Safe Assets, Collateralized Lending and Monetary Policy*

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Abstract

I study how quantities of safe bonds affect asset prices and lending volumes in financial markets. In a quantitative model, heterogeneous agents trade securities of different maturity and risk exposure. Risk-tolerant investors issue collateralized bonds to obtain leverage and to insure the risk-averse. Despite the presence of higher return assets, the most risk-tolerant hold long-maturity safe assets, which they value as good collateral. The value of collateralizability is high when safe bond quantities are low. Given measured variations in safe bond quantities between 1990 and 2015, the model replicates the dynamics of lending volumes and generates large, volatile credit spreads and excess return predictability. The model also predicts price effects of high-frequency changes of government debt quantities around tax due dates. In policy experiments, I use the model to study the effects of central bank asset purchases.

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1 Introduction

I study how quantities of safe bonds impact asset prices and lending volumes in financial markets. The paper is motivated by an empirical literature that finds a relation between the level of government debt and interest rates, and by the fact that the US central bank increased its holdings of government-backed securities by about $3.5 trillion between 2008 and 2014, with the explicit goal of lowering interest rates and raising asset prices. Standard theories with complete markets and no frictions do not feature any connection between asset prices and asset quantities. In such models, interest rates are driven by expectations about growth and inflation, and credit spreads reflect default risk. I move away from the frictionless benchmark and study an economy with incomplete markets and collateralized lending, building on the observation that long maturity safe bonds are held by highly leveraged investors like hedge funds, who use these assets as collateral.

To capture this observation, I develop a new quantitative model of financial markets with with heterogeneous investors, who trade safe and risky assets of different maturity. Moreover, they borrow and lend from each other against collateral. The model generates an endogenous segmentation of the market for safe assets. Short-maturity safe assets are held by the most risk-averse investors. Long-maturity safe assets are held by the most risk-tolerant investors, despite the presence of corporate bonds and stocks with higher expected returns, which can also be used as collateral. In the model, the exogenous quantities of safe assets determines the supply and demand for privately produced short safe assets. Risk-tolerant investors pay a premium for long safe assets because these assets are particularly useful as collateral. This "collateral premium" varies with safe bond quantities and generates additional volatility of long-maturity interest rates. Risk premia are time-varying because changes in the quantities of safe assets have quantitatively important effects on safe interest rates, but not on the returns of equities and corporate bonds.

I use the model in two quantitative applications. In the first application, I study the effects of measured variations in safe asset quantities over the business cycle. The goal of this exercise is to see whether the model can match the dynamics of collateralized lending in financial markets, and whether the variation of the collateral premium generates quantitatively important asset price effects. In the second application, I study high-frequency changes in the level of government debt around annual tax dates. I ask whether the model can predict the empirical asset price effects that occur due to that variation. Finally, I undertake counterfactual policy experiments to analyze the effects of large-scale asset purchases.

For the first application, I measure the evolution of the quantities of short-, medium- and long-maturity safe assets between 1990 and 2015 using data from the Flow of Funds. I also estimate the time series of the aggregate state of the economy using only macroeconomic data. Given these data, the model replicates the dynamics of lending volumes in the repurchase market over time. The time variation in the collateral premium provides a quantitatively important channel of interest rate volatility that creates excess return predictability, as documented in the empirical literature. The model also generates a large and volatile credit spread that matches the data. In line with previous research, the representative agent case of my model generates a low BAA credit spread volatility of 0.09%, while the full model predicts a value of 0.64%, closer to the volatility of 0.78% observed in the data.

In the high-frequency application, I study the response of asset prices to movements in the level of government debt around annual tax dates. Before April 15, the US Treasury issues short-maturity T-bills in anticipation of incoming tax receipts. After April 15, tax revenue repays outstanding debt. This seasonal variation generates an anticipated variation in the amount of outstanding short-maturity debt. Using issuance forecasts from the US Treasury, I show that there are also often large unanticipated tax inflows which lead to unexpected persistent changes in the level of government debt. The model replicates the observed asset price effects of the anticipated temporary and unanticipated persistent variation in government debt.

In both applications of the model, I find that safe asset quantities have quantitatively important effects on short- and long-maturity interest rates on safe bonds. This result has implications for fiscal and monetary policy. Not only does the US Treasury have a direct effect on the level and maturity structure of government debt, but also the Federal Reserve can change the quantity of safe assets through large-scale purchases of government-backed securities. The three rounds of quantitative easing (QE) after 2008, especially QE 2 and QE 3, had the explicit goal of changing the level of interest rates and asset prices; the central bank attempted to lower long rates to spur investment and economic activity.[1] While conventional monetary policy directly changes the short rate in the federal funds market, these unconventional market interventions targeted a much broader range of asset prices.

My framework allows me to study the transmission mechanisms of large-scale asset purchases because it incorporates private assets, like stocks and corporate bonds, as well as safe assets of different maturities. I can therefore analyze the differential effects of asset purchases on interest rates of all maturities, as well as on the prices of risky assets. In the quantitative model, a permanent reduction in the total value of long-maturity government-debt by 5% of consumption in 2012, approximately the magnitude of central bank purchases during QE 2, lowers the short-maturity rate by 28 basis points, or 0.28 percentage points, and increases the term spread by 8bp. An exchange of short safe assets for long safe assets in the same amount increases short rates by 17bp and reduces the term spread by 6bp. In both cases, the price effects on corporate bonds and equity are negligible.

The presence of collateralized lending is the key feature of the model to generate the above results. In the data, a central venue for collateralized lending is the repurchase market, a term

[1] As stated, for example, by Bernanke (2012).

that encompasses a system of several markets. This paper focuses on the repo system and in particular on its end-users: cash investors are lenders and collateral providers are borrowers in this market. A typical cash investor is a money market fund seeking short-maturity and safe investment opportunities, while a typical collateral provider is a hedge fund that borrows in order to hold a leveraged portfolio of long-maturity securities. Money market funds and hedge funds often do not interact directly, but connect through a dealer bank that acts as an intermediary. The model abstracts from the role of intermediaries, who maintain active repurchase markets among themselves and with hedge funds, and instead focuses on the relationship among end-users. Bu focusing on the repo market, the model only studies a single part of a larger financial system, ignoring in particular the role of banks and their role as producers of safe payment instruments. This limitation is important to keep in mind, when interpreting counterfactual model outcomes.

I study the role of collateralized lending markets in an infinite horizon endowment economy. Aggregate output is distributed through different assets that are in exogenous positive supply. These assets represent equity, defaultable corporate bonds, default-free nominal bonds of various maturities and labor income. The default-free bonds represent government debt and similarly safe assets. Output and the nominal price level are exogenous stochastic processes.

Investors differ in their risk-aversion which creates gains from trade because the risk-tolerant can insure the risk-averse. To do so, they issue one-period collateralized bonds. Risk-averse investors hold these collateralized bonds because they provide a safe and short-maturity investment opportunity. Risk-tolerant investors issue such bonds to maximize returns: by selling one-period bonds, they are borrowing funds at the low short interest rate. They use the borrowed funds to purchase long-maturity assets that earn, in expectation, a higher return. In the next period, they have to repay the bondholders/lenders at the low interest rate, but, on average, receive the higher returns on their portfolio of riskier, long-maturity securities.

Borrowers can only issue one-period bonds against collateral. Collateral constraints force the value of the provided collateral to cover the promised bond payments in each state of the world next period. Therefore, the lowest possible payoffs of the collateral assets determines how much an investor can borrow. While portfolios of corporate bonds and equity earn higher returns, they cannot guarantee large repayments because of their exposure to both aggregate and undiversified idiosyncratic risk. Holding such portfolios therefore limits leverage. Long safe bonds are the optimal collateral because they earn, on average, returns that are higher than the short rate. Moreover, long safe bonds are not affected by idiosyncratic risk and are only subject to limited aggregate risk, thus allowing for high leverage. I refer to this asset characteristic as collateralizability.[2] Risk-tolerant borrowers are willing to pay a premium for assets that earn an excess return and are highly collateralizable. Short- and medium-maturity safe assets are also useful as collateral, but

[2]The corporate finance literature calls it "debt-capacity"; see, for example, Shleifer and Vishny (1992).

they earn little excess return. They are therefore held by the most risk-averse investors who consider them as substitutes to collateralized bonds. In the absence of collateralized lending markets, the risk-averse would also hold long safe assets.

The resulting market segmentation has implications for the level and dynamics of term and credit spreads. In the model, there are fewer long safe bonds than the risk-tolerant would like to use as collateral, given his net-worth. The next best collateral assets are corporate bonds, which are priced by investors with intermediate risk-aversion who do not take on leverage. The risk-tolerant investors bid up the price of long safe assets until they are indifferent between holding a leveraged portfolio of long-maturity safe bonds or a leveraged portfolio of corporate bonds. The demand for collateral thus creates a large credit spread between long safe bonds and corporate bonds. This spread is not driven by default risk but by the collateral premium; it is therefore not present in the representative agent model nor in the absence of collateralized lending.

When the quantity of short-maturity safe assets falls, the short rate falls and the strategy of borrowing short-term to invest long-term becomes more attractive for the risk-tolerant. They then further bid up the price of long safe bonds; this increases the credit spread because the corporate bond price remains unchanged, since it is priced by the intermediate investors. The return on long-maturity safe assets will not decline as much as the short rate because the price of the next best collateral alternative, corporate bonds, does not change. The term spread therefore increases, ensuring that the risk-tolerant remain indifferent between the use of long safe bonds and corporate bonds as collateral. Changes in the quantities of safe assets thus generate additional variation in term and credit spreads.

This paper contributes to the literature on collateralized lending and builds on the idea that investors value assets, not only for their payoffs, but also for how useful they are as collateral. This is the first quantitative model within that literature, and I find that the time-varying collateral premium has sizable effects on long-maturity interest rates. Different from earlier papers, the model features several different assets that can all be used as collateral. Therefore, the model provides a microfoundation for how investors use collateral to trade with each other. Previous research studies the effects of large-scale asset purchases in the presence of exogenous market segmentation. My model generates an endogenous segmentation of the market for safe assets, and it therefore extends the "preferred habitat" literature by determining which asset characteristics set market segments, and why such segmentation plays an important role for safe assets but not riskier securities. The model replicates several findings of the empirical asset pricing literature, in particular, excess return predictability, as well as large and volatile credit spreads that are mostly unrelated to default risk. Section 7 discusses the related literature in more detail.

2 Safe asset holdings in the data

This section documents that short- and long-maturity safe assets are held by different types of investors in the data. The quantitative model presented in the next section will replicate this segmentation. In the model, more risk-averse investors hold short and medium safe assets, while risk-tolerant investors hold long safe assets, which they use as collateral to borrow. In the data, short safe assets are, to a large extent, held by money market funds who are regulated to only hold highly rated debt securities with short maturities.[3] Data on other market participants are more limited, but it is an uncontroversial claim that short safe assets are held for their safety and liquidity.

I argue that long-maturity safe assets are held by hedge funds and other leveraged investors based on three observations: First, in the Flow of Funds, the household sector, which includes hedge funds and similar investment firms, has large direct holdings of Treasury and agency debt. Second, data on general collateral repurchase markets show that almost all collateral assets are highly rated securities. Because borrowers provide their asset holdings as collateral, one can conclude that leveraged investors hold these safe assets. Third, the level of lending volumes in these markets is large, and it comoves with the total quantity of long safe assets. The remainder of this section discusses the three observations in more detail and provides a short introduction to collateralized lending in repurchase markets in the data.

Safe asset holdings in the Flow of Funds: I use Treasury holdings as a proxy for a sectoral prevalence to hold safe assets. I exclude foreign holdings, which are mostly held by foreign institutions like central banks. Between 1990 and 2015, the single largest group to hold Treasury debt were federal pension funds. During this time period, they owned on average about 30% of all domestically held marketable Treasury debt. Regulation forces federal pensions funds to hold only federal government debt. Private, local and state pension funds have more flexibility in choosing their portfolios. They held, on average, less than 14% of outstanding Treasury securities, even though they managed about five times as much net-worth as federal pension funds. Depository institutions, money market funds and mutual funds each held 7-8% of all Treasury debt. The second largest holder of Treasury securities is the household sector with an average direct holding share of 19%, a number that excludes non-marketable savings securities. I estimate a similar holding share for agency debt.

To interpret the large direct holdings of agency and Treasury debt of the households sector, it is important to note that the household sector aggregates all residual holdings that are unassigned to any other sector in the Flow of Funds. Therefore, the household sector also includes hedge funds and similar investment firms who do not file regulatory holding reports. In the data, hedge funds hold

[3] See rule 2a-7 of the Investment Company Act of 1940.

long safe assets when following an interest rate carry strategy, under which they borrow short-term to finance a leveraged portfolio of long-maturity securities. In the following, I discuss how hedge funds obtain such a leveraged portfolio, and I present evidence that the safe asset holdings of the household sector indeed represent portfolio holdings of such leveraged market participants.

Safe assets and collateralized lending: In the data, hedge funds and other leveraged investors can borrow funds in multiple ways, all of which require collateral. They can borrow from their prime broker, either through the means of a margin account or a bilateral repo agreement; they can borrow from a third party, usually through a bilateral repo agreement; or they can obtain synthetic leverage by entering derivative contracts. Even though such contracts also require collateral, I abstract in this paper from derivative trading. Including such instruments provides an interesting extension.

If a hedge fund obtains a loan from his prime broker or another security dealer, this lender will himself turn to cash-providing institutions to borrow the needed funds. To do so, the lender will rehypothecate the hedge fund's assets and provide them as collateral in a repo agreement of his own. A prime broker may also hold proprietary asset portfolios that are financed in repo markets. There are several such markets in the data which differ in the details of the contracts, in who participates and in which assets can be provided as collateral. In this paper, I am interested in how repo markets produce safe investment opportunities. Therefore, I do not include repo contracts among dealers, brokers and borrowing investors, and instead focus on their aggregate role as net-borrowers from cash providers.[4]

Cash providers, such as money market funds, interact with the repo system through two types of markets: Bilateral repo markets, in which the counterparties or their custodians are themselves responsible for settling and clearing the contracted transactions, and tri-party repo markets, in which a neutral third party provides clearing and settlement services. For a more detailed description of the institutional details see Baklanova, Copeland, and McCaughrin (2015).

Data on repo markets are limited. In terms of sample length, the best data source are quarterly regulatory filings by brokers and dealers. This data only contains repo transactions which include a registered broker or dealer as a counterparty. The data suffers from double-counting of repo transactions whenever both counterparties are brokers or dealers. Primary dealers have additional reporting requirements on a weekly frequency. The Federal Reserve Bank of New York publishes these data going back to 1998. Starting from 2010, there are additional data sources not based on the reporting market participant, but on the market venue itself. The Federal Reserve Bank of

[4] For the same reason, I focus on general collateral repo agreements that have the purpose of borrowing funds rather than borrowing specific securities. Special repo and security lending agreements function in the same way, but the main interest of the cash-providing counterparty is not to safely invest funds, but to obtain a specific security, often for short-selling purposes.

New York publishes monthly data on lending volumes in the tri-party repo market and the DTCC publishes daily data on the GCF market, a repo market only among dealers. Data on the bilateral repo market are at this point not available, and estimates on its size differ widely.[5] The tri-party repo data includes time series data on the assets borrowers use as collateral. Since 2010, about 80% of the collateral assets are government-backed securities. Since 2013, the primary dealer data also differentiates repo agreements by collateral type. The share of government-backed securities provided as collateral by primary dealers during that time is even higher, at nearly 90%.

These results show that leveraged institutions do indeed hold safe assets and that they provide them as collateral in the repurchase market. The lending volume between the repo system and cash providers could provide an estimate of how many such assets leveraged investors use as collateral to borrow funds. The broker and dealer data has the disadvantage that it both over- and understates lending volumes in collateralized lending markets. It understates lending volumes because dealers are just one set of market participants that borrow from cash providers. The data overstates net-borrowing in the repo market because it includes repo agreements among dealers, leading to both double-counting and an overstatement of what is borrowed from cash-providers, rather than within the dealer system. Krishnamurthy, Nagel, and Orlov (2014) estimate the size of the repo market based on regulatory filings by money market funds only, thereby estimating the actual cash provision from such cash investors. Gorton and Metrick (2012) argue that there are large unregulated cash pools which are not captured in regulatory filing data.

Figure 1 compares the total value of repo borrowing by brokers and dealers to an estimate of actively traded safe assets with a maturity of more than 3 years. The latter estimate is based on Flow of Funds data on domestic asset holdings and its derivation is described in more detail in Section 5.3. The quantity of safe assets comoves with lending volumes, and I interpret this as suggestive evidence that the household sector's holdings of Treasury debt largely represent holdings of long-maturity safe assets by leveraged institutions. This comovement is not present for short- or medium-maturity assets; the model will predict the use of long rather than short or medium assets as collateral.

3 Model

I study the interaction of safe asset quantities and collateralized lending in an infinite-horizon exchange economy. Time is discrete. Output is stochastic and exogenous, as is the inflation rate. The economy is populated by a continuum of investors with time-varying risk-aversion. In each period, there are M groups of investors that differ in their risk-aversion. These differences in risk-aversion create gains from trade. Claims on output are traded in the form of different types

[5] See, for example, Copeland, Davis, LeSueur, and Martin (2012a) and Gorton and Metrick (2012).

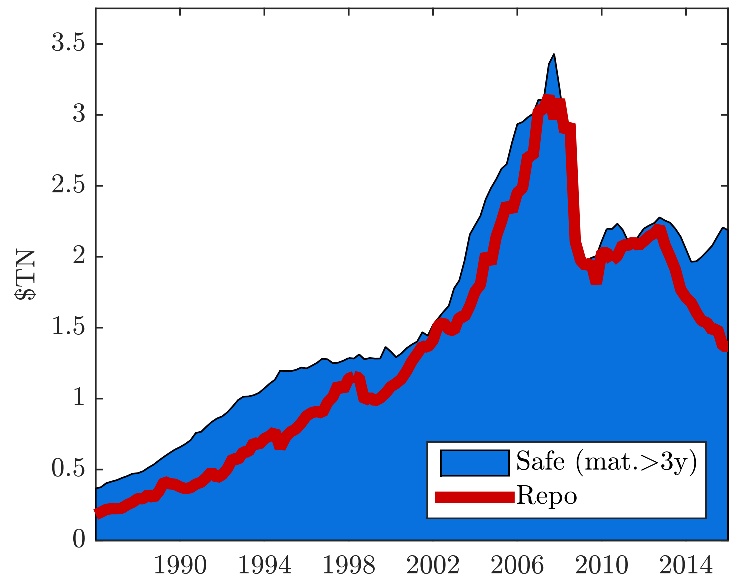

Figure 1: Comparison of long-maturity safe asset quantities with lending volume in repurchase markets. Safe asset quantities with maturity of 3 years or more are estimated from Flow of Funds data as described in Section 5.3. Repo volume data are repurchase agreement liabilities of brokers and dealers from the Flow of Funds. Section 2 discusses the repo data in more detail.

of assets: equity, corporate bonds, government debt of different maturities and a residual claim. Investors are, in general, short-sale constrained, but they can issue collateralized bonds in order to borrow. This section describes the model in detail. Section 4 discusses the individual assumptions and Section 5 presents the connection of the model's parameters and variables to the data.

3.1 Endowment and preferences

Aggregate state: Output growth g_t is exogenous and stochastic, and the process of aggregate output Y_t is

$$Y_t = \exp(g_t) Y_{t-1}.$$

The nominal price level P_t is also exogenous and grows at a stochastic inflation rate π_t:

$$P_t = \exp(\pi_t) P_{t-1}.$$

Growth and inflation are subject to transitory shocks and a common persistent component:

$$\begin{aligned} g_t &= \mu_g + \xi_g z_t + \eta_{g,t}, \\ \pi_t &= \mu_\pi + \xi_\pi z_t + \eta_{\pi,t}, \\ z_t &= \phi z_{t-1} + \eta_{z,t}, \end{aligned} \tag{1}$$

where μ_g and μ_π are the average growth and inflation rates, and ξ_g and ξ_π are the individual loadings on the persistent factor z_t. I assume that ξ_g is positive, so that the growth rate is increasing in z_t. The three shocks $\eta_{g,t}$, $\eta_{\pi,t}$ and $\eta_{z,t}$ have bounded support. Y_t, P_t and z_t summarize the aggregate state of the economy.

Preferences: There is a continuum of investors, $i \in [0,1]$, who derive utility from consumption $C_{i,t}$. Investors have recursive utility with time-varying risk-aversion $\gamma_{i,t}$. Risk-aversion can take $M = 4$ different values. At time t, the next period's risk-aversion $\gamma_{i,t+1}$ is unknown and will take value $\gamma_m \in \{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ with probability p_m. These probabilities are independent of time and state, and realizations of $\gamma_{i,t+1}$ are independent across investors. In the quantified model, investors will have a small probability of becoming very risk-tolerant with a low γ_1 or very risk-averse with high γ_3 and γ_4. The large majority of investors will have intermediate risk-aversion γ_2.

I denote by $s_t = [Y_t, P_t, z_t, \{\gamma_{i,t}\}_{i=0}^1]$ the current state of the economy, which collects the aggregate state and the current investor specific risk-aversion coefficients. The history of the economy up to period t is denoted s^t. Given a consumption plan $\{C_i(s^t)\}_{t=0}^\infty$, agent i's utility is defined recursively as

$$U_i(s^t) = \left((1-\beta)C_i(s^t)^{1-1/\sigma} + \beta \mathbb{E}_t \left[U_i(s^{t+1})^{1-\gamma_{i,t}}\right]^{\frac{1-1/\sigma}{1-\gamma_{i,t}}}\right)^{\frac{1}{1-1/\sigma}}, \tag{2}$$

where σ is the intertemporal elasticity of substitution.

3.2 Assets

Claims on aggregate output Y_t are traded in the form of six different assets: equity; default-free short-, medium- and long-maturity bonds; defaultable long-maturity bonds and a claim on residual payoffs. The net-payments to these asset classes add up to total output. The capital structure of the economy, meaning the way in which output is divided into payments to the different assets, is exogenous.

Equity: Equity pays a fraction α_t of the total tree payoff, such that dividends D_t are

$$D_t = \alpha_t Y_t.$$

A logistic function of the persistent component z_t in Equation 1 defines the dividend share α_t:

$$\alpha_t = \frac{\bar{\alpha}}{1 + \exp\left(-\kappa_D \cdot (z_t - \bar{z}_D)\right)}.$$

The dividend share varies between 0 and $\bar{\alpha}$. If the parameter κ_D is positive, the dividend share α_t is below $0.5\bar{\alpha}$ when z_t is lower than $\bar{z}_D$. The parameter κ_D governs the cyclicality of dividends. This specification makes dividend growth more volatile than aggregate consumption growth, because dividends are particularly low when current growth is low and vice versa. The real price of equity is Q_t^E.

Safe bonds: Bonds pay nominal coupons. Default-free bonds promise an infinite stream of geometrically declining coupon payments, as in Woodford (2001). There are three different types of such bonds: short-, medium and long bonds, indexed by $j \in \{S, M, L\}$. A unit of any safe bond purchased at time t promises to pay one nominal unit in period $t+1$, or $\frac{1}{P_{t+1}}$ in real terms. A deterministic fraction $(1-\delta_j)$ of the bond will disappear, while a fraction δ_j survives. Given this geometric decline, the coupon payment T periods from t is given by δ_j^{T-1}, or $\frac{1}{P_{t+T}}\delta_j^{T-1}$ in real terms. The parameter δ_j therefore governs the duration of a default-free bond, and the survival rates will be set such that $\delta_S < \delta_M < \delta_L$. I set $\delta_S = 0$, such that the short bond is a one-period bond.

The amount of outstanding default-free bonds $j \in \{S, M, L\}$ is exogenous and given as a fixed fraction of current nominal output:

$$\bar{A}_t^j = \bar{a}^j P_t Y_t.$$

A fraction δ_j of all bonds of type j that were issued in period t, survive in period $t+1$ and are therefore part of the new bond supply. Given a supply of $\bar{a}_{t-1}^j$ at $t-1$, an issuance of $\bar{a}_t^j - \delta_j \bar{a}_{t-1}^j$ achieves a new supply of $\bar{a}_t^j$. At each point in time t, previously issued bonds receive their coupon payments, and new bonds are issued to reach the exogenous level of supply. The part of output that is paid to default-free bonds is

$$\tau_t = \sum_{j=\{S,M,L\}} \left(\bar{A}_{t-1}^j \frac{1}{P_t} - \left(\bar{A}_t^j - \delta_j \bar{A}_{t-1}^j \right) Q_t^j \right),$$

which is the sum of coupon payments minus the revenue from the sales of newly issued bonds. One can interpret the net-payment τ_t as the output tax that an unmodeled government has to levy in order to finance its debt holdings. I will refer to default-free bonds as "government bonds", and call the one-period bond "T-bill".

Risky bond: The risky long-maturity bond represents a diversified portfolio of corporate bonds that can default. It has a similar structure as the default-free long bond, but its survival rate $\tilde{\delta}_t$ is stochastic. Similar to the dividend share, $\tilde{\delta}_t$ is a logistic function of the persistent factor z_t:

$$\tilde{\delta}_t = \frac{\delta_L}{1 + \exp(-\kappa_B \cdot (z_t - \bar{z}_B))}.$$

When $\bar{z}_B$ is far enough below 0, $\tilde{\delta}_t$ is close to δ_L whenever z_t is around or above its mean. When z_t is very low, the survival rate $\tilde{\delta}_t$ will be lower than δ_L, representing the defaults of a fraction of the bonds in the portfolio. The nominal coupon payment T periods from t is $\prod_{s=t+1}^{t+T-1} \tilde{\delta}_s$ and the real price of this bond is Q_t^B. As for the default-free bonds, the amount of outstanding defaultable bonds $\bar{a}_t^j$ is a fixed fraction of current nominal output,

$$\bar{A}_t^B = \bar{a}^B P_t Y_t.$$

The part of output paid to this type of bond is

$$\iota_t = \bar{A}_{t-1}^B \frac{1}{P_t} - \left(\bar{A}_t^B - \tilde{\delta}_t \bar{A}_{t-1}^B \right) Q_t^B,$$

which is the difference between coupon payments and the revenue from new bond issuance. Bond issuance relative to aggregate consumption is higher in bad times, because a smaller fraction of the outstanding bonds from last period survives. I will refer to the defaultable bond as "corporate bond".

Residual claim: At this point, three slices of total output have been assigned to financial assets: A slice $D_t = \alpha_t Y_t$ is paid as dividends to equity holders, a slice τ_t is paid to default-free bonds, and a slice ι_t is paid to defaultable bonds. The remainder $N_t = Y_t - \alpha_t Y_t - \tau_t - \iota_t$ is paid to a final asset, the residual claim. While all other assets are traded in a single competitive market, the residual claim can only be traded among investors with the same current risk-aversion coefficient γ_m. The asset is therefore traded in four separate markets at different real prices $Q_{m,t}^N$. This assumption ensures that every group of investors holds a fixed portion of the residual claim. I therefore label that asset "non-tradable". The assumption of tradability within each risk-aversion group ensures that the investor's optimization problem remains homogeneous in wealth.

3.3 Asset trading and collateralized lending

Investors trade the above assets in competitive markets. All trading is, in general, subject to short-sale constraints, but investors can borrow funds by issuing default-free one-period bonds S. By selling one unit of such a bond at time t, the borrower promises to pay one nominal unit to the buyer of the bond in the next period $t + 1$.

Borrowing is subject to a collateral constraint which states that the value of the provided collateral cannot fall below the promised repayment in any state in the next period. Since the promised repayment is in nominal terms, the relevant variable is the lowest possible nominal payoff of the collateral in the next period. All assets, except the residual claim, can be used as collateral, and I assume that the collateralizability of each asset is determined the by its individual payoff characteristics, not by the payoff risk of the overall asset portfolio provided as collateral. The collateral constraint is then

$$-a_{i,t}^{S} \leq \sum_{j\in\{M,L\}} a_{i,t+1}^{j} \min_{s_{t+1}} \left[P_{t+1}\Pi_{t+1}^{j}\right] + \zeta \sum_{j\in\{E,B\}} a_{i,t+1}^{j} \min_{s_{t+1}} \left[P_{t+1}\Pi_{t+1}^{j}\right] \tag{3}$$

where $a_{i,t}^{j}$ denotes agent i's holdings of asset j, and where I define the real payoffs of asset j as Π_{t+1}^{j}, so

$$\Pi_{t+1}^{E} = D_{t+1} + Q_{t+1}^{E}, \qquad \Pi_{t+1}^{B} = \frac{1}{P_{t+1}} + \tilde{\delta}_{t+1} Q_{t+1}^{B}, \qquad \Pi_{m,t+1}^{N} = N_{t+1} + Q_{m,t+1}^{N},$$
$$\Pi_{t+1}^{S} = \frac{1}{P_{t+1}}, \qquad \Pi_{t+1}^{M} = \frac{1}{P_{t+1}} + \delta_M Q_{t+1}^{M}, \qquad \Pi_{t+1}^{L} = \frac{1}{P_{t+1}} + \delta_L Q_{t+1}^{L}.$$

The parameter ζ allows for the possibility that the value of equity and corporate bond collateral is below the market value of the assets to incorporate unmodeled idiosyncratic risk in a reduced form. The constraint counts all of agent i's asset holdings as collateral. The assumption that borrowers can never default on their promised repayment implies that overcollateralization is irrelevant. I therefore assume that the borrower always provides his entire asset portfolio as collateral. Issuance of collateralized bonds ($a_{i,t}^{S} < 0$) is only possible if the lowest possible collateral value in the next period is strictly positive. The previous assumption of a bounded support of the shocks $\eta_{g,t}$, $\eta_{z,t}$, and $\eta_{\pi,t}$ ensures that such a positive lower bound exists.

Issuance of collateralized bonds is costly and the borrower has to pay an issuance fee F_t to an unmodeled intermediation sector outside of the economy. The fee is computed as a fraction f of the promised repayment, and hence the seller of a bond receives only $Q_t^S - f$ for each sold unit.

3.4 Investor problem and equilibrium

In the following, I denote with $J = \{S, M, L, E, B, N\}$ the set of all assets. Given a history of the economy s^t, as well as corresponding asset prices Q_t^j and investors' asset holdings $a_{i,t-1}^{j}$ of all assets $j \in J$, investor i with $\gamma_{i,t} = \gamma_m$ chooses a portfolio plan to maximize his utility defined in Equation 2. His choice is subject to his budget constraint

$$W_{i,t} \geq C_{i,t} + \sum_{j\in J\setminus\{N\}} a_{i,t}^{j} Q_t^{j} + a_{i,t}^{N} Q_{m,t}^{N} - F_t, \tag{4}$$

where the issuance fee is $F_t = \max\left[0, -f a_{i,t}^{0}\right]$ and where wealth $W_{i,t}$ is

$$W_{i,t} = \sum_{j\in J\setminus\{N\}} a_{i,t-1}^{j} \Pi_t^{j} + a_{i,t-1}^{N} \Pi_{m,t}^{j}.$$

Furthermore, the optimization is subject to the short-sale constraints

$$a_{i,t}^{j} \geq 0 \quad \forall j \in J \backslash \{S\}, \tag{5}$$

and subject to the collateral constraint (3). With the optimization problem at hand, I define the equilibrium as follows:

Definition 3.1 (Equilibrium) *Given initial levels of output Y_{-1}, price level P_{-1}, and of the persistent factor z_{-1}, as well as initial of asset holdings $a_{i,-1}^{j}$ for all investors i and assets j, and given a path of aggregate shocks $\{\eta_t\}_{t=0}^{\infty}$ and idiosyncratic realizations of risk aversion coefficients $\{\gamma_{i,t}\}_{t=0}^{\infty}$, the equilibrium is defined as the paths of*

- *asset prices $\{Q_t^E, Q_t^S, Q_t^M, Q_t^L, Q_t^B, Q_t^N\}_{t=0}^{\infty}$,*
- *consumption choices $\{C_{i,t}\}_{t=0}^{\infty}$,*
- *and asset holdings $\{a_{i,t}^E, a_{i,t}^S, a_{i,t}^M, a_{i,t}^L, a_{i,t}^B, a_{i,t}^N\}_{t=0}^{\infty}$,*

such that in each period $t \geq 1$

1. *Y_t, P_t and z_t evolve according to (1),*
2. *each investor maximizes his utility (2) subject to the constraints (3)-(5),*
3. *all common asset markets clear,*

$$\int_{i=0}^{1} a_{i,t}^{S} di = \bar{a}^{S} P_t Y_t, \qquad \int_{i=0}^{1} a_{i,t}^{M} di = \bar{a}^{M} P_t Y_t, \qquad \int_{i=0}^{1} a_{i,t}^{L} di = \bar{a}^{L} P_t Y_t,$$
$$\int_{i=0}^{1} a_{i,t}^{E} di = 1, \qquad \int_{i=0}^{1} a_{i,t}^{B} di = \bar{a}^{B} P_t Y_t,$$

4. *and the group-specific markets of the residual claim clear,*

$$\int_{i:\gamma_{i,t}=\gamma_m} a_{i,t}^{N} di = \int_{i:\gamma_{i,t}=\gamma_m} a_{i,t-1}^{N} di \quad \forall m \in \{1,2,3,4\} \text{ and with } \int_{i=0}^{1} a_{i,t-1}^{N} di = 1.$$

Solving for equilibrium prices and portfolio holdings is simpler than this definition suggests. The investors' maximization problem is homogeneous in wealth because of the homotheticity of the utility function and the fact that all constraints scale in wealth. Given a wealth distribution $W_{i,t}$, one can therefore aggregate all those agents that share the same risk aversion coefficient γ_m. The wealth of group m is $W_{m,t} = \int_{i:\gamma_{i,t}=\gamma_m} W_{i,t} di$ and the portfolio problem is only solved four times for each group $m \in \{1,2,3,4\}$.

Furthermore, the probability p_m of having risk aversion γ_m is constant and independent across time and investors. Invoking a law of large numbers, a fraction p_m of all those agents that had risk aversion γ_n in period $t-1$ will have risk aversion γ_m in period t. Given the holdings of the residual claim $a^N_{n,t-1}$ of each group n at the end of period $t-1$, the aggregate holding of all agents with risk aversion γ_m at time t are then

$$a^N_{m,t} = p_m \sum_{n=1}^{3} a^N_{n,t-1} = p_m,$$

where I have used the market clearing condition for the residual claim. By the same argument, the members of each group m enter period t with a fraction p_m of all other assets, such that their wealth is given as

$$W_{m,t} = p_m \left(Y_t + \sum_{j \in J \setminus \{N\}} \bar{a}^j_{t-1} Q^j_t \right) + p_m Q^N_{m,t}.$$

Independent of state and prices, group m therefore always holds a fixed fraction of tradable wealth (as given in the bracket) and a fixed fraction of the non-tradable residual claim.

In each state of the world, I only need to solve the maximization problem defined in equations (2)-(5) for the four different groups of investors $m \in \{1, 2, 3, 4\}$ who have constant tradable wealth shares p_m, fixed holdings p_m of the residual claim, and risk aversion coefficient γ_m. The state s_t reduces to the output level Y_t, the price level P_t and the level of the persistent factor z_t. Appendix A.1 further reduces the state space by defining an equivalent stationary economy in which only z_t remains as a state variable.

3.5 Assets in ϵ-supply

The slicing of aggregate output determines the span and supply of tradable assets. because short-sale constraints prohibit the issuance of additional assets, However, I introduce additional assets in net-supply ϵ, where ϵ is positive but arbitrarily close to 0. These assets leave the equilibrium unchanged, but are still priced by those investors with the highest marginal valuation. This allows me to not only study the prices of the three government bonds with δ_S, δ_M and δ_L, but to determine the prices $Q_t(\delta)$ for any δ. I use this to study the model implications for, for example, yields of δ_{3m}-bond that targets the duration of a 3-month T-Bill.

4 Discussion of the model

This section discusses the structure and assumptions of the model presented in the last section. The goal of the framework is to capture the price and quantity effects of the interaction between the supply of safe assets and collateralized lending markets. All assumptions are made in order to achieve that goal, while maintaining computational tractability.

4.1 Asset structure

Endowment economy: I study the effects of changes in the supply of safe assets in an endowment economy. The model therefore does not make any direct predictions on the real effects of such supply changes. Introducing a real sector provides an interesting extension to the current model. Such an extension does not change the decision problem faced by investors, but it endogenizes the supply of risky private assets. As the results will show, the supply of safe assets has, in this model, quantitatively small effects on the returns of corporate bonds and equity, because the intermediate investor who is neither a marginal investor of short- nor long-maturity safe assets, prices these riskier assets in equilibrium.

Capital structure: In a representative agent endowment economy, the supply of individual assets is irrelevant because any security is priced using a pricing kernel derived from aggregate consumption. In this heterogeneous agent framework, markets are incomplete because of short-sale and collateral constraints. The supply and characteristics of assets determine to what extent agents can share risk. I model the different assets by slicing aggregate consumption into different payoff streams. This approach is closely related to models that are populated by several Lucas trees, as in Santos and Veronesi (2006), Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2013). Different from these papers, the payoff shares are herein not each additional stochastic state variables, but are instead determined as functions of the aggregate state. This allows me to model a rich asset structure within a computationally tractable framework. Each asset in the model represents a diversified portfolio of the corresponding asset class in the data. The payoff functions are chosen to capture the payoff characteristics that determine pricing and collateralizability.

Equity: Early work on consumption-based asset pricing in an exchange economy, as in Lucas (1978) and Mehra and Prescott (1985), assumes that aggregate payoffs themselves represent dividend payments. However, in the data, dividend growth is more volatile than aggregate consumption. Some authors, for example Abel (1999), therefore assume that dividends are given by consumption raised to a power, or, relatedly, model dividend growth directly as subject to shocks with higher variance, as in Bansal and Yaron (2004). While tractable in a representative agent economy, these assumptions imply that the dividend share becomes non-stationary. In the present framework, in which the relative supplies of assets are of key interest, it is important that asset shares remain stationary. I therefore propose a new functional form that models the dividend share as a function of the aggregate state. Dividend growth is then more volatile because the dividend share is smaller in bad times and higher in good times. As discussed in more detail in Section 5, the functional form also allows the targeting of dividend growth skewness and kurtosis.

Safe bonds: Bond payments in the model are subject to nominal risk. In the data, the vast majority of issued bonds is denoted in nominal terms. In the model, I introduce three types of nominal default-free bonds in positive net-supply. All three bonds are infinitely lived and pay geometrically decaying nominal coupon payments, but they differ in their deterministic rate of survival δ_j. Bonds are usually finitely lived in the data, with fixed coupon payments and repayment of the principal at maturity. Introducing such bonds in the present framework is computationally unfeasible because it would require tracking each non-matured vintage. I borrow the concept of geometrically declining bonds from Woodford (2001). In Section 5, the parameter δ is chosen to match the Macaulay duration of representative bonds in the data, as in Hatchondo and Martinez (2009).

Corporate bonds: Geometrically decaying bonds are also used in the sovereign debt literature, as in Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012). While these frameworks study default, a default event triggers an immediate maturation of all outstanding bonds with limited repayment. In contrast, the present model does not study the default decision of a single issuer, but is interested in the credit risk of a diversified portfolio of corporate bonds. A small fraction of such a portfolio may default in each period, but never the whole set of bonds within it. I capture the partial reduction in duration and repayment through default by making the survival rate $\tilde{\delta}_t$ a stochastic function of the aggregate state. As discussed in more detail in Section 5, the functional form is well suited to capture the variance and skewness in default rates in the data.

Residual claim: The residual claim ensures that all payoffs are distributed despite the specific payoff models for stocks and bonds. In equilibrium, each agent will hold this asset because of the constraint that it can only be traded within risk aversion groups. The residual claim therefore captures background risk that investors have to hold, and it represents labor income in the later quantification.

4.2 Asset trading

Short-sale constraints: In the model, the structure and supply of assets matters because short-sale constraints prohibit the creation of additional assets, importantly also of those that are in zero-net supply. This restriction matters with respect to the possible gains from trade. In a complete market setting, risk-tolerant investors sell state-dependent insurance contracts to more risk-averse agents. They cannot do so in the present framework, but they can use short-maturity bonds as the second-best asset to provide that insurance. This relates to the data, in which the number of traded assets is limited, resulting in an incomplete payoff span.[6] In repurchase markets, nominal short bonds are the focal trading instrument between risk-averse investors, like money market funds, and more risk tolerant investors, such as hedge funds and investment banks. By relaxing the short-sale constraint for one-period bonds, the model captures the important role of such trades in the data. Extending the model to allow for collateralized short-sales of equity and bonds has limited quantitative effects, since such trades are largely orthogonal to the gains from trade arising from differences in risk aversion.

Limited commitment: In general, investors in the model are subject to short-sale constraints, but they can borrow funds to take on leverage by issuing collateralized bonds. The collateral constraint is necessary because investors have limited commitment with respect to promised repayments. In the data, collateral is used in numerous markets to limit credit risk exposure. For example, homeowners provide their house as collateral in order to obtain a mortgage, derivative traders post and demand cash collateral to ensure contractual payments, and borrowers provide collateral to credibly promise loan repayment in repurchase markets. The prevalence of collateral in credit relations highlights the importance of limited commitment in financial markets, which I therefore also assume by introducing collateral constraints. I assume no such constraint for the lender. In the tri-party repo market, collateral is held by the neutral third party, such that there is no opportunity for the lender to walk away with the collateral.

Default-free lending: I impose that collateralized lending contracts are default-free. While this is an endogenous outcome in binomial models, as shown by Geanakoplos (2010), there exists no such result for frameworks with more than two states. Endogenizing the default risk would complicate the model solution without obvious benefit. Given that the production of safety is a central goal of the lending agreement, and a primary focus of this paper in general, it is natural to assume the safety of collateralized loans. In the data, this safety is imposed with respect to a presumed worst-case scenario, which I capture by bounding the support of aggregate shocks.

Collateralizability: While I assume the production of safety in repurchase markets, the choice of collateral is an endogenous outcome of the model. Just as in the data, equity, corporate bonds and government-backed assets can be used as collateral. Given the assumption that there can be no default on collateralized loan, the model determines the collateralizability of each asset through the lowest possible nominal asset payoff in the next period. This stands in contrast to a specification in which haircuts are fixed and state-independent. The collateralizability of each asset class is calculated individually. This relates to the calculation of haircuts in the tri-party

[6]The question of why this is the case is outside of the scope of this paper. See for an introduction Duffie and Rahi (1995).

repo market in the data, where haircuts are defined for each class of collateral rather than each investor's portfolio. The model abstracts from the idiosyncratic risk of provided collateral. In the tri-party repo market, collateral provision is subject to concentration constraints that limit the idiosyncratic risk of collateral portfolios. The parameter ζ incorporates remaining idiosyncratic risk in a reduced form in the collateral constraint (3).

4.3 Investors

Time-varying risk aversion: The investors' preferences differ from the usual recursive utility framework of Epstein and Zin (1989) by assuming time-varying risk aversion. The differences in risk aversion generate gains from trade. For various reasons, including financial regulation, market participants in the data differ in their risk tolerance. This framework captures such differences in a reduced form. The risk aversion groups in the model represent financial institutions that are directly or indirectly held by households. The assumption of i.i.d. changes in risk aversion is one particular assumption of how earnings and portfolios are distributed across these institutions; it makes the model tractable by fixing the distribution of tradable wealth. Different from other work on collateralized lending, such as Brumm, Grill, Kubler, and Schmedders (2015) and Chabakauri (2014), my model abstracts from changes in the wealth distribution and instead highlights the quantitative importance of the asset supply for prices and lending volumes.

5 Quantification

I use the model to study how the supply of safe assets affects asset prices and portfolio decisions within the US financial market, with a particular focus on the repurchase market. This section discusses the quantification of the model parameters with respect to that goal. Section 6 presents two applications based on this calibration. All parameters, apart from those that govern the supply of safe bonds, remain fixed across these applications. The model is solved on a weekly frequency.

5.1 Growth and inflation process

To estimate the stochastic process defined in (1), I assume that the temporary shocks $\eta_{g,t}$ and $\eta_{\pi,t}$ are linear combinations of two serially uncorrelated i.i.d. shocks, $\epsilon_{1,t}$ and $\epsilon_{2,t}$:

$$\begin{bmatrix} \eta_{g,t} \\ \eta_{\pi,t} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}.$$

The persistent shock $\eta_{z,t}$ is also serially uncorrelated, and furthermore uncorrelated with the two temporary shocks. $\epsilon_{1,t}$, $\epsilon_{2,t}$ and $\eta_{z,t}$ are truncated standard-normally distributed. The truncation ensures that there is a lower bound of possible payoff risk that is taken into account when calculating the collateralizability of individual assets in the model.

To choose the truncation level, I use results from a survey among financial market participants that was undertaken by the Committee on the Global Financial System (2010), a committee of the Bank for International Settlements. According to the survey report, haircuts are usually calculated by estimating the value-at-risk over a 10-day liquidation period at a 95-99% confidence level. In the model, two periods represent 10 business days. Instead of calculating the value-at-risk over two periods, I estimate the value-at-risk using bounds within a single period that represent twice the size of the 99% confidence band. I therefore employ a symmetric truncation at ± 5.15 standard deviations. I assume that the worst case scenario is calculated individually over each shock. Allowing for a small idiosyncratic component in equity and corporate bond portfolios, the targeted haircuts are 2% for safe bonds, 6% for corporate bonds and 8% for equity, based on haircut data from the tri-party repo market.

Given these distributional assumptions, I estimate the stochastic process using quarterly consumption and price data from the National Income and Product Accounts (NIPA), provided by the BEA. The sample period is 1947:Q1 to 2016:Q2.[7] I use a method of simulated moments to directly fit data moments to the numerical simulation of the stochastic process. The process has eight parameters, and I estimate 25 target moments from the data: the average quarterly growth rates of consumption and price index, the respective standard deviations, the contemporaneous correlation and the first 10 autocorrelations of both consumption and price growth. Panel A in Table 1 and Figure 2 summarize the fit of the estimated process to the target moments, and Panel B reports the estimated parameters.

Having estimated the parameters of the process, I use a Kalman filter to estimate the aggregate state of the economy. This estimate $\hat{z}_t$ will serve as an input to the model when studying its conditional asset price implication for the time period 1990 to 2015. To match the collateralizability of equity in the data, I assume an additional rare payoff shock $\underline{\eta}_g = -5\%$ that occurs in expectation every 50 years.

5.2 Asset payoffs

The asset structure in the model is defined by the dividend payoff parameters $\bar{\alpha}$, κ_D and $\bar{z}_D$, the supply of short-, medium- and long-maturity government bonds, $\bar{a}^S$, $\bar{a}^M$ and $\bar{a}^L$, the corresponding duration parameters δ_M and δ_L, as well as the supply of corporate bonds $\bar{a}^B$ and their payoff parameters κ_B and $\bar{z}_B$.

[7] Appendix C.1 describes the used data and methodology in more detail.

Table 1: Stochastic process of consumption growth and inflation.

Panel A: Estimated moments

	Data	Model
$\bar{g}$	0.45%	0.45%
$\bar{\pi}$	0.85%	0.85%
σ_g	0.53%	0.53%
σ_π	0.68%	0.68%
$\rho_{g,\pi}$	−16.10%	−15.95%

Panel B: Estimated parameters

Parameters			
λ_{11}	1.44×10^{-3}	ξ_g	2.44×10^{-5}
λ_{12}	7.30×10^{-4}	ξ_π	-6.21×10^{-5}
λ_{21}	8.33×10^{-4}	μ_g	$1.82\%/52$
ϕ	$0.66^{1/52}$	μ_π	$3.37\%/52$

Note: Panel A: Empirical and estimated moments of quarterly consumption growth and inflation. Data: NIPA; Model: Numerical simulation of stochastic process on weekly frequency. Moments estimated based on aggregated quarterly growth and inflation rates. Panel B: Parameters of the stochastic process estimated using the numerical simulation and SMM.

Dividend parameters: The two parameters κ_D and $\bar{z}_D$ govern variance, skewness and kurtosis of dividend growth $g_d = \log(D_{t+1}/D_t)$. I use monthly data on stock market returns and T-bill rates from the Center for Research in Security Prices (CRSP) to estimate these three moments in the data. Following Hodrick (1992), I construct an annualized dividend series and calculate real dividend growth rates using the BEA CPI series. The sample period is December 1926 to December 2015. The left column of Table 2 presents the estimated moments. As before, I use a method of simulated moments to directly target moments of the numerical simulation of the stochastic process. The estimated model moments are in the right column of Table 2. The estimated parameters are $\kappa_D = -0.1915$ and $\bar{z}_D = 17.03$.

Table 2: Dividend growth in data and model.

	Data	Model
std(g_d)	11.22%	11.03%
skew(g_d)	−91.29%	−33.78%
kurt(g_d)	826.88%	827.89%

The estimates of κ_D and $\bar{z}_D$ are independent of the choice of the third dividend parameter $\bar{\alpha}$. Together with the endogenous payments to government bonds τ_t, and corporate bonds ι_t, this parameter determines what fraction of aggregate output is paid to the residual claim. Every investor holds in equilibrium a fixed share of the residual claim, and I therefore associate it with non-tradable labor income in the data. As is common in macroeconomics, I target a labor-share of 2/3. The bond payments τ_t and ι_t depend on several other parameters, in particular on the bond

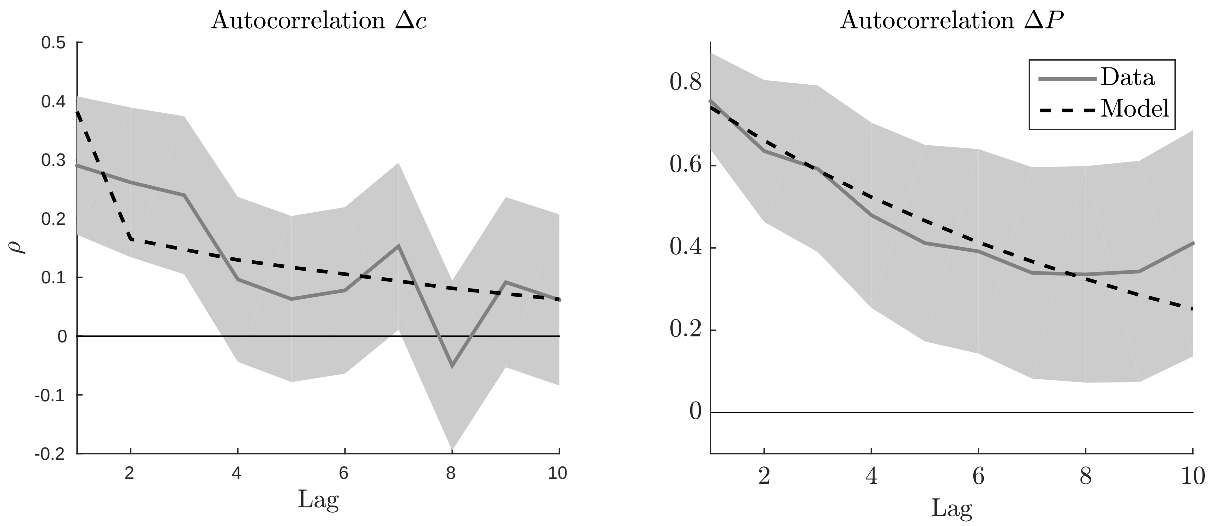

Figure 2: Empirical and estimated autocorrelations of quarterly consumption growth and inflation.

supply parameters $\bar{a}_t^j$, but for all used calibrations, a choice of $\bar{\alpha} = 0.33$ matches the labor-income share reasonably well.

Government bonds There are three types of safe bonds in positive net-supply: short-, medium- and long-maturity. The short bond represents default-free bonds in the data with a maturity of less than 6 months, the medium bond represents such bonds with a maturity of more than 6 months and less than 3 years, and the long bond represents all default-free bonds with a maturity greater than 3 years. Following Hatchondo and Martinez (2009), I choose δ_M and δ_L to target Macaulay duration measures. Given a stream of payments $\{\Pi_t\}_{t=1}^T$, the Macaulay duration is given as

$$MD = \frac{\sum_{t=1}^T t \cdot PV(\Pi_t)}{Q_j} = \frac{\sum_{t=1}^T te^{-yt}\Pi_t}{Q_j},$$

where Q_j is the price of the asset, and where I assume that the present value $PV(\Pi_t)$ is calculated using a constant yield y. I use the 10-year Treasury as the representative long-maturity bond in the data. Government bonds in the data pay semi-annual coupon payments and I assume that the coupon rate equals the yield, which I assume to be 5%. The Macaulay duration of the 10-year Treasury in the data is then 7.5 years. The duration of the geometrically declining bonds in the model is

$$MD(\delta) = \frac{\sum_{t=1}^\infty \frac{t}{52} e^{-\frac{y}{52}t}\delta^{t-1}}{\sum_{t=1}^\infty e^{-\frac{y}{52}t}\delta^{t-1}} = \frac{1}{52}\frac{1}{1-\delta e^{-\frac{y}{52}}}.$$

Given the annual yield of 5%, a choice of $\delta_L = 0.9986$ yields the empirical duration of 7.5 years.[8] I choose $\delta_M = 0.9881$ to target a duration of 1.5 years.

[8] To understand the importance of the assumed yield, consider an alternative yield of 4%. The duration of the 10-year bond is 8.33 years and the duration parameter is then $\delta_L = 0.9985$.

Corporate bond The difference between the default-free decay rate δ and the stochastic decay rate $\tilde{\delta}_t$ represents credit losses of the diversified corporate bond portfolio in the model. I choose lower investment grade corporate bonds as the corresponding bond category in the data, which are bonds that carry a BAA rating in Moody's rating scale, or a BBB rating in the rating scales of S&P and Fitch. In the data, credit losses occur when an issuer defaults and recovery rates are below 100%. I choose the parameters κ_B and $\bar{z}_B$ to match the level and distribution of loss rates in the data. Moody's (2016) provides historic transition and recovery rates in its "Annual Default Study". Using loss rates only within the BAA rating category would underestimate credit risk, because part of the credit risk of a BAA bond is the possibility of a downgrade to lower rating categories with higher loss rates. Instead, I use Moody's transition matrix to estimate the credit losses of a diversified portfolio of 20-year BAA bonds over its lifetime. I use a constant yield of 6.4% to calculate the present value of these losses, which is the average yield of the 10-year Treasury between 1962 and 2015.[9] The corresponding model moment is

$$E[\ell_\infty] = E_0\left[\sum_{t=1}^{\infty} e^{-yt}\left(\delta_L^{t-1} - \prod_{s=1}^{t-1}\tilde{\delta}_t\right)\right],$$

which defines the present value of credit losses relative to the default-free long-maturity bond.

Default rates in the data are highly skewed. In many years, no BAA rated bond defaults, but during economics downturns, default rates and downgrades spike. The skewness of annual loss rates within the BAA rating category is 1.69 between 1983 and 2015. I assume that this is a representative measure of the skewness, even though this measure does not capture the risk of downgrades. I define the annual loss rate in the model as

$$\ell_t = \log\left(\delta^{52}\right) - \log\left(\prod_{s=1}^{52}\tilde{\delta}_{t+s}\right),$$

and estimate its skewness using the numerical simulation. The estimated parameters are $\kappa_B = -0.0886$ and $\bar{z}_B = 105.51$. Table 3 reports the fit of the model for the two targeted moments and the untargeted kurtosis of the annual loss rate.

Table 3: Estimated moments of default risk in data and model.

	Data	Model
$E[\ell_\infty]$	4.15%	4.15%
skew(ℓ_t)	169.64%	169.64%
kurt(ℓ_t)	408.87%	484.75%

[9] Source: Constant maturity series, Table H.15, Board of Governors of the Federal Reserve Board.

5.3 Asset supply

Government bonds in the model represent nominal default-free securities in the data. I assume that all securities that are bearing explicit guarantees of the US government fall into this category, which includes all US Treasury debt as well as bonds issued by the Government National Mortgage Association (GNMA). Debt securities issued by other government-sponsored enterprises (GSEs) are not explicitly guaranteed by the US government, but at least until 2008, investors assumed an implicit government guarantee for these assets.[10] I therefore also consider all senior debt and mortgage-backed securities that are issued or guaranteed by GSEs as default-free. Only few corporate bond issuers bear an AAA rating, and I therefore abstract from default-free medium- and long-maturity corporate debt.[11] There are substantial amounts of short-maturity private debt that are considered default-free, such as highly rated commercial paper, and I therefore include a measure for these assets when estimating the supply of safe assets in the data. Before 2008, large amounts of asset-backed securities (ABS) were considered default-free, and I include an estimate of the supply AAA-rated ABS.

Given the above classification, I estimate the supply of the three default-free bonds in the data using Financial Accounts data provided by the Federal Reserve. I consider asset holdings by all domestic non-bank sectors. The Financial Accounts provide only limited detail on asset maturities, and I therefore create an estimate of the maturity distribution using data on outstanding Treasury debt from TreasuryDirect and CRSP.[12] I assume that agency debt not held by money market funds is equally divided between medium- and long-maturity debt, which approximates the average maturity structure of Treasury debt. Figure 3 summarizes the estimated values of safe assets relative to aggregate consumption. Because these estimates are based on the value of assets, they conflate changes in units and prices.

Figure 3 highlights that the asset supply of those three asset classes varies widely between 1990 and 2015. I subdivide this period into 13 episodes, each with a length of two years. When studying changes in the supply of safe assets over the business cycle in Section 6, I solve for the stochastic equilibrium separately for each episode. I set the asset supply parameters $\bar{a}_S$, $\bar{a}_M$ and $\bar{a}_L$ such that within each episode the average value of safe assets, conditional on the aggregated state, matches the estimated asset supply in the data. Connecting the episodes, I derive time-series implications for lending volumes, asset prices and portfolio holdings. Incorporating a stochastic model of the dynamics of the safe asset supply provides an interesting extension, but I cannot provide a confident estimate of a stationary stochastic process governing these supply changes. Investors in the data

[10] See, for example, Frame and White (2005).

[11] Since April 2016, there are only two AAA-rated corporations, Microsoft Corp. and Johnson & Johnson. See Ailworth and Hufford (2016).

[12] Using System Open Market Account (SOMA) holding data, I exclude Treasury securities held by the Federal Reserve.

face the same challenge, and I therefore assume that they expect the asset supply to stay constant as a share of nominal consumption. In the second quantitative application, I analyze the effects of a fully anticipated variation in the safe asset supply.

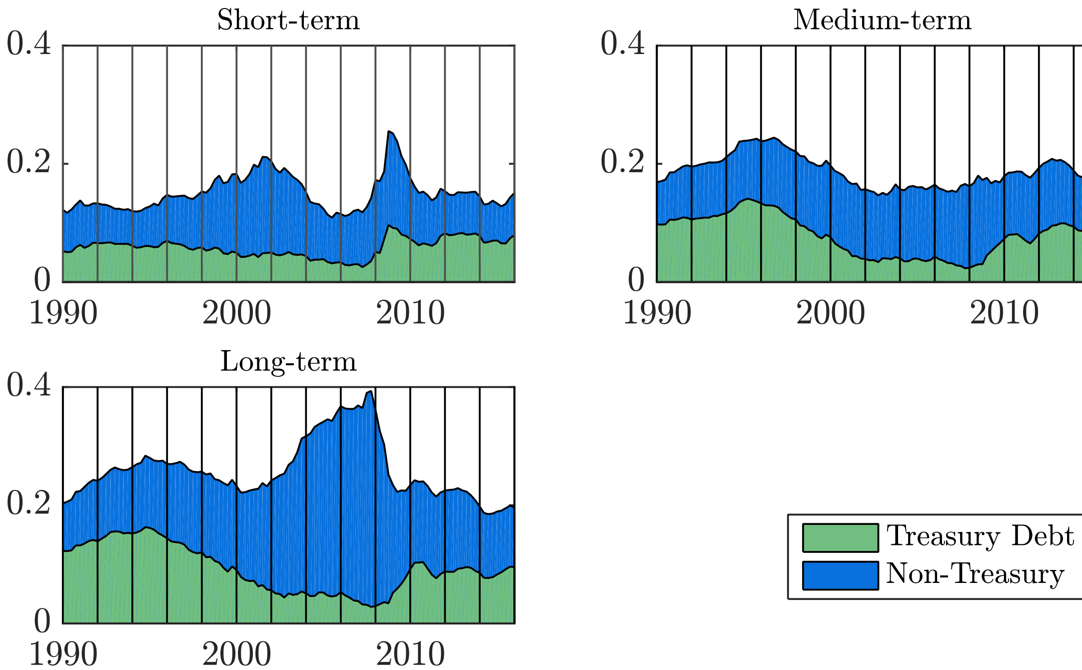

Figure 3: Estimated value of short-, medium- and long-maturity safe bonds relative to aggregate consumption of non-durables and services. Estimates are based on asset holdings of all US non-bank sectors (except federal pension funds) in the Financial Accounts. Safe bonds sum estimates for Treasury debt, agency debt, AAA-rated ABS, as well money market fund holdings of commercial paper.

To choose $\bar{a}_B$, I estimate the defaultable bond supply using Financial Accounts data on corporate and municipal bond holdings in all domestic non-bank sectors. I also include commercial paper not held by money market funds, because I abstract from a separate introduction of short-maturity risky debt. Between 1990 and 2015, the value of corporate bonds relative to aggregate consumption increases from 80% to 115%, and averages 95%. The increase is driven by changes in the Flow of Funds, in particular the reporting of municipal bonds, and by an increase in corporate bond issuance. Rather than adjusting $\bar{a}_B$ across episodes, which would also require a change in the dividend share parameter $\bar{\alpha}$ over time, I hold this parameter fixed and target an average value of 95% across episodes. Because both corporate bonds and equity will always be priced by the intermediate agent with γ_2, this assumption has little quantitative importance.

5.4 Preferences and Frictions

Collateralized lending The collateralizability of assets in period t depends on the endogenous distribution of payoffs in period $t+1$ as well as the parameter ζ, which determines what fraction of the next period's payoffs of equity and corporate bonds can be pledged as collateral. In the data, investors face concentration limits when providing portfolios of stocks or corporate bonds as collateral.[13] These constraints reduce idiosyncratic risk, but cannot fully eliminate it. I choose $\zeta = 0.97$ to match haircuts of corporate bonds and equity in the data. In the data, government bond portfolios are not subject to idiosyncratic risk and liquidity risk is small; I assume therefore that all payoffs of government bonds can be pledged as collateral. I choose the issuance fee f to be $3bp$. Given all other parameters, this choice matches the spread between the repo rate and the T-bill rate in the data.

Preference parameters There are nine preference parameters. The four risk aversion coefficients, γ_1, γ_2, γ_3 and γ_4, the discount factor β, the intertemporal elasticity of substitution ρ, as well as the three type probabilities p_1, p_2 and p_3. These parameters have no direct equivalent in the data, and I identify them through their effect on asset prices. To do so, I pick two episodes in the data, 1992-93 and 1994-95, and target average conditional interest rates in the model and data. I choose these two episodes because they provide some variation in the asset supply and because the estimated aggregate state is close to its steady state value during those years. While there is no one-to-one relation between a single parameter and a specific data moment, the risk aversion coefficients have a direct effect on the interest rates of both defaultable and and non-defaultable bonds. Table 4 summarizes the chosen risk aversion coefficients and related moments.

The intertemporal elasticity of substitution σ has a direct impact on the slope of the yield curve. As noted by Piazzesi and Schneider (2006), the average real yield curve is downward sloping in an expected utility model with persistent growth. Periods of low expected growth are associated with low real rates, and therefore high values of long-maturity bonds, making such securities a hedge against consumption risk. A low value of σ steepens the downward slope of the yield curve, because it increases the elasticity of real rates with respect to expected consumption growth. When estimating the stochastic process, I find, as Piazzesi and Schneider, that periods of high inflation are associated with low expected growth. This force counteracts the downward sloping real yield curve for nominal bonds because in bad times, inflation reduces the value of future nominal coupons payments. Given all other parameters, the latter effect induces an unconditionally upward sloping yield curve if the intertemporal elasticity of substitution is high enough. I find that a value of $\sigma = 1.5$ explains term spreads in the data well. There is substantial disagreement on the best choice of σ, but recent work on asset pricing with Epstein-Zin utility and time-varying consumption

[13] See Copeland, McLaughlin, Duffie, and Martin (2012b).

volatility suggests that the IES should be greater than 1.[14]

The type probabilities determine how much wealth is held by each investor group. I choose p_1 as the lowest value that still ensures that the most risk-tolerant investor is always able to purchase all long-maturity safe assets using leverage. Anticipating the application in the next section, this minimal value is chosen such that this criterion holds true given the asset supplies in all 13 episodes and given all values of the aggregate state within an interval of ± 2.5 standard deviations. The choice of p_2 determines the wealth of the group with intermediate risk aversion, which represents the large majority of investors. This value determines how limited the supply of safe assets is relative to the size of the risk-averse groups with γ_3 and γ_4, and it will therefore have an immediate impact on the price of the medium-maturity assets for which the marginal money market fund investor is sometimes, but not always, marginal. Finally, the parameter p_3 determines the wealth of the marginal money market investor with γ_3. I choose this parameter just large enough such that the investor group with γ_4 is never the marginal investor for any asset.

The time-discount factor β affects all interest rates. It also affects the price-dividend ratio in the model, because it determines the current valuation of future dividend payments. The model does not incorporate any force that induces large price-dividend volatility. A choice of $\beta = 0.97$ leads to an average PD ratio of about 30 across the two episodes, which is close to the average PD ratio of 32 between 1948 and 2015 that I measure using stock index return data from CRSP. Table 4 summarizes all chosen preference parameters and related moments, where it is important to note that all parameters affect all moments.

Table 4: Calibration of preference parameters.

Parameter			Moment	Data	Model
IES	σ	1.5	10y-3m spread (92-93)	3.1%	3.0%
time discount	β	0.97	PD ratio (48-15)	32.2	29.3
risk aversion	γ_1	0.25	10y-3m spread (94-95)	1.8%	2.3%
	γ_2	30	BAA (92-93)	8.4%	8.9%
	γ_3	80	3m T-bill (92-93)	3.3%	3.0%
	γ_4	120	3m T-bill (94-95)	4.9%	4.8%
type probabilities	p_1	0.1%	min. s.t. γ_1 holds all L	-	-
	p_2	93.2%	2y T-note (92-93)	4.3%	4.7%
	p_3	2.6%	min. s.t. γ_3 always holds some S	-	-

[14] For a summary of the debate on the right IES value, see section 4.6 in Bansal, Kiku, and Yaron (2012).

5.5 Numerical solution

The solution of the model poses several computational challenges. There are seven assets, one of them in endogenous supply. All prices are equilibrium objects that depend on the portfolio choices of four groups of agents who solve dynamic, constrained optimization problems. There are three stochastic shocks and the model is solved on a weekly frequency, which means that convergence is slow. It also implies that the stochastic process z has very high persistence, requiring a dense grid. The solution is simplified by the fact that the optimization problems are homogeneous in wealth, which allows me to aggregate agents with the same current risk aversion. The assumption that risk aversion type changes are i.i.d. further simplifies the problem by fixing the distribution of tradable wealth.

Given the high persistence of the state variable z, I solve the model on a grid with 36 nodes, covering ± 2.5 standard deviations of the unconditional process. Expectations are formed based on a three-dimensional quadrature rule, and I use linear inter- and extrapolation for points not on the grid. The quadrature rule takes the assumed truncation into account, following Burkardt (2014).

6 Results

Given the calibration described in the previous section, I use the model to study variations in the supply of safe assets in the data. In a first application, I analyze the effects of changes in the supply of short-, medium- and long-maturity safe assets over the period 1990-2015. In a second application, I study high-frequency variations around annual tax dates. Finally, I use the model to undertake counterfactual policy experiments in the time period after 2008.

6.1 Safe asset supply variation from 1990 to 2015

As depicted in Figure 3, the supply of safe assets varies over time and across maturities. These dynamics, together with the estimated time series of the aggregate state, provide a source of exogenous variation. In the model, the asset supply is determined by the exogenous parameters $\bar{a}_j$ and is fixed as a share of nominal output. To capture the variation in the data, I subdivide the sample period 1990-2015 into periods of two years. In each episode, I measure the average values of safe assets in the data. The quantity parameters $\bar{a}_j$ match the average supply of short-, medium- and long-maturity safe assets.

6.1.1 Lending volumes and asset holdings

The model generates a time series of lending volumes. Figure 4 compares collateralized lending in the model to the lending volume in repurchase markets in the data. Collateralized lending using long safe assets matches the dynamics of repo lending in the data. In the model, there is additional collateralized lending using corporate bonds. In the data, corporate bonds and stocks are rarely used as collateral in the general collateral repurchase markets. For a leveraged portfolio of such assets, hedge funds use margin accounts with their prime broker and bilateral repurchase agreements; such borrowing is therefore not captured in the data on brokers' and dealers' repo liabilities in the Flow of Funds.

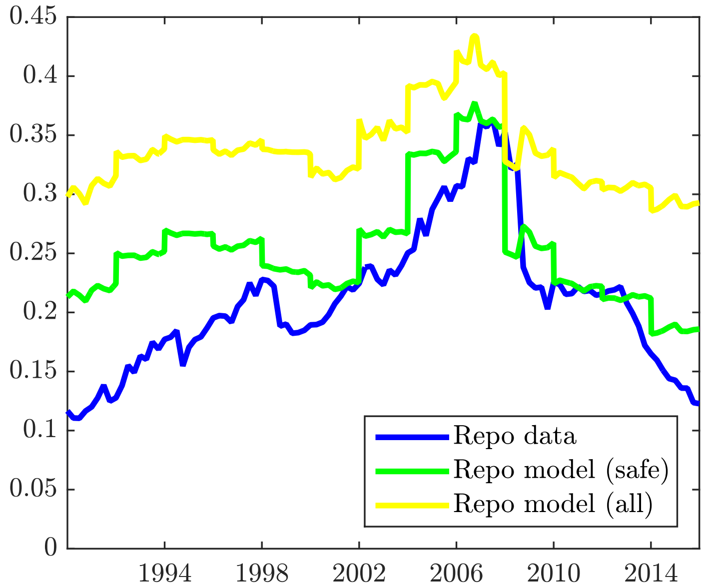

Figure 4: Comparison of collateralized lending volume in the model with volume in repurchase markets in the data. Total value relative to aggregate consumption. Data: Repurchase agreement liabilities of brokers and dealers from the Flow of Funds. Section 2 discusses the data in more detail.

As observed in the data, the model predicts a segmentation of the market for safe assets. Short-maturity safe assets are held by investors with the highest risk aversion, γ_3 and γ_4; long-maturity safe assets are held by the most risk-tolerant investors with γ_1, despite the higher expected returns of risky assets. The most risk-tolerant investor group with γ_1 is small, but leverage gives it a disproportionate importance in the market for safe assets: these investors hold all long-maturity safe assets and they produce a large fraction of short-maturity safe assets. The risk-tolerant group with γ_1 prefers to hold long safe assets because these assets have relatively high payoffs in all states and are therefore good collateral, which allows for high leverage. The most risk-averse agents lend funds by buying the safe one-period collateralized bonds. Collateralized lending produces all asset holdings of the most risk-averse group with γ_4 and, on average, 15% of the asset holdings of the second-most risk-averse group with γ_3. The intermediate agent with γ_2 holds equity and corporate bonds.[15]

6.1.2 Yield and spread volatility

In the model, short-maturity rates vary not only with the aggregate state z_t, but they are also affected by the supply of safe assets. If the exogenous supply of short safe assets falls, the marginal agent γ_2 holds a more risky portfolio and values safety more, such that the short rate falls. Even though long-maturity safe assets are, as in the data, held by a different group of agents, their supply also affects the short rate, because risk-tolerant agents use those assets in the production of short-term safety. The model therefore creates an additional channel for volatility of the short rate that is driven by the supply of all safe assets.

The additional short rate volatility is best seen in the first row of Table 5 which compares the yield volatility of the Treasury bill across data, the full model and the representative agent case for the period 1990 to 2015. For the representative agent case, I solve a single agent version of the model with risk aversion $\bar{\gamma} = \sum_{m=1}^{4} p_m \gamma_m = 34.5$. The volatility numbers of the representative agent are virtually the same as the volatilities in the full model with a fixed asset supply. As discussed in more detail below, the elasticity of the short rate is highest with respect to changes in the supply of short safe assets, because the collateral usage of long safe assets can be partially substituted by riskier securities like corporate bonds.

Table 5: Yield volatilities 1990-2015.

	Data	Model	Rep. A.
σ_{3m}	2.33%	1.13%	0.76%
σ_{10y}	1.81%	0.62%	0.22%
σ_{10-3m}	1.14%	0.66%	0.53%
σ_{BAA-10}	0.78%	0.64%	0.09%

Note: Row 1 based on constant maturity 3-month T-bill yield in the data, short bond in full and representative agent model. Row 2 based on yield spread between constant maturity 10-year Treasury yield in the data, long bond in full and representative agent model. Row 3 based on yield spread between constant maturity 10-year and 3-month Treasury yield in the data, yield spread between long and short bond in the model. Row 4 based on yield spread between Moody's BAA corporate bond yield and constant maturity 10-year yield in the data, yield spread between risky bond and safe long bond in the model. Representative agent has $\bar{\gamma} = \sum_{m=1}^{4} p_m \gamma_m$. All data: Selected Interest Rates (H.15), Federal Reserve Board, accessed through FRED.

The market segmentation also induces additional volatility for term and credit spreads. The risk-tolerant agents prefer long-maturity safe assets because they offer a small but positive term-

[15] Table 6 in Appendix B.1 reports the average portfolio holdings of each investor relative to their net-worth, as well as the average expected returns of each asset class.

premium and high collateralizability. However, there are less long safe assets than they can hold using leverage and given their net-worth, and the risk-tolerant agents therefore also hold corporate bonds. They bid up the price of long safe assets until they are indifferent between using long safe assets or corporate bonds as collateral. When the short rate falls, borrowing becomes more attractive and the risk-tolerant bid up the price of long safe assets even more, thereby reducing their yield. Corporate bonds are largely held (and therefore priced) by the intermediate agents, who do not take leverage. The price of corporate bonds is therefore unaffected by a change in the short rate, such that the spread between the corporate bond yield and the long safe yield increases. This mechanism leads to additional credit spread volatility that is not driven by default risk, but by the relative value of collateralizability. The fourth line of Table 5 compares the credit spread volatility between data, the full model and the representative agent case.

Because corporate bond rates are largely unaffected by changes in the short rate, they become a more attractive substitute for the risk-tolerant agent when borrowing rates fall. Long safe interest rates can therefore not fall as much as the short rate and the term spread therefore increases. This mechanism leads to additional volatility of term spreads, as seen in row 3 of Table 5.

6.1.3 Excess return predictability

Several authors, for example Fama and Bliss (1987), Campbell and Shiller (1991) and Cochrane and Piazzesi (2005), have noted that yield spreads and forward rates predict bond excess returns with a high R^2. This predictability violates the expectation hypothesis, which states that long rates represent the expected future path of short rates. The additional volatility in term spreads produced by the full model replicates this excess return predictability. I calculate expected excess returns in data and model as the realized expected excess return of a 2-year zero coupon bond relative to a 1 year zero coupon bond. I use CRSP data on zero coupon bond prices over the period 1964 to 2015. To derive zero coupon bond prices in the model, I use implied yields of geometrically decaying bonds with corresponding durations. I regress excess returns on the yield spread and find that in the model a 1% higher yield spread corresponds to 1.56% higher excess return over the next year. The R^2 of that regression is 11%. This corresponds well to the data, where I find a slope coefficient of 1.66 with an R^2 of 12%.

In the model, this excess return predictability is driven by changes in the relative value of collateralizability. When the supply of short-maturity safety is low, the short rate is low, because the marginal risk-averse investor holds a more risky asset portfolio and therefore values safety more. This implies that the short-term lending rate is lower and any long-short investment strategy becomes more attractive to a risk-tolerant investor. While he is now willing to pay a higher price for the long-maturity safe asset, he also more greatly values a leveraged portfolio of corporate bonds and equities. The latter assets are priced by the intermediate group of investors who are not using leverage. These assets will therefore not adjust in price, and hence become more attractive collateral assets. For the risk-tolerant investor to remain indifferent between corporate bonds and long-maturity safe bond, the expected return on long safe assets must not adjust as much as the short rate. This increases the term-spread and the expected excess return of long safe bonds. At the same time, excess returns of corporate bonds and equity are also higher, a feature that is further discussed in the following subsection.

6.1.4 Credit spreads in model and data

In the data, spreads between corporate bond rates and Treasury rates are large and volatile. Collin-Dufresne, Goldstein, and Martin (2001) study the empirical determinants of credit spreads and find that default risk has limited explanatory power. Similarly, Huang and Huang (2012) document that structural models of default can only explain a small fraction of the observed credit spread. Chen, Collin-Dufresne, and Goldstein (2009) replicate the results by Huang and Huang and note that this "credit spread puzzle" resembles the well known "equity premium puzzle". They study whether a pricing kernel reverse-engineered to explain the equity premium can also explain credit spreads. The authors find that a pricing kernel based on a utility function with habit formation, as in Campbell and Cochrane (1999), can do so. Key to this result are time-varying Sharpe ratios, which are not present in the long-run-risk literature with Epstein-Zin utility following Bansal and Yaron (2004).

My model proposes a new channel for large and volatile credit spreads that comove with expected excess returns. This channel is generated by the time-varying value of collateralizability. When the supply of short-maturity assets is low, the short rate is low, which makes a long-short investment strategy particularly attractive. Therefore, the price of long safe bonds rises, which lowers their yield and increases the credit spread. These are also periods of high expected excess returns on equity, because just as corporate bonds, equity is priced by intermediate agents who do not take leverage. Sharpe ratios are time-varying because excess returns are driven by the supply of safe assets. The supply of safe assets provides an additional source of variation that is separate from any variation in aggregate risk.

Figure 5 depicts the credit spread predicted by the model for the time period 1990 to 2015. The figure also reports the corresponding BAA spread in the data, as well as the credit spread predicted by the representative agent model. Confirming the "credit spread puzzle", the credit spread in the representative agent model is small and exhibits low volatility, while credit spreads in the data are large and volatile. The full model predicts the data well, except for the episode 2008-2009. It is important to highlight that both assets underlying this spread, the corporate and the long-maturity safe bond, are held and priced by the least risk-averse agent at all times. This agent has a risk aversion coefficient of 0.25, while the representative agent has a risk aversion coefficient of 34.5.

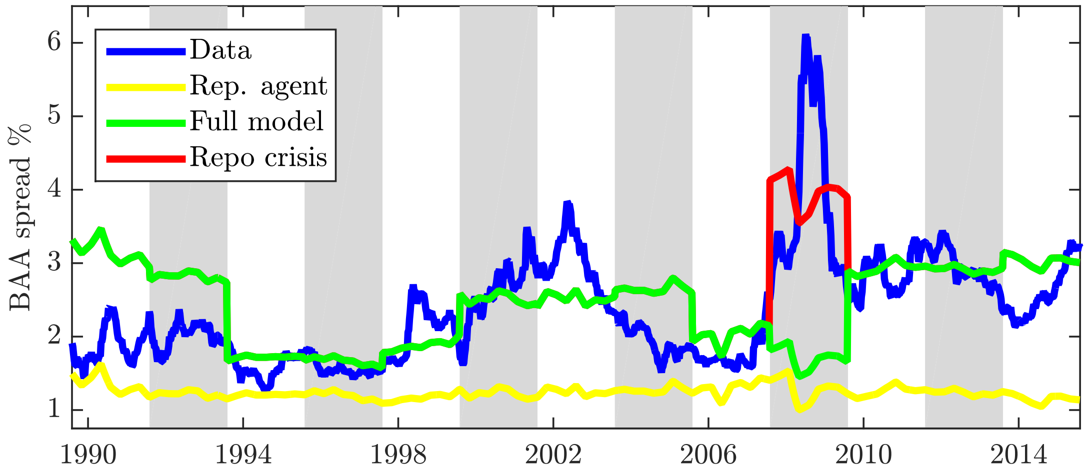

Figure 5: Comparison of BAA spreads in model and data. Repo crisis sets $\zeta = 0$ in 08-09.

The large and volatile spread in the full model is driven by the differential collateral value of the two assets, and not by default risk.

During the financial crisis of 2008, the functioning of repurchase markets was impaired. I test whether such a failure of repo markets can help us understand the large credit spreads observed during that period. To that extent, I assume that corporate bonds and equity are not collateralizable ($\zeta = 0$) in this episode. While this is a simplified view of the events of 2008, it captures the idea that collateralized borrowing using non-government-backed securities was limited during that period. The "Repo crisis" line in Figure 5 shows that such a repo failure can generate large credit spreads.

6.2 High-frequency supply variation around tax-due dates

The previous application used the model to study low-frequency changes in the supply of safe assets. The results showed that the model replicates the observed market segmentation and the dynamics in lending volumes over time. They also highlighted that the model provides a new and quantitatively important channel for generating yield and spread volatility, as well as time-varying risk premia. By the nature of macroeconomic time series data, the identification of these results is limited. I therefore turn to high-frequency supply variations that will not only better identify the studied mechanism, but also provide additional measures of the quantitative performance of the model.

6.2.1 Anticipated temporary deviations in the supply of T-bills

I exploit the fact that government debt varies around tax due dates, as documented in Greenwood, Hanson, and Stein (2015). Before tax-due dates, the Treasury issues short-maturity bills in anticipation of tax receipt inflows. After the due date, it uses tax income to repay Treasury bills. Figure 6 depicts this seasonal variation in the supply of T-bills around the mid-April tax date. The picture is based on auction data from TreasuryDirect. Based on that data, I measure the value of outstanding Treasury bills between 1986 and 2015. I compute the daily percentage deviation from the 120-day mean around the mid-April tax date in each year. The figure shows the average deviations over the time period. All other figures in this section are derived in the same manner.

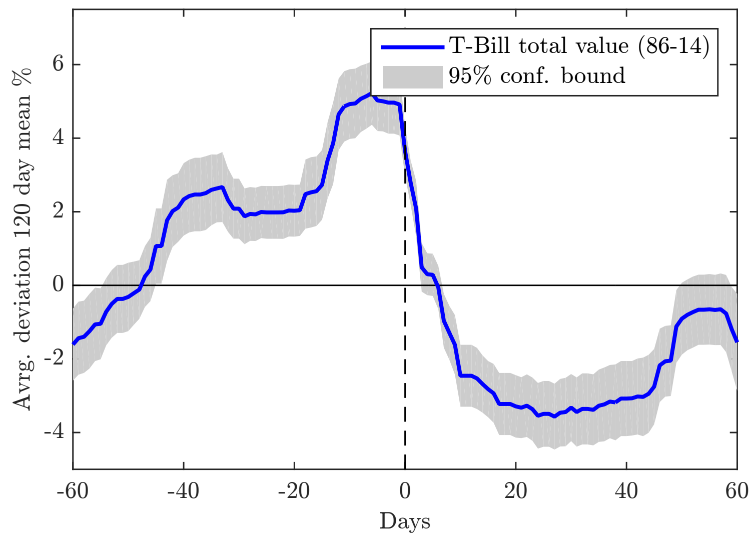

Figure 6: Average deviation from 120-day mean around mid-April tax date in percent.

There are other tax dates in the data, that create similar variations of smaller magnitude. In particular, corporate tax dates occur in mid-March, -June, -September and -December. The first two of those dates are visible in the aforementioned figure. These patterns are not present for Treasuries with longer maturity. I focus here on the mid-April tax date because it provides the largest variation in the data. The April variation in the supply of T-bills is a known phenomenon among market participants; see for example Skyrm (2013). Therefore, I study this effect as a fully anticipated variation in my model.

Effect on short-maturity Treasury yields: As documented by Greenwood et al. (2015), the variation in the supply of T-bills has an effect on short-term Treasury yields. Figure 7 depicts the average deviation of the 3-month T-bill rate from its mean during an 18 week window around the April tax date. The rate is high before the tax due date, when the supply of T-bills is high, and the rate falls as the period with a low T-bill supply approaches.

Figure 7 also depicts the same path of the average deviation of the 3m rate from its 18 week mean for model implied 3 month interest rate. I assume here that investors in the model anticipate a temporary deviation from of the period-specific asset supply which is equivalent in magnitude to the estimated average deviation in the data. To do so, I assume that investors in the model are the

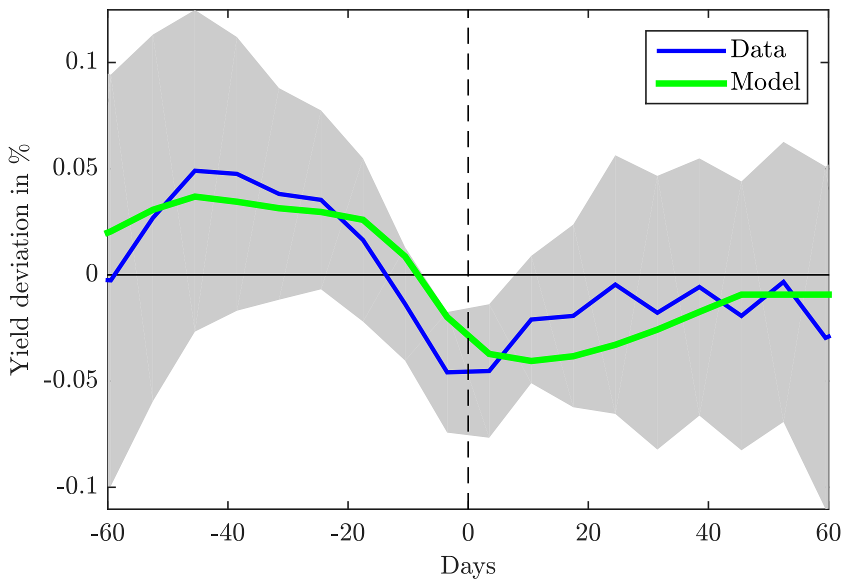

Figure 7: Treasury bill rates around the April tax date in data and model (1990-2015).

marginal investors in Treasury bills in the data, and therefore bear the total variation in absolute value observed in the data. The model creates yield deviations of the same order of magnitude and dynamic as the data.

Effect on collateralized lending rates: Using data from the GCF repo market, I also study the effects on collateralized lending rates. Because of the short sample, I remove noise by studying the spread between the collateralized GCF repo rate and the unsecured federal funds rate. As shown in Figure 8, the spread between the repo rate and the federal funds rate is high in the period before the tax date, when the supply of short-maturity safety is high. This higher spread suggests a lower premium for short-maturity safety. After April 15, market participants value the safety of secured lending relatively more than unsecured lending, so that the spread falls. These patterns are again well captured by the collateralized lending rate in the model.

6.2.2 Unanticipated government debt issuance

Even though there are no significant effects on long-maturity yields, I observe a significant variation in the corporate bond spread around April 15, as shown in Figure 9. This deviation represents large price changes because of the long maturity of the underlying assets. The temporary variation in the supply of T-bills cannot explain such large movements in the prices of long-maturity assets.

Using issuance forecasts of the US Treasury between 1992 and 2015, I document that there is substantial uncertainty in the total amount of tax receipts ahead of the tax due date. Unexpected tax inflows lead to a permanent change in government debt, and therefore a permanent shift in

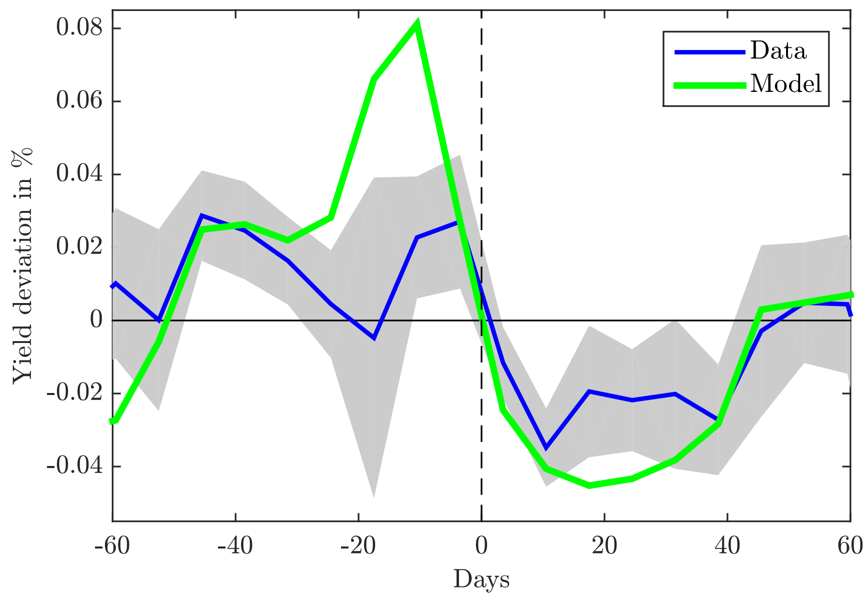

Figure 8: Collateralized lending rates around the mid-April tax date in data and model. The sample period is 2005 to 2015, excluding 2008 due to the disruptions in repo markets during that year.

the supply of safe assets. The model predicts that such changes have effects on the credit spread, because persistent variations in government debt change the collateral premium and therefore the relative value of good collateral, such as Treasuries, relative to less good collateral, such as corporate bonds. In the model, a permanent increase in the total value of short safe assets by 1% relative to total consumption reduces the credit on average by 4.1 basis points. This average is measured over the years 1992 to 2015 to capture the same time period as in the data. I regress changes in the BAA spread between the week before and after the tax due date on unexpected debt issuance. I find a significant negative relationship between the two series, as depicted in Figure 10. The slope of this relationship is 4.9 basis points.

6.3 Policy analysis

Between 2008 and 2014, the Federal Reserve implemented three rounds of quantitative easing during which it increased its holdings of US Treasuries, government agency debt and agency-insured mortgage-backed securities by about \$3.5 trillion. Between the second round of quantitative easing (QE 2, 2010-2011) and the third round (QE 3, 2012-2014), the US central bank sold short-maturity asset holdings and in exchange purchased assets with longer maturities. The latter policy is often referred to as “Operation Twist”. The explicit goal of these policies was to lower yields and raise asset prices.[16] There is a large empirical and a smaller theoretical literature studying the effects

[16] See, for example, Bernanke (2012).

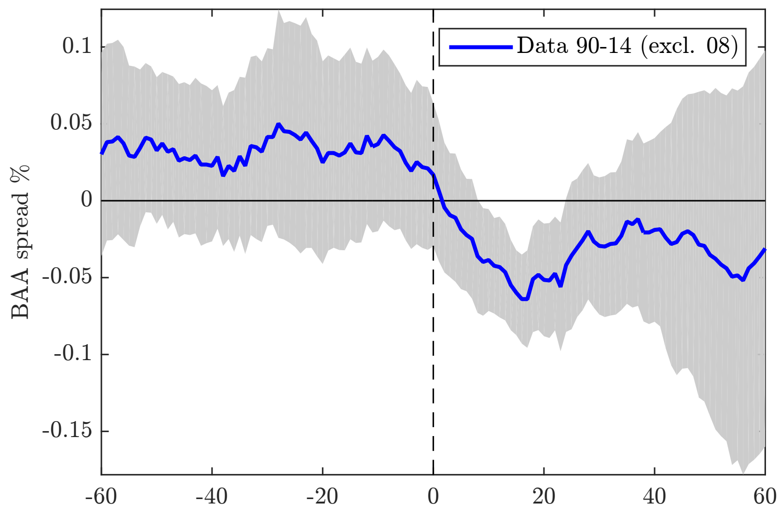

Figure 9: BAA spread around mid-April tax date.

of these market interventions, which I discuss in detail in the next section. In my framework, the supply of safe assets has effects on asset prices and, importantly, the supply of safe assets of different maturities has differential effects. Given the quantitative success documented in the previous sections, my model can be used for policy experiments to study the effects of changes in the supply of safe assets.

As an example of such policy experiments, I study the effect of changes in the supply of safe assets in the episode 2012-13. In a first experiment, I reduce the supply of long-maturity safe assets, such that the value of outstanding long safe assets falls by 5% of consumption. Such an experiment can be interpreted as a reduction in government debt and the chose quantity is similar in magnitude to the asset purchases during QE 2. Relative to the original yields, the T-bill yield falls by $28bp$, the long safe yield falls by $20bp$, while the corporate bond yield increases by $1bp$. The decline in the supply of long safe assets reduces the production of short-maturity safety by risk-tolerant investors. Because of the reduction in the endogenous supply of short-maturity safety, this intervention lowers short rates. It increases credit spreads through the increased value of collateralizability, and it also increases term spreads because of the partial substitutability of long safe assets and corporate bonds. The effect on the returns of risky assets is limited because they are priced by the large group of investors with intermediate risk aversion.

In a second experiment, I reduce the supply of long safe assets such that their outstanding value decreases by 5%, and increase the value of outstanding short safe assets by the same amount. This experiment represents an exchange of long-maturity government debt with short-maturity T-bills. Relative to the original yields, the T-bill yield now increases by $17bp$, the long safe yield rises by $11bp$, and the BAA yield falls by less than $1bp$. The effects of this experiment are reversed, because such an intervention actually increases the supply of short-maturity safety. While there are less

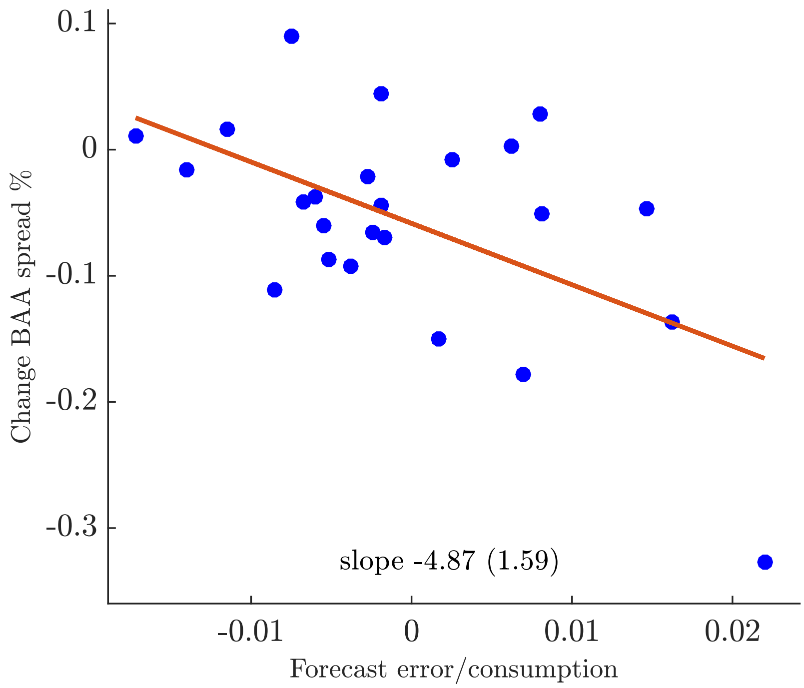

Figure 10: Unexpected debt issuance and changes in BAA spread before and after mid-April tax date.

long safe assets, there are now even more T-bills than the long safe assets previously produced in the collateralized lending market. Risk-tolerant investors substitute towards holding corporate bonds and produce additional short safety using those assets. It is unlikely that large-scale asset purchases of long safe assets in exchange for central bank reserves are equivalent to the above experiment in the short-term, because reserves can only be held by banks that have an account with the Federal Reserve. However, in the medium-term, reserves can be used to back deposits, which are short-maturity safe assets. The results in this section suggest that the effects of quantitative easing depend on specific implementation of such market interventions. A purchase of long safe assets not only reduces the supply of purchased long-maturity assets, but it also increases the supply of those assets that the central bank pays with. The nature of these assets determines not only the magnitude, but also the direction of the effects on asset prices.

7 Related Literature

My work is motivated by an empirical literature that documents a relation of the level of government debt and asset prices. Krishnamurthy and Vissing-Jorgensen (2012) show that the level of government debt is negatively correlated with yield spreads between corporate bonds and Treasury bonds. Greenwood and Vayanos (2014) find that a maturity-weighted measure of debt-to-GDP is positively correlated with the slope of the yield curve. Duffee (1996) finds a positive relationship between the supply of Treasury bills and the Treasury bill yield. This relation is further analyzed by Greenwood et al. (2015), who document that the supply of Treasury bills is positively correlated with deviations of the short-maturity yield from a yield predicted by a fitted yield curve model. The authors argue that the supply of short-maturity government debt has a larger effect on yields than changes in long-maturity government debt. They also highlight that private short-maturity debt in the form of commercial paper can substitute for short-maturity Treasury debt.[17]

My quantitative model replicates these findings. In the model, a low supply of safe assets induces high credit spreads because of the high collateral value of long safe bonds relative to risky bonds. The supply of short-maturity safe assets has a more direct effect on the level of short rates, because leveraged investors can also produce short-maturity safety using corporate bonds, instead of long safe assets as collateral. For the same reason, yield spreads and credit spreads are higher when the average maturity of government debt lengthens. My model features an endogenous substitution of safe assets with privately produced bonds. I study the same high-frequency, temporary variation of the T-bill supply around tax due dates, also analyzed by Greenwood et al. (2015). The model can explain the observed asset price effects on short rates documented by Greenwood et al. I also document additional price effects on collateralized lending rates and credit spreads. Using debt issuance forecasts by the US Treasury, I find that large unexpected tax inflows around tax due dates lead to permanent changes in the level of government debt. The model replicates the effects of these unanticipated, permanent shifts in the supply of safe assets on credit spreads.

My framework can be used to analyze the effects of large-scale purchases of safe assets by central banks. Estimates on the effects of such market interventions are either based on the empirical relations discussed above,[18] or on high-frequency studies that analyze price effects around the announcement of central bank purchases. Krishnamurthy and Vissing-Jorgensen (2011, 2013) find that QE 1 and QE 2 announcements lowered the yields of government-backed securities. Effects on corporate bonds were more limited, in particular during QE 2. Gagnon, Raskin, Remache, and Sack (2011) and D'Amico, English, Lopez-Salido, and Nelson (2012) also find that QE announcements lowered the yield of long-maturity treasuries. Swanson (2011) studies the effects of Operation Twist in 1961, a coordinated effort by the Treasury and the Federal Reserve to reduce the relative supply of long government debt. Also using a high-frequency event study approach, the author finds a moderate effect on long rates around announcement dates.

While such high-frequency studies provide additional identification, the results are based on a small number of observations with several concurrent policy announcements. It is also unclear to what extent short-term announcement studies can capture long-term effects of quantitative easing. My model complements the empirical results by providing a quantitative, structural model to study

[17] See also Greenwood, Hanson, and Stein (2010), Reinhart, Sack, and Heaton (2000), Cortes (2003) and Nagel (2016).

[18] See, for example, Hamilton and Wu (2012) and Li and Wei (2013).

the effects of large-scale asset purchases. The results are broadly consistent with the empirical literature, but my policy experiments highlight that the effects of large-scale asset purchases depend on how exactly the policy is implemented. In particular, a purchase of long-term safe assets in exchange for Treasury bills increases short rates, while a pure reduction in the supply of long-safe assets reduces short rates but increases the term spread.

There is smaller theoretical literature on the connection of the supply of safe assets and asset prices. With a representative agent, the supply of assets has no effect on asset prices unless it changes the aggregate consumption stream. For the particular case of government debt, aggregate consumption does not change because of the Ricardian equivalence formalized by Barro (1974). As discussed by Barro, this result can change when the government has some technological advantage or disadvantage in creating safe assets and collecting taxes. More recent work focuses on the effects of central bank asset purchases directly. In frameworks studied by Gertler and Karadi (2011), Cúrdia and Woodford (2011) and Moreira and Savov (2017), the central bank can alleviate balance sheet constraints by purchasing riskier securities in exchange for safe assets in times of financial distress. While these models help us understand the effects of central bank purchases of risky assets during a financial crisis, as arguably observed in the data during QE 1 (2008-09), these models predict no effects of purchases of long-maturity safe assets outside of a financial crises, as in the two later rounds QE 2 and QE 3.

Most theoretical frameworks studying purchases of government debt in normal times rely on exogenous market segmentation and are often labeled "preferred habitat" theories. The idea to model different investor groups with different exogenous portfolio preferences goes back to Tobin (1969). It has recently been revived by Andrés, López-Salido, and Nelson (2004) and Vayanos and Vila (2009). Applications based on these theories can be found in Hamilton and Wu (2012), Greenwood and Vayanos (2014) and Chen, Cúrdia, and Ferrero (2012). As noted by Krishnamurthy and Vissing-Jorgensen (2011), models of exogenous market segmentation need to specify along which asset characteristics investor preferences differ. A central bank purchase of the 10-year Treasury can have large effects in such models if there are investor groups with a strong preference for government debt with a maturity of 10 years, but it has small effects if investors have preferences for all long-maturity credit instruments, which is a much larger market. One also has to decide whether such preferences exist for all asset classes or just for government debt.

In contrast, this paper develops a theory of endogenous market segmentation. As explained above, the most risk-tolerant investors have an endogenous preference for long-maturity safe assets because of their high collateralizability. Surprisingly, riskier assets with higher expected returns are held by more risk-averse investors. This theory therefore complements the "preferred habitat" literature by determining which assets characteristics set market segments, and why such segmentation plays an important role for safe assets but not riskier securities.[19] The model also makes new predictions on how the supply in one market segment affects prices of other assets. For example, changes in the supply of long-maturity securities have an effect on the endogenous supply of safe short-maturity securities and, therefore, on their prices.

The model builds on previous theoretical work on collateralized lending, in particular on the idea that assets can be valued not only for their payoffs, but also for their usability as collateral. The corporate finance literature labels this asset characteristic "debt capacity"; see for example Shleifer and Vishny (1992). Holmström and Tirole (2001) derive the value of debt capacity or collateralizability in an asset pricing framework. Kiyotaki and Moore (1997) present a model of the real economy in which the interaction of collateral constraints and asset prices amplifies real shocks. Geanakoplos (1996, 2010) studies the determination of collateralized lending contracts in general equilibrium and analyzes the dynamics of prices and leverage. Brunnermeier and Pedersen (2009) analyze the interaction of endogenous variations in margin requirements and market liquidity.[20]

Within this strand of literature, my framework is most closely related to the work of Brumm et al. (2015), who study an environment in which agents differ in their risk aversion and trade a tree with stochastic payoffs. Agents trade collateralized loans, but only a fraction of tree shares can be used as collateral. The authors show that the collateralizable shares carry a collateralizability premium. The same premium is present in my framework, but different from previous work, there are multiple assets of different maturity and risk exposure and all assets can be used as collateral. These features allow me to apply the model to the data on repurchase markets and asset prices. To my knowledge, this is the first quantitative framework studying collateralized lending in financial markets.

This paper is related to Williamson (2016) who also studies the importance of government-backed assets as good collateral. In his framework agents use collateralized loans as forms of payment and short-maturity assets are the better collateral because they are subject to little price risk and therefore have a high degree of pledgeability. Closely related is the work of Venkateswaran and Wright (2014) and Rocheteau, Wright, and Xiao (2016). In my framework, collateral is used to produce short safe assets, and safe but long-maturity assets are the most desired form of collateral because they offer the ideal combination of excess return and collateralizability. In the data, long safe assets provide the most used form of collateral in repurchase agreements and, therefore, my framework is suited for the analysis of this central market for collateralized lending.

The importance of collateral in the creation of means of payment is also highlighted in the work of Piazzesi and Schneider (2015). In their framework, banks need collateral to back deposits which households use for payment transactions. Banks hold reserves and assets, trading off liquidity and

[19] For a different channel of endogenous market segmentation see Alvarez, Atkeson, and Kehoe (2002).

[20] See also Bernanke, Gertler, and Gilchrist (1996), Fostel and Geanakoplos (2008), Simsek (2013), Chabakauri and Han (2015) and Parlatore (2015).

collateral benefits. As in the above framework, assets are not only priced for their return but also for their collateralizability. The authors link the creation of money-like assets to the price level by incorporating an explicit end-user demand for payment transactions. Monetary policy affects asset prices and the price level by influencing the banks' portfolio choices. While my paper neglects the central role of safe short-maturity assets as means of payments, it complements the work of Piazzesi and Schneider by emphasizing the important role of safe but long-maturity securities in the creation of money-like assets.

My work shares some features with other papers analyzing the macroeconomic role of safe assets. Caballero and Farhi (2018) study an economy with two groups of agents, one being risk-neutral and the other infinitely risk-averse. Infinitely risk-averse agents can only hold risk-free claims issued by the risk-neutral investors who use aggregate payoff claims as collateral. When the collateralizability of the aggregate output claim falls, risk-neutral investors can borrow less and may have to scale down production. The introduction of a production margin within my framework provides an interesting extension. Interestingly, risk-tolerant investors hold mostly long-maturity government debt in my model. One of the main results of my quantitative analysis is that changes in the supply of safe assets have negligible price effects on risky assets, like equities and corporate bonds.

Barro and Mollerus (2014) study the safe asset share in an endowment economy with two investors of different risk aversion. In a complete market setting with two aggregate states, the authors determine the safe asset share as the value of risk-free one-period bonds that has to be sold by the more risk-tolerant investor to the more risk-averse investor to achieve the equilibrium risk allocation. Their work is motivated by the empirical work of Gorton, Lewellen, and Metrick (2012), who are finding a relatively stable share of safe assets in the economy because privately produced safe assets substitute government debt. My model also determines the private production of safe assets, but I incorporate an endogenous collateral constraint as well as short-sale constraints. These constraints and a real growth process subject to temporary and persistent shocks create a role for the capital structure of the economy. The exogenous supply of assets determines to what extent agents can share risk, and it therefore has implications for asset prices and allocations.

I study a dynamic equilibrium framework with endogenous asset prices. The model makes predictions not only on real rates, credit spreads and expected stock returns, but also on the level and slope of the nominal yield curve. I build on earlier work studying nominal yield curves in general equilibrium, as for example Piazzesi and Schneider (2006), Bansal and Shaliastovich (2013) or Swanson (2015). My work introduces heterogeneous investors and collateralized lending markets into a dynamic equilibrium framework. The quantitative solution replicates the observed dynamics of lending volumes in the data, and generates several asset pricing implications that have been documented in the data but that the standard representative agent model cannot generate. The time-varying collateral premium induces excess bond return predictability, as documented by Fama and Bliss (1987), Campbell and Shiller (1991) and Cochrane and Piazzesi (2005). The model also predicts sizable and volatile credit spreads that are largely unrelated to default risk, as documented in the data by Collin-Dufresne et al. (2001) and Huang and Huang (2012).

8 Conclusion

I study how quantities of safe bonds affect asset prices and lending volumes in financial markets. The answer to this question is important because central bank purchases of safe assets change the supply and maturity structure of safe assets, often with the explicit goal of changing the level of interest rates and asset prices. I argue that collateralized lending plays a central role in the market of both short- and long-maturity safe assets. Collateralized lending produces safe investment opportunity with short maturity for lenders. I document that borrowers hold leveraged portfolios of long safe assets which they provide as collateral to lenders. To capture this interaction of collateralized lending and the markets for safe assets, this paper develops a quantitative model of financial markets in which investors can borrow and lend against collateral. Claims on aggregate output are traded in the form of several assets of different maturity and risk exposure. Investors differ in their risk aversion, which creates gains from trade. Risk-tolerant investors prefer to hold long-maturity safe bonds, even though stocks and corporate bonds provide higher expected returns. This endogenous preference arises because long safe bonds earn a small term premium and because they are very collateralizable, allowing for high leverage. Risk-averse investors hold short- and medium-maturity safe assets, as well as collateralized bonds. Studying changes in the supply of safe assets between 1990 and 2015, the model not only replicates the observed segmentation of the market for safe assets, but it also matches the dynamics of lending volumes observed in the data. The endogenous market segmentation generates time-varying risk-premia, which induce excess return predictability as observed in the data. The model also generates large and volatile credit spreads that are mostly unrelated to default risk as documented in the empirical literature. I study high-frequency changes in the supply of government debt around the April tax due date in order to better identify the model mechanism. The model replicates the price effects of both anticipated temporary and unanticipated permanent supply shifts. The quantified framework provides a toolbox to study the effects of large-scale asset purchases. I find that the effects of such market interventions depend on the specific policy implementations and, in particular, on the characteristics of the asset that the central bank uses to pay for its purchases.

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Appendix A Model

A.1 Stationary equilibrium

As shown in section 3.4, the distribution of tradable wealth and the allocation of the non-tradable assets are constant across time and states because of the assumption that risk-aversion type changes are i.i.d. The state of the economy therefore reduces to $s_t = \{z_t, Y_t, P_t\}$. Y_t and P_t scale the real and nominal model outcomes, but do not affect allocations or relative prices. To derive a recursive, stationary equilibrium, I introduce the following definitions:

$$\begin{aligned} \tilde{C}_i(s_t) &= \frac{C_i(s_t)}{Y(s_t)} & \tilde{Q}^j(s_t) &= Q^j(s_t)P(s_t) \quad \forall j \in \{0, S, M, L\} \\ \tilde{Q}^E(s_t) &= \frac{Q^E(s_t)}{Y(s_t)} & \tilde{Q}^N_m(s_t) &= \frac{Q^N_m(s_t)}{Y(s_t)} \\ \tilde{W}_i(s_t) &= \frac{W_i(s_t)}{Y(s_t)} & \tilde{\Pi}^j(s_t) &= \frac{\Pi^j(s_t)}{Y(s_t)} \\ \tilde{a}^j_i(s_t) &= a^j_i(s_t) \quad \forall j \in \{E, B\} & \tilde{a}^j_i(s_t) &= \frac{a^j_i(s_t)}{P(s_t)Y(s_t)} \quad \forall j \in \{0, S, M, L\} \end{aligned}$$

Directly using the result that I can aggregate all investors with the same risk aversion type, I write the recursive optimization problem of group m with γ_m as

$$V_m(z) = \max \left((1-\beta)\tilde{C}_m(z)^{1-1/\sigma} + \beta \left[\sum_{n=1}^{4} p_n \mathbb{E}\left[(G(z')V_n(z'))^{1-\gamma_m} \right] \right]^{\frac{1-1/\sigma}{1-\gamma_m}} \right)^{\frac{1}{1-1/\sigma}},$$

subject to the budget constraint

$$\tilde{W}_m(z) \geq \tilde{C}_m(z) + \sum_{j \in J \setminus \{N\}} \tilde{a}^j_m(z)\tilde{Q}^j(z) + \tilde{a}^N_m(z)\tilde{Q}^N_{m,t}(z) - \tilde{F}(z),$$

where the issuance fee is $\tilde{F}(z) = -\min[0, f\tilde{a}^0_m(z)]$ and where wealth $\tilde{W}_m(z)$ is

$$\tilde{W}_m(z) = p_m \left(\sum_{j \in J \setminus \{N,E\}} \bar{a}^j \tilde{\Pi}^j(z) + \tilde{\Pi}^E(z) + \tilde{\Pi}^N_m(z) \right). \tag{6}$$

The optimization is furthermore subject to the short-sale constraints

$$\tilde{a}^j_m(z) \geq 0 \quad \forall j \in J \setminus \{0\}.$$

Equation (6) directly uses the fact that the wealth of each group is a deterministic function of z.

Definition A.1 (Recursive equilibrium) *A recursive equilibrium of the economy is defined by a stationary process z, and price functions $\tilde{Q}^j(z)$ for all assets j, as well as value functions $V_m(z)$ and portfolio policy functions $\tilde{a}_m^j(z)$ for all assets j and each investor group m, such that in each state z*

1. *the value functions and policy functions solve the above optimization problem, taking the prices $\tilde{Q}^j(z)$ as given,*

2. *all common asset markets clear,*

$$\sum_{m=1}^{4} \tilde{a}_m^0(z) = 0, \qquad \sum_{m=1}^{4} \tilde{a}_m^S(z) = \bar{a}^S, \qquad \sum_{m=1}^{4} a_m^M(z) = \bar{a}^M,$$
$$\sum_{m=1}^{4} \tilde{a}_m^L(z) = \bar{a}^L, \qquad \sum_{m=1}^{4} a_m^E(z) = 1, \qquad \sum_{m=1}^{4} a_m^B(z) = \bar{a}^B,$$

3. *and the group-specific markets of the residual claim clear,*

$$a_m^N(z) = p_m.$$

Appendix B Additional model results

B.1 Portfolio holdings

Table 6: Average portfolio holdings 1990-2015

	Risk aversion types				
Asset	γ_1	γ_2	γ_3	γ_4	$E[r]$
Repo	-3388%	0%	15%	100%	-0.5%
Short safe	0%	0%	36%	0%	0.1%
Medium safe	0%	0%	44%	0%	1.5%
Long safe	2609%	0%	0%	0%	2.7%
Equity	0%	92%	0%	0%	4.9%
Corp. bond	879%	8%	5%	0%	4.8%

Note: Average portfolio holdings of each investor group as a percentage of net-worth across all 13 episodes between 1990-2015. The right column reports the average conditional expected real returns of each asset class. Results are conditional on the estimated aggregate state $\hat{z}_t$.

Appendix C Quantification

C.1 Estimation of the stochastic process

The dynamics of the model are driven by two stochastic processes: the growth rate G_t, and the nominal price level P_t. I assume that the log-growth rate g_t is subject to a transitory and a permanent shock. In particular,

$$g_t = \mu_g + \xi_g x_t + \lambda_g \epsilon_t,$$

where

$$x_t = \phi x_{t-1} + \lambda_x \epsilon_t.$$

I define the growth rate of the price level as $\Pi_t = \frac{P_t}{P_{t-1}}$ and assume that its log-growth rate π_t is also driven by a transitory shock as well as the same permanent component x_t:

$$\pi_t = \mu_\pi + \xi_\pi x_t + \lambda_\pi \epsilon_t.$$

The vector of innovations, $\epsilon_t = [\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}]$, consists of three independently and identically distributed random variables with a truncated standard normal distribution. The truncation is symmetric and defined by the truncation parameter T, stating that the support of each element of ϵ_t is bounded between $-T$ and T. This assumption of a bounded support of ϵ_t ensures that the collateral constraint defined in Equation (3) yields a well-defined and positive borrowing limit.

Defining the vector of observables as $z_t = [g_t, \pi_t]'$, the stochastic model has the state-space representation

$$\begin{aligned} z_{t+1} &= \mu_z + \xi' x_t + \begin{bmatrix} \lambda_g \\ \lambda_\pi \end{bmatrix} \epsilon_{t+1}, \\ x_{t+1} &= \phi x_t + \lambda_x \epsilon_{t+1}. \end{aligned}$$

Without loss of generalization, I assume that

$$\begin{bmatrix} \lambda_g \\ \lambda_\pi \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ 0 & \lambda_{22} & \lambda_{23} \end{bmatrix} \quad \text{and} \quad \lambda_x = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

While the model is solved on a weekly frequency, I estimate the stochastic process with quarterly data, in particular consumption and price data from the National Income and Product Accounts (NIPA) from the BEA. I use data on the consumption of nondurables and services, and follow Piazzesi and Schneider (2006) in constructing a price index for that consumption bundle. The sample period is 1947:Q1 to 2016:Q2.

The above model has 10 parameters. Given that I use a numerical simulation when solving the model, I use a method of simulated moments to directly fit data moments to the numerical simulation of the stochastic process. To do so, I estimate 25 moments from the data: the average quarterly growth rates of consumption and price index, the respective standard deviations, the contemporaneous correlation, and the first 10 autocorrelations of both consumption and price growth. [21]

Since the model is overindentified, I need to choose a weighting matrix. I choose a diagonal matrix, where the diagonal elements are the inverse of the squared size of the respective 95% confidence bounds around the moments' estimates. This matrix has the desirable property of putting more weight on those moments that are estimated more precisely. I estimate a weekly AR coefficient of $\phi = 0.9937 = 0.973^{\frac{52}{12}}$.[22]

Despite its simplicity, the estimated process matches the empirical moments well. Table 1 summarizes the fit of growth rates, standard deviation and contemporaneous correlation. The estimation matches these five moments perfectly. Figure 2 compares the autocorrelation of data and model. The estimated process overstates the first order autocorrelation of consumption growth, but provides a good fit for the following lags. These latter autocorrelations are particularly important given the choice of Epstein-Zin preferences. The autocorrelations of inflation fit the data well. A more general process would allow for separate long-run components of consumption and inflation, as implemented by Piazzesi and Schneider.

C.1.1 Kalman filter

Given the estimated stochastic process, I use a Kalman filter to estimate z_t in the data. In the data, I observe growth and inflation on a quarterly frequency that aggregates 13 weeks. I denote with s the quarterly time index, so that $s = t/13$ for $t \in \{13, 26, 39, ...\}$. The quarterly growth rate in the data is $g^q_s = \sum_{t=(s-1)13+1}^{s13} g_t$ and similarly $\pi^q_s = \sum_{t=(s-1)13+1}^{s13} \pi_t$. For simplification, I assume instead that $g^q_s = 13g_s$. I define a new quarterly process as

$$[g^q_s, \pi^q_s]' = \left[\mu^q_g, \mu^q_\pi\right]' + \xi^q z_s + \Lambda^q \left[\epsilon_{1,s}, \epsilon_{2,s}\right]'$$
$$z_s = \rho z_{s-1} + \sigma^q_z \epsilon_{3,s},$$

with $\xi_q = 13\xi$, $\Lambda^q = \sqrt{13}\Lambda$, $\mu^q = 13\mu$, $\rho = \phi^{13}$ and $\sigma_z = \sqrt{\frac{1-\phi^{26}}{1-\phi^2}}$. As before,$\epsilon_{1,s}$, $\epsilon_{2,s}$ and $\epsilon_{3,s}$ are three independent truncated standard normally distributed shocks.

[21] Given the peculiarities of the population data noted by Piazzesi and Schneider, I estimate the moments using aggregate data and only afterwards, I adjust the consumption growth rate by the average population growth over the sample period.

[22] This is less persistent than the monthly AR coefficient of 0.979 used in Bansal and Yaron (2004) and the monthly estimate of 0.99 in Schorfheide, Song, and Yaron (2018).

I redefine a new state-space system as

$$
\begin{aligned}
y_s &= [g_s^q, \pi_s^q]' - \mu^q = \xi^q z_{s-1} + \Lambda_q \left[\epsilon_{1,s}, \epsilon_{2,s}\right]' \\
z_s &= \phi^q z_{s-1} + \sigma_z \epsilon_{3,s}
\end{aligned}
$$

The prediction equations are

$$
\begin{aligned}
\hat{z}_{s|s-1} &= \rho \hat{z}_{s-1|s-1} \\
\hat{y}_{s|s-1} &= \xi^q \hat{z}_{s|s-1}
\end{aligned}
$$

The variance of the prediction error is

$$
P_{s|s-1} = \rho^2 P_{s-1|s-1} + \sigma_z^2
$$

The updated prediction given y_s is

$$
\hat{z}_{s|s} = \hat{z}_{s|s-1} + P_{s|s-1}\xi_q' \left(P_{s|s-1}\xi_q\xi_q' + H\right)^{-1} (y_s - \xi \hat{z}_{s|s-1}),
$$

with $H = \Lambda_q \Lambda_q'$.

And its prediction error is

$$
P_{s|s} = P_{s|s-1} - P_{s|s-1}^2 \xi_q' \left(P_{s|s-1}\xi_q\xi_q' + H\right)^{-1} \xi_q
$$

I abstract from the truncation and treat $\epsilon_{1,s}$, $\epsilon_{2,s}$ and $\epsilon_{3,s}$ as standard normally distributed. I initialize the system by setting $\hat{z}_{0|0} = 0$ and $var(\hat{z}_{0|0}) = P_0 = \frac{\sigma_z^q}{1-\rho^2} = \frac{1}{1-\phi^2}$.