# Reflexivity in Credit Markets<sup>\*</sup>

Robin Greenwood Harvard University and NBER Samuel G. Hanson Harvard University and NBER

Lawrence J. Jin California Institute of Technology

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#### Abstract

Reflexivity is the idea that investor beliefs affect market outcomes, which in turn affect investor beliefs. We develop a behavioral model of the credit cycle featuring such a two-way feedback loop. In our model, investors form beliefs about firms' creditworthiness, in part, by extrapolating past default rates. Investor beliefs influence firms' actual creditworthiness because firms that can refinance maturing debt on attractive terms—even if fundamentals do not warrant such favorable terms—are less likely to default in the short-run. Our model is able to match many features of credit booms and busts, including the imperfect synchronization of credit cycles with the real economy, the negative relationship between past credit growth and the future return on risky bonds, and "calm before the storm" periods in which firm fundamentals have deteriorated but the credit market has not yet turned.

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### 1 Introduction

A central but underappreciated feature of the credit cycle is how disconnected credit growth can be from real economic growth. Panel A of Figure 1 plots the annual growth rate of U.S. GDP alongside the annual growth rate of outstanding debt at nonfinancial corporations, both expressed in real terms. In the upswing proceeding the 2008 financial crisis, GDP growth peaked in March 2005, but credit growth peaked more than two years later. This pattern of credit expansion at the end of an economic expansion is also apparent in the late 1990s, with credit growth rising only at the end of the business cycle. During downturns, the economy often recovers well before credit growth returns to normal rates. In the most recent economic recovery, real credit growth first reached 3% in 2013, several years after the economy began its recovery. Overall, the correlation between credit growth and GDP growth is only 43%. At short horizons, credit seems to have something of a life of its own.

This disconnect poses a challenge for most well-known models of the credit cycle, including Bernanke and Gertler (1989), Holmström and Tirole (1997), Bernanke, Gertler, Gilchrist (1999), and Bordalo, Gennaioli, and Shleifer (2018). Specifically, although credit market frictions amplify business cycle fluctuations, the business cycle and the credit cycle are essentially one and the same in these models.

While there is only a modest connection between credit growth and current macroeconomic fundamentals, credit growth is strongly correlated with measures of current credit market conditions, such as the credit spread. Panel B of Figure 1 plots credit growth against the Moody's Baa credit spread. The correlation between credit growth and the Baa credit spread is -37%. Credit growth is also correlated with other measures of credit market sentiment, such as the share of corporate bond issuance with a high-yield rating or the lending standards reported by bank loan officers. But what drives investor sentiment and credit supply, if not investors' perceptions of macroeconomic fundamentals?

In this paper, we present a new behavioral model of the credit cycle in which credit markets take on a life of their own in the short run, although they are ultimately tied down by fundamentals of the economy over the longer run. A key feature of our model is "reflexivity", which is the idea of a two-way feedback between investor perceptions and real outcomes. In finance, reflexivity is most prominently associated with the investor George Soros, who summarized the idea of reflexivity as follows:

Participants' view of the world is always partial and distorted. That is the principle of fallibility. ... These distorted views can influence the situation to which they relate because false views lead to inappropriate actions. That is the principle of reflexivity.—George Soros, *Financial Times*, October 26, 2009.

In credit markets, reflexivity arises because investors who overestimate the creditworthiness of

a borrower are likely to refinance maturing debt on more favorable terms, thereby making the borrower less likely to default and more likely to survive, at least in the short run. In our model, a firm invests in a series of short-term projects. Each project requires an upfront investment of capital, which the firm finances using short-term debt that it must refinance each period. Projects generate a random cash flow that varies exogenously according to the state of the economy. Debt financing is provided by investors whose beliefs are partly rational and forward-looking, but also partly extrapolative and backward-looking. To the extent that they are backward-looking, investors extrapolate the firm's recent repayment history to infer the probability that the firm will repay its debt in the next period. Following periods of low defaults, investors believe that debt is safe, and refinance maturing debt on attractive terms. Such investor behavior is consistent with Hyman Minsky's writings on the credit cycle:

Current views about financing reflect the opinions bankers ... hold about the uncertainties they must face. These current views reflect ... the recent past ... A history of success will tend to diminish the margin of safety ... bankers require ...; a history of failure will do the opposite.—Hyman Minsky, *Stabilizing an Unstable Economy*, 1986.

Because investors hold extrapolative beliefs based on defaults and not the fundamental cash flows directly, this leads to a two-way dynamic feedback loop between investor beliefs and future defaults. The feedback loop arises because investor beliefs depend on past defaults, but these beliefs also drive future defaults via the terms on which investors are willing to refinance debt. Figure 2 illustrates the feedback loop. During credit booms, default rates are low, so investors believe that future default rates will continue to be low. In the near term, these beliefs are self-fulfilling: the perception of low future defaults leads to elevated bond prices, which in turn, makes it easier for the firm to refinance their maturing debt. Holding constant the firm's cash flows, cheaper debt financing leads to slower debt accumulation and a near-term decline in future defaults, which further reinforces investor beliefs. If cash flow fundamentals deteriorate, the backward-looking nature of investors' beliefs may allow firms to skate by for some amount of time, a phenomenon that we refer to as the "calm before the storm." But eventually, the reality of poor cash flows catches up with the firm, and it defaults. The disconnect between investors' beliefs and financial reality is the greatest just before such an unexpected default.

Conversely, suppose that the economy has just been through a wave of defaults. Since investors over-extrapolate these recent outcomes, investors believe that the likelihood of future defaults is high. Investor beliefs turn out to be partially self-fulfilling in the short run: bearish credit market sentiment makes it harder for firms to refinance existing debts, leading to an increase in defaults in the short run. In some circumstances, this can lead to "default spirals" in which a default leads to further investor pessimism and an extended spell of further defaults.

In our model, transitions between credit booms and credit busts are ultimately caused by changes in fundamental cash flows. However, because investors extrapolate past defaults and not cash flows, changes in credit markets are not fully synchronized with changes in fundamental cash flows, and can be highly path-dependent. For example, as noted, our model generates periods of "calm before the storm" in which the fundamentals of the economy have turned, but credit markets are still buoyant. Such episodes are consistent with Krishnamurthy and Muir (2017), who show that credit spreads are typically low in the years preceding financial crises. But, because investors are also partially forward-looking, credit spreads will jump up on the eve of a crisis just as Krishnamurthy and Muir (2017) find.

The model is also useful for understanding how credit evolves following an exogenous shock to investor beliefs. For example, suppose that investors become more optimistic about firms' creditworthiness, perhaps instigated by a central bank lowering the short-term interest rate. In this case, firms are able to roll over debt at more attractive rates, which in turn makes default less likely in the near-term. For an investor looking back at past defaults, the debt now appears to have been safer, leading investors to become more optimistic, further reducing the credit spread. Over time, a shock to beliefs can be self-perpetuating. There is a limit to this self-perpetuation, however, because ultimately the firm will become over-leveraged and will default.

While the credit market investors in our model are not fully rational, their beliefs are often similar to those of fully rational agents. In part, this is due to reflexivity: when investors believe than default probabilities are low, these optimistic beliefs *cause* default probabilities to be low. Thus, while the investors in our model do make predictable mistakes, those mistakes need not be large in order to generate rich and realistic credit market dynamics.

The model matches a number of facts that researchers have documented in recent years about credit cycles. First, rapid credit growth appears to be quite useful for predicting future financial crises and business cycle downturns (Schularick and Taylor, 2012; Mian, Sufi, and Verner, 2017; López-Salido, Stein, and Zakrajšek, 2017), a result that is consistent with our model because outstanding credit will grow rapidly when sentiment is high but cash flow fundamentals are poor. Second, economies that have experienced high credit growth are more fragile, in the sense that they are vulnerable to shocks (Krishnamurthy and Muir, 2017). Third, high credit growth predicts low future returns on risky bonds (Greenwood and Hanson, 2013; Baron and Xiong, 2017), a result that obtains in our model because investors do not fully understand when credit is growing rapidly that they are quickly heading towards default. In the model as in historical U.S. data, credit growth. In fact, in our model, when sentiment is high, credit spreads reach their lowest point just before the economy enters a wave of defaults, consistent with the evidence of Krishnamurthy and Muir (2017) on credit spreads before financial crises.

Our paper has much in common with Austrian theories of the credit cycle, including Mises (1924) and Hayek (1925), as well as the accounts of booms, panics, and crashes by Minksy (1986) and Kindleberger (1978). More recently, the idea that investors may neglect tail risk in credit

markets was developed theoretically by Gennaioli, Shleifer, and Vishny (2012, 2015) and supported by numerous accounts of the financial crisis (Coval, Jurek, and Stafford, 2009; Gennaioli and Shleifer, 2018). We also draw on growing evidence that investors extrapolate cash flows, past returns, or past crash occurrences (Barberis, Shleifer, and Vishny, 1998; Greenwood and Shleifer, 2014; Barberis, Greenwood, Jin, and Shleifer, 2015; Jin, 2015; Greenwood and Hanson, 2015). Most related here is Jin (2015), who presents a model in which investors' perceptions of crash risk depend on recent experience.

Bordalo, Gennaioli, and Shleifer (2018) also provide a model of credit cycles in which extrapolative investor expectations play an important role and in which bond returns are predictable.<sup>1</sup> Their model is similar to ours in several respect, but extrapolative expectations in their model are based on the exogenous underlying fundamental cash flows rather than endogenous credit market outcomes as in our model. In their model, bond prices and bond defaults are perfectly tied to cash flow fundamentals. Extrapolative expectations serve as an amplification mechanism but not a propagation mechanism, so the credit cycle and the business cycle are fully synchronized in their model. The fact that investors extrapolate an endogenous outcome in our model leads to episodes—such as "calm before the storm" and "default spiral" episodes—in which the credit market can become quite disconnected from the underlying cash flow fundamentals. Thus, credit cycles acquire a life of their own in our model. Overall, compared to prior work, our central contribution is to explore how the interplay between extrapolative beliefs and the central role of refinancing—and the resulting potential for reflexivity—drive credit market dynamics.<sup>2</sup>

In Section 2, we summarize a number of stylized facts about the credit cycle, drawing on the papers cited above but also presenting some novel observations about the synchronicity of the credit cycle and the business cycle. In Section 3, we develop the baseline model featuring a single representative firm, and demonstrate the central feedback mechanism of the model. We then discuss how the model can match a number of features of credit cycles that researchers have documented in recent years, such as the predictability of returns and low credit spreads before crises. In Section 4, we discuss a model extension that include multiple firms and that generates more realistic default dynamics. Section 5 concludes.

## 2 Motivating facts about the credit cycle

We begin by summarizing a set of stylized facts about credit cycles. The first three facts are drawn from previous work; the fourth is based on some new empirical work of our own.

**Observation 1.** Credit growth predicts financial crises and business cycle downturns.

<sup>&</sup>lt;sup>1</sup>See also Gennaioli, Shleifer, and Vishny (2015) for a precursor to Bordalo, Gennaioli, and Shleifer (2018).

<sup>&</sup>lt;sup>2</sup>See also Coval, Pan, and Stafford (2014) who suggest that in derivatives markets, model misspecification only reveals itself in extreme circumstances, by which time it is too late. Bebchuk and Goldstein (2011) present a model in which self-fulfilling credit market freezes can arise because of interdependence between firms.

Schularick and Taylor (2012) show that, in a broad panel of 14 countries dating back to 1870, periods of rapid credit growth predict financial crises in an international panel. More recently, Mian, Sufi, and Verner (2017) show that rapid credit growth, and especially growth in household credit, predicts declines in GDP growth at a three year horizon in an international panel. López-Salido, Stein, and Zakrajšek (2017) show that frothy credit market conditions— proxied using declines in borrower quality and low credit spreads—predict low GDP growth in the U.S. data from 1929 to 2015. Schularick and Taylor (2012) interpret their evidence as suggesting that financial crises are episodes of credit booms "gone bust." López-Salido, Stein, and Zakrajšek (2017) attribute their findings to predictable reversals in credit market sentiment. Consistent with this view, using an international panel of 38 countries, Kirti (2018) shows that rapid credit growth that is accompanied by a deterioration in lending standard—i.e., by declining issuer quality—is associated with low future GDP growth. By contrast, when rapid credit growth is accompanied by stable lending standards, there is no such decline in future GDP growth.

#### **Observation 2.** Economies that have experienced high credit growth are more fragile.

Krishnamurthy and Muir (2017) argue that the natural way to interpret the findings of Schularick and Taylor (2012) about credit growth and financial crises is that credit growth creates fragility. When a more fragile system encounters a negative shock, such as a house price decline, this results in a financial crisis.

#### **Observation 3.** Rapid growth is correlated with low credit spreads and predicts low future returns.

In Figure 1, we have shown that there is a high degree of correlation between the pricing of credit and growth of credit: when credit spreads are low, credit growth is high. This correlation does not imply causality: it could imply that both credit spreads and credit growth reflect an abundance of safe investment opportunities. Alternatively, it could reflect credit growing quickly when investors are willing to supply it on favorable terms. Greenwood and Hanson (2013) and Baron and Xiong (2017) present evidence that conditional expected excess returns on risky bonds and bank stocks become reliably negative when credit markets appear to be overheated—i.e., when many low quality borrowers are able to obtain credit and when credit markets appear to be most overheated. These negative expected returns and the negative relationship between risk and return are difficult to square with rational risk-based models—even rational models with credit market frictions—and are powerful motivations for behavioral approach that we adopt in this paper.

In contrast to integrated-market models, Greenwood and Hanson (2013) and López-Salido, Stein, and Zakrajšek (2017) point out that variables that forecast credit returns are not strong predictors of equity returns and vice versa. This motivates approaches like the one in our paper and Barberis, Greenwood, Jin, and Shleifer (2015) where investors extrapolate outcomes that are specific to their market as opposed to firm cash flows. In other words, we need an approach that combines over-extrapolation and segmented markets—i.e., a localized version of extrapolative beliefs. Furthermore, the fact that investors extrapolate an endogenous equilibrium outcome—i.e., default—in our model gives rise to reflexive dynamics in which investor beliefs can actually impact future defaults.

Over the past several years, a number of authors have shown that periods of credit growth and deterioration in credit quality are associated with low future returns on risky bonds. Greenwood and Hanson (2013) develop a simple measure of credit market overheating based on the composition of corporate debt issuance. Their measure—the share of all corporate bond issuance from high-yield-rated firms—captures the intuition that when credit markets are overheated, low quality firms increase their borrowing to take advantage. Greenwood and Hanson (2013) show that declines in issuer quality are associated with concurrent growth in total corporate credit and that both quantity and quality predict low corporate bond returns. Adopting a similar intuition, Baron and Xiong (2017) show that bank credit expansion also predicts low bank equity returns in a large panel of countries.

Table 1 updates the data from Greenwood and Hanson (2013) through 2014 and also considers a set of additional proxies for credit market sentiment. The table shows regressions of the form:

$$rx_{t+k}^{HY} = a + b \cdot Sent_t + \epsilon_{t+k},\tag{1}$$

where  $rx_{t+k}^{HY}$  denotes the log excess return on high yield bonds over a k = 2- or 3-year horizon, and  $Sent_t$  is a proxy for credit market sentiment, measured using data through the end of year t. Excess returns are the difference between the return on the high yield bond index and the return on duration-matched Treasury bonds. All of our data begin in 1983.<sup>3</sup> Columns (1) and (5) show that the log high yield share significantly predicts reductions in subsequent excess high yield returns. A one standard deviation in the log high yield share is associated with an 8.3 percentage point reduction in log returns over the next two years, or 9.7 percentage points over the next three years.

Columns (2) and (6) of Table 1 show that the same forecasting results hold when credit market sentiment is measured as the growth in aggregate nonfinancial corporate credit. Aggregate nonfinancial corporate credit is the sum of nonfinancial corporate debt securities and loans from Table L103 of the Federal Reserve's Financial Accounts of the U.S. A one standard deviation increase in credit growth forecasts a 7.4 percentage point reduction in log returns over the next two years, or 9.3 percentage points over the next three years.

Table 1 supplements these forecasting results with regressions based on two additional measures of credit market sentiment. The first, *Loan Sentiment*, is a measure based on the Federal Reserve's

 $<sup>^{3}</sup>$ For results over different time horizons and with additional controls, see Greenwood and Hanson (2013) who compute alternate proxies for issuer quality that extend back as far as 1926.

Senior Loan Officer Survey, and the second is the excess bond premium (EBP) from Gilchrist and Zakrajšek (2012).<sup>4</sup> Table 1 shows that both of these additional measures of credit market sentiment forecast corporate bond returns in the expected direction.

To summarize, Table 1 confirms that periods of high credit market sentiment are associated with growth in total credit, a loosening of credit standards, and are followed by low subsequent returns.

#### **Observation 4.** The disconnect between the credit cycle and the business cycle.

Figure 3 illustrates the disconnect between the credit cycle and the business cycle in U.S. data. Here we provide additional perspective on the lack of synchronicity between the credit cycle and the business cycle. In particular, we show that credit growth tends to increase towards the end of a business cycle boom. In Panel A of Figure 3, we plot real GDP growth from trough to peak of the business cycle, by business cycle expansion quarter. As can be seen, GDP growth is low in the beginning of business cycle expansions, but after quarter five, it stabilizes and if anything, declines slightly in later quarters. In contrast, Panel B shows credit growth over the same periods. As the figure makes clear, credit expansion is particularly high in the latter part of the business cycle.

## 3 A model of credit market sentiment

In this section, we consider an infinite-horizon model with a representative firm and a set of identical, risk-neutral bond investors. Our assumption of a representative firm is made purely for simplicity and to most starkly illustrate the model's core implications. One should interpret a default by the representative firm as a "credit market bust" in which there is an economy-wide spike in corporate defaults. In Section 4, we introduce a continuum of firms which are subject to heterogeneous cash flow shocks.

We first describe the model setting and collect several preliminary results about investor beliefs. We then present a series of formal results and numerical simulations that trace out the model's key implications for credit market dynamics.

<sup>&</sup>lt;sup>4</sup>Every quarter, the Federal Reserve surveys senior loan officers of major domestic banks concerning their lending standards to households and firms. Officers report whether they are easing or tightening lending standards in the past quarter. We construct a measure of credit market sentiment, *Loan Sentiment*, by taking the three-year average percentage of banks that have reported easing credit standards in any given quarter. The idea behind this averaging procedure is that sentiment captures the level of beliefs about future creditworthiness, whereas the quarterly survey measures changes from the previous quarter. The senior loan officer opinion survey begins in the first quarter of 1990, so this measure of sentiment begins in December 1992. *Loan Sentiment* is 55% correlated with the high yield share and 68% correlated with the growth in aggregate corporate credit.

### 3.1 Model setting

Each period t, the representative firm invests in a one-period project that requires an upfront cost of c > 0. The next period, the project generates a random cash flow,  $x_{t+1}$ , that follows an AR(1) process

$$x_{t+1} - \bar{x} = \rho(x_t - \bar{x}) + \varepsilon_{t+1}, \qquad (2)$$

where  $\bar{x} \geq c$  and the fundamental cash flow shock  $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_{\varepsilon}^2)$  is *i.i.d.* over time.

The firm issues one-period bonds in order to finance these one-period projects. Each bond is a promise to pay back one dollar to investors in one period. At time t, the price of each bond is denoted  $p_t$ . The total face amount of debt outstanding at time t is  $F_t$ , meaning that the firm is obligated to repay  $F_t$  dollars to investors at time t + 1.

We now describe the evolution of the firm's outstanding debt,  $F_t$ . At time t, the firm must repay the face amount of debt issued the prior period  $F_{t-1}$ . The firm also must pay the cost cto begin a new project and receives the cash flow  $x_t$  from the prior period's project. Finally, the firm can issue new bonds at a price of  $p_t$ . Assuming the firm does not default and does not pay dividends to equity holders at time t, the total face amount of bonds outstanding at time t is

$$F_t = (F_{t-1} + c - x_t)/p_t,$$
(3)

which is obtained by equating sources and uses.<sup>5</sup> This law of motion is consistent with the fact that nonfinancial leverage is typically counter-cyclical (Kekre (2016)), falling in good times when  $x_t$  is high and rising in bad times when  $x_t$  is low.

We assume a simple mechanistic default rule. Specifically, if at any time t,  $F_{t-1} + c - x_t$  rises above a threshold of  $\overline{F}$ , the firm defaults. The existence of this threshold  $\overline{F}$  can be seen as a reduced form for informational or agency frictions that grow more severe as the amount of required external financing rises. Alternately, such a threshold may arise from the optimal exercise of the firm's default option by equity holders as in Leland (1994). Formally, letting  $D_t$  denote a binary variable indicating whether or not a default occurs at time t, we have

$$D_t = \mathbf{1}_{\{F_{t-1} + c - x_t \ge \bar{F}\}}.$$
 (4)

The "default boundary" is the line in  $(F_{t-1}, x_t)$  space where this default indicator switches on or off—i.e., the line  $F_{t-1} = (\bar{F} - c) + x_t$ .

In the event of default, the firm continues to operate, but writes off a fraction of its debt much

<sup>&</sup>lt;sup>5</sup>We assume that the firm always decides to invest, even when expected cash flows tomorrow do not cover the current cost—i.e., when  $c > \bar{x} + \rho(x_t - \bar{x})$ . There are various interpretations of this assumption. First, we could assume that the firm is operating a long-run technology that generates the stream  $\{x_t\}$ , that c is the cost of continuation each period, and that continuation is (almost) always efficient. Alternately, we could assume that managers receive private benefits from running the firm and will always choose continuation even if continuation is value destroying.

like under Chapter 11 of the U.S. Bankruptcy Code. Specifically, if the firm defaults, a fraction  $1 - \eta$  of the firm's debt is written off, generating losses for existing bondholders, and the remaining fraction  $\eta \in (0, 1)$  is refinanced at current market prices. Thus, if the firm defaults at time t, the amount of debt outstanding becomes

$$F_t = \eta (F_{t-1} + c - x_t) / p_t.$$
(5)

Finally, we assume that  $F_t = \underline{F}/p_t$  when  $F_{t-1} + c - x_t \leq \underline{F}$  where  $\underline{F} > 0$ . In this case, the firm pays the residual cash flow to equity holders as a dividend. The idea underlying the lower barrier  $\underline{F}$  for debt outstanding can be motivated via the pecking order theory of capital structure (Myers and Majluf, 1984). Firms only raise external finance in the form of debt. And when there is available free cash flow, the firm first uses this cash flow to retire existing debts. However, once the face value of debt reaches a sufficiently low level, the firm chooses to pay out all available free cash flow to its equity holders.

Taking  $F_t$  as given, it is straightforward to compute the fully-rational, forward-looking probability of a default at time t + 1, which we label  $\lambda_t^R$ . Given the cash flow process in equation (2) and the default rule in equation (4), a default will occur at time t + 1 if and only if  $F_t + c - \rho x_t - (1 - \rho)\bar{x} - \varepsilon_{t+1} \ge \bar{F}$ . Thus, at time t, the true probability of default on the promised bond payments at time t + 1 is

$$\lambda_t^R = \Phi\left(\frac{F_t - \bar{F} + c - \rho x_t - (1 - \rho)\bar{x}}{\sigma_{\varepsilon}}\right),\tag{6}$$

where  $\Phi(\cdot)$  denotes the cumulative normal distribution function.

Investors' beliefs at time t about the probability of a default at time t + 1 are denoted  $\lambda_t^C$ . We assume that investors' beliefs  $\lambda_t^C$  are a mixture of (i) an extrapolative and backward-looking component  $\lambda_t^B$  and (ii) the fully rational and forward-looking belief  $\lambda_t^R$ . We assume that fraction  $\theta \in [0, 1]$  of investors' beliefs are extrapolative and backward-looking and the remaining fraction  $1 - \theta$  are fully-rational and forward-looking. Thus, we have:

$$\lambda_t^C = \theta \lambda_t^B + (1 - \theta) \lambda_t^R = \lambda_t^R - \theta (\lambda_t^R - \lambda_t^B).$$
(7)

This formulation of beliefs in the spirit of Fuster, Laibson, and Mendel (2010) who argue that many agents have "natural expectations" which are a combination of fully-rational expectations and extrapolation expectations. Thus, equation (7) embeds the polar cases of fully-rational expectations ( $\theta = 0$ ) and fully-extrapolative expectations ( $\theta = 1$ ).

Our formulation of beliefs in equation (7) embeds two distinct notions of "credit market sentiment." First, one might say that credit market sentiment is strong when  $\lambda_t^B$  is low—i.e., when future defaults are perceived as being unlikely according to extrapolative component of beliefs. Alternately, one might say that credit market sentiment is strong when  $(\lambda_t^R - \lambda_t^B)$  is high—i.e., when investors underestimate the true likelihood of future default.

In a moment, we will detail precisely how  $\lambda_t^B$  is specified and how, when  $\theta < 1$ ,  $\lambda_t^R$  is pinned down in a rational-expectations equilibrium. For now, let us take  $\lambda_t^B$  and  $\lambda_t^R$  as given.

Since investors are risk-neutral in our model, the bond price at time t is simply

$$p_t = p(\lambda_t^B, \lambda_t^R) = (1 - \lambda_t^C) + \lambda_t^C \eta = \left[1 - (1 - \eta)\lambda_t^R\right] + (1 - \eta)\theta(\lambda_t^R - \lambda_t^B).$$
(8)

Thus, relative to the price of  $1 - (1 - \eta)\lambda_t^R$  in a fully-rational economy where  $\theta = 0$ , bond prices are elevated when  $\lambda_t^R - \lambda_t^B$  is high and investors are underestimating the true likelihood of a future default.

The default rule in equation (4) and the bond pricing equation (8) give rise to the following law of motion for the amount of debt outstanding:

$$F_{t} = f(F_{t-1}, \lambda_{t}^{B}, \lambda_{t}^{R}, x_{t}) = \begin{cases} \underline{F}/p(\lambda_{t}^{B}, \lambda_{t}^{R}) & \text{if} F_{t-1} + c - x_{t} \leq \underline{F} \\ (F_{t-1} + c - x_{t})/p(\lambda_{t}^{B}, \lambda_{t}^{R}) & \text{if} \underline{F} < F_{t-1} + c - x_{t} < \overline{F} \\ \eta(F_{t-1} + c - x_{t})/p(\lambda_{t}^{B}, \lambda_{t}^{R}) & \text{if} F_{t-1} + c - x_{t} \geq \overline{F} \end{cases}$$
(9)

Since  $p(\lambda_t^B, \lambda_t^R) \leq 1$ , it follows that we always have  $F_t \geq \underline{F}$ . Thus,  $\underline{F}$  is indeed a lower barrier for the amount of debt outstanding.

The model is fully characterized by equations (2), (8), and (9), together with the specifications for  $\lambda_t^B$  in equation (10) and the solution for  $\lambda_t^R$  in equation (12) which will be introduced below.

The extrapolative component of investor beliefs  $\lambda_t^B$ . We now introduce our specification for  $\lambda_t^B$ , the extrapolative, backward-looking component of investors' time t beliefs about the likelihood of a default at time t + 1. We assume that  $\lambda_t^B$  depends solely on past default realizations and past "sentiment" shocks that are unrelated to cash flow fundamentals. Specifically, we assume that the law of motion for this backward-looking component of beliefs is

$$\lambda_t^B = \max\left\{0, \min\left\{1, \beta \lambda_{t-1}^B + \alpha D_t + \omega_t\right\}\right\},\tag{10}$$

where  $0 < \beta < 1$  is a memory decay parameter,  $\alpha > 0$  measures the incremental impact of a default event on backward-looking beliefs, and  $\omega_t \sim \mathcal{N}(0, \sigma_{\omega}^2)$  is a random "sentiment" shock that is independent of the fundamental cash flow shocks  $\varepsilon_t$ . The min and max functions in equation (10) ensure that  $\lambda_t^B \in [0, 1]$  for all t. Assuming that  $\lambda_{t-j}^B \in (0, 1)$  for all  $j \ge 0$ , we have

$$\lambda_t^B = \sum_{j=0}^{\infty} \beta^j \left( \alpha D_{t-j} + \omega_{t-j} \right).$$
(11)

Thus, in this case, the extrapolative component of beliefs is just a geometric moving average of past defaults and past sentiment shocks.

The specification for extrapolative beliefs in equation (10) is similar to specifications in Barberis, Greenwood, Jin, and Shleifer (2015, 2018). Empirically, equation (10) is motivated by the findings in Greenwood and Hanson (2013) who present evidence that credit market investors tend to extrapolate recent credit market outcomes. Specifically, Greenwood and Hanson (2013) show that issuer quality tends to deteriorate following periods when default rates have fallen and the returns on high-yield bonds have been high. These results hold controlling for recent macroeconomic conditions or stock returns, suggesting that the recent experiences of credit market investors plays an important a role in shaping their current expectations. In other words, credit market investors appear to extrapolate recent credit market outcomes, and these may not be perfectly synchronized with aggregate macroeconomic outcomes. This evidence motivates the idea that bond investors form expectations about future defaults by extrapolating past defaults  $D_t$  (which are an endogenous outcome in our model) as opposed by extrapolating cash flow fundamentals  $x_t$ (which are exogenously given).

The following lemma explains how this extrapolative component of beliefs evolves over time.

**Lemma 1** Assume there are no sentiment shocks (i.e.,  $\omega_t = 0$  for all t), so the law of motion of for the extrapolative component of beliefs is simply  $\lambda_t^B = \max\{0, \min\{1, \beta\lambda_{t-1}^B + \alpha D_t\}\}$ .

- If there is no default at time t, then we always have  $\lambda_t^B \leq \lambda_{t-1}^B$  and  $\lambda_t^B < \lambda_{t-1}^B$  if  $\lambda_{t-1}^B > 0$  i.e., extrapolative beliefs always become more optimistic when there is no default.
- If there is a default at time t, there are two cases:
  - If  $\alpha \geq 1 \beta$ , then  $\lambda_t^B \geq \lambda_{t-1}^B$  and  $\lambda_t^B > \lambda_{t-1}^B$  if  $\lambda_{t-1}^B < 1$ —i.e., extrapolative beliefs always become more pessimistic following a default. As a result,  $\lambda_t^B$  will converge to 1 following a long sequence of defaults.
  - If  $\alpha < 1 \beta$ , then  $\lambda_t^B \ge \lambda_{t-1}^B$  as  $\lambda_{t-1}^B \le \alpha/(1-\beta)$ . As a result,  $\lambda_t^B$  will converge to  $\alpha/(1-\beta) < 1$  following a long sequence of defaults.

**Proof.** See the Appendix for all proofs.

Naturally, the dynamics of  $\lambda_t^B$  are governed by the incremental impact of a default on beliefs  $\alpha$  and the rate of memory decay  $(1 - \beta)$ . As we will see below, the potential for backward-looking beliefs to drive persistent default cycles is greatest when incremental belief impact  $\alpha$  is high and when memory decay  $(1 - \beta)$  is low. In this case, a default at time t will lead to a large, persistent increase in  $\lambda_t^B$  that makes it more difficult for firms to refinance maturing debt, raising the true likelihood of future defaults.

Solving for rational expectations equilibrium. We now explain how  $\lambda_t^R$  is pinned down in a rational expectations equilibrium when  $\theta < 1$ . According to equation (6),  $\lambda_t^R$  depends on  $F_t$ . However, equations (8) and (9) imply that  $F_t$  depends on  $\lambda_t^R$  when  $\theta < 1$ . Thus, when  $\theta < 1$ ,  $\lambda_t^R$ and  $F_t$  must be simultaneously determined in equilibrium.

The simultaneous determination of  $F_t$  and  $\lambda_t^R$  introduces the potential for multiple equilibria. The potential for equilibrium multiplicity reflects a straightforward self-fulfilling prophecy or "reflexive" intuition. If the rational component of investor beliefs about future default probabilities is low (high), then current bond prices are high (low). As a result, the face value of debt firms that must promise to repay tomorrow is low (high), leading to a true probability of default tomorrow that is indeed low (high).

Formally, combining equations (6) and (9), we see that the equilibrium value of  $\lambda_t^R$  must solve the following fixed-point problem when  $\theta < 1$ :

$$\lambda_t^R = g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t) \equiv \Phi\left(\frac{f(F_{t-1}, \lambda_t^B, \lambda_t^R, x_t) + c - \bar{F} - \rho x_t - (1 - \rho)\bar{x}}{\sigma_{\varepsilon}}\right).$$
(12)

Note from (9) that the bond price  $p(\lambda_t^B, \lambda_t^R)$  does not determine whether the firm defaults or pays dividends at time t; only  $F_{t-1}$  and  $x_t$  determine these outcomes. This means that  $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$ is a continuous and increasing function of  $\lambda_t^R$  for given values of  $(F_{t-1}, \lambda_t^B, x_t)$ . Also note that  $g(0|F_{t-1}, \lambda_t^B, x_t) > 0$  and  $g(1|F_{t-1}, \lambda_t^B, x_t) < 1$ . Therefore,  $g(\lambda_t^R|F_{t-1}, \lambda_t^B, x_t)$  is a continuous function that maps the unit interval into itself, so a fixed point always exists by Brouwer's fixed-point theorem.

Multiple equilibria are more likely to arise—i.e., there may be multiple solutions to equation (12)—(i) when investor beliefs are more rational and forward-looking (i.e., when  $\theta$  is low); (ii) when the configuration of  $(F_{t-1}, \lambda_t^B, x_t)$  means that the firm will be near the default boundary at time t + 1; and (iii) when cash flow volatility  $\sigma_{\varepsilon}$  is low. First, rational beliefs have a larger impact on current bond prices and hence on the likelihood of future defaults when  $\theta$  is low. Indeed, there is a single unique equilibrium when  $\theta = 1$  and beliefs are completely extrapolative. Second, multiple equilibria will only arise when the firm will be near the default boundary at time t + 1. If the firm is very far from the default boundary, then  $\partial g(\lambda_t^R|\cdot)/\partial \lambda_t^R$  is always small—there is no scope for self-fulfilling rational beliefs—and there is a unique equilibrium. Finally, when future cash flows are volatile (i.e., when  $\sigma_{\varepsilon}$  is high), the downside risk for the future cash flows is high which reduces the effect of self-fulfilling rational beliefs on future defaults. In this case, model has a unique equilibrium. Conversely, when future cash flows are not very volatile, self-fulfilling rational beliefs are not very volatile, self-fulfilling rational beliefs are unique equilibrium.

<sup>&</sup>lt;sup>6</sup>Formally,  $g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$  is an S-shaped function of  $\lambda_t^R$ , a property that it inherits from the normal cumulative density function  $\Phi(\cdot)$ . As we increase  $\sigma_{\varepsilon}$ ,  $g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$  becomes closer to a linear function of  $\lambda_t^R$ —i.e.,  $\partial^2 g(\lambda_t^R | \cdot) / \partial(\lambda_t^R)^2$  approaches zero—so it is harder to have multiple equilibria. As  $\sigma_{\varepsilon} \to 0$ ,  $g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$  converges to a step-function and it is easier to have multiple equilibria.

Figure 4 illustrates the existence of multiple equilibria in our model. In Figure 4, we assume that  $\bar{x} = 2.4, \rho = 0.8, \sigma_{\varepsilon} = 0.5, c = 2, \bar{F} = 5$ , and  $\eta = 0.5$ . We set  $F_{t-1} = 4$  and  $\lambda_t^B = 0.33$ . Thus, the default boundary at time t is  $x_t \leq F_{t-1} + c - \bar{F} = 1$ . The figure shows the  $g(\lambda_t^R)$  function for  $x_t \in \{1.9, 2, 2.1, 2.5\}$  and  $\sigma_{\varepsilon} \in \{0.5, 1.0, 0.01\}$ . When  $\sigma_{\varepsilon} = 0.5$ , beliefs about future defaults have a modest impact on the likelihood of future defaults. In this case,  $g(\lambda_t^R)$  is an S-shaped function of  $\lambda_t^R$  and there can be multiple equilibria. Specifically, there are three possible equilibria when  $x_t = 2$ . However, there is only a single equilibrium for other values of  $x_t$ . By contrast, when  $\sigma_{\varepsilon} = 1$ , beliefs have a much smaller impact on the likelihood of future defaults. In this case,  $g(\lambda_t^R)$  function is nearly linear and there is always a unique equilibrium. Finally, when  $\sigma_{\varepsilon} = 0.01$ , beliefs can have a very large impact on the likelihood of future defaults. In this case,  $g(\lambda_t^R)$  is close to being a step-function and multiple equilibria routinely arise.

How do we select amongst these equilibria when more than one exists? We focus on the smallest  $\lambda^R$  that solves  $\lambda^R = g(\lambda^R | \cdot)$ —i.e., the model's "best" stable equilibrium.<sup>7</sup> An equilibrium is "stable" if it is robust to a small perturbation in investors' beliefs regarding the likelihood of a default tomorrow. In our setting, if  $\partial g(\lambda^{R*} | \cdot) / \partial \lambda^R < 1$ , then  $\lambda^{R*}$  is stable; if  $\partial g(\lambda^{R*} | \cdot) / \partial \lambda^R > 1$ , then  $\lambda^{R*}_t$  is unstable. Since  $g(0|\cdot) > 0$  and  $g(1|\cdot) < 1$ , our model always has at least one stable equilibrium. Following the correspondence principle of Samuelson (1947), stable equilibria have local comparative statics that accord with common sense. For instance, at a stable equilibrium,  $\lambda^R_t$  is locally increasing in  $F_{t-1}$  and decreasing in  $x_t$ .<sup>8</sup>

The following lemma explains how the true probability of default  $\lambda_t^R$  is influenced by movements in  $F_{t-1}$ ,  $\lambda_t^B$ , and  $x_t$ .

**Lemma 2** First, assume that the economy is not near the default boundary  $F_{t-1} = (\bar{F} - c) + x_t$ at time t, so small changes in  $F_{t-1}$  and  $x_t$  do not affect whether or not there is a default at time t. Then a small increase in  $F_{t-1}$  raises  $\lambda_t^R$  when  $\underline{F} < F_{t-1} + c - x_t$ , a small increase in  $\lambda_t^B$ always raises  $\lambda_t^R$ , and a small increase in  $x_t$  always reduces  $\lambda_t^R$ . When  $\theta = 1$ ,  $\lambda_t^R$  is everywhere a continuous function of  $F_{t-1}$ ,  $\lambda_t^B$ , and  $x_t$ . By contrast, when  $\theta < 1$ ,  $\lambda_t^R$  can be discontinuous in  $F_{t-1}$ ,  $\lambda_t^B$ , and  $x_t$ , jumping discretely in response to small changes in these variables when the smallest solution to equation (12) jumps—we call these jumps "equilibrium discontinuity points." However,  $\lambda_t^R$  is continuous and differentiable in these variables almost everywhere when  $\theta < 1$ .

Next, assume that the economy is near the default boundary at time t, so small changes in  $F_{t-1}$  and  $x_t$  can affect whether or not there is a default at time t. Near the default boundary, a small increase in  $F_{t-1}$  can trigger a default at time t, resulting in a discrete downward jump in the probability of a default at t + 1,  $\lambda_t^R$ . Similarly, near the default boundary, a small increase in

<sup>&</sup>lt;sup>7</sup>We obtain very similar simulation results if we instead focus on the the largest  $\lambda^R$  that solves  $\lambda^R = g(\lambda^R | \cdot)$ . i.e., the model's "worst" stable equilibrium.

<sup>&</sup>lt;sup>8</sup>By contrast, unstable equilibria have local comparative statics with the opposite signs, which run contrary to common sense. For instance, at an unstable equilibrium,  $\lambda_t^R$  is locally decreasing in  $F_{t-1}$  and increasing in  $x_t$ .

 $x_t$  can avert a default at time t, resulting in a discrete upward jump in  $\lambda_t^R$ . However, it is still the case that a small increase in  $\lambda_t^B$  always raises  $\lambda_t^R$ .

Figure 4 illustrates one of the equilibrium discontinuity points mentioned in Lemma 2. Specifically, when  $\sigma_{\varepsilon} = 0.5$ , we see that the number of solutions to equation (12) jumps from one to three as  $x_t$  increases from 1.9 to 2. As a result, the smallest solution to equation (12) jumps discretely downward as  $x_t$  increases from 1.9 to 2.

**Reflexivity.** Our model captures George Soros' notion of reflexivity which is the idea that incorrect beliefs can impact reality. And, paradoxically, incorrect beliefs have the potential to become partially self-fulfilling. Specifically, when investors are partially extrapolative ( $\theta > 0$ ), our model incorporates an important feedback loop that arises from extrapolative, backward-looking beliefs. Past defaults affect investors' beliefs about future defaults via equation (10). These beliefs then feed back into bond prices via equation (8). Finally, since bond prices influence the ease with which the firm can refinance its existing debt, they in turn affect the evolution of debt outstanding via equation (9) and hence the true probability of future defaults in equation (6).

While this feedback loop is always present, there are times when the strength of this feedback loop—i.e., when the degree of reflexivity—is stronger and other times when this feedback loop is weaker. Specifically, we say that the economy in a "highly reflexive state" when  $\partial \lambda_t^R / \partial \lambda_t^B$  is large. While we always have  $\partial \lambda_t^R / \partial \lambda_t^B > 0$ , there are "non-reflexive regions" of our models' state-space  $(x_t, F_{t-1}, \lambda_t^B)$  where  $\partial \lambda_t^R / \partial \lambda_t^B$  is small. However, there are also "highly reflexive regions" where  $\partial \lambda_t^R / \partial \lambda_t^B$  is large: here a change in the extrapolative component of beliefs  $\lambda_t^B$ —whether due to a current default or a sentiment shock  $\omega_t$ —will have a large impact on the true probability of default  $\lambda_t^R$ . As we will see, these highly reflexive regions play an important role in driving credit market dynamics in our model.

### **3.2** Model implications

In this section, we provide a set of formal results and simulations to illustrate the key implications of the model. In particular, we lay out three main implications of the model: the "calm before the storm" phenomenon, the "default spiral" phenomenon, and the predictability of corporate bond returns. As we emphasize, these three novel implications reflect the interaction between default extrapolation and the reflexive nature of credit markets. In other words, these three results arise because (i) investors hold beliefs that are (partially) backward-looking—i.e., they extrapolate past defaults when forming beliefs about future defaults—and (ii) beliefs about future defaults are (partially) self-fulfilling. We also use the model to draw impulse-response functions which show how shocks to cash flow fundamentals and investor beliefs impact credit markets.

#### 3.2.1 Model parameters

We use the following set of baseline parameters throughout:

- Cash flow dynamics:  $\bar{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ .
- Investment cost: c = 2.
- Default and dividend barriers:  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ .
- Write-off parameter:  $\eta = 0.5$ .
- Belief dynamics:  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ .
- Belief mix:  $\theta = 0.5$ .

While these parameters are only illustrative, they have a number of desirable properties derived from long sample simulations:

- 1. The unconditional default probability is realistic. Here the unconditional probability of default is 12%. As noted above, one should interpret a default by our representative firm as a "credit market bust" in which there is an economy-wide spike in corporate defaults. Thus, these parameters imply that roughly one in ten years corresponds to such a bust.
- 2. The unconditional means of  $\lambda_t^B$  and  $\lambda_t^R$  are similar. Here the average of  $\lambda_t^R$  is 12% and  $\lambda_t^B$  is 15%. Thus, the behavioral component of beliefs is reasonable on average. As a result, means of  $(\lambda_t^R \lambda_t^B)$  and  $r_{t+1}$  are small. Here the mean of  $(\lambda_t^R \lambda_t^B)$  is -3% and the average return is 0.3%.
- 3. The time-series correlation between  $\lambda_t^B$  and  $\lambda_t^R$  is meaningful. Thus, while clearly imperfect, investors' beliefs are reasonable over time. Specifically, we have  $Corr(\lambda_t^B, \lambda_t^R) =$ 0.58. Thus, the backward-looking component of investors beliefs is strongly correlated with fully-rational beliefs over time. And, investor's combined beliefs  $\lambda_t^C = \theta \lambda_t^B + (1 - \theta) \lambda_t^R$  are close to the fully-rational ideal:  $Corr(\lambda_t^C, \lambda_t^R) = 0.93$ .
- 4. Relation of  $\alpha$  and  $\beta$ . The strength of default spiral mechanism is increasing in both  $\alpha$ and  $\beta$ . Specifically, if  $\alpha > (1 - \beta)$  then  $\lambda_t^B$  always rises when  $D_t = 1$  and  $\omega_t = 0$ . However, if  $\alpha \leq 1 - \beta$  then  $\lambda_t^B$  can actually fall when  $D_t = 1$  and  $\omega_t = 0$ . Since  $\alpha = 1 - \beta$  is this calibration, default spirals are possible.

Figure 5 shows a typical sample path of simulated data using these parameters. Notice that the time-series distribution of  $\lambda_t^R$  is bimodal when  $\theta = 0.5$ :  $\lambda_t^R$  is typically either close to zero or 1. This bimodal distribution is largely a function of the short-term nature of debt in our model.

Short-term debt is extremely safe until the moment that it is not; and at that point, short-term debt often becomes quite risky. However, the partially forward-looking nature of beliefs also contributes to the bimodal distribution of  $\lambda_t^R$ . Specifically, when  $\theta < 1$ , the model admits multiple equilibria and the smallest stable equilibrium will often discretely jump from  $\lambda_t^R \approx 0$  to  $\lambda_t^R \approx 1$  as the economy approaches the default boundary. This effect is diminished when we increase  $\theta$ , so the distribution of  $\lambda_t^R$  becomes less bimodal as beliefs become more backward-looking.

#### 3.2.2 The "calm before the storm" phenomenon

An elevated level of credit market sentiment—i.e., a lower level of  $\lambda_t^B$ —slows down the accumulation of debt in the face of deteriorating cash flows fundamentals, thereby delaying or even preventing future defaults. We term this phenomenon the "calm before the storm." Below we provide a formal result regarding this phenomenon.

**Proposition 1 Calm before the storm.** Assume that  $\theta > 0$ . For any initial level of debt outstanding  $F_{t-1}$  and cash flow  $x_t$ , lowering the initial extrapolative component of investor beliefs  $\lambda_t^B$  weakly delays the next default path by path—i.e., for any given time series of future cash flow and sentiment shocks—and strictly delays the next default in expectation.

To illustrate this "calm before the storm" phenomenon, Figure 6 depicts a sample path of the model using our baseline set of parameters. The cash flow fundamental  $x_t$  is initially set to  $x_0 = 1.5 < 2 = c$  and debt is set to  $F_0 = 3.5$ . We assume that all of the subsequent shocks are zero ( $\varepsilon_t = \omega_t = 0$ ). Figure 6 plots cash flow  $x_t$ , debt outstanding  $F_t$ , the default indicator  $D_t$ , bond price  $p_t$ , rational beliefs beliefs  $\lambda_t^R$ , and backward-looking beliefs  $\lambda_t^B$ . We compare the model dynamics starting from a low initial value  $\lambda_0^B(Low) = 0.15$  and a high initial value  $\lambda_0^B(High) = 0.30$  of backward-looking component of beliefs. We separately plot these dynamics for each value of  $\theta \in \{0.25, 0.5, 0.75, 1\}$ .

When  $\theta = 0.25$  or 0.50, the firm defaults at time 3 when  $\lambda_0^B = \lambda_0^B (High)$  and at time 4 and  $\lambda_0^B = \lambda_0^B (Low)$ . Consistent with Proposition 1, more optimistic initial beliefs have the potential to delay default in the face of poor fundamental cash flows. This effect becomes stronger as  $\theta$  rises and beliefs become more backward-looking. Specifically, when  $\theta = 0.75$ , the firm defaults at time 3 when  $\lambda_0^B = \lambda_0^B (High)$  and at time 5 when  $\lambda_0^B = \lambda_0^B (Low)$ . Finally, when  $\theta = 1$ , the firm defaults at t = 3 when  $\lambda_0^B = \lambda_0^B (High)$ . However, when  $\lambda_0^B = \lambda_0^B (Low)$ , the firm is just able to skate by, narrowly averting default. Intuitively, when  $\theta = 1$  and  $\lambda_0^B = \lambda_0^B (Low)$ , bond prices state high for long enough that the firm is able to continue refinancing its debt until fundamentals rise back above c.

This calm before the storm phenomenon is consistent with recent findings on credit cycles. Specifically, when  $\theta \in (0, 1)$  so beliefs are neither fully forward-looking nor fully-backward looking, credit spreads will typically be low in the run-up to a default and but will jump up on the eve of a default. This behavior is consistent with the evidence in Krishnamurthy and Muir (2017) who examine the behavior of credit spreads around a large sample of financial crises in developed countries. The calm before the storm phenomenon helps make sense of what Gennaioli and Shleifer (2018) have dubbed the "quiet period" of the 2008 global financial crisis—the stretch of time between the initial disruptions in housing and credit markets in the summer of 2007 and onset of a full-blown financial crisis in the fall of 2008 with the collapse of Lehman Brothers. Indeed, as Gennaioli and Shleifer (2018) argue, if investors were fully forward-looking ( $\theta = 0$ ), one might have have expected a more rapid deterioration of financial conditions in late 2007 rather than the gradual slide into crisis that was witnessed.

#### 3.2.3 The "default spiral" phenomenon

Once the storm hits the credit market, default extrapolation can generate a "default spiral": extrapolative, backward-looking beliefs lead to a form of default persistence that is absent when beliefs are fully rational and forward-looking. Specifically, investor beliefs typically become more pessimistic following a default according to equation (10). This pushes down bond prices, raising debt outstanding, and increasing the likelihood of future defaults. In particular, persistent default spirals can arise even when fundamental cash flows are strong  $(x_t > c)$  if (i)  $\theta$  is sufficiently large, (ii) the increment  $\alpha$  is large relative to the decay rate of extrapolative beliefs  $(1 - \beta)$ , (iii) the initial debt level is sufficiently high, and (iv) the initial backward-looking component of beliefs is sufficiently pessimistic. We formalize this observation in the following proposition.

**Proposition 2 Default spirals.** Assume that (i)  $F_{t-1} + c - x_t \ge \overline{F}$ , so there is a default at time t  $(D_t = 1)$ ; (ii)  $\alpha > (1 - \beta)$  and  $\omega_t = 0$ , so extrapolative beliefs necessarily become more pessimistic following this default; (iii) that extrapolative beliefs are initially relatively pessimistic  $(\lambda_{t-1}^B \ge \lambda_{t-1}^R)$ ; and (iv) that  $x_t = x_{t-1} = x > c$ . Let  $p_t(\theta)$ ,  $F_t(\theta)$ , and  $\lambda_t^R(\theta)$  denote the time t price, amount of outstanding debt, and true probability of default when fraction  $\theta$  of beliefs are backward looking. Although default leads to a reduction in debt—i.e.,  $F_t(\theta) < F_{t-1}$  for any  $\theta$ ,  $p_t(\theta)$  is decreasing in  $\theta$ . And  $F_t(\theta)$  and  $\lambda_t^R(\theta)$  are increasing in  $\theta$ . Thus, a larger extrapolative component of beliefs lowers prices and slows the process of debt discharge in the event of default, increasing the likelihood of a future default.

Proposition 2 says that the credit cycle can experience a persistent run of defaults due to a negative feedback loop induced by default extrapolation. Said differently, the backward-looking nature of investor beliefs may make the financial recovery from a crises slower and more protracted than in a world with fully forward-looking investors. This result further highlights the potential disconnect between the endogenous credit cycle and the exogenous business cycle that is at the heart of our model.

Figure 7 illustrates these ideas. We assume use our baseline parameters from above. We consider a sample path starting from  $F_0 = 6$  and  $x_0 = 2.25$ . We assume that  $\rho = 1$  and set  $\varepsilon_t = \omega_t = 0$  for all t > 0, so cash flow fundamentals remain constant at  $x_t = 2.25 > 2 = c$ . Since  $F_0 + c - x_1 = 5.75 > 5 = \overline{F}$  there is a default at t = 1. Indeed, since  $x_t$  is constant at 2.25 a default occurs at time t whenever  $F_{t-1}$  rises above 5.25.

We show the dynamics for different values of  $\theta$  and  $\lambda_0^B$  to trace out the roles of backwardlooking beliefs ( $\theta$ ) and initial beliefs ( $\lambda_0^B$ ) on the resulting dynamics. Specifically, we show the dynamics for  $\theta \in \{0.1, 0.5, 0.9, 1\}$  and for  $\lambda_0^B \in \{0.50, 0.95\}$ .

When investors beliefs are largely forward-looking (i.e., when  $\theta = 0.1$ ), the debt writedown that occurs upon default at t = 1 leads to an immediate decrease in the rationally-expected default rate  $\lambda_t^R$ . In this case, bond prices immediately recover following the default at t = 1, the firm rapidly repays its outstanding debt (since  $x_t = 2.25 > 2 = c$  for all t here) with debt quickly reaching the lower bound of  $\underline{F} = 1.5$ , and default never occurs again after t = 2.

By contrast, if investor beliefs are highly backward-looking ( $\theta = 0.9$  or  $\theta = 1$ ), then default extrapolation keeps bond prices low and the debt level high for many periods. As a result, there is a lengthy sequence of recurring default when investors are initially highly pessimistic ( $\lambda_0^B = 0.95$ ). And, recurring defaults even arise when investors are only moderately pessimistic at t = 0 ( $\lambda_0^B = 0.5$ ).

In between these two polar cases, when  $\theta = 0.5$ , the economy experiences a recurring wave of defaults when investors are initially very pessimistic, but not when investors are moderately pessimistic.

This default spiral dynamic suggests that a moderate improvement in cash flows can be insufficient to "rescue" credit markets from a depressed state. Moreover, the likely timing of the recovery is influenced by the extent of backward-looking extrapolation ( $\theta$ ) and the initial pessimism of investor beliefs ( $\lambda_0^B$ ): one needs a large improvement in cash flows to ensure a recovery when both  $\theta$  and  $\lambda_0^B$  are high. The crucial role that investor beliefs play in driving default spirals suggests that a favorable sentiment shock (i.e., a large negative draw of  $\omega_t$ ) coming from a policy intervention may also be an effective way to help the credit market recover.

#### 3.2.4 Bond return predictability

In this subsection, we examine the model's implications for bond return predictability. Naturally, the returns on bonds in our model are predictable whenever  $\theta > 0$ —i.e., whenever beliefs are partially extrapolative and backward-looking. However, what is most interesting is that changes in investor sentiment—i.e., movements in  $\lambda_t^B$ —have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets. Specifically, holding fixed expected future debt repayments, more bearish investor sentiment (higher values of  $\lambda_t^B$ ) lowers bond prices, thereby raising expected bond returns. However, since investor beliefs about defaults are partially self-

fulfilling there is a competing effect: more bearish investor sentiment makes it more difficult for firms to refinance maturing debt, raising the true probability of default. And, in highly reflexive states where beliefs have a large impact on the true likelihood of default—i.e., where  $\partial \lambda_t^R / \partial \lambda_t^B$  is large—the latter effect can outweigh the former. As a result, more bearish sentiment can actually reduce expected returns in highly reflexive states.

To see the model's implications for bond returns, note that investor who buy bonds for a price of  $p_t = p(\lambda_t^B, \lambda_t^R)$  at time t will receive a payment of  $1 - (1 - \eta)D_{t+1}$  at time t + 1. Thus, the realized return on bonds from time t to t + 1 is

$$r_{t+1} = \frac{1 - (1 - \eta)D_{t+1}}{p(\lambda_t^B, \lambda_t^R)} - 1.$$
(13)

At any time t, the risk-neutral investors in our model believe that  $\mathbb{E}_t^C[D_{t+1}] = \lambda_t^C$  and bond prices are  $p(\lambda_t^B, \lambda_t^R) = 1 - (1 - \eta)\mathbb{E}_t^C[D_{t+1}]$ . Thus, by construction, investors always perceive a zero expected return from time t to t + 1—i.e., we have  $\mathbb{E}_t^C[r_{t+1}] = 0$ . However, since  $\mathbb{E}_t^R[D_{t+1}] = \lambda_t^R$ from the vantage point of a rational econometrician, the true rationally-expected return is

$$\mathbb{E}_{t}^{R}[r_{t+1}] = \frac{1 - (1 - \eta)\lambda_{t}^{R}}{p(\lambda_{t}^{B}, \lambda_{t}^{R})} - 1 = \frac{-(1 - \eta)\theta(\lambda_{t}^{R} - \lambda_{t}^{B})}{1 - (1 - \eta)\lambda_{t}^{R} + (1 - \eta)\theta(\lambda_{t}^{R} - \lambda_{t}^{B})},$$
(14)

which is decreasing in  $\lambda_t^R - \lambda_t^B$  whenever  $\theta > 0$ . For instance, in a "calm before the storm" scenario where cash flow fundamentals have deteriorated but our partially-extrapolative investors remain bullish because they haven't witnessed a default in a long time, we will have  $(\lambda_t^R - \lambda_t^B) > 0$  and  $\mathbb{E}_t^R[r_{t+1}] < 0$ . Conversely, in a "default spiral" scenario it is likely that investors will be over-estimating the likelihood of future defaults since they have just experienced a default, we will have  $(\lambda_t^R - \lambda_t^B) < 0$  and  $\mathbb{E}_t^R[r_{t+1}] > 0$ .

More generally, using equation (14) and Lemma 1, we can ask how small changes in  $F_{t-1}$  and  $x_t$  impact expected bond returns. When  $\theta > 0$ , an increase in  $\lambda_t^R$  is associated with a decline in expected returns holding fixed  $\lambda_t^B$ :

$$\frac{\partial \mathbb{E}_{t}^{R}\left[r_{t+1}\right]}{\partial \lambda_{t}^{R}} = -\frac{\theta\left(1-\eta\right)\left(1-\lambda_{t}^{B}\left(1-\eta\right)\right)}{\left[p(\lambda_{t}^{B},\lambda_{t}^{R})\right]^{2}} < 0.$$

$$(15)$$

Thus, assuming we are not at the default boundary, Lemma 1 implies that a small increase in  $F_{t-1}$  leads to decline in  $\mathbb{E}_t^R[r_{t+1}]$  and a small increase in  $x_t$  leads to an increase in  $\mathbb{E}_t^R[r_{t+1}]$ . Intuitively, when investors are extrapolative, holding fixed the extrapolative component of beliefs  $\lambda_t^B$ , worse cash flow fundamentals and higher levels of leverage predict lower future bond returns, all else equal.

We now turn to the role of the extrapolative component of beliefs  $\lambda_t^B$ . Surprisingly, a small increase in  $\lambda_t^B$  has an ambiguous impact on  $\mathbb{E}_t^R[r_{t+1}]$ . Holding fixed expected future debt repay-

ments, increasing  $\lambda_t^B$  lowers bond prices which raises the expected return on bonds. This is the familiar intuition that, all else equal, one wants to buy securities when investor sentiment turns bearish. However, there is a potentially offsetting effect here due to credit market reflexivity: movements in extrapolative beliefs impact the true probability of a default at time t + 1. Specifically, we know from Lemma 1 that a small increase in  $\lambda_t^B$  raises  $\lambda_t^R$ , lowering expected future debt repayments. As a result, the total impact is ambiguous: depending on which effect dominates, a small increase in  $\lambda_t^B$  can either lead  $\mathbb{E}_t^R [r_{t+1}]$  to rise or fall.

For instance, assuming we are at an equilibrium continuity point where  $\partial \lambda_t^R / \partial \lambda_t^B$  exists,<sup>9</sup> we have

$$\frac{\partial \mathbb{E}_{t}^{R}[r_{t+1}]}{\partial \lambda_{t}^{B}} = \frac{\partial \mathbb{E}_{t}^{R}[r_{t+1}]}{\partial \lambda_{t}^{B}} |_{\lambda_{t}^{R}=\text{Constant}} + \frac{\partial \mathbb{E}_{t}^{R}[r_{t+1}]}{\partial \lambda_{t}^{R}} \times \frac{\partial \lambda_{t}^{R}}{\partial \lambda_{t}^{B}} = \frac{\theta \left(1-\eta\right)}{\left[p(\lambda_{t}^{B},\lambda_{t}^{R})\right]^{2}} \left[ \left(1-\lambda_{t}^{R}\left(1-\eta\right)\right) - \left(1-\lambda_{t}^{B}\left(1-\eta\right)\right) \frac{\partial \lambda_{t}^{R}}{\partial \lambda_{t}^{B}} \right].$$
(16)

The sign of  $\partial \mathbb{E}_t^R[r_{t+1}] / \partial \lambda_t^B$  is ambiguous. Specifically, we have  $\partial \mathbb{E}_t^R[r_{t+1}] / \partial \lambda_t^B \leq 0$  as  $\partial \lambda_t^R / \partial \lambda_t^B \geq [1 - \lambda_t^R(1 - \eta)] / [1 - \lambda_t^B(1 - \eta)]$ . In other words, we are more likely to have  $\partial \mathbb{E}_t^R[r_{t+1}] / \partial \lambda_t^B < 0$  when the economy is in a highly reflexive state where current beliefs have a large impact on future outcomes—i.e., where  $\partial \lambda_t^R / \partial \lambda_t^B$  is large.

Figure 8 plots  $\mathbb{E}_{t}^{R}[r_{t+1}]$  and  $\lambda_{t}^{R}$  versus  $\lambda_{t}^{B}$  using our baseline parameter values for  $(x_{t}, F_{t-1}) = (1.6, 3.4)$ , which is a highly reflexive state. For  $\lambda_{t}^{B}$  less than 0.26,  $\lambda_{t}^{R}$  rises gradually with  $\lambda_{t}^{B}$  and  $\mathbb{E}_{t}^{R}[r_{t+1}]$  is increasing as the negative effect of  $\lambda_{t}^{B}$  on price outweighs the positive effect on  $\lambda_{t}^{R}$ . For  $\lambda_{t}^{B}$  between 0.26 and 0.32,  $\lambda_{t}^{R}$  rises more rapidly with  $\lambda_{t}^{B}$  and  $\mathbb{E}_{t}^{R}[r_{t+1}]$  is decreasing as the negative effect on price. At  $\lambda_{t}^{B} = 0.33$ ,  $\lambda_{t}^{R}$  jumps discretely up—the low default probability equilibrium disappears—and expected returns fall significantly. Returns continue falling until  $\lambda_{t}^{B}$  reaches 0.35 after which they are again increasing.

We collect these observations in Proposition 3.

**Proposition 3 Return predictability.** If investor beliefs are fully rational, then bond returns cannot be predicted. Formally, if  $\theta = 0$ , then  $\mathbb{E}_t^R[r_{t+1}] = 0$ .

If investor beliefs are partially extrapolative, then bond returns are predictable. Specifically, when  $\theta > 0$ ,  $\mathbb{E}_t^R[r_{t+1}]$  is decreasing in  $\lambda_t^R - \lambda_t^B$  and is equal to zero when  $\lambda_t^R - \lambda_t^B = 0$ .

• If the economy is not at the default boundary at time t, then, all else equal,  $\mathbb{E}_t^R[r_{t+1}]$  is increasing in  $x_t$  and is decreasing in  $F_{t-1}$ . However, these relationships flip signs when the economy is at the default boundary.

<sup>&</sup>lt;sup>9</sup>As explained in Lemma 1, the derivative  $\partial \lambda_t^R / \partial \lambda_t^B > 0$  only exists at equilibrium continuity points. At equilibrium discontinuity points,  $\lambda_t^R$  jumps up discretely in response to a small increase in  $\lambda_t^B$ .

- A small increase in  $\lambda_t^B$  has an ambiguous effect on  $\mathbb{E}_t^R[r_{t+1}]$ :
  - In non-reflexive states—where a small increase in  $\lambda_t^B$  has a small effect on  $\lambda_t^R$ —a small increase in  $\lambda_t^B$  leads to an increase in  $\mathbb{E}_t^R [r_{t+1}]$ .
  - In highly reflexive states—where a small increase in  $\lambda_t^B$  has a large effect on  $\lambda_t^R$ —a small increase in  $\lambda_t^B$  leads to a decline in  $\mathbb{E}_t^R[r_{t+1}]$ .

Finally, we can use Proposition 3 to provide a heuristic understanding of the relationship between past changes in debt outstanding  $\Delta F_t = F_t - F_{t-1}$  and future expected returns  $\mathbb{E}_t^R[r_{t+1}]$ in the model. Assuming that  $\underline{F} < F_{t-1} + c - x_t < \overline{F}$ , the change is debt outstanding at time t is

$$\Delta F_t = \frac{F_{t-1} + c - x_t}{p_t} - F_{t-1} = \frac{c - x_t}{p_t} + \frac{1 - p_t}{p_t} F_{t-1}.$$

Using Lemma 2, it is straightforward to show that  $\partial \Delta F_t / \partial x_t < 0$ ,  $\partial \Delta F_t / \partial F_{t-1} > 0$ , and  $\partial \Delta F_t / \partial \lambda_t^B > 0$ . Combing these results with those in Proposition 3, one would expect large values of  $\Delta F_t$  to predict low future values of  $r_{t+1}$  in data simulated from the model. Specifically, changes in  $x_t$  and  $F_{t-1}$  always have opposing effects on  $\mathbb{E}_t^R [r_{t+1}]$  and  $\Delta F_t$ . And, changes in  $\lambda_t^B$  will have opposing effects on  $\Delta F_t$  and  $\mathbb{E}_t^R [r_{t+1}]$  in reflexive states where  $\partial \lambda_t^R / \partial \lambda_t^B$  is large. Thus,  $\mathbb{E}_t^R [r_{t+1}]$  and  $\Delta F_t$  will generally move in opposite directions, leading to a negative relationship between  $\Delta F_t$  and  $\mathbb{E}_t^R [r_{t+1}]$  in data simulated from the model.

Forecasting returns and defaults in model simulations. To further explore the model's implications on return and default predictability, we first simulate the model for 100,000 periods.<sup>10</sup> We then examine the return and default forecasting regressions using current variables such as credit growth and sentiment.

Table 2 shows that the model is able to match a number of facts that researchers have documented about the credit cycle. First, credit growth forecasts future defaults: regressing defaults  $(D_{t+1})$  over the next year on the debt growth over the prior four years  $(F_t - F_{t-4})$ , the model produces a coefficient of 0.11 with an *R*-squared of 22%. Furthermore, credit growth forecasts defaults particularly when the credit cycle and the business cycle become disconnected. Regressing defaults over the next year  $(D_{t+1})$  on both prior debt growth  $(F_t - F_{t-4})$  and the current level of cash flows  $(x_t)$ , the model generates coefficients of 0.07 and -0.14, respectively, with an *R*squared of 31%. Notably, compared to the univariate regression, the multivariate regression leads to a much higher *R*-squared. In our model, the "calm before the storm" period is accompanied by growing credit and low growth in fundamentals. These conditions move the economy towards default.

 $<sup>^{10}</sup>$ We choose the parameter values such that default occurs infrequently in the model; in this example, defaults occurs about 5% of the time.

Second, credit growth forecasts future bond returns: regressing returns over the next year  $(r_{t+1})$  on the debt growth over the prior four years  $(F_t - F_{t-4})$ , the model generates a coefficient of -0.04 with an *R*-squared of 33%. In addition, credit growth predicts bond returns better when credit growth becomes disconnected with growth in fundamentals. Regressing returns over the next year  $(r_{t+1})$  on both prior debt growth  $(F_t - F_{t-4})$  and the current level of cash flows  $(x_t)$ , the model generates coefficients of -0.04 and 0.01, respectively, with an *R*-squared of 34%.

Third, sentiment  $\lambda_t^R - \lambda_t^B$  strongly predicts future returns and future defaults. Regressing bond returns over the next year on the current level of sentiment, the model generates a coefficient of -0.34 with an *R*-squared of 82%. Regressing next year defaults on the current level of sentiment, the coefficient is 0.90, with an *R*-squared of 52%. Towards the end of the "calm before the storm" period, sentiment rises because true default likelihood increases while the backward-looking extrapolative expectations of default remain low. This is when defaults are about to occur.

Fourth, since  $\mathbb{E}_t^R[r_{t+1}] \approx -\text{Constant} \times (1 - \eta)\theta(\lambda_t^R - \lambda_t^B)$ ,  $(\lambda_t^R - \lambda_t^B)$  is a strong univariate predictor of returns: a regression of  $r_{t+1}$  on  $(\lambda_t^R - \lambda_t^B)$  delivers a coefficient of -0.33 with a R-square of 82%. However,  $\lambda_t^B$  is a not a strong univariate predictor of future returns. This owes the fact that  $\lambda_t^R$  moves nearly one-for-one with  $\lambda_t^B$ —the coefficient from a regression of  $\lambda_t^R$  on  $\lambda_t^B$ is 0.86—which reflects both the reasonableness of the backward-looking component of beliefs and the reflexivity effect  $(\partial \lambda_t^R / \partial \lambda_t^B > 0)$  at the heart of our model. As a result, there is not a strong univariate relationship between  $\mathbb{E}_t^R[r_{t+1}] \approx -\text{Constant} \times (1 - \eta)\theta(\lambda_t^R - \lambda_t^B)$  and  $\lambda_t^B$ .

Finally, the debt level in the model is more persistent than cash flows. The one-year autocorrelation of debt level is 0.89, whereas the one-year autocorrelation of cash flows is 0.80. This result again confirms that our model allows for periods during which the credit cycle and the business cycle are disconnected.

Model-implied impulse-response functions. Given the highly non-linear nature of model, impulse-response functions (IRFs) are potentially (i) highly asymmetric and (ii) highly dependent on initial conditions. Figure 9 shows the impulse-response functions for a shock to cash flows  $x_t$ (an  $\varepsilon_t$  shock) and for a shock to beliefs  $\lambda_t^B$  (an  $\omega_t$  shock). To compute these IRFs, we shock  $\varepsilon_t$ and  $\omega_t$  up or down at t = 1 and then generate 10,000 random paths following this shock. We then plot the average paths following each shock.<sup>11</sup>

The initial condition in Figure 9 is  $x_t = 2.25$ ,  $F_{t-1} = 2$ , and  $\lambda_t^B = 0.25$ . As shown below, the impulse-responses are asymmetric. Bad shocks to either fundamentals (a downward shock to cash

<sup>&</sup>lt;sup>11</sup>Let  $\mathbf{z}_t = (x_t, F_{t-1}, \lambda_t^B)$  denote the model's state vector and consider some model quantity  $y_t$ . The response of  $y_{t+j}$  following an impulse  $\varepsilon_t = s_{\varepsilon}$  to cash flow fundamentals is  $\Phi_y(j, \mathbf{z}_{t-1}, \varepsilon_t = s_{\varepsilon}) = \mathbb{E}[y_{t+j}|\mathbf{z}_{t-1}, \varepsilon_t = s_{\varepsilon}] - \mathbb{E}[y_{t+j}|\mathbf{z}_{t-1}, \varepsilon_t = 0]$ . Similarly, the response of  $y_{t+j}$  following an impulse  $\omega_t = s_{\omega}$  to investor sentiment is  $\Phi_y(j, \mathbf{z}_{t-1}, \omega_t = s_{\omega}) = \mathbb{E}[y_{t+j}|\mathbf{z}_{t-1}, \omega_t = s_{\omega}] - \mathbb{E}[y_{t+j}|\mathbf{z}_{t-1}, \omega_t = 0]$ . Due to the nonlinear nature of the model, these impulse response functions are asymmetric in the sense that, for example,  $\Phi_y(j, \mathbf{z}_{t-1}, \omega_t = -s_{\omega}) \neq -\Phi_y(j, \mathbf{z}_{t-1}, \omega_t = s_{\omega})$ . The impulse response functions are also state-contingent in the sense that, for example,  $\Phi_y(j, \mathbf{z}_{t-1}, \omega_t = s_{\omega})$  depends on the initial condition  $\mathbf{z}_{t-1}$ .

flows) or investors beliefs (an upward shock to the perceived default likelihood) have larger and more persistent effects on credit market outcomes. The saw-tooth patterns following shocks to sentiment arise, even in expectation, because of the jaggedness of debt outstanding in individual sample paths due to our mechanistic default rule.

Figure 10 shows the same impulse-response functions starting from an initial condition of  $x_t = 1.6$ ,  $F_{t-1} = 3.4$ , and  $\lambda_t^B = 0.2$ —which is a highly reflexive region of the parameter space. Consider first an impulse to cash flows  $x_t$  at time 1—i.e., an unexpected jump in  $\varepsilon_1$ . In this region, a positive shock to fundamentals at time 1 typically helps averts what would otherwise be a near inevitable default at time 2. However, since the firm is already in dire straits a negative shock has a far less pivotal effect on the future path.

Next consider an unexpected shocks to beliefs at time 1—i.e., a large draw of  $\omega_1$ . In this reflexive region, a similarly sized shock to beliefs has far larger impact on debt accumulation and defaults than in the less reflexive region shown in Figure 9. However, now the asymmetry between positive and negative belief shocks is generally smaller than it is in the non-reflexive region as shown in Figure 10.

### 4 Extension with multiple firms

In this section, we consider an extension of the model that incorporates multiple firms who face idiosyncratic cash flow shocks and study its implications in brief. This extension addresses a limitation of baseline model which is that defaults are binary events since the baseline economy has a single representative firm. The one additional assumption of this extension is that investors price all firms' bonds identically even though firms have heterogeneous cash flows. We also examine belief contagion, the notion that investors may update their beliefs about future defaults if *any* firm in the economy defaults (in the real world, downgrades might serve a similar purpose).

We assume that there are M firms, i = 1, 2, ..., M; we will focus on the limiting case when M grows large. Furthermore, we assume that the cash flow of firm  $i, x_{it}$ , consists of two components:

$$x_{it} = x_t + z_{it},\tag{17}$$

where the systematic component  $x_t$  evolves according to equation (2) and the mean-zero, firmspecific component  $z_{it}$  follows

$$z_{it} = \psi \cdot z_{it-1} + \xi_{it},\tag{18}$$

where  $\xi_{it} \sim \mathcal{N}(0, \sigma_{\xi}^2)$  is *i.i.d.* over time, independent across firms, and independent of the systematic cash flow shock  $(\varepsilon_t)$  and the sentiment shock  $(\omega_t)$ .

Although firms are heterogeneous, we assume that investors pay the price the debt of all firms. The idea is that investors cannot perfectly observe each firm's cash flow  $x_{it}$  and leverage  $F_{it-1}$  and treat some class of firms as a homogeneous category. The rule under which each firm defaults is similar to the rule in the base case model: if at any time t,  $F_{it-1} + c - x_{it}$  goes above a threshold of  $\overline{F}$ , then firm *i* defaults. Thus, the law of motion for each firms outstanding bonds  $F_{it}$  is similar to the baseline model. Specifically, we have

$$F_{it} = f\left(F_{it-1}, \lambda_t^B, \lambda_t^R, x_{it}\right) = \begin{cases} \frac{F}{p(\lambda_t^B, \lambda_t^R)} & \text{if } F_{it-1} + c - x_{it} \leq \underline{F} \\ (F_{it-1} + c - x_t)/p(\lambda_t^B, \lambda_t^R) & \text{if } \underline{F} < F_{it-1} + c - x_{it} < \overline{F} \\ \eta(F_{it-1} + c - x_t)/p(\lambda_t^B, \lambda_t^R) & \text{if } F_{it-1} + c - x_{it} \geq \overline{F} \end{cases}$$
(19)

where

$$p(\lambda_t^B, \lambda_t^R) = \left[1 - (1 - \eta)\lambda_t^R\right] + (1 - \eta)\theta(\lambda_t^R - \lambda_t^B).$$
(20)

With multiple firms, the beliefs  $\lambda_t^B$  and  $\lambda_t^R$  are specified as follows. Suppose at time t, after the realization of  $\{x_{it}\}_{i=1}^M$ ,  $M_t$  out of M firms default—these firms have outstanding debt that  $F_{it-1} + c - x_{it} \ge \bar{F}$ —then

$$\lambda_t^B = \max\left\{0, \min\left\{1, \beta \lambda_{t-1}^B + \alpha D_t + \omega_t\right\}\right\},\tag{21}$$

where  $D_t \equiv M_t/M$  is the realized default rate and

$$\lambda_{t}^{R} = g(\lambda_{t}^{R}|\{F_{it-1}\}_{i=1}^{M}, \lambda_{t}^{B}, x_{t}, \{z_{it}\}_{i=1}^{M})$$

$$\equiv \frac{1}{M} \sum_{i=1}^{M} \Phi\left(\frac{f(F_{it-1}, \lambda_{t}^{B}, \lambda_{t}^{R}, x_{it}) - \bar{F} + c - \rho x_{t} - (1-\rho)\bar{x} - \psi z_{it}}{\sqrt{\sigma_{\varepsilon}^{2} + \sigma_{\xi}^{2}}}\right).$$
(22)

Equation (22) defines  $\lambda_t^R$  as the expected default rate averaged across firms.

Here we make three observations. First, equations (19) to (22) imply that the right hand side of (22) can be viewed as a continuous function of  $\lambda_t^R$ , just as in the base case model. More important, the distribution of  $\{F_{it-1}\}_{i=1}^M$ , the distribution of  $\{z_{it}\}_{i=1}^M$ , and the level of  $x_t$  jointly determine  $\lambda_t^R$ . Second, both (21) and (22) reflect belief contagion: past defaults and future defaults of each firm affect the bond price that applies to all firms. Third, one aspect of having firms with different debt levels is that, at each point in time, only a fraction of firms is close to default. Thus, many firms will not default immediately. This makes it more difficult for the multiple equilibria described in Section 3 to arise; rational investors do not anticipate a high  $\lambda_t^R$  when a significant fraction of firms have their debt level far below the default barrier.

This extended model yields similar implications compared to the base case model, but with more realistic dynamics for firm defaults. Figure 11 reports a sample path of the model with M = 100 firms. As a comparison, we also plot the sample path using the same cash flow shocks and sentiment shocks but with a single representative firm. In this example, after staying above the long-run mean  $\bar{x}$  for many periods, cash flow fundamentals  $x_t$  begin to deteriorate in period 32. The actual default rate stays lows for three more periods—a "calm before the storm" period and then starts to rise in period 34. Furthermore, there is a clear lead-lag structure between the rational and the behavioral components of investor beliefs:  $\lambda_t^R$  responds to deteriorating market fundamentals in period 34 while  $\lambda_t^B$  only responds several periods later. Similarly,  $\lambda_t^R$  responds to improving market fundamentals in period 41 while  $\lambda_t^B$  stays high for several more periods. Overall, the presence of multiple firms makes the rational and the behavioral components of investor beliefs more synchronized: in this example, the time-series correlation between  $\lambda_t^R$  and  $\lambda_t^B$  increases from 30% in the single firm case (Panel A) to 69% in the multiple firm case (Panel B).

### 5 Conclusion

We present a model of credit market sentiment in which investors extrapolate past defaults. Our key contribution is to model reflexivity in credit markets, an endogenous two-way feedback between credit market sentiment and credit market outcomes. This feedback mechanism is unique to credit markets because firms must return to the market to refinance maturing debts and the terms on which debt is refinanced will impact the likelihood of future default.

This combination of extrapolative beliefs and reflexive dynamics can lead to large disconnects between cash flow fundamentals and credit market outcomes, including "calm before the storm" and "default spiral" episodes. Extrapolative beliefs naturally lead to bond return predictability. But what is most striking here is that changes in investor sentiment can have an ambiguous impact on expected bond returns due to the reflexive nature of credit markets.

Our analysis leaves open at least two areas for further analysis. First, we have not allowed conditions in credit markets to explicitly affect the underlying cash flow fundamentals of the economy. The relationship between credit market conditions and macroeconomic fundamentals plays a major role in Austrian accounts of credit cycles: as the credit boom grows, increasing amounts of capital are devoted to poor quality projects. Relatedly, as demonstrated by a growing macro-finance literature, the inability to access credit on reasonable terms following a credit market bust may exacerbate an incipient economic downturn. Incorporating these features into our model would likely further strengthen the feedback loop between investor sentiment and credit market outcomes.

Second, we have been silent on issues of welfare and optimal policy, even though our model suggests a potential role. During credit booms, high sentiment can prevent defaults from occurring in the near future, which can be welfare-improving if fundamentals recover soon enough. Nonetheless, self-fulfilling beliefs during default spirals can be welfare-reducing, both because these deteriorating beliefs accelerate future default realizations and because they lead to a slow recovery in the presence of improving fundamentals. Accepting this at face value suggests a role for policy in moderating investor beliefs.

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### A Proofs

**Proof of Lemma 1:** Since  $\beta < 1$ ,  $\lambda_t^B$  weakly declines if there is no default at time t and the decline is strict if  $\lambda_{t-1}^B > 0$ .

How do extrapolative beliefs typically react to a default at time *t*—i.e., if  $D_t = 1$  and  $\omega_t = 0$ ? In this case,  $\lambda_t^B = \min \{1, \beta \lambda_{t-1}^B + \alpha\} > 0$ . If  $\alpha \ge (1 - \beta)$ ,  $\lambda_t^B$  weakly increases following a default and the increase is strict if  $\lambda_{t-1}^B < 1$ . Specifically, if  $\lambda_t^B < 1$ , then  $\lambda_t^B - \lambda_{t-1}^B = \alpha - (1 - \beta) \lambda_{t-1}^B > 0$  for all  $\lambda_{t-1}^B \in [0, 1)$  since  $\alpha \ge (1 - \beta)$ . By contrast, if  $\lambda_t^B$  then we trivially have  $\lambda_t^B - \lambda_{t-1}^B > 0$  for all  $\lambda_{t-1}^B \in [0, 1)$ . Thus, if  $\alpha \ge (1 - \beta)$ , extrapolative beliefs will converge to  $\lambda_t^B = 1$  following a long sequence of defaults.

By contrast, if  $\alpha < (1 - \beta)$ , extrapolative beliefs will not always become more pessimistic following a default. Specifically, if  $D_t = 1$  and  $\omega_t = 0$ , then we have  $\lambda_t^B \ge \lambda_{t-1}^B$  as  $\lambda_{t-1}^B \le \alpha/(1 - \beta)$ and extrapolative beliefs will converge to  $\lambda_t^B = \alpha/(1 - \beta) < 1$  following a long sequence of defaults.

**Proof of Lemma 2:** First, assume that the economy is not near the default boundary  $F_{t-1} + c - x_t = \overline{F}$ , so small changes in  $F_{t-1}$  and  $x_t$  do not affect whether there is a default or the firm pays dividends at time t. Suppose that we are at an equilibrium continuity point where the smallest solution to  $\lambda_t^R = g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$  is a continuous and differentiable function of  $F_{t-1}, \lambda_t^B, x_t$ .  $(g(\lambda_t^R | F_{t-1}, \lambda_t^B, x_t)$  is continuous, but not differentiable in  $F_{t-1}$  at the dividend payout boundary  $\underline{F} = F_{t-1} + c - x_t$ ). At such an continuity point, for any  $z_t \in \{F_{t-1}, \lambda_t^B, x_t\}$ , we have  $\lambda_t^R / \partial z_t = [\partial g(\cdot) / \partial z_t] / [1 - \partial g(\cdot) / \partial \lambda_t^R]$ . At a stable equilibrium we have  $\partial g(\cdot) / \partial \lambda_t^R < 1$ , so this has the same sign as  $\partial g(\cdot) / \partial z_t$ . This argument shows that  $\partial \lambda_t^R / \partial F_{t-1} > 0$ ,  $\partial \lambda_t^R / \partial \lambda_t^B > 0$ , and  $\partial \lambda_t^R / \partial x_t < 0$ . There are also equilibrium discontinuity points where the number of solutions to the fixed-point problem changes and the smallest solution discretely jumps. Although  $\lambda_t^R$  is not a continuous function of  $F_{t-1}$ ,  $\lambda_t^B$ ,  $x_t$  at these equilibrium discontinuity points, the signs of discrete jumps in  $\lambda_t^R$  at these points will have the same sign as the partial derivatives at equilibrium continuity points. For instance, an small increase in  $x_t$  shifts the  $g(\lambda_t^R | \cdot)$  function down for all  $\lambda_t^R$ . At an equilibrium continuity point where the relevant partial derivative is well-defined, this results in a small decline in  $\lambda_t^R$ . At an equilibrium discontinuity point where the relevant partial derivative is not well-defined, this results in a discrete downward jump in  $\lambda_t^R$ .

Second, assume that we are near the default boundary. At the default boundary the derivatives with respect to  $F_{t-1}$  and  $x_t$  are undefined. Near the default boundary, a small increase in  $F_{t-1}$  can trigger a default at time t, resulting in a discrete downward jump in  $\lambda_t^R$ . Similarly, a small increase in  $x_t$  can avert a default at time t, resulting in a discrete upward jump in  $\lambda_t^R$ .

**Proof of Proposition 1 (Calm Before the Storm):** We compare two sample paths, denoted L and H, that differ only in their initial levels of  $\lambda_t^B$ . Specifically, suppose that  $\lambda_t^B(L) < \lambda_t^B(H)$ . Because shocks to cash flows and sentiment are exogenous, we have have  $x_{t+j}(L) = x_{t+j}(H)$  and  $\omega_{t+j}(L) = \omega_{t+j}(H)$  for all  $j \ge 0$ . Because  $\lambda_t^R$  and  $F_t$  are always increasing in  $\lambda_t^B$ , we have  $\lambda_t^R(L) < \lambda_t^R(H)$  and  $F_t(L) < F_t(H)$ . Since  $F_t(L) < F_t(H)$ , if there is a default at time t+1 in the H path, then there is also a default at time t+1.

Assume that there is no default at time t + 1 along either the L or H paths. Then we have  $\lambda_{t+1}^B(L) \leq \lambda_{t+1}^B(H)$  by equation (10) and the equality is strict so long as  $0 < \lambda_{t+1}^B(H)$ . Since  $\lambda_{t+1}^R$  and  $F_{t+1}$  are increasing in  $\lambda_{t+1}^B$  and  $F_t$ , it also follows that  $\lambda_{t+1}^R(L) \leq \lambda_{t+1}^R(H)$  and  $F_{t+1}(L) \leq F_{t+1}(H)$  and these inequalities are strict when  $0 < \lambda_{t+1}^B(H)$ . Since  $F_{t+1}(L) \leq F_{t+1}(H)$ , if the first default occurs at time t + 2 in the H path, then first default also occurs at time t + 2 in the L path. However, we can have default in the H path, but not in the L path at t + 2.

Proceeding inductively in this fashion, we see that, so long as there is no default along either path by time t + j, we have  $\lambda_{t+j}^B(L) \leq \lambda_{t+j}^B(H)$  and  $F_{t+j}(L) \leq F_{t+j}(H)$  and these inequalities are strict when  $\lambda_{t+j}^B(H) > 0$ . Thus, lowering the default rate  $\lambda_t^B$  weakly delays the next future default stochastic path by stochastic path. And, averaging across these paths, lowering the default rate  $\lambda_t^B$  strictly delays the next default in expectation.

**Proof of Proposition 2 (Default Spiral):** Since  $p_t \ge \eta$ , if there is a default at time t (i.e.,  $D_t = 1$ ) we have  $F_t = \eta (F_{t-1} + c - x_t) / p_t \le F_{t-1} + c - x_t$ . Thus, if  $D_t = 1$  and  $x_t > c$ , we always have  $F_t < F_{t-1}$ . By contrast, if  $D_t = 1$  and  $x_t < c$ , we have  $F_t > F_{t-1}$  if  $(c - x_t) / (p_t/\eta - 1) > F_{t-1}$  and  $F_t < F_{t-1}$  if  $(c - x_t) / (p_t/\eta - 1) < F_{t-1}$ .

Next, note that

$$\lambda_t^R = \Phi\left(\frac{\frac{O_t \underline{F} + (1 - O_t)(1 - D_t(1 - \eta))(F_{t-1} + c - x_t)}{1 - (1 - \eta)\theta\lambda_t^B - (1 - \eta)(1 - \theta)\lambda_t^R} + c - \bar{F} - \rho x_t - (1 - \rho)\bar{x}}{\sigma_{\varepsilon}}\right)$$

where  $D_t = \mathbf{1}_{\{F_{t-1}+c-x_t \ge \bar{F}\}}$  and  $O_t = \mathbf{1}_{\{F_{t-1}+c-x_t \le \bar{F}\}}$ . Thus, we have

$$\frac{\partial \lambda_t^R}{\partial \theta} = \frac{\partial g\left(\lambda_t^R | \cdot\right) / \partial \theta}{1 - \partial g\left(\lambda_t^R | \cdot\right) / \partial \lambda_t^R} = \frac{\frac{O_t \underline{F} + (1 - O_t)(1 - D_t(1 - \eta))(F_{t-1} + c - x_t)}{\left[1 - (1 - \eta)\theta\lambda_t^B - (1 - \eta)(1 - \theta)\lambda_t^R\right]^2}}{1 - \partial g\left(\lambda_t^R | \cdot\right) / \partial \lambda_t^R} \frac{(1 - \eta)\left(\lambda_t^B - \lambda_t^R\right)}{\sigma_{\varepsilon}} \propto \left(\lambda_t^B - \lambda_t^R\right)$$

Thus,  $\lambda_t^R$  is increasing in  $\theta$  when  $\lambda_t^B - \lambda_t^R > 0$ .

We have assumed that (i)  $F_{t-1} + c - x_t \ge \overline{F}$ , so  $D_t = 1$ ; (ii)  $\alpha > (1 - \beta)$  and  $\omega_t = 0$ ; (iii)  $\lambda_{t-1}^B \ge \lambda_{t-1}^R$ ; and  $x_t = x_{t-1} = x > c$ . Since  $\alpha > (1 - \beta)$ ,  $D_t = 1$ , and  $\omega_t = 0$ , we have  $\lambda_t^B \ge \lambda_{t-1}^B$ . Since  $p_t(\theta) \ge \eta$  and  $x_t > c$  we have  $F_t(\theta) = \eta(F_{t-1} + c - x_t)/p_t(\theta) \le F_{t-1} + c - x_t < F_{t-1}$ . Thus, since  $x_t = x_{t-1}$ , we have

$$\lambda_t^R(\theta) = \Phi\left(\frac{F_t(\theta) - \bar{F} + c - \rho x_t - (1 - \rho)\bar{x}}{\sigma_{\varepsilon}}\right) < \Phi\left(\frac{F_{t-1} - \bar{F} + c - \rho x_{t-1} - (1 - \rho)\bar{x}}{\sigma_{\varepsilon}}\right) = \lambda_{t-1}^R$$

irrespective of the value of  $\theta \in [0, 1]$ . Thus, we have  $\lambda_t^B \ge \lambda_{t-1}^B \ge \lambda_{t-1}^R \ge \lambda_t^R(\theta)$  irrespective of  $\theta$ , so we have  $\partial \lambda_t^R(\theta) / \partial \theta > 0$  and

$$\frac{\partial \lambda_t^C(\theta)}{\partial \theta} = \left(\lambda_t^B - \lambda_t^R(\theta)\right) + (1 - \theta) \frac{\partial \lambda_t^R(\theta)}{\partial \theta} > 0.$$

In other words, both the rational component and the combined belief become more pessimistic as the fraction of backward-looking beliefs rises.

Since  $\partial \lambda_t^C(\theta) / \partial \theta > 0$ , it then follows that  $\partial p_t(\theta) / \partial \theta < 0$  and  $\partial F_t(\theta) / \partial \theta > 0$ . Thus, a larger extrapolative component of beliefs lowers prices and slows the process of debt discharge in the event of default, increasing the chances of subsequent defaults. Indeed, for any  $\theta > 0$ , we have  $\lambda_t^R(\theta) > \lambda_t^R(0)$ .

**Figure 1. The credit market cycle.** Panel A plots the year-over-year growth in real GDP and the year-over-year growth in real credit outstanding (defined as the sum of loans and bonds) to nonfinancial corporate businesses from the Federal Reserve's Financial Accounts of the United States. Panel B plots real year-over-year credit growth versus the corporate credit spread, measured as the yields on Moody's seasoned Baa corporate bond yield minus the 10-year constant maturity Treasury yields.



Panel A: Credit growth and GDP growth

Panel B: Corporate credit growth and credit spreads







**Figure 3. Real GDP growth and credit growth as a function of business cycle expansion quarter.** This figure plots real GDP growth and real credit growth—the growth in real nonfinancial corporate loans and bonds—as a function of NBER business cycle expansion quarter.



Panel A: Real GDP growth as a function of business cycle expansion quarter



Panel B: Real credit growth as a function of business cycle expansion quarter

**Figure 4. Multiple equilibria.** This figure illustrates the potential for multiple equilibria in our model when investors are partially forward-looking ( $\theta < 1$ ), plotting the  $g(\lambda_t^R | \cdot)$  function versus  $\lambda_t^R$  as we vary  $x_t$  and  $\sigma_{\varepsilon}$ . The figures use the following set of common parameters throughout:  $\overline{x} = 2.4$ ,  $\rho = 0.8$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\theta = 0.5$ ,  $F_{t-1} = 4$ , and  $\lambda_t^R = 0.33$ . The first panel shows the  $g(\lambda_t^R | \cdot)$  function when  $\sigma_{\varepsilon} = 0.5$ ; the second panel when  $\sigma_{\varepsilon} = 1.0$ ; and the third panel when  $\sigma_{\varepsilon} = 0.01$ . In each panel, we separately plot the  $g(\lambda_t^R | \cdot)$  function for  $x_t \in \{1.9, 2.0, 2.1, 2.5\}$ .









.3 .4 .5 .6 .7 .8 .9

.2

.2 .3 .4 .5 .6

.7



Figure 5. Simulated data using baseline parameter values (partially backward-looking beliefs). This figure shows a typical path of simulated data using our baseline set of parameter values in which beliefs are partially backward-looking and partially forward-looking ( $\theta = 0.5$ ). Specifically, the baseline parameters are  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ . We plot the evolution of cash flow ( $x_t$ ), debt outstanding ( $F_t$ ), the default indicator ( $D_t$ ), bond prices ( $p_t$ ), rational forward-looking beliefs about future defaults ( $\lambda_t^R$ ), and extrapolative backward-looking beliefs about future defaults ( $\lambda_t^R$ ).



**Figure 6. Calm before the storm.** This figure illustrates the calm before the storm phenomenon. The figure depicts sample paths of the model with cash flows initially set to  $x_0 = 1.5 < 2 = c$  and debt initially set to  $F_0 = 3.5$ . We compare the model dynamics starting from a low initial value of  $\lambda_0^B(L) = 0.15$  and a high initial value  $\lambda_0^B(H) = 0.30$ . We assume all subsequent shocks are zero ( $\varepsilon_t = \omega_t = 0$ ). We separately plot the dynamics for various values of  $\theta \in \{0.25, 0.5, 0.75, 1\}$ . Otherwise, the model parameters are the same as those in Figure 5. Specifically, we set  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ .



**Figure 7. Default spirals.** This figure illustrates the default spiral phenomenon. The figure depicts sample paths of the model with cash flows initially set to  $x_0 = 2.25 > 2 = c$  and debt initially set to  $F_0 = 6$ . We compare the model dynamics starting from a low initial value of  $\lambda_0^B(L) = 0.50$  and a high initial value  $\lambda_0^B(H) = 0.95$ . We assume  $\rho = 1$  and that subsequent shocks are zero ( $\varepsilon_t = \omega_t = 0$ ), so  $x_t = 2.25$  for all *t*. We separately plot the dynamics for various values of  $\theta \in \{0.1, 0.5, 0.9, 1\}$ . Otherwise, the model parameters are the same as those in Figure 5. Specifically, we set  $\overline{x} = 2.4$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ , and  $\sigma_{\omega} = 0.05$ .



Figure 8. Impact of backward-looking beliefs on the true default probability and expected returns. This figure plots the true default probability  $\lambda_t^R$  (in blue line) and rationally-expected returns  $\mathbb{E}_t^R[r_{t+1}]$  (in red line) against backward-looking beliefs  $\lambda_t^R$  in a highly reflexive region of the state space—i.e., a region where  $\partial \lambda_t^R / \partial \lambda_t^B$  is large so changes in beliefs have a large impact on future defaults. Specifically, we set  $x_t = 1.6 < 2 = c$  and  $F_{t-1} = 3.4$ . The other model parameters are the same as those in Figure 5:  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ .



**Figure 9. Model-implied impulse response functions in a non-reflexive region.** The top panel shows the responses following a 0.5 up or down impulses to cash flows ( $x_t$ ) at t = 1. The top panel shows the responses following a 0.15 up or down impulses to backward-looking beliefs ( $\lambda_t^B$ ) at t = 1. The initial condition in both cases is  $x_0 = 2.25$ ,  $F_0 = 2$ , and  $\lambda_0^B = 0.25$ . The model parameters are the same as those in Figure 5. Specifically, the model parameters are  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ .



**Figure 10. Model-implied impulse response functions in a reflexive region.** The top panel shows the responses following a 0.5 up or down impulses to cash flows ( $x_t$ ) at t = 1. The top panel shows the responses following a 0.15 up or down impulses to backward-looking beliefs ( $\lambda_t^B$ ) at t = 1. The initial condition in both cases is  $x_0 = 1.6$ ,  $F_0 = 3.4$ , and  $\lambda_0^B = 0.33$ . The model parameters are the same as those in Figure 5. Specifically, the model parameters are  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ .



Figure 11. Simulated data with multiple firms. The top panel shows a typical path of simulated data using our baseline set of parameter values for a representative firm. The bottom panel show the analogous simulation for multiple firms (M = 100), using the exact same aggregate cash flow shocks and sentiment shocks identical as those in the top panel. For the top panel, the initial state of the economy is  $\lambda_0^B = 0.2$ ,  $x_0 = 2$ , and  $F_0 = 4$ . For the bottom panel, the initial state of the economy is  $\lambda_0^B = 0.2$ ,  $x_0 = 2$ , and  $F_{i0} = 4$ . For the bottom panel, the initial state of the economy is  $\lambda_0^B = 0.2$ ,  $x_0 = 2$ ,  $z_0 = 0$ , and  $F_{i0} = 4$  for all *i*. In both panels, the model parameters are  $\overline{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ . In the bottom panel, we also assume that  $\psi = 0.8$  and  $\sigma_{\xi} = 0.25$ .

Panel A: Representative firm



Panel B: Multiple firms (M = 100)





#### Table 1. Credit market sentiment. Time-series regressions of the form

$$rx_{t+k}^{HY} = a + b \cdot Sent_t + \epsilon_t$$

where *Sent*<sub>t</sub> denotes investor sentiment in year t. The dependent variable is the cumulative k = 2- or 3-year excess return on high-yield bonds. *HYS* is the fraction of nonfinancial corporate bond issuance with a high-yield rating from Moody's from Greenwood and Hanson (2013). The percentage change in corporate credit is computed using Table L103 from the Flow of Funds. *Loan Sentiment* is the three-year average of the percentage of loan officers reporting a loosening of commercial lending standards. *Excess Bond Premium* is the excess bond premium from Gilchrist and Zakrajšek (2012). t-statistics for k-period forecasting regressions (in brackets) are based on Newey-West (1987) standard errors, allowing for serial correlation up to k-lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	2-у	ear future e	3-year future excess returns					
log(HYS)	-15.95 [-3.06]				-18.63 [-3.78]			
Growth of Corporate Credit		-126.50 [-2.28]				-158.02 [-2.86]		
Loan Sentiment			-0.57 [-2.12]				-0.80 [-4.03]	
Excess Bond Premium				19.29 [2.43]				24.72 [4.79]
Constant	-15.67	11.46	-0.27	3.25	-17.54	14.93	-1.26	4.51
	[-2.18]	[3.85]	[-0.06]	[0.94]	[-2.29]	[4.42]	[-0.21]	[1.15]
Ν	29	29	20	29	28	28	19	28
R-squared	0.20	0.11	0.20	0.20	0.25	0.16	0.38	0.32

**Table 2. Return and default forecasting results via model simulations.** This table report univariate and multivariate forecasting regressions for cumulative returns (1 through 5 years) and cumulative number of defaults (1 through 5 years). The model parameters are the same as those in Figure 5. Specifically, the model parameters are  $\bar{x} = 2.4$ ,  $\rho = 0.8$ ,  $\sigma_{\varepsilon} = 0.5$ , c = 2,  $\underline{F} = 1.5$ ,  $\overline{F} = 5$ ,  $\eta = 0.5$ ,  $\beta = 0.8$ ,  $\alpha = 0.2$ ,  $\sigma_{\omega} = 0.05$ , and  $\theta = 0.5$ .

		Panel A: Return forecasting				Panel B: Default forecasting					
	Univariate forecasting	1-yr	2-yr	3-yr	4-yr	5-yr	1-yr	2-yr	3-yr	4-yr	5-yr
(1)	Cashflow $(x_t)$	0.04 12%	0.07 18%	0.09 19%	0.11 18%	0.12 16%	-0.19 24%	-0.37 30%	-0.53 32%	-0.69 32%	-0.84 32%
(2)	Debt face value $(F_t)$	-0.02 15%	-0.02 13%	-0.03 10%	-0.03 7%	-0.03 5%	0.13 65%	0.23 69%	0.32 68%	0.41 65%	0.49 63%
(3)	Debt growth $(F_t - F_{t-4})$	-0.04 33%	-0.06 32%	-0.07 30%	-0.08 27%	-0.09 24%	0.11 22%	0.17 17%	0.22 16%	0.28 14%	0.33 13%
(4)	Price $(p_t)$	0.37 21%	0.38 11%	0.39 7%	0.35 4%	0.29 2%	-2.49 83%	-4.06 76%	-5.61 72%	-7.01 68%	-8.29 64%
(5)	Sentiment $(\lambda_t^R - \lambda_t^B)$	-0.34 82%	-0.37 49%	-0.42 40%	-0.46 33%	-0.48 27%	0.90 52%	1.10 26%	1.37 20%	1.61 17%	1.84 15%
(6)	Rational beliefs $(\lambda_t^R)$	-0.22 51%	-0.23 29%	-0.25 21%	-0.25 15%	-0.25 10%	1.00 96%	1.51 74%	2.03 67%	2.51 62%	2.94 57%
(7)	Extrapolative beliefs $(\lambda_t^B)$	0.02 0%	0.04 0%	0.07 1%	0.12 2%	0.18 2%	0.86 32%	1.68 42%	2.43 44%	3.11 43%	3.73 42%
(1)	$\frac{Multivariate forecasting}{Cashflow (x_t)}$ Debt face value (F <sub>t</sub> )	<b>1-yr</b> 0.02 -0.01 18%	<b>2-yr</b> 0.05 -0.01 20%	<b>3-yr</b> 0.08 -0.01 20%	<b>4-yr</b> 0.10 -0.01 18%	<b>5-yr</b> 0.11 0.00 16%	<b>1-yr</b> -0.03 0.13 66%	<b>2-yr</b> -0.10 0.21 70%	<b>3-yr</b> -0.16 0.29 70%	<b>4-yr</b> -0.22 0.36 68%	<b>5-yr</b> -0.29 0.42 65%
(2)	Cashflow $(x_t)$ Debt growth $(F_t - F_{t-4})$	0.01 -0.04 34%	0.03 -0.05 35%	0.05 -0.06 34%	0.06 -0.06 31%	0.06 -0.07 28%	-0.14 0.07 31%	-0.30 0.08 33%	-0.46 0.09 34%	-0.61 0.10 34%	-0.75 0.12 33%
(3)	Cashflow $(x_t)$ Debt face value $(F_t)$ Extrapolative beliefs $(\lambda_t^B)$	0.01 -0.04 0.34 42%	0.04 -0.05 0.44 41%	0.06 -0.06 0.56 40%	0.08 -0.06 0.66 38%	0.09 -0.07 0.76 35%	-0.02 0.14 -0.12 66%	-0.10 0.19 0.25 71%	-0.18 0.24 0.53 71%	-0.25 0.29 0.77 69%	-0.32 0.34 0.97 66%