Stock Market Spillovers via the Global Production Network: Transmission of U.S. Monetary Policy*

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Abstract
We quantify the role of global production linkages in explaining spillovers of the U.S. monetary policy shocks to stock returns of 54 sectors in 26 countries. A simple open-economy production network model predicts a spillover pattern consistent spatial autoregression framework, which allows us to decompose the overall impact of the U.S. monetary policy on stock returns into a direct and a network effect. We find that approximately 60% of the total impact of the U.S. monetary policy shocks on average country-sector stock returns are due to the network effect of global production linkages. We further show that U.S. monetary policy shocks have a direct impact predominantly on U.S. sectors and then propagates to the rest of the world through global production network. Our results are robust to extending our sample to 2000-16 and to controlling for other correlates of the global financial cycle.

Keywords: Global production network, asset prices, monetary policy shocks

JEL Codes: G15, F10, F36

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1 Introduction

The recent era of globalization has led to (i) greater cross-country integration via real linkages as firms’ production chains have spread across the world (e.g., Hummels et al., 2001), and (ii) stock markets returns becoming more correlated across countries (e.g., Dutt and Mihov, 2013). We study the relationship between these two phenomena by analyzing the importance of global production linkages in propagating U.S. monetary policy shocks across international financial markets.

We conduct our analysis in three steps. First, we construct a simple open-economy production network model. The model is based on the basic closed-economy setup (Carvalho, 2010; Acemoglu et al., 2012), and introduces money via a cash-in-advanced assumption (Ozdagli and Weber, 2017). The model predicts that the shock transmission pattern follows a spacial autoregression (SAR) process. Next, we construct a novel dataset that combines production linkages information from the World Input-Output Database (WIOD, Timmer et al., 2015) with firm-level stock returns worldwide. Using these data, we document a relationship between production linkages and stock market co-movements across countries at the industrial sector level. Third, using a panel SAR we empirically analyze the transmission of U.S. monetary policy shocks, through the global production network. Our analysis allows us to quantify the role of the global production network in the transmission of shocks across countries’ stock markets.

Using monthly stock return data at the country-sector level, we find that the propagation of the U.S. monetary policy shock through the global production network is statistically significant and accounts for more than half of the total impact of the shock on stock returns. Specifically, average monthly stock returns increase by 0.10 percentage points in response to one percent expansionary surprise in the U.S. monetary policy rate, and 60% this stock return increase is due to the spillover via global production linkages. U.S. monetary shocks are felt most prominently by U.S. domestic sectors. The shocks then propagate from domestic stock returns to stock returns abroad via the global production network. This finding is robust to different time periods and to controlling for other variables that may drive a common financial cycle across markets, such as the VIX, 2-year Treasury rate, and the U.S. dollar Broad Index.

Our theoretical framework is a multi-country production network model, in which firms combine labor with domestic and foreign intermediate goods with decreasing returns to scale technology to produce intermediate and final goods output. All goods are traded with iceberg trade costs creating a wedge between domestic and foreign prices. We follow the simple closed-economy setup of Ozdagli and Weber (2017), where firms are competitive but production has decreasing returns and requires fixed costs, which generates positive profits. Furthermore, we also introduce money into the model

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1 Johnson and Noguera (2012, 2017) show that as trade barriers declined in recent decades, the share of value added in trade dropped, also indicating longer supply chains.

2 Both Cobb-Douglas and CES production technology assumptions give the same qualitative results.
via cash-in-advance constraint and exogenous domestic money supplies in each country. In such a
model, profits of all firms in all countries will be affected by a monetary shock in one country in
proportion to their production linkages with the rest of the firms and the importance of intermediate
products in their production function. Unlike Herscovic (2018), we take the input-output matrix
as given, both in the model and in our empirical analysis. Finally, to close the model we assume
that trade is balanced in each country, that wages are preset, and that sector prices follow the law
of on price cum an iceberg trade cost.

To conduct our regression analysis we make use of the 2016 version of WIOD, which provides
domestic and global input-output linkages for 56 sectors across 43 countries and a rest of the world
aggregate. We obtain firm-level stock prices, market capitalization, and firms’ sector classification
from Thompson Reuters Datastream. Using the market capitalization as a weight, we construct
our own country-sector stock market indexes by aggregating firm-level information to the same in-
dustrial sector level as WIOD for 26 of the countries available in WIOD. The final merged dataset
covers 2000–14, with monthly country-sector stock returns and annual input-output matrices. Finally,
our baseline analysis uses the 30-minute window U.S. monetary policy shocks from Jarociński
and Karadi (2020). Because of the global trade collapse in 2008–09 followed by the period of un-
conventional monetary policy, we limit our analysis to 2000–07 for the baseline analysis. However,
our results are robust to other periods.

By studying raw data we find that country-sector cells that are more closely connected in
the global production network have also more correlated stock returns. This observation remains
true even if we exclude same-country cross-sector correlations from this analysis. This observation
suggests that input-output linkages can be a quantitatively important channel of the financial
shock transmission. Importantly, it appears that markets participants understand the importance
of global production linkages as was demonstrated by larger stock market reaction to Brexit vote
outcome for sectors and firms that relied more heavily on UK-EU trade linkages (Breinlich et al.,
2018; Davies and Studnicka, 2018).

The model predicts a SAR structure for our empirical analysis (LeSage and Pace, 2009), where
the spatial distance is represented by the coefficients in the global input-output matrix. Our
specific case of the SAR, however, is different from a standard one in two ways. First, in addition
to the spatial dimension, country-sector in our case, we have a time dimension. Thus, we have a

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3These countries cover a majority of world production and trade. See Appendix A for details.
4We extend stock returns data through 2016.
5This is consistent with Curcuru et al. (2018) finding that international spillovers of the U.S. monetary policy did
not change substantially during the zero lower bound period.
6Because input-output coefficients do not change much over time, we use a static, beginning-of-period input-output
matrix. We are implicitly assuming that market participants react on the intensive margin of production networks,
rather than to the expected changes in production linkages. This assumption is arguably more justifiable at the sector
than the firm level. However, trade patterns have changed over time, so we also experiment by varying the weighting
matrix for different time periods in our empirical analysis and find that results are not sensitive to these changes.
panel spatial autoregression. Second, the model points us to the SAR with country-sector-specific coefficients on the direct shock impact, while forces outside the model suggest that stock market shock transmission through production linkages may encounter resistance that varies by country and sector. Thus, we estimate country-sector specific coefficients, which is possible thanks to the time dimension in our panel setting. We estimate such a heterogeneous-coefficient panel SAR using maximum likelihood methodology in Aquaro et al. (2019) and approximate standard errors using a wild bootstrap procedure.

We find a very robust and quantitatively important role of the global production network in the transmission of the U.S. monetary policy shocks across countries and sectors. Quantitatively, about 60% the total effect of the U.S. monetary policy shock on global stock returns is due to input-output linkages, while the rest is a direct impact effect. This finding is consistent with Acemoglu et al. (2016) study that shows that the network-based shock propagation can be larger than a direct effect, as well as to what Ozdagli and Weber (2017) find for the response of U.S. stock returns to monetary policy shocks. Both of these studies focus only on the U.S. in a closed-economy setting, while ours incorporates global value chains. By separating the estimates for sectors in the U.S. from those of foreign sectors, we show that the direct impact of the U.S monetary policy shock is mostly affecting U.S. sectoral stock returns. The shocks to U.S. stock returns then propagate through the global production network to stock returns in foreign countries. The magnitude of the direct impact of the U.S monetary policy on foreign stock returns is small and only marginally statistically significant.

Our results are not sensitive to the choice of a specific time period, especially if we exclude 2008 from the sample. We also show that the year at which the input-output matrix is sampled does not affect the result, suggesting very limited, if any, endogenous response of global supply chains to monetary and financial shocks. This result justifies the assumption of an exogenous trade structure in our theoretical framework. Our results are also robust to controlling for correlates of the global financial cycle. While we find that the effect of the U.S. monetary policy shocks is smaller when we control for VIX, the pattern of the shock propagation is the same. Furthermore, while the 2-year Treasury rate and U.S. dollar Broad Index have a statistically significant effect on global stock returns, controlling for these shocks does not affect the impact of the U.S. monetary policy shocks.

Our finding of the quantitative importance of the global production network in international transmission of U.S. monetary policy shocks to global stock returns at the sector level contributes to various strands of literature. The closest is the fast growing literature on the international transmission of shocks through production linkages. For example, Burstein et al. (2008), Bems et al. (2010), Johnson (2014), and Eaton et al. (2016), among others, model and quantify international shock transmission through input trade. Baqae and Farhi (2019b) and Huo et al. (2020) develop theoretical and quantitative treatments of the international input network model. 

Boehm et al.
(2019) and Carvalho et al. (2016) use a case study of the Tōhoku earthquake to provide evidence of real shock transmission through global and domestic supply chains, while di Giovanni et al. (2018) show the importance of firms’ international trade linkages in driving cross-country GDP comovement.

Our paper also contributes to broader literature on international spillovers of financial shocks by documenting and quantifying the importance of real linkages. Most recent papers in this literature focus on bank lending channel (see, among others, Bruno and Shin, 2015b; Avdjiev et al., 2017; di Giovanni et al., 2017) and a survey by Claessens (2017). Much less attention has been devoted to real channels, such as international and domestic input-output linkages. Yet we know that real linkages across sectors play an important role in the domestic shock transmission (see, among others, Carvalho, 2010; Foerster et al., 2011; Acemoglu et al., 2012; Atalay, 2017; Grassi, 2017; Baqaee and Farhi, 2019a). A recent paper by Ozdagli and Weber (2017), to which our paper is most closely related, shows that input-output linkages are quantitatively important for monetary policy transmission in the United States. Bigio and La’O (2019) and Alfaro et al. (2020) show the importance of production linkages in transmitting sectoral shocks and financial frictions to the aggregate economy. We bridge the gap between these literatures by showing the importance of production linkages in the international transmission of financial shocks via real linkages.

Finally, by focusing on the U.S. monetary policy shocks, our paper also contributes to the burgeoning literature on the global transmission of the U.S. monetary policy. This literature shows the importance of the U.S. monetary policy in driving global financial cycle, beginning with Miranda-Agrippino and Rey (2020), which showed that U.S. monetary policy shocks induce comovements in international equity markets. More recently, a number of papers, including Iacoviello and Navarro (2019), demonstrated that U.S. interest rates affect real economic activity in foreign countries. Most analysis of the spillover channels focus on bank lending and, more generally, global bank activity (see, among others, Cetorelli and Goldberg, 2012; Bruno and Shin, 2015b; Avdjiev et al., 2018; Temesvary et al., 2018; Buch et al., 2019; Morais et al., 2019). Another large group of papers study, more generally, the impact of the U.S monetary policy on international capital flows (see, among others, Bruno and Shin, 2015a; Burger et al., 2015; Avdjiev and Hale, 2019). Our paper adds to this literature by showing, on the global scale, the importance of the trade channel in transmitting the U.S. monetary policy shocks, and providing a quantitative estimate of its contribution as well as transmission pattern: from U.S. monetary policy directly to domestic stock returns and through production network to the rest of the world.

We present a stylized global production model with cross-country monetary policy shock transmission in Section 2, which motivates the empirical model outlined in Section 3. We then describe

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our data in Section 4, before presenting our empirical results in Section 5. Section 6 concludes.

2 Theoretical Framework

In this section we provide a simple framework to motivate our estimation strategy for studying the transmission of U.S. monetary policy shocks to stock returns internationally via production linkage. The core model is based on the static closed-economy model of sectoral linkages of Carvalho (2010) and Acemoglu et al. (2012). In addition, we incorporate three features in order to study the impact of monetary policy shocks on stock returns, as in Ozdagli and Weber (2017): (i) firms produce with decreasing returns to scale and face fixed costs of production, (ii) wages are preset and do not adjust given monetary shocks, and (iii) consumers have cash-in-advanced constraints.

We take the technology and trade structure as fixed since we are studying the short run. We make two further assumptions to solve the model analytically. First, we assume that trade is balanced across countries. Second, we assume that prices in a given sector are equal across countries after adjusting for an iceberg trade cost, which varies at the sector and country-pair level.

The world is comprised of $N$ countries and $J$ sectors. Countries are denoted by $m$ and $n$, and sectors by $i$ and $j$. The notation follows the convention that for trade between any two country-sectors, the first two subscripts always denotes exporting (source) country-sector, and the second subscript the importing (destination) country-sector.

2.1 Model Setup

Households. There is a representative household in each country $n$, which consumes a bundle of goods across all sectors $i$ produced across countries $m$, and supplies labor in country $n$, $l_n$. Its maximization problem is

$$\max_{\{c_{mi,n}\}, l_n} \sum_{i=1}^{J} \sum_{m=1}^{N} b_{mi,n} \log c_{mi,n} - l_n$$

s.t.

$$\sum_{i=1}^{J} \sum_{m=1}^{N} p_{mi,n} c_{mi,n} = w_n l_n + \pi_n + f_n,$$

where $b_{mi,n}$ is a preference parameter for which we assume $\sum_{i=1}^{J} \sum_{m=1}^{N} b_{mi,n} = 1$. Besides wage income, the domestic household’s income includes aggregate profits, $\pi_n$ and aggregate fixed costs, $f_n$, which firms must pay to produce. Note that in writing the budget constraint we assume balanced trade, because in each country total income is equal to total consumption. We also assume that aggregate labor supply, profits, and fixed costs are additive across sectors: $l_n = \sum_{j=1}^{J} l_{nj}$, $\pi_n = \sum_{j=1}^{J} \pi_{nj}$, $f_n = \sum_{j=1}^{J} f_{nj}$.
The first-order conditions are:

\[
\frac{b_{mi,n}}{c_{mi,n}} = \theta p_{mi,n} \quad \forall \, mi,n \\
\theta = \frac{1}{w_n},
\]

where \(\theta\) is the Lagrange multiplier. Combining the two FOCs we have:

\[
b_{mi,n}w_n = p_{mi,n}c_{mi,n} \quad \forall \, mi,n.
\]

Technology. There are \(j = 1, \ldots, J\) sectors in each country \(n = 1, \ldots, N\). Firms in country-sector \(nj\) face the following Cobb-Douglas production function:

\[
y_{nj} = z_{nj} l_{nj}^{\alpha_{nj}} X_{nj}^{\lambda_{nj}},
\]

where \(z_{nj}\) is a Hicks-neutral technology term, \(l_{nj}\) is labor, \(X_{nj}\) is a composite intermediate good, and \(\alpha_{nj} + \lambda_{nj} < 1\) implying decreasing returns to scale. Given our focus on monetary policy shocks, we simplify notation by assuming that \(z_{nj} = 1 \forall \, nj\).

The composite intermediate good is a Cobb-Douglas aggregate of intermediate goods sourced both domestically and abroad from all sectors. Specifically:

\[
X_{nj} = \prod_{i=1}^{J} \prod_{m=1}^{N} x_{mi,nj}^{\omega_{mi,nj}},
\]

where \(x_{mi,nj}\) is the amount of sector \(i\)'s good produced in country \(m\) used by country-sector \(nj\) in final production, and \(\omega_{mi,nj}\) is the associated input-output coefficient for country-sector \(nj\) usage of the intermediate good from country-sector \(mi\) in the aggregate intermediate good, where \(\sum_{i=1}^{J} \sum_{m=1}^{N} \omega_{mi,nj} = 1\).

Given a competitive market structure with wages preset and prices taken as given by each firm, profit maximization for country-sector \(nj\) is

\[
\max_{l_{nj},\{x_{mi,nj}\}} p_{nj} y_{nj} - \sum_{i=1}^{J} \sum_{m=1}^{N} p_{mi,n} x_{mi,nj} - w_n l_{nj} - f_{nj} \quad \text{s.t.} \quad (4), (5),
\]

where \(p_{nj}\) is the price of the good produced by sector \(j\) in country \(n\), \(\{p_{mi,n}\}\) is a vector of prices of goods sold in country \(n\), \(w_n\) is the wage in country \(n\), and \(f_{nj}\) is a fixed cost of production. We do not model these costs but they may include access to credit or bureaucratic costs, for example. Further, we do not differentiate between fixed costs of production and fixed costs of accessing foreign markets, as is common in the international trade literature.

\[8\text{We have also solved the model assuming a CES production structure in labor and the aggregate intermediate good, as well as as CES aggregator underlying intermediate goods. The main results needed to motivate the empirical approach setup do not change qualitatively. The model solution is available upon request.}\]
Solving the maximization problem we arrive at the following sets of first-order conditions for any given country-sector pair:

\[ \lambda_{nj} \omega_{mi,nj} R_{nj} = p_{mi,n} x_{mi,nj}, \]  
\[ \alpha_n R_{nj} = w_n l_{nj}, \]

where total revenue \( R_{nj} = p_{nj} y_{nj} \).

We combine the two FOCs with the profit function to solve for profits as a function of total revenue and the fixed costs:

\[ \pi_{nj} = (1 - \lambda_{nj} - \alpha_n) R_{nj} - f_{nj}. \]

**Goods Market Clearing.** Global goods market clearing condition for any good \( m_i \) is given by

\[ y_{mi} = \sum_{n=1}^{N} c_{mi,n} + \sum_{j=1}^{J} \sum_{n=1}^{N} x_{mi,nj}, \]

where the first term captures final consumption of good \( m_i \) across \( n \) destination countries, and the second term captures intermediate consumption across \( nj \) country-sector destinations. To simplify the market clearing condition we first use the household first-order condition (1) and its budget constraint to express consumption as

\[ c_{mi,n} = \frac{b_{mi,n} \sum_{j=1}^{J} (1 - \lambda_{nj}) p_{nj} y_{nj}}{p_{mi,n}}. \]

We then use (6) and (10) to express (9) as

\[ y_{mi} = \sum_{j=1}^{J} \sum_{n=1}^{N} b_{mi,n} (1 - \lambda_{nj}) R_{nj} + \sum_{j=1}^{J} \sum_{n=1}^{N} \lambda_{nj} \omega_{mi,nj} R_{nj}. \]

Next, multiplying (11) by \( p_{mi} \), and assuming iceberg trade costs \( \tau_{mi,n} \) that vary by sector and country pair \( (p_{mi,n} = \tau_{mi,n} p_{mi}, \text{where } \tau_{mi,n} \geq 1) \), we express revenues in country-sector \( m_i \) as:

\[ R_{mi} = \sum_{j=1}^{J} \sum_{n=1}^{N} \frac{b_{mi,n} (1 - \lambda_{nj})}{\tau_{mi,n}} R_{nj} + \sum_{j=1}^{J} \sum_{n=1}^{N} \frac{\lambda_{nj} \omega_{mi,nj}}{\tau_{mi,n}} R_{nj}. \]

The above equation characterizes a recursive relationship between sectors’ revenues across countries, as well as the role of different parameters in the model. Note that we are implicitly assuming that these revenues are denominated in a common currency. While we do not incorporate the exchange rate explicitly in this framework, we address this issue in our regression analysis.

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\( ^9 \)Note that \( \tau_{mi,n} \) may differ depending on the direction of trade; i.e., \( \tau_{mi,n} \) need not equal \( \tau_{ni,m} \). However, given our empirical definition of trade costs described in Section 4.1, the constructed trade costs are in fact symmetric and are equal to one for trade within the same country.
Stacking (12) across country-sectors leads to a matrix formulation of the global system of country-sector revenues:

\[(I - \tilde{\Omega}\Lambda)\mathbf{R} = \sum_{j=1}^{J} \sum_{n=1}^{N} \frac{b_{mi,n}(1 - \lambda_{nj})}{\tau_{mi,n}} R_{nj},\]  

where \( \mathbf{R} \equiv (R_{11}, \ldots, R_{NJ})', \) \( \mathbf{R} \in \mathbb{R}^{NJ \times 1}, \)

\( \Lambda \equiv \text{diag} \{\{\lambda_{nj}\}\}, \)

\( \Omega \equiv \begin{pmatrix} 
\omega_{11,11} & \cdots & \omega_{11,NJ} \\
\vdots & \ddots & \vdots \\
\omega_{NJ,11} & \cdots & \omega_{NJ,NJ} 
\end{pmatrix}, \)

\( \tilde{\tau} \equiv \begin{pmatrix} 
\left(\frac{1}{\tau_{11,1}}\right) \circ 1_{1 \times J} & \cdots & \left(\frac{1}{\tau_{11,N}}\right) \circ 1_{1 \times J} \\
\vdots & \ddots & \vdots \\
\left(\frac{1}{\tau_{NJ,1}}\right) \circ 1_{1 \times J} & \cdots & \left(\frac{1}{\tau_{NJ,N}}\right) \circ 1_{1 \times J} 
\end{pmatrix}, \)

\( \tilde{\Omega} \equiv \tilde{\tau} \circ \Omega, \)

where \( \circ \) represents the Hadamard product, and \( \Omega \) is the global input-output matrix, where each element of the matrix, \( \omega_{mi,nj} \), is the associated input-output coefficient for country-sector \( nj \) usage of the intermediate good from country-sector \( mi \) in \( nj \)'s aggregate output.

Money Supply. We introduce money by assuming that consumers face a cash-in-advance constraint as in Ozdagli and Weber (2017); they justify this approach by assuming that firms enter into trade credit relationships, and thus there is no such constraint in the trade of intermediate goods.\(^{10}\) Specifically, for a given economy \( n \) total final consumption is given by

\[
\sum_{i=1}^{J} \sum_{m=1}^{N} p_{mi,n} c_{mi,n} = \sum_{i=1}^{J} \sum_{m=1}^{N} b_{mi,n} \sum_{j=1}^{J} (1 - \lambda_{nj}) R_{nj} = \mathcal{M}_n,
\]

where \( \mathcal{M}_n \) is the domestic money supply in country \( n \) and we again see the result of our assumption of balanced trade. Recalling that \( \sum_{i=1}^{J} \sum_{m=1}^{N} b_{mi,n} = 1 \), we can re-write the cash-in-advance constraints for country \( n \) as

\[
\sum_{j=1}^{J} (1 - \lambda_{nj}) R_{nj} = \mathcal{M}_n. \tag{14}
\]

We can next substitute (14) into (13) to arrive at

\[(I - \tilde{\Omega}\Lambda)\mathbf{R} = \tilde{b}\mathcal{M}, \tag{15}\]

\(^{10}\)This assumption may be more tenuous in the open-economy context given potential frictions in international trade credit. Given the differences in these frictions across sectors and countries, they are partly incorporated in our iceberg trade costs (Antrás and Foley, 2015; Caballero et al., 2018; Niepmann and Schmidt-Eisenlohr, 2017). The remaining part, not reflected in the model, gives us heterogeneity across countries and sectors in our regression analysis.
where $\tilde{b}$ is a $NJ \times N$ matrix composed of elements $\{\tilde{b}_{mi,n}\}$, where $\tilde{b}_{mi,n} \equiv \frac{b_{mi,n}}{\bar{y}_{mi,n}}$, and $\mathcal{M} \equiv (M_1, \ldots, M_N)'$.

2.2 Network Effects of Money Shocks on Global Stock Returns

To determine the impact of money shocks on global stock returns we will examine deviations of firm/sector profits around their deterministic steady state and only consider a shock to the money supply of one country $n$ (the U.S.).\(^{11}\)

In particular, for any variable $x$, define the log deviation from steady-state $\tilde{x} = \log(x) - \log(\bar{x})$ so that $x = \bar{x} \exp(\tilde{x}) \approx \bar{x}(1 + \tilde{x})$, where $\bar{x}$ is the steady-state value of $x$. Further define $\pi$ to be a $NJ \times 1$ vector composed of elements $\{\pi_{mi}\}$, $\lambda$ to be a $NJ \times 1$ vector composed of elements $\{\lambda_{mi}\}$, $\alpha$ to be a $NJ \times 1$ vector composed of elements $\{\alpha_{mi}\}$, and $f$ to be a $NJ \times 1$ vector composed of elements $\{f_{mi}\}$. We can stack country-sector profits in (8) to express them as:

$$\pi = (1 - \lambda - \alpha) \circ R - f.$$ \hspace{1cm} (16)

Log-linearizing (16) and using (15), we arrive at:

$$\hat{\pi} = \left( I - \bar{\Omega} \Lambda \right)^{-1} \beta \tilde{\mathcal{M}},$$ \hspace{1cm} (17)

where $\beta \equiv \text{diag}\left(\left\{\frac{1-\lambda_{nj}}{\pi_{nj}} \tilde{b}_{mi,n}\right\}\right)$ is a $NJ \times N$ matrix.

Allowing for shocks only to the U.S. monetary supply, we can write (17) as

$$\hat{\pi} = \left( I - \bar{\Omega} \Lambda \right)^{-1} \beta_{US} \tilde{\mathcal{M}}_{US},$$ \hspace{1cm} (18)

where $\beta_{US} \equiv \text{diag}\left(\left\{\frac{1-\lambda_{USj}}{\pi_{USj}} \tilde{b}_{mi,US}\right\}\right)$ is a $NJ \times 1$ vector.

3 Regression Framework

Under the efficient markets hypothesis, a change in stock returns reflects expected change in profits. Thus, the model predicts that a monetary policy shock affects all stock returns in the amount proportional to their input-output distance (scaled by trade costs) from the source of the shock. The empirical counterpart to this propagation pattern is a spatial autoregression.

Specifically, holding the parameters of the model fixed, and defining $\mathbf{W} \equiv \bar{\Omega} \Lambda$, the empirical counterpart of Equation (18) for a given country-sector observation is

$$\pi_{mi,t} = (I - \rho \mathbf{W})^{-1} \beta_{mi} \hat{\mathcal{M}}_{US,t},$$ \hspace{1cm} (19)

\(^{11}\)In equating stock returns with changes in profits, we apply efficient market hypothesis. Appendix B derives the solution for real output, both in the flexible-wage and in sticky wage equilibria.
where the subscript $t$ is for the year-month in which a monetary policy shock occurs. $\rho$ and $\beta_{mi}$ are coefficients that will be estimated. While we can derive $\beta_{mi}$ from the model, we cannot measure it directly. Moreover, the estimate of $\beta_{mi}$ can be affected by factors that are outside of the scope of the model, such as financial openness, level of financial developments, sector’s dependence on external financing, and institutional factors. Such factors may also add resistance to the shock transmission through the production network. While the system of equations (18) predicts the pass-through of monetary policy shocks to stock returns perfectly ($\rho = 1$), this need not be the case in practice, which is why we let the data determine the empirical estimate of $\rho$.

Equation (19) is a representation of a spatial autoregressive process, and can be written in the following vector form:

$$\pi_t = \beta \hat{M}_{US,t} + \rho W \pi_t,$$

(20)

or, adding an error term,

$$\pi_t = \beta \hat{M}_{US,t} + \rho W \pi_t + \varepsilon_t,$$

(21)

where $\rho$ is the spatial autoregressive coefficient, and $\beta$ is a vector of $\beta_{mi}$'s.

To allow for barriers to shock propagation to vary across sectors and countries, we extend the SAR model to allow for heterogeneity in the autoregressive coefficient. In particular, like $\beta$, we can allow $\rho$ to vary at the $mi$ level:

$$\pi_t = \beta \hat{M}_{US,t} + \rho W \pi_t + \varepsilon_t,$$

(22)

where $\beta$ and $\rho$ are $NJ \times 1$ vectors of the coefficients to estimate and $\varepsilon$ is the $NJ \times 1$ vector of error terms. The time dimension of our data allows us to estimate individual parameters for every country-sector pair.

**Additional Controls.** The panel SAR model (22) can be extended to include additional controls:

$$\pi_t = \beta_1 \hat{M}_{US,t} + \beta_2 X_t + \rho W \pi_t + \varepsilon_{mi,t},$$

(23)

where $X_t$ is matrix of additional independent variables. This specification assumes that additional shocks may also impact stock returns both directly and via the global input-output matrix. We use this specification to examine the robustness of results by including variables related to the global financial cycle that have been found to both be correlated with U.S. monetary policy shocks and drive global asset prices.

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$^{12}$FOMC announcements do not occur every month, and at times multiple times within a month. We only include in our sample months with FOMC announcements, but the results are robust to including all months. For months with multiple announcements, we aggregate all announcement by adding up measures of monetary policy shock.
Inference. Because of the recursive nature of the spatial autoregression model, coefficient \( \beta \) is not equal to the marginal impact of the monetary shock \( \hat{\mathcal{M}}_{US,t} \) on stock returns \( \hat{\pi}_{mi,t} \). Instead, from (19), the \( NJ \times 1 \) vector of marginal effects is given by

\[
\text{Total} = (I - \rho W)^{-1} \beta.
\]

Following LeSage and Pace (2009) this marginal effect for each \( mi \) can be decomposed into a direct effect of the shock and the network effect as

\[
\text{Direct} = \text{diag}(I - \rho W)^{-1} \beta, \quad (25)
\]

\[
\text{Network} = \text{Total} - \text{Direct} \quad (26)
\]

where Direct, Network are \( NJ \times 1 \) vectors.

Reporting and standard errors. We present our results by reporting simple average values of \( \beta, \rho \), direct, and network effects across all country-sectors. We also examine the cross-country transmission of monetary policy shocks by splitting the effects into the domestic and international components. Specifically, we compute international direct and network effects as averages of elements of Direct and Network across all the non-U.S. country-sectors. We take averages of elements of Direct and Network over only U.S. sectors in order to compute the U.S.-only direct and network effects.

We compute standard errors for each element of \( \beta, \rho \), Direct, and Network as well as their overall, international, and U.S. average values using a wild bootstrap procedure proposed by Mammen (1993). To do so, for each iteration \( k \) of the 500 repetitions we replace our dependent variable with a synthetic one that is equal to the fitted values from the main estimation plus a random perturbation \( \nu \) of the fitter error term:

\[
\hat{\pi}_{mi,t}^k = \hat{\beta}_{mi} \hat{\mathcal{M}}_{US,t} + \hat{\rho}_{mi} W \pi_t + v_{mi,t}^k \varepsilon_{mi,t}.
\]

We use continuous distribution from which we draw perturbations

\[
v_{mi,t}^k = \frac{u_{mi,t}}{\sqrt{2}} + \frac{1}{2} \left[ (v_{mi,t}^k)^2 - 1 \right],
\]

where \( u \) and \( v \) are drawn from independent standard normal distributions. We then estimate our regression model replacing true dependent variable with synthetic one and retain estimation results. Standard deviations of each estimated parameter across 500 repetitions are reported as standard errors.
4 Data

We source data from two main datasets: the global production network are from the World Input-Output Database (WIOD), and the stock market information is from Thompson-Reuters Datastream (TREI). The WIOD provides annual data for input-output linkages across 56 sectors and 43 countries and a rest of the world aggregate for 1996–2014. For our analysis, we limit the data to 26 countries with active stock markets and 54 sectors that are connected to each others.\footnote{The remaining two sectors, household production (“T” in WIOD codes) and extraterritorial organization (“U”) are not sufficiently connected to the rest of the network.}

From TREI, we obtain end-of-period monthly stock prices, stock market capitalization, and industrial classification for individual companies. We then construct our own stock return indexes for the same sector definitions as used in WIOD, using stock market capitalization of the firm as a weight. This is not straightforward, given that the TREI sector classification is using Thomson Reuters Business Classification (TRBC), while the World Input-Output Tables are constructed under ISIC Revision 4. Fortunately, in addition to TRBC, TREI also reports NAICS 2007 sector codes for each firm, which we use to create a crosswalk to ISIC 4. This then allows us to aggregate firms’ stock market indices into WIOD-based sectors.\footnote{Even with these data, there is not always 1-to-1 correspondence between the TREI and WIOD codes, and we rectify such instances in a variety of ways as described in Appendix A.}

For each of the resulting country-sector cells we construct monthly stock returns as a log change in weighted average of stock prices of all firms in that country-sector cell.

Table A1 presents cross-country sector coverage of monthly returns for the months where there are monetary surprise shocks over 2000–14. Given cross-country differences in size, industrial specialization patterns, and stock market depth we see that larger countries (e.g., the United States) have a larger coverage of sectors, while some countries only cover a few sectors (e.g., Portugal and Russia). These differences motivate a flexible empirical approach, where we allow for country-sector fixed effects as well as country-sector specific coefficients for the effect of monetary policy surprise variable.

4.1 Input-Output Coefficient Construction

The construction of the global input-output matrix using WIOD data is standard and follows from the literature. Denote countries as $m, n \in [1; N]$ and sectors as $i, j \in [1; J]$. WIOD provides information of output produced in a given country-sector and where it flows to – both geographical and what sector of the economy (including government and households). We first use this information to build a matrix $W$, which is $NJ \times NJ$, where each element $w_{mi,nj}$ represents the use of inputs from country $m$ sector $i$ as a share of total output of sector $j$ in country $n$:

$$ w_{mi,nj} = \frac{Sales_{mi \rightarrow nj}}{Sales_{nj}}. $$
In network terminology, $W$ is the adjacency matrix that gives us direct linkages between each pair of country-sector cells. Because by construction $w_{mi,nj} \in [0;1]$ and $w_{mi,nj} \neq w_{nj,mi}$, the network is weighted and directed. Note that we use all countries and sectors when constructing the adjacency matrix, but only exploit the sub-matrix where we have stock returns in the estimation below. This requires a re-normalization of the matrix for estimation purposes, but all preliminary statistics are based on manipulating the adjacency matrix without this re-normalization.

The $W$ matrix differs from the model-based input-output matrix, $\Omega$, because $\Omega$ is constructed using sectors’ total input usage rather than total sales. In particular, each element of the matrix $\Omega$ is

$$\omega_{mi,nj} = \frac{Sales_{mi \rightarrow nj}}{Input_{nj}} = \frac{w_{mi,nj}}{\lambda_{nj}},$$

where recall that $\lambda_{nj}$ is a country-sector’s input share used in production. In other words, $W = \Omega \Lambda$, which is the theoretical and empirical weighting matrices (ignoring trade costs) in (18) and (19).

Figure 1 presents the empirical counter cumulative distribution function (CCDF) of the weighted outdegree of $W$ for WIOD data, where we use the average input-output coefficients over the sample period 2000–14. The weighted distribution for a given country-sector pair $mi$ is defined as:

$$out_{mi} = \sum_{n=1}^{N} \sum_{j=1}^{J} w_{mi,nj}.$$

The weighted outdegree measures how important a given country-sector’s inputs are for production use across all possible country-sector pairs. It is informative to look at this distribution, since a skewed one implies the potential for shocks to propagate and amplify across the production network (Acemoglu et al., 2012). Panel (a) plots the distribution using all possible input-output linkages in the world including both domestic and international linkages in computing the weighted outdegree, while panel (b) exploits only the international linkages. As can be seen in both figures, the distributions are very skewed. The curves were fitted using a Pareto distribution and as can be seen the slopes of the tail are steep, implying that the distributions are fat-tailed. This finding is along the lines of what Carvalho (2014) shows for the U.S. economy using detailed input-output tables from the BEA. In comparing panels (a) and (b), it’s worth noting that the x-axis are on two different scales. In particular, the international weighted outdegree measures tend to be smaller on average than those using the full world input-output table (which includes domestic linkages) as several country-sector cells are not used as intermediate inputs (or in very tiny amounts) abroad.

**Trade Costs.** We construct a matrix of trade costs using the methodology of Head and Ries (2001), which relies on observed trade flows. In particular, the index is constructed based on total trade – intermediate and final consumption goods – for a given sector between two countries. The Head-Reis index is a bilateral measure that imposes symmetry in trade costs between countries.
Figure 1. Distribution of Weighted Outdegree for WIOD

Notes: This figure plots the counter cumulative distribution function of the weighted outdegree using the average of the WIOD annual database over 2000–14. The panel with World Linkages is based on the full WIOD table, while the International Linkages panel uses only internationally connected country-sector cells (i.e., we omit the domestic-only linkages across sectors) in constructing the weighted outdegree measure.

Specifically, using the notation from Section 2, we define bilateral iceberg trade costs of good \( i \) between countries \( m \) and \( n \) as

\[
\tau_{mi,n} = \sqrt{\frac{X_{mi,n} \times X_{ni,m}}{X_{mi,m} \times X_{ni,n}}},
\]

where \( X_{mi,n} \) is \( m \)'s exports to \( n \) of good \( i \), and \( X_{mi,m} \) is \( m \)'s internal trade of good \( i \). Similarly for exports from country \( n \).

We calculate \( \tau_{mi,n} \) for every country-sector pair in WIOD and create trade cost matrix \( \tau \), which we adjust the input-output matrix (\( W \)) by to create the final weighting matrix for the spatial autoregressions. We use the WIOD trade data for the sample period in constructing both the \( \tau \) and \( W \) matrices. Further, note that to eliminate some outliers in \( \tau \), we winsorize the final sample matrix at the one percent level.

4.2 Returns Data

We next explore our data and show that there is a relationship between stock return correlations and input-output linkages. As described previously, a unit of observation in our data is monthly stock returns in country \( m \) and sector \( i \) in a given month. Because not all sectors are present in all countries, we have stock indexes for 671 out of possible 1404 country-sector cells for each month from January 2000 through December 2014.\(^{15}\) Figure 2 presents the distribution of pairwise

\(^{15}\)Recall that we have potentially a maximum of 54 sectors and 26 countries.
Figure 2. Correlation of Stock Returns over the Entire Sample

Notes: This figure plots the distribution of pairwise correlations of monthly stock returns over 2000–14 across 26 countries and 54 sectors.

correlations between each possible pair of the 671 time series of stock returns. We can see that most correlations are positive and that the mass of the distribution is between 0 and 0.5.

Returns and the Input-Output Network. Our main goal is to explore whether these stock market correlations are associated with production linkages. To do so, we first compute a measure of distance between each pair of cells. The concept of distance is better defined for binary networks. Thus, for illustrative purposes, we replace \( w_{mi,nj} < 0.05 \) with 0, and the rest of the cells with 1, converting our network into binary one. In a such a network, the distance between two cells is defined as the length of the shortest path (geodesic).

We this concept of for each pair of country-sector cells and compare it to the correlation of stock returns for this pair of country-sector cells. Figure 3 plots this relationship. Even though the diameter, the longest distance, of the input-output network averaged over time is 23, we only plot distances up to 8 because for any distances longer than that the decline in stock price correlation levels off.

Panel (a), which uses the full set of country-sector cells, we can see that pairs most closely connected through input-output linkages exhibit the highest correlation of stock returns (correlation coefficient of 0.45). The larger is the distance, the lower is the correlation. We can see that it tapers out just below 0.25 for any distance over 4. Panel (b) shows that similar patter holds when we
Figure 3. WIOD Network Distance and Correlation of Stock Returns: Supplier Linkages

Notes: This figure plots correlations of monthly stock returns over 2000–14 across 26 countries and 54 sectors on the y-axis, across network distance bins based on the direct bilateral supply linkage using the average of the WIOD annual database over 2000–14. The elements of IO matrix are defined as country-sector $m_i$’s usage of country-sector $n_j$’s good as an intermediate divided by $m_i$’s gross output. The panel with World Linkages is based on the full WIOD table, while the International Linkages panel extracts the correlation and distance variable for only internationally connected country-sector cells (i.e., we omit the domestic-only linkages across sectors).

exclude from the analysis all domestic sector pairs are omitted. This alleviates a concern that our results are driven entirely by domestic input-output linkages and stock return correlations. We can see that even excluding domestic linkages, the country-sector cells that are most highly connected exhibit a strong correlation of stock returns (correlation coefficient of 0.33).

These two figures provide prima facie evidence that two sectors which rely more heavily on each other for the supply of inputs in productions also have more strongly correlated stock returns. However, these bilateral correlations may be driven by other transmission channels and are silent how shocks are transmitted via the overall network.

4.3 Monetary Policy Shocks and Global Financial Cycle correlates

Our baseline measure of U.S. monetary policy shocks is sourced from Jarociński and Karadi (2020). They construct a measure of an interest rate surprise as the change in the 3-month federal funds future rate, which they interpret as the expected federal funds rate following the next policy meeting. The change in the futures rate is calculated in the 30-minute window around the time of the FOMC press release, which is 2pm East Coast time on the day of a regular FOMC meeting.\textsuperscript{16}

We explore robustness to controlling for other correlates of the global financial cycle, namely VIX, 2-year U.S. Treasury rate, and Broad Dollar Index. VIX is obtained from Federal Reserve

\textsuperscript{16}This measure of monetary surprise shocks is common in the literature, and follow the work of Gertler and Karadi (2015).
5 Empirical Results

5.1 OLS results

To establish a benchmark, we estimate a simple OLS regression that ignores any spatial network effects:

\[ \pi_{mi,t} = \alpha + \beta_{OLS} \tilde{M}_{US,t} + \varepsilon_{mi,t}, \]  
(27)

where \( \alpha \) represents either a constant or different sets of fixed effects.

The results of the estimation for the Jarociński and Karadi (2020) shock for 2000–07 sample period are reported in Table 1.\(^{17}\) The simple OLS estimate in column (1) implies that a one percentage point surprise in the monetary policy shock results in a 0.1 percentage points rise in the average country-sector monthly stock return. The standard errors increase substantially when we cluster them at the monthly (\( t \)) level, as reported in column (2), which should be expected given that the monetary policy shock is being repeated for each country-sector return in a given time period of the panel. The magnitude of the effect does not change much whether we control for country, sector, or country-sector fixed effects (column (3)).\(^{18}\) We use the country-sector fixed effect specification as our benchmark for linear regression.

Keeping in mind that the model predicts different \( \beta \)'s for each \( mi \), we also allow for the \( \beta \)'s to vary across country-sectors. This is possible because of the time dimension of our data. First, we estimate a random coefficients model with \( \beta \)'s varying across country-sector panels. We find that the coefficient estimate declines slightly, as shown in column (4). Second, we use a Mean Group estimator (Pesaran and Smith, 1995) with groups defined as country-sector. In this case, the average \( \beta \) is nearly identical to the OLS estimate (column (5)).

Finally, we aggregate stock returns returns at the country level and estimate a country fixed effects linear regression, reported in column (6). We find that the coefficient for this country-time panel specification is slightly larger.

Table 2 reports the same sets of regressions, splitting the samples to all foreign countries (Panel A) and only the United States (Panel B). The overall point estimate for the international sample in similar to the baseline estimates using the whole sample of Table 1. However, the point estimates for the United States (Panel B) are substantially larger. The fixed effect coefficient in column

\(^{17}\)The results for other monetary shock measures and other time periods are nearly identical and can be obtained from the authors upon request. The exception is including 2008, which lowers the magnitude of the effect. Because the dependent variable is stock return, including lagged dependent variable in these regression does not alter the results.

\(^{18}\)Only results with country-sector fixed effects are reported.
Table 1. Linear Regression Estimation Results, Full Sample

\[
\pi_{mi,t} = \alpha + \beta_{OLS} M_{US,t} + \varepsilon_{mi,t}
\]

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP shock</td>
<td>-0.102***</td>
<td>-0.102**</td>
<td>-0.103***</td>
<td>-0.083***</td>
<td>-0.098***</td>
<td>-0.136***</td>
</tr>
<tr>
<td>(\beta_{OLS})</td>
<td>(0.008)</td>
<td>(0.044)</td>
<td>(0.044)</td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.010***</td>
<td>0.010**</td>
<td>0.010*</td>
<td>0.010***</td>
<td>0.010***</td>
<td>0.010*</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.005)</td>
</tr>
</tbody>
</table>

Estimator       | OLS   | OLS   | LS    | Random coeffs | Mean Group | LS - country m |
Fixed effects    | None  | None  | mi    | Random        | mi          | m               |
St. errors       | Regular | Clustered on t | Conventional | Group-specific | Clustered on t |                 |

Notes: This table reports coefficients from linear regressions where the dependent variable \(\pi_{mi,t}\) is the country-sector monthly stock return (country average in column (6)) over 2000-07 in month with FOMC announcements, and the independent variable \(M_{US,t}\) is the measure of the monetary policy shock taken from Jarociński and Karadi (2020). There are 49,667 observations in columns (1)-(5), and 1,716 observations in column (6). Standard errors are in parentheses with *, **, and *** denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

(3) implies that a one percentage point surprise in monetary loosening is associated with a 0.17 percentage point increase in the average monthly returns across U.S. sectors.\(^{19}\)

The OLS estimation does not allow for the network structure and therefore \(\beta_{OLS}\) combines both direct and network effects. We next turn to spatial autoregression (SAR) to be able to measure these two effects separately.

5.2 SAR results

We now allow for network effects by estimating spatial autoregression model (SAR). Effectively, it removes the restriction, imposed by OLS, of independent panels, i.e. \(\rho = 0\) in Equation (21).

The baseline results of the estimation of the spatial autoregression model with heterogeneous coefficients Equation (22) are presented in Table 3. We allow for country-sector fixed effects following Elhorst (2014). We estimate the regression with maximum likelihood and bootstrap standard errors for all parameters as well as for decompositions, using wild panel bootstrap with 500 repetitions.

Panel A of Table 3 shows the average values of \(\beta\), \(\rho\), Direct, Network, and share of Network in Total across country-sectors. We report averages across all country-sectors, for country-sectors outside of the U.S., and for the U.S. sectors only. The full distribution of these estimates are reported in Figure ???. In addition to our benchmark, which accounts for trade costs \(\tau\), we estimate an alternative specification, in which \(\tau\) is set to one. Effectively, the second specification uses

\(^{19}\)Note that this point estimate is substantially smaller than the implied impact in Ozdagli and Weber (2017). We believe this is due to higher level of aggregation in our data (fewer industries) and possibly attenuation due to our use of monthly frequency data, rather than looking at the returns around the 30-minute window of the FOMC announcement.
Table 2. Linear Regression Estimation Results, International and United States Sub-Samples

\[ \pi_{mi,t} = \alpha + \beta_{OLS} \tilde{M}_{US,t} + \varepsilon_{mi,t} \]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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<th>(3)</th>
<th>(4)</th>
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<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Excluding the U.S.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP shock ( \beta_{OLS} )</td>
<td>-0.097***</td>
<td>-0.097**</td>
<td>-0.134**</td>
<td>-0.076***</td>
<td>-0.092***</td>
<td>-0.134***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.045)</td>
<td>(0.045)</td>
<td>(0.012)</td>
<td>(0.010)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.010 ***</td>
<td>0.010 **</td>
<td>0.010 *</td>
<td>0.010 ***</td>
<td>0.010 ***</td>
<td>0.010 *</td>
</tr>
<tr>
<td></td>
<td>(0.000)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Panel B: U.S. only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP shock ( \beta_{OLS} )</td>
<td>-0.171***</td>
<td>-0.171***</td>
<td>-0.171***</td>
<td>-0.156***</td>
<td>-0.179***</td>
<td>-0.173***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.040)</td>
<td>(0.040)</td>
<td>(0.029)</td>
<td>(0.023)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.007***</td>
<td>0.007*</td>
<td>0.007***</td>
<td>0.007***</td>
<td>0.007***</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.004)</td>
<td>(0.004)</td>
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<table>
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<tr>
<th>Estimator</th>
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<th>LS</th>
<th>Random coeffs</th>
<th>Mean Group</th>
<th>LS - country</th>
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<tr>
<td>Fixed effects</td>
<td>None</td>
<td>None</td>
<td>mi</td>
<td>Random</td>
<td>mi</td>
<td>m</td>
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<tr>
<td>St. errors</td>
<td>Regular</td>
<td>Clustered on t</td>
<td>Conventional</td>
<td>Group-specific</td>
<td>Clustered on t</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports coefficients from linear regressions where the dependent variable \( \pi_{mi,t} \) is the country-sector monthly stock return (country average in column (6)) over 2000–07 in month with FOMC announcements, and the independent variable \( \tilde{M}_{US,t} \) is the measure of the monetary policy shock taken from Jarociński and Karadi (2020). Panel A includes all countries but the United States (25 countries in total, 46,357 observations in columns (1)-(5), 1,650 observations in column (6)), and Panel B includes only the United States (3,310 observations in columns (1)-(5), 66 observations in column (6)). Standard errors are in parentheses with *, **, and *** denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

We find that for the full sample about 40% of the average total effect is due to the direct impact of the U.S. monetary policy shock while the rest is due to the production network shock transmission. This is due to high coefficient of shock propagation \( \rho \), which is on average 0.68. As we would expect, average \( \rho \) is less than one, as implied by the model, due to unmodelled resistance to transmission of stock market shocks the global production network.

Computing the averages for foreign country-sectors and for the U.S. sectors separately, we can see the pattern of transmission of the U.S. monetary policy shock to stock returns globally. We can see a much stronger (2.5 times stronger) direct effect of U.S monetary policy shock on U.S. sectors, which is expected. This direct effect is then propagated through production network, both globally and domestically. The share of the production network effect for U.S. sectors is only 42%, while for foreign country-sectors it is 62%. In fact, the direct effect of the U.S. monetary policy shock on stock returns in foreign countries is only marginally statistically significant. These results are very
Table 3. Spatial Autoregression Panel Estimation Results

\[ \pi_t = \beta \tilde{\lambda}_{US,t} + \rho W \pi_t + \varepsilon_{mi,t} \]

<table>
<thead>
<tr>
<th></th>
<th>Average ( \beta )</th>
<th>Average ( \rho )</th>
<th>Avg. Direct</th>
<th>Avg. Network</th>
<th>Network/Total</th>
</tr>
</thead>
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<tr>
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<td>(1)</td>
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</table>

Panel A: \( W = (\tilde{\tau} \circ \Omega) \Lambda \)

<p>| | | | | | |</p>
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<thead>
<tr>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>-0.027*</td>
<td>0.675***</td>
<td>-0.035**</td>
<td>-0.053***</td>
<td>60%***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.157)</td>
<td>(0.020)</td>
<td>(0.012)</td>
<td>(0.164)</td>
</tr>
<tr>
<td>International</td>
<td>-0.023</td>
<td>0.681***</td>
<td>-0.031*</td>
<td>-0.052****</td>
<td>62%***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.158)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>USA</td>
<td>-0.080***</td>
<td>0.600***</td>
<td>-0.087***</td>
<td>-0.065**</td>
<td>42%***</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.154)</td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.008)</td>
</tr>
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</table>

Panel B: \( W_{\tau=1} = \Omega \Lambda \)

<p>| | | | | | |</p>
<table>
<thead>
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<th></th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>-0.019</td>
<td>0.748***</td>
<td>-0.026*</td>
<td>-0.093***</td>
<td>78%***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.179)</td>
<td>(0.020)</td>
<td>(0.018)</td>
<td>(0.197)</td>
</tr>
<tr>
<td>International</td>
<td>-0.016</td>
<td>0.746***</td>
<td>-0.023</td>
<td>-0.091***</td>
<td>80%***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.179)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.084)</td>
</tr>
<tr>
<td>USA</td>
<td>-0.056*</td>
<td>0.768***</td>
<td>-0.066**</td>
<td>-0.122***</td>
<td>65%***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.212)</td>
<td>(0.033)</td>
<td>(0.035)</td>
<td>(0.047)</td>
</tr>
</tbody>
</table>

**Notes:** This table reports results from heterogeneous coefficient spatial panel autoregressions where the dependent variable is the country-sector monthly stock return over 2000–07 over month with FOMC announcements, and the independent variable is the measure of the monetary policy shock taken from Jarośiński and Karadi (2020). There are 44,286 observations total comprised of 671 country-sectors over 66 months. In Panel B autoregressive weighting matrix \( W \) is replaced with the one that sets all trade costs \( \tilde{\tau} \) to 1. Standard errors (in parentheses) are obtained via wild bootstrap with 500 repetitions and *, **, and *** denote coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

intuitive and show that production linkages are very important in transmitting financial shocks at the sector level.\(^{20}\)

Panel B shows that setting all \( \tau \) to 1 increases the autoregressive coefficient and lowers the direct effect overall as well as for international and U.S. subsamples. That is, not accounting explicitly for trade costs exaggerates the share of shock transmission that is due to the global production network on average. By looking at the distribution of direct and network effects across country-sectors for both sets of estimates reported in panels A and B, as shown in Figure 4, we can see that the amplification of the network effect is due to larger proportion of country-sectors with negative network effects in case when \( \tau \) is set to 1.

We conjecture that there are two potential reasons that the network effects decline when including trade costs in the spatial weighting matrix. First, the trade costs place greater weights on

\(^{20}\)We will show that the direct effect on foreign sectors declines further when we explicitly allow for other financial shocks to affect foreign stock returns.
Figure 4. Distribution of Direct and Network Effects across country-sectors

Notes: This figure plots the distribution of Direct and Network across $m_i$ from the estimation of equation

$$\pi_t = \beta \tilde{M}_{US,t} + \rho W \pi_t + \varepsilon_t$$

for 2000–07, using Jarocinski and Karadi (2020) monetary policy shocks for $\tilde{M}_{US}$. The averages of these distributions are reported in Table 3.

countries that have larger bilateral trade in a given sector with respect to their total output – i.e., a measure of bilateral sectoral integration. This integration may not match up to how intensely intermediate goods are used for total production, and may therefore dampen the input-output weights. Second, the trade costs are symmetric for a given sector, while the input-output weights are asymmetric. Therefore, introducing the trade costs may create some noise, which will attenuate the estimated impact of the production network.

5.3 Sensitivity to Time Period

So far we limited our analysis to the 2000–07 time period. Our benchmark estimates are through 2007 for three reasons: first, this period includes full cycle of monetary policy actions but exclude the effective lower bound period; second, this period ends well prior to the Great Trade Collapse
Table 4. Spatial Autoregression Panel Estimation Results: variation over time

<table>
<thead>
<tr>
<th>Time period</th>
<th>Observations</th>
<th>Year for W</th>
<th>Share of network effect</th>
<th>Full sample</th>
<th>International</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000-07</td>
<td>44,286</td>
<td>Average 2000-07</td>
<td>59%</td>
<td>(0.141)</td>
<td>(0.013)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>2000-16</td>
<td>92,598</td>
<td>2000</td>
<td>74%</td>
<td>(0.349)</td>
<td>(0.240)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>2000-16</td>
<td>92,598</td>
<td>Average 2000-14</td>
<td>77%</td>
<td>(0.364)</td>
<td>(0.202)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>2000-07,09-16</td>
<td>87,230</td>
<td>2000</td>
<td>65%</td>
<td>(0.181)</td>
<td>(0.101)</td>
<td>(0.108)</td>
</tr>
<tr>
<td>2000-07,09-16</td>
<td>87,230</td>
<td>Average 2000-14</td>
<td>63%</td>
<td>(0.172)</td>
<td>(0.099)</td>
<td>(0.087)</td>
</tr>
</tbody>
</table>

Notes: This table reports results from heterogeneous coefficient spatial panel autoregressions where the dependent variable is the country-sector monthly stock return over 2000–07 over month with FOMC announcements, and the independent variable is the measure of the monetary policy shock taken from Jarociński and Karadi (2020). There are 44,286 observations total comprised of 671 country-sectors over 66 months. In Panel B autoregressive weighting matrix $W$ is replaced with the one that sets all trade costs $\tau$ to 1. Standard errors (in parentheses) are obtained via wild bootstrap with 500 repetitions and *, **, and *** denote coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

that occurred during the Global Financial Crisis in 2008:H2–2009:H1; third, this period does not include dramatic decline in global stock prices that followed the collapse of Lehmann Brothers. In our benchmark analysis, as in our model, we take global production network as given, and therefore we use the input-output coefficients from 2000. It is possible, however, that rapid increase in trade globalization and the lengthening of global supply chains in the early 2000s may affect our results. Here we want to explore the evolution of our results as we vary the time period and the year from which we sample matrix $W$.

Table 4 reports just the share of network effect across different variations of the sample for our benchmark regression reported in Panel A of Table 3. We can see that replacing $W$ measured in 2000 with the average $W$ for 2000–07 does not change the results. This is not surprising given that elements of $W$ are driven by production technologies and trade structure that do not change very fast.

Next we extend our time period through 2016.\textsuperscript{21} We can see that the share of network effect increases dramatically in this extended sample, especially for foreign sectors. However, we can tell that this is driven by the coincidence of monetary policy shocks, stock market crash, and global trade collapse in 2008 – once we exclude 2008 from the sample, our results become very similar to

\textsuperscript{21}While WIOD is only available through 2014, we gather information on all other variables through the end of 2016. To compute average $W$ for 2000–16 we simply replicate 2014 WIOD for 2015 and 2016.
Table 5. Least-Squares Panel Estimation Results: Other Shocks

\[ \pi_{mi,t} = \beta_{MP} M_{US,t} + \beta_X X_{US,t} + \rho W' \pi_t + \varepsilon_{mi,t} \]

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>International</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP shock</td>
<td>-0.061</td>
<td>-0.117**</td>
<td>-0.076</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>VIX</td>
<td>-0.162***</td>
<td>-0.146***</td>
<td>-0.148***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>T2y</td>
<td>0.146*</td>
<td>0.091*</td>
<td>0.09*</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>(0.047)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>USD</td>
<td>-0.546</td>
<td>-0.338</td>
<td>-0.332</td>
</tr>
<tr>
<td></td>
<td>(0.363)</td>
<td>(0.290)</td>
<td>(0.297)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.060</td>
<td>0.030</td>
<td>0.02</td>
</tr>
<tr>
<td>Observations</td>
<td>49,667</td>
<td>46,357</td>
<td>3,310</td>
</tr>
</tbody>
</table>

Notes: This table reports results from baseline the least-squares with \( m_i \) fixed effects (LS-\( m_i \)) and heterogeneous coefficient (HSAR) spatial autocorrelation panel regressions where the dependent variable is the country-sector monthly stock return over 2000–07 over month with FOMC meetings (an off-month meetings), and the independent variable is the measure of the monetary policy shock taken from Jarociński and Karadi (2020) in columns (1)-(3), and Ozdagli and Weber (2017) in column (4). Panel A presents the OLS and average of the estimated country-sector coefficients for the HSAR model, while Panel B presents the decomposition of the total effect based on these coefficients. The 2000–07 sample period regressions of columns (1), (2) and (4) use 49,667 observations. Column (3) use observations for 2000–14, excluding 2008, and has 76,380 observations. Bootstrapped standard errors, adjusted for clustering at the monthly (\( t \)) level, are in parentheses with *, **, and *** denoting coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

There is clear evidence in the literature, starting with Miranda-Agrippino and Rey (2020) that global stock prices respond to the global financial cycle. Some movements of the global financial cycle are due to changes in the U.S. monetary policy, while others are market-driven. Here we show the robustness of our results to controlling for such shocks. In our analysis we focus on three variables that are not highly correlated with each other and are easily available: the VIX, the U.S. 2-year Treasury rate, and the U.S. dollar Broad Index. We conduct both LS and SAR analysis and include these variables one at a time and then all together.

Table 5 shows the results of the fixed effects least square regressions for full sample as well as for subsamples of foreign country-sectors and for the U.S. only. In the interest of space we only present the results with all three additional control variables included for the subsamples — the results do not vary much if we include them individually.\(^\text{22}\)

\(^{22}\) Full set of regressions is available upon request.
VIX is shown to be highly correlated with the global financial cycle (Bruno and Shin, 2015a; Miranda-Agrippino and Rey, 2020) and is therefore likely to affect global stock returns. To the extent that some movements in VIX are correlated with monetary policy shock, our benchmark regressions may be attributing some of the effect of VIX to the effect of monetary policy shock. Indeed, when we include VIX in the regression, we find that the impact of the monetary policy shock is smaller than in the benchmark and is no longer statistically significant for full sample or for foreign country-sectors. The effect of monetary policy shock does remain significant for the U.S. sectors. Consistent with the literature, increase in VIX lowers stock market returns worldwide, and by about the same amount in the U.S. and in foreign countries.

Monetary policy can affect stock returns through surprises but it may also have an effect through the level of interest rates, which would not be necessarily reflected in monetary policy shocks. This second effect is likely to be reflected in capital flows (Avdjiev and Hale, 2019). According to the authors, an increase in the policy rate during the lending boom is likely to increase capital flows worldwide, which would imply increase in stock returns globally. Indeed, we find that an increase in the 2-year Treasury rate increases stock returns during our sample period of 2000-07, which corresponds to a lending boom. Controlling for the 2-year Treasury rate, however, does not change much the impact of the monetary policy shock, relative to benchmark.

In our benchmark analysis we assumed away the explicit effect of exchange rates. Given that the value of the dollar can be affected by monetary policy shocks (Inoue and Rossi, 2019), we want to separate the impact of monetary policy surprises that is orthogonal to exchange rate changes from reaction to the change in the value of the dollar. To do so, we control for the U.S. dollar broad index. We find that the value of the dollar does not have an effect on global stock returns and that controlling for the dollar index does not change our benchmark results. Combining the three additional control variables produces results that are similar to the regression with VIX only, showing, consistent with the literature, that VIX is the dominant driver of the global financial cycle when it comes to global stock returns.

The least square analysis, as before, does not allow us to separate direct impact from the effect of the global production chain. We would expect that the drivers of the global financial cycle, when omitted, may appear as the direct effect of the U.S. monetary policy. Thus, we include these additional control variables in our benchmark spatial autoregression. The results of this analysis are reported in Table 6. In the interest of space, we only show decomposition into foreign and U.S. sectors for the regression that includes all three controls at once. We also only report Direct and Network estimates.

When we control for VIX, we find that both direct and network effect of monetary policy shock are reduced and that the impact of VIX is roughly equally split between direct and network effects. This implies that (a) some of the impact of monetary policy shock on global stock prices is due
### Table 6. Spatial Autoregression Panel Estimation Results: Other Shocks

\[
\pi_{mi,t} = \beta_{MP} \tilde{M}_{US,t} + \beta_X X_{US,t} + \rho W' \pi_t + \varepsilon_{mi,t}
\]

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>International</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct effect of MP</td>
<td>-0.021**</td>
<td>-0.041***</td>
<td>-0.026**</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.019)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Network effect of MP</td>
<td>-0.027**</td>
<td>-0.060***</td>
<td>-0.054***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.012)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Direct effect of VIX</td>
<td>-0.067***</td>
<td>-0.062***</td>
<td>-0.062***</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.021)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Network effect of VIX</td>
<td>-0.072***</td>
<td>-0.063***</td>
<td>-0.064***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Direct effect of T2y</td>
<td>0.052**</td>
<td>0.034***</td>
<td>0.033***</td>
</tr>
<tr>
<td></td>
<td>(0.028)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>Network effect of T2y</td>
<td>0.078***</td>
<td>0.042***</td>
<td>0.042***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.013)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Direct effect of USD</td>
<td>-0.157**</td>
<td>-0.099**</td>
<td>-0.089*</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.057)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>Network effect of USD</td>
<td>-0.257***</td>
<td>-0.096*</td>
<td>-0.098</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.077)</td>
<td>(0.078)</td>
</tr>
</tbody>
</table>

**Notes:** This table reports direct and network effects from heterogeneous coefficient spatial panel autoregressions where the dependent variable is the country-sector monthly stock return over 2000–07 over month with FOMC announcements, and the independent variable is the measure of the monetary policy shock taken from Jarociński and Karadi (2020). There are 44,286 observations total comprised of 671 country-sectors over 66 months. Standard errors (in parentheses) are obtained via wild bootstrap with 500 repetitions and *, **, and *** denote coefficients significantly different from zero at the 1, 5 and 10% levels, respectively.

Controlling for the 2-year Treasury rate and Broad U.S. dollar Index does not alter our benchmark results, even though both direct and network effects of these controls are statistically significant. As with the linear regression, we find that including all three variables at once produces results similar to those with VIX only. As in our benchmark, we continue to find that for foreign country-sectors most of the monetary policy shock transmission is due to the production network, while for the U.S. sectors the role of direct effect is larger. Interestingly, this is not true for VIX – the total effect of VIX is split equally between direct and network effects and is, in fact, smaller for U.S. sectors. This highlights, once again, the importance of VIX in driving global asset prices.

Overall, we find that, while there is clearly some contamination of our benchmark results that arises from omitting correlates of the global financial cycle, especially VIX, our description of the

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\(^{23}\)Note that we only estimate one autoregression coefficient \(\rho\) for each country-sector, which them implies transmission of all shocks through the production network.
pattern of the monetary policy shock transmission through the global production network remains unchanged.

5.5 Heterogeneity of Estimates

We next explore drivers of heterogeneity in the importance of network effects across countries and sectors. Our approach is to analyze country-sector cross-section of the decomposition of the total effect into direct and network components.

By comparing average direct and network effects for advanced and emerging economies separately, we find that both components are substantially larger for emerging economies. On average, direct impact effect is more than twice as large for emerging economies as it is for advanced economies, while the network effect is 30-50% larger for emerging economies, depending on the decomposition.

We observe that large direct and network effects are not concentrated in specific sectors or specific countries. Thus, we consider possible sources of heterogeneity that include size, trade openness, exchange rate regimes, bilateral trade agreements, measures of financial openness and financial development for possible explanation of cross-country differences, while controlling for sector fixed effects. Alternatively, we consider a variety of sector characteristics to explore cross-sector differences, while controlling for country fixed effects.

To be completed.

5.6 Robustness Tests

To be completed: other measures of MP shocks, real \( y_{mi} \)

6 Conclusion

In this paper we quantitatively evaluate the propagation of the U.S. monetary policy shocks to stock returns worldwide through global production network. Basing our analysis on a multi-country production network model, we estimate a spatial autoregression in a panel setting, which allows for coefficients to vary across countries and sectors. The model predicts country-sectors that are more closely linked to the U.S. via supply linkages will be more affected by U.S. monetary policy shocks.

We find a very robust and quantitatively important role of the production network — over 60% of total impact of U.S. monetary policy shocks on global stock returns is due to production linkages. Among U.S. sectors, the share of network is smaller and the magnitude of the direct effect is substantially larger than for foreign sectors. Our findings suggest that U.S. monetary policy shocks directly affect predominantly domestic stock returns and the resulting changes in stock returns propagate globally mainly through production linkages.
These findings contribute to the vast and growing literature on the spillovers of the U.S. monetary policy internationally by documenting and quantifying the role of real linkages in global transmission of financial shocks. The pattern we uncover is not affected by allowing for financial channel of U.S. monetary policy shock transmission studied in the literature, namely the global financial cycle.
References


Appendix A  Linking sector classifications

TREIs data are available under Thomson Reuters Business Classification (TRBC), but the World Input-Output Tables (WIOT) have been constructed under ISIC Revision 4.

We take advantage of the fact that TREI reports both 10-digit TRBC activity codes and 6-digit NAICS 2007 codes for all equity prices. With this information one can use a concordance from NAICS 2007 to ISIC Rev. 4 to match each firm’s information to WIOT codes. In the next step, one can use the firm-level information from TREI data to construct alternative sector-specific stock price indices that are consistent with WIOT sector definitions.

However, a mapping from NAICS2007 to WIOT16 codes (2-digit ISIC Rev 4) is not perfect, as there can be many-to-many correspondences between NAICS 2007 and ISIC Rev. 4 codes. The following figure shows an example of a possible ‘rear’ overlapping of NAICS2007 sectors (3-digit code) in a WIOT2016 code.

In this example, the WIOT2016 Code B (Mining and quarrying) besides mining and oil sectors, it also contains the NAICS2007-Food Manufacturing sector. This occurs because the NAICS2007 sector “311942-Spice and Extract Manufacturing” from the Food Manufacturing includes the “mining and processing of table salt” activity, that is classified as a Mining activity in ISIC Rev. 4.

A.1 A reduced version of the NAICS 2007 to ISIC Rev. 4 correspondence

To limit similar occurrences as in the one in the previous example, a new version of the NAICS 2007 to ISIC Rev. 4 correspondence is constructed. The objective is to reduce the number of very different 4-digit ISIC Rev. 4 sectors per each 6-digit NAICS 2007 sector. With that in mind, the next steps were followed:

1. Work only on the set of 6-digit NAICS 2007 codes that (i) have more than one 2-digit ISIC 4 sector, and/or (ii) have more than one WIOT16 sector.

2. For a single 6-digit NAICS 2007 code, compute the frequency of its corresponding multiple 4-digit ISIC 4 sectors. When possible, the following principles were taken into consideration to assign one single NAICS 2007 code to a single 2-digit sector, the predominant sector.

3. Frequency criteria: If a 2-digit ISIC 4 sector represents more than 60 percent of the 6-digit NAICS 2007 sector in consideration, it is the called the predominant sector.
Example: The following example shows the corresponding multiple ISIC 4 codes for the single 6-digit NAICS 2007 sector “Paper (except Newsprint) Mill”:

<table>
<thead>
<tr>
<th>naics2007</th>
<th>naics2007_name</th>
<th>type_match</th>
<th>isic4</th>
<th>isic4_name</th>
</tr>
</thead>
<tbody>
<tr>
<td>32211</td>
<td>Paper (except Newsprint) Mills</td>
<td>keep</td>
<td>1709</td>
<td>Manufacture of other articles of paper and paperboard</td>
</tr>
<tr>
<td></td>
<td></td>
<td>keep</td>
<td>1701</td>
<td>Manufacture of pulp, paper and paperboard</td>
</tr>
<tr>
<td></td>
<td></td>
<td>keep</td>
<td>1702</td>
<td>Manufacture of corrugated paper and paperboard and of containers of paper and paperboard</td>
</tr>
<tr>
<td></td>
<td></td>
<td>delete</td>
<td>3999</td>
<td>Manufacture of other non-metallic mineral products n.e.c. (for paper made in paper mills)</td>
</tr>
</tbody>
</table>

The frequency of the 2-digit ISIC 4 sector “17-Manufacture of paper and paper products” is 75 percent and it is the predominant sector. The other 2-digit ISIC 4 sector, “23- Manufacture of other non-metallic mineral products”, is not predominant and its deleted from the concordance. Note that for this sector its 2-digit ISIC 4 meaning is very different from the 3-digit NAICS 2007 meaning too (“322-Paper Manufacturing”).

Closest sector criteria: When the frequency criteria is not sufficient, the predominant sector is chosen by a comparison of meanings between the single 6-digit NAICS 2007 code and its corresponding 4-digit ISIC 4 codes. Then, the ISIC 4 sector with the closest meaning to the NAICS 2007 sector is selected as the predominant sector. The meaning of aggregate codes (3-digit NAICS 2007 and 2-digit ISIC 4) helped also to decide, when the comparison of 6-digit NAICS and 4-digits ISIC 4 meanings were not clear enough to reach a decision.

Example: The following example shows the corresponding multiple 4-digit ISIC 4 codes for the single 6-digit NAICS 2007 sector “Carbon and Graphite Product Manufacturing”

<table>
<thead>
<tr>
<th>naics2007</th>
<th>isic4</th>
<th>naics2007_3digit</th>
<th>isic4_2digit</th>
</tr>
</thead>
<tbody>
<tr>
<td>335991</td>
<td>2299</td>
<td>Manufacture of other electrical equipment</td>
<td>Manufacture of other electrical equipment</td>
</tr>
<tr>
<td></td>
<td>2399</td>
<td>Manufacture of other non-metallic mineral products n.e.c.</td>
<td>Manufacture of other non-metallic mineral products n.e.c.</td>
</tr>
</tbody>
</table>

Although by frequency the two 4-digit (and 2-digit) ISIC 4 sectors are equally representative for this NAICS 2007 code, their sector meanings are different. In fact, the 6-digit NAICS 2007 “335991-Carbon and Graphite Product Manufacturing” is closest to the 4-digit ISIC 4 “2399-Manufacture of other non-metallic mineral products n.e.c.” than to the 4-digit ISIC 4 “2790-Manufacture of other electrical equipment” sector. Then, the 2-digit ISIC 4 “27- Manufacture of electrical equipment” is denominated the predominant sector.

There was only one exception, NAICS 2007 “337920-Blind and Shade Manufacturing”. As it can be observed below, none of the previous criteria worked; and it was hard coded arbitrarily based on its 3-digit NAICS 2007 meaning, “Furniture and Related Product Manufacturing”, to the 2-digit ISIC 4 “3100-Manufacture of furniture” sector.
Once this new NAICS 2007 to ISIC 4 concordance was finished, it was easy to go from NAICS 2007 to WIOT16. In the final NAICS 2007-WIOT16 concordance:

- 1020 correspondences were tagged based on the official NAICS 2007-ISIC 4 concordance.
- 37 correspondences were tagged based on the frequency criteria.
- 122 correspondences were tagged based on the closest sector criteria.
- 1 correspondence was arbitrarily hard coded.

Table A1 presents cross-country sector coverage of monthly returns for the months where there are monetary surprise shocks over 2000–14. Given cross-country differences in size, industrial specialization patterns, and stock market depth we see that larger countries (e.g., the United States) have a larger coverage of sectors, while some countries only cover a few sectors (e.g., Portugal and Russia). These differences motivate a flexible empirical approach, where we allow for country-sector fixed effects as well as country-sector specific coefficients for the effect of monetary policy surprise variable.

Table A2 presents coverage of of monthly returns for the months where there are monetary surprise shocks along the sector dimension. This table shows how the distribution of sector returns varies across countries. For example, all countries have returns for the ‘Construction,’ ‘Telecommunication,’ and ‘Financial service activities, except insurance and pension funding’ sectors. Meanwhile, sectors like ‘Forestry and logging,’ ‘Fishing and aquaculture,’ and ‘Repair and installation of machinery and equipment’ have sparse stock returns coverage across countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>No. Industries</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>38</td>
<td>5,141</td>
</tr>
<tr>
<td>Austria</td>
<td>15</td>
<td>2,157</td>
</tr>
<tr>
<td>Brazil</td>
<td>17</td>
<td>3,237</td>
</tr>
<tr>
<td>Canada</td>
<td>38</td>
<td>5,099</td>
</tr>
<tr>
<td>China</td>
<td>47</td>
<td>5,935</td>
</tr>
<tr>
<td>Germany</td>
<td>28</td>
<td>4,224</td>
</tr>
<tr>
<td>Denmark</td>
<td>17</td>
<td>2,201</td>
</tr>
<tr>
<td>Spain</td>
<td>24</td>
<td>3,303</td>
</tr>
<tr>
<td>Finland</td>
<td>22</td>
<td>2,982</td>
</tr>
<tr>
<td>France</td>
<td>38</td>
<td>4,870</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>40</td>
<td>5,225</td>
</tr>
<tr>
<td>Greece</td>
<td>10</td>
<td>1,703</td>
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<td>Indonesia</td>
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<tr>
<td>India</td>
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<td>Japan</td>
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<td>Korea</td>
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<td>5,348</td>
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<td>Mexico</td>
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<tr>
<td>Netherlands</td>
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<td>Poland</td>
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<td>Portugal</td>
<td>8</td>
<td>1,049</td>
</tr>
<tr>
<td>Russia</td>
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<td>1,211</td>
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<td>Sweden</td>
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<td>Turkey</td>
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<tr>
<td>Taiwan</td>
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<tr>
<td>United States</td>
<td>50</td>
<td>6,166</td>
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</tbody>
</table>

Notes: This table presents information on the number of sectors and observation of monthly sector returns per country for dates where there are monetary surprise shocks (FOMC meetings or off-cycle meetings) over 2000–14. The data are constructed by merging stock returns data from TREI with the WIOD classification of sectors.

<table>
<thead>
<tr>
<th>Industry</th>
<th>WIOD code</th>
<th>No. countries</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crop and animal production, hunting and related service activities</td>
<td>A01</td>
<td>13</td>
<td>1,406</td>
</tr>
<tr>
<td>Forestry and logging</td>
<td>A02</td>
<td>3</td>
<td>300</td>
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<tr>
<td>Fishing and aquaculture</td>
<td>A03</td>
<td>6</td>
<td>530</td>
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<tr>
<td>Mining and quarrying</td>
<td>B</td>
<td>19</td>
<td>2,281</td>
</tr>
<tr>
<td>Manufacture of food products, beverages and tobacco products</td>
<td>C10-C12</td>
<td>23</td>
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</tr>
<tr>
<td>Manufacture of textiles, wearing apparel and leather products</td>
<td>C13-C15</td>
<td>16</td>
<td>1,911</td>
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<tr>
<td>Manufacture of wood and of products of wood and cork, etc</td>
<td>C16</td>
<td>10</td>
<td>1,036</td>
</tr>
<tr>
<td>Manufacture of paper and paper products</td>
<td>C17</td>
<td>19</td>
<td>2,200</td>
</tr>
<tr>
<td>Printing and reproduction of recorded media</td>
<td>C18</td>
<td>8</td>
<td>906</td>
</tr>
<tr>
<td>Manufacture of coke and refined petroleum products</td>
<td>C19</td>
<td>20</td>
<td>2,303</td>
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<tr>
<td>Manufacture of chemicals and chemical products</td>
<td>C20</td>
<td>25</td>
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<tr>
<td>Manufacture of basic pharmaceutical products and pharmaceutical preparations</td>
<td>C21</td>
<td>20</td>
<td>2,191</td>
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<tr>
<td>Manufacture of rubber and plastic products</td>
<td>C22</td>
<td>18</td>
<td>2,082</td>
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<tr>
<td>Manufacture of other non-metallic mineral products</td>
<td>C23</td>
<td>18</td>
<td>2,196</td>
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<tr>
<td>Manufacture of basic metals</td>
<td>C24</td>
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<tr>
<td>Manufacture of fabricated metal products, except machinery and equipment</td>
<td>C25</td>
<td>14</td>
<td>1,500</td>
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<tr>
<td>Manufacture of computer, electronic and optical products</td>
<td>C26</td>
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<tr>
<td>Manufacture of electrical equipment</td>
<td>C27</td>
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<tr>
<td>Manufacture of machinery and equipment n.e.c.</td>
<td>C28</td>
<td>19</td>
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<tr>
<td>Manufacture of motor vehicles, trailers and semi-trailers</td>
<td>C29</td>
<td>20</td>
<td>2,388</td>
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<tr>
<td>Manufacture of other transport equipment</td>
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<tr>
<td>Manufacture of furniture; other manufacturing</td>
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<td>17</td>
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<tr>
<td>Repair and installation of machinery and equipment</td>
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<td>1</td>
<td>56</td>
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<td>Electricity, gas, steam and air conditioning supply</td>
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<td>22</td>
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<tr>
<td>Water collection, treatment and supply</td>
<td>E36</td>
<td>6</td>
<td>644</td>
</tr>
<tr>
<td>Sewerage; waste collection, treatment and disposal activities; etc</td>
<td>E37-E39</td>
<td>9</td>
<td>967</td>
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<tr>
<td>Construction</td>
<td>F</td>
<td>26</td>
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<tr>
<td>Wholesale and retail trade and repair of motor vehicles and motorcycles</td>
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<tr>
<td>Wholesale trade, except of motor vehicles and motorcycles</td>
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</tr>
<tr>
<td>Retail trade, except of motor vehicles and motorcycles</td>
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<td>2,752</td>
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<tr>
<td>Land transport and transport via pipelines</td>
<td>H49</td>
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<tr>
<td>Water transport</td>
<td>H50</td>
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</tr>
<tr>
<td>Air transport</td>
<td>H51</td>
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<tr>
<td>Warehousing and support activities for transportation</td>
<td>H52</td>
<td>19</td>
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</tr>
<tr>
<td>Postal and courier activities</td>
<td>H53</td>
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<td>659</td>
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<tr>
<td>Accommodation and food service activities</td>
<td>I</td>
<td>19</td>
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<tr>
<td>Publishing activities</td>
<td>J58</td>
<td>18</td>
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<tr>
<td>Motion picture, video and television programme production, etc</td>
<td>J59-J60</td>
<td>16</td>
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<tr>
<td>Telecommunications</td>
<td>J61</td>
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<tr>
<td>Computer programming, consultancy and related activities; info; etc</td>
<td>J62-J63</td>
<td>21</td>
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<tr>
<td>Financial service activities, except insurance and pension funding</td>
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<tr>
<td>Insurance, reinsurance and pension funding, except compulsory social security</td>
<td>K65</td>
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<tr>
<td>Activities auxiliary to financial services and insurance activities</td>
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<tr>
<td>Real estate activities</td>
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<tr>
<td>Legal and accounting activities; activities of head offices; etc</td>
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<tr>
<td>Architectural and engineering activities; technical testing and analysis</td>
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<td>Advertising and market research</td>
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<td>Other professional, scientific and technical activities; veterinary activities</td>
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<td>Administrative and support service activities</td>
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<tr>
<td>Other service activities</td>
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<td>17</td>
<td>1,765</td>
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</tbody>
</table>

Notes: This table presents information on the number of sectors and observation of monthly sector returns per sector for dates where there are monetary surprise shocks (FOMC meetings or off-cycle meetings) over 2000–14. The data are constructed by merging stock returns data from TREI with the WIOD classification of sectors.
Appendix B  Solving for Equilibrium Output

Solving for the Price Level  We first derive the price level of a country-sector pair in two-stages given a firm’s minimization problem, where the nested intermediate goods allow us to do this. The top-level minimization problem is:

\[
\min_{l_{nj},X_{nj}} P_{nj} X_{nj} + w_{nl_{nj}} \quad \text{s.t.} \quad (4) = 1,
\]

where \(P_{nj}\) is an aggregate price level of the underlying country-sector intermediates source by \(nj\), which will be solved for in the second step.

The first-order-conditions are, given a Lagrangian multiplier, \(\mu\):

\[
w_n = \mu \alpha_{nj} \omega^\alpha_{nj} X_{nj}^{\frac{\lambda_{nj}}{\omega^\alpha_{nj}}}, \quad (B.1)
\]

\[
P_{nj} = \mu \lambda_{nj} \omega^\lambda_{nj} X_{nj}^{\frac{\lambda_{nj}}{\omega^\lambda_{nj}}}, \quad (B.2)
\]

\[
y_{nj} = l_{nj}^{\frac{\alpha_{nj}}{\omega^{\alpha_{nj}}}} X_{nj}^{\frac{\lambda_{nj}}{\omega^{\lambda_{nj}}}}. \quad (B.3)
\]

Dividing (B.1) by (B.2) and re-arranging we have:

\[
\frac{w_n l_{nj}}{P_{nj} X_{nj}} = \frac{\alpha_{nj}}{\lambda_{nj}},
\]

\[
\Rightarrow X_{nj} = \left( \frac{w_n}{P_{nj}} \right) \left( \frac{\lambda_{nj}}{\alpha_{nj}} \right) l_{nj}.
\]

Substituting \(X_{nj}\) into the production function we have:

\[
y_{nj} = \left[ \left( \frac{w_n}{P_{nj}} \right) \left( \frac{\lambda_{nj}}{\alpha_{nj}} \right) \right]^{\lambda_{nj}},
\]

which solving for labor yields:

\[
l_{nj} = \frac{y_{nj}^{\lambda_{nj}+\lambda_{nj}}}{\left( \frac{P_{nj}}{w_n} \right)^{\frac{\lambda_{nj}}{\alpha_{nj}}} \left( \frac{\lambda_{nj}}{\alpha_{nj}} \right)^{\frac{\lambda_{nj}}{\alpha_{nj}+\lambda_{nj}}}} \quad (B.4)
\]

and plugging this value into the production function to solve out for the intermediate good:

\[
X_{nj} = \left( \frac{w_n}{P_{nj}} \right)^{\frac{\alpha_{nj}}{\omega^{\alpha_{nj}}}} \left( \frac{\lambda_{nj}}{\omega^{\lambda_{nj}}} \right)^{\frac{\lambda_{nj}}{\omega^{\lambda_{nj}}+\lambda_{nj}}} X_{nj}^{\frac{\lambda_{nj}}{\omega^{\lambda_{nj}}+\lambda_{nj}}} \quad (B.5)
\]

Plugging (B.4) and (B.5) into the cost minimization function we have, where set \(y_{nj} = 1\):

\[
C(l_{nj}, X_{nj}) = \frac{1}{y_{nj}^{\lambda_{nj}+\lambda_{nj}}} \left( \frac{w_n}{P_{nj}} \right)^{\frac{\alpha_{nj}}{\omega^{\alpha_{nj}}}} \left( \frac{\lambda_{nj}}{\omega^{\lambda_{nj}}} \right)^{\frac{\lambda_{nj}}{\omega^{\lambda_{nj}}+\lambda_{nj}}} X_{nj}^{\frac{\lambda_{nj}}{\omega^{\lambda_{nj}}+\lambda_{nj}}} + \left( \frac{\lambda_{nj}}{\omega^{\lambda_{nj}}} \right)^{\frac{\alpha_{nj}}{\omega^{\alpha_{nj}}+\lambda_{nj}}}. \quad (B.6)
\]

The next step is to solve for \(P_{nj}\) as a function of the prices of the underlying intermediate goods. We do this by minimizing the cost of building on unit of the composite intermediate, \(X_{nj}\). I.e.,:

\[
\min_{\{x_{mi,nj}\}} \sum_{i=1}^{J} \sum_{m=1}^{N} p_{mn,i} x_{mi,nj} \quad \text{s.t.} \quad (5) = 1.
\]
The first-order-condition for every good $x_{mi,nj}$ given a Lagrange multiplier, $\mu$, is:

$$p_{mn,i} = \mu \omega_{mi,nj} x_{mi,nj}^{-1} \left( \prod_{k=1}^{J} \prod_{l=1}^{N} x_{kl,nj}^{\omega_{kl,nj}} \right)_{lk \neq mi}$$  \hspace{1cm} (B.7)

$$X_{nj} = \prod_{i=1}^{J} \prod_{m=1}^{N} x_{mi,nj}^{\omega_{mi,nj}}.$$  \hspace{1cm} (B.8)

Taking the ratio between (B.7) for $p_{mn,i}$ and $p_{ln,k}$ (as an example), we have:

$$\frac{p_{mn,i}}{p_{ln,k}} = \frac{\omega_{mi,nj}}{\omega_{mk,nj}} x_{ln,kj} x_{mn,ij},$$

Substituting $x_{mn,ij}$ into the intermediate aggregate function we have:

$$X_{nj} = \frac{p_{ln,k} x_{lk,nj}}{\omega_{lk,nj}} \prod_{i=1}^{J} \prod_{m=1}^{N} \left( \frac{\omega_{mi,nj}}{p_{mn,i}} \right)^{\omega_{mi,nj}},$$

which solving for the input $x_{lk,nj}$ yields:

$$x_{lk,nj} = X_{nj} \frac{\omega_{lk,nj}}{p_{ln,k}} \prod_{i=1}^{J} \prod_{m=1}^{N} \left( \frac{\omega_{mi,nj}}{p_{mn,i}} \right)^{-\omega_{mi,nj}}.$$

We then multiply (B.9) by its respective price level and sum over all $lk$ pairs to solve for the price-level (cost) of one unit of $X_{nj}$:

$$P_{nj} = \sum_{k=1}^{J} \sum_{l=1}^{N} p_{lk,n} x_{lk,nj}$$

$$= \prod_{i=1}^{J} \prod_{m=1}^{N} \left( \frac{p_{mn,i}}{\omega_{mi,nj}} \right)^{\omega_{mi,nj}}.$$  \hspace{1cm} (B.10)

Finally, plugging (B.10) into (B.6) we have the final cost function, which equals marginal cost:

$$C(l_{nj}, \{x_{mi,nj}\}) = g(\alpha_{nj}, \lambda_{nj}) \left( \frac{1}{y_{nj}} \right) \left( \frac{\alpha_{nj}}{w_{n}} \right)^{\alpha_{nj}} \left[ \prod_{i=1}^{J} \prod_{m=1}^{N} \left( \frac{p_{mn,i}}{\omega_{mi,nj}} \right)^{\omega_{mi,nj}} \right]^{\alpha_{nj}}.$$  \hspace{1cm} (B.11)

where $g(\alpha_{nj}, \lambda_{nj}) = \left[ \frac{\alpha_{nj}}{\lambda_{nj}} \right]^{\lambda_{nj}} + \left[ \frac{\lambda_{nj}}{\alpha_{nj}} \right]^{\alpha_{nj}}$.

This implies that marginal costs are

$$MC_{nj} = \tilde{g}(\alpha_{nj}, \lambda_{nj}) \left( \frac{1-\alpha_{nj}-\lambda_{nj}}{y_{nj}} \right) \left( \frac{\alpha_{nj}}{w_{n}} \right)^{\alpha_{nj}} \left[ \prod_{i=1}^{J} \prod_{m=1}^{N} \left( \frac{p_{mn,i}}{\omega_{mi,nj}} \right)^{\omega_{mi,nj}} \right]^{\lambda_{nj}},$$  \hspace{1cm} (B.12)

where $\tilde{g}(\alpha_{nj}, \lambda_{nj}) = \frac{1}{\alpha_{nj}+\lambda_{nj}} g(\alpha_{nj}, \lambda_{nj})$.

Firms in sector $nj$ will set their price to equal (B.12). Further, given the assumption we make below on relative prices across countries, $p_{mn,i} = \tau_{mn,i} p_{mi}$, it follows that there will be $N \times J$ prices to solve for in equilibrium, along with $N - 1$ wages.
Solving for Output  First, set the price of a firm’s good, \( p_{nj} \) equal to log of its marginal cost (B.12):
\[
\ln p_{nj} = B_{nj} + \frac{1 - \alpha_{nj} - \lambda_{nj}}{\alpha_{nj} + \lambda_{nj}} \ln y_{nj} + \frac{\alpha_{nj}}{\alpha_{nj} + \lambda_{nj}} \ln w_n + \frac{\lambda_{nj}}{\alpha_{nj} + \lambda_{nj}} \sum_{i=1}^{J} \sum_{m=1}^{N} \omega_{mi,nj} \tau_{mn,i} \ln p_{mi},
\]
where \( B_{nj} = \ln \tilde{g}(\alpha_{nj}, \lambda_{nj}) + \frac{\lambda_{nj}}{\alpha_{nj} + \lambda_{nj}} \ln \left( \sum_{i=1}^{J} \sum_{m=1}^{N} \omega_{mi,nj} \right) \), and we’ve applied the pricing assumption to relate prices across markets cum an iceberg trade cost.

Writing this expression in matrix form across all \( nj \) we have:
\[
\ln p = B + \Gamma \ln y + \Phi \ln w + \Psi \tilde{\Omega}' \ln p,
\]
(B.13)
\[
\ln p \equiv (\ln p_{11}, \ldots, \ln p_{NJ})', \quad NJ \times 1,
\]
\[
B \equiv (B_{11}, \ldots, B_{NJ})', \quad NJ \times 1,
\]
\[
\Gamma \equiv \text{diag} \left( \left\{ \frac{1 - \alpha_{nj} - \lambda_{nj}}{\alpha_{nj} + \lambda_{nj}} \right\} \right), \quad NJ \times NJ,
\]
\[
\ln y \equiv (\ln y_{11}, \ldots, \ln y_{NJ})', \quad NJ \times 1,
\]
\[
\Phi \equiv \text{diag} \left( \left\{ \frac{\alpha_{nj}}{\alpha_{nj} + \lambda_{nj}} \right\} \right), \quad NJ \times NJ,
\]
\[
\ln w \equiv (1_{1 \times J} \odot \ln w_1, \ldots, 1_{1 \times J} \odot \ln w_N)', \quad NJ \times 1,
\]
\[
\Psi \equiv \text{diag} \left( \left\{ \frac{\lambda_{nj}}{\alpha_{nj} + \lambda_{nj}} \right\} \right), \quad NJ \times NJ,
\]
\[
\tilde{\Omega}' = \tau \odot \Omega', \quad NJ \times NJ,
\]
\[
\tau \equiv \left( \begin{array}{ccc}
\{ \tau_{11,} \}_{1 \times J} & \cdots & \{ \tau_{N1,} \}_{1 \times J} \\
\vdots & \ddots & \vdots \\
\{ \tau_{1N,} \}_{1 \times J} & \cdots & \{ \tau_{NN,} \}_{1 \times J}
\end{array} \right), \quad NJ \times NJ,
\]
where \( \tau_{mn,i} \equiv (\tau_{mn,1}, \ldots, \tau_{mn,J}) \).

Solving for the price level in (B.13) we have
\[
\ln p = (I - \Psi \tilde{\Omega}')^{-1} [B + \Gamma \ln y + \Phi \ln w].
\]
(B.14)

Next, re-write (15) in logs as
\[
\ln p + \ln y = \ln \left[ (I - \tilde{\Omega})^{-1} \tilde{b} \mathcal{M} \right].
\]
Substituting (B.14) into this expression and solving for output yields
\[
\ln y = \ln \left[ (I - \tilde{\Omega})^{-1} \tilde{b} \mathcal{M} \right] - (I - \Psi \tilde{\Omega}')^{-1} [B + \Gamma \ln y + \Phi \ln w],
\]
which re-arranging gives
\[
\ln y = \left[ I + (I - \Psi \tilde{\Omega}')^{-1} \Gamma \right]^{-1} \left\{ \ln \left[ (I - \tilde{\Omega})^{-1} \tilde{b} \mathcal{M} \right] - (I - \Psi \tilde{\Omega}')^{-1} [B + \Phi \ln w] \right\}.
\]
(B.15)

Equations (B.14) and (B.15) highlight the interdependence of output on wages across countries that still needs to be solved for.
Fixed-Wage Solution  In the case when wages are preset, the second term of (B.15) is simply and elaborate constant as wages will not adjust to shocks to the money supply, and breaking up the firm terms there are also numerous constants that are functions of parameters of the model. Ignoring these constants, we re-write the vector of country-sector outputs as

\[ \ln y_{fix} = \left[ I + (I - \Psi\hat{\Omega}')^{-1} \Gamma \right]^{-1} \ln \mathcal{M} + \text{constant}. \]  

Taking the derivative of (B.16) with respect to changes in the money supply around the steady-state yields

\[ \hat{y}_{fix} = \bar{\theta} \circ \left( \left[ I + (I - \Psi\hat{\Omega}')^{-1} \Gamma \right]^{-1} \tilde{\mathcal{M}} \right), \]  

where \( \bar{\theta} \) is a \( NJ \times 1 \) vector of the ratio of the steady-state country-sector output to money supply, \( \bar{\theta}_{nj} = \frac{\mathcal{M}_n}{y_{nj}} \).

Flex-Wage/Steady-State Solution  To solve for output when wages are allowed to adjust, we need to utilize additional first-order conditions and impose an additional assumption to solve for output in equilibrium as a function of the money supply and other parameters of the model. To proceed, we use the market clear condition (9), but rather than solving for consumption as a function of revenues using the consumption first-order conditions alone as in Section 2, we use the FOC (3) to re-write the goods market clearing condition in terms of wages:

\[ y_{mi} = \sum_{n=1}^{N} b_{mi,n} w_n + \sum_{j=1}^{J} \sum_{n=1}^{N} \lambda_{nj} \omega_{mi,nj} R_{nj}. \]  

(B.18)

As before, define \( R_{mi} = p_{mi} y_{mi} \) and following our second assumption to re-write import prices as \( p_{mi,n} = \tau_{mi,n} p_{mi} \), and multiplying (B.18) we express revenues in sector \( i \) of country \( m \) as:

\[ R_{mi} = \sum_{n=1}^{N} \frac{b_{mi,n}}{\tau_{mi,n}} w_n + \sum_{j=1}^{J} \sum_{n=1}^{N} \frac{\lambda_{nj} \omega_{mi,nj}}{\tau_{mi,n}} R_{nj}. \]  

(B.19)

Define \( \phi_{mi,n} = \frac{b_{mi,n}}{\tau_{mi,n}} \) then (B.19) can be re-written in vector notation using other definitions above as

\[ (I - \bar{\Omega}\Lambda)R = \phi w, \]  

(B.20)

where

\[ \phi = \begin{pmatrix} \phi_{11,1} & \cdots & \phi_{11,N} \\ \vdots & \ddots & \vdots \\ \phi_{NJ,1} & \cdots & \phi_{NJ,N} \end{pmatrix}, \quad NJ \times N, \]

\[ w = (w_1, \ldots, w_N)', \quad N \times 1. \]

Inspecting (15) and (B.20) it is immediately apparent that

\[ \phi w = \tilde{b} \mathcal{M}. \]
Further note that $\phi = \tilde{b}$, so

$$w = M.$$ 

That is the money supply defines the nominal wage level in each country. Given this solution, we then substitute for the wage in (B.15) to solve for output as a function of countries’ money supplies:

$$\ln y_{\text{flex}} = \left[ I + (I - \Psi \tilde{\Omega})^{-1} \Gamma \right]^{-1} \left\{ \ln \left[ (I - \tilde{\Omega})^{-1} \tilde{b} M \right] - (I - \Psi \tilde{\Omega})^{-1} [B + \Phi \ln M] \right\}. \quad (B.21)$$

Taking the derivative of (B.21) delivers:

$$d \ln y_{\text{flex}} = \left[ I + (I - \Psi \tilde{\Omega})^{-1} \Gamma \right]^{-1} \left[ I - (I - \Psi \tilde{\Omega})^{-1} \Phi \right] d \ln M. \quad (B.22)$$

Given the second term of (B.22) may be positive or negative, the relationships between changes in real output at the country-sector level and changes in money supply (of any country) is ambiguous. In particular, while relative outputs may vary given a change in money supply, changes will offset each other within a country so that aggregate real GDP remains unchanged.