiv a_t + b_t\bar{i}_1, \\
\mathrm{MB}_t &= (\Delta u)^2(-\frac{dp_t}{d\bar{i}_1}) + 2\Delta u\,(-\frac{dp_t}{d\bar{i}_1})\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} \;=\; (\Delta u)^2(-\frac{dp_t}{d\bar{i}_1}) + 2\Delta u\,(-\frac{dp_t}{d\bar{i}_1})\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}\bar{i}_1 \\
&\equiv c_t + e_t\bar{i};
\end{aligned}$$

this way defining the coefficients $a_t$, $b_t$, $c_t$, and $e_t$ for $t \geq 1$. Then we can write,

$$\begin{aligned}
\mathrm{NMC}_t &= (a_t - c_t) + (b_t - e_t)\bar{i}_1 \\
&\equiv f_t + g_t\bar{i}_1; \qquad \text{(I.4)}
\end{aligned}$$

this way defining the coefficients $f_t$ and $g_t$ for $t \geq 1$.

By (I.2) and (I.4), we are thus looking for the $\bar{i}_1$ that solves

$$\sum\nolimits_{t=1}^{\infty} \delta^{t-1} f_t + \sum\nolimits_{t=1}^{\infty} \delta^{t-1} g_t\,\bar{i}_1 = 0,$$

that is,

$$\bar{i}_1 = -\sum\nolimits_{t=1}^{\infty} \delta^{t-1} f_t \Big/ \sum\nolimits_{t=1}^{\infty} \delta^{t-1} g_t\ .$$

Given that I only know the coefficients $f_t$ and $g_t$ for $1 \leq t \leq 40$, I will approximate by just solving for

$$\bar{i}_1 = -\sum\nolimits_{t=1}^{40} \delta^{t-1} f_t \Big/ \sum\nolimits_{t=1}^{40} \delta^{t-1} g_t\ .$$

It turns out that the constrained-optimal policy in this case, with the numbers above and a quarterly discount factor of $\delta = 0.995$ (an annual discount factor of 0.98), is $\bar{i}_1 = -0.83$ percentage point. That is, given the constraints specified above, the optimal policy is to *reduce* the policy rate for quarters 1–4 below the baseline by about 0.8 percentage points and then let it return to the baseline, as shown by the grey line in figure I.1.

The blue line show the corresponding discounted net marginal cost, $\delta^{t-1}\mathrm{NMC}_t$ for $1 \leq t \leq 40$. By (I.2), they sum to zero for $1 \leq t \leq 40$. That is, the sum of the magnitude of the negative discounted net marginal costs up to quarter 10 equals the sum of the positive discounted net marginal costs from quarter 11.

The yellow and red lines show, respectively, the constrained-optimal (undiscounted) expected unemployment rate taking the possibility of a crisis into account and expected non-crisis unemployment rate; both are shown relative to the expected benchmark unemployment rate. We see

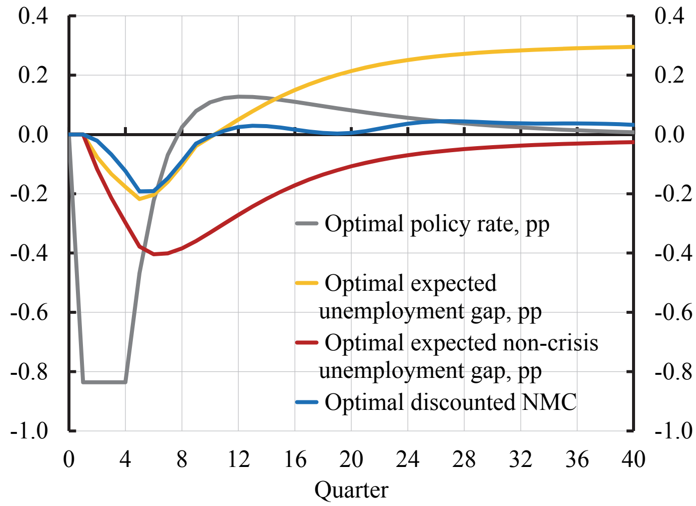


Figure I.1: The constrained-optimal policy rate, discounted net marginal cost, expected non-crisis unemployment gap, and expected unemployment gap. (Source: Schularick and Taylor (2012), Sveriges Riksbank, and own calculations.)

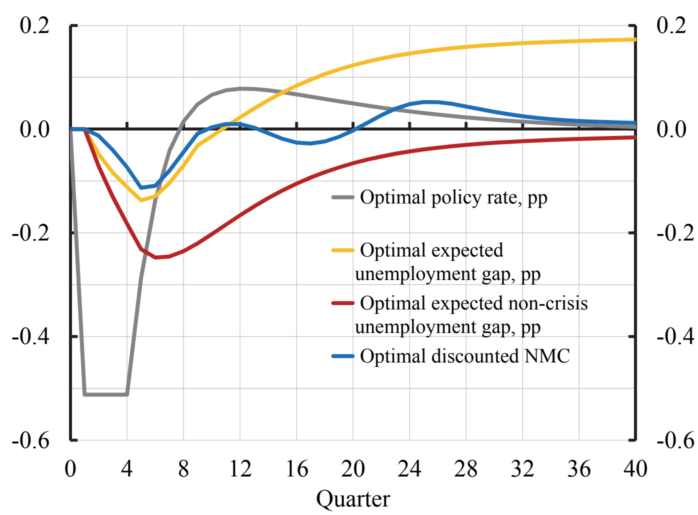


Figure I.2: The constrained-optimal policy rate, discounted net marginal cost, expected non-crisis unemployment gap, and expected unemployment gap. (Source: IMF staff estimates, Sveriges Riksbank, and own calculations.)

that the expected unemployment rate including the possibility of a crisis is below the benchmark unemployment rate up to quarter 10 and above the benchmark from quarter 11. Furthermore, the expected non-crisis unemployment rate is below but approaches the benchmark unemployment rate at quarter 40 and beyond, meaning that the expected unemployment rate taking into account the possibility of a crisis approaches a level 0.32 percentage points above the benchmark unemployment rate.

We may ask why the optimal policy is not to keep the expected unemployment rate including the possibility of a equal to the benchmark far into the future rather than allowing it to exceed the benchmark by a substantial margin. The reason is the sever constraint on policy here, that we are only considering changes in the policy rate during quarters 1–4. By figure 2.1, which shows the derivative $d\mathrm{E}_1 u_t^{\mathrm{n}}/d\bar{\imath}_1$, this constraint implies that the derivative is so small for quarters far into the future that it is not possible to affect the expected non-crisis unemployment rate far into the future without affecting the expected non-crisis unemployment rate in the near future much more. Policy is constrained to scaling the expected non-crisis unemployment rate in figure 2.1 by positive or negative factors. Given this, the best policy is to scale it by $\bar{\imath}_1 = -0.83$.

As a robustness test, assume that the policymaker would disregard the marginal cost and benefit beyond quarter 24, when the marginal benefit turns negative. This can also be seen as assuming some non-neutrality of monetary policy, such that there is a long-run effect on the real debt level. Here it means solving for the optimal $\bar{\imath}_1$ when the discounted terms $f_t$ and $g_t$ are summed up to quarter 24,

$$\bar{\imath}_1 = -\sum_{t=1}^{24} \delta^{t-1} f_t \Big/ \sum_{t=1}^{24} \delta^{t-1} g_t \ .$$

The constrained-optimal policy rate is then given by $\bar{\imath}_1 = -0.68$, that is, it consists of lowering the policy rate in quarters 1–4 by 0.68 percent, somewhat less than when the marginal cost and benefit in all quarters up through quarter 40 are taken into account.

Figure I.2 shows the constrained-optimal policy for the IMF staff estimates for the Laeven and Valencia (2012) dataset discussed in section 7. In this case the optimal $\bar{\imath}_1 = -0.51$, that is, the constrained-optimal policy is to lower the policy rate in quarters 1–4 by 0.51 percentage points, somewhat less than for the Schularick and Taylor (2012) dataset. If the marginal benefit and cost beyond quarter 24 is disregarded, the constrained-optimal policy is to lower the policy rate in quarters 1–4 by somewhat less, 0.40 percentage points.

In summary, regardless of whether or not the marginal cost and benefit of beyond quarter 24 are taken into account, and regardless of whether the datasets of Schularick and Taylor (2012) or

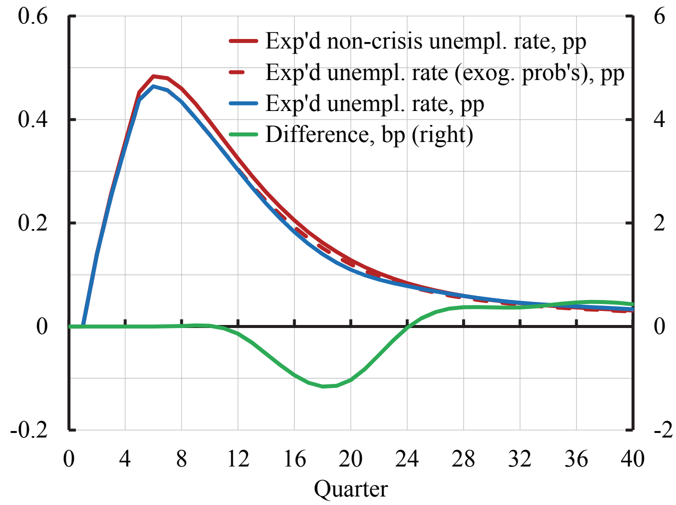


Figure J.1: For a fixed cost of a crisis, the effect on the expected unemployment gap and its component of a 1 percentage point higher policy rate during quarters 1–4, when the expected unemployment gap is zero.

Laeven and Valencia (2012) are used, the constrained-optimal policy is to lower the policy rate in quarters 1–4 and thus lean *with* the wind, not against.

# J The alternative assumption of a fixed cost of a crisis

This appendix further examines the unrealistic case of a fixed cost of a crisis, discussed in section 3.4. Under that assumption, the expected crisis unemployment gap equals $\Delta u$,

$$\mathrm{E}_1\tilde{u}_t^{\mathrm{c}} = \Delta u,$$

regardless of the expected non-crisis unemployment gap,

The expected unemployment gap is then given by

$$\mathrm{E}_1\tilde{u}_t = (1-p_t)\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} + p_t\Delta u \tag{J.1}$$

and the effect of the policy rate is

$$\frac{d\mathrm{E}_1\tilde{u}_t}{d\bar{i}_1} = (1-p_t)\frac{d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}}{d\bar{i}_1} + (\Delta u - \mathrm{E}_1\tilde{u}_t^{\mathrm{n}})\frac{dp_t}{d\bar{i}_1}. \tag{J.2}$$

Figure J.1 shows the effect on the expected non-crisis unemployment gap ($d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}/d\bar{i}_1$, the red solid line), the expected unemployment gap for an exogenous probability of a crisis ($(1-p_t)d\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}/d\bar{i}_1$, the red dashed line), and the expected unemployment gap ($d\mathrm{E}_1\tilde{u}_t/d\bar{i}_1$, the blue line).

The green line shows the difference between the last two, that is, the difference due to the effect on the probability of a crisis, when the expected non-crisis unemployment gap is zero ($\Delta u dp_t/d\bar{i}_1$, measured along the right in basis points). The difference is the same as in figure 2.3 and thus equally small.

The quarter-$t$ the net marginal cost of leaning against the wind, is then

$$\begin{aligned}\mathrm{NMC}_t \equiv \frac{d\mathrm{E}_1 L_t}{d\bar{i}_1} &= (1-p_t)2\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} - [(\Delta u)^2 - \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2](-\frac{dp_t}{d\bar{i}_1}) \\ &\equiv \mathrm{MC}_t - \mathrm{MB}_t,\end{aligned}$$

where the marginal cost and marginal benefit of leaning against the wind satisfy

$$\mathrm{MC}_t \equiv 2(1-p_t)\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1}, \tag{J.3}$$

$$\mathrm{MB}_t \equiv [(\Delta u)^2 - \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2](-\frac{dp_t}{d\bar{i}_1}). \tag{J.4}$$

Furthermore, for (3.7),

$$\mathrm{MC}_t = 0, \tag{J.5}$$

$$\mathrm{MB}_t = (\Delta u)^2(-\frac{dp_t}{d\bar{i}_1}). \tag{J.6}$$

That is, if the expected non-crisis unemployment gap is zero, the marginal cost is now zero, not positive as in (4.6), whereas at the marginal benefit is the same as in (4.7) and positive if the probability of a crisis is decreasing in the policy rate.

We also note that the loss increase in a crisis, the term in the squared bracket in (J.4), is decreasing in $(\mathrm{E}_1\tilde{u}_t^{\mathrm{n}})^2$. It is at its maximum when the expected non-crisis unemployment gap is zero and becomes negative when the expected non-crisis unemployment gap exceeds $\Delta u$, $\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} > \mathrm{E}_1\tilde{u}_t^{\mathrm{c}} = \Delta u$. Of course, in the (unlikely) situation in which the expected non-crisis unemployment gap is greater than the expected crisis unemployment gap, it is better to be in a crisis (under the maintained assumption that the conditional variance of the crisis unemployment gap is not larger than that of the non-crisis unemployment gap).

Figure J.2 shows the marginal cost, marginal benefit, and net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero the loss in a crisis is fixed. Because the marginal cost is zero, the net marginal cost is simply the negative of the marginal benefit and thus negative for quarters 1–23 and positive for quarter 24 and beyond. Because of the neutrality of monetary policy, the accumulated marginal benefit and net marginal cost over a

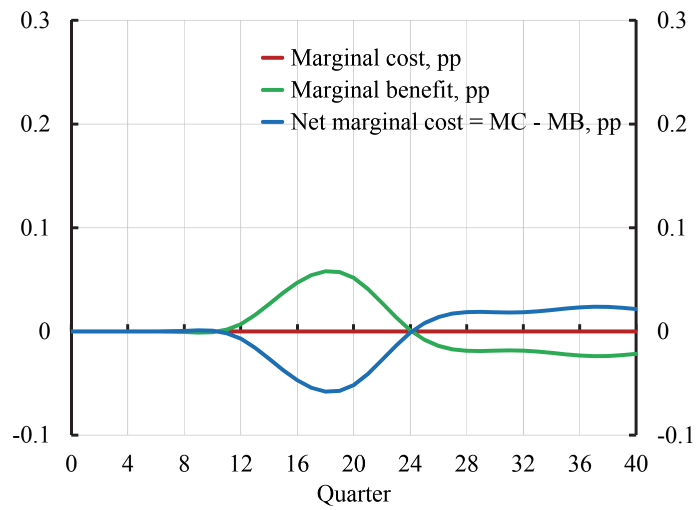


Figure J.2: For a fixed cost of a crisis, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is zero.

long horizon are approximately zero. However, because the marginal benefit is positive earlier and negative later, the sum of the discounted marginal benefits would be positive, whereas the sum of the discounted marginal costs would be zero. Then a small amount of leaning against the wind would be optimal.

If the expected non-crisis unemployment gap is not zero but positive, by (J.3) the marginal cost of leaning against the wind is no longer zero but positive. In figure J.3, the dashed lines show the marginal cost, marginal benefit, and net marginal cost when the expected non-crisis unemployment gap is 0.25 percentage points for all quarters. The solid lines show the same, when the expected non-crisis unemployment gap is zero for all quarters. There is a substantial increase in the marginal cost but no noticeable change in the marginal benefit (because in (J.4) the term $(\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}})^2 = 0.25^2 = 0.0625$ is so small relative to the term $(\Delta u)^2 = 25$). Clearly, sum of discounted net marginal costs is positive. This shows that a small positive expected unemployment gap removes any justification for leaning against the wind, also in the case when the cost of a crisis is fixed. We realize that this would be the case also if the expected non-crisis unemployment gap would be 0.25 percentage points just for the first 12 quarters (or even for the first 8 quarters) and then zero after.

This indicates that any optimal leaning against the wind is very small, definitely much smaller than that resulting in an expected non-crisis unemployment gap of 0.25 percentage point. Let me examine the optimal leaning against the wind further.

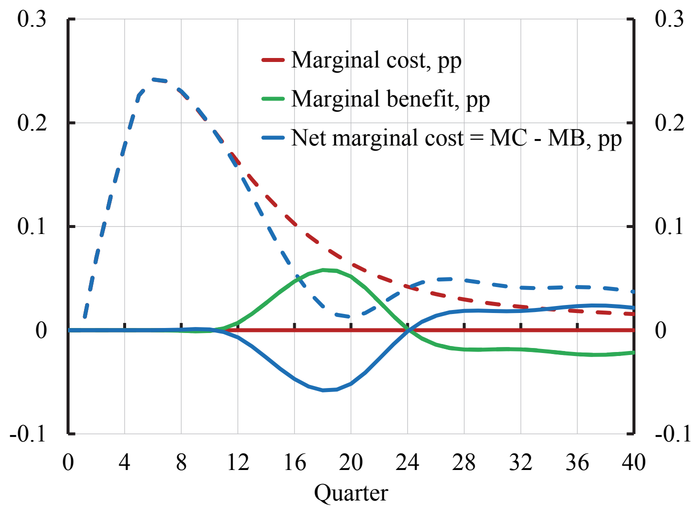


Figure J.3: For a fixed cost of a crisis, the marginal cost, the marginal benefit, and the net marginal cost of leaning against the wind, when the expected non-crisis unemployment gap is positive and equal to 0.25 percentage points for all quarters.

## J.1 The optimal leaning against the wind

What is the optimal policy? We can simplify this question a bit by just asking, given the above relations, what is the optimal policy rate $\bar{i}_1$ and the optimal non-crisis unemployment gap, $\tilde{u}_t \equiv u_t^{\mathrm{n}} - u_t^*$? That is, if policy is constrained to choosing the same constant policy rate $\bar{i}_1$ for quarters 1–4 and return the policy rate to its baseline level after that, what level of the policy rate $\bar{i}_1$ is then optimal. Thus, as in appendix I, we are considering a particular constrained optimal policy, not an unconstrained policy.

Proceeding as in appendix I, by (J.3) and (J.4) we have

$$\begin{aligned} \mathrm{MC}_t &= 2(1-p_t)\mathrm{E}_1\tilde{u}_t^{\mathrm{n}}\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1} \;=\; 2(1-p_t)(\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1})^2\bar{i}_1 \\ &\equiv f_t\bar{i}_1 \qquad\qquad (\mathrm{J.7}) \\ \mathrm{MB}_t &= [(\Delta u)^2 - \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2](-\frac{dp_t}{d\bar{i}_1}) \;=\; (\Delta u)^2(-\frac{dp_t}{d\bar{i}_1}) + (\frac{d\mathrm{E}_1 u_t^{\mathrm{n}}}{d\bar{i}_1})^2\frac{dp_t}{d\bar{i}_1}(\bar{i}_1)^2 \\ &= g_t + h_t(\bar{i}_1)^2. \qquad\qquad (\mathrm{J.8}) \end{aligned}$$

Given these definitions of $f_t$, $g_t$, and $h_t$ for $t \geq 1$, we have

$$\mathrm{NMC}_t = f_t\bar{i}_1 - g_t - h_t(\bar{i}_1)^2.$$

We are thus looking for a solution to the quadratic equation

$$\sum_{t=1}^{40} \delta^{t-1}\mathrm{NMC}_t = \sum_{t=1}^{40} \delta^{t-1}[-h_t(\bar{i}_1)^2 + f_t\bar{i}_1 - g_t] = A(\bar{i}_1)^2 + 2B\bar{i}_1 + C = 0,$$

where

$$A \equiv \sum_{t=1}^{40} -\delta^{t-1}h_t, \quad B \equiv \frac{1}{2}\sum_{t=1}^{40} \delta^{t-1}f_t, \quad C \equiv \sum_{t=1}^{40} -\delta^{t-1}g_t. \tag{J.9}$$

It follows that the optimal $\bar{i}_1$ will be given by one of the two solutions,

$$\bar{i}_1 = -\frac{B}{A} \pm \sqrt{\frac{B^2}{A^2} - \frac{C}{A}}. \tag{J.10}$$

## J.2 Stacking the cards further in favor of leaning against the wind

Before solving for the optimal policy, let me stack the cards further in favor of leaning against the wind. Suppose the policymaker only considers a horizon of 24 quarters. That is, the policymaker disregards the negative marginal benefit beyond quarter 24 in figure J.2. Then the discounted marginal benefit is positive while the discounted marginal cost is zero when the expected unemployment gap is zero, and some leaning will be optimal.

Thus, suppose the policy maker solves for the optimal constant quarter 1–4 policy rate $\bar{i}_1$, taking only quarters 1–24 into account. This means solving for the optimal $\bar{i}_1$ when sums in (J.9) run from 1 to 24 instead of from 1 to 40. Then $A$ and $B$ will be positive and $C$ will be negative. The relevant solution in (J.10) is the one with a plus before the square root. The solution is shown in figure J.4, for an annual discount factor of 0.98.

It is striking how small the optimal leaning is, with a policy-rate increase of only 0.11 percentage points (the grey line). The expected non-crisis unemployment gap (the red line) is only about 0.05 percentage points at its maximum (in quarter 6), thus, very close to zero. The maximum reduction in the probability of a crisis is only 0.025 percentage points (in quarter 18), a small reduction compared to the steady-state probability of a crisis of 6.4 percent. This optimal leaning of the wind only reduces the expected intertemporal loss is only reduced by 0.07 percent of its baseline value. This is in spite of my having stacked the cards in favor of leaning against the wind, by assuming a fixed cost of a crisis so the marginal cost of leaning is zero at an initial zero expected non-crisis unemployment gap, and by restricting the horizon to 24 quarters so as to maximize the accumulated marginal benefit by disregarding the negative marginal benefit beyond that horizon.

Figure J.5 shows more detail, not only the discounted net marginal cost but also the discounted marginal cost and marginal benefit. The sum of net marginal cost from quarter 1 to quarter 24 is

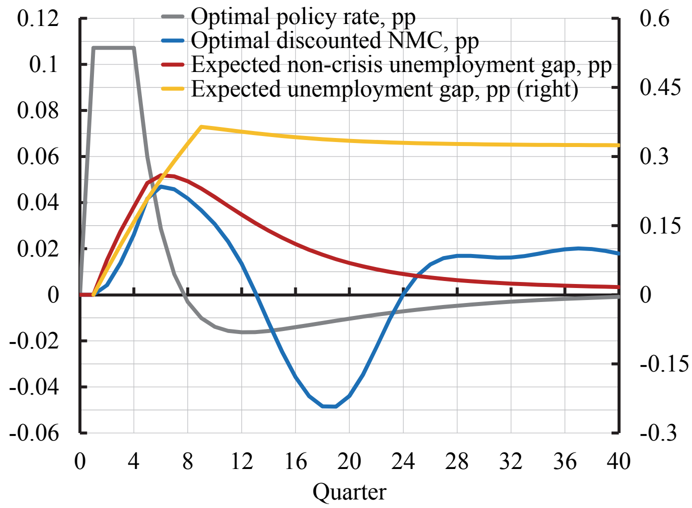


Figure J.4: For a fixed cost of a crisis and a horizon of 24 quarters, the optimal policy rate and the corresponding discounted net marginal cost, expected non-crisis unemployment gap (all left axis), and expected unemployment gap (right axis)

zero. The net marginal cost is composed mainly of the marginal cost up to quarter 12 and of the negative of the marginal benefit from quarter 12 to quarter 24.

Clearly, the main message is that, even if the cards are stacked in favor of leaning against the wind, the optimal leaning against the wind and the reduction in the intertemporal loss is insignificant, and the obvious policy conclusion is that even then, leaning against the wind is not justified and worth bothering about.

### J.3 The "quarterly-optimal" expected non-crisis unemployment gap

We can illustrate how small the optimal leaning against the wind is in a different way. Consider the quarter-$t$ marginal cost with respect to an increase in the expected non-crisis unemployment gap (rather than with respect to an increase in the policy rate, therefore the subindex $u$),

$$\mathrm{MC}_{ut} = 2(1 - p_t)\mathrm{E}_1 \tilde{u}_t^{\mathrm{n}},$$

and the corresponding marginal benefit from an increase in the non-crisis unemployment gap,

$$\mathrm{MB}_{ut} = [(\Delta u)^2 - \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2](-\frac{dp_t}{d\mathrm{E}_1 u_t^{\mathrm{n}}}).$$

Here

$$\frac{dp_t}{d\mathrm{E}_1 u_t^{\mathrm{n}}} \equiv \frac{dp_t / d\bar{i}_1}{d\mathrm{E}_1 u_t^{\mathrm{n}} / d\bar{i}_1}$$

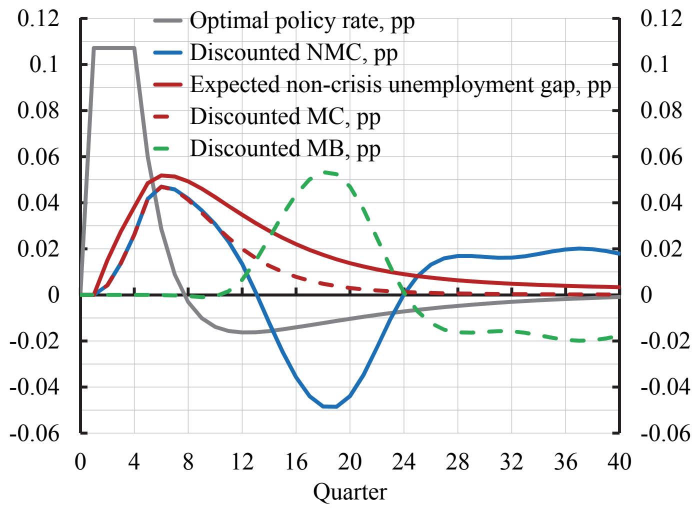


Figure J.5: For a fixed cost of a crisis and a horizon of 24 quarters, the optimal policy rate and the corresponding discounted net marginal cost, marginal cost, and marginal benefit

denotes the decrease in the probability of a crisis in quarter $t$ associated with an increase in the expected quarter-$t$ unemployment gap. We can think of this as a measure of the tradeoff between a higher expected non-crisis unemployment rate and a lower probability of a crisis, the marginal transformation of a higher expected unemployment rate into a lower probability of a crisis.

Let the *quarterly-optimal* expected non-crisis unemployment gap be the unemployment gap that equalizes the quarter-$t$ marginal cost and benefit, that is, the expected non-crisis unemployment gap that is optimal when quarter $t$ is considered in isolation. This is the solution to this second-order equation,

$$2(1-p_t)\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = [(\Delta u)^2 - \mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2](-\frac{dp_t}{d\mathrm{E}_1 u_t^{\mathrm{n}}}).$$

However, the term $\mathrm{E}_1(\tilde{u}_t^{\mathrm{n}})^2$ will be very small relative to $(\Delta u)^2$ and can be disregarded. (Because this means slightly increasing the marginal benefit, it will slightly increase, and therefore be an upper bound of, the quarterly-optimal expected non-crisis unemployment gap.) Then the quarterly-optimal expected non-crisis unemployment gap is given by

$$\mathrm{E}_1\tilde{u}_t^{\mathrm{n}} = \frac{(\Delta u)^2}{2(1-p_t)}(-\frac{dp_t}{d\mathrm{E}_1 u_t^{\mathrm{n}}}). \tag{J.11}$$

Figure J.6 shows for each quarter $1 \leq t \leq 40$ the reduction in the probability of a crisis per increase in the expected non-crisis unemployment gap and the quarterly-optimal expected non-crisis unemployment gap. We see that the most favorable probability-unemployment tradeoff occurs for quarter 20, and equals a probability reduction of 1.6 percentage points for an 1 percentage point

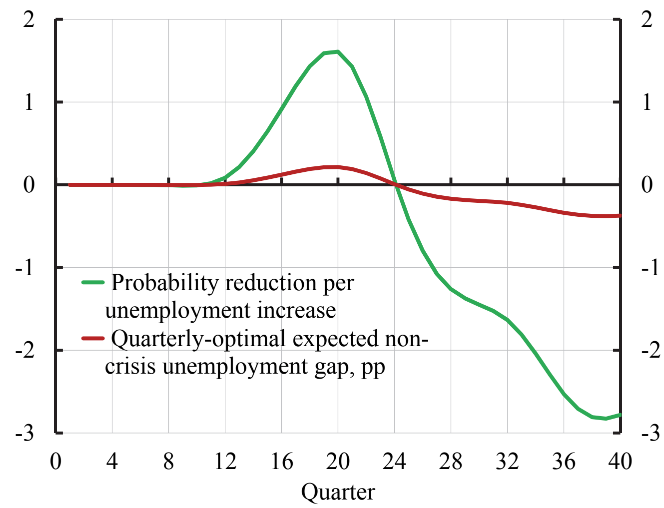


Figure J.6: The reduction in the probability of a crisis per increase in the expected non-crisis unemployment gap and, for a fixed cost of a crisis, the quarterly-optimal expected non-crisis unemployment gap

expected non-crisis unemployment increase. That is, the probability of a crisis falls by 0.016 for an increase of 1 percentage point in the expected non-crisis unemployment gap. Given this, together with $\Delta u = 5$ percentage points and $p_{20} = 6.4$ percent, the quarterly-optimal expected non-crisis unemployment gap in (J.11) equals a modest 0.22 percentage points for the quarter when it is the largest. Again, it is striking how small the maximum quarterly expected non-crisis unemployment gap is. (Because $0.22^2 \approx 0.05$ is a small fraction of $(\Delta u)^2 = 25$, the above approximation is justified.)