

# Credit Spreads and Monetary Policy: Technical Appendix\*

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## 1 Equilibrium Conditions and Parameter Values

This section describes the complete model of credit frictions.<sup>1</sup> The first subsection contains all the non-linear equations and objective welfare function, the second presents the steady state, the third the log-linearized equations, and the fourth presents a detailed description of the parameter values used for the numerical exercises.

### 1.1 Full set of non-linear equilibrium conditions

The objective:

$$\tilde{U}_t = \pi_b \frac{(\lambda_t^b)^{1-\sigma_b} \bar{C}_t^b}{1-\sigma_b^{-1}} + (1-\pi_b) \frac{(\lambda_t^s)^{1-\sigma_s} \bar{C}_t^s}{1-\sigma_s^{-1}} - \frac{\psi}{1+\nu} \left( \frac{\tilde{\lambda}_t}{\tilde{\Lambda}_t} \right)^{-\frac{1+\nu}{\nu}} \bar{H}_t^{-\nu} \left( \frac{Y_t}{Z_t} \right)^{1+\omega_y} \Delta_t \quad (1.1)$$

The equations describing the economy are summarized below:

$$0 = (1+i_t^d)(1+\omega_t)\beta E_t \left[ [\delta + (1-\delta)\pi_b] \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + (1-\delta)(1-\pi_b) \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \right] - \lambda_t^b \quad (1.2)$$

$$0 = (1+i_t^d)\beta E_t \left[ (1-\delta)\pi_b \frac{\lambda_{t+1}^b}{\Pi_{t+1}} + [\delta + (1-\delta)(1-\pi_b)] \frac{\lambda_{t+1}^s}{\Pi_{t+1}} \right] - \lambda_t^s \quad (1.3)$$

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\*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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<sup>1</sup>For details on the derivations please refer to Cúrdia and Woodford (2009) and its technical appendix.

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$$0 = \Lambda (\lambda_t^b, \lambda_t^s) \mu^p (1 + \omega_y) \psi \mu_t^w \tilde{\lambda} (\lambda_t^b, \lambda_t^s)^{-1} \bar{H}_t^{-\nu} \left( \frac{Y_t}{Z_t} \right)^{1+\omega_y} + \alpha \beta E_t \left[ \Pi_{t+1}^{\theta(1+\omega_y)} K_{t+1} \right] - K_t \quad (1.4)$$

$$0 = \Lambda (\lambda_t^b, \lambda_t^s) (1 - \tau_t) Y_t + \alpha \beta E_t \left[ \Pi_{t+1}^{\theta-1} F_{t+1} \right] - F_t \quad (1.5)$$

$$0 = \pi_b (1 - \pi_b) B (\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t) - \pi_b b_t^g \quad (1.6)$$

$$+ \delta \left[ b_{t-1} (1 + \omega_{t-1}) + \pi_b b_{t-1}^g \right] \frac{1 + i_{t-1}^d}{\Pi_t} - (1 + \pi_b \omega_t) b_t$$

$$0 = \pi_b \bar{C}_t^b (\lambda_t^b)^{-\sigma_b} + (1 - \pi_b) \bar{C}_t^s (\lambda_t^s)^{-\sigma_s} + \tilde{\Xi}_t b_t^\eta + G_t - Y_t \quad (1.7)$$

$$0 = \alpha \Delta_{t-1} \Pi_t^{\theta(1+\omega_y)} + (1 - \alpha) \left( \frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta(1+\omega_y)}{\theta-1}} - \Delta_t \quad (1.8)$$

$$0 = \frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} - \left( \frac{F_t}{K_t} \right)^{\frac{\theta-1}{1+\omega_y \theta}} \quad (1.9)$$

$$0 = 1 + (1 + \varkappa) \tilde{\chi}_t b_t^\varkappa + \eta \tilde{\Xi}_t b_t^{\eta-1} - (1 + \omega_t) \quad (1.10)$$

Auxiliary:

$$B (\lambda_t^b, \lambda_t^s, Y_t, \Delta_t; \xi_t) \equiv \bar{C}_t^b (\lambda_t^b)^{-\sigma_b} - \bar{C}_t^s (\lambda_t^s)^{-\sigma_s} \quad (1.11)$$

$$- \left[ \left( \frac{\lambda_t^b}{\psi_b} \right)^{\frac{1}{\nu}} - \left( \frac{\lambda_t^s}{\psi_s} \right)^{\frac{1}{\nu}} \right] \left( \frac{\lambda_t}{\psi} \right)^{-\frac{1+\nu}{\nu}} \mu_t^w \bar{H}_t^{-\nu} \left( \frac{Y_t}{Z_t} \right)^{1+\omega_y} \Delta_t$$

$$\Lambda (\lambda_t^b, \lambda_t^s) \equiv \pi_b \lambda_t^b + (1 - \pi_b) \lambda_t^s \quad (1.12)$$

$$\tilde{\lambda} (\lambda_t^b, \lambda_t^s) \equiv \psi \left[ \pi_b \left( \frac{\lambda_t^b}{\psi_b} \right)^{\frac{1}{\nu}} + (1 - \pi_b) \left( \frac{\lambda_t^s}{\psi_s} \right)^{\frac{1}{\nu}} \right]^\nu \quad (1.13)$$

$$\tilde{\Lambda} (\lambda_t^b, \lambda_t^s) \equiv \psi^{\frac{1}{1+\nu}} \left[ \pi_b \psi_b^{-\frac{1}{\nu}} (\lambda_t^b)^{\frac{1+\nu}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}} (\lambda_t^s)^{\frac{1+\nu}{\nu}} \right]^{\frac{\nu}{1+\nu}} \quad (1.14)$$

$$c_t^b = \bar{C}_t^b (\lambda_t^b)^{-\sigma_b} \quad (1.15)$$

$$c_t^s = \bar{C}_t^s (\lambda_t^s)^{-\sigma_s} \quad (1.16)$$

## 1.2 Zero inflation steady state

We consider the solution to steady state in which we simply assume zero inflation. We use notation  $\bar{x}$  as denoting the steady state value of generic variable  $x$ , unless otherwise noted.

For simplification of the analysis consider the following definitions

$$\begin{aligned}
 s_c &\equiv \pi_b s_b + (1 - \pi_b) s_s, \\
 s_b &\equiv \bar{c}^b / \bar{Y}, \\
 s_s &\equiv \bar{c}^s / \bar{Y}, \\
 s_c^{bs} &\equiv s_b / s_s, \\
 \bar{\sigma} &\equiv \pi_b s_b \sigma_b + (1 - \pi_b) s_s \sigma_s, \\
 \sigma_{bs} &\equiv \sigma_b / \sigma_s, \\
 \rho_b &\equiv \bar{b} / \bar{Y}, \\
 s_{\Xi} &\equiv \bar{\Xi}(\bar{b}) / \bar{Y}, \\
 \rho_b^g &\equiv \bar{b}^g / \bar{Y}, \\
 s_g &\equiv \bar{G} / \bar{Y}, \\
 \psi_{bs} &\equiv \psi_b / \psi_s.
 \end{aligned}$$

Without any loss of generality we calibrate the following values:

$$\begin{aligned}
 \bar{Y} &= 1, \\
 \psi &= 1.
 \end{aligned}$$

The values of  $s_c$ ,  $s_b/s_s$  and  $\sigma_b/\sigma_s$  are set according to the calibration described in the paper.

For the interest rate we have:

$$1 + \bar{r}^d = \beta^{-1} \frac{(\delta + 1) + \bar{\omega} [\delta + (1 - \delta) \pi_b] - \sqrt{\{(\delta + 1) + \bar{\omega} [\delta + (1 - \delta) \pi_b]\}^2 - 4\delta(1 + \bar{\omega})}}{2\delta(1 + \bar{\omega})}. \quad (1.17)$$

(Note that if  $\bar{\omega} = 0$ , this reduces to  $1 + \bar{r}^d = \beta^{-1}$ .) We use this steady-state relation to calibrate  $\beta$ , given assumed values for  $\delta$ ,  $\pi_b$ ,  $\bar{\omega}$  and  $\bar{r}^d$ .

The nominal deposit rate is equal to the real rate:

$$1 + \bar{r}^d = 1 + \bar{r}^d. \quad (1.18)$$

The markup is calibrated so that  $\bar{\chi}$  and  $\bar{\Xi}$  insure that the equation defining  $\bar{\omega}$  is satisfied. We consider that in steady state a positive spread is the result of the financial intermediation costs, so that we set  $\bar{\chi} = 0$ , implying that  $\bar{\Xi}'(\bar{b}) = \bar{\omega}$ ,  $s_{\Xi} = \frac{\bar{\omega}}{\eta} \rho_b$  and  $\bar{\Xi} = \bar{\omega} / (\eta \bar{b}^{\eta-1})$ .

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The two marginal utilities of consumption are related through:

$$\bar{\lambda}^b = \bar{\Omega} \bar{\lambda}^s, \quad (1.19)$$

with

$$\bar{\Omega} \equiv \frac{1 - (1 + \bar{r}^d) \beta [\delta + (1 - \delta) (1 - \pi_b)]}{(1 + \bar{r}^d) \beta (1 - \delta) \pi_b}. \quad (1.20)$$

Given the assumption that we calibrate  $\psi_{bs}$  and  $\psi$ , we can then write

$$\psi_s = \psi \left[ \pi_b \psi_{bs}^{-\frac{1}{\nu}} + (1 - \pi_b) \right]^\nu,$$

and  $\psi_b = \psi_{bs} \psi_s$ . This implies that, given  $\psi_b$  and  $\psi_s$ , we get

$$\Lambda(\bar{\lambda}^b, \bar{\lambda}^s) = [\pi_b \bar{\Omega} + (1 - \pi_b)] \bar{\lambda}^s, \quad (1.21)$$

$$\tilde{\lambda}(\bar{\lambda}^b, \bar{\lambda}^s) = \psi \left[ \pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}} \right]^\nu \bar{\lambda}^s, \quad (1.22)$$

$$\tilde{\Lambda}(\bar{\lambda}^b, \bar{\lambda}^s) = \psi^{\frac{1}{1+\nu}} \left[ \pi_b \psi_b^{-\frac{1}{\nu}} \bar{\Omega}^{\frac{1+\nu}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}} \right]^{\frac{\nu}{1+\nu}} \bar{\lambda}^s. \quad (1.23)$$

Using  $\bar{F} = \bar{K}$ ,

$$(1 - \bar{\tau}) = \mu^p (1 + \omega_y) \psi \mu_t^w \tilde{\lambda}(\bar{\lambda}^b, \bar{\lambda}^s)^{-1} \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}},$$

hence

$$\bar{\lambda}^s = \frac{\mu^p (1 + \omega_y) \mu_t^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1 - \bar{\tau}) \left[ \pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}} \right]^\nu}. \quad (1.24)$$

Given our calibration of  $s_c$  and  $s_c^{bs}$ , we can write:

$$s_s = \frac{s_c}{\pi_b s_c^{bs} + (1 - \pi_b)}, \quad (1.25)$$

and  $s_b = s_c^{bs} s_c$ .

The resources constraint implies

$$1 - s_c - s_g = \frac{\bar{\omega}}{\eta} \rho_b. \quad (1.26)$$

Zero inflation implies that

$$\bar{\Delta} = 1. \quad (1.27)$$

The debt equation is

$$[1 + \pi_b \bar{\omega} - \delta (1 + \bar{\omega}) (1 + \bar{r}^d)] \rho_b = \pi_b (1 - \pi_b) \frac{B(\bar{\lambda}^b, \bar{\lambda}^s, \bar{Y}, \bar{\Delta}; 0)}{\bar{Y}} - \pi_b \rho_b^g [1 - \delta (1 + \bar{r}^d)],$$

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with

$$\frac{B\left(\bar{\lambda}^b, \bar{\lambda}^s, 1, 1; 0\right)}{\bar{Y}} = s_b - s_s - \frac{\bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} - \psi_s^{-\frac{1}{\nu}}}{\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}}} \frac{1 - \bar{\tau}}{\mu^p (1 + \omega_y)},$$

implying that

$$\rho_b = \frac{\pi_b (1 - \pi_b) \left( s_b - s_s - \frac{\bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} - \psi_s^{-\frac{1}{\nu}}}{\pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}}} \frac{1 - \bar{\tau}}{\mu^p (1 + \omega_y)} \right) - \pi_b \rho_b^g [1 - \delta (1 + \bar{r}^d)]}{1 + \pi_b \bar{\omega} - \delta (1 + \bar{\omega}) (1 + \bar{r}^d)}, \quad (1.28)$$

which is then used to solve for real debt according to  $\bar{b} = \rho_b \bar{Y}$ . Given  $\bar{b}$  and  $s_c$ , the real resources equation (1.26) determines  $s_g = 1 - s_c - \frac{\bar{\omega}}{\eta} \rho_b$ .

Furthermore, we set  $\bar{\sigma}$ , hence

$$\sigma_s = \frac{\bar{\sigma}}{\pi_b s_b \sigma_{bs} + (1 - \pi_b) s_s}, \quad (1.29)$$

$$\sigma_b = \sigma_{bs} \sigma_s. \quad (1.30)$$

Finally,

$$\bar{C}^b = s_b \left( \bar{\lambda}^b \right)^{\sigma_b}, \quad (1.31)$$

$$\bar{C}^s = s_b \left( \bar{\lambda}^s \right)^{\sigma_s}, \quad (1.32)$$

$$\bar{K} = \frac{\Lambda \left( \bar{\lambda}^b, \bar{\lambda}^s \right) \mu^p (1 + \omega_y) \psi \mu^w \tilde{\lambda} \left( \bar{\lambda}^b, \bar{\lambda}^s \right)^{-1} \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{1 - \alpha\beta}, \quad (1.33)$$

$$\bar{F} = \frac{\Lambda \left( \bar{\lambda}^b, \bar{\lambda}^s \right) (1 - \bar{\tau})}{1 - \alpha\beta}. \quad (1.34)$$

We further set  $\psi_b/\psi_s$  such that the labor supply is the same in steady state, which implies that

$$\frac{\bar{\lambda}^b}{\psi_b} = \frac{\bar{\lambda}^s}{\psi_s} \Leftrightarrow \frac{\bar{\lambda}^b}{\bar{\lambda}^s} = \frac{\psi_b}{\psi_s} \Rightarrow \frac{\psi_b}{\psi_s} = \bar{\Omega}, \quad (1.35)$$

hence

$$\psi_s = \left[ \pi_b \bar{\Omega}^{-\frac{1}{\nu}} + (1 - \pi_b) \right]^\nu, \quad (1.36)$$

$$\psi_b = \bar{\Omega} \psi_s, \quad (1.37)$$

$$\bar{\lambda}^s = \frac{\mu^p (1 + \omega_y) \mu^w \frac{\bar{H}^{-\nu}}{\bar{Z}^{1+\omega_y}}}{(1 - \bar{\tau}) \left[ \pi_b \bar{\Omega}^{\frac{1}{\nu}} \psi_b^{-\frac{1}{\nu}} + (1 - \pi_b) \psi_s^{-\frac{1}{\nu}} \right]^\nu}, \quad (1.38)$$

$$\bar{\lambda}^b = \bar{\Omega} \bar{\lambda}^s, \quad (1.39)$$

$$\Lambda \left( \bar{\lambda}^b, \bar{\lambda}^s \right) = \pi_b \bar{\lambda}^b + (1 - \pi_b) \bar{\lambda}^s. \quad (1.40)$$

### 1.3 Log-linear equilibrium conditions

In this section we present all the log-linear relations of the model, in which we linearize around the zero inflation steady state. We start by simply presenting the equilibrium conditions in log-linear form without any simplifications, so as to exactly match the set of non-linear equations. Then we proceed to present a simplified set of equations and the exact definitions of the natural rate of output and the natural interest rate used in the policy rules considered.

#### Full system

The full system of log-linear equation is given by:

$$\hat{\lambda}_t^b = \hat{i}_t^d + \hat{\omega}_t - E_t \pi_{t+1} + \chi_b E_t \hat{\lambda}_{t+1}^b + (1 - \chi_b) E_t \hat{\lambda}_{t+1}^s, \quad (1.41)$$

$$\hat{\lambda}_t^s = \hat{i}_t^d - E_t \pi_{t+1} + (1 - \chi_s) E_t \hat{\lambda}_{t+1}^b + \chi_s E_t \hat{\lambda}_{t+1}^s, \quad (1.42)$$

$$\begin{aligned} \hat{K}_t = & (1 - \alpha\beta) \left[ \hat{\Lambda}_t - \hat{\lambda}_t + \hat{\mu}_t^w - \nu \bar{h}_t + (1 + \omega_y) (\hat{Y}_t - z_t) \right] \\ & + \alpha\beta E_t \left[ \theta (1 + \omega_y) \pi_{t+1} + \hat{K}_{t+1} \right], \end{aligned} \quad (1.43)$$

$$\hat{F}_t = (1 - \alpha\beta) \left[ \hat{\Lambda}_t - \hat{r}_t + \hat{Y}_t \right] + \alpha\beta E_t \left[ (\theta - 1) \pi_{t+1} + \hat{F}_{t+1} \right], \quad (1.44)$$

$$\begin{aligned} (1 + \pi_b \bar{\omega}) \hat{b}_t = & \pi_b (1 - \pi_b) \rho_b^{-1} \hat{B}_t - \pi_b (1 + \bar{\omega}) \hat{\omega}_t \\ & + \delta (1 + \bar{r}^d) [(1 + \bar{\omega}) + \pi_b \rho_b^g / \rho_b] (\hat{i}_{t-1}^d - \pi_t) \\ & + \delta (1 + \bar{r}^d) (1 + \bar{\omega}) (\hat{b}_{t-1} + \hat{\omega}_{t-1}) \\ & - \pi_b \rho_b^{-1} \left[ \hat{b}_t^g - \delta (1 + \bar{r}^d) \hat{b}_{t-1}^g \right], \end{aligned} \quad (1.45)$$

$$\hat{Y}_t = \pi_b s_b \left( \bar{c}_t^b - \sigma_b \hat{\lambda}_t^b \right) + (1 - \pi_b) s_s \left( \bar{c}_t^s - \sigma_s \hat{\lambda}_t^s \right) + \hat{\Xi}_t + \eta s_{\Xi} \hat{b}_t + \hat{G}_t, \quad (1.46)$$

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1}, \quad (1.47)$$

$$\pi_t = \frac{1 - \alpha}{\alpha} \frac{1}{1 + \omega_y \theta} \left( \hat{K}_t - \hat{F}_t \right), \quad (1.48)$$

$$\hat{\omega}_t = \frac{(1 + \varkappa) \bar{\chi} \bar{b}^\varkappa}{1 + \bar{\omega}} \left( \frac{\zeta_\chi}{\bar{\chi}} \hat{\lambda}_t + \varkappa \hat{b}_t \right) + \frac{\eta \bar{\Xi} \bar{b}^{\eta-1}}{1 + \bar{\omega}} \left( \frac{\zeta_{\Xi}}{\bar{\Xi}} \hat{\Xi}_t + (\eta - 1) \hat{b}_t \right). \quad (1.49)$$

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Auxiliary equations:

$$\hat{B}_t = s_b (\hat{c}_t^b - \sigma_b \hat{\lambda}_t^b) - s_s (\hat{c}_t^s - \sigma_s \hat{\lambda}_t^s) \quad (1.50)$$

$$\begin{aligned} & -\psi \bar{\lambda}^{\nu-1} \bar{\mu}^w \bar{H}^{-\nu} \left( \frac{\bar{Y}}{\bar{Z}} \right)^{1+\omega_y} \bar{Y}^{-1} \frac{1}{\nu} \left[ \left( \frac{\psi \bar{\lambda}^b}{\psi_b \bar{\lambda}} \right)^{\frac{1}{\nu}} (\hat{\lambda}_t^b - \hat{\lambda}_t) - \left( \frac{\psi \bar{\lambda}^s}{\psi_s \bar{\lambda}} \right)^{\frac{1}{\nu}} (\hat{\lambda}_t^s - \hat{\lambda}_t) \right] \\ & -\psi \bar{\lambda}^{\nu-1} \bar{\mu}^w \bar{H}^{-\nu} \left( \frac{\bar{Y}}{\bar{Z}} \right)^{1+\omega_y} \bar{Y}^{-1} \left[ \left( \frac{\psi \bar{\lambda}^b}{\psi_b \bar{\lambda}} \right)^{\frac{1}{\nu}} - \left( \frac{\psi \bar{\lambda}^s}{\psi_s \bar{\lambda}} \right)^{\frac{1}{\nu}} \right] \times \\ & \times \left[ \hat{\mu}_t^w - \nu \bar{h}_t - \hat{\lambda}_t + (1 + \omega_y) (\hat{Y}_t - z_t) + \hat{\Delta}_t \right], \end{aligned} \quad (1.51)$$

$$\hat{\lambda}_t = \pi_b \left( \frac{\psi \bar{\lambda}^b}{\psi_b \bar{\lambda}} \right)^{\frac{1}{\nu}} \hat{\lambda}_t^b + (1 - \pi_b) \left( \frac{\psi \bar{\lambda}^s}{\psi_s \bar{\lambda}} \right)^{\frac{1}{\nu}} \hat{\lambda}_t^s, \quad (1.52)$$

$$\hat{\Lambda}_t = \pi_b \frac{\bar{\lambda}^b}{\bar{\lambda}} \hat{\lambda}_t^b + (1 - \pi_b) \frac{\bar{\lambda}^s}{\bar{\lambda}} \hat{\lambda}_t^s, \quad (1.53)$$

$$\hat{\tilde{\Lambda}}_t = \pi_b \frac{\psi_b^{-\frac{1}{\nu}} (\bar{\lambda}^b)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}} \bar{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}} \hat{\lambda}_t^b + (1 - \pi_b) \frac{\psi_s^{-\frac{1}{\nu}} (\bar{\lambda}^s)^{\frac{1+\nu}{\nu}}}{\psi^{-\frac{1}{\nu}} \bar{\tilde{\Lambda}}^{\frac{1+\nu}{\nu}}} \hat{\lambda}_t^s, \quad (1.54)$$

$$\hat{c}_t^b = \bar{c}_t^b - \sigma_b \hat{\lambda}_t^b, \quad (1.55)$$

$$\hat{c}_t^s = \bar{c}_t^s - \sigma_s \hat{\lambda}_t^s. \quad (1.56)$$

The exogenous variables all follow an AR(1) process as follows:

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t \quad (1.57)$$

In the above equations we consider the following definitions

$$\hat{i}_t^\tau \equiv \ln \left( (1 + i_t^\tau) / (1 + \bar{i}_t^\tau) \right), \quad (1.58)$$

$$\hat{\omega}_t \equiv \ln \left( (1 + \omega_t) / (1 + \bar{\omega}) \right), \quad (1.59)$$

$$\pi_t \equiv \ln \Pi_t, \quad (1.60)$$

$$\bar{\lambda}_t^\tau \equiv \ln (\lambda_t^\tau / \bar{\lambda}^\tau), \quad (1.61)$$

$$\hat{Y}_t \equiv \ln (Y_t / \bar{Y}), \quad (1.62)$$

$$\hat{F}_t \equiv \ln (F_t / \bar{F}), \quad (1.63)$$

$$\hat{K}_t \equiv \ln (K_t / \bar{K}), \quad (1.64)$$

$$\hat{b}_t \equiv \ln (b_t / \bar{b}), \quad (1.65)$$

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$$\bar{h}_t \equiv \ln (\bar{H}_t / \bar{H}), \quad (1.66)$$

$$z_t \equiv \ln (Z_t / \bar{Z}), \quad (1.67)$$

$$\hat{\tau}_t \equiv -\log ((1 - \tau_t) / (1 - \bar{\tau})), \quad (1.68)$$

$$\hat{b}_t^g \equiv (b_t^g - \bar{b}) / \bar{Y}, \quad (1.69)$$

$$\bar{c}_t^\tau \equiv \ln (\bar{C}_t^\tau / \bar{C}^\tau), \quad (1.70)$$

$$\hat{\mu}_t^w \equiv \ln (\mu_t^w / \bar{\mu}^w), \quad (1.71)$$

$$\hat{G}_t \equiv (G_t - \bar{G}) / \bar{Y}, \quad (1.72)$$

$$\hat{\Xi}_t \equiv \frac{\bar{b}^\eta}{\bar{Y}} (\tilde{\Xi}_t - \bar{\Xi}), \quad (1.73)$$

$$\hat{\chi}_t \equiv (1 + \varkappa) \bar{b}^\varkappa (\tilde{\chi}_t - \bar{\chi}), \quad (1.74)$$

and

$$\chi_\tau \equiv \beta (1 + \bar{r}^\tau) [\delta + (1 - \delta) \pi_\tau]. \quad (1.75)$$

**Simplified log-linear system of equilibrium conditions**

We can write the required equations as

$$\hat{i}_t^{avg} = \hat{i}_t^d + \pi_b \hat{\omega}_t, \quad (1.76)$$

$$\hat{\Omega}_t = \hat{\omega}_t + \delta E_t \hat{\Omega}_{t+1}, \quad (1.77)$$

$$\begin{aligned} \hat{Y}_t = & E_t \hat{Y}_{t+1} - \bar{\sigma} (\hat{i}_t^{avg} - E_t \pi_{t+1}) - E_t \Delta g_{t+1} - E_t \Delta \hat{\Xi}_{t+1} \\ & - \bar{\sigma} s_\Omega \hat{\Omega}_t + \bar{\sigma} (s_\Omega + \psi_\Omega) E_t \hat{\Omega}_{t+1}, \end{aligned} \quad (1.78)$$

$$\begin{aligned} \pi_t = & \beta E_t \pi_{t+1} + u_t + \kappa \left( \hat{Y}_t - (\omega_y + \bar{\sigma}^{-1})^{-1} [\bar{\sigma}^{-1} g_t + \nu \bar{h}_t + (1 + \omega_y) z_t] \right) \\ & - \xi \bar{\sigma}^{-1} \hat{\Xi}_t + \xi (s_\Omega + \pi_b - \gamma_b) \hat{\Omega}_t, \end{aligned} \quad (1.79)$$

$$\hat{\omega}_t = \omega_b \hat{b}_t + \omega_\chi \hat{\chi}_t + \omega_\Xi \hat{\Xi}_t, \quad (1.80)$$

$$\begin{aligned} \hat{b}_t = & \varrho_r (\hat{i}_{t-1}^d - \pi_t) + \varrho_Y \hat{Y}_t + \varrho_\Omega \hat{\Omega}_t + \varrho_\omega \hat{\omega}_t + \varrho_b (\hat{b}_{t-1} + \hat{\omega}_{t-1}) \\ & + \varrho_\xi \left[ \pi_b (1 - \pi_b) s_c \bar{c}_t + B_\lambda \bar{\sigma}^{-1} (g_t + \hat{\Xi}_t) - \tilde{B}_u [\hat{\mu}_t^w - \nu \bar{h}_t - (1 + \omega_y) z_t] \right] \\ & - \pi_b \varrho_\xi \left[ \hat{b}_t^g - \delta (1 + \bar{r}^d) \hat{b}_{t-1}^g \right], \end{aligned} \quad (1.81)$$

with

$$s_c \bar{c}_t = \pi_b s_b \bar{c}_t^b + (1 - \pi_b) s_s \bar{c}_t^s, \quad (1.82)$$

$$g_t = s_c \bar{c}_t + \hat{G}_t, \quad (1.83)$$

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$$u_t \equiv \xi (\hat{\mu}_t^w + \hat{\tau}_t), \quad (1.84)$$

$$\Delta g_t \equiv g_t - g_{t-1}, \quad (1.85)$$

$$\Delta \hat{\Xi}_t \equiv \hat{\Xi}_t - \hat{\Xi}_{t-1}, \quad (1.86)$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t. \quad (1.87)$$

and

$$\hat{\delta} \equiv \chi_b + \chi_s - 1, \quad (1.88)$$

$$\psi_\Omega \equiv \pi_b (1 - \chi_b) - (1 - \pi_b) (1 - \chi_s), \quad (1.89)$$

$$s_\Omega \equiv \pi_b (1 - \pi_b) \frac{s_b \sigma_b - s_s \sigma_s}{\bar{\sigma}}, \quad (1.90)$$

$$\xi \equiv \frac{1 - \alpha}{\alpha} \frac{1 - \alpha \beta}{1 + \omega_y \theta}, \quad (1.91)$$

$$\gamma_b \equiv \pi_b \left( \frac{\psi \bar{\lambda}^b}{\psi_b \bar{\lambda}} \right)^{\frac{1}{\nu}}, \quad (1.92)$$

$$\kappa \equiv \xi (\omega_y + \bar{\sigma}^{-1}), \quad (1.93)$$

$$\omega_b \equiv \frac{\varkappa \bar{\chi}'(\bar{b}) + \eta (\eta - 1) \frac{s_\Xi}{\rho_b}}{1 + \bar{\omega}}, \quad (1.94)$$

$$\omega_\chi \equiv \frac{1}{1 + \bar{\omega}}, \quad (1.95)$$

$$\omega_\Xi \equiv \frac{1}{1 + \bar{\omega}} \frac{\eta}{\rho_b}, \quad (1.96)$$

$$\tilde{B}_\Omega \equiv \psi \bar{\lambda}^{-1} \bar{\mu}^w \bar{H}^{-\nu} \left( \frac{\bar{Y}}{\bar{Z}} \right)^{1 + \omega_y} \bar{Y}^{-1} \frac{1}{\nu} \gamma_b (1 - \gamma_b), \quad (1.97)$$

$$\tilde{B}_u \equiv \psi \bar{\lambda}^{-1} \bar{\mu}^w \bar{H}^{-\nu} \left( \frac{\bar{Y}}{\bar{Z}} \right)^{1 + \omega_y} \bar{Y}^{-1} (\gamma_b - \pi_b), \quad (1.98)$$

$$B_\Omega \equiv s_\Omega \pi_b - s_b \sigma_b \pi_b (1 - \pi_b) - \tilde{B}_\Omega + \tilde{B}_u (\gamma_b - \pi_b), \quad (1.99)$$

$$B_\lambda \equiv \tilde{B}_u - s_\Omega, \quad (1.100)$$

$$\varrho_r \equiv \frac{\delta (1 + \bar{r}^d)}{1 + \pi_b \bar{\omega}} \left[ (1 + \bar{\omega}) + \pi_b \frac{\rho_b^g}{\rho_b} \right], \quad (1.101)$$

$$\varrho_Y \equiv -\varrho_\xi \left( \tilde{B}_u (1 + \omega_y) + B_\lambda \bar{\sigma}^{-1} \right), \quad (1.102)$$

$$\varrho_\Omega \equiv \varrho_\xi (B_\Omega - B_\lambda s_\Omega), \quad (1.103)$$

$$\varrho_\omega \equiv -\frac{\pi_b (1 + \bar{\omega})}{1 + \pi_b \bar{\omega}}, \quad (1.104)$$

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$$\varrho_\xi \equiv \frac{1}{\rho_b (1 + \pi_b \bar{\omega})}, \quad (1.105)$$

$$\varrho_b \equiv \frac{\delta (1 + \bar{r}^d) (1 + \bar{\omega})}{1 + \pi_b \bar{\omega}}. \quad (1.106)$$

### Natural rates

In the policy rules considered, unless otherwise noted, it is assumed that the interest rate responds to the output gap,  $\hat{Y}_t - \hat{Y}_t^n$ , and the natural rate of interest,  $\hat{r}_t^n$ . It is important to notice that in order to be transparent about the role of the response to the financial variables we exclude from this definition any changes in the financial intermediation frictions, implying that neither the natural rate of output nor the natural interest rate respond to changes in the financial frictions. Therefore we consider these two variables as solving the flexible price equilibrium of this economy when the intermediation frictions remain at their steady state levels. This means that the natural rate of output is given by

$$\hat{Y}_t^n = (\omega_y + \bar{\sigma}^{-1})^{-1} [\bar{\sigma}^{-1} g_t - \hat{\tau}_t - \hat{\mu}_t^w + \nu \bar{h}_t + (1 + \omega_y) z_t], \quad (1.107)$$

and the natural interest rate is defined as the rate at which the Euler equation is satisfied when output is at its natural level,

$$\hat{r}_t^n = \bar{\sigma}^{-1} \left( E_t \hat{Y}_{t+1}^n - \hat{Y}_t^n - E_t \Delta g_{t+1} \right). \quad (1.108)$$

## 1.4 Parameter values

The paper discusses the strategy for the calibration. Here we present the exact values for all the parameters.

Notice that unless otherwise mentioned, all exogenous disturbances follow an AR(1) process with autocorrelation coefficient equal to  $\rho_\xi$ , which is discussed in the main text and is assumed to take different values.

### Linear financial intermediation technology

The financial intermediation technology is linear, implying that the spread is exogenous, i. e.  $\varkappa = 0$ ,  $\bar{\chi} = 0$ ,  $\bar{\Xi} = \bar{\omega}$  and  $\eta = 1$ . The full list of parameters is:

$\phi^{-1}$	0.75	$1 + \bar{\omega}$	$(1.02)^{1/4}$	$s_\Xi$	0.0159	$\bar{\sigma}^{-1}$	0.16
$\alpha$	0.66	$\delta$	0.975	$\rho_b^g$	0	$\sigma$	8.9286
$\omega_y$	0.473	$\pi_b$	0.5	$s_c$	0.7	$\sigma_b$	13.8019
$\nu$	0.1048	$\rho_b$	3.2	$s_b$	0.7821	$\sigma_s$	2.7604
$(\theta - 1)^{-1}$	0.15	$\eta$	1	$s_s$	0.6179	$\sigma_b/\sigma_s$	5
$\mu^p$	1.15	$\varkappa$	0	$s_b/s_s$	1.2657	$\bar{\lambda}_b/\bar{\lambda}_s$	1.2175
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	$\psi_b$	1.1492	$\bar{Z}$	1
$\beta$	0.9874	$\bar{\tau}$	0.2	$\psi_s$	0.9439	$\bar{H}$	1
$\bar{Y}$	1	$s_g$	0.2841	$\psi_b/\psi_s$	1.2175		

**Convex financial intermediation technology**

The financial intermediation technology is convex, implying that the spread is endogenous, i.e.  $\eta > 1$ ,  $\varkappa = 0$  and  $\bar{\chi} = 0$ . The full list of parameters is:

$\phi^{-1}$	0.75	$1 + \bar{\omega}$	$(1.02)^{1/4}$	$s_{\Xi}$	0.0003	$\bar{\sigma}^{-1}$	0.16
$\alpha$	0.66	$\delta$	0.975	$\rho_b^g$	0	$\sigma$	8.9286
$\omega_y$	0.473	$\pi_b$	0.5	$s_c$	0.7	$\sigma_b$	13.8019
$\nu$	0.1048	$\rho_b$	3.2	$s_b$	0.7821	$\sigma_s$	2.7604
$(\theta - 1)^{-1}$	0.15	$\eta$	51.623	$s_s$	0.6179	$\sigma_b/\sigma_s$	5
$\mu^p$	1.15	$\varkappa$	0	$s_b/s_s$	1.2657	$\bar{\lambda}_b/\bar{\lambda}_s$	1.2175
$\bar{r}^d$	0.01	$\bar{\mu}^w$	1	$\psi_b$	1.1492	$\bar{Z}$	1
$\beta$	0.9874	$\bar{\tau}$	0.2	$\psi_s$	0.9439	$\bar{H}$	1
$\bar{Y}$	1	$s_g$	0.2997	$\psi_b/\psi_s$	1.2175		

## 2 Taylor Rules With No Natural Rate Adjustments

The main text presents several alternative Taylor rules, all of which consider that the interest rate responds to the natural rate of interest and to deviations of output from its natural level, as described in the baseline rule,

$$i_t^d = r_t^n + \phi_{\pi}\pi_t + \phi_y \left( \hat{Y}_t - \hat{Y}_t^n \right). \quad (2.1)$$

In this section we shall consider a different version of the interest rate rule, in which the interest rate does not respond to the natural interest rate and it responds to deviations of output from steady state,

$$i_t^d = \phi_{\pi}\pi_t + \phi_y \hat{Y}_t. \quad (2.2)$$

This shall be labeled as the "basic" Taylor rule, due to its simplicity.<sup>2</sup>

This rule has the advantage of being much simpler to implement than the rule implied by (2.1), in the sense that there is no need to evaluate what is the natural interest rate nor the natural level of output at each period. However this policy rule is usually farther way from optimal than the one presented in the main text. That is true in the standard New-Keynesian model and it is also true in the current model. This is well illustrated in the case of a productivity shock, shown in Figure 1. This figure shows impulse responses to a one percent increase in productivity for the case of a convex financial intermediation technology. It is clear from the figure that the responses of the variables are much closer to the optimal policy (shown as a solid line) for the case of policy rule (2.1), with adjustment for the natural interest rate and responding to the output gap – shown as a dashed line – than in the case of the basic Taylor rule in (2.2) – shown as a dashed line with "+" markers. Similar results can be shown for the other exogenous shocks considered in the full model.

<sup>2</sup>The units quoted here are the ones used by Taylor, in which the inflation rate and interest rates are annualized rates. If instead these are quarterly rates, as in the model equations expounded here, the value of  $\phi_y$  is instead  $0.5/4 = 0.125$ .

The rest of this section considers adjustments to the basic Taylor rule, (2.1), by incorporating an interest rate response to the level of spreads or a response to the level of credit.

## 2.1 Spread-Adjusted Taylor Rules

Let us first consider generalizations of (2.1) of the form

$$i_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t - \phi_\omega \hat{\omega}_t, \quad (2.3)$$

for some coefficient  $0 \leq \phi_\omega \leq 1$ . Like the rule (2.4) in the main text, these rules reflect the idea that the funds rate should be lowered when credit spreads increase, so as to prevent the increase in spreads from "effectively tightening monetary conditions" in the absence of any justification from inflation or high output relative to potential, except that now we consider a rule without any response to the natural interest rate or the natural rate of output.

We now consider the consequences of alternative values for  $\phi_\omega > 0$ ; and compare the equilibrium responses to shocks under this kind of policy to those under Ramsey policy (i.e., an optimal policy commitment). Figures 2-8 present numerical responses in the case of several different types of exogenous disturbances, when the model is calibrated in the same way as in the previous section, for the case of a convex intermediation technology.

Figure 2 shows the responses of endogenous variables to a "financial shock" for variant monetary policy rules of the form (2.3). The figure shows the responses in the case of five different possible values of  $\phi_\omega$ , ranging between 0 and 1. The response of each variable under the Ramsey policy is also shown (as a solid blue line). We observe that adjusting the intercept of the Taylor rule in response to changes in the credit spread can indeed largely remedy the defects of the simple Taylor rule, in the case of a shock to the economy of this kind. And the optimal degree of adjustment is close to 100 percent, as proposed by McCulley and Toloui and by Taylor. To be more precise, both inflation and output increase a little more under the 100 percent spread adjustment than they would under Ramsey policy; but the optimal responses of both variables are between the paths that would result from a 75 percent spread adjustment and the one that results from a 100 percent spread adjustment. If we optimize our welfare criterion over policy rules with alternative values of  $\phi_\omega$ ; assuming that this type of disturbance is the only kind that ever occurs, the welfare maximum is reached when  $\phi_\omega = 0.82$ , as shown in Table 1.

It is interesting to observe in Figure 2 that, while a superior policy involves a reduction in the policy rate relative to what the unadjusted Taylor rule would prescribe, this does not mean that under such a policy the central bank actually cuts its interest rate target more sharply in equilibrium. The size of the fall in the policy rate (shown in the middle left panel) is about the same regardless of the value of  $\phi_\omega$ ; but when  $\phi_\omega$  is near 1, output and inflation no longer have to decline in order to induce the central bank to accept an interest-rate cut of this size, and in equilibrium they do not decline. (In fact, the nominal policy rate does fall a little more, and since expected inflation does not fall, the real interest rates faced by both savers and borrowers fall more substantially when  $\phi_\omega$  is near 1.) The contraction of private credit in equilibrium is also virtually the same regardless of the value of  $\phi_\omega$ . Nonetheless,

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Table 1: Optimal value of the spread-adjustment coefficient  $\phi_\omega$  in policy rule (2.3), in the case of a convex intermediation technology. Each column indicates a different type of disturbance, for which the policy rule is optimized; each row indicates a different possible degree of persistence for the disturbance.

	$Z_t$	$\mu_t^w$	$\tau_t$	$G_t$	$b_t^g$	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	5.65*	5.65*	5.29	2.86	0.62	5.65*	-0.01	2.01	0.86	0.64
$\rho_\xi = 0.50$	5.65*	5.65*	5.65*	3.93	0.71	5.65*	0.19	1.00	0.84	0.72
$\rho_\xi = 0.90$	5.65*	5.65*	5.65*	-1.22	0.74	5.65*	1.50	-0.33	0.82	0.75
$\rho_\xi = 0.99$	5.65*	5.65*	5.65*	-8.12	0.65	5.65*	5.65*	-3.75	0.70	0.66

\* higher number leads to indeterminacy

aggregate expenditure falls much less when  $\phi_\omega$  is positive; the expenditure of borrowers no longer has to be cut back so much in order to reduce their borrowing, because their labor income no longer falls in response to the shock, and there is an offsetting increase in the expenditure of savers.

The figure is very similar in the case of an exogenous shock to the marginal resource cost of intermediation (an exogenous increase in the multiplicative factor  $\tilde{\Xi}_t$ , not shown). As indicated in Table 1, in this case the optimal response coefficient is only slightly smaller, 0.75. The other comments about the shock to  $\tilde{\chi}_t$  apply equally to this case. The responses to a purely financial shock, both under the modified Taylor rules and under optimal policy, do depend greatly on the assumed persistence of the disturbance. Figure 3 shows the responses to a shock to the default rate (an exogenous increase in the factor  $\tilde{\chi}_t$ ) of the same magnitude as in Figure 2, but under the assumption that the disturbance lasts for only one quarter. Under the unadjusted Taylor rule, such a shock again contracts output and reduces inflation in the quarter of the disturbance; but both output and inflation then overshoot their long-run levels in the quarter following the shock, as a consequence of the reduced level of private indebtedness. A spread-adjusted Taylor rule leads both to smaller immediate declines in output and inflation (or even to increases, in the case of a sufficiently large spread adjustment), and to smaller subsequent increases in output and inflation (or even to decreases in both variables in the quarter following the disturbance, if the spread adjustment is large enough). One observes that the responses of output and inflation under optimal policy again lie between those resulting from the simple rules with  $\phi_\omega = 0.75$  and with  $\phi_\omega = 1.0$ ; as Table 1 shows, the optimal value of the response coefficient is actually 0.86. In fact, as Table 1 shows, the optimal value of  $\phi_\omega$  in the case that shocks of this kind are the only disturbance to the economy is fairly similar, regardless of the persistence of the shocks; a nearly complete (though not quite complete) offset for the spread variation is optimal in each of the cases considered. Again, we obtain broadly similar conclusions in the case of disturbances to  $\tilde{\Xi}_t$ ; the optimal values of  $\phi_\omega$  (as shown in Table 1) are slightly smaller, but in all cases greater than 0.5.

However, our conclusions about the optimal spread adjustment are considerably more varied when we consider other kinds of disturbances. (In the model with an endogenous

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credit spread,  $\eta > 0$ , a spread adjustment in the Taylor affects the economy's equilibrium response to disturbances of all types, and not just disturbances originating in the financial sector.) Even if we restrict our attention to disturbances that with a serial correlation  $\rho = 0.9$  (the third row of Table 1), we see from the table that the optimal spread adjustment is quite different for different disturbances. In the case of an exogenous disturbance to the level of government debt  $b_t^g$ , the optimal spread adjustment is again a large fraction of 1, regardless of the degree of persistence; this is because in our model, the effects of a government debt shock are essentially equivalent to a disturbance to the financial intermediation technology (as government borrowing crowds out private borrowing). But the results for other disturbances are much less similar.

Figure 4 shows the equilibrium responses to an exogenous increase in the productivity factor  $Z_t$ , again for the case  $\rho = 0.9$ . We observe that equilibrium responses under the unadjusted Taylor rule are quite different than those under optimal policy: output does not increase nearly as much as would be optimal, and inflation sharply declines, while under optimal policy it would not decline (and indeed would very slightly increase). (Essentially, this is because the Taylor rule reacts to a productivity-driven boom by raising interest rates, unless inflation falls sharply enough for this no longer to be required, as it does in equilibrium owing to the monetary tightening.) Because such a boom is associated with a credit expansion, the credit spread rises in the case of an endogenous spread; hence a spread adjustment  $\phi_\omega > 0$  adjusts policy in the right direction, accommodating the boom to a somewhat greater extent. But as shown in the figure, even a 100 percent spread adjustment is not nearly enough of a modification of the baseline Taylor rule to correct this problem; monetary policy remains much too contractionary in response to such a shock. The optimal response, in the case that shocks of this kind were the only disturbances in the model, would be a value of  $\phi_\omega$  much greater than 1. If we optimize over the value of  $\phi_\omega$ , imposing only the constraint that the policy rule (2.3) must lead to a determinate rational-expectations equilibrium,<sup>3</sup> we find that welfare is maximized by making  $\phi_\omega$  as large as is possible given the determinacy constraint. (In the case of our calibrated parameter values, determinacy requires that we restrict attention to values  $\phi_\omega \leq 5.65$ ) The same conclusion is reached in the case of a shock to attitudes toward labor supply  $\bar{H}_t$ , a shock to the wage markup  $\mu_t^w$ , or a shock to the tax rate  $\tau_t$ .

In other cases, the optimal spread adjustment is less extreme, but still greater than 100 percent of the increase in the credit spread. Figure 5 shows the equilibrium responses to an exogenous increase in the factor  $\bar{C}_t^b$ , representing an increase in the spending opportunities of type  $b$  households, again under the assumption that  $\rho = 0.9$ . As in the case of a productivity shock, the unadjusted Taylor rule tightens policy in response to an output increase that is actually efficient, and so is too contractionary. Because this kind of boom is associated with a credit boom, the credit spread increases, and a spread adjustment  $\phi_\omega > 0$  modifies the policy rule in a desirable direction. But again, even a 100 percent spread adjustment is insufficient. As shown in Table 1, the optimal spread adjustment coefficient would be greater

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<sup>3</sup>For further discussion of this requirement, and its relevance to the choice of a monetary policy rule, see, for example, Woodford (2003, chap. 4).

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than 1.5. (It is much less than in the case of the productivity shock, however, because this kind of disturbance leads to more procyclical credit.)

Our conclusions are quite different, however, if one considers instead a disturbance to the spending opportunities of type  $s$  households (increase in  $\bar{C}_t^s$ ). As shown in Figure 6, the unadjusted Taylor rule is too contractionary in this case as well. But because the shock results in counter-cyclical variation in private credit, and hence in the credit spread, a spread adjustment  $\phi_\omega > 0$  changes the Taylor rule in the wrong direction: as shown in the figure, this would make monetary policy even more excessively contractionary in response to this kind of disturbance. In fact, as shown in Table 1, the optimal spread adjustment would have the opposite sign ( $\phi_\omega = -0.33$ ). We obtain a similar conclusion (for essentially the same reason) in the case of a shock to government purchases  $G_t$  (again assuming  $\rho = 0.9$ ), as shown in Figure 7. This is another example of an expansionary shock that reduces private credit (because government purchases crowd out mainly the spending of type  $b$  households, which is the more interest-sensitive kind of private expenditure) and so reduces the equilibrium credit spread; a spread adjustment then modifies the baseline Taylor rule in the wrong direction. (As shown in Table 1 the optimal adjustment would actually be  $\phi_\omega = -1.22$ .)

Many of these results are also quite sensitive to the assumed degree of persistence of the disturbance. For example, in the case of shocks to government purchases, if the disturbance has a coefficient of serial correlation  $\rho = 0.5$ ; as assumed in Figure 8, the optimal spread adjustment is positive. The reason is that in this case, unlike the one shown in Figure 7, monetary policy is too expansionary under the baseline Taylor rule; hence welfare is improved by a positive spread adjustment, which in this case would raise the policy rate owing to the decline in the credit spread. In fact, the optimal spread adjustment is much larger than 100 percent ( $\phi_\omega = 3.93$ ). Instead, in the case of a higher degree of persistence (for example, the case  $\rho = 0.9$  shown in Figure 7), policy is too tight under the baseline Taylor rule, and the optimal spread adjustment is negative. In fact, if the serial correlation is instead  $\rho = 0.99$ , the optimal spread adjustment is not only negative, but also very large ( $\phi_\omega = -8.12$ ).

Thus in the endogenous-spread case, we certainly cannot say in general that a positive spread adjustment is necessarily an improvement upon the baseline Taylor rule, let alone that the appropriate adjustment will generally be of about the size of the increase in the credit spread. The optimal spread adjustment is quite different in the case of different types of disturbances (including disturbances of different degrees of persistence). It is not possible to offer a general statement about the optimal spread adjustment without reaching a view about the quantitative importance of the different types of theoretically possible disturbances in practice.

When doing this, it is important to consider not only the optimal spread adjustment in the case of a given type of disturbance, but also the size of the change in welfare achieved by a spread adjustment in each case. Table 2 reports the welfare change (relative to the baseline Taylor rule) for each of the types of shocks, for each of several different possible sizes of spread adjustment (the same four values of  $\phi_\omega$  considered in the figures). The first part of the table shows results for the case of disturbances with zero persistence, the second part for the case of disturbances with  $\rho = 0.9$ . In the case of each type of disturbance, the

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Table 2: Welfare consequences of increasing  $\phi_\omega$  in policy (2.3), in the case of different disturbances. Each column indicates a different type of disturbance, while each row corresponds to a given degree of spread adjustment.

	$Z_t$	$\mu_t^w$	$\tau_t$	$G_t$	$b_t^g$	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
No persistence ( $\rho_\xi = 0$ )										
$\phi_\omega = 0.25$	1.1	0.7	0.5	6.4	26.9	1.1	-4.4	22.2	38.4	27.7
$\phi_\omega = 0.50$	2.2	1.4	0.9	12.7	40.8	2.2	-17.8	41.8	65.0	42.5
$\phi_\omega = 0.75$	3.2	2.0	1.4	18.8	40.4	3.2	-41.3	58.7	78.4	43.1
$\phi_\omega = 1.00$	4.3	2.7	1.9	24.9	24.2	4.3	-76.2	72.8	77.1	27.7
Persistence ( $\rho_\xi = 0.9$ )										
$\phi_\omega = 0.25$	16.5	15.4	14.8	-20.8	55.1	16.5	60.9	-33.7	61.6	55.5
$\phi_\omega = 0.50$	33.1	30.8	29.7	-46.2	89.5	33.1	112.2	-87.3	102.9	90.3
$\phi_\omega = 0.75$	49.6	46.2	44.5	-76.4	101.1	49.6	153.4	-161.9	121.9	102.3
$\phi_\omega = 1.00$	66.1	61.6	59.3	-111.7	87.6	66.1	183.6	-258.7	116.2	89.4

amplitude of the shock is normalized so that the standard deviation of fluctuations in output around trend will be one percentage point, in the case that that disturbance is the only kind that exists.

When considering the overall advantage of a given increase in the spread adjustment, it is necessary to consider the implications for the way in which the economy will respond to all of the different types of disturbances to which it is subject at different times. It is possible to determine this, however, by looking across a given row of the table. For example, suppose that in a given economy, 50 percent of output fluctuations (relative to trend) are due to productivity shocks, 25 percent are due to variations in the level of government purchases, and 25 percent are due to credit spread variations resulting from shocks to the default rate. Suppose furthermore that each of the three types of disturbances that occur have serial correlation coefficient  $\rho = 0.9$  (so that the second part of Table 2 applies), and that the three disturbances are independent of one another (so that we can simply sum the contributions of the three disturbances to our quadratic loss function).<sup>4</sup> Then a change in the value of  $\phi_\omega$  will raise welfare if and only if raises  $W^{tot} = 0.5W_Z + 0.25W_G + 0.25W_{\tilde{\chi}}$ , where  $W_Z$  is the welfare measure reported in the  $Z_t$  column of Table 2,  $W_G$  is the welfare measure reported in the  $G_t$  column, and so on. For example, in the case of an increase in  $\phi_\omega$  from 0.50 to 0.75, the table indicates that  $W_Z$  and  $W_{\tilde{\chi}}$  both increase, while  $W_G$  falls. However, the increases in  $W_Z$  and  $W_{\tilde{\chi}}$  are larger than the decline in  $W_G$ . If we use the weights just proposed,  $W^{tot}$  increases by a net amount of 5.45, so that the increase to a 75

<sup>4</sup>Of course, there is no reason why these disturbances are necessarily distributed independently of one another. For example, the preferences of type b households and of type ss households need not fluctuate independently of one another. But to deal with this possibility, we would need additional information beyond that reported in Table 2. In effect, we would have to consider additional types of disturbances besides those reported in the table: a disturbance that raises  $\bar{C}_t^b$  and  $\bar{C}_t^s$  in the same proportion, a disturbance that raises  $\tau_t$  by half the amount of the increase in  $G_t$ , and so on.

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percent adjustment would be beneficial in welfare terms, despite the fact that it leads to a less optimal response to one of the types of disturbances.<sup>5</sup> Among the cases considered in the table,  $\phi_\omega = 0.75$  achieves the highest value of  $W^{tot}$ . In fact,  $W^{tot}$  would be maximized by setting  $\phi = 0.72$ . (This represents a compromise among the values that would be optimal for each of the three types of disturbance individually, as reported in Table 1.)

This result, however, is quite dependent on which types of disturbances are thought to account for greater shares of the variance decomposition of output fluctuations. If, for example, one supposes that 50 percent of output fluctuations are due to productivity shocks and 50 percent to fluctuations in  $\tilde{C}_t^s$  (again, assuming  $\rho = 0.9$  for both disturbances and that they are independent of one another), then the appropriate welfare measure would be  $W^{tot} = 0.5W_Z + 0.5W_{C^s}$ . One observes that this measure decreases when one moves from  $\phi_\omega = 0$  to 0.25 ( $W_Z$  increases by less than  $W_{C^s}$  falls). In fact, in this case,  $W^{tot}$  would be maximized by  $\phi_\omega = 0.16$ . If we allow for the fact that disturbances with many different degrees of persistence are also possible (the cases in Table 1 representing only a few of the simplest possibilities), then the range of possible conclusions about the optimal spread adjustment are even larger.

The considerations involved in judging the optimal spread adjustment are simpler in the case that we assume a linear intermediation technology (along with our maintained assumption in the above calculations that  $\chi_t(b)$  is linear). In this case, the credit spread is an exogenous process, so that a spread adjustment to the Taylor rule has no consequences (in our log-linear approximation) for the economy's response to disturbances other than purely financial disturbances (shocks to  $\tilde{\chi}_t$  or to  $\tilde{\Xi}_t$ , the two determinants of the credit spread). Moreover, the consequences of a spread adjustment are quite similar in the case of these two types of financial disturbances; so it might seem that we should be able to choose the spread adjustment so as to optimize the response to a single type of shock. However, as shown in Table 3, the optimal spread adjustment is quite different depending on the degree of persistence of the financial disturbances. It is positive and even greater than 1, in the case of either type of disturbance, if the degree of persistence is  $\rho = 0.5$  or less. But the optimal degree of spread adjustment is much smaller (on the order of 0.25, for either type of disturbance) if instead we assume  $\rho = 0.9$ . In the case of even more persistent financial disturbances, the optimal spread adjustment changes sign. If, for example, we assume  $\rho = 0.99$ , the optimal spread adjustment is more negative than -2, for either type of disturbance.

To summarize, while under many assumptions welfare can be improved by a spread adjustment  $\phi_\omega > 0$ , the optimal size of spread adjustment need not be even approximately one-for-one (as suggested by discussions such as those of Taylor and of McCulley and Toloui), and depends on which kinds of disturbances are most important as sources of aggregate instability. In the case of a convex intermediation technology parameterized to imply a sharply rising marginal cost of intermediation (our baseline case), a spread-adjustment coefficient that is a large fraction of 1 can be justified in the case that the most important disturbances are ones that affect the economy primarily by affecting the efficiency of intermediation (the

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<sup>5</sup>This assumes, of course, that only policy rules within the restricted family (2.3) are considered.

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Table 3: Optimal value of the spread-adjustment coefficient  $\phi_\omega$  in policy rule (2.3), as in Table 1, but for the case of a linear intermediation technology.

	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	1.84	1.30
$\rho_\xi = 0.50$	1.62	1.40
$\rho_\xi = 0.90$	0.26	0.28
$\rho_\xi = 0.99$	-2.43	-2.36

$\tilde{\chi}_t$ ,  $\tilde{\Xi}_t$  and  $b_t^g$  shocks). A spread adjustment that is positive but much larger than one-for-one is instead preferred if "supply shocks" (the  $Z_t$ ,  $\bar{H}_t$ ,  $\mu_t^w$  or  $\tau_t$  shocks) are the main source of instability. To the extent that "demand shocks" are instead important, our results are more complex; the optimal size and even the optimal sign of the spread adjustment depends both on which type of demand disturbance is more important and on the degree of persistence of these disturbances. Nonetheless, unless one thinks that aggregate fluctuations are driven mainly by (certain types of) real "demand shocks" (highly persistent disturbances to  $G_t$  or  $\bar{C}_t^s$ ), a modestly positive value of  $\phi_\omega$  is almost certainly an improvement over the unadjusted Taylor rule, though the optimal degree of adjustment can easily be less than one-for-one.

## 2.2 Responding to Variations in Aggregate Credit

As mentioned in the main text, a common recommendation is that monetary policy should be used to help to stabilize aggregate private credit, by tightening policy when credit is observed to grow unusually strongly and loosening policy when credit is observed to contract. We now propose to replace (2.2) by a rule of the form

$$i_t^d = \phi_\pi \pi_t + \phi_y \hat{Y}_t + \phi_b \hat{b}_t, \quad (2.4)$$

for some coefficient  $\phi_b$ , the sign of which we shall not prejudge, much like the analysis shown in the main text, except that now we consider a rule without any response to the natural interest rate or the natural rate of output. Figure 9 illustrates the consequences of alternative degrees of response (of either sign) to credit variations, in the case of the same kind of increase in government purchases as in Figure 7, again in an economy with a convex intermediation technology, and with  $\phi_\pi$  and  $\phi_y$  set at the Taylor values.

Because in the case of a convex intermediation technology (and in the absence of "purely financial" disturbances) the credit spread  $\omega_t$  is a monotonic function of the aggregate volume of private credit  $b_t$  (and in our log-linear approximation,  $\hat{\omega}_t$  is a linear function of  $\hat{b}_t$ ), any rule of the form (2.4) is actually equivalent to a particular rule of the form (2.3), as far as our model's predictions about the responses to all non-financial shocks are concerned. Under our calibration, a rule of the form (2.4) with a coefficient  $\phi_b$  is equivalent to a rule of the form (2.3) with coefficient  $\phi_\omega = -\phi_b$ . Hence the results shown in Figure 9 (at least for the two cases with  $\phi_b < 0$ ) are actually the same as those in Figure 7 (for the corresponding values of  $\phi_\omega > 0$ ). As noted before, the optimal spread adjustment in this case would actually be

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Table 4: Optimal value of the response coefficient  $\phi_b$  in policy rule (2.4), for the same set of possible disturbances as in Table 1, and a convex intermediation technology.

	$Z_t$	$\mu_t^w$	$\tau_t$	$G_t$	$b_t^g$	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	-5.65*	-5.65*	-5.29	-2.86	-0.62	-5.65*	0.01	-2.01	1.14	0.97
$\rho_\xi = 0.50$	-5.65*	-5.65*	-5.65*	-3.93	-0.71	-5.65*	-0.19	-1.00	0.42	0.40
$\rho_\xi = 0.90$	-5.65*	-5.65*	-5.65*	1.22	-0.74	-5.65*	-1.50	0.33	0.06	0.06
$\rho_\xi = 0.99$	-5.65*	-5.65*	-5.65*	8.12	-0.65	-5.65*	-5.65*	3.75	0.00	0.00

\* lower number leads to indeterminacy

negative; this means that a positive coefficient  $\phi_b$  would similarly increase welfare, as the equilibrium responses are moved somewhat closer to those associated with Ramsey policy, as shown in the figure. In fact, the optimal adjustment in the case of this one type of disturbance would be  $\phi_b = 1.22$ .

Table 4 reports the optimal value of  $\phi_b$  in the rule (2.4), in the case of each of the types of individual disturbances considered in Table 1, using the same format as the earlier table.<sup>6</sup> The results for disturbances other than  $\tilde{\chi}_t$  and  $\tilde{\Xi}_t$  all follow directly from the results in Table 1. As before, our most important finding is that both the sign and magnitude of the optimal response coefficient depends on which types of disturbances one is concerned with. However, to the extent that our previous results provided some support for the view that a positive value of  $\phi_\omega$  is likely to be beneficial (even in the case of non-financial disturbances), this would correspond to a preference for a negative value of  $\phi_b$ , rather than a positive value as assumed in most discussion of this proposal.

However, the results in Table 1 according to which it is desirable for  $\phi_\omega$  to be positive in the case of "purely financial" disturbances do not imply that it is optimal for  $\phi_b$  to be negative, since these disturbances shift the equilibrium relation between aggregate credit and the credit spread. In fact, Table 4 shows that the optimal  $\phi_b$  in the case of either of the two types of purely financial disturbances is at least slightly positive. As in our discussion of the spread adjustment, we find that it is desirable to loosen policy in response to a shock that increases  $\omega_t(\bar{b})$ , to a greater extent than would occur under the unadjusted Taylor rule; but because credit contracts in response to such a disturbance (at the same time that the credit spread increases), this is achieved by setting  $\phi_b > 0$ . Nonetheless, the table shows that except when the disruption of financial intermediation is quite transitory, the optimal response coefficient is quite small. Figure 10 shows how alternative sizes of responses to aggregate credit change the equilibrium responses to an increase in the default rate with persistence  $\rho = 0.9$ , the same kind of disturbance considered in Figure 2. One sees that responses to credit of either sign make the economy's equilibrium response farther from

<sup>6</sup>The coefficients in the table indicate the desired increase in the policy rate target, expressed in percentage points per year, per percentage point increase in real aggregate credit. Thus  $\phi_b = 1.22$  means that a one percent greater volume of aggregate credit raises the operating target for the policy rate by 1.22 percentage points per year, in the absence of any change in inflation or output. If, in equation (2.4),  $\hat{z}_t^d$  and  $\pi_t$  are understood to be quarterly rates, then the coefficient on  $\hat{b}_t$  in that equation should be written as  $\phi_b = 4$ .

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what would occur under optimal policy, when the responses are of moderate size (the sizes of response that would be optimal in the case of other types of disturbance).

We can make two general observations about these results. First, there is little support for the idea that responding to variations in aggregate credit in a way that "leans against the wind" (i.e., with a coefficient  $\phi_b > 0$ ) would increase welfare, in a model of the kind that we consider here. (Of course, one might argue that the benefits of such a policy depend on mechanisms that are simply not present in our model.) When  $\rho = 0.5$  or less, Table 4 shows that the optimal response coefficient is not positive in the case of any type of non-financial disturbance,<sup>7</sup> and only moderately positive in the case of financial disturbances; when  $\rho = 0.9$  or more, it is positive only for two types of non-financial disturbances (the  $G_t$  and  $\bar{C}_t^s$  shocks, that are not obviously major sources of aggregate fluctuations in practice), and is only very slightly positive even in the case of the financial disturbances. And second, it is even harder to find a policy within the class (2.4) that is reasonably good regardless of the type of disturbance affecting the economy than it is to find a robust rule within the class (2.3). The robustness properties of the two types of rules is the same, if we are concerned only with non-financial disturbances; the question is whether the type of response that is desirable in the case of non-financial disturbances is also desirable in the case of financial disturbances. In the case of the spread-adjusted rules, the optimal sign of  $\phi_\omega$  is positive for most non-financial disturbances, and also for financial disturbances; in the case of the rules that respond to credit, the optimal sign of  $\phi_b$  is negative for most non-financial disturbances, but at least slightly) positive in the case of financial disturbances. If one must choose a policy rule from one of these two classes, one will in many cases do better by choosing the best rule from the family (2.3), because there is less tension between the goals of achieving desirable responses to the different types of disturbances.

In the case of a linear intermediation technology, rules in the family (2.4) are no longer equivalent to any rules in the family (2.3), in the case of non-financial disturbances. It is then a less trivial question to ask what might be achieved by allowing a non-zero value of  $\phi_b$ . However, the answer is that this lowers welfare, regardless of the sign of the response, in almost all cases. Table 5 reports the optimal value of  $\phi_b$  for each of the types of disturbance considered in Table 4, but for the case of the linear technology ( $\eta = 1$ ). The optimal coefficient is close to zero in all cases. The reason that the dynamic response of credit to the various shocks makes it not a useful indicator of the way in which monetary policy needs to be adjusted, regardless of the sign with which one responds to it (assuming that one responds only to the contemporaneous volume of credit).

This is illustrated in Figure 11, which shows the responses to a productivity disturbance of the kind considered in Figure 4 under alternative rules in the family (2.4). The reason that the optimal  $\phi_b$  is near zero is not that the unadjusted Taylor rule is already optimal; as discussed earlier, the unadjusted Taylor rule is quite sub-optimal in the case of this kind of disturbance, as the central bank tightens policy too much in response to an output increase that should instead be accommodated. One might think that since credit expands in response to the disturbance, a coefficient  $\phi_b < 0$  would move policy in the right direction.

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<sup>7</sup>It is very slightly positive only for the  $\bar{C}_t^b$  shock in the case that the persistence is reduced to zero.

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Table 5: Optimal value of the response coefficient  $\phi_b$  in policy rule (2.4), for the same set of possible disturbances as in Table 4, but a linear intermediation technology.

	$Z_t$	$\mu_t^w$	$\tau_t$	$G_t$	$b_t^g$	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	-0.09	-0.09	-0.08	-0.03	0.00	-0.09	0.00	-0.01	0.02	0.01
$\rho_\xi = 0.50$	-0.08	-0.08	-0.08	-0.02	0.00	-0.08	0.00	0.00	0.01	0.01
$\rho_\xi = 0.90$	-0.08	-0.08	-0.08	0.01	0.00	-0.08	-0.01	0.00	0.00	0.00
$\rho_\xi = 0.99$	-0.07	-0.07	-0.07	0.02	0.00	-0.07	-0.03	0.01	-0.01	-0.01

But the figure shows that while a response to credit of that kind can mitigate the excessively disinflationary effect of the disturbance on impact, it is at the price of making policy be too inflationary later, as credit continues to surge.

The problem is that the time path of the response of credit (only slightly positive in the quarter of the shock, and then growing larger over the next several years) is very different from the time path of the distortions that need to be corrected (greatest in the quarter of the shock, and decaying substantially over the next few years). This is a fairly general problem with the usefulness of aggregate credit as an indicator for monetary policy in the model with a linear intermediation technology: the aggregate volume of credit is a stock that reflects the cumulative effects of disturbances over many previous years, rather than recent (and still relevant) disturbances alone. Once again, we find that responding to credit spreads provides a more useful rule of thumb than responding to the volume of credit. (Under this parameterization of our model, a spread adjustment cannot improve the economy's response to non-financial disturbances, but at least it does not cause worse responses to those shocks, either; and a positive spread adjustment does improve the economy's response to financial disturbances, regardless of the type and of the degree of persistence of the disturbance.)

### 3 Taylor Rules with Natural Rate Adjustments

The main text presents several alternative Taylor rules in which the interest rate responds to the natural interest rate and to the output gap as mentioned previously. In particular the text discusses two type of rules:

$$i_t^d = r_t^n + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \hat{Y}_t^n) - \phi_\omega \hat{\omega}_t, \quad (3.1)$$

and

$$i_t^d = r_t^n + \phi_\pi \pi_t + \phi_y (\hat{Y}_t - \hat{Y}_t^n) + \phi_b \hat{b}_t, \quad (3.2)$$

where the first allows for adjustments to the interest rate in response to changes in the spread between deposit and borrowing interest rates, while the second allows for adjustments in response to changes in the level of aggregate credit. In this section we further the discussion of the policy rule of the latter type with adjustment for changes in credit. However here we offer some additional detail on the case of linear financial intermediation technology.

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Table 6: Optimal value of the response coefficient  $\phi_b$  in policy rule (3.2), for the same set of possible disturbances as in Table 5, but with response to natural variables.

	$Z_t$	$\mu_t^w$	$\tau_t$	$G_t$	$b_t^g$	$\bar{H}_t$	$\bar{C}_t^b$	$\bar{C}_t^s$	$\tilde{\chi}_t$	$\tilde{\Xi}_t$
$\rho_\xi = 0.00$	-0.01	0.01	0.02	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00
$\rho_\xi = 0.50$	0.00	0.00	0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00
$\rho_\xi = 0.90$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\rho_\xi = 0.99$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

### 3.1 Responding to Variations in Aggregate Credit

In the case of a linear intermediation technology, rules in the family (3.2) are no longer equivalent to any rules in the family (3.1), in the case of non-financial disturbances. It is then a less trivial question to ask what might be achieved by allowing for a non-zero value of  $\phi_b$ . However, the answer is that this lowers welfare, regardless of the sign of the response, in almost all cases. Table 6 reports the optimal value of  $\phi_b$  for each of the types of disturbance considered in Table 5, but for the policy rule described in (3.2). The optimal coefficient is close to zero in all cases. The reason that the dynamic response of credit to the various shocks makes it not a useful indicator of the way in which monetary policy needs to be adjusted, regardless of the sign with which one responds to it (assuming that one responds only to the contemporaneous volume of credit).

This is illustrated in Figure 12, which shows the responses to a productivity disturbance of the kind considered in Figure 11, but under alternative rules in the family (3.2). This figure shows that for this class of policy rules setting  $\phi_b = 0$  gets the economy very close to the optimal policy, and any value of this coefficient will move the economy away from the optimal equilibrium (for all variables). The problem is that the time path of the response of credit (only slightly positive in the quarter of the shock, and then growing larger over the next several years) is very different from the time path of the distortions that need to be corrected (greatest in the quarter of the shock, and decaying substantially over the next few years). This is a fairly general problem with the usefulness of aggregate credit as an indicator for monetary policy in the model with a linear intermediation technology: the aggregate volume of credit is a stock that reflects the cumulative effects of disturbances over many previous years, rather than recent (and still relevant) disturbances alone. Once again, we find that responding to credit spreads provides a more useful rule of thumb than responding to the volume of credit. (Under this parameterization of our model, a spread adjustment cannot improve the economy's response to non-financial disturbances, but at least it does not cause worse responses to those shocks, either; and a positive spread adjustment does improve the economy's response to financial disturbances, regardless of the type and of the degree of persistence of the disturbance.)

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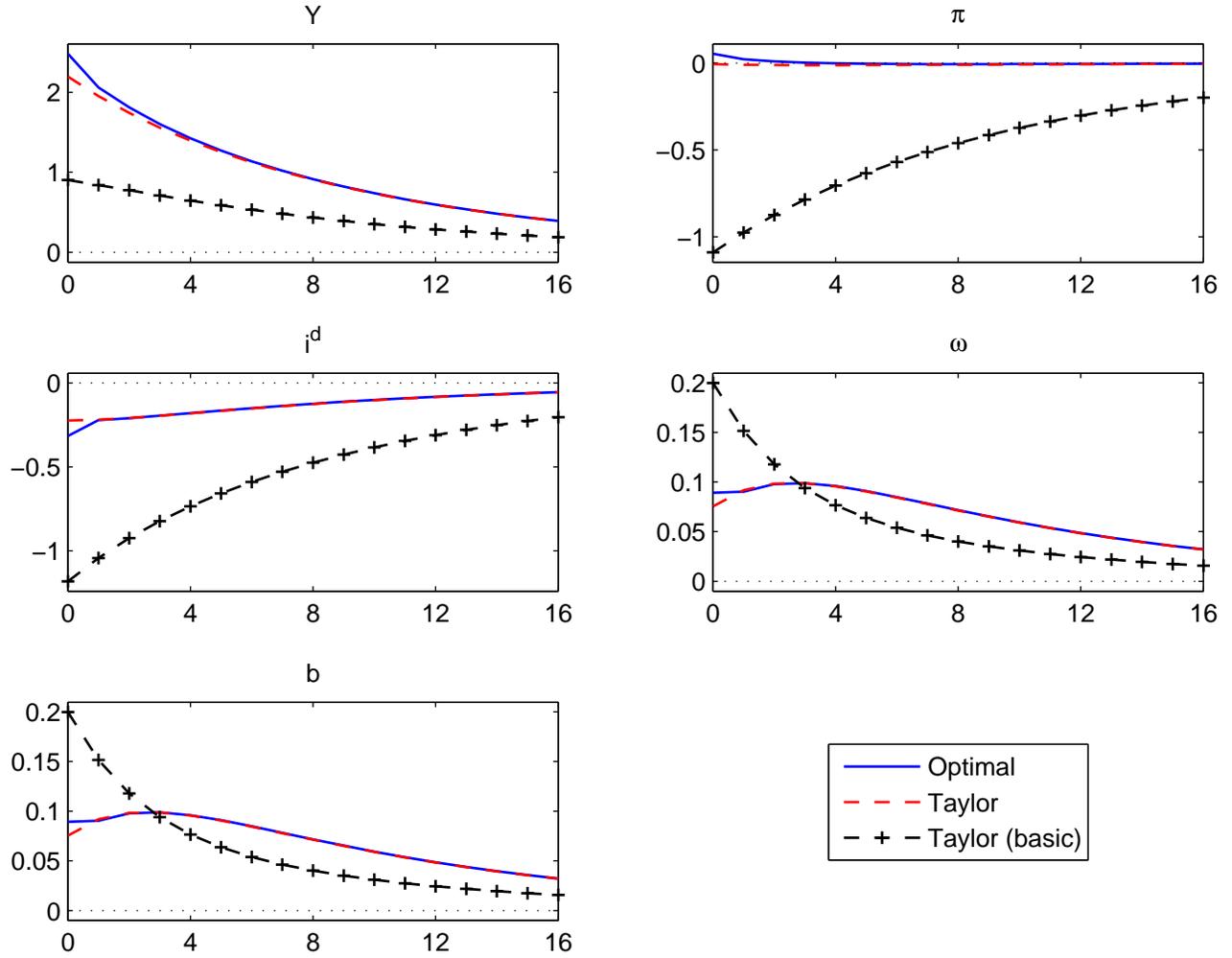


Figure 1: Impulse responses to a 1 percent shock to  $Z_t$ , with a convex intermediation technology, under three alternative monetary policy rules.

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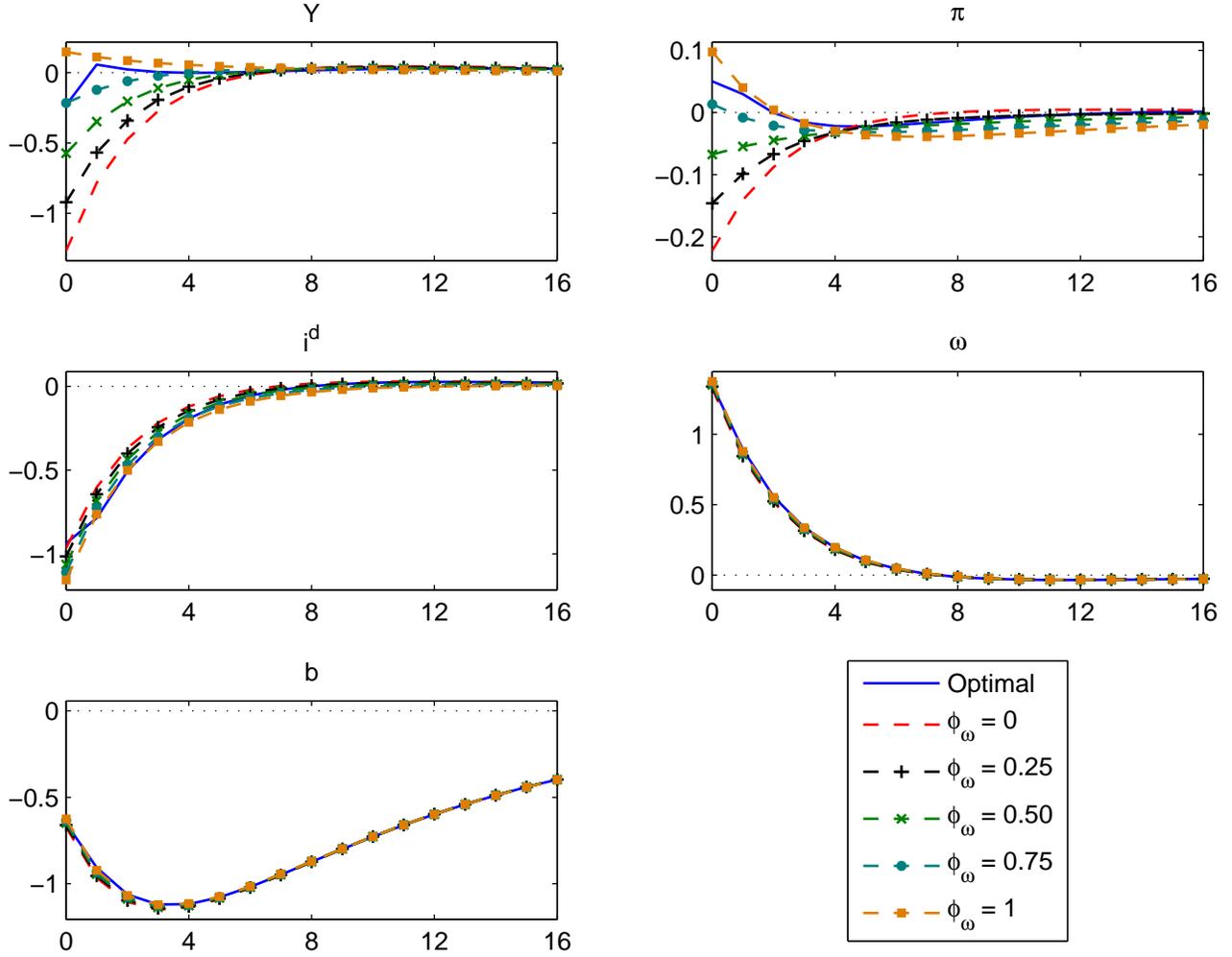


Figure 2: Impulse responses to a shock to  $\tilde{\chi}_t$  that increases  $\omega_t(\bar{b})$  initially by 4 percentage points (annualized), with persistence  $\rho = 0.9$ , under alternative degrees of spread adjustment.

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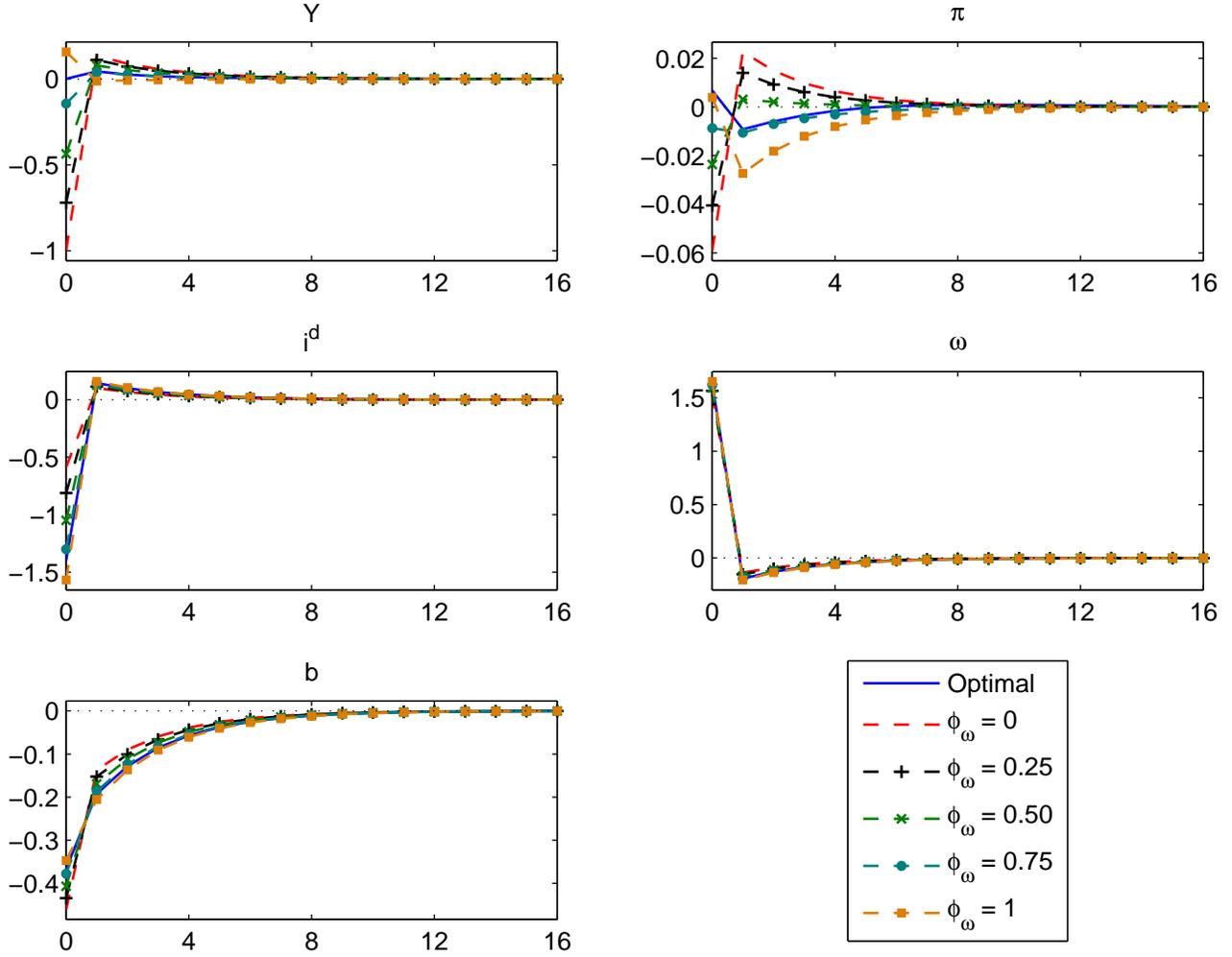


Figure 3: Impulse responses to a shock to  $\tilde{\chi}_t$  that increases  $\omega_t(\bar{b})$  initially by 4 percentage points (annualized), assuming no persistence ( $\rho = 0$ ), under alternative degrees of spread adjustment.

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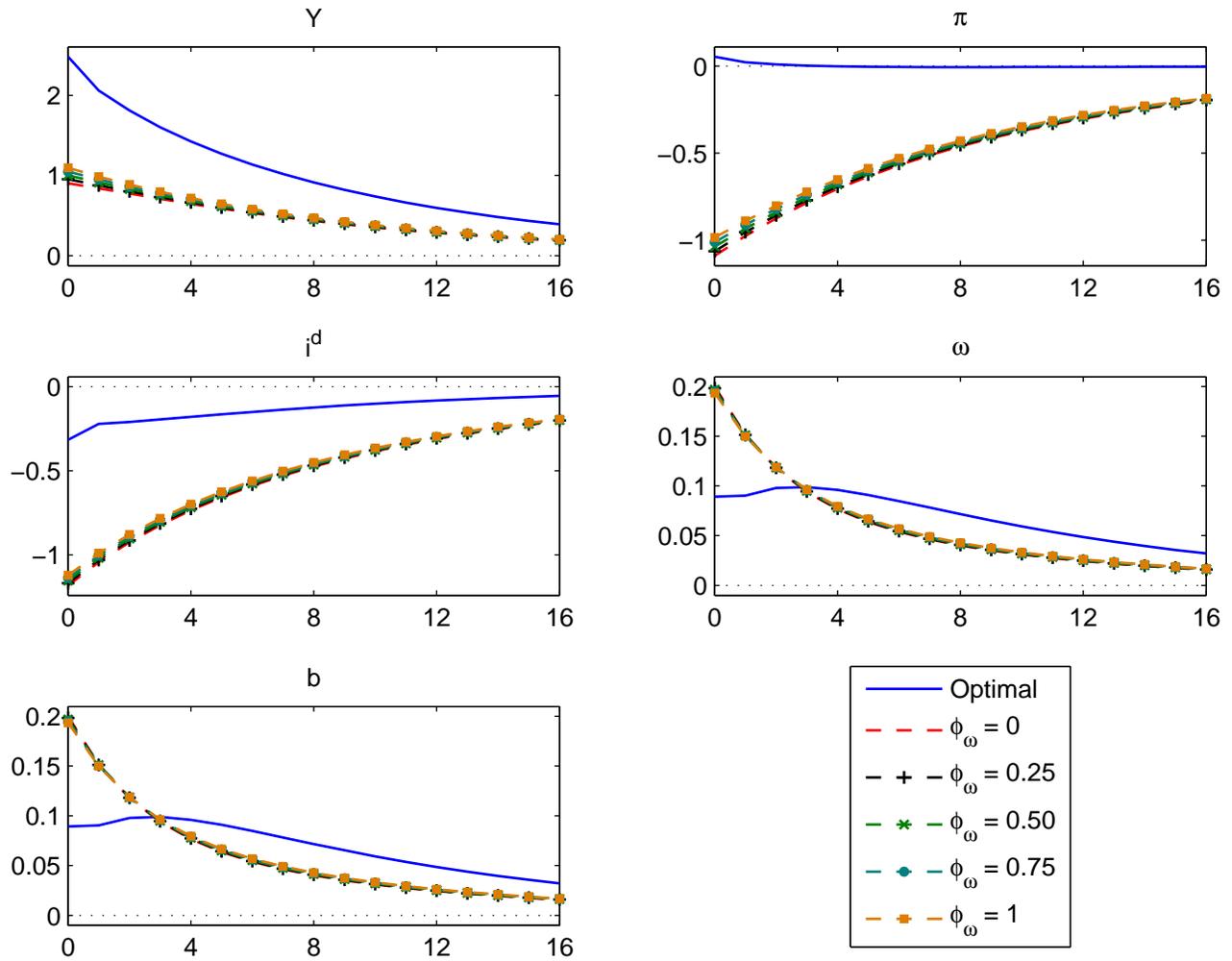


Figure 4: Impulse responses to a 1 percent shock to  $Z_t$ , with persistence  $\rho = 0.9$ , under alternative degrees of spread adjustment.

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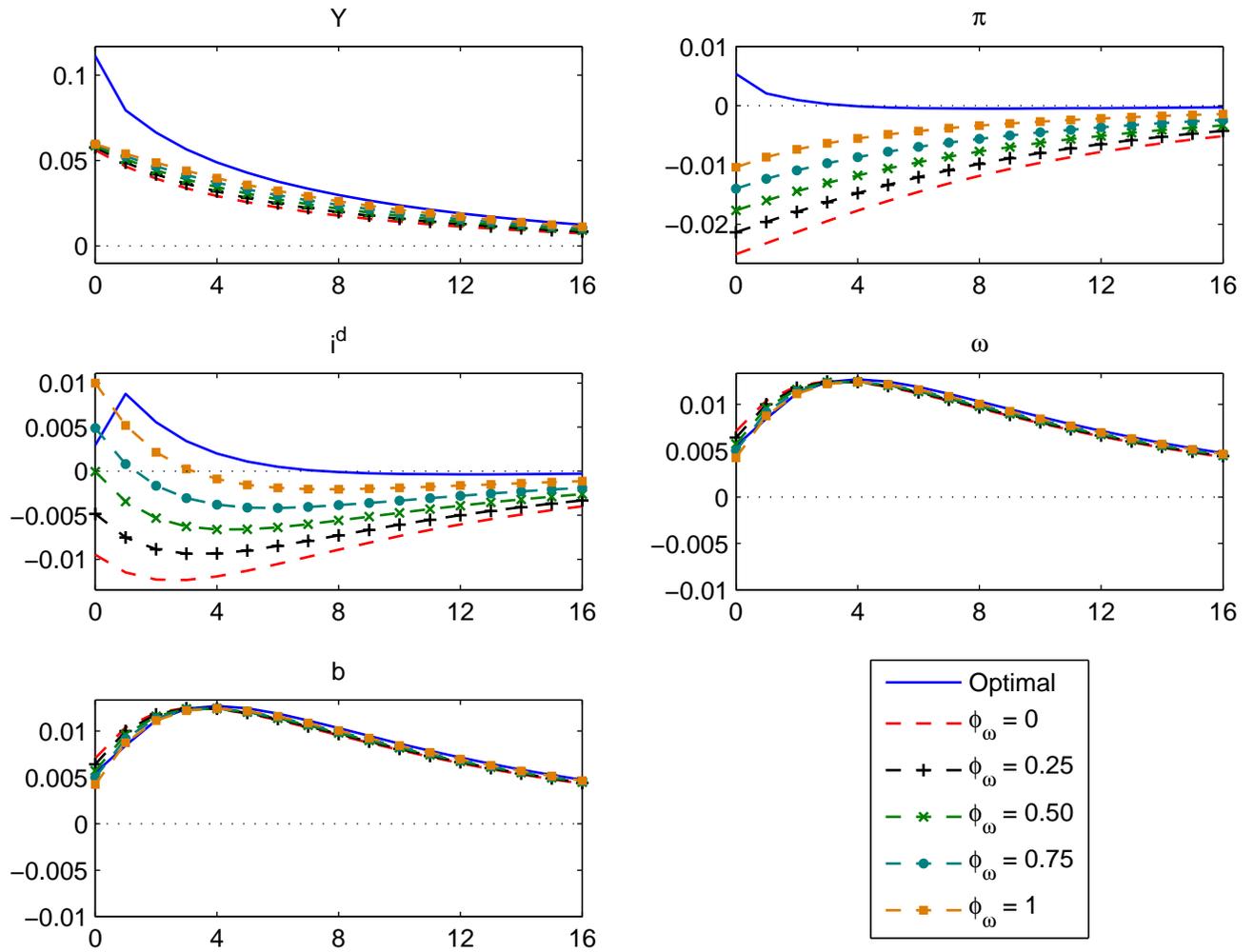


Figure 5: Impulse responses to a 1 percent shock to type  $b$  expenditure, with persistence  $\rho = 0.9$ , under alternative degrees of spread adjustment.

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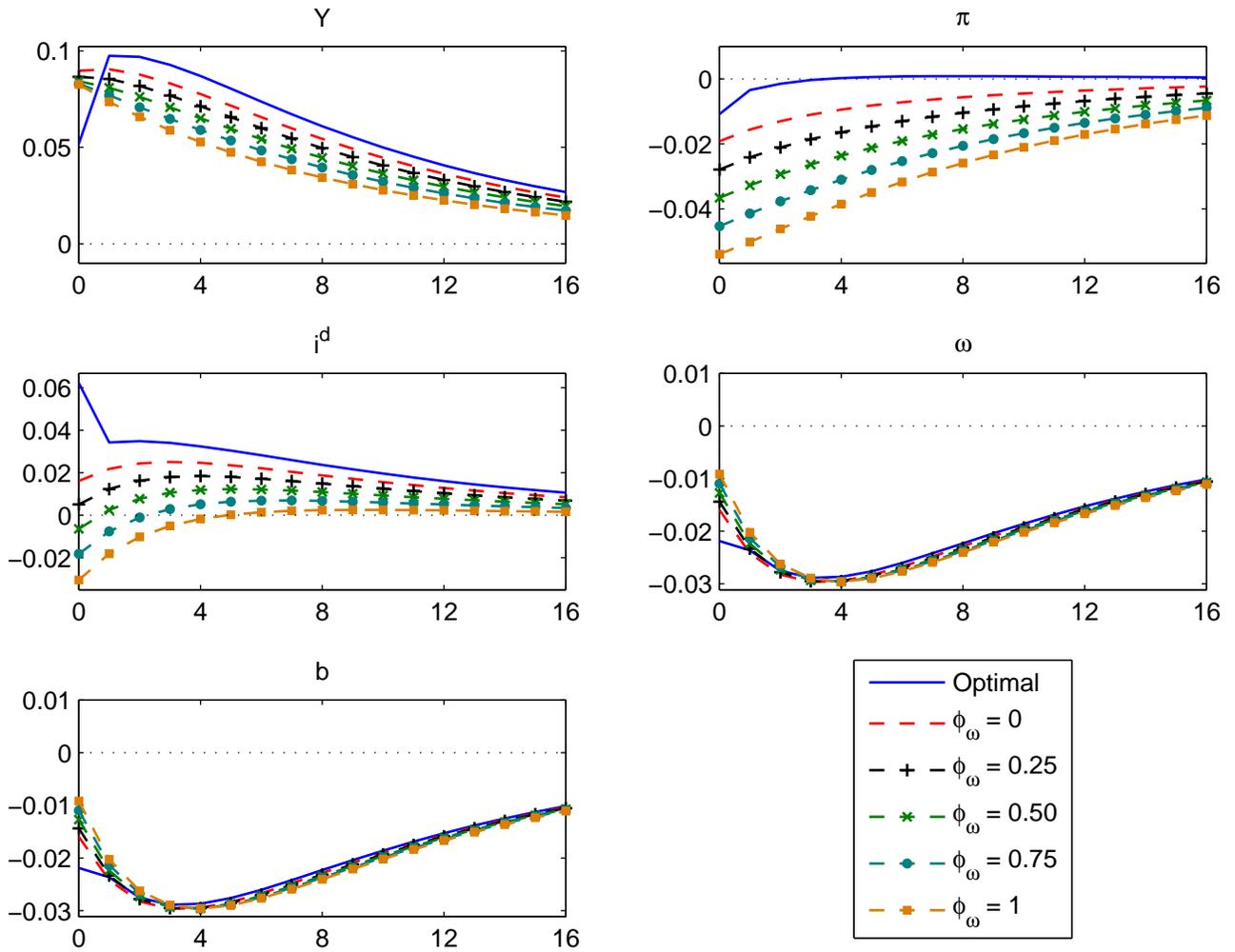


Figure 6: Impulse responses to a 1 percent shock to type  $s$  expenditure, with persistence  $\rho = 0.9$ , under alternative degrees of spread adjustment.

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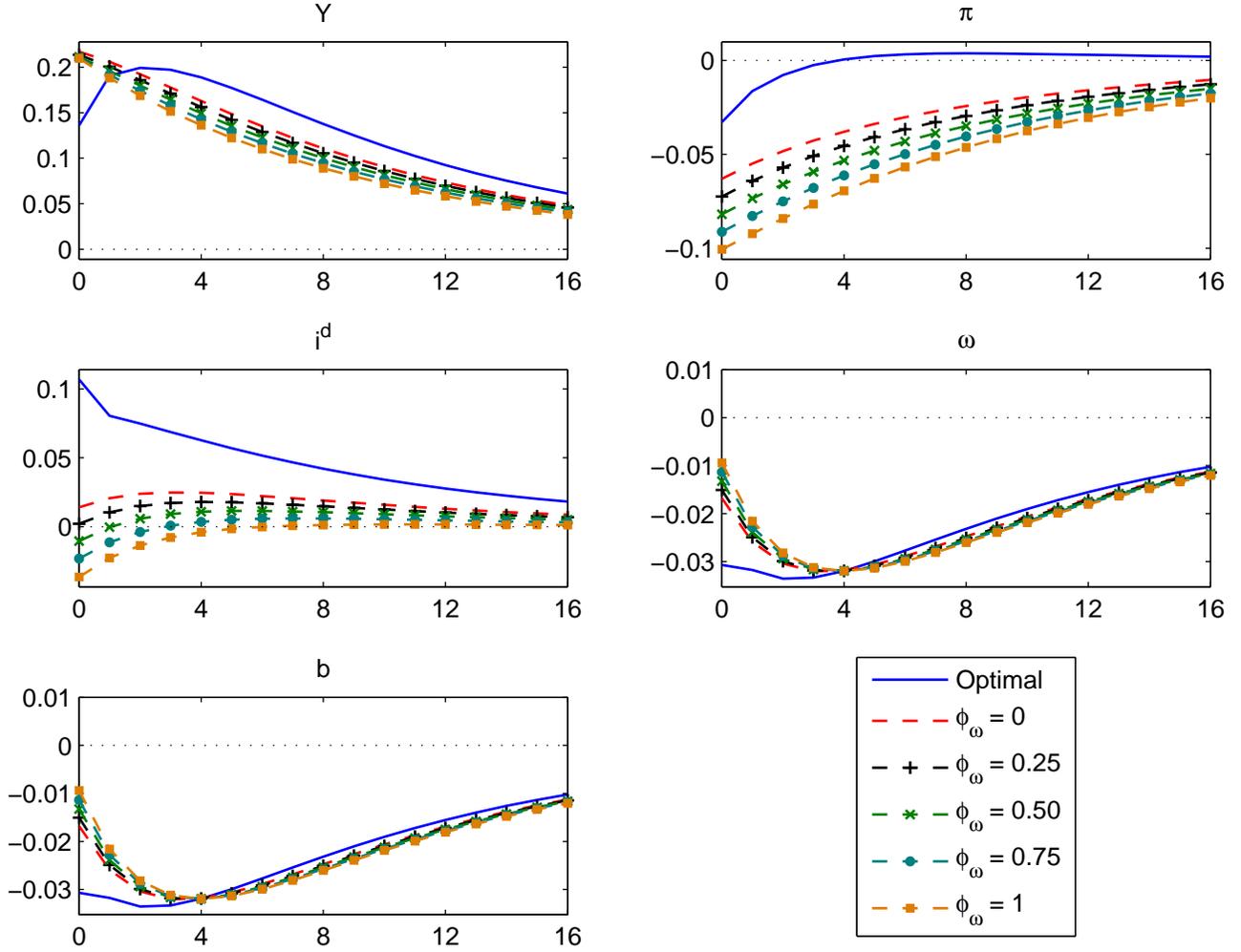


Figure 7: Impulse responses to a shock to  $G_t$  equal to 1 percent of steady-state output, with persistence  $\rho = 0.9$ , under alternative degrees of spread adjustment.

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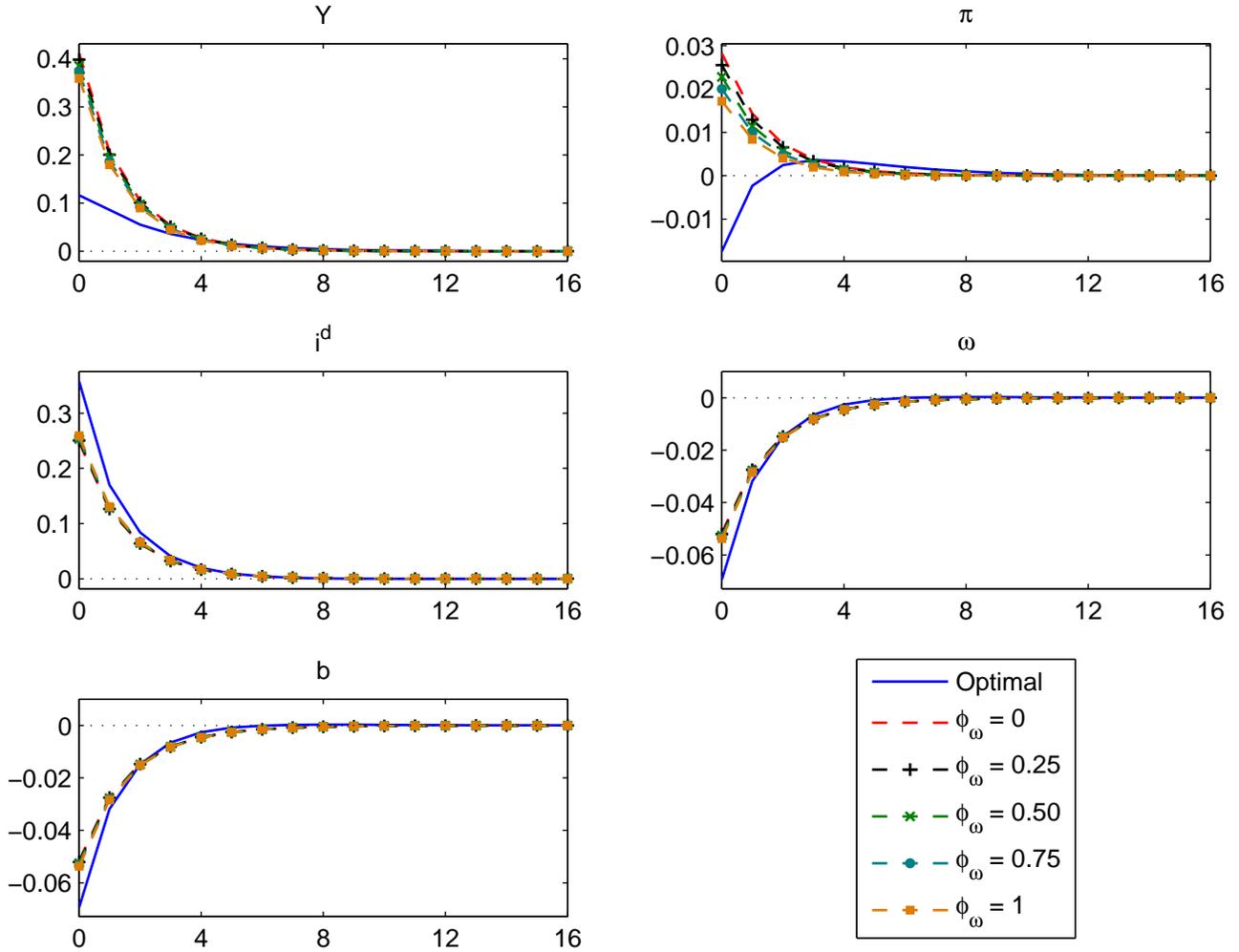


Figure 8: Impulse responses to a shock to  $G_t$  equal to 1 percent of steady-state output, but with persistence  $\rho = 0.5$ , under alternative degrees of spread adjustment.

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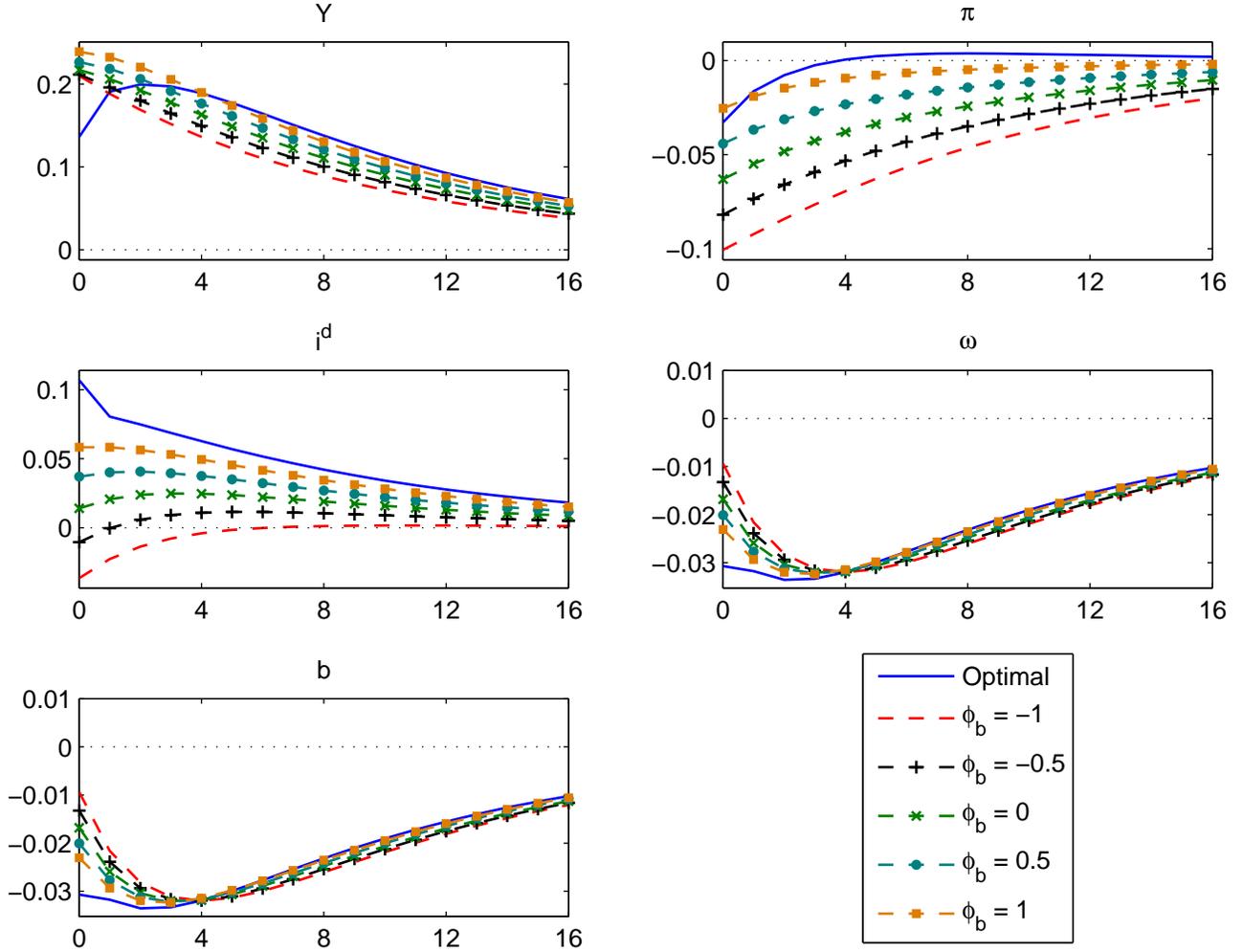


Figure 9: Impulse responses to a shock to  $G_t$  equal to 1 percent of steady-state output, under alternative degrees of response to aggregate credit.

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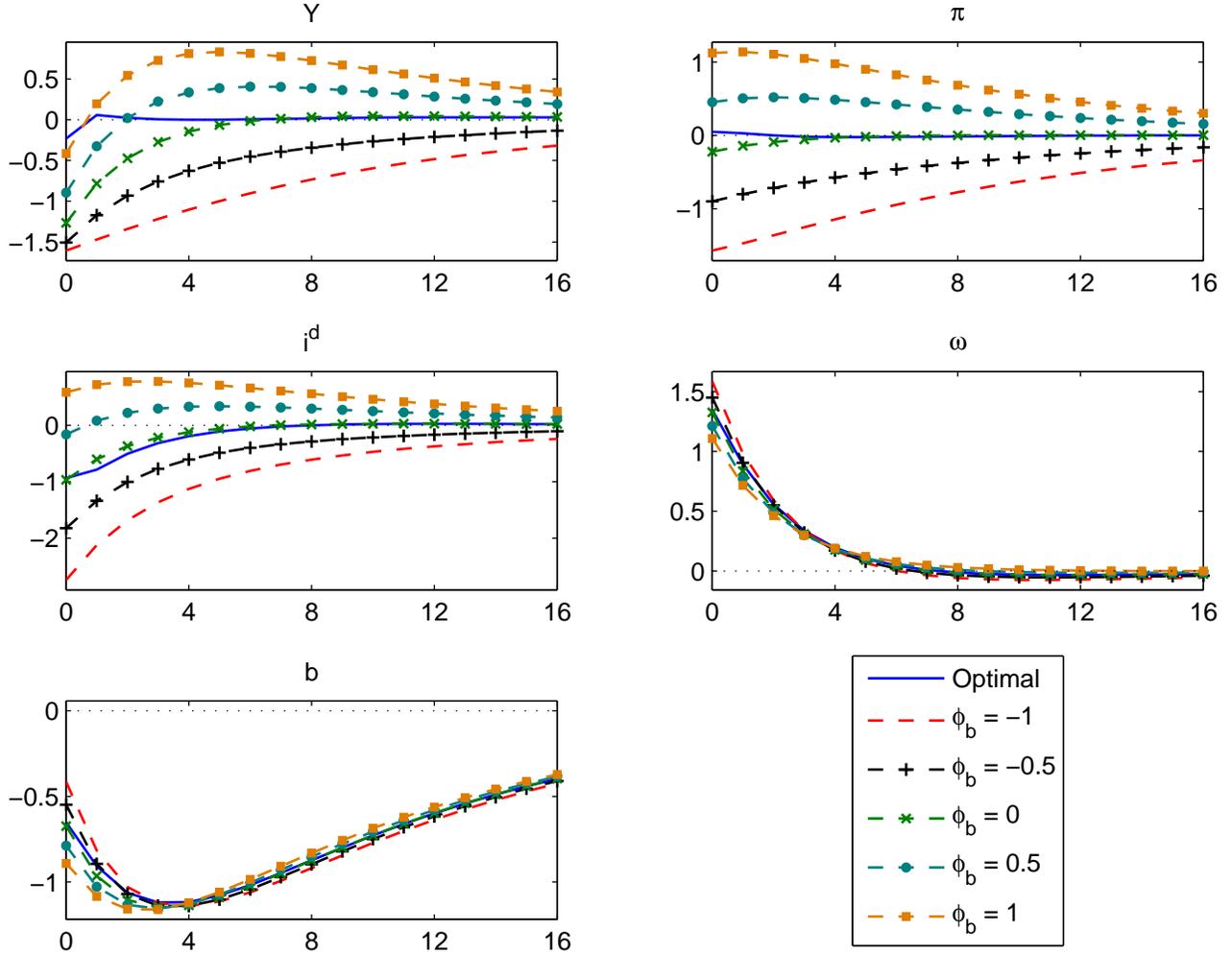


Figure 10: Impulse responses to a shock to  $\tilde{\chi}_t$  that increases  $\omega_t(\bar{b})$  temporarily by 4 percentage points (annualized), under alternative degrees of response to aggregate credit.

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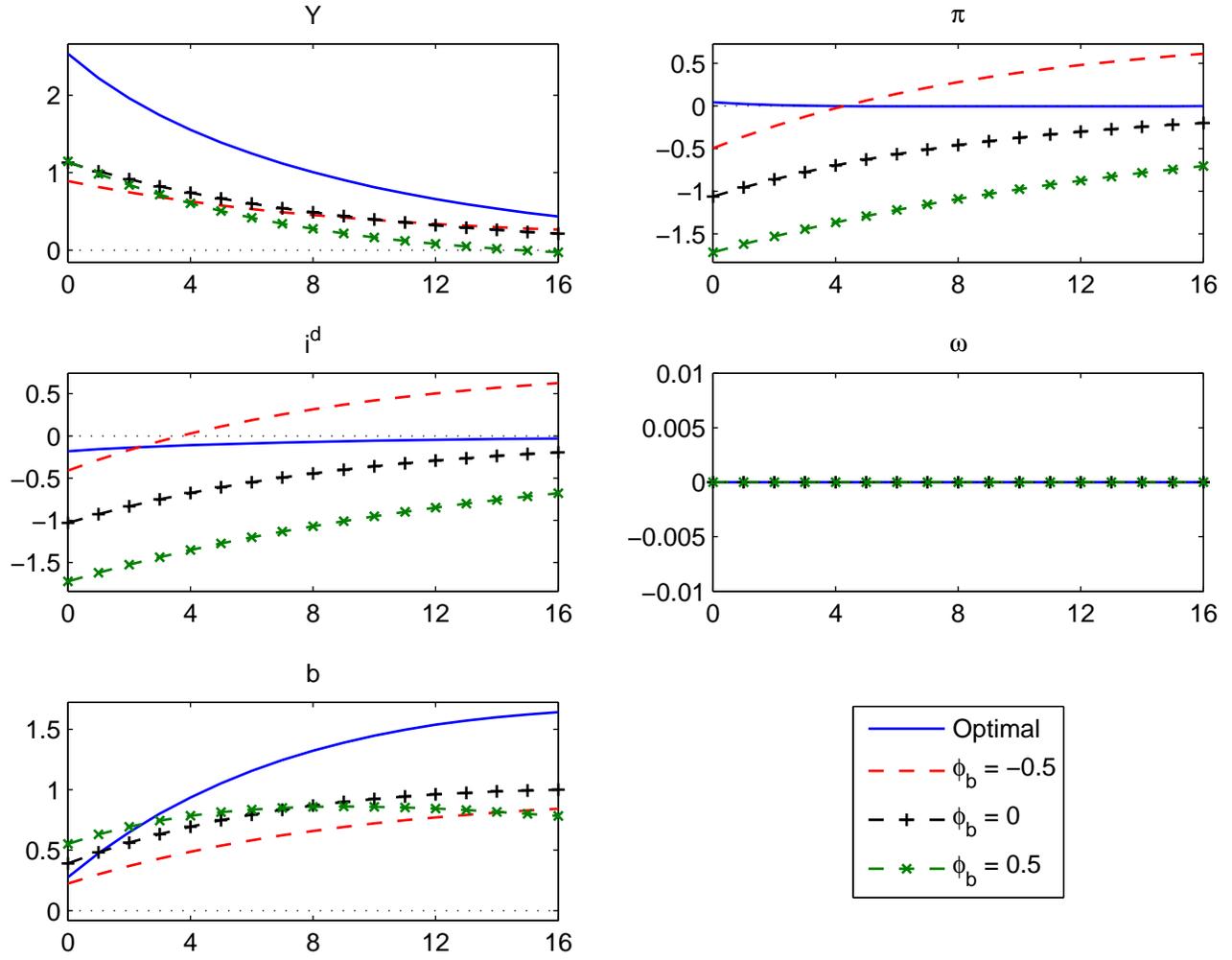


Figure 11: Impulse responses to a 1 percent shock to  $Z_t$ , under alternative degrees of response to aggregate credit, in the case of a linear intermediation technology.

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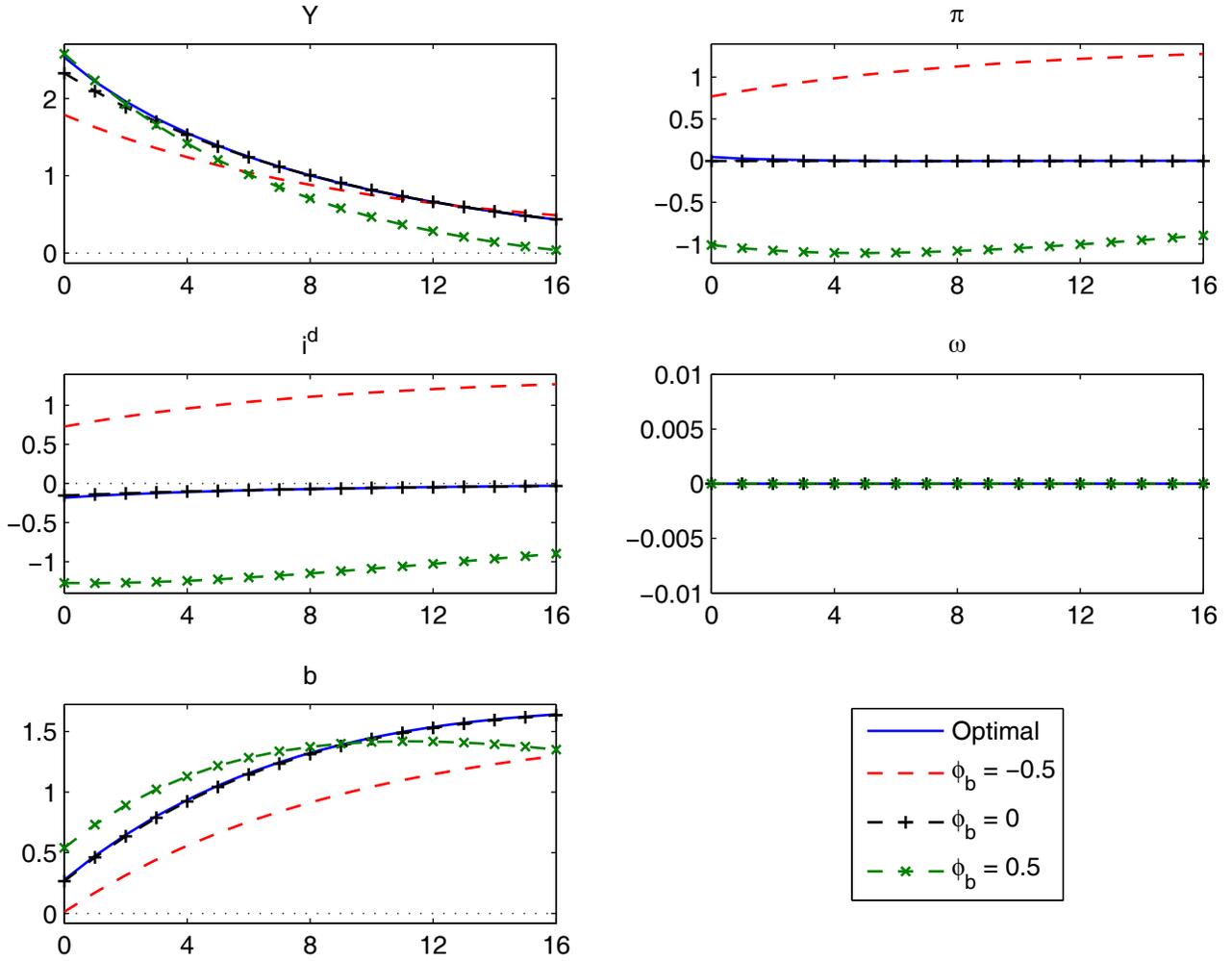


Figure 12: Impulse responses to a 1 percent shock to  $Z_t$ , under alternative degrees of response to aggregate credit, in the case of a linear intermediation technology, as in Figure 11, but when the Taylor rule takes account of changes in the natural rates.