

Optimal Monetary Policy in Production Networks*

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Abstract

This paper studies the optimal conduct of monetary policy in a multi-sector economy in which firms buy and sell intermediate goods over a production network. We first provide a necessary and sufficient condition for the monetary policy's ability to implement flexible-price equilibria in the presence of nominal rigidities and show that, generically, no monetary policy can implement the first-best allocation. We then characterize the constrained-efficient policy in terms of the economy's production network and the extent and nature of nominal rigidities. Our characterization result yields general principles for the optimal conduct of monetary policy in the presence of input-output linkages: it establishes that optimal policy stabilizes a price index with higher weights assigned to larger, stickier, and more upstream industries, as well as industries with less sticky upstream suppliers but stickier downstream customers. In a calibrated version of the model, we find that implementing the optimal policy can result in quantitatively meaningful welfare gains.

Keywords: monetary policy, production networks, nominal rigidities, misallocation.

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1 Introduction

Optimal monetary policy in the canonical New Keynesian framework is well-known and takes a particularly simple form: as long as there are no missing tax instruments, it is optimal to stabilize the nominal price level. Price stability neutralizes the effects of nominal rigidities, implements flexible-price allocations, and, in the absence of inefficient shocks, restores productive efficiency. In the language of the New Keynesian literature, the “divine coincidence” holds: price stabilization simultaneously eliminates inflation and the output gap.¹

The ability of monetary policy to replicate flexible-price allocations in the textbook New Keynesian models, however, relies critically on the assumption that all firms are technologically identical: as long as all firms employ the same production technology, a price stabilization policy that implements zero relative price dispersion across firms equalizes firms’ marginal products and hence achieves productive efficiency, regardless of the extent of nominal rigidities (Correia, Nicolini, and Teles, 2008). But once there are technological differences across firms—say, in a multi-sector economy with input-output linkages—monetary policy may lose its ability to replicate flexible-price allocations: while productive efficiency would dictate movements in relative quantities across different producers in response to producer-specific shocks, monetary policy may not be able to induce the corresponding relative price movements across the various sticky-price producers.

In view of the above, it is not readily obvious what principles should guide the conduct of monetary policy when firms employ heterogeneous production technologies, rely on a host of different intermediate goods and services produced by other firms in the economy, and are subject to various degrees of nominal rigidities.

In this paper, we address these questions by studying the optimal conduct of monetary policy in a multi-sector New Keynesian framework while allowing for inter-firm trade over a production network. We first provide a necessary and sufficient condition for the monetary policy’s ability to implement flexible-price equilibria in the presence of nominal rigidities and show that, generically, no monetary policy can implement the first-best allocation. We then characterize the constrained-efficient policy in terms of the economy’s production network and the extent of nominal rigidities. Our characterization result yields general principles for the optimal conduct of monetary policy in the presence of input-output linkages.

We develop these results in the context of a static multi-sector general equilibrium model à la Long and Plosser (1983) and Acemoglu et al. (2012) in which firms are linked to one another via input-output linkages and are subject to industry-level productivity shocks.² As in the New-Keynesian tradition, we also assume that firms are subject to nominal rigidities. More specifically, we assume that firms make their nominal pricing decisions under incomplete information about the productivity shocks. As a result, nominal prices respond to changes in productivities only to the extent that such changes

¹While divine coincidence holds in the case of efficient shocks or in the presence of inefficient shocks neutralized by tax instruments, it may fail more generally when inefficient cost-push shocks are introduced and state-contingent tax instruments that may counteract these shocks are assumed away. In this case, monetary policy becomes an imperfect instrument to fight inefficient cost-push shocks. As a result, the policymaker faces a trade-off between minimizing the output gap and productive inefficiency due to inflation.

²Throughout, we assume that these productivity shocks are the only payoff-relevant shocks in the economy, thus abstracting away from inefficient markup, or cost-push, shocks.

are reflected in the firms' information sets. This nominal friction opens the door to potential price distortions throughout the production network, as well as a role for monetary policy in shaping real allocations.

Within this framework, we start by characterizing the entire sets of sticky- and flexible-price equilibria, defined as the sets of allocations that can be implemented as an equilibrium in the presence and absence of nominal rigidities, respectively.³ We show that while both sets of allocations are characterized by similar sets of conditions relating the marginal rates of substitution between goods and their marginal rates of transformation, the conditions characterizing sticky-price allocations exhibit an additional collection of wedges that depend on the interaction between the conduct of monetary policy and the firms' information sets. Using these characterizations, we then provide the exact conditions on the firms' technologies and information sets under which monetary policy can implement flexible-price equilibria and hence restore productive efficiency. As an important byproduct of this result, we also show that these conditions are violated for a generic set of information structures, thus concluding that, generically, monetary policy cannot achieve productive efficiency.

Having established the failure of monetary policy to implement the first-best allocation, we then turn to the study of optimal monetary policy, i.e., the policy that maximizes household welfare over the set of all possible sticky-price-implementable allocations. In order to obtain closed-form expressions for the optimal policy, we impose a number of functional form assumptions by assuming that all firms employ Cobb-Douglas production technologies and that all signals are normally distributed.

We establish three sets of results. First, we show that firms' optimal pricing decisions can be recast as a generalized version of a "beauty contest" game à la [Morris and Shin \(2002\)](#), in which firms face heterogenous strategic complementarities in their price-setting decisions due to interdependencies arising from the production network. Second, we demonstrate that the constrained-efficient policy faces a trade-off between three sources of welfare losses: misallocation due to price dispersion within sectors, misallocation arising from pricing errors across sectors, and output gap volatility. Third, building on our previous results, we derive the monetary policy that optimally trades off these three components.

Our characterization of the optimal policy yields general principles for the conduct of monetary policy in the presence of input-output linkages. In particular, we establish that, all else equal, optimal policy stabilizes a price index with higher weights assigned to (i) larger industries as measured by their Domar weights, (ii) stickier industries, (iii) more upstream industries, and (iv) industries with less sticky upstream suppliers but with stickier downstream customers.

We then use our theoretical results to undertake a quantitative exercise and determine the optimal monetary policy for the U.S. economy as implied by the model. Matching input-output tables constructed by the Bureau of Economic Analysis with industry-level data on nominal rigidities provided to us by [Pasten, Schoenle, and Weber \(2019\)](#), we compute the weights corresponding to the optimal price-stabilization index and quantify the resulting welfare loss due to the presence of nominal rigidities. We find that the optimal policy generates a welfare loss equivalent to a 2.98% loss in quarterly consumption relative to the flexible-price equilibrium, with the overwhelming fraction

³Our approach is thus similar to [Correia et al. \(2008, 2013\)](#) and [Angeletos and La'O \(2020\)](#), who apply the primal approach to optimal taxation of [Atkinson and Stiglitz \(2015\)](#) and [Diamond and Mirrlees \(1971\)](#) to New Keynesian models.

of this loss arising from misallocation within and across industries. We then provide a comparison between the performance of the optimal policy and four alternative, non-optimal, price-stabilization policies. We find that, in our calibration, the welfare differences between the optimal policy and a policy that stabilizes the output gap are minuscule, amounting to less than 0.01 percentage points of quarterly consumption. In contrast, moving from price stabilization targets based on industries' shares in the household's consumption basket (akin to CPI or PCE), industry size, or sectoral stickiness to the optimal price index would result in quantitatively meaningful welfare gains.

Related Literature. Our paper is part of the growing literature that studies the role of production networks in macroeconomics. Building on the multi-sector model of [Long and Plosser \(1983\)](#), [Acemoglu et al. \(2012\)](#) investigate whether input-output linkages can transform microeconomic shocks into aggregate fluctuations.⁴ We follow this line of work by focusing on a multi-sector general equilibrium economy with nominal rigidities and investigating the interaction between monetary policy and the economy's production network. Within this literature, our paper builds on the works of [Jones \(2013\)](#), [Bigio and La'O \(2020\)](#), and [Baqae and Farhi \(2020\)](#), who study misallocation in economies with non-trivial production networks. However, in contrast to these papers, which treat markups and wedges as exogenously-given model primitives, we focus on an economy in which wedges are determined endogenously as the result of firms' individually-optimal price-setting decisions and monetary policy. We investigate the monetary authority's ability to shape these wedges using available policy instruments, and use this characterization to derive the optimal policy.

In a series of recent papers, [Pasten, Schoenle, and Weber \(2019, 2020\)](#) and [Ozdagli and Weber \(2020\)](#) study the production network's role as a possible transmission mechanism of monetary policy shocks. We differ from these papers by providing a closed-form characterization of the optimal monetary policy as a function of the economy's underlying production network and the extent of nominal rigidities. Also related is the recent work of [Wei and Xie \(2020\)](#), who study the role of monetary policy in the presence of global value chains. Whereas they focus on an open economy in which production occurs over a single production chain, we provide a characterization of optimal policy for a general production network structure in a closed economy.

More closely related to our work is the independent and contemporaneous work of [Rubbo \(2020\)](#), who also studies monetary policy in a production network economy, but with a different focus. In particular, Rubbo mainly focuses on how the production network shapes the economy's Phillips curve. In contrast, we provide a series of analytical results that distill the role of the various forces that shape the optimal policy. Our normative analysis thereby yields general principles for the optimal conduct of monetary policy. In addition, we obtain necessary and sufficient conditions under which flexible-price allocations are implementable as sticky-price equilibria.⁵

⁴Other papers in this line of work include [Carvalho \(2010\)](#), [Foerster, Sarte, and Watson \(2011\)](#), [Acemoglu et al. \(2016, 2017\)](#), and [Atalay \(2017\)](#). See [Carvalho \(2014\)](#) and [Carvalho and Tahbaz-Salehi \(2019\)](#) for surveys of the theoretical and empirical literature on production networks.

⁵Also see [Castro \(2019\)](#), who studies the quantitative relevance of sectoral heterogeneity in production networks. He finds that, compared to the standard one-sector model, aggregate inflation in the multi-sector economy reacts 30% less to the output gap, rendering monetary policy less effective and inducing a welfare cost of trend inflation that is larger by one order of magnitude.

Our paper also belongs to a small strand of the New Keynesian literature that studies optimal monetary policy in multi-sector economies. In one of the earliest examples of this line of work, [Aoki \(2001\)](#) shows that in a two-sector economy with one sticky and one fully flexible industry (but no input-output linkages), a policy that stabilizes the price of the sticky industry implements the first-best allocation. [Mankiw and Reis \(2003\)](#), [Woodford \(2003b, 2010\)](#), [Benigno \(2004\)](#), and [Eusepi, Hobijn, and Tambalotti \(2011\)](#) generalize this insight to multi-sector economies with varying degrees of price stickiness and establish that the monetary authority should stabilize a price index that places greater weights on industries with stickier prices.⁶ While most of this literature has abstracted from input-output linkages, [Huang and Liu \(2005\)](#) consider a two-sector economy with a final and an intermediate good and show that the second-best policy stabilizes a combination of the price of the two goods. To the best of our knowledge, the prior literature has not studied optimal monetary policy with a general production network.

The importance of strategic complementarities in firms' price-setting behavior in the presence of nominal rigidities has a long history. Investigating "in-line" and "roundabout" production structures, [Blanchard \(1983\)](#) and [Basu \(1995\)](#) emphasize the role of intermediate inputs in creating strategic complementarities in firms' price-setting behavior. We build on these papers by providing a closed-form characterization of how strategic complementarities arising from input-output linkages shape equilibrium nominal prices, quantities, and the optimal conduct of monetary policy. Furthermore, we show that our micro-founded general equilibrium macro model is isomorphic to the "beauty contest" games such as [Morris and Shin \(2002\)](#), [Bergemann, Heumann, and Morris \(2017\)](#), and [Golub and Morris \(2018\)](#) that explore the consequences of strategic complementarities in more reduced-form settings.⁷

Finally, our approach in modeling nominal rigidities as an informational constraint on the firms' price-setting decisions follows the extensive literature that proposes informational frictions as an appealing substitute to Calvo frictions and menu costs on theoretical ([Mankiw and Reis, 2002](#); [Woodford, 2003a](#); [Mackowiak and Wiederholt, 2009](#); [Nimark, 2008](#)) and empirical ([Coibion and Gorodnichenko, 2012, 2015](#)) grounds. We follow this approach not only because we find informational frictions to be a priori more plausible than other alternatives, but also because, in the context of our exercise, they lend themselves to a more transparent analysis. Importantly, this modeling feature does not upset the key normative lessons of the New Keynesian paradigm; in particular, price stability remains optimal insofar as monetary policy need not substitute for missing tax instruments ([Angeletos and La'O, 2020](#)).

Outline. The rest of the paper is organized as follows. Section 2 sets up the environment and defines the sticky- and flexible-price equilibria in our context. Section 3 characterizes these equilibria and

⁶This principle of "sticky-price stabilization" was first proposed by [Goodfriend and King \(1997\)](#) and later formalized in the above-mentioned papers. Also see [Erceg, Henderson, and Levin \(2000\)](#), who study an economy with both nominal price and wage rigidities.

⁷A smaller strand of this literature, such as [Ballester, Calvó-Armengol, and Zenou \(2006\)](#), [Candogan, Bimpikis, and Ozdaglar \(2012\)](#), and [Galeotti, Golub, and Goyal \(2020\)](#), studies how various policies can impact aggregate action or welfare in games of strategic complementarities over networks. In a departure from this literature, which models network interactions, payoffs, and policy instruments in a reduced-form manner, we focus on a micro-founded general equilibrium economy and provide a characterization of optimal policy in terms of primitives such as preferences, technologies, and information sets.

provides necessary and sufficient conditions for the monetary policy's ability to implement the first-best allocation. Section 4 contains our closed-form characterization of the optimal policy in terms of the economy's production network and the extent of nominal rigidities. We present the quantitative analysis of the model in Section 5. All proofs and some additional mathematical details are provided in the Appendix.

2 Framework

Consider a static economy consisting of n industries indexed by $i \in I = \{1, 2, \dots, n\}$. Each industry consists of two types of firms: (i) a unit mass of monopolistically-competitive firms, indexed by $k \in [0, 1]$, producing differentiated goods and (ii) a competitive producer whose sole purpose is to aggregate the industry's differentiated goods into a single sectoral output. The output of each industry can be either consumed by the households or used as an intermediate input for production by firms in other industries. In addition to the firms, the economy consists of a representative household as well as a government with the ability to levy industry-specific taxes and control aggregate nominal demand.

The monopolistically-competitive firms within each industry use a common constant-returns-to-scale technology to transform labor and intermediate inputs into their differentiated products. More specifically, the production function of firm $k \in [0, 1]$ in industry i is given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \dots, x_{in,k}),$$

where y_{ik} is the firm's output, l_{ik} is its labor input, $x_{ij,k}$ is the amount of sectoral commodity j purchased by the firm, z_i is an industry-specific productivity shock, and F_i is homogenous of degree 1. Throughout, we assume that labor is an essential input for the production technology of all goods, in the sense that $F_i(0, x_{i1,k}, \dots, x_{in,k}) = 0$ and that $dF_i/dl_{ik} > 0$ whenever all other inputs are used in positive amounts.

The nominal profits of the firm are given by

$$\pi_{ik} = (1 - \tau_i) p_{ik} y_{ik} - w l_{ik} - \sum_{j=1}^n p_j x_{ij,k}, \quad (1)$$

where p_{ik} is the nominal price charged by the firm, p_j is nominal price of the industry j 's sectoral output, w denotes the nominal wage, and τ_i is an industry-specific revenue tax (or subsidy) levied by the government.

The competitive producer in industry i transforms the differentiated products produced by the unit mass of firms in that industry into a uniform sectoral good using a constant-elasticity-of-substitution (CES) production technology

$$y_i = \left(\int_0^1 y_{ik}^{(\theta_i-1)/\theta_i} dk \right)^{\theta_i/(\theta_i-1)} \quad (2)$$

with elasticity of substitution $\theta_i > 1$. This producer's profits are thus given by $\pi_i = p_i y_i - \int_0^1 p_{ik} y_{ik} dk$, where p_i is the price of the aggregated good produced by industry i . We include this producer—which

has zero value added—only for expositional purposes: it ensures that a homogenous good is produced by each industry, while at the same time allowing for monopolistic competition among firms within the industry.

The preferences of the representative household are given by

$$W(C, L) = U(C) - V(L), \quad (3)$$

where C and L denote the household's final consumption basket and total labor supply, respectively. We impose the typical regularity conditions on U and V : they are strictly increasing, twice differentiable, and satisfy $U'' < 0$, $V'' > 0$, and the Inada conditions. The final consumption basket of the household is given by $C = \mathcal{C}(c_1, \dots, c_n)$, where c_i is the household's consumption of the good produced by industry i and \mathcal{C} is a homogenous function of degree 1. The representative household's budget constraint is thus given by

$$PC = \sum_{j=1}^n p_j c_j \leq wL + \sum_{i=1}^n \int_0^1 \pi_{ik} dk + T,$$

where $P = \mathcal{P}(p_1, \dots, p_n)$ is the nominal price of the household's consumption bundle. The left-hand side of the above inequality is the household's nominal expenditure, whereas the right-hand side is equal to the household's total nominal income, consisting of wage income, dividends from owning firms, and lump sum transfers from the government.⁸

In addition to the firms and the representative household, the economy also consists of a government with the ability to set fiscal and monetary policies. The government's fiscal instrument is a collection of industry-specific taxes (or subsidies) on the firms, with the resulting revenue then rebated to the household as a lump sum transfer. Therefore, the government's budget constraint is given by

$$T = \sum_{i=1}^n \tau_i \int_0^1 p_{ik} y_{ik} dk,$$

where τ_i is the revenue tax imposed on firms in industry i and T is the net transfer to the representative household. Finally, to model monetary policy, we sidestep the micro-foundations of money and, instead, impose the following cash-in-advance constraint on the household's total expenditure:

$$PC = m, \quad (4)$$

where we assume that m —which can be interpreted as either money supply or nominal aggregate demand—is set directly by the monetary authority.

2.1 Nominal Rigidities and Information Frictions

We model nominal rigidities by assuming that firms do not observe the realized productivity shocks $z = (z_1, \dots, z_n)$ and, instead, make their nominal pricing decisions under incomplete information.

⁸Since sectoral aggregators are competitive, they make zero profits in equilibrium. Hence, we only need to account for dividends from the monopolistically-competitive firms.

This assumption implies that nominal prices respond to changes in productivities only to the extent that such changes are reflected in the firms' information sets.

Formally, we assume that each firm k in industry i receives a signal $\omega_{ik} \in \Omega_{ik}$ about the economy's aggregate state. The aggregate state includes not only the vector of realized productivity shocks, but also the realization of all signals, that is,

$$s = (z, \omega),$$

where $\omega = (\omega_1, \dots, \omega_n) \in \Omega$ denotes the realized cross-sectional distribution of signals in the economy and $\omega_i = (\omega_{ik})_{k \in [0,1]}$ denotes the realized cross-sectional distribution of signals in industry i . We use $\mu(\cdot)$ to denote the unconditional distribution of the aggregate state over the set $S = \mathbb{R}_+^n \times \Omega$.

Since ω_{ik} is the only component of state s that is observable to firm ik , the nominal price set by this firm has to be measurable with respect to ω_{ik} . We capture this measurability constraint by denoting the firm's price by $p_{ik}(\omega_{ik})$. Similarly, we write $p_i(\omega_i)$ and $P(\omega)$ to capture the fact that the nominal prices of sectoral good i and the consumption bundle have to be measurable with respect to the profile of signals in industry i and in the entire economy, respectively.⁹

A few remarks are in order. First, note that the above formulation implies that state $s = (z, \omega)$ not only contains all payoff-relevant shocks, but also contains shocks to the aggregate profile of beliefs. Therefore, our framework can accommodate the possibility of higher-order uncertainty, as ω_{ik} may contain information about other firms' (first or higher-order) beliefs, as in [Angeletos and La'O \(2013\)](#). Second, our framework is flexible enough to nest models with "sticky information" ([Mankiw and Reis, 2002](#); [Ball, Mankiw, and Reis, 2005](#)) as a special case by assuming that a fraction of firms in industry i set their nominal prices under complete information ($\omega_{ik} = z$), whereas the rest of the firms in that industry observe no informative signals ($\omega_{ik} = \emptyset$) and hence set their nominal prices based only on the prior beliefs. Finally, note that while all firms set nominal prices under incomplete information, we do not impose any form of wage rigidities, thus allowing the nominal wage to depend on the entire state s . While this assumption simplifies the exposition, it is without loss of generality in our multi-sector framework.

In summary, we can represent the economy's price system by the collection of nominal prices and nominal wage at any given state:

$$\varrho = \left\{ \left((p_{ik}(\omega_{ik}))_{k \in [0,1]}, p_i(\omega_i) \right)_{i \in I}, P(\omega), w(s) \right\}_{s \in S}.$$

While nominal prices are set under incomplete information, we assume that firms and the household make their quantity decisions *after* observing the prices and the realization of productivities. As a result, quantities may depend on the entire state s . We thus represent an allocation in this economy by

$$\xi = \left\{ \left((y_{ik}(s), l_{ik}(s), x_{ik}(s))_{k \in [0,1]}, y_i(s), c_i(s) \right)_{i \in I}, C(s), L(s) \right\}_{s \in S},$$

where $l_{ik}(s), x_{ik}(s) = (x_{i1,k}(s), \dots, x_{in,k}(s))$, and $y_{ik}(s)$ denote, respectively, the labor input, material input, and output of firm k in industry i , $y_i(s)$ is the output of industry i , $c_i(s)$ is the household's

⁹More specifically, the sectoral good producer's CES technology implies that $p_i(\omega_i) = \left(\int_0^1 p_{ik}^{1-\theta_i}(\omega_{ik}) dk \right)^{1/(1-\theta_i)}$, whereas the consumption good's price index is given by $P(\omega) = \mathcal{P}(p_1(\omega_1), \dots, p_n(\omega_n))$.

consumption of sectoral good i , and $C(s)$ and $L(s)$ are the household's consumption and labor supply, respectively.

We conclude this discussion by specifying how government policy depends on the economy's aggregate state. Recall from equations (1) and (4) that the fiscal and monetary authorities can, respectively, levy taxes and control the nominal demand. We assume that while the fiscal authority has the ability to levy industry-specific taxes τ_i , these taxes cannot be contingent on the economy's aggregate state. In contrast, the monetary authority can set the nominal demand as an arbitrary function $m(s)$ of the economy's aggregate state s . This is equivalent to assuming that the monetary authority has the ability to commit, *ex ante*, to a policy that can, in principle, depend on the realized productivities and the profile of beliefs throughout the economy. Government policy can thus be summarized as

$$\vartheta = \{(\tau_1, \dots, \tau_n), m(s)\}_{s \in S}. \quad (5)$$

Note that our formulation of government's policy instruments in (5) allows for non-state-contingent taxes to undo steady-state distortions due to monopolistic markups, while ruling out state-contingent taxes. Were we to allow for a full set of state-contingent taxes, such taxes could be used to undo the real effects of nominal rigidities and implement the first-best allocation under any monetary policy. We also remark that while it may be far-fetched to assume that the monetary authority can commit to a policy that is contingent not just on the payoff-relevant shocks, z , but also on the entire profile of beliefs, ω , we nonetheless make this assumption to illustrate the limit to monetary policy's ability to implement allocations even under maximum policy flexibility.

2.2 Equilibrium Definition

We now define our notions of sticky- and flexible-price equilibria. To this end, first note that the market-clearing conditions for labor and commodity markets are given by

$$L(s) = \sum_{i=1}^n \int_0^1 l_{ik}(s) dk \quad (6)$$

$$y_i(s) = c_i(s) + \sum_{j=1}^n \int_0^1 x_{ji,k}(s) dk = \left(\int_0^1 y_{ik}(s)^{(\theta_i-1)/\theta_i} dk \right)^{\theta_i/(\theta_i-1)} \quad (7)$$

for all $i \in I$ and all $s \in S$, whereas the production technology of firm k in industry i requires that

$$y_{ik}(s) = z_i F_i(l_{ik}(s), x_{i1,k}(s), \dots, x_{in,k}(s)) \quad (8)$$

for all $s \in S$. Given the above, the definition of a sticky-price equilibrium is straightforward:

Definition 1. A *sticky-price equilibrium* is a triplet $(\xi, \varrho, \vartheta)$ of allocations, prices, and policies such that

- (i) firms set prices $p_{ik}(\omega_{ik})$ to maximize expected real value of profits given their information;
- (ii) firms optimally choose inputs to meet realized demand;
- (iii) the representative household maximizes her utility;

- (iv) the government budget constraint is satisfied;
- (v) all markets clear.

We next define our notion of flexible-price equilibria by dropping the measurability constraint on prices imposed on the sticky-price equilibrium in Definition 1. More specifically, we assume that, in contrast to the sticky-price firms, flexible-price firms make their nominal pricing decisions based on complete information of the aggregate state. We can capture this scenario in our framework by simply considering the special case in which all firm-level prices are measurable in the aggregate state, $p_{ik}(s)$. Accordingly, we also adjust our notation for the nominal price of sectoral goods and the consumption bundle by expressing them as $p_i(s)$ and $P(s)$, respectively.

Definition 2. A *flexible-price equilibrium* is a triplet (ξ, δ, ϑ) of allocations, prices, and policies that satisfy the same conditions as those stated in Definition 1, except that all prices are measurable with respect to the aggregate state s .

While not the main focus of our study, the set of flexible-price-implementable allocations serves as a benchmark to which we will contrast equilibria in the presence of nominal rigidities. We conclude with one additional definition, whose meaning is self-evident.

Definition 3. An allocation ξ is *feasible* if it satisfies resource constraints (6), (7), and (8).

3 Sticky- and Flexible-Price Equilibria

In this section, we provide a characterization of the set of all allocations that can be implemented as part of flexible- and sticky-price equilibria. We then use our characterization results to establish that, except for a non-generic set of specifications, the two sets of allocations never intersect, thus implying that, in our multi-sector framework, monetary policy cannot undo the effects of nominal rigidities.

3.1 First-Best Allocation

We start by focusing on the first-best allocation that maximizes household welfare among all feasible allocations. Note that, by symmetry, a planner who maximizes social welfare dictates that all firms within an industry choose the same input, labor, and output quantities.¹⁰ The planner's problem thus reduces to maximizing (3) state-by-state subject to resource constraints (6)–(8). The equations characterizing the planner's optimum are straightforward, summarized in the following lemma:

Lemma 1. *The first-best optimal allocation is a feasible allocation that satisfies*

$$V'(L(s)) = U'(C(s)) \frac{dC}{dc_i}(s) z_i \frac{dF_i}{dl_i}(s) \quad (9)$$

$$\frac{dC/dc_j}(s) = z_i \frac{dF_i}{dx_{ij}}(s) \quad (10)$$

for all pairs of industries $i, j \in I$ and all states s .

¹⁰Specifically, the planner chooses, $x_{ij,k} = x_{ij}$, $l_{ik} = l_i$, and $y_{ik} = y_i$ for all firms k in industry i .

Equation (9) states that, for any good, it is optimal to equate the marginal rate of substitution between consumption of that good and labor with the marginal rate of transformation. In particular, the planner equates the household's marginal disutility of labor on the left-hand side of (9) with its marginal social benefit on the right-hand side, which itself consists of three multiplicative components: the marginal product of labor in the production of commodity i , the marginal product of good i in the production of the final good, and the marginal utility of consumption of the final good.

The second condition (10) similarly indicates that the planner finds it optimal to equate the marginal rate of substitution between two goods to their marginal rate of transformation. The marginal rate of substitution on the left-hand side of equation (10) is the ratio of marginal utilities from consumption of the two goods, whereas the marginal rate of transformation is simply the marginal product of good i in the production of good j , as shown on the right-hand side of this condition.

3.2 Flexible-Price Equilibrium

We now turn to the set of allocations that are implementable as flexible-price equilibria. Since the tax instruments (τ_1, \dots, τ_n) are industry-specific and, in a flexible-price equilibrium, all firms in the same industry have identical information sets, we can once again drop the firm index k .

Proposition 1. *A feasible allocation is part of a flexible-price equilibrium if and only if there exists a set of positive scalars $(\chi_1^f, \dots, \chi_n^f)$ such that*

$$V'(L(s)) = \chi_i^f U'(C(s)) \frac{dC}{dc_i}(s) z_i \frac{dF_i}{dl_i}(s) \quad (11)$$

$$\frac{dC/dc_j}{dC/dc_i}(s) = \chi_i^f z_i \frac{dF_i}{dx_{ij}}(s) \quad (12)$$

for all pairs of industries $i, j \in I$ and all states s .

The conditions in Proposition 1 are almost identical to those characterizing the first-best allocation in Lemma 1, aside from the set of scalars $(\chi_1^f, \dots, \chi_n^f)$. The first condition (11) indicates that for any good, the marginal rate of substitution between consumption and labor is equal to the marginal rate of transformation, modulo a non-state-contingent wedge χ_i^f . Similarly, the second condition equates the marginal rate of substitution between two goods to their marginal rate of transformation, again subject to the wedge χ_i^f . This non-stochastic wedge, which is given by

$$\chi_i^f = (1 - \tau_i) \frac{\theta_i - 1}{\theta_i}, \quad (13)$$

consist of two terms: the tax or subsidy levied by the government and the markup that arises due to monopolistic competition among firms within each industry. As a result, the scalars $(\chi_1^f, \dots, \chi_n^f)$ parameterize the power of the fiscal authority. In particular, with sectoral taxes or subsidies, the fiscal authority can move allocations by inducing wedges in conditions (11) and (12).

Another immediate consequence of Proposition 1 is that the first-best allocation is implementable as a flexible-price equilibrium. This follows from the observation that equations (11) and (12) reduce to (9) and (10) whenever $\chi_i^f = 1$ for all i . Consequently, the first-best allocation can be implemented as a flexible-price equilibrium with industry-specific subsidies $\tau_i = 1/(1 - \theta_i)$. This, of course, is not

surprising: the only distortion in the economy without nominal rigidities arises from monopolistic competition. Therefore, it is optimal for the government to set industry-specific subsidies that are invariant to the economy's aggregate state and undo the monopolistic markups.

3.3 Sticky-Price Equilibrium

With the above preliminary results in hand, we are now ready to characterize the set of equilibrium allocations in the presence of nominal rigidities.

Proposition 2. *A feasible allocation is implementable as a sticky-price equilibrium if and only if there exist positive scalars $(\chi_1^s, \dots, \chi_n^s)$, a policy function $m(s)$, and firm-level wedge functions $\varepsilon_{ik}(s)$ such that*

(i) *the allocation, the scalars $(\chi_1^s, \dots, \chi_n^s)$, and the set of wedge functions $\varepsilon_{ik}(s)$ jointly satisfy*

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s)) \frac{dC}{dc_i}(s) z_i \frac{dF_i}{dl_{ik}}(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i} \quad (14)$$

$$\frac{dC/dc_j}{dC/dc_i}(s) = \chi_i^s \varepsilon_{ik}(s) z_i \frac{dF_i}{dx_{ij}}(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i} \quad (15)$$

for all firms k , all pairs of industries i and j , and all states s ;

(ii) *the policy function $m(s)$ induces the wedge functions $\varepsilon_{ik}(s)$ given by*

$$\varepsilon_{ik}(s) = \frac{mc_i(s) \mathbb{E}_{ik}[v_{ik}(s)]}{\mathbb{E}_{ik}[mc_i(s) v_{ik}(s)]} \quad (16)$$

for all firms k , all industries i , and all states s , where

$$mc_i(s) = m(s) \frac{V'(L(s))}{C((s)) U'(C(s))} \left(z_i \frac{dF_i}{dl_i}(s) \right)^{-1} \quad (17)$$

$$v_{ik}(s) = U'(C(s)) \frac{dC}{dc_i}(s) y_i(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{(\theta_i-1)/\theta_i} \quad (18)$$

are, respectively, the firm's nominal marginal cost and stochastic discount factor.

Proposition 2 provides a characterization of the set of sticky-price-implementable allocations in terms of model primitives and the monetary policy instrument $m(s)$. It is straightforward to see that conditions (14) and (15) are identical to their flexible-price counterparts (11) and (12) in Proposition 1, except for a new wedge $\varepsilon_{ik}(s)$. Also, as in Proposition 1, industry-specific wedges $(\chi_1^s, \dots, \chi_n^s)$ are given by (13) and capture the fiscal authority's ability to influence allocations via tax instruments.

The new wedge $\varepsilon_{ik}(s)$ in equations (14) and (15), which is firm-specific and depends on the economy's aggregate state, represents an additional control variable for the government, one that encapsulates the power of monetary policy over real allocations in the presence of nominal rigidities. Similar to the fiscal authority's ability to influence the allocation by setting taxes, the monetary authority can implement different allocations by moving the wedges $\varepsilon_{ik}(s)$ in (14) and (15). This power is non-trivial, but it is also restrained by conditions (16) and (17): unlike the fiscal authority's full control over $(\chi_1^s, \dots, \chi_n^s)$, the monetary authority's choice of the single policy instrument $m(s)$ pins down all wedges $\varepsilon_{ik}(s)$ at the same time.

The constraint on the monetary policy's ability in shaping real allocations can also be seen by the fact that equation (16) implies $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s) - 1)] = 0$. This means the wedge $\varepsilon_{ik}(s)$ cannot be moved around in an unconstrained manner, as it has to be equal to 1 in expectation irrespective of the policy. This is because these wedges arise only due to “mistakes” by the sticky-price firms in setting their nominal prices. But since firms set their prices optimally given their information sets, they do not make any pricing errors in expectation.

3.4 The Power of Monetary Policy

As illustrated in Proposition 2, the monetary authority can use monetary policy to implement different allocations by moving around the wedge $\varepsilon_{ik}(s)$ as a function of the economy's aggregate state. This leads to the natural question of whether monetary policy can fully undo the effect of nominal rigidities. Our first main result provides an answer to this question by characterizing the set of flexible-price allocations that can be implemented as sticky-price equilibria.

To state this result, let $\sigma(\omega_{ik})$ denote the σ -field generated by the signal ω_{ik} . Also let $g_i(z_1, \dots, z_n)$ denote the marginal product of labor in the production of commodity i (as a function of productivity shocks) under the first-best allocation. Note that g_i only depends on the firms' production technologies and is independent of household preferences, policy, and the firms' information structure. We have the following result:

Theorem 1. *A flexible-price allocation indexed by $(\chi_1^f, \dots, \chi_n^f)$ is implementable as a sticky-price equilibrium if and only if there exists a nominal wage function $w(s)$ such that*

$$\frac{1}{w(s)} \cdot g_i(\chi_1^f z_1, \dots, \chi_n^f z_n) \in \sigma(\omega_{ik}) \quad (19)$$

for all firms $k \in [0, 1]$ in all industries i , where $w(s)$ is pinned down by the monetary policy via $w(s) = m(s)V'(L(s))/C(s)U'(C(s))$.

The above result thus provides a joint restriction on the firms' technology, information structure, and monetary policy under which a flexible-price allocation can be implemented as a sticky-price equilibrium.¹¹ More specifically, it establishes that monetary policy can implement a given flexible-price allocation if there exists a policy-induced function $w(s)$ that can make the product $1/w(s) \cdot g_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$ measurable with respect to the information set of all firms in industry i , simultaneously for all industries. To see the intuition underlying Theorem 1, note that the left-hand side of (19) coincides with the nominal marginal cost of firms in industry i under complete information. Therefore, condition (19) simply states that there exists a policy that can make all firms' nominal marginal costs measurable with respect to their corresponding information sets. This guarantees that all firms set their nominal prices as if they had complete information about their nominal marginal costs, hence neutralizing the effect of nominal rigidities.

We can now use the characterization in Theorem 1 to obtain the following corollary:

¹¹Note that $g_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$ only depends on the firms' production technologies and productivity shocks z , $\sigma(\omega_{ik})$ is only a function of the information structure ω , and $w(s)$ is controlled by the monetary policy.

Corollary 1. *Let \mathcal{X}^f and \mathcal{X}^s denote the set of allocations that are implementable as flexible-price and sticky-price equilibria, respectively. Then, $\mathcal{X}^f \cap \mathcal{X}^s = \emptyset$ for a generic set of information structures.*

That is, in general, any allocation implementable as an equilibrium under flexible prices cannot be implemented as an equilibrium under sticky prices with any monetary policy. Interpreted through the lens of Theorem 1, the above result is an immediate consequence of the observation that, given a generic information structure, it is impossible to satisfy condition (19) for all firms simultaneously. The following result is then immediate.

Corollary 2. *In a multi-sector economy with given preferences and technologies, the first-best allocation is not implementable as a sticky-price equilibrium for a generic set of information structures.*

The intuition behind Corollary 2 is straightforward. Consider the planner’s optimal allocation which can itself be implemented as a flexible-price equilibrium with appropriate industry-level subsidies. The planner would like relative quantities across industries to move efficiently with productivity shocks, while at the same time ensuring that all firms within each industry produce the same quantity. In order to implement this under flexible prices, relative prices across industries should move with relative productivities, while prices across firms within each industry should be identical. This specific pattern of price movements with productivity shocks is necessary for flexible-price allocations and in particular for the first-best allocation.

However, inducing this pattern of price movements is in general impossible under sticky prices in a multi-sector economy. In order to ensure that prices are uniform within a particular industry, the monetary authority must target price stability for that industry. This is the typical first-best policy in one-sector New Keynesian models as it implements zero price dispersion and hence productive efficiency within that particular industry. But when there are multiple industries, if monetary policy is used to achieve price stability within one particular industry, it cannot, in general, be used to target price stability in any other industry. That is, monetary policy cannot stabilize prices in all industries at once. And even if it could—for example, because the information structure is such that all firms in any given industry set the same exact price—it is still not sufficient for achieving the first best: in general, no monetary policy can induce relative prices of all pairs of industries to move with their corresponding productivity shocks.

Another consequence of condition (19) and Corollaries 1 and 2 is that monetary policy may not be able to implement the first-best allocation even if all firms in the economy have perfect information about their own productivities: incomplete information about productivity shocks to other far away industries generically results in flexible-price allocations that are not sticky-price implementable.¹² Furthermore, even if all firms know the realization of the entire vector $z = (z_1, \dots, z_n)$, the first-best allocation may still remain unattainable as long as this information is not common knowledge among all firms in the economy, as higher-order uncertainty also induces nominal rigidities that cannot be neutralized by any monetary policy.

While Corollaries 1 and 2 illustrate the limitation of monetary policy in a generic multi-sector

¹²Condition (19) implies that a sufficient condition for sticky-price implementability of flexible-price allocations is that firms in industry i have complete information about productivity shocks to all their (direct and indirect) upstream industries.

economy, there are some non-generic, yet important, special cases in which the monetary authority can implement the first-best allocation.

Corollary 3. *If there is a single sticky-price industry i , any flexible-price equilibrium allocation can be implemented as a sticky-price equilibrium with a monetary policy that stabilizes industry i 's price.*

This result is an immediate consequence of Theorem 1: if there is a single sticky-price industry i , then setting $w(s) = Mg_i(\chi_1^f z_1, \dots, \chi_n^f z_n)$ for any constant $M > 0$ ensures that the left-hand side of (19) is measurable with respect to the information set of all firms in industry i . Importantly, such a policy stabilizes the nominal marginal cost and hence the nominal price of firms in the sticky-price industry: $p_{ik}(\omega_{ik}) = M$, which is independent of the economy's aggregate state.

Though focused on a non-generic class of economies, Corollary 3 nests two important economies as special cases. The first special case is the textbook single-sector New Keynesian model with no markup shocks. As is well-known (and in line with Corollary 3), the first-best allocation can always be implemented by a combination of (i) price stabilization and (ii) an industry-level subsidy that eliminates monopolistic markups. Importantly, the above result establishes that such a policy mix is optimal irrespective of the nature and extent of information frictions, thus generalizing the insights of Correia, Nicolini, and Teles (2008) to a broad class of nominal rigidities; also see Angeletos and La'O (2020). The second special case is the two-sector model of Aoki (2001), who considers an economy consisting of one flexible industry and one sticky industry subject to Calvo frictions and shows that stabilizing the price of the sticky industry can implement the first-best allocation. Corollary 3 establishes that, as long as there is a single sticky-price industry, Aoki's result generalizes to a multi-sector economy with input-output linkages and an arbitrary form of pricing friction.

4 Optimal Monetary Policy

Our results in Corollaries 1 and 2 establish that, in general, monetary policy cannot implement the first-best allocation as a sticky-price equilibrium. In view of these results, we now turn to the study of optimal monetary policy, i.e., the policy that maximizes household welfare over the set of all possible sticky-price-implementable allocations.

In order to obtain closed-form expressions for the optimal policy, we impose a number of functional form assumptions on preferences, technologies, and the nature of price stickiness in the economy. More specifically, we assume that firms in each industry employ Cobb-Douglas technologies to transform labor and intermediate goods into their differentiated products, with the production technology of a firm k in industry i given by

$$y_{ik} = z_i F_i(l_{ik}, x_{i1,k}, \dots, x_{in,k}) = z_i \zeta_i l_{ik}^{\alpha_i} \prod_{j=1}^n x_{ij,k}^{\alpha_{ij}}. \quad (20)$$

In the above expression, l_{ik} denotes the amount of labor hired by the firm, $x_{ij,k}$ is the quantity of good j used as an intermediate input, $\alpha_i \geq 0$ denotes the share of labor in industry i 's production technology, z_i is a Hicks-neutral productivity shock that is common to all firms in industry i , and ζ_i is a normalization constant, the value of which only depends on model parameters and is independent

of the shocks.¹³ The exponent $a_{ij} \geq 0$ in (20) captures the fact that firms in industry i may rely on the goods produced by other industries as intermediate inputs for production. Note that, for all technologies to exhibit constant returns, it must be the case that $\alpha_i + \sum_{j=1}^n a_{ij} = 1$.

Input-output linkages in this economy can be summarized by matrix $\mathbf{A} = [a_{ij}]$, which, with some abuse of terminology, we refer to as the economy's *input-output matrix*. As is customary in the literature, these input-output linkages can also be represented by a weighted directed graph on n vertices, known as the economy's *production network*. Each vertex in this graph corresponds to an industry, with a directed edge with weight $a_{ij} > 0$ present from vertex j to vertex i if industry j is an input supplier of industry i . We also define the economy's *Leontief inverse* as $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$, whose (i, j) element captures the role of industry j as a direct or indirect intermediate input supplier to industry i .

As in Section 2, we assume a monopolistically-competitive market structure within each industry, with the output of firms in industry i aggregated into a sectoral good according to the CES production function (2). We further assume that sector-specific taxes/subsidies in (1) are set to $\tau_i = 1/(1 - \theta_i)$ for all i . As discussed in Section 3, this choice undoes the effect of monopolistic markups and guarantees that the flexible-price equilibrium is efficient.

The consumption basket is also a Cobb-Douglas aggregator of the sectoral goods given by

$$C(c_1, \dots, c_n) = \prod_{i=1}^n (c_i/\beta_i)^{\beta_i},$$

where c_i is the amount of good i consumed and the constants $\beta_i \geq 0$ measure various goods' shares in the household's consumption basket, normalized such that $\sum_{i=1}^n \beta_i = 1$. In addition, we assume the representative household's preferences (3) are homothetic, with

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad , \quad V(L) = \frac{L^{1+1/\eta}}{1+1/\eta},$$

where recall that C and L denote the household's consumption and labor supply, respectively.

To specify firms' information structure and the resulting nominal rigidities, we assume all productivity shocks are drawn independently from a log-normal distribution,

$$\log z_i \sim \mathcal{N}(0, \delta^2 \sigma_z^2). \quad (21)$$

Each firm k in industry i then receives a collection of private signals $\omega_{ik} = (\omega_{i1,k}, \dots, \omega_{in,k})$ about the realized productivities given by

$$\omega_{ij,k} = \log z_j + \epsilon_{ij,k} \quad , \quad \epsilon_{ij,k} \sim \mathcal{N}(0, \delta^2 \sigma_{ik}^2), \quad (22)$$

where the noise terms $\epsilon_{ij,k}$ are independent from one another and the productivity shocks. In this formulation, $\delta > 0$ is a normalization constant, σ_z^2 measures firms' (common) prior uncertainty about the shocks, and σ_{ik}^2 parametrizes the quality of information available to firm k in industry i .¹⁴ Hence,

¹³In what follows, we set the value of this constant to $\zeta_i = \alpha_i^{-\alpha_i} \prod_{j=1}^n a_{ij}^{-a_{ij}}$. This choice has no bearing on the results, as the sole purpose of this constant is to simplify the analytical expressions.

¹⁴Throughout, we assume that all productivity shocks are drawn from a distribution with the same variance. This is a normalization assumption to ensure that the optimal policy is driven by our main objects of study (i.e., the economy's input-output structure and the extent of nominal rigidities) as opposed to differences in shock volatilities. In addition, we also assume that signals $\{\omega_{i1,k}, \dots, \omega_{in,k}\}$ observed by firm k in industry i have the same precision. This assumption allows us to capture the degree of price stickiness/flexibility of each firm by a single scalar parameter σ_{ik}^2 . A simple generalization of this framework can accommodate heterogeneity in shock volatilities and signal precisions.

an increase in σ_{ik}^2/σ_z^2 corresponds to an increase in the extent of nominal rigidity faced by firm k in industry i . In contrast, the extreme case that $\sigma_{ik}^2 = 0$ for all k and all i corresponds to an economy with fully flexible prices. More generally, it is straightforward to verify that

$$\begin{aligned}\mathbb{E}[\log z_j|\omega_{ik}] &= \phi_{ik}\omega_{ij,k} \\ \text{var}[\log z_j|\omega_{ik}] &= (1 - \phi_{ik})\text{var}[\log z_j],\end{aligned}\tag{23}$$

where

$$\phi_{ik} = \sigma_z^2/(\sigma_z^2 + \sigma_{ik}^2).\tag{24}$$

Equation (23) indicates that an increase in ϕ_{ik} corresponds to a reduction in the firm's uncertainty about the payoff-relevant productivity shocks. We thus refer to $\phi_{ik} \in [0, 1]$ as the *degree of price flexibility of firm k in industry i* . Similarly, we define the *degree of price flexibility of industry i* as

$$\phi_i = \int_0^1 \phi_{ik} dk.$$

Note that in the special case that $\sigma_{ik}^2 \in \{0, \infty\}$ for all firms, $\phi_{ik} \in \{1, 0\}$ for all i and k , in which case our framework reduces to an economy that is subject to “sticky information” pricing frictions à la [Mankiw and Reis \(2002\)](#): firms in each industry can either set their prices flexibly with no frictions or face full nominal rigidity.

To keep the analysis tractable, we work with the log-linearization of the above economy as $\delta \rightarrow 0$, where recall from equations (21) and (22) that $\delta > 0$ simultaneously parametrizes the firms' prior uncertainty about (log) productivity shocks and the noise in their private signals. This specific parametrization leads to two desirable features. First, the fact that $\text{var}(\log z_i) = \delta^2\sigma_z^2$ means that our small- δ approximation is akin to focusing on small departures from the economy's steady-state, as is typical in the New Keynesian literature. Second, scaling $\text{var}(\epsilon_{ij,k})$ with δ^2 ensures that the degree of price flexibility ϕ_{ik} in equation (24) remains independent of δ .

As a final remark, we note that the independence assumption imposed on the noise shocks $\epsilon_{ij,k}$ implies that aggregate uncertainty in this economy is solely driven by the productivity shocks $z = (z_1, \dots, z_n)$. As a result, without loss of generality, we can restrict our attention to monetary policies of the form $m(z)$ that only depend on the productivity shocks, as opposed to the entire state of the economy $s = (z, \omega)$.

4.1 Strategic Complementarities and Monetary Non-Neutrality

In this subsection, we use Proposition 2 to obtain a set of preliminary results that will serve as the basis of our characterization of the optimal policy. We first illustrate the central role of strategic complementarities in firms' price-setting decisions in our production network economy. We then show how the interaction of such strategic complementarities with nominal rigidities impacts nominal prices and marginal costs and shapes the extent of monetary non-neutrality in the economy.

Lemma 2. *The nominal price set by firm k in industry i satisfies*

$$\log p_{ik} = \mathbb{E}_{ik}[\log mc_i] + o(\delta) \quad (25)$$

$$= \alpha_i \mathbb{E}_{ik}[\log w] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n a_{ij} \mathbb{E}_{ik}[\log p_j] + o(\delta) \quad (26)$$

up to a first-order approximation as $\delta \rightarrow 0$.

Equation (25), which is a consequence of Proposition 2, establishes that, up to a first-order approximation, each firm sets its nominal price equal to its expected marginal cost, given its information set. This is a consequence of monopolistic competition and the assumption that sector-specific taxes/subsidies are set to $\tau_i = 1/(1 - \theta_i)$ to eliminate monopolistic markups.¹⁵

Lemma 2 also illustrates that our multi-sector New Keynesian model is isomorphic to a “beauty contest” game over the production network (Bergemann, Heumann, and Morris, 2017; Golub and Morris, 2018). In particular, equation (26) is identical to the first-order conditions of a network game of incomplete information in which firms in industry i choose their log nominal prices to match an industry-specific “fundamental” (given by $\alpha_i \log w - \log z_i$), while simultaneously coordinating with a linear combination of the (log) prices set by their supplier industries (given by $\sum_{j=1}^n a_{ij} \log p_j$).¹⁶ This coordination motive is the consequence of strategic complementarities in firms’ price-setting behavior in the presence of input-output linkages: all else equal, an increase in the price set by firms in an industry increases the incentive of its downstream customers to also raise their prices.

To capture the implications of the interaction between such strategic complementarities and nominal rigidities, we next define the following concept:

Definition 4. The *upstream (price) flexibility* of industry i is given by

$$\rho_i = \alpha_i + \sum_{j=1}^n a_{ij} \phi_j \rho_j, \quad (27)$$

where ϕ_j is the degree of price flexibility of industry j and α_i is the labor share of industry i .

A few observations are in order. First, note that while the definition in (27) is recursive, the vector of upstream flexibilities has a simple closed-form representation given by $\rho = (\mathbf{I} - \mathbf{A}\Phi)^{-1}\alpha$, where $\Phi = \text{diag}(\phi_1, \dots, \phi_n)$ is the diagonal matrix of (own) price flexibilities and $\alpha = (\alpha_1, \dots, \alpha_n)'$ is the vector of industry labor shares. Second, it is straightforward to verify that $\rho_i \in [0, 1]$ for all i . Third, and more importantly, the recursive nature of (27) implies that upstream flexibility of industry i is strictly increasing in the own (ϕ_j) and upstream (ρ_j) price flexibilities of any of i ’s supplier industries j , with these terms weighted by the importance of j in i ’s production technology, a_{ij} . As a result, ρ_i decreases if any of i ’s direct or indirect upstream suppliers are subject to more pricing frictions, whereas ρ_i takes its maximum value of 1 if none of its direct and indirect upstream suppliers are subject to any pricing

¹⁵The relationship between the firm’s marginal cost and its nominal price is, in general, more complicated. As we show in the proof of Proposition 2, the nominal price set by the firm is equal to $p_{ik} = \mathbb{E}_{ik}[\log mc_i \cdot v_{ik}] / \mathbb{E}_{ik}[v_{ik}]$, where v_{ik} is given by (18). However, in the limit as $\delta \rightarrow 0$, this relationship reduces to (25).

¹⁶More specifically, equation (26) coincides with the first-order conditions of a game in which the payoff of firms in industry i is given by $u_i(\log p_{ik}, \log p, \log w, \log z) = -(\log p_{ik} - (\alpha_i \log w - \log z_i))^2 - (\log p_{ik} - \sum_{j=1}^n a_{ij} \log p_j)^2$.

friction. This observation clarifies the sense in which ρ_i serves as a summary statistic for the extent of nominal rigidities in industries upstream to i .

We now use Definition 4 and the representation in Lemma 2 to characterize equilibrium nominal prices and marginal costs as a function of model primitives and the nominal wage.

Proposition 3. *Equilibrium log nominal marginal costs and prices are, respectively, given by*

$$\log mc_i = \rho_i \log w - \sum_{j=1}^n h_{ij} \log z_j + o(\delta) \quad (28)$$

$$\log p_i = \phi_i \rho_i \log w - \phi_i \sum_{j=1}^n h_{ij} \log z_j + o(\delta) \quad (29)$$

up to a first-order approximation as $\delta \rightarrow 0$, where h_{ij} is the (i, j) element of matrix $\mathbf{H} = (\mathbf{I} - \mathbf{A}\Phi)^{-1}$.

Proposition 3 illustrates that the interaction between nominal rigidities and the strategic complementarities that arise from the economy's production network amplifies the sluggishness of the response of nominal variables to real and monetary shocks. In particular, it is immediate from (28) and (29) that the pass-through of changes in the nominal wage to i 's nominal marginal cost and nominal price are given by

$$\frac{d \log mc_i}{d \log w} = \rho_i \quad , \quad \frac{d \log p_i}{d \log w} = \phi_i \rho_i, \quad (30)$$

respectively, both of which are increasing in industry i 's upstream flexibility, ρ_i .¹⁷

An important consequence of this observation is that, in our production network economy, the observed frequency and magnitude of price changes by any given firm reflect not just the pricing friction faced by that firm, but also the nominal rigidities faced by any of its direct and indirect upstream suppliers. This is in contrast to the standard one-sector models, in which one can simply treat the frequency of price changes as the exogenous price flexibility parameter.

We conclude our set of preliminary results in this subsection by characterizing how the strategic complementarities arising from the economy's production network shape the extent of monetary non-neutrality. To this end, let

$$\rho_0 = \sum_{j=1}^n \beta_j \phi_j \rho_j, \quad (31)$$

where β_j is the share of good j in the household's consumption basket. In view of (30), ρ_0 captures the pass-through of changes in the log nominal wage to the price of the household consumption basket.¹⁸ Recall from our previous discussion that this quantity has a closed-form representation in terms of model primitives given by $\rho_0 = \beta' \Phi (\mathbf{I} - \mathbf{A}\Phi)^{-1} \alpha$. Furthermore, as expected, ρ_0 is increasing in the extent of price flexibilities (ϕ_1, \dots, ϕ_n) of all industries in the economy, regardless of whether they sell directly to the household or not. We have the following result:

¹⁷This is in line with Blanchard (1983), Basu (1995), and Nakamura and Steinsson (2010), who, in simpler settings, argue that the strategic complementarities arising from the presence of input-output linkages amplify the effect of nominal rigidities and increase the sluggishness of the response of nominal variables to shocks.

¹⁸Alternatively, and in view of (27), ρ_0 can also be interpreted as the degree of (upstream) price flexibility faced by the household.

Proposition 4. *The degree of monetary non-neutrality is*

$$\Xi = \frac{d \log C}{d \log m} = \frac{1 - \rho_0}{1 + (\gamma - 1 + 1/\eta)\rho_0}, \quad (32)$$

where C is the household's consumption and m is the nominal demand.

Proposition 4, which generalizes the result of [Pasten, Schoenle, and Weber \(2019\)](#), characterizes how nominal rigidities interact with the economy's production network to generate monetary non-neutrality. As a first observation, note that Ξ is monotone decreasing in ρ_0 , which itself is decreasing in the degree of price stickiness of all industries in the economy. Therefore, as expected, an increase in nominal rigidities anywhere in the economy results in a higher degree of monetary non-neutrality. Furthermore, the recursive nature of equation (27) underscores how the strategic complementarities arising from the economy's production network amplify monetary non-neutrality: an increase in the degree of price stickiness of industry j (i.e., a decrease in ϕ_j) not only decreases ρ_0 directly as is evident from (31), but also does so indirectly by making the marginal cost of any industry i that rely on j more sluggish (thus reducing ρ_0 by reducing ρ_i).

4.2 Welfare Loss and Policy Objective

In the remainder of this section, we use our preliminary results in Subsection 4.1 to obtain a closed-form expression for the optimal monetary policy, which maximizes the expected welfare of the representative household over the set of all sticky-price equilibrium allocations.

We express the household's welfare relative to a benchmark with no nominal rigidities, which corresponds to the first-best allocation. More specifically, let W and W^* denote the representative household's welfare in the presence and absence of nominal rigidities, respectively. Similarly, let $\varrho = (p_{ik}, p_i, w)$ and $\varrho^* = (p_{ik}^*, p_i^*, w^*)$ denote the nominal price systems under the two scenarios. Given the indeterminacy of prices in the flexible-price equilibrium, we normalize the nominal wage such that $w^* = w$. We also use $e_{ik} = \log p_{ik} - \log p_{ik}^*$ to denote the "pricing error" of firm k in industry i in the sticky-price economy relative to the benchmark with no nominal rigidities. The cross-sectional average and dispersion of pricing errors within industry i are thus given by

$$\bar{e}_i = \int_0^1 e_{ik} dk \quad (33)$$

$$\vartheta_i = \int_0^1 e_{ik}^2 dk - \left(\int_0^1 e_{ik} dk \right)^2. \quad (34)$$

We have the following result:

Proposition 5. *The welfare loss due to the presence of nominal rigidities is given by*

$$W - W^* = -\frac{1}{2} \left[\sum_{i=1}^n \lambda_i \theta_i \vartheta_i + (\gamma + 1/\eta) \Sigma + \sum_{i=1}^n \lambda_i \text{var}_i(\bar{e}_1, \dots, \bar{e}_n) + \text{var}_0(\bar{e}_1, \dots, \bar{e}_n) \right] + o(\delta^2) \quad (35)$$

up to a second-order approximation as $\delta \rightarrow 0$, where $\lambda_i = p_i y_i / PC$ is the steady-state Domar weight of

industry i , ϑ_i is the dispersion of pricing errors in industry i defined in (34),

$$\Sigma = (\log C - \log C^*)^2 = \frac{1}{(\gamma + 1/\eta)^2} \left(\sum_{j=1}^n \beta_j \bar{e}_j \right)^2 + o(\delta^2) \quad (36)$$

is the volatility of output gap, and

$$\begin{aligned} \text{var}_i(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^n a_{ij} \bar{e}_j^2 - \left(\sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 \\ \text{var}_0(\bar{e}_1, \dots, \bar{e}_n) &= \sum_{j=1}^n \beta_j \bar{e}_j^2 - \left(\sum_{j=1}^n \beta_j \bar{e}_j \right)^2 \end{aligned} \quad (37)$$

are the inter-industry dispersion of pricing errors from the perspectives of industry i and the household, respectively.

Proposition 5 thus generalizes the well-known expression for welfare loss in single-sector New Keynesian models (e.g., Woodford (2003b) and Galí (2008)) to our multi-sector economy with input-output linkages.¹⁹ In particular, equation (35) illustrates that the loss in welfare due to the presence of nominal rigidities manifests itself via four separate terms.

The first term, $\lambda_i \theta_i \vartheta_i$ measures welfare losses due to price dispersion *within* each industry i and is the counterpart of welfare loss due to inflation in the textbook New Keynesian models: relative price dispersion ϑ_i in industry i translates into output dispersion and hence misallocation of resources, with the extent of this misallocation increasing in the elasticity of substitution θ_i between firms in that industry. This term vanishes if all firms in industry i make their nominal pricing decisions under the same information. Not surprisingly, the loss due to price dispersion in industry i is weighted by the industry's Domar weight, λ_i .

The second term on the right-hand side of (35) is proportional to the volatility of output gap $\Sigma = (\log C - \log C^*)^2$. This term, which is also present in the benchmark New Keynesian models and vanishes as the Frisch elasticity of labor supply $\eta \rightarrow 0$, captures loss of welfare due to inefficient supply of labor by the household, i.e., the aggregate labor wedge. Equation (36) characterizes how output gap volatility relates to industry-level pricing errors in our multi-sector economy. This equation also illustrates that, unlike textbook New Keynesian models—which assume exogenous cost-push shocks—an output gap emerges endogenously in this multi-sector economy with only efficient shocks.

In contrast to the first two terms, the third and fourth terms on the right-hand side of (35) only appear in multi-sector economies and correspond to welfare losses arising from misallocation of resources *across* industries. To see this, consider the expression $\text{var}_i(\bar{e}_1, \dots, \bar{e}_n)$ in equation (37). This term measures the dispersion in the average pricing error of i 's supplier industries, with higher weights

¹⁹It also generalizes the corresponding expressions in Woodford (2003b, 2010) for a two-sector economy with no input-output linkages and that of Huang and Liu (2005) when the economy consists of two industries forming a vertical production chain.

assigned to industries that are more important input-suppliers to i . To be even more specific, suppose industry i has two suppliers indexed j and r such that $a_{ij} + a_{ir} = 1$. In this case, it is immediate that

$$\text{var}_i(\bar{e}_j, \bar{e}_r) = a_{ij}a_{ir}(\bar{e}_j - \bar{e}_r)^2 = a_{ij}a_{ir}\left(\log(p_j/p_r) - \log(p_j^*/p_r^*)\right)^2$$

simply measures the extent to which nominal relative prices of i 's inputs diverge from the relative prices that would have prevailed under the flexible-price (and hence efficient) allocation. Finally, note that $\text{var}_i(\bar{e}_1, \dots, \bar{e}_n) = 0$ whenever industry i has only a single input supplier j with $a_{ij} = 1$, as this corresponds to a scenario in which there is no room for misallocation between i 's input suppliers.²⁰

In summary, Proposition 5 indicates that, in a multi-sector economy, the monetary authority faces an inherent trade-off between minimizing the various losses captured by equation (35). Importantly, as we already established in Corollaries 1 and 2, this trade-off cannot be circumvented, in the sense that, generically, there is no policy that can simultaneously eliminate all forms of welfare loss. Finally, note that, unlike the textbook New Keynesian models, the various trade-offs faced by the monetary authority arises from the structural properties of the economy's production network as opposed to ad hoc markup shocks.

4.3 Optimal Monetary Policy

We are now ready to present our main result of this section, which characterizes the optimal policy as a function of the economy's production network and the extent of nominal rigidities.

Theorem 2. *The optimal monetary policy is a price-stabilization policy of the form $\sum_{s=1}^n \psi_s^* \log p_s = 0$, with the weight assigned to industry s in the target price index given by*

$$\psi_s^* = \psi_s^{\text{o.g.}} + \psi_s^{\text{within}} + \psi_s^{\text{across}}, \quad (38)$$

where

$$\psi_s^{\text{o.g.}} = (1/\phi_s - 1)\lambda_s \left(\frac{1 - \rho_0}{\gamma + 1/\eta} \right) \quad (39)$$

$$\psi_s^{\text{within}} = (1 - \phi_s)\lambda_s\theta_s\rho_s \quad (40)$$

$$\psi_s^{\text{across}} = (1/\phi_s - 1) \left[(\rho_0 - \rho_s)\lambda_s + \sum_{i=1}^n (1 - \phi_i)\lambda_i\rho_i\ell_{is} \right] \quad (41)$$

and ρ_i is the degree of upstream price stickiness in (27).

Theorem 2 provides a characterization of the optimal policy in terms of model primitives, with each term on the right-hand side of (38) aimed at minimizing a specific source of welfare loss in (35). Recall from Subsection 4.2 that the monetary authority faces a trade-off between minimizing the various sources of allocational inefficiencies due to nominal rigidities. Not surprisingly then, the optimal policy in (38) consists of three different terms corresponding to the relative importance of each of these misallocations for household welfare: the first term on the right-hand side of (38) aims

²⁰The interpretation for the term $\text{var}_0(\bar{e}_1, \dots, \bar{e}_n)$ in equation (35) is identical, with the household replacing industry i as purchaser of various goods.

to minimize the welfare loss induced by the labor wedge (or equivalently, output gap volatility), the second term arises due to the policymakers' concern about within-industry price dispersion, and the last term is in response to misallocation across industries.²¹ Note that the three terms constituting the optimal policy in (38) are in general not proportional to one another, thus indicating that the monetary authority faces a real trade-off between the corresponding misallocation losses.

The above result also illustrates that the optimal policy is shaped by how nominal rigidities interact with the economy's production network. In particular, the weight ψ_i^* assigned to any given industry i in the optimal policy depends not just on that industry's size (as measured by its Domar weight λ_i) and price flexibility (as parametrized by ϕ_i), but also on i 's position in the production network and the nominal rigidities faced by other industries in the economy. This dependence is a consequence of strategic complementarities in firms' price-setting decisions discussed in Subsection 4.1.

As a final remark, we note that in deriving the above result, we did not restrict our attention to the class of price-stabilization policies to begin with. In particular, the monetary authority can choose the nominal demand, $m(z)$, as an arbitrary function of productivities in an unrestricted manner, including policies that do not stabilize any specific price index.²² Nonetheless, Theorem 2 establishes that the optimal policy stabilizes the price index $\sum_{s=1}^n \psi_s^* \log p_s$ with weights given by (38). This reflects the fact that the underlying flexible-price economy is efficient: even though our multi-sector economy gives rise to an endogenous output gap, the absence of inefficient cost-push shocks means that there are no forces driving optimal policy away from price stabilization.

4.4 Comparative Statics

In what follows, we present a series of comparative static results to distill the role of the various forces that shape the optimal policy (38) in a transparent manner. These results allow us to obtain general insights on how optimal policy depends on the production network and the distribution of pricing frictions throughout the economy. In particular, we establish that, all else equal, optimal policy stabilizes a price index with higher weights assigned to (i) larger industries as measured by their Domar weights, (ii) stickier industries, (iii) more upstream industries, (iv) industries with less sticky upstream suppliers but with stickier downstream customers, and (v) industries whose customers exhibit higher upstream flexibilities.

We start with a definition.

Definition 5. Industries i and j are *upstream symmetric* if $a_{ir} = a_{jr}$ for all industries r . They are *downstream symmetric* if $a_{ri} = a_{rj}$ for all industries r and $\beta_i = \beta_j$.

While upstream symmetry means that i and j have the same production technology, downstream symmetry means that the two industries take identical roles as input suppliers of other firms in the economy as well as in household preferences. Consequently, if i and j are upstream symmetric, then they have the same degree of upstream flexibilities ($\rho_i = \rho_j$), whereas if they are downstream symmetric, they have the same steady-state Domar weights ($\lambda_i = \lambda_j$). We have the following result:

²¹Alternatively, the expressions in (39), (40), and (41) correspond to the optimal price-stabilization policies of a planner who only intends to minimize welfare losses arising from output gap volatility, within-industry price dispersion, and inter-industry price dispersion, respectively.

²²Lemma A.1 in the appendix characterizes the set of policies $m(z)$ that can be implemented as a price-stabilization policy.

Proposition 6. *Suppose industries i and j are upstream and downstream symmetric. Also suppose $\theta_i = \theta_j$. Then, $\psi_i^* > \psi_j^*$ in the optimal policy if and only if $\phi_i < \phi_j$.*

This result thus extends what Eusepi, Hobijn, and Tambalotti (2011) refer to as the “stickiness principle” to our multi-sector economy with input-output linkages: all else equal, the monetary authority should stabilize a price index that places larger weights on producers with stickier prices.

To see the intuition underlying Proposition 6, note that, by the upstream symmetry assumption, mc_i and mc_j have the same ex ante distribution. Yet, firms in the stickier of the two industries respond more sluggishly to changes in their realized marginal costs. Therefore, by targeting the industry that is subject to more nominal rigidities for price stabilization, the optimal policy reduces the need for price adjustments by firms in that industry and thus reduces the overall level of within-industry price dispersion. In fact, equation (40) in Theorem 2 implies that, due to upstream symmetry, if $\phi_i < \phi_j$, then $\psi_i^{\text{within}} > \psi_j^{\text{within}}$. At the same time, the downstream symmetry assumption implies that a policy that targets the stickier industry more also reduces output gap volatility and the inter-industry price dispersion faced by i and j 's common customers. In fact, from equations (39) and (41), it is easy to see that whenever $\phi_i < \phi_j$, it must be the case that $\psi_i^{\text{O.G.}} > \psi_j^{\text{O.G.}}$ and $\psi_i^{\text{across}} > \psi_j^{\text{across}}$. Putting these inequalities together then guarantees that $\psi_i^* > \psi_j^*$,

By assuming that i and j take symmetric positions in the production network, Proposition 6 effectively assumes away how differences in input-output linkages may matter for optimal policy. In our subsequent results, we instead assume that the two industries are equally sticky and focus on the role of network connections in shaping the optimal policy.

Proposition 7. *Suppose i and j are downstream symmetric. Also suppose $\phi_i = \phi_j < 1$ and $\theta_i = \theta_j$. Then, $\psi_i^* > \psi_j^*$ if and only if $\rho_i > \rho_j$.*

This result encapsulates the second general principle that emerges from our characterization of optimal policy: all else equal, the optimal target price index places a higher weight on the industry with less sticky upstream suppliers, as summarized by a higher degree of upstream flexibility ρ .

The intuition underlying Proposition 7 is straightforward. Since i and j are downstream symmetric, stabilizing the price of either industry would have the same exact effect on the labor wedge and the extent of inter-industry relative price distortions perceived by their (direct and indirect) customers. As a result, any differential treatment of i and j by the optimal policy is solely driven by concerns for within-industry misallocation.²³ At the same time, the fact that $\rho_i > \rho_j$ means that industry j 's marginal cost responds more sluggishly to shocks, making the lack of complete information about the realized shocks less material for price-setting by firms in j compared to those in i . As a result, all else equal, the within-industry price dispersion would be lower in j than in i , despite the fact that both industries are equally sticky. Not surprisingly then, the optimal price-stabilization target assigns a lower weight to the industry with stickier suppliers, which has an already more stabilized marginal cost.

²³In fact, it is easy to verify that (39) and (41) imply that $\psi_i^{\text{O.G.}} = \psi_j^{\text{O.G.}}$ and $\psi_i^{\text{across}} = \psi_j^{\text{across}}$.

Proposition 8. *Suppose industries i and j are upstream symmetric. Also suppose $\phi_i = \phi_j < 1$, $\theta_i = \theta_j$, and $\lambda_i = \lambda_j$. Then, $\psi_i^* > \psi_j^*$ if and only if*

$$\sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}) > 0. \quad (42)$$

The above result highlights yet another important dimension along which the production network structure shapes the optimal policy. In particular, it establishes that, all else equal, industry i receives a higher weight in the optimal price-stabilization index if (i) it is a more important supplier to stickier industries and (ii) its customers have a higher degree of upstream flexibility. To see these from inequality (42), recall that expression $\ell_{si} - \ell_{sj}$ captures the differential importance of i and j as direct or indirect input suppliers to any industry s . Therefore, the left-hand side of (42) is positive if, relative to j , industry i is a more important supplier of industries with lower degree of price flexibility ϕ_s but higher degree of upstream price flexibility ρ_s .

Why is it optimal to stabilize the price of the industry with stickier downstream customers? This is because such a policy would reduce the need for the firms in the customer industry to adjust their nominal price. Therefore, the stickier are those customers, the higher is the welfare gain of stabilizing their marginal cost by assigning a higher weight on their suppliers in the target price-stabilization index.

The argument for why optimal policy places a larger weight on the industry whose customers have a higher degree of upstream flexibility ρ is also similar. Recall that, all else equal, a higher degree of upstream flexibility ρ_s means that firms in industry s face a more volatile nominal marginal cost and hence, on average, have to adjust their nominal price by more. Therefore, stabilizing the price of one of their inputs, i , would reduce the need for such price adjustment and hence reduce the welfare loss arising from nominal rigidities.

Proposition 9. *Suppose j is the sole input-supplier of i and i is the sole supplier of j . Also, suppose $\phi_i = \phi_j < 1$ and $\theta_i = \theta_j$. Then, $\psi_i^* < \psi_j^*$.*

The assumption that i and j are, respectively, each other's only customer and supplier and have identical stickiness and substitution elasticities is meant to ensure that the difference between the two industries is solely in their respective positions in the chain of production, with industry j taking an unambiguously upstream position vis-à-vis industry i . As such, Proposition 9 establishes that, all else equal, the optimal policy assigns a higher weight to more upstream industries. The differential treatment of the two industries by the optimal policy is driven purely by concerns about within-industry misallocation.

Taken together, our results presented as Propositions 6–9 yield general principles for the optimal conduct of monetary policy in the presence of input-output linkages. In particular, they establish that, all else equal, optimal policy stabilizes a price index with higher weights assigned to industries that are stickier (Proposition 6), have more flexible upstream suppliers (Proposition 7) but more sticky downstream customers (Proposition 8), have downstream customers with a higher degree of upstream flexibility (Proposition 8), and are themselves more upstream (Proposition 9).

4.5 Examples

We conclude this section by providing a series of examples to further clarify the dependence of optimal policy on model primitives.

Example 1 (vertical production network). Consider the economy depicted in Figure 1(a), in which each industry $i \neq n$ depends on the output of a single other industry as its input for production ($a_{i,i+1} = 1$), industry n only uses labor ($\alpha_n = 1$), and the household, labeled as vertex 0 in the figure, only consumes the good produced by industry 1 ($\beta_1 = 1$ and $\beta_i = 0$ for all $j \neq 1$).

Given the vertical nature of production, it is immediate that in this economy—a variant of which is studied by [Huang and Liu \(2005\)](#) and [Wei and Xie \(2020\)](#)—there is no room for inter-industry misallocation. Indeed, the expressions in (37) corresponding to welfare losses arising from inter-industry misallocation are equal to zero, irrespective of the extent of nominal rigidities. It is therefore not surprising that this source of welfare loss is immaterial for the design of optimal policy: equation (41) in Theorem 2 implies that $\psi_s^{\text{across}} = 0$ for all s .

There is, however, room for within-industry misallocation, as firms' incomplete information about productivity shocks may result in a non-trivial price dispersion within each industry. Indeed, equation (40) implies that the corresponding component of optimal policy is non-zero and is given by

$$\psi_s^{\text{within}} = (1 - \phi_s)\theta_s\rho_s,$$

where $\rho_s = \phi_{s+1}\phi_{s+2}\dots\phi_n$ is the degree of upstream flexibility of industry s . Note that due to the presence of strategic complementarities, ρ_s is smaller for industries further downstream: $\rho_s \leq \rho_{s+1}$.

Combining the above with equation (39)—which captures the component of optimal policy that aims at reducing welfare losses arising from the labor wedge—implies that the optimal price-stabilization target is given by

$$\psi_s^* = \psi_s^{\text{within}} + \psi_s^{\text{o.g.}} = (1 - \phi_s)\theta_s\phi_{s+1}\phi_{s+2}\dots\phi_n + \frac{1 - \phi_1\phi_2\dots\phi_n}{\gamma + 1/\eta}.$$

Consequently, the optimal policy assigns a larger weight to (i) industries with higher degrees of price stickiness (i.e., lower ϕ_s), (ii) those with higher elasticities of substitution θ_s , and (iii) more upstream industries. This latter observation is of course consistent with the prescription of Proposition 9.

Example 2 (horizontal production network). Next, consider the economy depicted in Figure 1(b), in which all industries only rely on labor as their input for production, i.e., $\alpha_i = 1$ for all i . This economy is therefore similar to the multi-sector economies with no input-output linkages that were studied in the prior literature, such as [Mankiw and Reis \(2003\)](#), [Benigno \(2004\)](#), and [Woodford \(2010\)](#).

Unlike the economy in Example 1, nominal rigidities in the horizontal economy result in misallocation not only within but also across industries: while efficiency requires relative prices across industries to move with corresponding productivities, such movements are in general not possible. This observation means that the component of optimal policy that targets inter-industry misallocation losses is non-zero. In particular, equation (41) in Theorem 2 implies that

$$\psi_s^{\text{across}} = (1/\phi_s - 1)\beta_s \sum_{i=1}^n \beta_i(\phi_i - \phi_s).$$

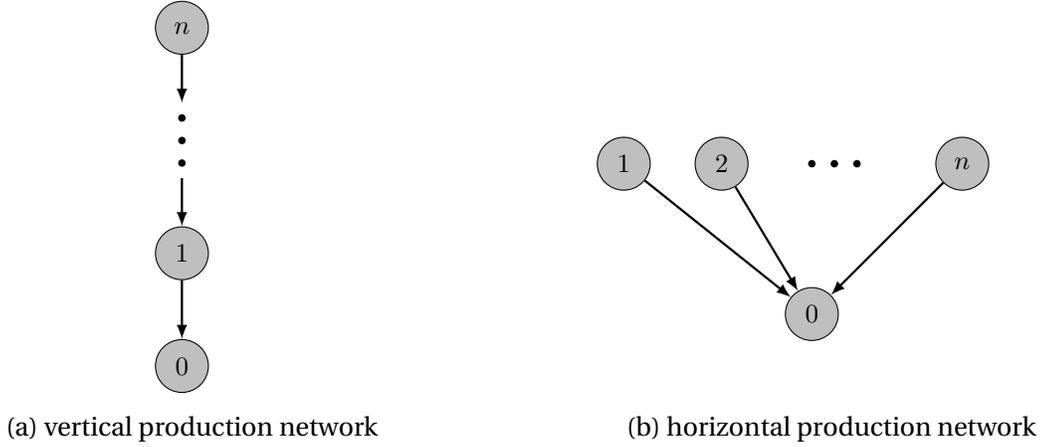


Figure 1. Horizontal and vertical production networks. Each vertex corresponds to an industry, with a directed edge present from one vertex to another if the former is an input-supplier to the latter. The vertex indexed 0 represents the household.

Next, note that all industries in this economy are upstream symmetric, and in fact, since no industry has a sticky-price supplier, $\rho_s = 1$ for all s . As a result, (40) reduces to

$$\psi_s^{\text{within}} = (1 - \phi_s)\beta_s\theta_s,$$

whereas (39) implies that

$$\psi_s^{\text{o.g.}} = (1/\phi_s - 1)\beta_s \left(\frac{1 - \sum_{i=1}^n \beta_i \phi_i}{\gamma + 1/\eta} \right).$$

Taken together, the above expressions imply that, consistent with the results of [Benigno \(2004\)](#) and [Woodford \(2010\)](#), industries with (i) higher levels of price stickiness and (ii) higher shares in the household's consumption basket receive a larger weight in the optimal price-stabilization policy.

Example 3. Finally, consider the economy depicted in [Figure 2](#), in which industries s and t only rely on labor as their input for production ($\alpha_s = \alpha_t = 1$) and are in turn input-suppliers to industries 1 and 2, with $a_{1s} = a_{2t} = a < 1$. To isolate the role of the network structure, we in addition impose the symmetry assumptions that $\phi_s = \phi_t = \phi$ and $\theta_s = \theta_t = \theta$ and that s and t have the same steady-state Domar weight, i.e., $\lambda_s = \lambda_t = \lambda$. Therefore, any heterogeneity across s and t only comes from the stickiness of their respective customers 1 and 2 and the latter firms' other suppliers 3 and 4.

As a first observation, note that since the two industries are identical in size and stickiness and are upstream symmetric, they contribute equally to welfare losses arising from output gap volatility and within-industry misallocation. Not surprisingly then equations (39) and (40) imply that the weights in the optimal policy corresponding to output gap and within-industry misallocation losses are also equal, i.e., $\psi_s^{\text{o.g.}} = \psi_t^{\text{o.g.}}$ and $\psi_s^{\text{within}} = \psi_t^{\text{within}}$.

The contributions of s and t to across-industry misallocation, on the other hand, depend on their downstream supply chains, which are not necessarily identical. In particular, an immediate application of equation (41) in [Theorem 2](#) implies that $\psi_s^{\text{across}} > \psi_t^{\text{across}}$ if and only if

$$(1 - \phi_1)\rho_1 > (1 - \phi_2)\rho_2. \tag{43}$$

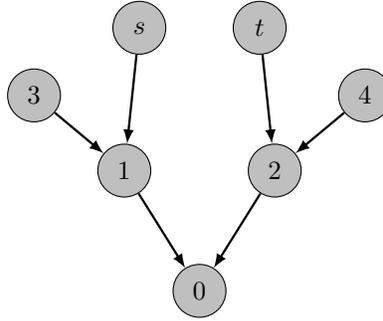


Figure 2. Each vertex corresponds to an industry, with a directed edge present from one vertex to another if the former is an input-supplier to the latter. The vertex indexed 0 represents the household.

To further clarify the nature of optimal policy, suppose that industries 3 and 4 are equally sticky, so that $\rho_1 = \rho_2$. In such a case, the above inequality implies that $\psi_s^* > \psi_t^*$ if and only if $\phi_1 < \phi_2$. In other words, all else equal, the optimal policy places a larger weight on the industry whose downstream customer is stickier, as prescribed by Proposition 8. This is because by stabilizing the industry with the stickier customer, the policy can reduce the need for the firms in the customer industry to adjust their nominal price, thus effectively reducing the variance of pricing errors by more. A similar argument also illustrates that if firms in industry 1 and 2 are equally sticky (so that $\phi_1 = \phi_2$), then $\psi_s^* > \psi_t^*$ if and only if $\phi_3 > \phi_4$, which guarantees that $\rho_1 > \rho_2$.

5 Quantitative Analysis

In this section, we use our results in Section 4 to determine the optimal monetary policy for the U.S. economy as implied by the model.

Our analysis relies on two sources of data. As our first source, we use the 2018 input-output tables constructed by the Bureau of Economic Analysis (BEA) to determine the intermediate input expenditure by various industries. The BEA tables also detail total compensation of employees for each industry as well as each industry’s contribution to final uses.

The second source of data is provided to us by [Pasten, Schoenle, and Weber \(2019\)](#), who use the confidential micro data underlying the Bureau of Labor Statistics producer price index (PPI) to calculate the frequency of price adjustment at the industry level. The PPI measures changes in producers’ selling prices of all goods-producing industries including mining, manufacturing, gas and electricity, as well as retail and service sectors. [Pasten et al. \(2019\)](#) calculate the frequency of producer price changes as the ratio of the number of price changes to the number of sample months. We use this data to obtain proxies for each industry’s degree of price flexibility.

We merge price-adjustment data with the BEA input-output data at the 3-digit North American Industry Classification System (NAICS) level, while excluding sectors corresponding to finance, insurance, and real estate. This results in a matched dataset consisting of 44 industries. The mean and median price change frequency across industries are 23.4% and 19.4%, implying a mean and median price duration of 3.75 months and 4.64 months, respectively. These frequencies and implied durations are in line with prior estimates of price frequencies and durations produced by [Bils and Klenow \(2004\)](#)

and Klenow and Kryvtsov (2008).²⁴

Calibration. We interpret each period as a quarter. We calibrate the input-output matrix, \mathbf{A} , and labor expenditure shares, α , so as to match the intermediate good expenditure shares and compensation of employees by industry in the BEA input-output data, respectively. We similarly construct the vector of consumption shares, β , to match final uses of each industry's output.

In order to calibrate the model's vector of price flexibilities ϕ , recall from our discussion in Subsection 4.1 that, due to the presence of strategic complementarities in firms' price-setting decisions, the observed frequency and magnitude of price changes may differ from the underlying degrees of price flexibilities. In particular, price adjustments by firms in industry i depend not just on ϕ_i but also on nominal price adjustments by i 's suppliers, who themselves are subject to pricing frictions. To account for this endogeneity, we thereby treat the observed price change frequencies measured by Pasten et al. (2019) as the empirical counterpart of $\phi_i \rho_i$, where ρ_i is the degree of upstream flexibility of industry i in Definition 4. We then back out the implied vector of price flexibilities ϕ .

Next, we calibrate the volatility of the idiosyncratic productivity shocks in order to match the average absolute size of price changes in micro data. Klenow and Kryvtsov (2008) find that conditional on the occurrence of a price change, the average absolute size of a price change is 11.5%. We thus set the standard deviation of sectoral shocks such that the average standard deviation of price changes match this number under complete information in our model. This procedure yields a standard deviation of sectoral productivity equal to 8.8%.²⁵

We set the within-industry elasticity of substitution, θ , equal to 6. This number is consistent with values commonly used in the New Keynesian literature, typically set to match steady-state levels of markups. For example, Coibion, Gorodnichenko, and Wieland (2012) set the elasticity of substitution equal to 7 in order to match steady-state markups of 17% as estimated by Burnside (1996) and Basu and Fernald (1997). Similarly, McKay, Nakamura, and Steinsson (2016) and Christiano, Eichenbaum, and Rebelo (2011) set the elasticity of substitution equal to 6, consistent with steady-state markups of 20%. While we allow for a subsidy in our model that eliminates all steady-state markups, we keep the value of this elasticity in line with the rest of the literature.

Finally, the preference parameters η and γ are chosen as follows. We set the Frisch elasticity of labor supply to $\eta = 2$; this value is consistent with "macro" elasticities of labor supply (Hall, 2009). As for γ , note that risk aversion and intertemporal substitution play no role in our setting as all choices are static and the representative household faces no uncertainty. Therefore, the parameter γ merely controls the household's wealth effect on labor supply. We thus set $\gamma = 0.1$; this value essentially minimizes the income effect on labor supply, similar to using GHH preferences (Greenwood, Hercowitz, and Huffman, 1988).

²⁴In particular, Bils and Klenow (2004) report a median duration of prices across all categories to be 4.3 months for posted prices and 5.5 months for regular prices. For posted prices, Klenow and Kryvtsov (2008) report the mean and the median to be 6.8 and 3.7 months, respectively.

²⁵When they exclude sale-related price changes, Klenow and Kryvtsov (2008) find that the average absolute size of price changes falls to 9.7%. It is unclear whether one should exclude or include sales; we choose to target the number that includes sales. Furthermore, we calibrate σ_z to match the average size of price changes under complete information rather than under incomplete information, as this yields a smaller standard deviation for z_i .

Table 1. Welfare Loss under Various Policies

	optimal policy (1)	output-gap stabilization (2)	CPI targeting (3)	Domar weighted (4)	stickiness weighted (5)
Welfare loss (percent consumption)	2.98	2.99	3.51	3.75	3.22
within-industry misallocation	2.66	2.67	3.00	3.16	2.80
across-industry misallocation	0.32	0.32	0.40	0.42	0.36
output gap volatility	10^{-5}	0	0.11	0.17	0.05
Cosine similarity to optimal policy	1	0.9957	0.5181	0.5929	0.6260

Notes: The table reports the welfare loss due to the presence of nominal rigidities under various monetary policies as a percentage of quarterly consumption in the flexible-price economy. The various components of welfare loss are defined in accordance with the decomposition in equation (35). The last row reports the cosine similarity between each policy and the optimal policy defined as $\cos(\psi, \psi^*) = \psi' \psi^* / (\|\psi\|_2 \cdot \|\psi^*\|_2)$, where ψ and ψ^* denote the vector of weights in target price index of the policy and the optimal policy, respectively.

Optimal Monetary Policy. With the calibrated model in hand, we use our characterization results in Section 4 to obtain the optimal monetary policy implied by the model as well as the associated welfare loss. In particular, we use equation (38) in Theorem 2 to calculate the weights corresponding to the optimal price-stabilization index, while using equation (35) to obtain the welfare loss due to the presence of nominal rigidities under the optimal policy. This equation also provides us with a decomposition of the welfare loss in terms of the various sources of misallocation as well as the labor wedge. We measure the welfare loss in our sticky-price economy as a percentage of consumption in its flexible-price counterpart.

The first column of Table 1 reports the results. We find that the optimal policy generates a welfare loss equivalent to a 2.98% loss in quarterly consumption relative to the flexible-price equilibrium, with the largest component due to misallocation within industries accounting for 2.66 percentage points of loss in consumption. The second largest component is due to misallocation across industries, which accounts for another 0.32 percentage points loss in consumption. Finally, as the table indicates, under the optimal policy, there is nearly zero welfare loss due to the third component: volatility of the output gap.

Table 1 also provides a comparison between the performance of the optimal policy and four alternative, non-optimal, price-stabilization policies. The first of these is the policy that minimizes the volatility of output gap. Recall from equation (39) that such a policy stabilizes a price index with weights given by $\psi_s^{\text{O-G}} \propto (1/\phi_s - 1)\lambda_s$ for all s , where ϕ_s is the degree of price flexibility of industry s and λ_s is the corresponding Domar weight. As the second column of the table indicates, the policy that minimizes the volatility of output gap generates a welfare loss that is equivalent to a 2.99% fall in quarterly consumption. This welfare loss is incredibly similar to that under the optimal policy both in magnitude as well as in decomposition.

The third and fourth columns of Table 1 present welfare losses corresponding to two policies that weigh sectors by size. The first of these policies, which is akin to targeting CPI or PCE, stabilizes the

household’s consumption price index, with weights that are equal to consumption shares: $\psi_s^{\text{CPI}} = \beta_s$. The second policy weighs sectors not by consumption shares but by their sales shares: $\psi_s^{\text{Domar}} \propto \lambda_s$. As is evident from the table, and in contrast to the output-gap-stabilization policy, these policies result in materially larger welfare losses compared to the optimal policy: consumption-weight-based price stabilization leads to a welfare loss equivalent to a 3.51% fall in quarterly consumption, while the Domar-weights-based price stabilization results in a welfare loss equivalent to a 3.75% reduction in quarterly consumption. Further note that, unlike the output-gap stabilization policy, these two policies generate a non-zero welfare loss due to volatility of the output gap.

Finally, the last column of Table 1 reports the composition of welfare losses under a policy that targets a price index with weights proportional to the sectors’ degree of price stickiness. The rationale behind this policy—which applies the stickiness principle by placing greater weights on sticky-price sectors and less weight on more flexible ones—is the same rationale that underlies the targeting of core price indices. For this policy, we set the weights in the target price index based on the sectors’ “observed” degree of price stickiness, as this is the typical estimate of price stickiness without taking the input-output linkages into consideration, i.e., $\psi_s^{\text{sticky}} \propto 1 - \phi_s \rho_s$ for all s . We find that such a policy generates a welfare loss equivalent to 3.22% of quarterly consumption. Thus, moving from the price-stickiness policy to the optimal policy reduces welfare losses by 0.76 percentage points in quarterly consumption.

All four alternative, sub-optimal policies we consider are within the class of price stabilization policies, in which $\sum_{s=1}^n \psi_s \log p_s = 0$ for some vector of weights (ψ_1, \dots, ψ_n) . One could, however, consider policies that lie outside of this class. Quantitative explorations suggest that policies that are not in the form of price stabilization result in significantly larger welfare loss relative to the five policies presented in Table 1. This is not surprising given that the underlying flexible-price economy is efficient. In the absence of inefficient cost-push shocks, there are no forces driving optimal policy away from price stabilization; therefore any such non-price-stabilization policy only moves the economy even further away from the efficient allocation.

Approximate Optimality of Output-Gap Stabilization. As already discussed, the welfare differences between the optimal and output-gap-stabilization policies in our calibration are minuscule, amounting to less than 0.01 percentage points of quarterly consumption. Crucially, the industry-specific weights in the implied target price indices of the two policies are also very similar. This similarity can be seen from Figure 3, which plots the weights corresponding to the two policies side by side (normalized such that the weights in each policy add up to 1).

To quantify the similarity between various policies, the last row of Table 1 reports the cosine similarities between the optimal policy and each of the four alternative policies.²⁶ According to this metric, the optimal and the output-gap-stabilization policies are more than 99% similar—that is, they are virtually identical. It is thereby no surprise that the difference in welfare between the two is

²⁶We define the cosine similarity between two policies as the cosine of the angle between with vectors representing the weights in the policies’ corresponding target price indices ψ and $\hat{\psi}$. More specifically, $\cos(\psi, \hat{\psi}) = \psi' \hat{\psi} / \|\psi\|_2 \|\hat{\psi}\|_2$. The cosine similarity between two policies is always between -1 and 1 , and is equal to 1 if and only if ψ and $\hat{\psi}$ are (positively) proportional. If ψ and $\hat{\psi}$ are element-wise non-negative, then $\cos(\psi, \hat{\psi})$ is always between 0 and 1 .

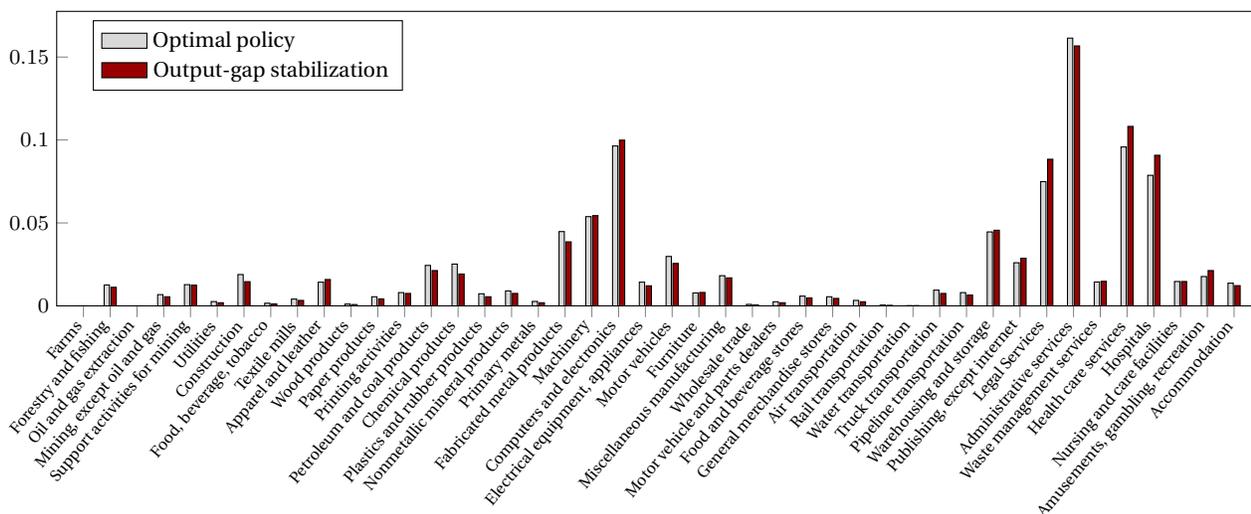


Figure 3. Industry Weights under Optimal and Output-Gap-Stabilization Policies

Notes: The figure plots the weights assigned to each industry in the price indices corresponding to the optimal and output-gap-stabilization policies. The weights for each policy are normalized such that they add up to 1.

negligible. For comparison, the cosine similarity between the optimal policy and any of the other three policies never exceeds 63%.

It is important to note that, while the optimal and output-gap-stabilization policies are nearly identical, this does not mean that welfare losses due to within- and across-industry misallocation are immaterial. For one thing, as Table 1 illustrates, the bulk of the loss is due to these terms irrespective of the policy.²⁷ Furthermore, these losses are also not policy-invariant, as the magnitude of the welfare loss due to misallocation between and across industries changes depending on the policy. Rather, the similarity between the optimal and output-gap-stabilization policies is driven by the fact that, in our calibration, the three components that contribute to the optimal policy weights in equation (38) are more or less aligned, with cosine similarities that exceed 90%. In particular, $\cos(\psi^{o.g.}, \psi^{within}) = 99.08\%$, $\cos(\psi^{o.g.}, \psi^{across}) = 92.07\%$, and $\cos(\psi^{across}, \psi^{within}) = 90.01\%$.

We conclude by noting that, as indicated by Figure 3, both the optimal and the output-gap-stabilization policies place the greatest weight on certain service sectors—e.g. administrative, health care, legal services, and hospitals—as well as manufacturing sectors such as computers and electronics, machinery, fabricated metal products, and motor vehicles. The reason these particular sectors command such large weights is due to the fact that they exhibit both large Domar weights but low price flexibility. The service and health care sectors in particular comprise a generous portion of the economy but are relatively sticky.²⁸ At the same time, many of the largest sectors in the data exhibit very flexible prices—examples include wholesale trade, oil and gas extraction, and petroleum, coal, and chemical products. These sectors, despite being large and important sectors in the economy,

²⁷One can also measure the contribution of each of the three components to the optimal policy weights using equation (38). We find that while, on average, the term corresponding to output gap volatility ($\psi^{o.g.}$) accounts for 18.6% of the optimal weight ψ^* , the terms corresponding to within- and across-industry misallocations are, on average, responsible for 75% and 6.4% of optimal weights, respectively.

²⁸See [Bils and Klenow \(2004\)](#), [Dhyne et al. \(2006\)](#), and [Gorodnichenko and Weber \(2016\)](#) for more evidence on the stickiness of health care and service sectors in the cross-section (both in the U.S. and in Europe).

command less weight under the optimal policy than their size alone would dictate. Conversely, sectors like apparel, printing, accommodation, and waste management are relatively sticky but are small; as a result, stabilizing the prices of these sectors also becomes less of a priority.

6 Conclusion

In this paper, we study the optimal conduct of monetary policy in a multi-sector economy in which firms buy and sell intermediate goods over a production network. We introduce nominal rigidities into a rather canonical multi-sector, input-output model along the lines of [Long and Plosser \(1983\)](#) and [Acemoglu et al. \(2012\)](#) by assuming that firms make nominal pricing decisions under incomplete information about the aggregate state.

Within the context of this model, we make two theoretical contributions. First, we obtain necessary and sufficient conditions on the economy's disaggregated production structure and the nature of nominal rigidities under which monetary policy can implement flexible-price equilibria and hence restore productive efficiency. As an important byproduct of this result, we also show that these conditions are violated for a generic set of information structures, thus concluding that, generically, monetary policy cannot achieve productive efficiency. This is in stark contrast to the canonical one-sector New Keynesian model in which, in the absence of inefficient shocks, the efficient allocation can be implemented with price stability.

Given that the first-best allocation cannot be attained generically, our second theoretical contribution is thereby to characterize the constrained-optimal policy and to provide general principles for the optimal conduct of monetary policy in the presence of input-output linkages. In particular, we show that the optimal policy faces a trade-off between three components of welfare loss: misallocation across sectors, misallocation within sectors, and volatility of the output gap. We find that the optimal monetary policy is a price-index-stabilization policy with higher weights assigned to larger, stickier, and more upstream industries, as well as industries with less sticky upstream suppliers but stickier downstream customers.

Finally, in a quantitative application of our framework, we determine the optimal price index for the U.S. economy and find that welfare gains (as measured as percentages of first best consumption) from moving to the optimal policy is rather large. In particular, moving from CPI targeting to the optimal price index results in a welfare gain equivalent to more than 0.5 percentage points of quarterly consumption. At the same time, we also find that, in our calibration, the difference in welfare loss under the optimal policy and under the policy that stabilizes the output gap is rather negligible.

Our theoretical and quantitative results can inform the policy debate around the appropriate price level the central bank should target ([Mishkin, 2007](#); [Bullard, 2011](#); [Thornton, 2011](#)). There are numerous measures of the aggregate price level; the indices most often considered by policymakers are overall measures of consumer prices (the CPI or the PCE), measures of consumer prices that exclude food and energy categories (core CPI or core PCE), as well as measures of producer prices (the PPI). On the theoretical side, our results provide a formal framework to account for the disaggregated nature of production in designing the proper target index. On the quantitative side, the near optimality of the output-gap-stabilization policy indicate that core inflation measures that discount

flexible price sectors (such as wholesale trade and energy) but also weigh sectors by their sales shares are desirable stabilization targets.

We view our paper as a step towards exploring the implications of the disaggregated nature of production for the transmission and the optimal conduct of monetary policy. Several important issues, however, remain open for future research. First, as emphasized throughout the paper, we assumed that the underlying flexible-price allocation in our economy is efficient. While this was a conscious modeling decision made in order to isolate how the multi-sector, input-output feature of our economy fundamentally changes the policy prescriptions of one-sector New Keynesian models, the role of monetary policy would be more complicated in an economy with an inefficient steady-state, as the monetary policy faces an additional trade-off between stabilizing prices and substituting for missing tax instruments. Exploring the implications of such a trade-off for the conduct of monetary policy would be a natural next step.

Second, a growing empirical literature has documented the propagation of real shocks —such as natural disasters (Carvalho et al., 2016), trade shocks (Huneus, 2019), and demand shocks (Acemoglu et al., 2016)—over input-output linkages. We believe that similar empirical investigations on the production network's role as a monetary transmission mechanism, along the lines of Ozdagli and Weber (2020), would shed further light on how monetary policy can shape real economic outcomes.

A Proofs

Proof of Proposition 2

First, we establish that if an allocation is implementable as a sticky-price equilibrium, then it satisfies equations (14) and (15). To this end, first note that in any sticky-price equilibrium, the first-order conditions corresponding to household's optimization problem are given by

$$V'(L(s)) = \lambda_c(s)w(s) \quad (\text{A.1})$$

$$U'(C(s)) \frac{dC}{dc_i}(s) = \lambda_c(s)p_i(\omega_i) \quad (\text{A.2})$$

for all i , where $\lambda_c(s)$ is the Lagrange multiplier corresponding to the household's budget constraint. From the above it is therefore immediate that

$$V'(L(s)) = \frac{w(s)}{p_i(\omega_i)} U'(C(s)) \frac{dC}{dc_i}(s)$$

for all i . Furthermore, the fact that $\sum_{i=1}^n p_i(\omega_i)c_i(s) = m(s)$ for all $s \in S$ implies that the nominal wage satisfies

$$w(s) = m(s) \frac{V'(L(s))}{C(s)U'(C(s))}, \quad (\text{A.3})$$

where we are using that the consumption aggregator C is homogenous of degree 1.

Next, consider the firms' price setting problem. In any sticky price equilibrium, the nominal price set by firm k in industry i is the solution to the optimization problem

$$\max_{p_{ik}} \mathbb{E}_{ik} [\mathcal{M}(s) ((1 - \tau_i) p_{ik}(\omega_{ik})y_{ik}(s) - mc_i(s)y_{ik}(s))] \quad (\text{A.4})$$

$$\text{s.t. } y_{ik}(s) = (p_{ik}(\omega_{ik})/p_i(\omega_i))^{-\theta_i} y_i(s), \quad (\text{A.5})$$

where the expectation is taken with respect to the firm's information set and \mathcal{M} denotes the household's nominal stochastic discount factor. Note that we are using the fact that the realized marginal cost of all firms in the same industry are identical, i.e., $mc_{ik}(s) = mc_i(s)$ for all $s \in S$ and all firms k in industry i . Plugging the constraint into the objective function and taking first-order conditions implies that

$$\mathbb{E}_{ik} \left[\mathcal{M}(s)y_i(s) \left(\frac{p_{ik}(\omega_{ik})}{p_i(\omega_i)} \right)^{-\theta_i} \left((1 - \tau_i) \left(\frac{\theta_i - 1}{\theta_i} \right) p_{ik}(\omega_{ik}) - mc_i(s) \right) \right] = 0. \quad (\text{A.6})$$

On the other hand, note that $U'(C(s))dC/dc_i(s) - \mathcal{M}(s)p_i(\omega_i) = 0$ for any given industry i , thus implying that the household's nominal stochastic discount factor is given by

$$\mathcal{M}(s) = \frac{1}{p_i(\omega_i)} U'(C(s)) \frac{dC}{dc_i}(s).$$

Plugging the above into equation (A.6), we obtain

$$\mathbb{E}_{ik} \left[U'(C(s)) \frac{dC}{dc_i}(s) y_i(s) \left(\frac{p_{ik}(\omega_{ik})}{p_i(\omega_i)} \right)^{1-\theta_i} \left((1 - \tau_i) \left(\frac{\theta_i - 1}{\theta_i} \right) - \frac{mc_i(s)}{p_{ik}(\omega_{ik})} \right) \right] = 0.$$

Using the demand function $y_{ik}(s) = y_i(s) (p_{ik}(\omega_{ik})/p_i(\omega_i))^{-\theta_i}$ one more time, we get

$$\mathbb{E}_{ik} \left[U'(C(s)) \frac{dC}{dc_i}(s) y_i(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{(\theta_i-1)/\theta_i} \left((1-\tau_i) \left(\frac{\theta_i-1}{\theta_i} \right) - \frac{mc_i(s)}{p_{ik}(\omega_{ik})} \right) \right] = 0.$$

As a result, the nominal price $p_{ik}(\omega_{ik})$ set by firm k in industry i is given by

$$p_{ik}(\omega_{ik}) = \left[(1-\tau_i) \left(\frac{\theta_i-1}{\theta_i} \right) \right]^{-1} \frac{\mathbb{E}_{ik}[v_{ik}(s) mc_i(s)]}{\mathbb{E}_{ik}[v_{ik}(s)]},$$

where $v_{ik}(s)$ is given by

$$v_{ik}(s) = U'(C(s)) \frac{dC}{dc_i}(s) y_i(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{(\theta_i-1)/\theta_i}.$$

Consequently, the nominal price set by the firm can be written as

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s \varepsilon_{ik}(s)} mc_i(s), \quad (\text{A.7})$$

where $\chi_i^s = (1-\tau_i)(\theta_i-1)/\theta_i$ is a wedge arising due to government taxes/subsidies and monopolistic markups and

$$\varepsilon_{ik}(s) = \frac{mc_i(s) \mathbb{E}_{ik}[v_{ik}(s)]}{\mathbb{E}_{ik}[mc_i(s) v_{ik}(s)]} \quad (\text{A.8})$$

is the pricing error due to nominal rigidities. On the other hand, note that the firm's cost minimization implies that

$$mc_i(s) = w(s) (z_i \cdot dF_i/dl_{ik}(s))^{-1}. \quad (\text{A.9})$$

Therefore, replacing $mc_i(s)$ in the above equation into (A.8) and using (A.3) thus establishes equation (16).

Next, note that

$$V'(L(s)) = U'(C(s)) \frac{dC}{dc_i}(s) \frac{w(s)}{p_i(\omega_i)} = U'(C(s)) \frac{dC}{dc_i}(s) \frac{w(s)}{p_{ik}(\omega_{ik})} \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i},$$

where the second equality follows from the fact that the demand faced by firm k in industry i satisfies (A.5). Now replacing for $p_{ik}(\omega_{ik})$ from (A.7) implies that

$$V'(L(s)) = \chi_i^s \varepsilon_{ik}(s) U'(C(s)) \frac{dC}{dc_i}(s) \frac{w(s)}{mc_i(s)} \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i}.$$

On the other hand, recall that the firm's cost satisfies (A.9). Therefore, replacing for $mc_i(s)$ from (A.9) into the above equation establishes (14).

The proof is therefore complete once we establish (15). Recall that the household's first-order condition requires that (A.2) is satisfied for all pairs of industries. As a result, for any pairs of industries i and j , we have

$$\frac{dC}{dc_j}(s) = \frac{p_j(\omega_j)}{p_i(\omega_i)} \frac{dC}{dc_i}(s) = \chi_i^s \varepsilon_{ik}(s) \frac{dC}{dc_i}(s) \frac{p_j(\omega_j)}{mc_i(s)} \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i},$$

where once again we are using (A.5) and (A.7). On the other hand, whenever industry j is an input-supplier of industry i , the cost minimization of firm ik implies that

$$mc_i(s) = p_j(\omega_j) (z_i \cdot dF_i/dx_{ij,k}(s))^{-1}.$$

The juxtaposition of the last two equations now establishes (15) and completes the proof. \square

Proof of Proposition 1

We establish Proposition 1 as a special case of Proposition 2. Recall from Definitions 1 and 2 that a flexible-price equilibrium is sticky-price equilibrium if all productivity shocks are common knowledge. As a result, the right-hand sides of equations (17) and (18) are both measurable with respect to ω_{ik} for all k and all i . Consequently, (16) implies that $\varepsilon_{ik}(s) = 1$ for all $k \in [0, 1]$, all $i \in I$, and all $s \in S$. Plugging this into equations (14) and (15) then immediately establishes (11) and (12), thus completing the proof. \square

Proof of Theorem 1

Suppose that there exists some feasible allocation that is implementable as an equilibrium under both flexible and sticky prices. By Propositions 1 and 2, this allocation must simultaneously satisfy equations (11)–(12) and (14)–(15). As a result,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s) \left(\frac{y_{ik}(s)}{y_i(s)} \right)^{-1/\theta_i}$$

for all firms $(i, k) \in I \times [0, 1]$ and all states $s \in S$, where the wedge function $\varepsilon_{ik}(s)$ satisfies equation (16). Since in any flexible-price allocation, the output of all firms in the same industry coincide, we have $y_{ik}(s) = y_i(s)$ for all $k \in [0, 1]$, and as a result,

$$\chi_i^f = \chi_i^s \varepsilon_{ik}(s)$$

for all $(i, k) \in I \times [0, 1]$ and all $s \in S$. Note that, by assumption, the scalars χ_i^f and χ_i^s representing the fiscal policy are assumed to be invariant to the state s and do not depend on the firm index k . Therefore, the above equation implies that $\varepsilon_{ik}(s)$ is also independent of s and k for all i , i.e., $\varepsilon_{ik}(s) = \varepsilon_i$.

On the other hand, note that equation (16) guarantees that $\mathbb{E}_{ik}[v_{ik}(s)(\varepsilon_{ik}(s) - 1)] = 0$, where $v_{ik}(s)$ is given by equation (18). As a result,

$$(\varepsilon_i - 1)\mathbb{E}_{ik}[v_{ik}(s)] = 0$$

for all i . But note that $v_{ik}(s) > 0$ for all $s \in S$ in any feasible allocation, which guarantees that $\varepsilon_i = 1$ for all i . Also, recall from the proof of Proposition 2, that in any sticky-price equilibrium, each firm's price and marginal cost satisfy equation (A.7). Therefore,

$$p_{ik}(\omega_{ik}) = \frac{1}{\chi_i^s} \text{mc}_i(s). \quad (\text{A.10})$$

The above observation has three implications. First, given that the right-hand side of the above expression is independent of k , it implies that all firms within the same industry set the same nominal price. Thus, we can write $p_{ik}(\omega_{ik}) = p_i(\omega_i)$, with the understanding that $p_i(\omega_i)$ is measurable with respect to the information set of any individual firm k in industry i . Second, equation (A.10) also implies that the marginal cost of industry i is measurable with respect to the information set of all firms in that industry. Finally, it establishes that, whenever an allocation can be implemented as both a

sticky- and a flexible-price equilibrium, all firms employ constant markups to set their nominal prices. Consequently, we can write i 's nominal price as a function of industry i 's input prices as

$$p_i(\omega_i) = \frac{1}{\chi_i^s z_i(s)} C_i(w(s), p_1(\omega_1), \dots, p_n(\omega_n)),$$

where C_i is homogenous of degree 1 and denotes the cost function of firms in industry i . Dividing both sides of the above equation by the nominal wage, we obtain

$$p_i(\omega_i)/w(s) = (\chi_i^s z_i)^{-1} C_i(1, p_1(\omega_1)/w(s), \dots, p_n(\omega_n)/w(s)). \quad (\text{A.11})$$

We thus obtain a system of n equations and n unknowns, relating all industries' nominal prices relative to the wage to the productivity shocks and the vector of wedges χ^s . Since we have assumed that labor is an essential input for the production technology of all industries, Theorem 1 of [Stiglitz \(1970\)](#) guarantees that there exists a unique collection of relative prices that solves system of equations (A.11). Hence, for any given industry i , there exists a function g_i such that the nominal price set by firms in industry i can be written as

$$p_i(\omega_i) = w(s)/g_i(\chi_1^s z_1, \dots, \chi_n^s z_n),$$

where the collection of functions (g_1, \dots, g_n) only depend on the production technologies (F_1, \dots, F_n) and are independent of the economy's information structure. But recall that the left-hand side of the above equation is measurable with respect to the information set of all firms in industry i . Therefore, a feasible allocation is implementable as an equilibrium under both flexible and sticky prices only if there exists a function $w(s)$ such that

$$w(s)/g_i(\chi_1^s z_1, \dots, \chi_n^s z_n) \in \sigma(\omega_{ik})$$

simultaneously for all firms k in all industries i , a relationship that is equivalent to condition (19). Finally, equation (A.3) guarantees that the nominal wage and the nominal demand are related to one another via $w(s) = m(s)V'(L(s))/C(s)U'(C(s))$. \square

Proof of Corollary 3

Suppose all firms in all industries $j \neq i$ set their prices under complete information. This means that the aggregate state s is measurable with respect to ω_{jk} for all $k \in [0, 1]$ and all $j \neq i$. Hence, by equation (16), the wedge functions of firms in industry $j \neq i$ are equal to

$$\varepsilon_{jk}(s) = \frac{\text{mc}_j(s)\mathbb{E}_{jk}[v_{jk}(s)]}{\mathbb{E}_{jk}[\text{mc}_j(s)v_{jk}(s)]} = \frac{\text{mc}_j(s)v_{jk}(s)}{\text{mc}_j(s)v_{jk}(s)} = 1 \quad (\text{A.12})$$

for all s . Let the monetary policy function $m(s)$ be given by

$$m(s) = M z_i \frac{U'(C(s))C'(s)}{V'(L(s))} \frac{dF_i}{dl_i}(s), \quad (\text{A.13})$$

for some constant M that does not depend on the aggregate state. By equation (17), such a policy induces $\text{mc}_i(s) = M$ for all s . As a result, equation (16) implies that $\varepsilon_{ik}(s) = 1$ for all firms $k \in [0, 1]$.

This, alongside equation (A.12), therefore establishes that the above policy can eliminate all wedges arising from nominal rigidities, thus reducing equations (14)–(15) to (11)–(12). In other words, any flexible-price-implementable allocation can be implemented as part of a sticky-price equilibrium.

The proof is therefore complete once we show that the policy in (A.13) stabilizes the price of industry i . As we already established, such a policy induces $mc_i(s) = M$ for all s . Thus, by equation (A.7), $p_{ik}(\omega_{ik}) = M/\chi_i^s$, which means that the nominal price set by the firms in industry i is invariant to the economy's aggregate state. \square

Proof of Lemma 2

Recall from equations (A.7) and (A.8) in the proof of Proposition 2 that the nominal price set by firm k in industry i is given by

$$p_{ik} = \frac{\mathbb{E}_{ik}[\text{mc}_i \cdot v_{ik}]}{\mathbb{E}_{ik}[v_{ik}]},$$

where v_{ik} is given by equation (18) and we are using the fact that $\chi_i^s = 1$. Consequently,

$$\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log \mathbb{E}_{ik} \left[e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i]} \right] - \log \mathbb{E}_{ik} \left[e^{\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}]} \right].$$

Since the standard deviations of log productivity shocks in (21) and noise shocks in (22) scale linearly in δ , it must be the case that $\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] = o(\delta)$ and $\log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i] = o(\delta)$ as $\delta \rightarrow 0$. As a result,

$$\begin{aligned} \log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] &= \log \left(1 + \mathbb{E}_{ik} \left[\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] + \log \text{mc}_i - \mathbb{E}_{ik}[\log \text{mc}_i] \right] + o(\delta) \right) \\ &\quad - \log \left(1 + \mathbb{E}_{ik} \left[\log v_{ik} - \mathbb{E}_{ik}[\log v_{ik}] \right] + o(\delta) \right), \end{aligned}$$

which in turn implies that $\log p_{ik} - \mathbb{E}_{ik}[\log \text{mc}_i] = \log(1 + o(\delta)) - \log(1 + o(\delta))$. Hence,

$$\log p_{ik} = \mathbb{E}_{ik}[\log \text{mc}_i] + o(\delta)$$

as $\delta \rightarrow 0$. Finally, the fact that all firms in industry i have Cobb-Douglas production technologies, as in (20), implies that their marginal cost is given by $\log \text{mc}_i = \alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$, thus completing the proof. \square

Proof of Proposition 3

As a first observation, we note that since noise shocks $\epsilon_{ij,k}$ in firms' private signals are idiosyncratic and of order δ , the log-linearization of all industry-level and aggregate variables only depends on the productivity shocks as $\delta \rightarrow 0$. We thus let

$$\log w = \sum_{j=1}^n \kappa_j \log z_j + o(\delta) \tag{A.14}$$

$$\log p_i = \sum_{j=1}^n b_{ij} \log z_j + o(\delta) \tag{A.15}$$

denote, respectively, the log-linearization of the nominal wage and the nominal price of sectoral good i as $\delta \rightarrow 0$. Furthermore, recall from Lemma 2 that the nominal price set by firm k in industry i is given by (26) up to a first-order approximation as $\delta \rightarrow 0$. Therefore,

$$\begin{aligned}\log p_{ik} &= \alpha_i \sum_{j=1}^n \kappa_j \mathbb{E}_{ik}[\log z_j] - \mathbb{E}_{ik}[\log z_i] + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \mathbb{E}_{ik}[\log z_r] + o(\delta) \\ &= \alpha_i \sum_{j=1}^n \kappa_j \phi_{ik} \omega_{ij,k} - \phi_{ik} \omega_{ii,k} + \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \phi_{ik} \omega_{ir,k} + o(\delta),\end{aligned}\tag{A.16}$$

where ϕ_{ik} is the degree of price flexibility of firm k in industry i given by (24). Integrating both sides of the above equation over all firms k in industry i implies that

$$\log p_i = \phi_i \alpha_i \sum_{j=1}^n \kappa_j \log z_j - \phi_i \log z_i + \phi_i \sum_{j=1}^n \sum_{r=1}^n a_{ij} b_{jr} \log z_r + o(\delta),$$

where $\phi_i = \int_0^1 \phi_{ik} dk$ is the degree of price flexibility of industry i and we are using the facts that $\phi_{ik} \in [0, 1]$ and $\log p_i = \int_0^1 \log p_{ik} dk + o(\delta)$. The juxtaposition of the above equation with equation (A.15) therefore implies that $b_{ij} = \phi_i \alpha_i \kappa_j - \phi_i \mathbb{I}_{\{j=i\}} + \phi_i \sum_{r=1}^n a_{ir} b_{rj}$, which can be written in matrix form as $\mathbf{B} = \Phi \alpha \kappa' - \Phi + \Phi \mathbf{A} \mathbf{B}$, where $\Phi = \text{diag}(\phi)$. Solving for matrix \mathbf{B} and using the fact that $\alpha = (\mathbf{I} - \mathbf{A})\mathbf{1}$ lead to

$$\mathbf{B} = (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) (\mathbf{1} \kappa' - \mathbf{L}).\tag{A.17}$$

Multiplying both sides by $\log z$ and using equations (A.14) and (A.15) then establishes that industry-level prices satisfy (29).

To establish (28), note that $\log mc_i = \alpha_i \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$, which can be written in vector form as $\log mc = \alpha \log w - \log z + \mathbf{A} \log p$. Using (29) therefore implies that

$$\log mc = (\mathbf{I} + \mathbf{A}(\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi) (\alpha \log w - \log z),$$

which reduces to (28). \square

Proof of Proposition 5

We prove this result in three steps. First, we solve for household welfare in terms of nominal prices and the nominal wage. We then compare the result to welfare under the first-best allocation to obtain an expression for welfare loss, given all nominal prices. Finally, we provide a quadratic log-approximation to the welfare loss in terms of the distribution of firm-level pricing errors.

Expressing welfare in terms of nominal prices: As our first step, we obtain an expression for welfare as a function of all nominal prices and the wage.

Recall from equation (A.5) that the output of firm k in industry i is given by $y_{ik} = y_i (p_{ik}/p_i)^{-\theta_i}$, whereas cost minimization implies that the firm's demand for sectoral good j is $x_{ij,k} = a_{ij} y_{ik} mc_i / p_j$.

Therefore, total demand for sectoral good j by firms in industry i is $\int_0^1 x_{ij,k} dk = a_{ij} mc_i / p_j \int_0^1 y_{ik} dk = a_{ij} p_i y_i \varepsilon_i / p_j$, where

$$\varepsilon_i = mc_i p_i^{\theta_i - 1} \int_0^1 p_{ik}^{-\theta_i} dk. \quad (\text{A.18})$$

Hence, market clearing (7) for sectoral good i implies that $p_i y_i = p_i c_i + \sum_{j=1}^n a_{ji} p_j y_j \varepsilon_j$. Dividing both sides by aggregate nominal demand PC and using the fact that $p_i c_i = \beta_i PC$, we obtain

$$\lambda_i = \beta_i + \sum_{j=1}^n a_{ji} \lambda_j \varepsilon_j, \quad (\text{A.19})$$

where λ_i is the Domar weight of industry i . On the other hand, the household's budget constraint is given by

$$PC = wL + \sum_{i=1}^n \int_0^1 \pi_{ik} dk = wL + \sum_{i=1}^n \left(p_i y_i - mc_i \int_0^1 y_{ik} dk \right) = wL + \sum_{i=1}^n p_i y_i (1 - \varepsilon_i),$$

thus implying that $PC = wL / (1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i))$. Furthermore, note that the household's optimal labor supply requires that $L^{1/\eta} = C^{-\gamma} w / P$. Therefore, solving for household's aggregate consumption and aggregate labor supply from the last two equations, we obtain

$$\begin{aligned} C &= (w/P)^{\frac{1+\eta}{1+\eta\gamma}} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1}{1+\eta\gamma}} \\ L &= (w/P)^{\frac{\eta(1-\gamma)}{1+\eta\gamma}} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{\frac{\eta\gamma}{1+\eta\gamma}}. \end{aligned} \quad (\text{A.20})$$

Plugging the above into (3) now characterizes household welfare as a function of all nominal prices and the nominal wage via the expression

$$W = \frac{1}{1-\gamma} (w/P)^{\frac{(1+\eta)(1-\gamma)}{1+\eta\gamma}} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1-\gamma}{1+\eta\gamma}} \left(1 - \frac{\eta(1-\gamma)}{1+\eta} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) \right), \quad (\text{A.21})$$

where ε_i is given by (A.18) and only depends on nominal prices.

Welfare loss: We now use the above expression to determine the first-best welfare W^* under flexible prices, which we then use to calculate the welfare loss arising from nominal rigidities.

Recall that, in the flexible-price equilibrium, all firms in industry i set identical prices and charge no markups, i.e., $mc_i^* = p_{ik}^* = p_i^*$. Therefore, equation (A.18) implies that $\varepsilon_i^* = 1$ for all industries i . Plugging this back into (A.21) leads to $W^* = \frac{\gamma+1/\eta}{(1-\gamma)(1+1/\eta)} (w/P^*)^{\frac{(1+\eta)(1-\gamma)}{1+\eta\gamma}}$, where recall that, by assumption, $w = w^*$. Hence, we can rewrite (A.21) as

$$W = W^* (P/P^*)^{\frac{(1+\eta)(\gamma-1)}{1+\eta\gamma}} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1-\gamma}{1+\eta\gamma}} \left(1 + \frac{\eta(1-\gamma)}{1+\eta\gamma} \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right). \quad (\text{A.22})$$

Similarly, we can use (A.20) to relate aggregate output in the sticky-price equilibrium to that in the flexible-price equilibrium in the form of

$$C = C^* (P/P^*)^{-\frac{1+\eta}{1+\eta\gamma}} \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right)^{-\frac{1}{1+\eta\gamma}}. \quad (\text{A.23})$$

The juxtaposition of the last two equations now implies that welfare in the sticky- and flexible-price equilibria are related to one another via

$$W = W^* (C/C^*)^{1-\gamma} \left(1 + \frac{\eta(1-\gamma)}{1+\eta\gamma} \left(1 - (C/C^*)^{-(1+\eta\gamma)} (P/P^*)^{-(1+\eta)} \right) \right). \quad (\text{A.24})$$

Second-order approximations: We next derive quadratic log approximations of our main equations, (A.18), (A.19), (A.23), and (A.24), as $\delta \rightarrow 0$.

First consider equation (A.18). Taking logarithms from both sides and using the fact that $\log mc_i = \alpha \log w - \log z_i + \sum_{j=1}^n a_{ij} \log p_j$ implies that

$$\log \varepsilon_i = \sum_{j=1}^n a_{ij} (\log p_j - \log p_j^*) + (\theta_i - 1) (\log p_i - \log p_i^*) + \log \int_0^1 (p_{ik}/p_i^*)^{-\theta_i} dk.$$

Consequently, under a second-order approximation,

$$\log \varepsilon_i = \sum_{j=1}^n a_{ij} \bar{e}_j - \bar{e}_i + \frac{1}{2} \sum_{j=1}^n a_{ij} (1 - \theta_j) \vartheta_j + \frac{1}{2} (2\theta_i - 1) \vartheta_i + o(\delta^2), \quad (\text{A.25})$$

where \bar{e}_i and ϑ_i are the cross-sectional average and dispersion of pricing errors in industry i defined in equations (33) and (34), respectively.

Next, consider equation (A.19). Recall that $\varepsilon_i^* = 1$ in the flexible-price equilibrium. As a result, Domar weights in the flexible-price equilibrium satisfy $\lambda_i^* = \beta_i + \sum_{j=1}^n a_{ji} \lambda_j^*$ and a first-order approximation of Domar weights in the sticky-price equilibrium as $\delta \rightarrow 0$ is given by

$$\lambda_i = (1 - \log \varepsilon_i) \lambda_i^* + \sum_{j=1}^n \ell_{ji} \lambda_j^* \log \varepsilon_j + o(\delta), \quad (\text{A.26})$$

where ℓ_{ji} is the (j, i) element of the economy's Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$. As a result,

$$\begin{aligned} \log \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) &= \sum_{i=1}^n \lambda_i \log \varepsilon_i + \frac{1}{2} \sum_{i=1}^n \lambda_i \log^2 \varepsilon_i - \frac{1}{2} \left(\sum_{i=1}^n \lambda_i \log \varepsilon_i \right)^2 + o(\delta^2) \\ &= \sum_{i=1}^n \lambda_i^* \log \varepsilon_i + \sum_{i=1}^n \sum_{j=1}^n \lambda_j^* \ell_{ji} \log \varepsilon_j \log \varepsilon_i - \frac{1}{2} \sum_{i=1}^n \lambda_i^* \log^2 \varepsilon_i - \frac{1}{2} \left(\sum_{i=1}^n \lambda_i^* \log \varepsilon_i \right)^2 + o(\delta^2), \end{aligned}$$

where the second equality follows from replacing λ_i by its first-order approximation in (A.26). By also replacing $\log \varepsilon_i$ by its first-order approximation in (A.25) and simplifying the result, we obtain the following second-order approximation:

$$\begin{aligned} \log \left(1 - \sum_{i=1}^n \lambda_i (1 - \varepsilon_i) \right) &= -(\log P - \log P^*) + \frac{1}{2} \sum_{i=1}^n \lambda_i \theta_i \vartheta_i \\ &\quad + \frac{1}{2} \sum_{i=1}^n \lambda_i \bar{e}_i^2 - \frac{1}{2} \sum_{i=1}^n \lambda_i \left(\sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 - \frac{1}{2} \left(\sum_{i=1}^n \beta_i \bar{e}_i \right)^2 + o(\delta^2). \end{aligned} \quad (\text{A.27})$$

where we are using the fact that $\log P - \log P^* = \sum_{i=1}^n \beta_i \bar{e}_i + \frac{1}{2} \sum_{i=1}^n \beta_i (1 - \theta_i) \vartheta_i + o(\delta^2)$.

Next, consider equation (A.23). A second-order log-approximation leads to

$$\begin{aligned} \log C - \log C^* = & -\frac{1}{\gamma + 1/\eta} (\log P - \log P^*) - \frac{1}{2(1 + \eta\gamma)} \left[\sum_{i=1}^n \lambda_i \theta_i \vartheta_i \right. \\ & \left. + \sum_{i=1}^n \lambda_i \left(\bar{e}_i^2 - \left(\sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 \right) - \left(\sum_{i=1}^n \beta_i \bar{e}_i \right)^2 \right] + o(\delta^2), \end{aligned} \quad (\text{A.28})$$

where we are using (A.27) and . Note that the above expression also implies that, up to a first-order approximation,

$$\log C - \log C^* = -\frac{1}{\gamma + 1/\eta} (\log P - \log P^*) + o(\delta). \quad (\text{A.29})$$

Finally, consider the second-order approximation of for welfare loss in (A.24) as $\delta \rightarrow 0$. We have

$$\begin{aligned} \log(W/W^*) = & (1 - \gamma)(1 + \eta) \left[(\log C - \log C^*) + \frac{1}{\gamma + 1/\eta} (\log P - \log P^*) - \frac{1}{2} \eta (\log C - \log C^*)^2 \right. \\ & \left. - \frac{1}{2} \eta \frac{(1 + \eta)^2}{(1 + \eta\gamma)^2} (\log P - \log P^*)^2 - \frac{1 + \eta}{\gamma + 1/\eta} (\log C - \log C^*) (\log P - \log P^*) \right] + o(\delta^2). \end{aligned}$$

To simplify the above, replace for $\log P - \log P^*$ in the last two terms from its first-order approximation in (A.29) to obtain

$$\log(W/W^*) = (1 - \gamma)(1 + \eta) \left(\log C - \log C^* + \frac{1}{\gamma + 1/\eta} (\log P - \log P^*) - \frac{1}{2\eta} \Sigma \right) + o(\delta^2),$$

where $\Sigma = (\log C - \log C^*)^2$ is the volatility of output gap. Finally, replacing for $\log C - \log C^*$ in terms of its second-order approximation in (A.28) leads to

$$\log(W/W^*) = -\frac{1}{2} \frac{(1 - \gamma)(1 + \eta)}{1 + \eta\gamma} \left[\sum_{i=1}^n \lambda_i \theta_i \vartheta_i + \sum_{j=1}^n \lambda_j \bar{e}_j^2 - \sum_{i=1}^n \lambda_i \left(\sum_{j=1}^n a_{ij} \bar{e}_j \right)^2 - \left(\sum_{i=1}^n \beta_i \bar{e}_i \right)^2 + \frac{1 + \eta\gamma}{\eta} \Sigma \right] + o(\delta^2).$$

Further simplification reduces the above to

$$\begin{aligned} \log(W/W^*) = & -\frac{1}{2} \frac{(1 - \gamma)(1 + 1/\eta)}{(\gamma + 1/\eta)} \left[\sum_{i=1}^n \lambda_i \theta_i \vartheta_i + (\gamma + 1/\eta) \Sigma \right. \\ & \left. + \sum_{i=1}^n \lambda_i \text{var}_i(\bar{e}_1, \dots, \bar{e}_n) + \text{var}_0(\bar{e}_1, \dots, \bar{e}_n) \right] + o(\delta^2) \end{aligned}$$

On the other hand, observe that $W - W^* = W^* \log(W/W^*) + o(\delta^2)$ and $W^* = \frac{\gamma + 1/\eta}{(1 - \gamma)(1 + 1/\eta)} + O(\delta)$. The juxtaposition of these observations with the above equation establishes (35). \square

Proof of Proposition 4

We start by stating and proving a lemma, which we will also use in proof in the proof of Theorem 2. Statement (a) of the lemma establishes that even though in our model the policy instrument is the nominal demand $m(z)$, as long as no industry is perfectly sticky, there is an isomorphism between setting the nominal demand and the nominal wage $w(z)$. Statement (b) of the lemma then provides conditions under which a policy can be implemented as a price-stabilization policy.

Lemma A.1. Suppose $\phi_i > 0$ for all i . Then, up to a first-order approximation,

- (a) an allocation is implementable by setting the nominal demand if and only if it is implementable by setting the nominal wage;
- (b) if vector $\kappa = (\kappa_1, \dots, \kappa_n)'$ satisfies $\kappa' \alpha = 1$, then the nominal wage $\log w(z) = \sum_{i=1}^n \kappa_i \log z_i$ can be implemented by a price-stabilization policy of the form $\sum_{i=1}^n \psi_i \log z_i = 0$ for some vector $\psi = (\psi_1, \dots, \psi_n)'$.

Proof of part (a) Is sufficient to show that, as long as $\phi_i > 0$ for all industries i , there exists a one-to-one correspondence between the nominal wage $w(z)$ and nominal demand $m(z)$ for all realizations of productivity shocks z . Since $m = PC$, it is immediate that

$$\log m = \left(1 - \frac{1}{\gamma + 1/\eta}\right) (\log P - \log P^*) + \log P^* + \log C^*,$$

where P^* and C^* are, respectively, the consumption price index and aggregate output in the flexible-price economy. In deriving the above expression we are using equation (A.29) in the proof of Proposition 5 to write the output gap in terms of distortion in the consumption price index.

Next, recall from Proposition 3 that industry-level nominal prices satisfy (29). Therefore, the vector of average pricing errors defined in (33) is given by

$$\bar{e} = (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\alpha \log w - \log z) - (\mathbf{I} - \mathbf{A})^{-1} (\alpha \log w - \log z) + o(\delta),$$

which can be rewritten as

$$\bar{e} = \mathbf{Q}(\mathbf{L} - \mathbf{1}\kappa') \log z + o(\delta), \quad (\text{A.30})$$

where $\mathbf{Q} = (\mathbf{I} - \Phi \mathbf{A})^{-1}(\mathbf{I} - \Phi)$. Therefore, the fact that $\log P - \log P^* = \beta' \bar{e}$ implies that

$$\log m = \left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q}(\mathbf{L} \log z - \mathbf{1} \log w) + \log P^* + \log C^* + o(\delta).$$

It is also immediate to verify that, in the flexible-price economy, the consumption price index and aggregate output are given by $\log P^* = \log w - \lambda' \log z$ and $\log C^* = \frac{1+1/\eta}{\gamma+1/\eta} \lambda' \log z$, respectively. As a result, up to a first-order approximation, the nominal wage and the nominal demand are related to one another via the following relationship:

$$\log m = \left[1 - \left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q} \mathbf{1}\right] \log w + \left[\left(1 - \frac{1}{\gamma + 1/\eta}\right) \beta' \mathbf{Q} \mathbf{L} - \frac{\gamma - 1}{\gamma + 1/\eta} \lambda'\right] \log z + o(\delta). \quad (\text{A.31})$$

The above expression thus establishes a one-to-one correspondence between $w(z)$ and $m(z)$ as long as $\left(1 - \frac{1}{\gamma+1/\eta}\right) \beta' \mathbf{Q} \mathbf{1} \neq 1$. The proof is therefore complete once we show that this condition is indeed satisfied. To this end, note that it is sufficient to show that $0 \leq \beta' \mathbf{Q} \mathbf{1} < 1$. The fact that $\beta' \mathbf{Q} \mathbf{1} \geq 0$ is a straightforward implication of the fact that $\mathbf{H} = (\mathbf{I} - \Phi \mathbf{A})^{-1}$ is an inverse M-matrix and hence is element-wise non-negative. To show that $\beta' \mathbf{Q} \mathbf{1} < 1$, note that

$$1 - \beta' \mathbf{Q} \mathbf{1} = \beta' (\mathbf{I} - \Phi \mathbf{A})^{-1} \phi (\mathbf{I} - \mathbf{A}) \mathbf{1} = \sum_{i,j=1}^n \beta_i h_{ij} \phi_j \alpha_j,$$

where $\mathbf{H} = (\mathbf{I} - \Phi \mathbf{A})^{-1}$. Once again, the fact that \mathbf{H} is an inverse M-matrix implies that the right-hand side of the above equation is non-negative. To show that it is in fact strictly positive, suppose to the contrary that it is equal to zero. This means that

$$\beta_i h_{ij} \phi_j \alpha_j = 0$$

for all pairs of industries i and j . But for any industry i , there exists at least one industry j (which may coincide with i) such that $\alpha_j > 0$ and $h_{ij} > 0$. This coupled with the fact that $\sum_{i=1}^n \beta_i = 1$ and the assumption that $\phi_j > 0$ for all j leads to a contradiction. Therefore, it must be the case that $\beta' \mathbf{Q} \mathbf{1} < 1$, which completes the proof. \square

Proof of part (b) Let $\kappa = (\kappa_1, \dots, \kappa_n)'$ satisfy $\kappa' \alpha = 1$. We show that stabilizing the price index $\psi' \log p$ with weights given by

$$\psi' = \kappa' \Phi^{-1} (\mathbf{I} - \Phi \mathbf{A}) \tag{A.32}$$

induces a nominal wage given by $\log w = \kappa' \log z + o(\delta)$. To this end, note that the juxtaposition of $\psi' \log p = 0$ and equation (29) implies that $\psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) (\mathbf{1} \log w - \mathbf{L} \log z) = o(\delta)$. Consequently,

$$\log w = \frac{1}{\psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi (\mathbf{I} - \mathbf{A}) \mathbf{1}} \psi' (\mathbf{I} - \Phi \mathbf{A})^{-1} \Phi \log z + o(\delta).$$

Replacing for ψ from (A.32) into the above implies that $\log w = \frac{1}{\kappa' \alpha} \kappa' \log z + o(\delta)$. Using the assumption that $\kappa' \alpha = 1$ then completes the proof. \square

Proof of Proposition 4: With Lemma A.1 in hand, we are now ready to prove the proposition. First, note that the identity $PC = m$ implies that degree of monetary non-neutrality satisfies

$$\Xi = 1 - \frac{d \log P}{d \log m} = 1 - \frac{d \log P}{d \log w} \frac{d \log w}{d \log m}.$$

On the other hand, the fact that $\log P = \sum_{i=1}^n \beta_i \log p_i$ coupled with equation (29) implies that

$$\frac{d \log P}{d \log w} = \sum_{i=1}^n \beta_i \phi_i \rho_i = \rho_0,$$

where the second equality is a consequence of definition of ρ_0 in (31). The juxtaposition of the above equations with (A.31) then leads to

$$\Xi = 1 - \rho_0 \left[1 - \left(1 - \frac{1}{\gamma + 1/\eta} \right) \beta' \mathbf{Q} \mathbf{1} \right]^{-1},$$

where $\mathbf{Q} = (\mathbf{I} - \Phi \mathbf{A})^{-1} (\mathbf{I} - \Phi)$. To simplify the above further, note that $\mathbf{Q} \mathbf{1} = \mathbf{1} - \Phi \rho$, which implies that $\beta' \mathbf{Q} \mathbf{1} = 1 - \rho_0$. Consequently,

$$\Xi = \frac{1 - \rho_0}{1 + (\gamma - 1 + 1/\eta) \rho_0},$$

which coincides with (32). \square

Proof of Theorem 2

By statement (a) of Lemma A.1, any allocation that is implementable by setting the nominal demand $m(z)$ is also implementable by setting the nominal wage $w(z)$. Therefore, we first determine the optimal policy by characterizing how the wage should optimally respond to productivity shocks. More specifically, we characterize the vector of optimal weights $\kappa = (\kappa_1, \dots, \kappa_n)$ in $\log w = \sum_{j=1}^n \kappa_j \log z_j$ that minimizes the expected welfare loss in (35). We then determine the price-stabilization policy that implements such a nominal wage.

To calculate the expected welfare loss in (35), we need to determine the average pricing and dispersion of pricing errors of each industry, defined by (33) and (34), respectively. In the proof of Lemma A.1, we already established that the vector of average pricing errors is given by (A.30). To determine the within-industry dispersion of pricing errors, note that since all firm-level nominal prices within the same industry coincide with one another in the flexible-price economy, it is immediate that the dispersion of pricing errors in industry i satisfies

$$\vartheta_i = \int_0^1 e_{ik}^2 dk - \left(\int_0^1 e_{ik} dk \right)^2 = \int_0^1 (\log p_{ik} - \log p_i)^2 dk.$$

Furthermore, recall from the proof of Proposition 3 that the nominal price of firm k in industry i in the sticky-price equilibrium is given by (A.16), which implies that

$$\log p_{ik} = \frac{\phi_{ik}}{\phi_i} \sum_{j=1}^n b_{ij} \omega_{ij,k},$$

where matrix \mathbf{B} is given by (A.17). Consequently,

$$\log p_{ik} - \log p_i = \left(\frac{\phi_{ik}}{\phi_i} - 1 \right) \sum_{j=1}^n b_{ij} \log z_j + \frac{\phi_{ik}}{\phi_i} \sum_{j=1}^n b_{ij} \epsilon_{ij,k}.$$

Therefore, the expected dispersion of pricing errors in industry i is given by

$$\begin{aligned} \mathbb{E}[\vartheta_i] &= \frac{\delta^2}{\phi_i^2} \sum_{j=1}^n b_{ij}^2 \left(\sigma_z^2 \int_0^1 (\phi_{ik} - \phi_i)^2 dk + \int_0^1 \phi_{ik}^2 \sigma_{ik}^2 dk \right) \\ &= \sigma_z^2 \frac{\delta^2}{\phi_i^2} \sum_{j=1}^n b_{ij}^2 \left(\int_0^1 (\phi_{ik} - \phi_i)^2 dk + \int_0^1 \phi_{ik} (1 - \phi_{ik}) dk \right), \end{aligned}$$

where the second equality is a simple consequence of the definition of ϕ_{ik} in (24). Hence,

$$\mathbb{E}[\vartheta_i] = \sigma_z^2 \delta^2 \left(\frac{1 - \phi_i}{\phi_i} \right) \sum_{j=1}^n b_{ij}^2.$$

With the expected within-industry price dispersion and the industry-level pricing errors in (A.30) in hand, we now minimize the expected welfare loss by optimizing over the vector $\kappa = (\kappa_1, \dots, \kappa_n)$, where recall that $\log w = \sum_{j=1}^n \kappa_j \log z_j$. Taking expectations from both sides of (35), differentiating it

with respect to κ_s , and setting it equal to zero implies that

$$\begin{aligned} & \sigma_z^2 \delta^2 \sum_{i=1}^n \lambda_i \theta_i \left(\frac{1 - \phi_i}{\phi_i} \right) \sum_{j=1}^n b_{ij} \frac{db_{ij}}{d\kappa_s} + \frac{1}{\gamma + 1/\eta} \sum_{i=1}^n \sum_{j=1}^n \beta_i \beta_j \mathbb{E} \left[\bar{e}_i \frac{d\bar{e}_j}{d\kappa_s} \right] \\ & + \sum_{i=0}^n \lambda_i \left(\sum_{j=1}^n a_{ij} \mathbb{E} \left[\bar{e}_j \frac{d\bar{e}_j}{d\kappa_s} \right] - \sum_{j=1}^n \sum_{r=1}^n a_{ij} a_{ir} \mathbb{E} \left[\bar{e}_r \frac{d\bar{e}_j}{d\kappa_s} \right] \right) = 0, \end{aligned}$$

with the convention that $\lambda_0 = 1$ and $a_{0j} = \beta_j$ for all j . To simplify the above, note that (A.30) implies that $d\bar{e}_j/d\kappa_s = -\log z_s \sum_{r=1}^n q_{jr}$, while equation (A.17) implies that $db_{ij}/d\kappa_s = 0$ if $j \neq s$. As a result,

$$\begin{aligned} & \sum_{i=1}^n \lambda_i \theta_i \left(\frac{1 - \phi_i}{\phi_i} \right) \frac{db_{is}}{d\kappa_s} b_{is} - \frac{1}{\gamma + 1/\eta} \left(\sum_{i=1}^n \sum_{j=1}^n \beta_i q_{ij} (\ell_{js} - \kappa_s) \right) \left(\sum_{i=1}^n \sum_{j=1}^n \beta_i q_{ij} \right) \\ & - \sum_{i=0}^n \sum_{j=1}^n \lambda_i a_{ij} \left(\sum_{r=1}^n q_{jr} (\ell_{rs} - \kappa_s) \right) \left(\sum_{m=1}^n q_{jm} \right) + \sum_{i=0}^n \lambda_i \left(\sum_{j=1}^n \sum_{r=1}^n a_{ij} q_{jr} \right) \left(\sum_{j=1}^n \sum_{r=1}^n a_{ij} q_{jr} (\ell_{rs} - \kappa_s) \right) = 0. \end{aligned}$$

Since the above first-order condition has to hold for all industries s , it can be rewritten in matrix form as

$$\begin{aligned} & \left[\lambda' \text{diag}(\theta) (\mathbf{I} - \Phi) \Phi^{-1} (\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1})) (\mathbf{I} - \mathbf{Q}) + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1}) \beta' \mathbf{Q} \right. \\ & \left. + \lambda' \mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1}) \mathbf{Q} + \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1}) \mathbf{A}\mathbf{Q} + \beta' \text{diag}(\mathbf{Q}\mathbf{1}) \mathbf{Q} + (\beta' \mathbf{Q}\mathbf{1}) \beta' \mathbf{Q} \right] (\mathbf{L} - \mathbf{1}\kappa') = 0, \end{aligned}$$

where we are using the fact that $\mathbf{B} = (\mathbf{I} - \mathbf{Q})(\mathbf{1}\kappa' - \mathbf{L})$. Solving for κ then implies that the vector of weights in the optimal wage-setting policy is given by $\kappa' = \iota' / (\iota' \alpha)$, where

$$\begin{aligned} \iota' &= \lambda' \text{diag}(\theta) (\mathbf{I} - \Phi) \Phi^{-1} (\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1})) (\mathbf{I} - \mathbf{Q}) \mathbf{L} + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1}) \beta' \mathbf{Q} \mathbf{L} \\ &+ \lambda' \mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1}) \mathbf{Q} \mathbf{L} - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1}) \mathbf{A} \mathbf{Q} \mathbf{L} + \beta' \text{diag}(\mathbf{Q}\mathbf{1}) \mathbf{Q} \mathbf{L} - (\beta' \mathbf{Q}\mathbf{1}) \beta' \mathbf{Q} \mathbf{L}. \end{aligned} \quad (\text{A.33})$$

Having determined the nominal wage that minimizes expected welfare loss, we next determine the price-stabilization policy that implements such a nominal wage. First, note that since $\kappa' = \iota' / (\iota' \alpha)$, we have $\kappa' \alpha = 1$. As a result, we can apply statement (b) of Lemma A.1. In particular, the lemma implies that the above nominal wage can be induced by a price-stabilization policy $\psi' \log p = 0$, with weights given by (A.32), i.e., $\psi' = \kappa' \Phi^{-1} (\mathbf{I} - \Phi \mathbf{A})$. Hence,

$$\begin{aligned} \psi' &= \lambda' \text{diag}(\theta) (\mathbf{I} - \Phi) \Phi^{-1} (\mathbf{I} - \text{diag}(\mathbf{Q}\mathbf{1})) + \frac{1}{\gamma + 1/\eta} (\beta' \mathbf{Q}\mathbf{1}) \beta' \mathbf{L} (\mathbf{I} - \Phi) \Phi^{-1} \\ &+ (\lambda' \mathbf{A} \text{diag}(\mathbf{Q}\mathbf{1}) - \lambda' \text{diag}(\mathbf{A}\mathbf{Q}\mathbf{1}) \mathbf{A}) \mathbf{L} (\mathbf{I} - \Phi) \Phi^{-1} + (\beta' \text{diag}(\mathbf{Q}\mathbf{1}) - (\beta' \mathbf{Q}\mathbf{1}) \beta') \mathbf{L} (\mathbf{I} - \Phi) \Phi^{-1}, \end{aligned}$$

where we are using matrix identities $\mathbf{Q}\mathbf{L}\Phi^{-1}(\mathbf{I} - \Phi\mathbf{A}) = \mathbf{L}(\mathbf{I} - \Phi)\Phi^{-1}$ and $(\mathbf{I} - \mathbf{Q})\mathbf{L}\Phi^{-1}(\mathbf{I} - \Phi\mathbf{A}) = \mathbf{I}$. Next, note that the definition of ρ in (27) implies that $\mathbf{Q}\mathbf{1} = \mathbf{1} - \Phi\rho$. Therefore,

$$\begin{aligned} \psi' &= \lambda' (\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left(\frac{1 - \beta' \Phi \rho}{\gamma + 1/\eta} \right) \lambda' (\Phi^{-1} - \mathbf{I}) \\ &+ \lambda' (\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho) - \text{diag}(\mathbf{A}\mathbf{1} - \mathbf{A}\Phi\rho) \mathbf{A}) \mathbf{L} (\Phi^{-1} - \mathbf{I}) + ((\beta' \Phi \rho) \beta' - \beta' \Phi \text{diag}(\rho)) \mathbf{L} (\Phi^{-1} - \mathbf{I}). \end{aligned} \quad (\text{A.34})$$

To simplify the above further, note that $\mathbf{A}\Phi\rho = \rho - (\mathbf{I} - \mathbf{A})\mathbf{1}$, which implies that

$$\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho) - \text{diag}(\mathbf{A}\mathbf{1})\mathbf{A} + \text{diag}(\mathbf{A}\Phi\rho)\mathbf{A} = \text{diag}(\rho)\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho).$$

As a result, equation (A.34) simplifies to

$$\begin{aligned} \psi' &= \lambda'(\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left(\frac{1 - \beta'\Phi\rho}{\gamma + 1/\eta} \right) \lambda'(\Phi^{-1} - \mathbf{I}) \\ &\quad + \lambda'(\text{diag}(\rho)\mathbf{A} - \mathbf{A}\Phi \text{diag}(\rho)) \mathbf{L}(\Phi^{-1} - \mathbf{I}) + ((\beta'\Phi\rho)\beta' - \beta'\Phi \text{diag}(\rho)) \mathbf{L}(\Phi^{-1} - \mathbf{I}). \end{aligned}$$

Noting that $\lambda'\mathbf{A} = \lambda' - \beta'$ and $\rho_0 = \beta'\Phi\rho$, we can further simplify the above equation as

$$\psi' = \lambda'(\mathbf{I} - \Phi) \text{diag}(\theta) \text{diag}(\rho) + \left(\frac{1 - \rho_0}{\gamma + 1/\eta} \right) \lambda'(\Phi^{-1} - \mathbf{I}) + \lambda'((\mathbf{I} - \Phi) \text{diag}(\rho)\mathbf{L} + \rho_0\mathbf{I} - \text{diag}(\rho))(\Phi^{-1} - \mathbf{I}).$$

The above expression therefore implies that the weight assigned to the price of industry s in the optimal price-stabilization policy satisfies

$$\psi_s = (1/\phi_s - 1) \left[\lambda_s \phi_s \theta_s \rho_s + \lambda_s \left(\frac{1 - \rho_0}{\gamma + 1/\eta} \right) + \sum_{i=1}^n (1 - \phi_i) \lambda_i \rho_i \ell_{is} + (\rho_0 - \rho_s) \lambda_s \right],$$

which coincides with (38). \square

Proof of Proposition 6

Since i and j are upstream symmetric, it is immediate that $\rho_i = \rho_j$. Furthermore, the fact that they are downstream symmetric implies that they have identical steady-state Domar weights, i.e., $\lambda_i = \lambda_j$. As a result, equations (39)–(41) in Theorem 2 imply that if $\phi_i < \phi_j$, then $\psi_i^{\text{o.g.}} > \psi_j^{\text{o.g.}}$, $\psi_i^{\text{across}} > \psi_j^{\text{across}}$, and $\psi_i^{\text{within}} > \psi_j^{\text{within}}$. Putting the three inequalities together then guarantees that $\psi_i^* > \psi_j^*$. \square

Proof of Proposition 7

Since i and j are downstream symmetric, they have identical steady-state Domar weights, i.e., $\lambda_i = \lambda_j = \lambda$. In addition, recall that by assumption $\phi_i = \phi_j = \phi$. Therefore, equation (39) implies that $\psi_i^{\text{o.g.}} = \psi_j^{\text{o.g.}}$. Furthermore, equation (41) implies that

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}) = 0,$$

where once again we are using the assumptions that i and j are downstream symmetric and that $\phi_i = \phi_j = \phi$. Finally, equation (40) and the assumption that $\theta_i = \theta_j = \theta$ implies that

$$\psi_i^{\text{within}} - \psi_j^{\text{within}} = (1 - \phi) \lambda \theta (\rho_i - \rho_j).$$

Thus, if $\rho_i > \rho_j$, then $\psi_i^{\text{within}} > \psi_j^{\text{within}}$ and hence, $\psi_i^* > \psi_j^*$. \square

Proof of Proposition 8

By assumption, $\theta_i = \theta_j$, $\phi_i = \phi_j$, and $\lambda_i = \lambda_j$. Furthermore, the assumption that i and j are upstream symmetric implies that $\rho_i = \rho_j$. Therefore, equations (39) and (40) in Theorem 2 imply that $\psi_i^{\text{0.g.}} = \psi_j^{\text{0.g.}}$ and $\psi_i^{\text{within}} = \psi_j^{\text{within}}$. Turning to the dimension of policy targeting across-industry misallocation, equation (41) implies that

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{s=1}^n (1 - \phi_s) \lambda_s \rho_s (\ell_{si} - \ell_{sj}),$$

where once again we are using the fact that $\phi_i = \phi_j = \phi$. It is now immediate that $\psi_i^* > \psi_j^*$ if and only if inequality (42) is satisfied. \square

Proof of Proposition 9

Suppose $\theta_i = \theta_j = \theta$ and $\phi_i = \phi_j = \phi < 1$. Also suppose industry j is the sole input-supplier of industry i and i is the sole customer of j . This implies that both industries have identical Domar weights in the economy's efficient steady-state, i.e., $\lambda_i = \lambda_j$. Therefore, by (39), $\psi_i^{\text{0.g.}} = \psi_j^{\text{0.g.}}$.

The fact that j is the sole supplier of i also implies that $\rho_i = \phi_j \rho_j < \rho_j$, where we are using the definition of upstream flexibility in (27). As a result, equation (40) implies that, in the optimal policy, $\psi_i^{\text{within}} < \psi_j^{\text{within}}$.

Finally, note that the proposition's main assumption guarantees that $\ell_{sj} = \ell_{si} + \mathbb{I}_{\{s=j\}}$ for all s . As a result, equation (41) implies that

$$\psi_i^{\text{across}} - \psi_j^{\text{across}} = (1/\phi - 1) \sum_{t=0}^n \lambda_t \sum_{s=1}^n a_{ts} (\rho_t - \phi_s \rho_s) (\ell_{si} - \ell_{sj}) = (1/\phi - 1) \sum_{t=0}^n \lambda_t a_{tj} (\phi_j \rho_j - \rho_t).$$

Note that, by assumption, $a_{tj} = \mathbb{I}_{\{t=i\}}$. As a result, the right-hand side of the above equation is equal to $(1/\phi - 1)(\phi_j \rho_j - \rho_i)$. Now the fact that $\rho_i = \phi_j \rho_j$ guarantees that $\psi_i^{\text{across}} = \psi_j^{\text{across}}$. \square

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