Optimal Monetary Policy in Production Networks

Jennifer La’O      Alireza Tahbaz-Salehi

March 26, 2021
In the canonical New Keynesian model,

- optimal policy: stabilize the aggregate price level

- why? price stability preserves productive efficiency and implements the first best

  - price stability minimizes both inflation and the “output gap”

- target is straightforward in the model: aggregate price level = average price across firms
But the real world is much more complex.

- multiple, heterogeneous sectors that interact in a network of intermediate good trade

- how should the aggregate price index depend on:
  - whether sectors produce final goods or intermediate inputs? e.g. CPI vs. PPI?
  - the relative position of sectors in the input-output network?
  - differences in the relative price flexibility of sectors?
  - changes in the relative size of sectors? e.g. healthcare and services
Our Question

How does the multi-sector, input-output structure of the economy affect the optimal conduct of monetary policy?
Our Framework

  - input-output network of intermediate good trade across sectors
  - sectoral productivity shocks $\rightarrow$ underlying flex-price economy is efficient

- firms face nominal rigidities
  - must set nominal prices before observing demand
  - informational friction, à la Woodford (2003), Mankiw Reis (2002), Angeletos La’O (2020)
Our Results

- Divine Coincidence is non-generic
  - efficient allocation cannot be implemented under sticky prices

- Optimal policy stabilizes an optimal price index with greater weight on:
  - larger sectors (as measured by Domar weights, i.e. sales shares of GDP)
  - stickier sectors
  - more upstream sectors, sectors with stickier customers, sectors with more flexible suppliers

- Quantitative welfare improvements from adopting the optimal policy
  - we calibrate the model: BEA US input-output tables + data on price stickiness
  - CPI stabilization $\rightarrow$ optimal policy $\approx$ welfare gain of .5 percentage point of quarterly consumption
Related Literature

- **production networks**
  - **markups and misallocation:** Jones (2013), Bigio and La’O (2020), Baqae and Farhi (2020)

- **monetary policy in multi-sector New Keynesian models**
  - **multi-sector:** Mankiw and Reis (2003), Eusepi, Hobijn, Tambalotti (2011)
  - **w/intermediate good trade:** Basu (1995), Huang and Liu (2005)

- **informational frictions as nominal rigidities**
The Environment
The Environment

- static environment

- production: \( n \) sectors indexed by \( i \in I \equiv \{1, \ldots, n\} \)
  - input-output network of intermediate good trade across sectors

- continuum of identical firms within a sector, indexed by \( k \in [0, 1] \)
  - firms produce differentiated goods \( \rightarrow \) monopolistic competitors
  - firm managers make nominal pricing decision under incomplete info
Technology

- CRS production function of firm $k$ in sector $i$
  \[
y_{ik} = z_i F_i(\ell_{ik}, x_{i1,k}, \ldots, x_{in,k}) = z_i \ell_{ik}^{\alpha_i} \prod_{j \in I} x_{ij,k}^{a_{ij}}
\]

  ▶ input-output matrix $A = [a_{ij}]$

- nominal profits
  \[
  \pi_{ik} = (1 - \tau_i) p_{ik} y_{ik} - w \ell_{ik} - \sum_{j=1}^{n} p_j x_{ij,k}
  \]

  for every $i \in I$, perfectly-competitive CES aggregator firm

  \[
y_i = \left( \int_0^1 y_{ik}^{\frac{\theta_i - 1}{\theta_i - 1}} \, dk \right)^{\frac{\theta_i}{\theta_i - 1}}
  \]

  ▶ output may be either consumed or used as an intermediate good
Representative Household

- preferences

\[ U(C) - V(L) \]

\[ C = C(c_1, \ldots, c_n) = \prod_{i \in I} (c_i/\beta_i)^{\beta_i} \]

- budget set

\[ \sum_{i \in I} p_i c_i \leq wL + \sum_{i \in I} \int_0^1 \pi_{ik} dk + T \]
The Government and Market Clearing

- government has full commitment, fiscal budget set

\[ T = \sum_{i \in I} \tau_i \int_0^1 p_{ik} y_{ik} dk \]

- monetary authority controls aggregate nominal demand

\[ m = PC = \sum_{i \in I} p_i c_i \]

- market clearing

\[ y_j = c_j + \sum_{i \in I} \int x_{ij,k} dk \quad \forall j \in I, \quad \text{and} \quad L = \sum_{i \in I} \int \ell_{ik} dk \]
Nominal Rigidity = Informational Friction

- sectoral technology shocks

\[ \log z_i \sim \mathcal{N} \left( 0, \delta^2 \sigma^2 z \right) \text{ i.i.d.} \]

- Gaussian information set: vector of signals about technology shocks

\[ \omega_{ik} = (\omega_{i1,k}, \ldots, \omega_{in,k}) \]

\[ \omega_{ij,k} = \log z_j + \epsilon_{ij,k}, \quad \text{with} \quad \epsilon_{ij,k} \sim \mathcal{N} \left( 0, \delta^2 \sigma^2 \right) \]

- aggregate state

\[ s = (z, \omega) \in S \]

- vector of sectoral productivities \( z = (z_1, \ldots, z_n) \)

- entire distribution of information sets \( \omega \)
Nominal Rigidity $\equiv$ Informational Friction

1. Firms’ nominal pricing decisions made under incomplete info

$$p_{ik}(\omega_{ik})$$

- nominal rigidity $\equiv$ measurability constraint on the nominal price

2. All other market outcomes, allocations adjust to the aggregate state

- household chooses consumption
- inputs must adjust so that supply $\equiv$ demand (but input mix chosen optimally)

$$y_{ik}(s), \ell_{ik}(s), x_{ij,k}(s)$$

- monetary policy contingent on $s$, but sectoral taxes are non-contingent
Nominal Rigidity = Informational Friction

1. Firms’ nominal pricing decisions made under incomplete info

   \[ p_{ik}(\omega_{ik}) \]

   - nominal rigidity = measurability constraint on the nominal price

2. All other market outcomes, allocations adjust to the aggregate state

   - household chooses consumption
   - inputs must adjust so that supply = demand (but input mix chosen optimally)

   \[ y_{ik}(s), \ell_{ik}(s), x_{ij,k}(s) \]

   - monetary policy contingent on \( s \), but sectoral taxes are non-contingent
The first-best allocation $\xi^*$ is the unique feasible allocation which satisfies

$$V'(L(s)) = U'(C(s)) \frac{dC(s)}{dc_i} z_i(s) \frac{dF_i(s)}{d\ell_i}, \quad \forall i, k, s$$

$$\frac{dC(s)}{dc_j} = \frac{dC(s)}{dc_i} z_i(s) \frac{dF_i(s)}{dx_{ij}}, \quad \forall i, j, k, s$$

- Efficiency requires zero dispersion in quantities within sectors
  
  $$\ell_i(s) = \ell_{ik}(s), \quad x_{ij}(s) = x_{ij,k}(s), \quad y_i(s) = y_{ik}(s), \quad \forall k \in [0, 1]$$

  but movement in relative quantities across sectors
Equilibrium
A sticky price equilibrium is a set of allocations, prices, and policies such that:

(i) prices $p_{ik}(\omega_{ik})$ maximize the firm’s expected real value of profits given information set $\omega_{ik}$;

(ii) firms optimally choose inputs to meet realized demand;

(iii) the representative household maximizes her utility;

(iv) the government budget constraint is satisfied; and

(v) markets clear.

A flexible price equilibrium is a set of allocations, prices, and policies such that:

same as above, but

$p_{ik}(s)$
Proposition

A feasible allocation is implementable as a flexible-price equilibrium iff

\[ V'(L(s)) = \chi_U' \left( C(s) \right) z_i(s) \frac{dC(s)}{dc_i} \frac{dF_i(s)}{d\ell_i}, \quad \forall i, k, s \]

\[ \frac{dC(s)}{dc_j} = \chi_i \frac{dC(s)}{dc_i} z_i(s) \frac{dF_i(s)}{dx_{ij}}, \quad \forall i, j, k, s \]

where \( \chi_i \equiv (1 - \tau_i) \left( \frac{\theta_i - 1}{\theta_i} \right) \).

Proposition

The first best allocation \( \xi^* \) can be implemented under flexible prices with \( \chi_i = 1, \forall i \).
Proposition

A feasible allocation is implementable as a *sticky-price equilibrium* iff

\[
V'(L(s)) = \chi_i \varepsilon_{ik}(\omega_{ik}, s) U'(C(s)) \frac{dC(s)}{dc_i} \left( \frac{y_{ik}(\omega_{ik}, s)}{y_i(s)} \right)^{-1/\theta_i} \frac{dF_i(s)}{d\ell_i}, \quad \forall i, k, s
\]

\[
\frac{dC(s)}{dc_j} = \chi_i \varepsilon_{ik}(\omega_{ik}, s) \frac{dC(s)}{dc_i} \left( \frac{y_{ik}(\omega_{ik}, s)}{y_i(s)} \right)^{-1/\theta_i} \frac{dF_i(s)}{dx_{ij}}, \quad \forall i, j, k, s,
\]

with stochastic wedges (due to pricing errors):

\[
\varepsilon_{ik}(\omega_{ik}, s) \equiv \frac{mc_i(s) \mathbb{E}[v_{ik}(s)|\omega_{ik}]}{\mathbb{E}[v_{ik}(s)mc_i(s)|\omega_{ik}]},
\]
Flexible Price allocations are unattainable

- let $X^f$ denote the entire set of flexible-price allocations
- let $X^s$ denote the entire set of sticky-price allocations

**Theorem**

The sets $X^f$ and $X^s$ are generically disjoint

$$X^f \cap X^s = \emptyset$$
Divine Coincidence is non-generic

Corollary

The first best allocation cannot generically be implemented under sticky prices:

\[ \xi^* \notin X^s \]

- impossible for any monetary policy to simultaneously achieve:
  - productive efficiency within sectors (zero price dispersion within each sector)
  - efficient relative price movement across sectors
When can you implement first best?

**Proposition**

*If there is a single sticky-price industry $i$, then*

\[ X_f \subset X^s \]

*and as a result,*

\[ \xi^* \in X^s. \]

- nests special cases:
  - canonical NK model
  - Aoki (2001): two-sector model with one flex-price sector, one sticky-price sector
  - Erceg, Henderson, Levin (1999): either wage flexibility or price flexibility
Optimal Monetary Policy
Gaussian Priors and Posteriors

\[ \mathbb{E}[\log z_j|\omega_{ik}] = \phi_i \omega_{ij,k} \]
\[ \text{var}[\log z_j|\omega_{ik}] = (1 - \phi_i)\text{var}[\log z_j] \]

- \( \phi_i \in [0, 1] \) is the degree of price flexibility of industry \( i \)

\[ \phi_i = \frac{\sigma_{z^2}}{\sigma_{z^2}^2 + \sigma_i^2} \]

- lower \( \phi_i \) is greater “price stickiness”
- \( \phi_i = 1 \) is full price flexibility
Welfare Loss Decomposition

Theorem

Let $\mathcal{W}^*$ denote the first-best level of welfare. Up to a second order approximation,

$$\mathcal{W} \propto \mathcal{W}^* \exp \{-\Delta\}$$

$\Delta$ denotes welfare losses from first best:

$$\Delta \equiv \frac{1}{1/\eta + \gamma} \mathbb{V} + \mathbb{L}_{acr} + \mathbb{L}_{wi}$$

- $\mathbb{V}$ is the volatility of the (endogenous) output gap
- $\mathbb{L}_{acr}$ is productive inefficiency: misallocation across sectors
- $\mathbb{L}_{wi}$ is productive inefficiency: misallocation within sectors
Theorem

The optimal monetary policy is a price index stabilization policy:

\[ \sum_{i \in I} \psi_i^* \log p_i = 0 \quad \text{with} \quad \sum_{i \in I} \psi_i = 1, \]

with optimal weights \((\psi_1^*, \ldots, \psi_n^*)\) given by

\[ \psi_i^* \propto \frac{1}{1/\eta + \gamma} \psi^\text{og}_i + \psi^\text{wi}_i + \psi^\text{acr}_i \]

- \(\psi^\text{og}_i\) is the policy that minimizes volatility of the output gap
- \(\psi^\text{wi}_i\) is the policy that minimizes within-industry misallocation
- \(\psi^\text{acr}_i\) is the policy that minimizes across-industry misallocation
Optimal Monetary Policy

Theorem

(i) The policy that minimizes volatility of the output gap is given by

$$\psi_{i}^{og} \propto \lambda_{i}(1/\phi_{i} - 1), \quad \text{where} \quad \lambda_{i} \equiv \frac{Piy_{i}}{PC} \quad \text{is the Domar weight}$$

(ii) The policy that minimizes within-industry misallocation is given by

$$\psi_{i}^{wi} \propto \lambda_{i}(1 - \phi_{i})\theta_{i}\rho_{i}, \quad \text{where} \quad \rho_{i} \equiv \frac{d \log mc_{i}(s)}{d \log w(s)}$$

(iii) The policy that minimizes across-industry misallocation is given by

$$\psi_{i}^{acr} \propto \lambda_{i}(1/\phi_{i} - 1) \left[ \rho_{0} - \rho_{i} + \sum_{j \in I} (1 - \phi_{j})\lambda_{j}\rho_{j}\ell_{ji}/\lambda_{i} \right]$$
General Principles for Monetary Policy

- the optimal price index places greater weight on:
  - larger sectors as measured by Domar weights $\lambda_i$
  - stickier sectors (low $\phi_i$)
  - more upstream sectors
  - sectors with stickier downstream customers
  - sectors with more flexible upstream suppliers
Quantitative Illustration

- what would be the welfare gains from adopting the optimal policy?
- we calibrate the model to the U.S. input-output tables and data on price stickiness
- we use model to quantify the welfare gains of the optimal policy relative to CPI stabilization
Welfare Loss relative to the first best

<table>
<thead>
<tr>
<th>Table 1. Welfare Loss under Various Policies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Welfare loss (percent consumption)</td>
</tr>
<tr>
<td>optimal policy</td>
</tr>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>output-gap stabilization</td>
</tr>
<tr>
<td>(2)</td>
</tr>
<tr>
<td>CPI targeting</td>
</tr>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>Domar</td>
</tr>
<tr>
<td>weighted</td>
</tr>
<tr>
<td>(4)</td>
</tr>
<tr>
<td>stickiness</td>
</tr>
<tr>
<td>weighted</td>
</tr>
<tr>
<td>(5)</td>
</tr>
<tr>
<td>2.98</td>
</tr>
<tr>
<td>2.99</td>
</tr>
<tr>
<td>3.51</td>
</tr>
<tr>
<td>3.75</td>
</tr>
<tr>
<td>3.22</td>
</tr>
<tr>
<td>2.66</td>
</tr>
<tr>
<td>2.67</td>
</tr>
<tr>
<td>3.00</td>
</tr>
<tr>
<td>3.16</td>
</tr>
<tr>
<td>2.80</td>
</tr>
<tr>
<td>0.32</td>
</tr>
<tr>
<td>0.32</td>
</tr>
<tr>
<td>0.40</td>
</tr>
<tr>
<td>0.42</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.11</td>
</tr>
<tr>
<td>0.17</td>
</tr>
<tr>
<td>0.05</td>
</tr>
<tr>
<td>0.32</td>
</tr>
<tr>
<td>0.32</td>
</tr>
<tr>
<td>0.40</td>
</tr>
<tr>
<td>0.42</td>
</tr>
<tr>
<td>0.36</td>
</tr>
<tr>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0.11</td>
</tr>
<tr>
<td>0.17</td>
</tr>
<tr>
<td>0.05</td>
</tr>
<tr>
<td>0.9957</td>
</tr>
<tr>
<td>0.5181</td>
</tr>
<tr>
<td>0.5929</td>
</tr>
<tr>
<td>0.6260</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Cosine similarity to optimal policy
Conclusion

- Divine Coincidence is non-generic. In equilibrium, welfare loss arises from:
  - volatility of the output gap
  - misallocation both within and across sectors

- Optimal Policy: price index stabilization with greater weight on:
  - larger (in Domar weights) & stickier sectors
  - more upstream sectors, sectors with stickier customers, sectors with more flexible suppliers

- Quantitative welfare improvements from the adopting optimal policy
  - optimal policy relative to CPI stabilization $\approx$ half percentage point of quarterly consumption
  - output gap stabilization is approximately optimal